SIGNAL PROCESSING

LAB – 4 REPORT

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Question 1

Question - 1

Part (a)

Given $p[n] = \cos\left(2\pi \frac{f_0}{f_s} n\right)$

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} \cos\left(2\pi \frac{f_0}{f_s} n\right) e^{-j\omega n}$$

$$\Rightarrow \sum_{-\infty}^{\infty} e^{-j\omega n} \cdot \left(\frac{e^{\frac{j2\pi f_0 n}{f_s}} + e^{-j\frac{2\pi f_0}{f_s} n}}{2}\right)$$

$$P(e^{j\omega}) = \frac{2\pi}{2}\left(\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \delta\left(\omega + \frac{2\pi f_0}{f_s}\right)\right)$$

(b) location of peak_1 is at $\omega = \frac{2\pi f_0}{f_s}$.

location of peak_2 is at $\omega = -\frac{2\pi f_0}{f_s}$.

$\therefore$ peak_1 and peak_2 will be separated by a distance of $\frac{4\pi f_0}{f_s}$

- Also, the magnitude of the peaks will be equal.

(c) To find DFT of $x[n] = p[n] \cdot w[n]$, we use

$$\boxed{x_1[n] \cdot x_2[n] = X_1(e^{j\omega}) * X_2(e^{j\omega})}$$

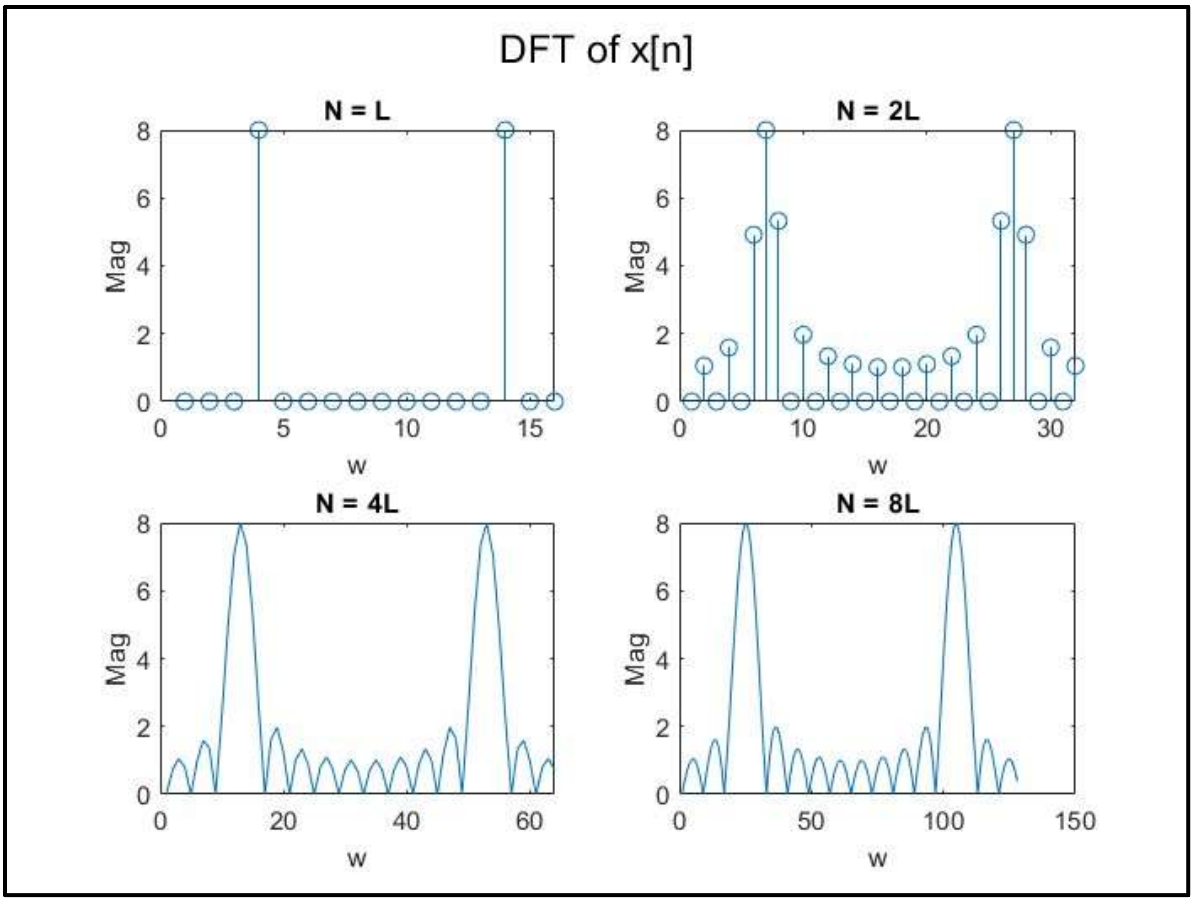
$$P(e^{j\omega}) = \pi\left(\delta\left(\omega - \frac{2\pi f_0}{f_s}\right) + \delta\left(\omega + \frac{2\pi f_0}{f_s}\right)\right)$$

