

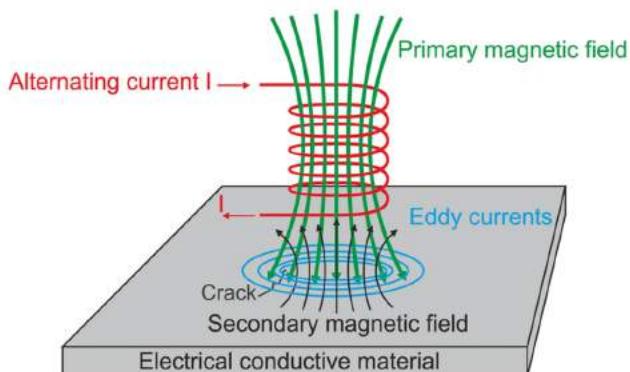
Chapter 6

Electromagnetic Induction

Eddy Currents

What are Eddy Currents?

- An eddy current is a current set up in a conductor in response to a changing magnetic field. They flow in closed loops in a plane perpendicular to the magnetic field. By Lenz law, the current swirls in such a way as to create a magnetic field opposing the change; for this to occur in a conductor, electrons swirl in a plane perpendicular to the magnetic field.
- Because of the tendency of eddy currents to oppose, eddy currents cause a loss of energy. Eddy currents transform more useful forms of energy, such as kinetic energy, into heat, which isn't generally useful.



Formation of Eddy Currents

Some Practical Applications

In the Brakes of Trains

During braking, the brakes expose the metal wheels to a magnetic field which generates eddy currents in the wheels. The magnetic interaction between the applied field and the eddy currents acts to slow the wheels down. The faster the wheels spin, the stronger is the effect, meaning that as the train slows the braking force is reduced, producing a smooth stopping motion.

Electromagnetic damping

- Used to design deadbeat galvanometers. Usually, the needle oscillates a little about its equilibrium position before it comes to rest. This causes a delay in taking the reading so to avoid this delay, the coil is wound over a non-magnetic metallic frame.
- As the coil is deflected, eddy currents set up in the metallic frame and thus, the needle comes to rest almost instantly.
- Thus, the motion of the “coil is damped”. Certain galvanometers have a fixed core made up of nonmagnetic metallic material. When the coil oscillates, the eddy currents that generate in the core oppose the motion and bring the coil to rest.

Electric Power Meters

The shiny metal disc in the electric power meter rotates due to eddy currents. The magnetic field induces the electric currents in the disc. You can also observe the shiny disc at your house.



Electric power meters

Induction Furnace

In a rapidly changing magnetic fields, due to a large emf produced, large eddy currents are set up. Eddy currents produce temperature. Thus a large temperature is created. So a coil is wound over a constituent metal which is placed in a field of the highly oscillating magnetic field produced by high frequency.



Industrial induction furnace

The temperature produced is enough to melt the metal. This is used to extract metals from ores. Induction furnace can be used to prepare alloys, by melting the metals at a very high temperature.

Speedometers

To know the speed of any vehicle, these currents are used. A speedometer consists of a magnet which keeps rotating according to the speed of our vehicle. Eddy currents are been produced in the drum. As the drum turns in the direction of the rotating magnet, the pointer attached to the drum indicates the speed of the vehicle



Speedometer

Ques: Eddy currents are produced in a metallic conductor when

- (a) The magnetic flux linked with it changes
- (b) It is placed in the changing magnetic field
- (c) Placed in the magnetic field
- (d) Both A and B

Ans: D

Solution: They are produced when the magnetic flux passing through the metal object continuously changes. This may happen due to many reasons.

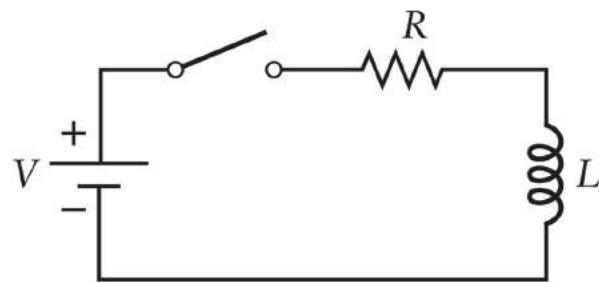
$$\nabla^2 \mathbf{H} = \sigma \frac{\partial \mathbf{B}}{\partial t}.$$

By definition, $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, where \mathbf{M} is the magnetization of the material and μ_0 is the vacuum permeability. The diffusion equation therefore is

$$\nabla^2 \mathbf{H} = \mu_0 \sigma \left(\frac{\partial \mathbf{M}}{\partial t} + \frac{\partial \mathbf{H}}{\partial t} \right).$$

L.R. Circuit

As the switch S is closed in given figure, current in



circuit wants to rise upto $\frac{V}{R}$ in no time but inductor

opposes it $(\frac{di}{dt} \rightarrow \frac{V}{L})$

hence at time $t = 0$ inductor will behave as an open circuit

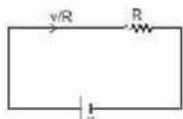
at $t = 0$

As the time passes, i in the circuit rises and $\frac{di}{dt}$ decreases. At any instant t .

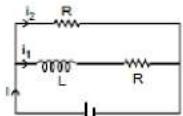
$$\frac{L di}{dt} + iR = V$$

current reaches the value $\frac{V}{R}$ at time $t = \infty$ or we can say, inductor will behave as a simple wire.

at $t = \infty$



Example 1. Find value of current i , i_1 and i_2 in given figure at

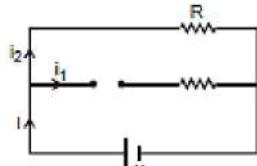


- (a) time $t = 0$
 (b) time $t = \infty$

Ans. (a) At time $t = 0$ inductor behaves as open circuit

$$i = v/R$$

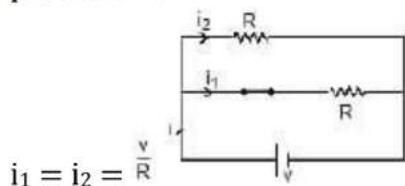
$$i_1 = 0$$



$$i_2 = i = v/R$$

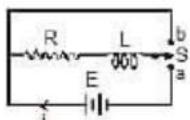
(b) At time $t = \infty$. Inductor will behave as simple wire

$$i = \frac{v}{(R/2)} - \frac{2v}{R}$$



Growth and Decay of Current in L-R Circuit

Consider a circuit containing a resistance R , an inductance L , a two way key and a battery of e.m.f E connected in series as shown in figure. When the switch S is connected to a , the current in the circuit grows from zero value. The inductor opposes the growth of the current. This is due to the fact that when the current grows through inductor, a back e.m.f. is developed which opposes the growth of current in the circuit. So the rate of growth of current is reduced. During the growth of current in the circuit, let i be the current in the circuit at any instant t . Using Kirchhoff's voltage law in the circuit, we obtain



$$E - L \frac{di}{dt} = Ri \text{ or } E - Ri = L \frac{di}{dt} \text{ or } \frac{di}{E - Ri} = \frac{dt}{L}$$

