16-642: Manipulation Estimation and Control
Problem Set 2

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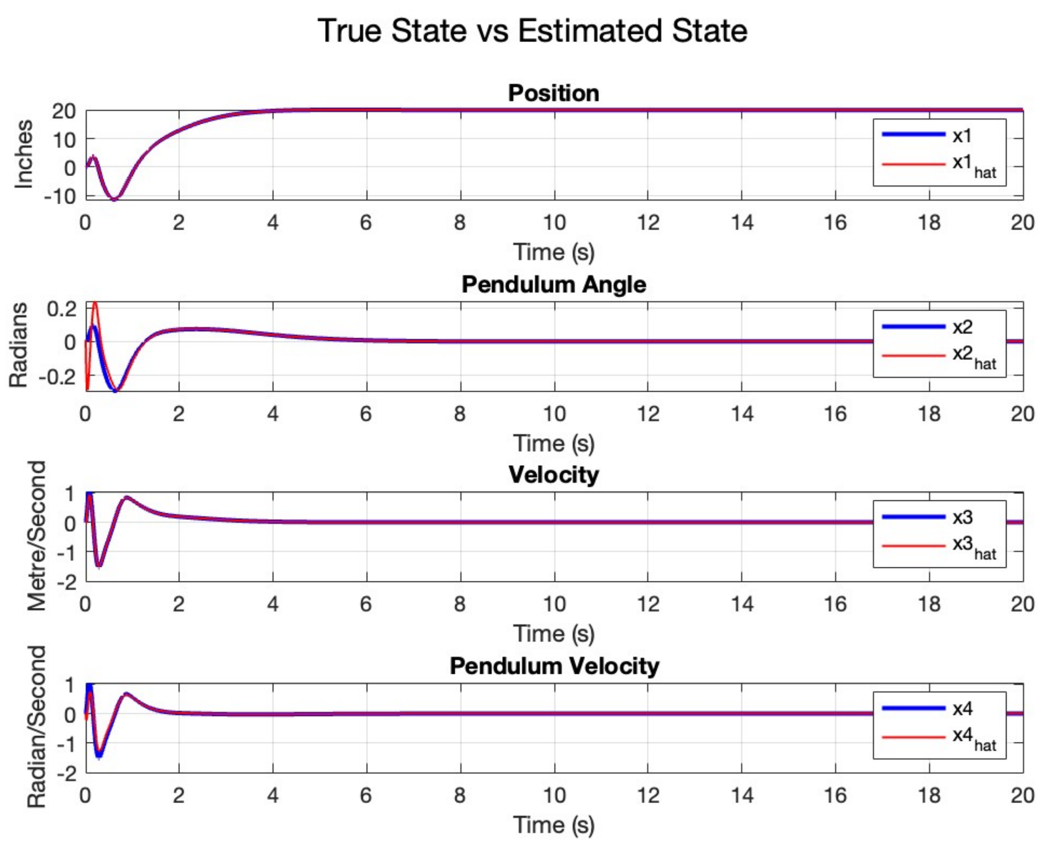
1

The MATLAB script with the observer added is attached as *Pendulum_on_a_Cart_Non_Linear_observer.m*
It can be observed that estimated position converges immediately while estimations of pendulum angle, velocity and pendulum velocity converge soon after.



2

2.a

Given plant model in time domain:

$$\ddot{y}(t) + 13\dot{y}(t) + 78y(t) = \ddot{u}(t) + 4\dot{u}(t) + 80u(t)$$

Applying Laplace transform, the same model in frequency domain:

$$s^2Y(s) + 13sY(s) + 78Y(s) = s^2U(s) + 4sU(s) + 80U(s)$$
$$\implies Y(s) = \frac{s^2 + 4s + 80}{s^2 + 13s + 78}U(s)$$

The transfer function G(s): U(s) $\rightarrow$ Y(s) is hence:

$$G(s) = \frac{s^2 + 4s + 80}{s^2 + 13s + 78}$$

2.b

Defining unity negative feedback as -Y(s), The plant is now given by:

$$[U(s) - Y(s)]G(s) = Y(s)$$
$$\implies Y(s)[1 + G(s)] = U(s)G(s)$$
$$\implies Y(s) = \frac{G(s)}{1 + G(s)}U(s)$$

Hence, the closed loop transfer function is:

$$T(s) = \frac{Y(s)}{U(s)} = \frac{G(s)}{1 + G(s)}$$
$$T(s) = \frac{\frac{s^2+4s+80}{s^2+13s+78}}{1 + \frac{s^2+4s+80}{s^2+13s+78}}$$
$$T(s) = \frac{s^2 + 4s + 80}{2s^2 + 17s + 158}$$

2.c

The poles and zeros determined using the MATLAB function *pole* and *zero* are:

```
>> plant_2
Poles:
  -4.2500 + 7.8062i
  -4.2500 - 7.8062i

Zeros:
  -2.0000 + 8.7178i
  -2.0000 - 8.7178i
```