$$W(e^{j\omega}) = 2\pi\,\delta(\omega)$$

$$W(e^{j(\omega-\omega_0)}) = 2\pi\,\delta(\omega-\omega_0)$$

$$X(e^{j\omega}) = \frac{1}{\cancel{2\pi}}\int_{-\pi}^{\pi} P(e^{j\omega_0}) \cdot \cancel{2\pi}\,\delta(\omega-\omega_0)\,d\omega_0 \Rightarrow \boxed{X(e^{j\omega}) = P(e^{j\omega})}$$

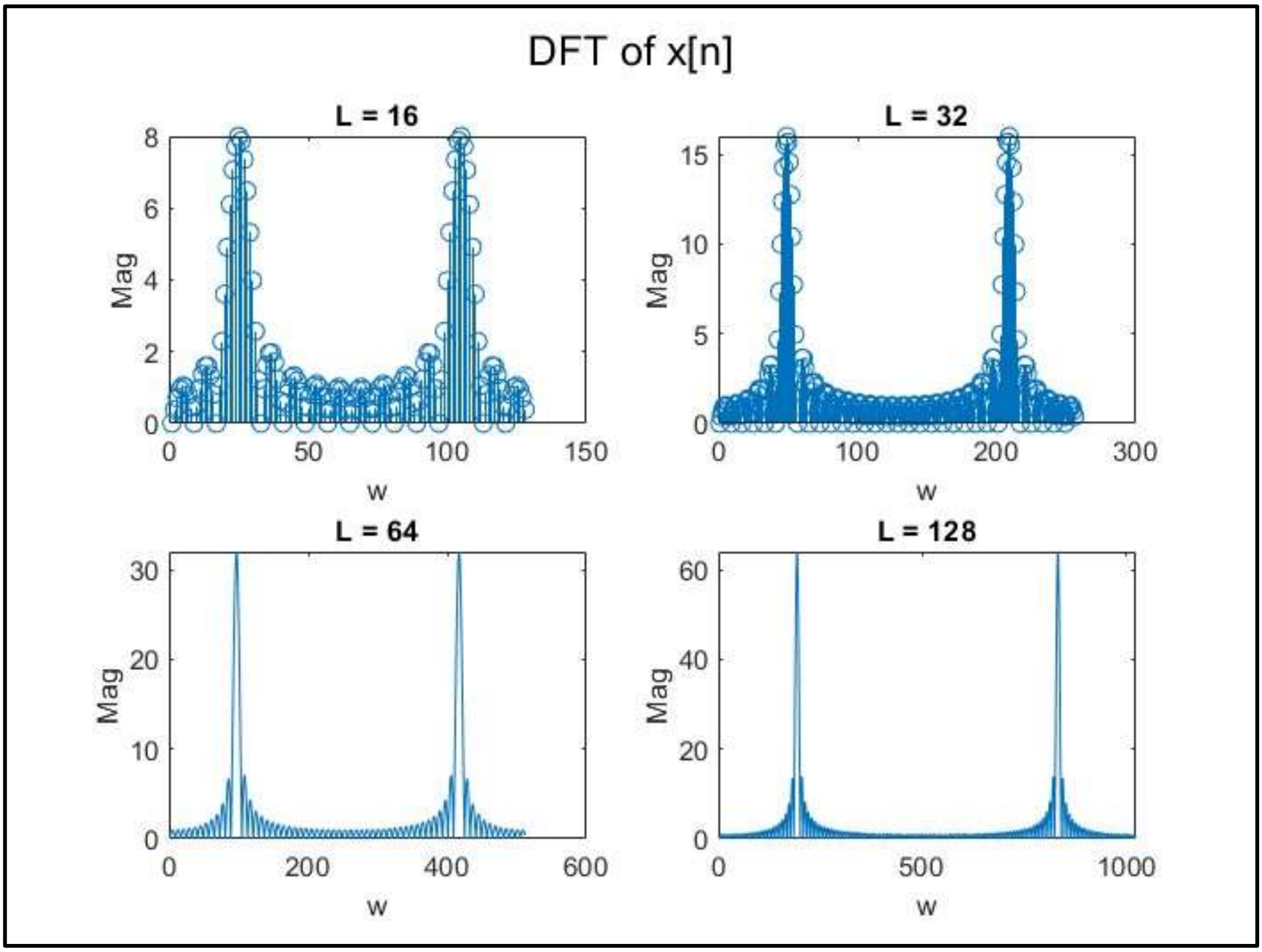
(d) Computing DFT of the signal x[n] for different values of N (sampling rate).