Multiplying by $-R$ on both the sides, we get

$$\frac{-R di}{E - Ri} = \frac{-R dt}{L}$$

Thus after an interval of (L/R) second, the current reaches to a value which is 63% of the maximum current. The value of (L/R) is known as time constant of the circuit and is represented by t . Thus the time constant of a circuit may be defined as the time in which the current rises from zero to 63% of its final value. In terms of t ,

$$i = i_0 \left(1 - e^{-\frac{t}{t}} \right)$$

(ii) The rate of growth of current (di/dt) is given by

$$\frac{di}{dt} = \frac{d}{dt} \left[i_0 \left[1 - \exp \left(-\frac{R}{L} t \right) \right] \right] \Rightarrow \frac{di}{dt} = i_0 \left(\frac{R}{L} \right) \exp \left(-\frac{R}{L} t \right) \dots (4)$$

$$\text{From equation (3), } \exp \left(-\frac{R}{L} t \right) = \frac{i_0 - i}{i_0}$$

$$\text{Therefore, } \frac{di}{dt} = i_0 \left(\frac{R}{L} \right) \left(\frac{i_0 - i}{i_0} \right) = \frac{R}{L} (i_0 - i) \dots (5)$$

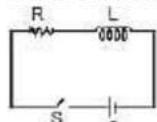
This shows that the rate of growth of the current decreases as i tends to i_0 . For any other value of current, it depends upon the value of R/L . Thus greater is the value of time constant, smaller will be the rate of growth of current.

Note:

- Final current in the circuit = $\frac{s}{R}$, which is independent of L .
- After one time constant, current in the circuit = 63% of the final current (verify yourself)
- More time constant in the circuit implies slower rate of change of current.
- If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.

$$L_1 i_1 = L_2 i_2$$

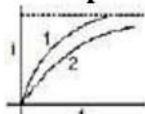
Example 2. At $t = 0$ switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made h times lesser than its initial value, find out instant current just after the operation.



Ans. Using above result (note 4)

$$L_1 i_1 = L_2 i_2 \Rightarrow i_2 = \frac{i_1}{h}$$

Example 3. Which of the two curves shown has less time constant.



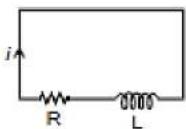
Ans. curve 1

Decay of Current

Let the circuit be disconnected from battery and switch S is thrown to point b in the figure. The current now begins to fall. In the absence of inductance, the current would have fallen from maximum i_0 to zero almost instantaneously. But due to the presence of inductance, which opposes the decay of current, the rate of decay of current is reduced.

Suppose during the decay of current, i be the value of current at any instant t . Using Kirchhoff's voltage law in the circuit, we get

$$-L \frac{di}{dt} - Ri \text{ or } \frac{di}{dt} + \frac{R}{L}i$$



Integrating this expression, we get

$$\log_e i = -\frac{R}{L}t + B$$

where B is constant of integration. The value of B can be obtained by applying the condition that when $t = 0$, $i = i_0$

Therefore, $\log_e i_0 = B$

Substituting the value of B , we get

$$\log_e i = -\frac{R}{L}t + \log_e i_0$$

$$\text{or } \log_e \frac{i}{i_0} = -\frac{R}{L}t$$

$$\text{or } (i/i_0) = \exp\left(-\frac{R}{L}t\right) \dots (6)$$

$$\text{or } i = i_0 \exp\left(-\frac{R}{L}t\right) = i_0 \exp\left(-\frac{t}{\tau}\right)$$

where $t = L/R =$ inductive time constant of the circuit.

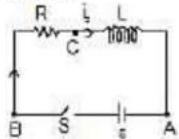
It is obvious from equation that the current in the circuit decays exponentially as shown in figure.

We observe the following points

- After $t = L/R$, the current in the circuit is given by

$$i = + \frac{\varepsilon}{3R} \left(1 - e^{-\frac{3Rt}{L}} \right)$$

Example 5. Figure shows a circuit consisting of an ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at $t = 0$. Suppose at $t = 0$ current in the inductor is i_0 then find out equation of current as a function of time



Ans: Let an instant t current in the circuit is i which is increasing at the rate di/dt . Writing KVL along the circuit, we have

$$\begin{aligned} \varepsilon - L \frac{di}{dt} - iR &= 0 \Rightarrow L \frac{di}{dt} = \varepsilon - iR \\ \Rightarrow \int \frac{i}{\varepsilon - iR} \frac{di}{dt} &= \int \frac{dt}{L} \quad \Rightarrow \ln \left(\frac{\varepsilon - iR}{\varepsilon - i_0 R} \right) = - \frac{Rt}{L} \\ \Rightarrow \varepsilon - iR &= (\varepsilon - i_0 R) e^{-Rt/L} \Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0 R) e^{-Rt/L}}{R} \end{aligned}$$

Self-Inductance

6. Self induction

Self induction is induction of emf in a coil due to its own current change. Total flux

$N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$ where L is called coefficient of self induction or inductance. The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt , the average emf induced in the coil is given as

$$\varepsilon = - \frac{\Delta(N\phi)}{\Delta t} = - \frac{\Delta(LI)}{\Delta t} = - \frac{LI}{\Delta t}$$

The instantaneous emf is given as

$$\varepsilon = - \frac{d(N\phi)}{dt} = - \frac{d(LI)}{dt} = - \frac{Ldi}{dt}$$

S.I unit of inductance is wb/amp or Henry (H)

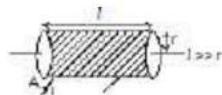
L - self inductance is +ve Quantity.

