SUDOKU SOLVER

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A Project by

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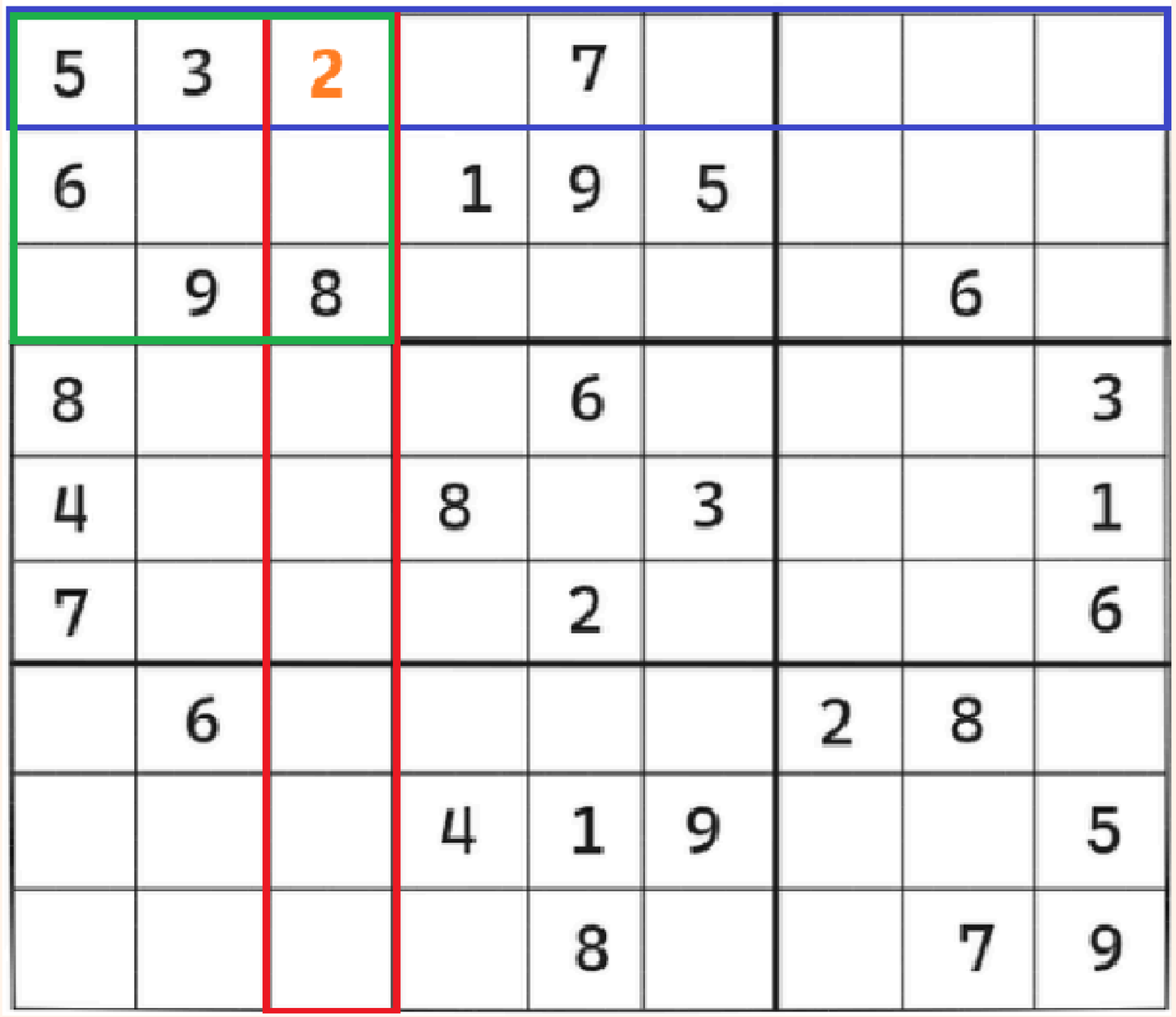
INITIAL STEPS