The above obtained plots represent the DFTs for N = {1,2,4,8} * L.

- We can observe that there is spectral leakage when the sampling rate is greater than the length of the signal.
- Spectral leakage occurs when a non-periodic sample of the signal is sent into the function. Using a value of N = L gives us proper samples in the DFT.
- Spectral leakage can be reduced by using the correct window size to obtain sharp results and to find frequencies easily.
- Here N = L is the appropriate window size.

(e) Plotting the DFT of x[n] while changing the value of the length of the signal.



We can observe that the number of samples is accurate as our results have samples until N = 8*L for respective L (length of signal).

As L increases the peaks get sharper and sharper, this is due to increases number of samples which results in a higher resolution as L takes values {16,32,64,128}.

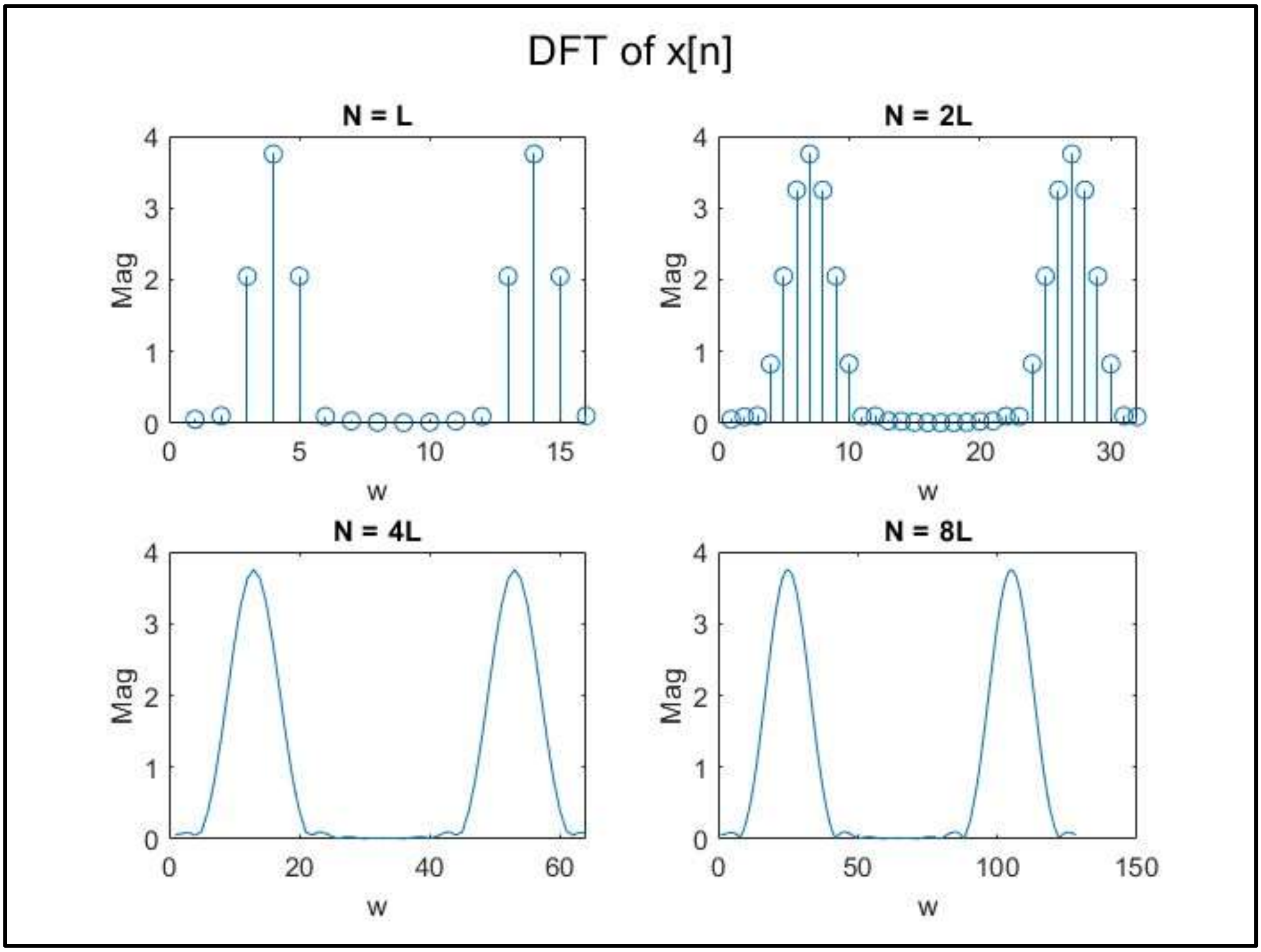
(g) A hanning window is a window function used to perform Hann smoothing.[1] The function, with length L and amplitude 1/L is given by:

$$w_0(x) \triangleq \left\{ \begin{array}{cc} \frac{1}{L}\left(\frac{1}{2}+\frac{1}{2}\cos\left(\frac{2\pi x}{L}\right)\right) = \frac{1}{L}\cos^2\left(\frac{\pi x}{L}\right), & |x| \leq L/2 \\ 0, & |x| > L/2 \end{array} \right\}$$

Using this we get our signal x[n]

x[n] = p[n] x w[n]

Finding DFT of a signal after using a hanning window :