L depends on: (1) Geometry of loop
 (2) Medium in which it is kept. L does not depend upon current. L is a scalar quantity.

Brain Teaser

If a circuit has large self-inductance, what inference can you draw about the circuit.

6.1 Self Inductance of solenoid



Let the volume of the solenoid be V , the number of turns per unit length be n . Let a current I be flowing in the solenoid. Magnetic field in the solenoid is given as $B = \mu_0 n l$. The magnetic flux through one turn of solenoid $\Phi = \mu_0 n l A$.

$$\Phi$$

$$\begin{aligned} \text{The total magnetic flux through the solenoid} &= N \\ &= N \mu_0 n l A \\ &= \mu_0 n^2 l A I \\ \text{Therefore, } L &= \mu_0 n^2 l A = \mu_0 n^2 V \end{aligned}$$

$$\begin{aligned} \Phi &= \mu_0 n i \pi r^2 (n l) \\ L &= \frac{\Phi}{i} = \mu_0 n^2 \pi r^2 l \\ \text{Inductance per unit volume} &= \mu_0 n^2 \end{aligned}$$

Ex.38 The current in a coil of self-inductance $L = 2\text{H}$ is increasing according to the law $i = 2 \sin t^2$. Find the amount of energy spent during the period when the current changes from 0 to 2 ampere.

Sol. Let the current be 2 amp at $t = \tau$

$$\text{Then } 2 = 2 \sin \tau^2 \Rightarrow \tau = \sqrt{\frac{\pi}{2}}$$

When the instantaneous current is i , the self induced emf is $L \frac{di}{dt}$. If the amount of charge that is displaced in time dt is $d\theta$, then the elementary work done

$$= L \left(\frac{di}{dt} \right) dq = L \frac{di}{dt} idt = L i di$$

$$W = \int_0^T L di = \int_0^T L(2 \sin t^2) d(2 \sin t^2)$$

$$W = \int_0^T 8L \sin t^2 \cos t^2 (2t) dt = 4L \int_0^T \sin 2t^2 (2t) dt$$

Let $\theta = 2t^2$

Differentiating $d\theta = 4t dt$

$$\text{Therefore, } W = \frac{4L}{4} \int \frac{\sin \theta}{4} d\theta$$

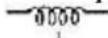
$$= L (-\cos \theta) = -L \cos 2t^2$$

$$W = -L [\cos 2t^2]_0^{4t^2}$$

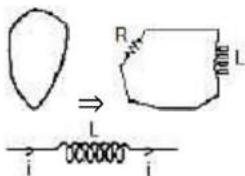
$$= 2L = 2 \times 2 = 4 \text{ joule}$$

7. Inductor

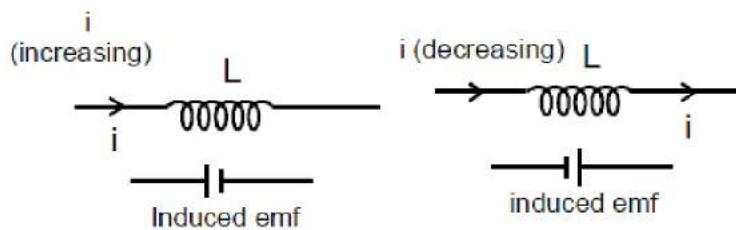
It is represented by



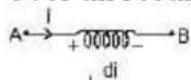
electrical equivalence of loop



If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current. If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.



Overall result



$$V_A - L \frac{di}{dt} = V_B$$

Note

➤ If there is a resistance in the inductor (resistance of the coil of inductor) then :

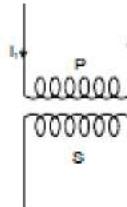
10. MUTUAL INDUCTANCE

Consider two coils P and S placed close to each other as shown in the figure. When the current passing through a coil increases or decreases, the magnetic flux linked with the other coil also changes and an induced e.m.f. is developed in it. This phenomenon is known as mutual induction. This coil in which current is passed is known as primary and the other in which e.m.f. is developed is called as secondary. Let the current through the primary coil at any instant be i_1 . Then the magnetic

flux ϕ_1 in the secondary at any time will be proportional to i_1 i.e., ϕ_1 is directly proportional to i_1

Therefore the induced e.m.f. in secondary when i_1 changes is given by

$$e = -\frac{d\phi_1}{dt} \text{ i.e., } e \propto -\frac{di_1}{dt}$$



$$\therefore e = -M \frac{di_1}{dt} = -\frac{dM i_1}{dt} \Rightarrow \phi_2 = M i_1$$

where M is the constant of proportionality and is known as mutual inductance of two coils. It is defined as the e.m.f. induced in the secondary coil by unit rate of change of current in the primary coil. The unit of mutual inductance is henry (H).

10.1 Mutual Inductance of a Pair of Solenoids one Surrounding the other coil

Figure shows a coil of N_2 turns and radius R_2 surrounding a long solenoid of length l_1 , radius R_1 and number of turns N_1 .



To calculate mutual inductance M between them, let us assume a current i_1 through the inner solenoid S_1 .

There is no magnetic field outside the solenoid and the field inside has magnitude,

$$B = \mu_0 \left(\frac{N_1}{l_1} \right) i_1$$

and is directed parallel to the solenoid's axis. The magnetic flux ϕ_{B_1} through the surrounding coil is, therefore,

$$\phi_{B_1} = B(\pi R_1^2) = \frac{\mu_0 N_1 i_1}{l_1} \pi R_1^2$$

$$\text{Now, } M = \frac{N_2 \phi_{B_1}}{i_1} = \left(\frac{N_2}{i_1} \right) \left(\frac{\mu_0 N_1 i_1}{l_1} \right) \pi R_1^2 \Rightarrow \frac{\mu_0 N_1 N_2 \pi R_1^2}{l_1}$$

Notice that M is independent of the radius R_2 of the surrounding coil. This is because solenoid's magnetic field is confined to its interior.

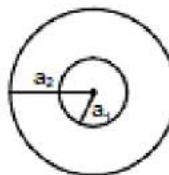
Brain Teaser

What is the meaning of the statement "The coefficient of mutual inductance for a pair of coils is large"?

Note: $M \leq \sqrt{L_1 L_2}$.