Regular backtracking, O(n) search time

Regular backtracking using hash arrays, O(1) search

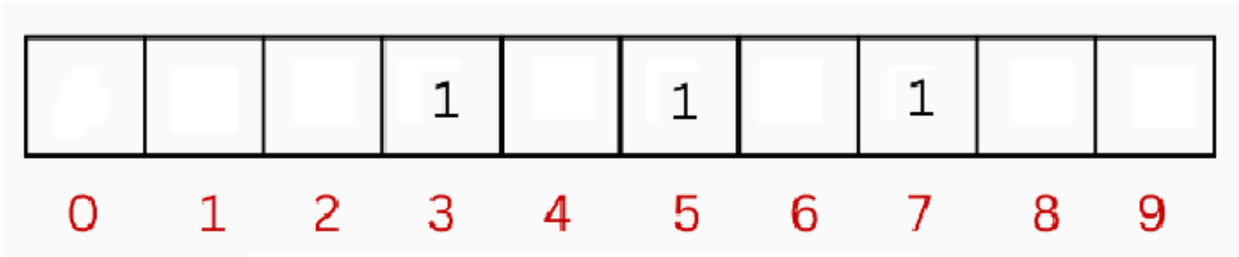
Regular backtracking, hashing with bits.

Looking for faster algorithms...


5 3 2 7
6 1 9 5
9 8 6
8 6 3
4 8 3 1
7 2 6
6 2 8
4 1 9 5
8 7 9

ARRAY HASHING

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9



BIT HASHING

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

0 0 1 0 1 0 1 0 0 0

9 8 7 6 5 4 3 2 1 0

$$1 \ll val$$

FASTER ALGORITHMS

Sudoku, being NP-complete, can be converted into other NP-complete problems. Efficient algorithms that reduce the search space for these problems exist.

The exact cover problem is one such problem, and Donald Knuth’s ‘Algorithm X’ implemented using ‘Dancing Links’ is a known efficient algorithm used to solve it.

THE EXACT COVER PROBLEM

Given a set S and another set where each element is a subset to S, is it possible to select a set of subsets such that every element in S exist in exactly one of the selected sets?

$$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$S^* = \{\{1, 2, 5, 7\}, \{3, 4, 6\}, \{1, 5, 9\}, \{1, 3, 4, 5, 6, 8, 9\}, \{2, 7, 8\}\}$$