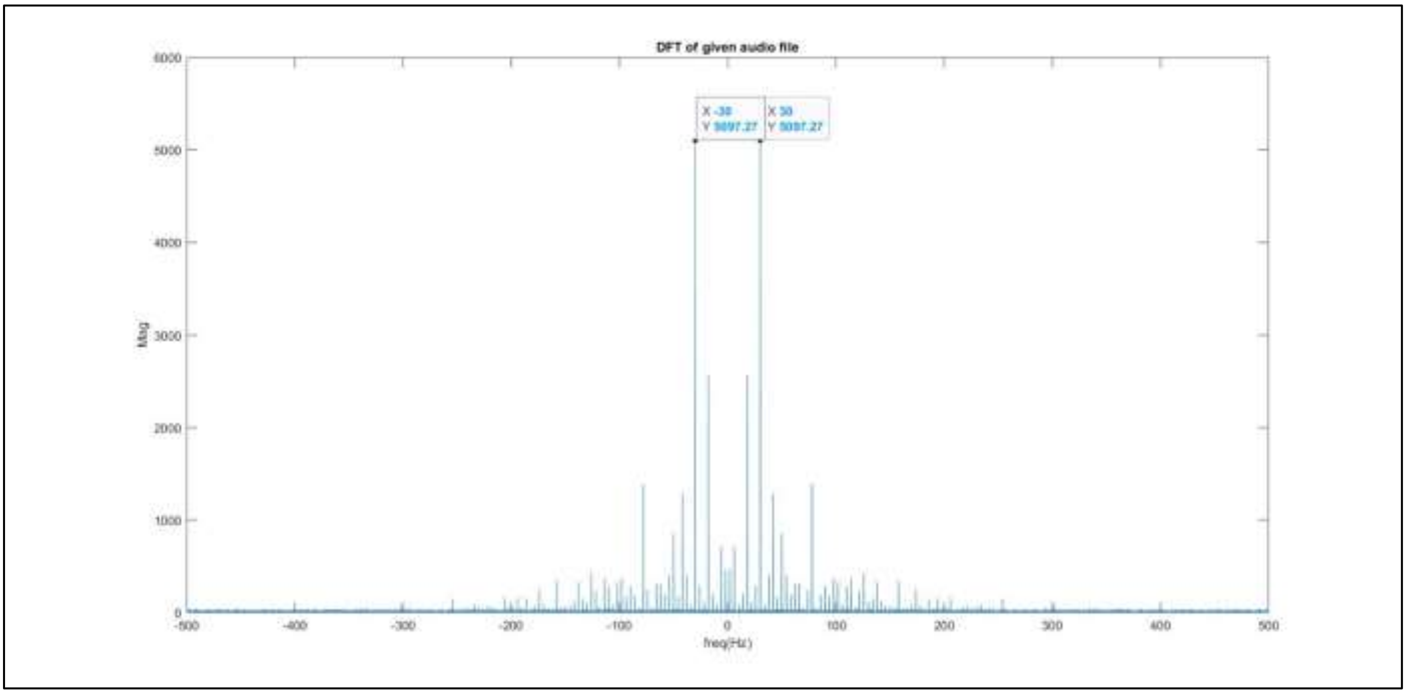
The main lobe width is the width of the fattest peak.

For a hanning window the width is double that of the width of the peak of the rectangular window DFT. The hanning window prevents spectral leakage, there is lesser leakage than that of a rectangular window DFT.

(i) Using the given audio file, DFT:

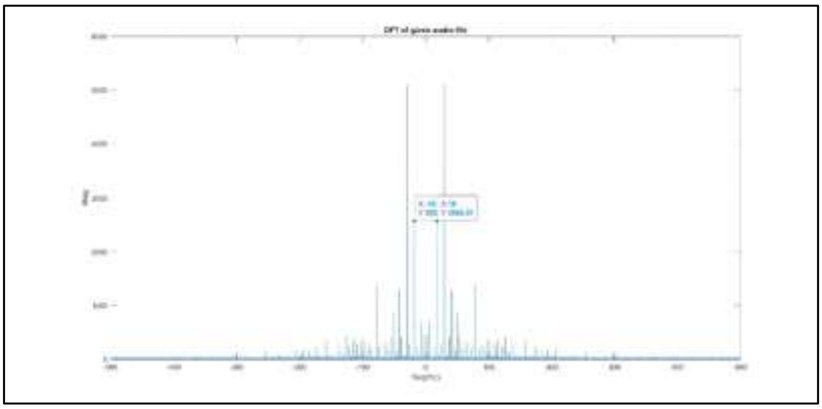
Here we are plotting a shifted output using the fftshift function in matlab. We do this since the fft function outputs the DFT for 0-4pi, shifting it plots the signal for -2pi – 2pi which is what we need. Also changing indexing for it to match frequency.