- For two coils in series if mutual inductance is considered then
 $L_{\text{eq}} = L_1 + L_2 \pm 2M$

Ex.48 Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 \ll a_2$) if the planes of coils are same.



Sol. Let a current i flow in coil of radius a_2 .

Magnetic field at the centre of coil =

$$\text{or } M i = \frac{\mu_0 i}{2a_2} \pi a_1^2 \quad \text{or} \quad M = \frac{\mu_0 \pi a_1^2}{2a_2}$$

Ex.49 Solve the above question, if the planes of coil are perpendicular.

Sol. Let a current i flow in the coil of radius a_1 . The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence $M = 0$

Ex.50 Solve the above problem if the planes of coils make θ angle with each other.

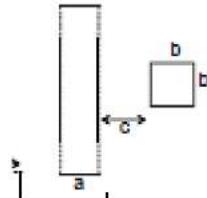
Sol. If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

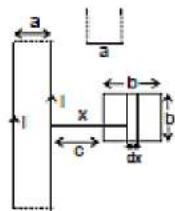
$$\text{Thus flux} = \frac{\mu_0 i}{2a_2} \cdot \pi a_1^2 \cos \theta$$

$$\text{or } M = \frac{\mu_0 \pi a_1^2 \cos \theta}{2a_2}$$

Ex.51 Find the mutual inductance between two rectangular loops, shown in figure.



Sol. Let current i flow in the loop having ∞ -by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

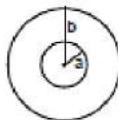


$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)} \right] b dx$$

$$\Rightarrow \phi = \int_{-\infty}^{+\infty} \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)} \right] b dx$$

$$= \frac{\mu_0 j b}{2\pi} \left[\ln \frac{c+b}{c} - \ln \frac{a+b+c}{a+c} \right]$$

Ex.52 Figure shows two concentric coplanar coils with radii a and b ($a \ll b$). A current $i = 2t$ flows in the smaller loop. Neglecting self inductance of larger loop



- (a) Find the mutual inductance of the two coils
- (b) Find the emf induced in the larger coil
- (c) If the resistance of the larger loop is R find the current in it as a function of time

Sol. (a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be

flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 j}{2b}$.

flux through the smaller coil = $\frac{\mu_0 j}{2b} \pi a^2$

Therefore, $M = \frac{\mu_0}{2b} \pi a^2$

(ii) |emf induced in larger coil| = $M \left[\left(\frac{di}{dt} \right) \text{in smaller coil} \right]$

$$= \frac{\mu_0}{2b} \pi a^2 (2) = \frac{\mu_0 \pi a^2}{b}$$

(iii) current in the larger coil = $\frac{\mu_0 \pi a^2}{bR}$

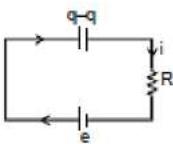
Ex.53 If the current in the inner loop changes according to $i = 2t^2$ then, find the current in the capacitor as a function of time.



$$\text{Sol. } M = \frac{\mu_0}{2b} \pi a^2$$

|emf induced in larger coil| = $M \left[\left(\frac{di}{dt} \right) \text{in smaller coil} \right]$

$$e = \frac{\mu_0 \pi a^2}{2b} (4t) = \frac{2\mu_0 \pi a^2 t}{b}$$

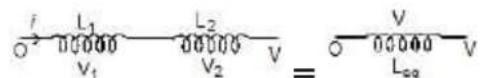


Applying KVL: -

$$+e - \frac{q}{c} - iR = 0 \Rightarrow \frac{2\mu_0 \pi a^2 t}{b} - iR = 0$$

$$\text{differentiate wrt time: } \frac{2\mu_0 \pi a^2}{b} - \frac{i}{c} - \frac{di}{dt} R = 0 \text{ on solving it } i = \frac{2\mu_0 \pi a^2 C}{b} [1 - e^{-t/Rc}]$$

11. Series Combination of Inductors

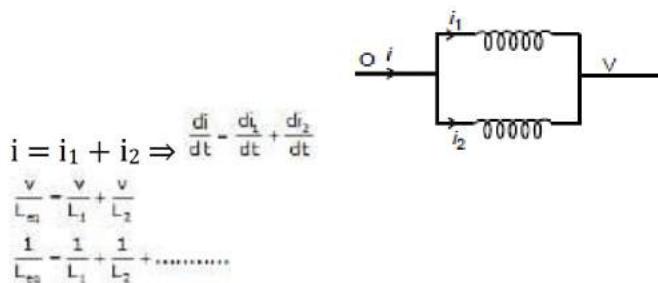


$$\text{Therefore, } V = V_1 + V_2$$

$$L_{eq} = \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + \dots$$

Parallel Combination of inductor



Faraday's Law of Electromagnetic Induction & Lenz's Law

1. MAGNETIC FLUX

Consider a closed curve enclosing an area A (as shown in the figure). Let there be a uniform magnetic field \vec{B} in that region. The magnetic flux through the area \vec{A} is given by

...(i)

$$\varphi = BA \cos \theta$$

where θ is the angle which the vector B makes with the normal to the surface. If \vec{B} is perpendicular to \vec{A} , then the flux through the closed area \vec{A} is zero. SI unit of magnetic flux is weber (Wb).

Notes

- Area vector is perpendicular to the surface
- For open surface choose one direction as the area vector direction and stick to it for the whole problem.
- For closed surfaces outward normal is taken as area vector direction
- Flux is basically count of number of lines crossing a surface
- Because magnetic field lines exists in closed loop.

Ex.1 Find flux passing through Area?



Sol. Since \vec{n} is \perp to \vec{B}

$$\varphi = \vec{B} \cdot \vec{A} = 0$$

2. FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

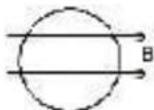
Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes an emf is produced in the loop. The emf is given by
... (ii)

where the flux of magnetic field through the area.

The emf so produced drives an electric current through the loop. If the resistance of the loop is R , then the current

$$i = \frac{\epsilon}{R} = -\frac{1}{R} \frac{d\varphi}{dt} \quad \dots \text{(iii)}$$

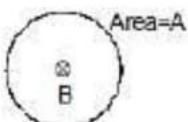
Ex.2 A coil is placed in a constant magnetic field. The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



Sol. $\epsilon = 0$ (always) since area is perpendicular to magnetic field.