$$A = \{1, 2, 5, 7\}$$

$$B = \{3, 4, 6\}$$

$$C = \{1, 5, 9\}$$

$$D = \{1, 3, 4, 5, 6, 8, 9\}$$

$$E = \{2, 7, 8\}$$

BINARY MATRIX REPRESENTATION

$$A = \{1, 2, 5, 7\}$$
$$B = \{3, 4, 6\}$$
$$C = \{1, 5, 9\}$$
$$D = \{1, 3, 4, 5, 6, 8, 9\}$$
$$E = \{2, 7, 8\}$$

*

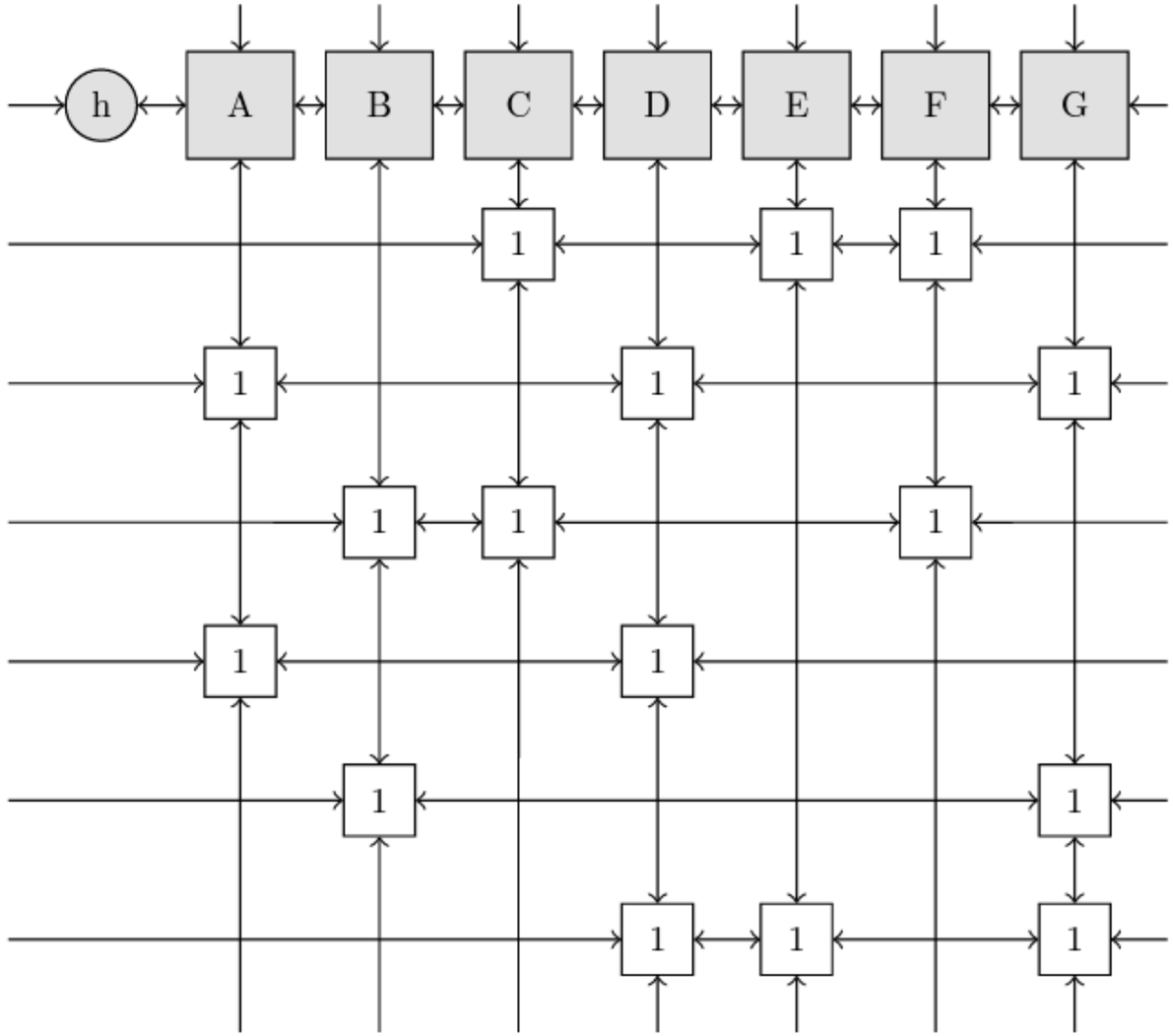
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

ALGORITHM X*

$$
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{c}
\begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\
\left(\begin{array}{ccccccccc}
1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}\right)
\end{array}
$$

TOROIDAL LINKED LIST *

$$\begin{array}{ccccccc} A & B & C & D & E & F & G \\ \left(\begin{array}{ccccccc} 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \end{array}$$