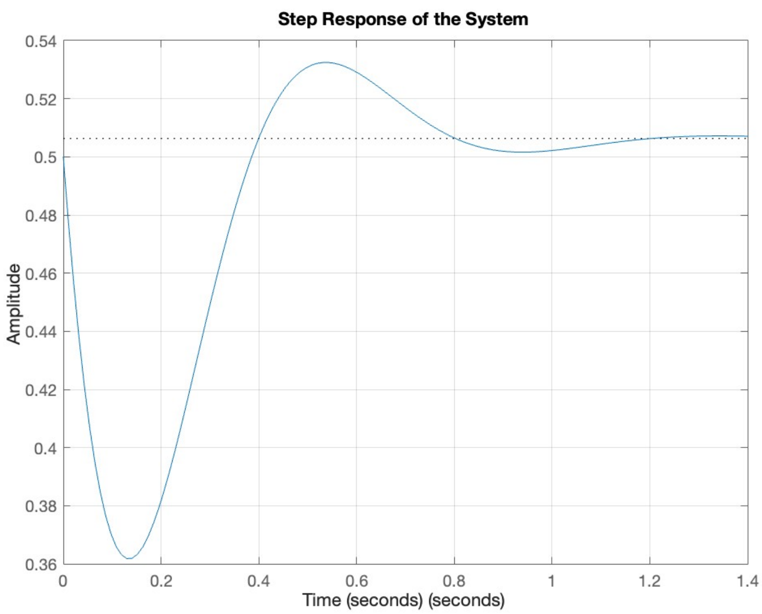
2.d

In all poles of T(s), the real part is < 0, which means the function would be asymptotically stable. The poles also have a non-zero imaginary part, meaning the system would oscillate. So, when a step input is given, it would oscillate but converge to the equilibrium point.

Zeros do not affect stability but influence transient response of the system. Since the zeros lie on the left of the imaginary axis in the complex plane and they are close to the poles, they would result in a 'high' overshoot M_p.

2.e

The step response of T(s) plotted using the step function:



2.f

To find the steady state value, applying $\lim_{s\to 0}$ to T(s):

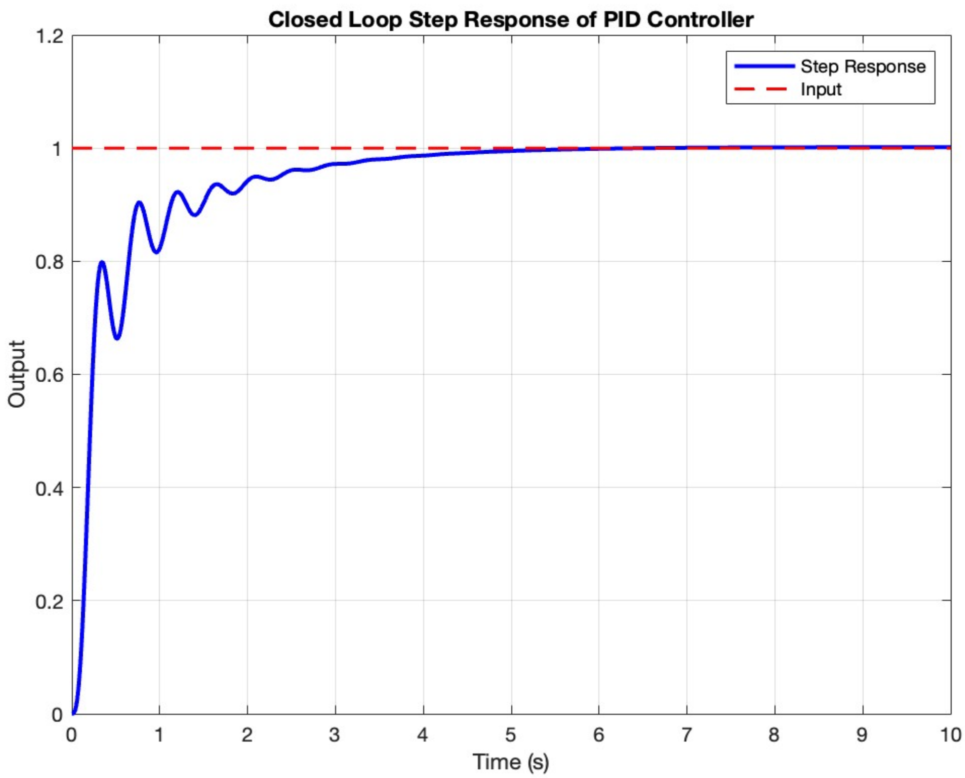
$$\lim_{s\to 0} T(s) = \lim_{s\to 0} \frac{s^2 + 4s + 80}{2s^2 + 17s + 158} = \frac{40}{79}$$

Hence, the steady state value is $\frac{40}{79}$.

NOTE: The MATLAB script for this question is attached as *plant_2.m*.

3

The PID controller has been implemented in attached MATLAB script *plant_3.m*.
Gains Used: $K_p = 40, K_i = 6, K_d = 0$.
Closed-loop step response:



Transient Response of the System:

```
>> plant_3
Steady-State Error: 0.00
Rise Time: 0.6458 seconds
Percentage Overshoot: 0.00%
```

—THE END—