Therefore, $\text{emf} = 0$

Ex.3 Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



Sol. $f = BA$ (always) = const.

Therefore, $\text{emf} = 0$

Ex.4 Show that if the flux of magnetic induction through a coil changes from f_1 to f_2 , then the charge θ that flows through the circuit of total resistance R is given by θ

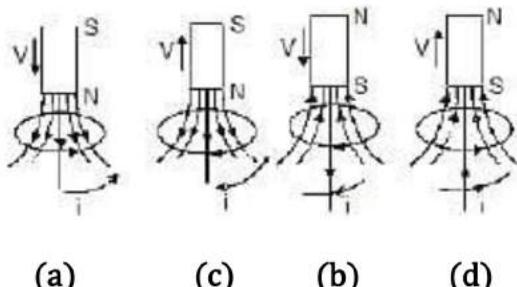
$$= \frac{\phi_2 - \phi_1}{R}, \text{ where } R \text{ is the resistance of the coil.}$$

Sol. Let f be the instantaneous flux. Then $\frac{df}{dt}$ is the instantaneous rate of change of flux which is equal to the magnitude of the instantaneous emf. so the current in the circuit $|i| = \frac{1}{R} \left(\frac{df}{dt} \right)$, since the current is the rate of flow of charge, that is, $i = \frac{dq}{dt}$
 $\theta = \int i dt$ or $\theta = \int_{t=0}^{t=t} \left(\frac{1}{R} \cdot \frac{df}{dt} \right) dt$
where t is the time during which change takes place. but at $t = 0$, $f = f_1$, and at $t = t$, $f = f_2$

$$\text{Therefore, } \theta = \frac{1}{R} \int_{t=0}^{t=t} d\phi = \frac{\phi_2 - \phi_1}{R}$$

3. LENZ'S LAW

The effect of the induced emf is such as to oppose the change in flux that produces it.



In figure (a & b) as the magnet approaches the loop, the flux through the loop increases. The induced current sets up an induced magnetic field B_{ind} whose flux

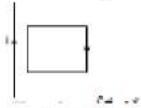
opposes this change. The direction of B_{ind} is opposite to that of external field B_{ext} due to the magnet.

In figure (c & d) the flux through the loop decreases as the magnet moves away from the loop, the flux due to the induced magnetic field tries to maintain the flux through the loop. The direction of B_{ind} is same as that of B_{ext} due to magnet.

Lenz's law is closely related to the law of conservation of energy and is actually a consequence of this general law of nature. As the north pole of the magnet moves towards the loop an induced current is produced. This opposes the motion of N-pole of the bar magnet. Thus, in order to move the magnet toward the loop with a constant velocity an external force is to be applied. The work done by this external force gets transformed into electric energy, which induces current in the loop.

There is another alternative way to find the direction of current inside the loop which is described below.

Figure shows a conducting loop placed near a long, straight wire carrying a current i as shown. If the current increases continuously, then there will be an emf induced inside the loop. Due to this induced emf, an electric current is induced. To determine the direction of current inside the loop we put an arrow as shown. The right hand thumb rule shows that the normal to the loop is going into the plane. Again the same rule shows that the magnetic field at the site of the loop is also going into the plane of the diagram.

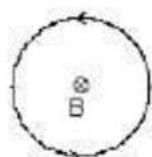


Thus \vec{B} and $d\vec{A}$ are in the same direction. Therefore $\int \vec{B} \cdot d\vec{A}$ is positive if i increases, the magnitude of flux Φ increases. Since magnetic flux Φ is positive and its magnitude increases, $\frac{d\Phi}{dt}$ is positive. Thus ϵ is negative and hence the current is negative. Thus the current induced is opposite, to that of arrow.

Brain Teaser

Two identical coaxial circular loops carry equal currents circulating in the same direction. What will happen to the current in each loop if the loops approach each other?

Ex.5 Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



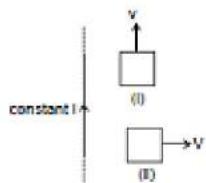
Sol. Inward flux is increasing with time. To oppose it outward magnetic field should be induced. Hence current will flow in anticlockwise.

Ex.6 Figure shows a long current carrying wire and two rectangular loops moving with velocity v .

Find the direction of current in each loop.

Sol. In loop (i) no emf will be induced because there is no flux change.

In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.



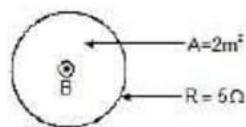
4. CALCULATION OF INDUCED EMF

As we know that magnetic flux (ϕ) linked with a closed conducting loop = $BA \cos \theta$ where B is the strength of the magnetic field, A is the magnitude of the area vector and θ is the angle between magnetic field vector and area vector.

Hence flux will be affected by change in any of them, which is discussed in the next page.

4.1 By changing the magnetic field

Ex.7 Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil. The magnetic field is decreasing at a rate of 10 T/s . Find out current in magnitude and direction



Sol. $\Phi = BA$

$$\text{emf} = A \cdot \frac{dB}{dt} = 2 \times 10 = 20 \text{ V}$$

Therefore, $i = 20/5 = 4 \text{ amp}$. From Lenz's law direction of current will be anticlockwise.

Ex.8 The magnetic flux (ϕ_2) in a closed circuit of resistance 20 W varies with time (t) according to the equation $f = 7t^2 - 4t$ where f is in weber and t is in seconds. The magnitude of the induced current at $t = 0.25$ s is
 (A) 25 mA (B) 0.025 mA (C) 47 mA (D) 175 mA

Sol. $\phi = 7t^2 - 4t$

$$\Rightarrow \text{Induced emf: } |e| = \frac{d\phi}{dt} = 14t - 4$$

⇒ Induced current:

$$i = \frac{|e|}{R} = \frac{|14t - 4|}{20} = \frac{|14 \times 0.25 - 4|}{20} \quad (\text{at } t = 0.25 \text{ s}) \\ = \frac{0.5}{20} = 2.5 \times 10^{-2} \text{ A}$$

Therefore, (A)

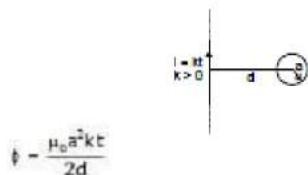
Ex.9 Consider a long infinite wire carrying a time varying current $i = kt$ ($k > 0$). A circular loop of radius a and resistance R is placed with its centre at a distance d from the wire ($a \ll d$). Find out the induced current in the loop?