CONVERTING BETWEEN THE TWO

Convert

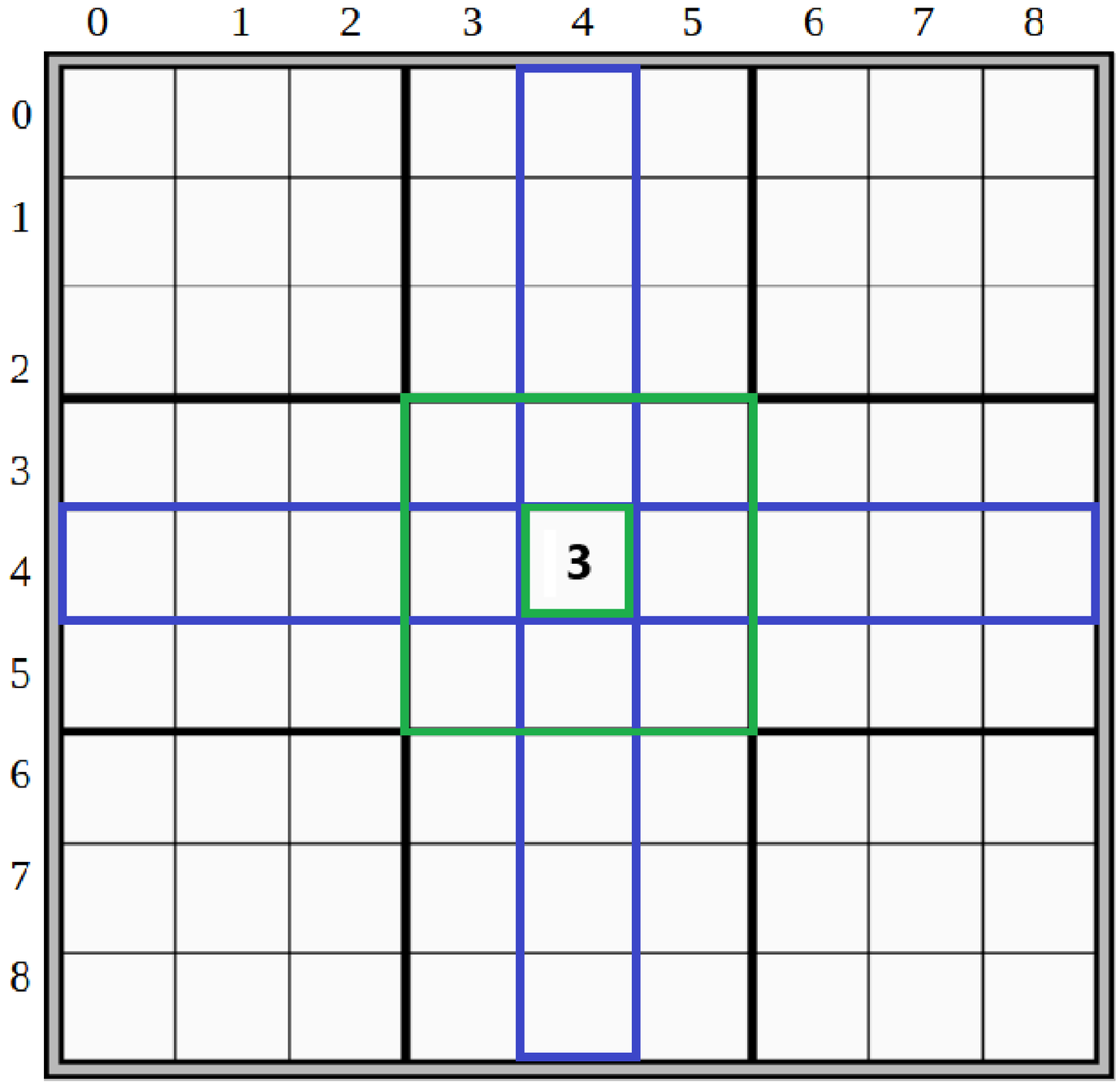
Convert Sudoku into the exact cover problem

Solve

Identify constraints and solve using Knuth’s DLX

Convert back

Convert solution into solved Sudoku format

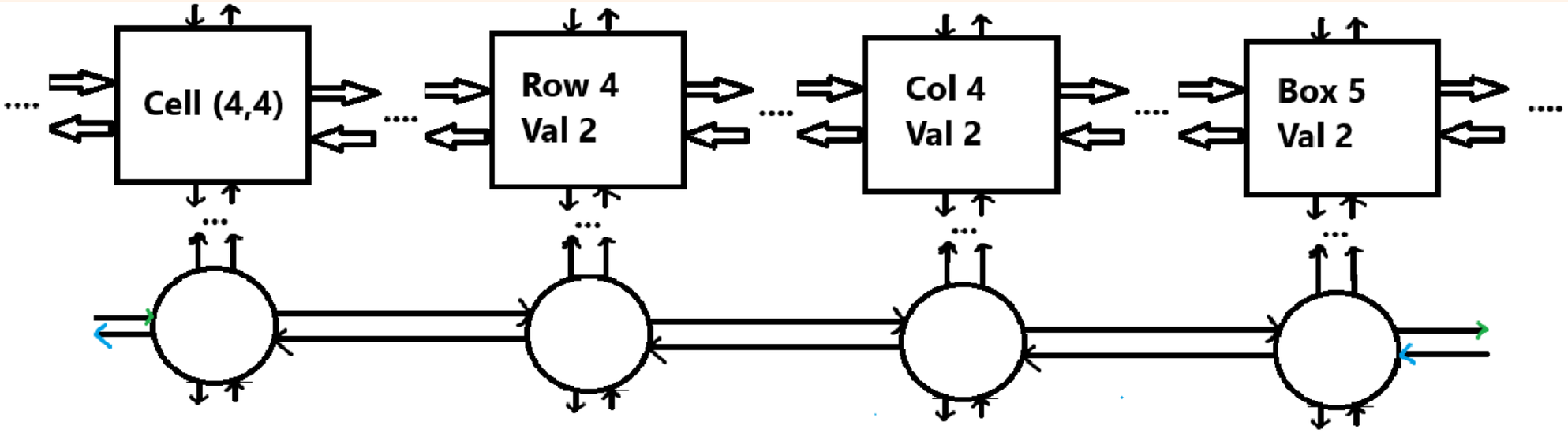


Row 4, Col 4
Row 4, value 2
Col 4, value 2
Box 5, value 2

[row] [col] [val-1]
[4][4][2]