Sol. Since current in the wire is continuously increasing therefor we conclude that magnetic field due to this wire in the region is also increasing.

Magnetic field B due to wire $= \frac{\mu_0 i}{2\pi d}$ going into and perpendicular to the plane of the paper

Flux through the circular loop,

$$\phi = \frac{\mu_0 i}{2\pi d} \times \pi a^2$$



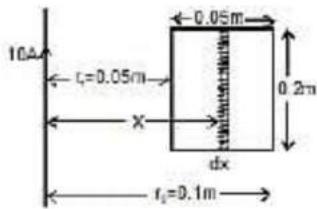
Induced e.m.f. in the loop

$$e = -\frac{d\phi}{dt} = \frac{-\mu_0 a^2 k}{2d}$$

$$\text{Induced current in the loop } i = \frac{|e|}{R} = \frac{\mu_0 a^2 k}{2dR}$$

Direction of induced current in the loop is anticlockwise.

Ex.10 Two concentric coplanar circular loops have diameters 20 cm and 2 m and resistance of unit length of the wire = 10^{-4} W/m. A time -dependent voltage $V = (4 + 2.5 t)$ volts is applied to the larger as shown. The current in the smaller loop is



Sol. A current, $i = 10 \text{ A}$ is flowing in the long straight wire. Consider a small rectangular strip (in the rectangular wire frame) of width dx at a distance x from the straight wire.

The magnetic flux at the location of the strip,

$$B_x = \frac{\mu_0 i}{2\pi x}$$

The flux linked with the infinitesimally small rectangular strip

$$\begin{aligned} &= B_x \times \text{Area of the strip} = d\phi_x = \frac{\mu_0 i}{2\pi x} I dx \\ &\text{where } I \text{ is the length of the rectangular wire circuit} \\ &= 2 \times 10^{-1} \text{ m} \end{aligned}$$

$$\text{or } d\phi_x = (m_0 i / 2\pi) (dx/x)$$

Hence, the total magnetic flux linked with the rectangular frame

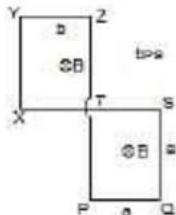
$$\begin{aligned} &= \int d\phi_x = \phi = \frac{\mu_0 i}{2\pi} [\log_e x]_1^2 \\ &\text{or } \phi = \frac{\mu_0 i}{2\pi} [\log_e r_2 - \log_e r_1] = \frac{\mu_0 i}{2\pi} \log_e \left(\frac{r_2}{r_1} \right) \end{aligned}$$

Substituting values, we get

$$\begin{aligned} \phi &= 2 \times 10^{-7} \times 10 \times 2 \times 10^{-1} \times \log_e 2 \\ &= 2.772 \times 10^{-7} \text{ Wb} \end{aligned}$$

$$\begin{aligned} \text{Induced e.m.f. } &| \varepsilon | = \frac{d\phi}{dt} = \frac{\mu_0 / \log_e \left(\frac{r_2}{r_1} \right)}{2\pi} \frac{di}{dt} \\ &= (2 \times 10^{-7} \times 2 \times 10^{-1} \log_e 2) \frac{10}{0.2} \\ &= 1.386 \times 10^{-6} \text{ V} = 1.386 \text{ mV} \end{aligned}$$

Ex.12 Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B = bt$, where b is a positive constant. Resistance per unit length of the wire is l . Find the current induced in the wire and draw its electrical equivalent diagram.



Sol. Induced emf in part PQST = $b a^2$ (in anticlockwise direction, from Lenz's Law)
Similarly induced emf in part TXYZ = $b b^2$ (in anticlockwise direction, from Lenz's Law)

Total resistance of the part PQST = $l 4a$

Total resistance of the part TXYZ = $l 4b$.

The equivalent circuit it is shown in the diagram.
writing KVL along the current flow

$$\beta b^2 - \beta a^2 - \lambda 4ai - \lambda 4bi = 0$$

$$i = \frac{\beta}{4\lambda} (b - a)$$

Brain Teaser:

A copper ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring. Will the acceleration of the falling magnet be equal to, greater than or lesser than the acceleration due to gravity ?

4.2 BY CHANGING THE AREA

Solved Examples:

Ex.13 A space is divided by the line AD into two regions. Region I is field free and the region II has a uniform magnetic field B directed into the paper. ACD is a semicircular conducting loop of radius r with centre at O, the plane of the loop being in the plane of the paper. The loop is now made to rotate with a constant angular velocity w about an axis passing through O, and perpendicular to the plane of the paper in the clockwise direction. The effective resistance of the loop is R .

- (a) Obtain an expression for the magnitude of the induced current in the loop.
- (b) Show the direction of the current when the loop is entering into the region II.

(c) Plot a graph between the induced emf and the time of rotation for two periods of rotation.

Sol. (a) As in time t , the arc swept by the loop in the field, i.e., region II.

$$A = \frac{1}{2} r(\theta) - \frac{1}{2} r^2 \omega t$$

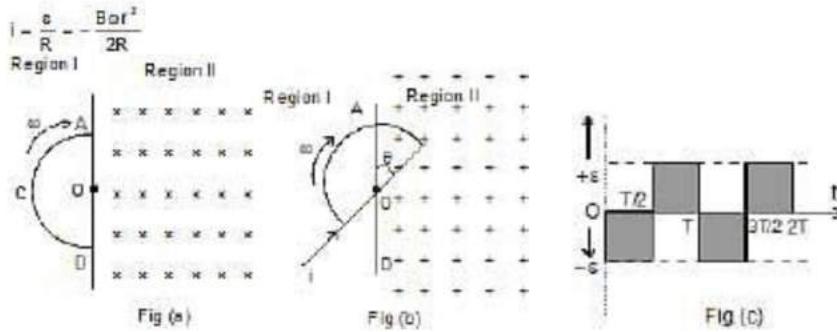
So the flux linked with the rotating loop at time t ,

$$\Phi = BA = \frac{1}{2} B \omega r^2 t [B = \omega t]$$

and hence the induced emf in the loop,

$$e = -\frac{d\Phi}{dt} = -\frac{1}{2} B \omega r^2 = \text{constant.}$$

And as the resistance of the loop is R , the induced current in it,



(b) When the loop is entering the region II, i.e., the field figure (b), the inward flux linked with it will increase, so in accordance with Lenz's law an anticlockwise current will be induced in it.

(c) Taking induced emf to the negative when flux linked with the loop is increasing and positive when decreasing, the emf versus time graph will be, as shown in figure (c)

Ex.14 Two parallel, long, straight conductors lie on a smooth plane surface. Two other parallel conductors rest on them at right angles so as to form a square of side a initially. A uniform magnetic field B exists at right angles to the plane containing the conductors. Now they start moving out with a constant velocity (v). (a) Will the induced emf be time dependent? (b) Will the current be time dependent?

Sol. (a) Yes, Φ (instantaneous flux) = $B(a + 2vt)^2$

$$\text{Therefore, } e = \frac{d\Phi}{dt} = 4Bv(a + 2vt)$$

(b) No,

$$(\text{instantaneous current}) i = \frac{e}{R}$$

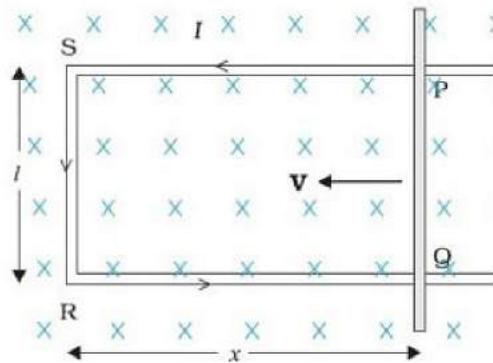
Now $R = 4(a + 2vt)r$ where r = resistance per unit length

$$\text{Therefore, } i = \frac{4Bv(a+2vt)}{4r(a+2vt)} = \frac{Bv}{r} \text{ (a constant)}$$

The current will be time independent.

Motional Electromotive Force

An emf induced by the motion of the conductor across the magnetic field is a motional electromotive force. The equation is given by $E = -vLB$. This equation is true as long as the velocity, field, and length are mutually perpendicular. The minus sign associated with the Lenz's law.



For us to understand the motional electromotive force, let us make a particular setup. Let us take a rectangular coil, a metal rod of length L , moving with velocity V , through a magnetic field B . There is a magnetic field at some location. Length, velocity and magnetic field should always be at a right angle with each other. The direction of the magnetic field is going inside. Assume the metal rod is frictionless that means there is no loss of energy due to friction and we apply a uniform magnetic field. The conductor rod is moved with a constant velocity and placed in the magnetic field.

Browse more Topics under Electromagnetic Induction

$$\Phi_B = Blx$$

- AC Generator
- Eddy Currents
- Energy Consideration: A Quantitative Study
- Faraday's and Lenz's Law
- Inductance

But 'x' changes with time,

$$E = - \frac{d\Phi_B}{dt} = - \frac{d}{dt} (Blx) = -Bl \frac{dx}{dt}$$

$$E = Blv$$

The induced emf Blv is motion electromotive force. So we produce emf by moving a conductor inside the uniform magnetic field. The power required to move a conductor rod in a magnetic field is,

$$P = \frac{B^2 l^2 v^2}{R}$$

Where

- B is the magnetic field,
- l is the length of the conductor
- v is the velocity of the conductor
- R is the resistance

The magnetic flux associated with the coil is given by $\Phi = BA \cos \theta$. We know that $\cos \theta = 0$, so $\Phi = BA$. The motion of electromotive force can be further explained by Lorentz force which acts on free charge carriers. The Lorentz force on charge is:

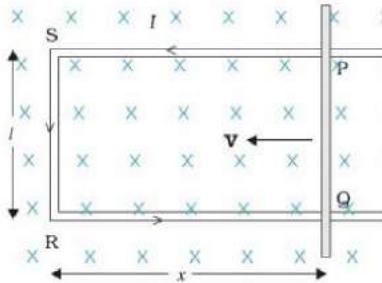
$$F = qVB$$

Energy Consideration: A Quantitative Study

Energy Consideration

We all know that Lenz's law is consistent with the law of conservation of energy. Lenz's Law states that, when you induce a current in a wire via a changing magnetic field, the current flows through the wire in such a direction so that its magnetic field opposes the change that produced the current.

Now let us understand energy consideration in a better way



Suppose there is a rectangular conductor. now from the above figure, we can say that the sides of the sides of the rectangular conductor are PQ, RS, QR, and SP. Now in this rectangular conductor, the three sides are fixed, while one of it's side that is the side PQ is set free.

Let r be that movable resistance of the conductor. So the resistance of the other remaining sides of the rectangular conductor that is the resistance of side RS, SP and QR is very small as compared to this movable resistance. In a constant magnetic field, if we change the flux, an emf is induced. i.e $E = d\Phi/dt$

If there is induced emf E and a movable resistance r in the conductor then, we can say that $I = Blv/R$. As the magnetic field is present, there will also be a force F acting, as $F = ILB$. This force is directed outwards in the direction opposite to the velocity of the rod, given by $F = B^2l^2v/R$

$$\text{Power} = \text{force} \times \text{velocity} = B^2l^2v/R$$

Now here the work done is mechanical and this mechanical energy is dissipated as Joule heat. This is given as $PJ = I^2R = B^2l^2v/R$. Further, the mechanical energy converts into electrical energy and finally into thermal energy. From the Faraday's law, we have learned that, $|E| = \Delta\Phi_B/\Delta t$

So we get, $|E| =$

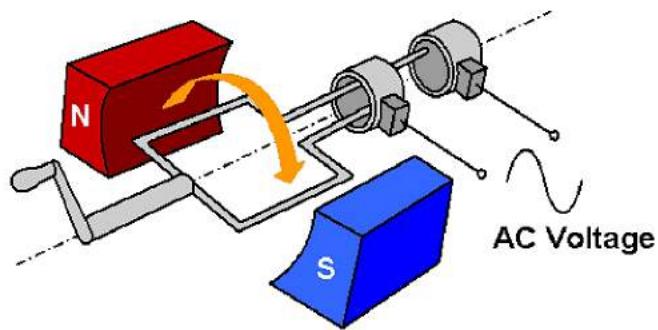
$$IR = \frac{\Delta Q}{\Delta t} R$$

Hence we get,

$$\Delta Q = \frac{\Delta\Phi_B}{R}$$

AC Generator

An AC generator is an electric generator that converts mechanical energy into electrical energy in form of alternative emf or alternating current. AC generator works on the principle of Electromagnetic Induction".