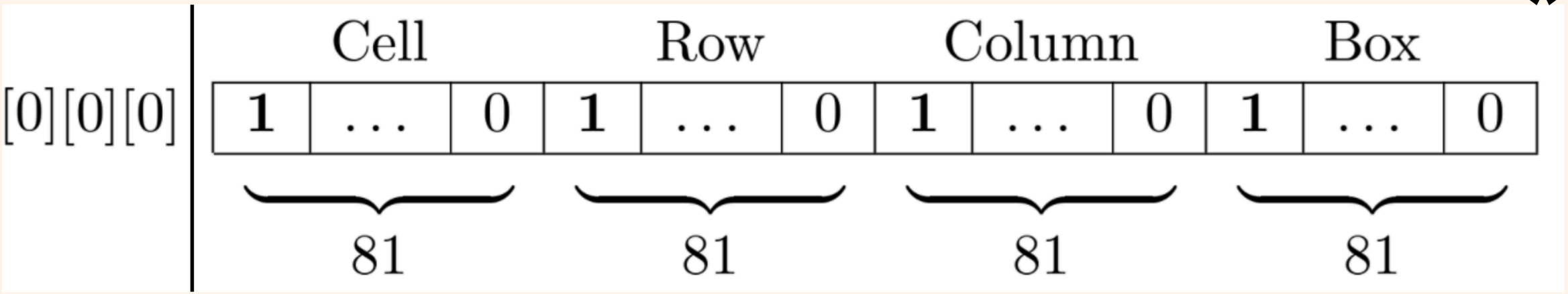
$$
\begin{array}{lll}
Cell\ constraints: & \{R0C0, R0C1, R0C2, \ldots\ldots, R8C8\} & 81 \\
Row\ constraints: & \{R0V0, R0V1, R0V2, \ldots\ldots, R8V8\} & 81 \\
Column\ constraints: & \{C0V0, C0V1, C0V2, \ldots\ldots, C8V8\} & 81 \\
Box\ constraints: & \{B0V0, B0V1, B0V2, \ldots\ldots, B8V8\} & 81
\end{array}
$$

[4][4][2]



CONVERSION TO EXACT COVER

Represent a choice as [Row index][Column index][Value - 1]



*

```
RCidx = row      * n + col                , //constr 0: cell
RVidx = row      * n + val + n * n        , //constr 1: row-val
CVidx = col      * n + val + 2 * n * n    , //constr 2: col-val
GVidx = gridnum  * n + val + 3 * n * n ; //constr 3: grid-val
```

CONVERTING BACK TO SUDOKU

```
constraintType  = (solutionRowNode->col->index)/(n*n);

if(constraintType == 0)      cell_index = solutionRowNode->col->index;
else                         val = (solutionRowNode->col->index)%(this->n) + 1;
```

All the following measurements are for an $n \times n$ Sudoku

Time Complexity

The generalized Sudoku, being an NP-complete problem, cannot have an accurate time complexity estimation, but the theoretical upper bound would be $O(n^m)$ where m denotes the number of unfilled cells in the given puzzle, with $0 \leq m \leq n^2$.

Space Complexity

$O(n^3)$, taken by the toroidal linked list.

Data Structures

Toroidal Linked List: Doubly, circularly linked(across 2 dimensions) list.
std::vector and *std::array* as linear data structures where necessary

Algorithm used

Donald Knuth's Algorithm X, implemented using the Toroidal Linked List mentioned above. Known to drastically reduce the search space for solving the exact cover problem.

Supported Operations

The current implementation can solve any Sudoku upto 25×25 in size, but this can easily be expanded to 64×64 sized ones (maximum integer limit in *C++* is *64 bits* without having to import complex libraries).

It is able to output all the solutions of any given Sudoku, but we've limited it to 10 currently.

IMAGE CREDITS

All images with a ‘*’ on the top right were taken from reference 2 mentioned on our GitHub repository.

The rest were made by us.

THANK YOU