Parts of an AC Generator

An Ac generator consists of two poles i.e is the north pole and south pole of a magnet so that we can have a uniform magnetic field. There is also a coil which is rectangular in shape that is the armature. These coils are connected to the slip rings and attached to them are carbon brushes.

The slip rings are made of metal and are insulated from each other. The brushes are carbon brushes and one end of each brush connects to the ring and other connects to the circuit. The rectangular coils rotate about an axis which is perpendicular to the magnetic field. There is also a shaft which rotates rapidly.

Working of an AC Generator

When the armature rotates between the poles of the magnet upon an axis perpendicular to the magnetic field, the flux which links with the armature changes continuously. Due to this, an emf is induced in the armature. This produces an electric current through the galvanometer and the slip rings and brushes.

The galvanometer swings between the positive and negative values. This indicates that there is an alternating current flowing through the galvanometer.

Emf induced in an AC generator

If the coil of N turn and area A is rotated at v revolutions per second in a uniform magnetic field B, then the motional emf produced is $e = NBA(2\pi v) \sin(2\pi v t)$, where we assume that at time $t = 0$ s, the coil is perpendicular to the field. The direction of the induced emf is given by Fleming's right-hand rule or the Lenz's law.

Fleming's right-hand rule states that, stretch the forefinger, the middle finger and the thumb of the right hand such that they are manually perpendicular to each other. If the forefinger indicates the direction of the magnetic field, thumb indicates the direction of the motion of the conductor. The middle finger indicates the direction of the induced current in the conductor.

3-Phase AC generator

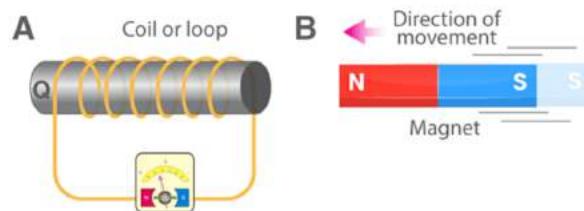
In a symmetric three-phase power supply system, three conductors each carry an alternating current of the same frequency and voltage amplitude relative to a common reference but with a phase difference of one third the period. The common reference usually connects to ground and often to a current-carrying conductor that is neutral.

Due to the phase difference, the voltage on any conductor reaches its peak at one-third of a cycle after one of the other conductors and one-third of a cycle before the remaining conductor. This phase delay gives constant power transfer to a balanced linear load. It also makes it possible to produce a rotating magnetic field in an electric motor and generate other phase arrangements using transformers.

Experiments of Faraday & Henry

In this section, we will learn about the experiments carried out by Faraday and Henry that are used to understand the phenomenon of electromagnetic induction and its properties.

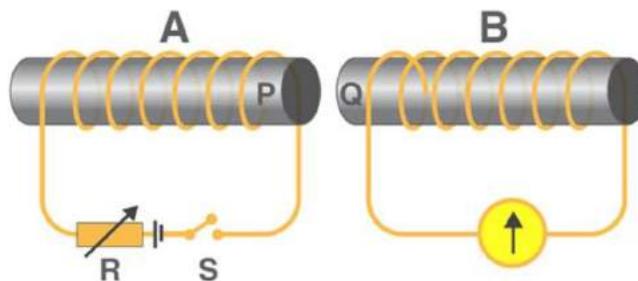
Experiment 1



In this experiment, Faraday connected a coil to a galvanometer, as shown in the figure above. A bar magnet was pushed towards the coil, such that the north pole is pointing towards the coil. As the bar magnet is shifted, the pointer in the galvanometer gets deflected, thus indicating the presence of current in the coil under consideration. It is observed that when the bar magnet is stationary, the pointer shows no deflection and the motion lasts only till the magnet is in motion. Here, the direction of the deflection of the pointer depends upon the direction of motion of the bar magnet. Also, when the south pole of the bar magnet is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed with the north-pole for similar movements. Apart from this, the deflection of the pointer is larger or smaller depending upon the speed with which it is pulled towards or away from the coil. The same effect is observed when instead of the bar magnet, the coil is moved and the magnet is held stationary. This shows that

only the relative motion between the magnet and the coil are responsible for the generation of current in the coil.

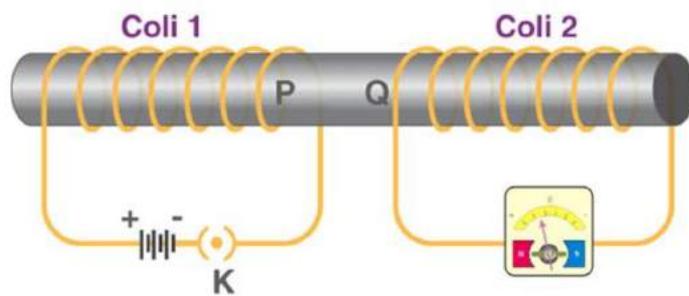
Experiment 2



In the second experiment, Faraday replaced the bar magnet by a second current-carrying coil that was connected to a battery. Here, the current in the coil due to the connected battery produced a steady magnetic field, which made the system analogous to the previous one. As we move the second coil towards the primary coil, the pointer in the galvanometer undergoes deflection, which indicates the presence of the electric current in the first coil.

Similar to the above case, here too, the direction of the deflection of the pointer depends upon the direction of motion of the secondary coil towards or away from the primary coil. Also, the magnitude of deflection depends upon the speed with which the coil is moved. All these results show that the system in the second case is analogous to the system in the first experiment.

Experiment 3



From the above two experiments, it was concluded by Faraday that the relative motion between the magnet and the coil resulted in the generation of current in the primary coil. But another experiment conducted by Faraday proved that the relative motion between the coils was not really necessary for the current in the primary to be generated.

In this experiment, he placed two stationary coils and connected one of them to the

galvanometer and the other to a battery, through a push-button. As the button was pressed, the galvanometer in the other coil showed a deflection, indicating the presence of current in that coil. Also, the deflection in the pointer was temporary and if pressed continuously, the pointer showed no deflection and when the key was released, the deflection occurred in the opposite direction.