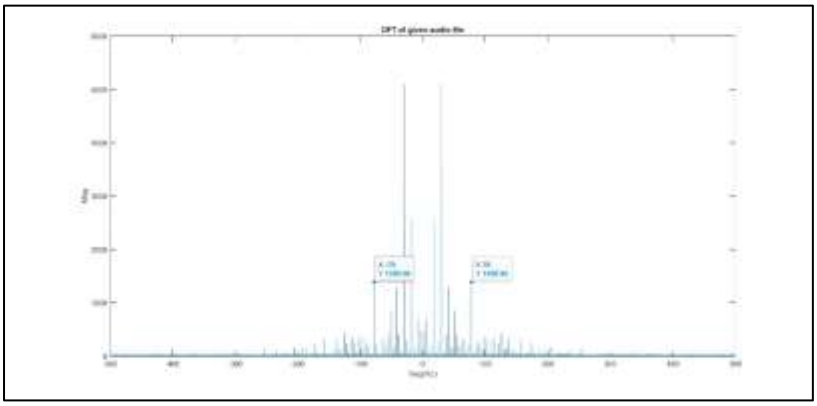
Obtained output :



We can observe that the biggest frequency component corresponds to frequency 30 Hz and -30 Hz.

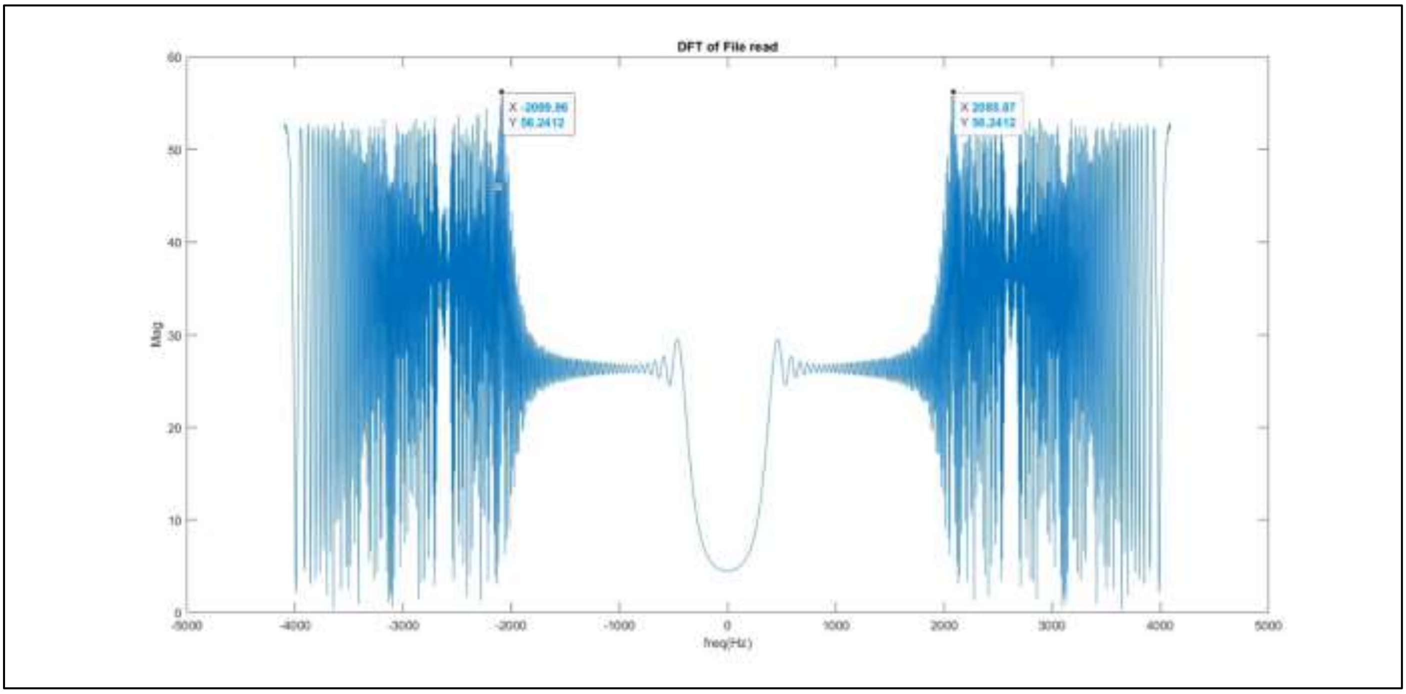
Similarly, the second highest corresponds to 18 and -18. And the third occurs at w = 78 and – 78.





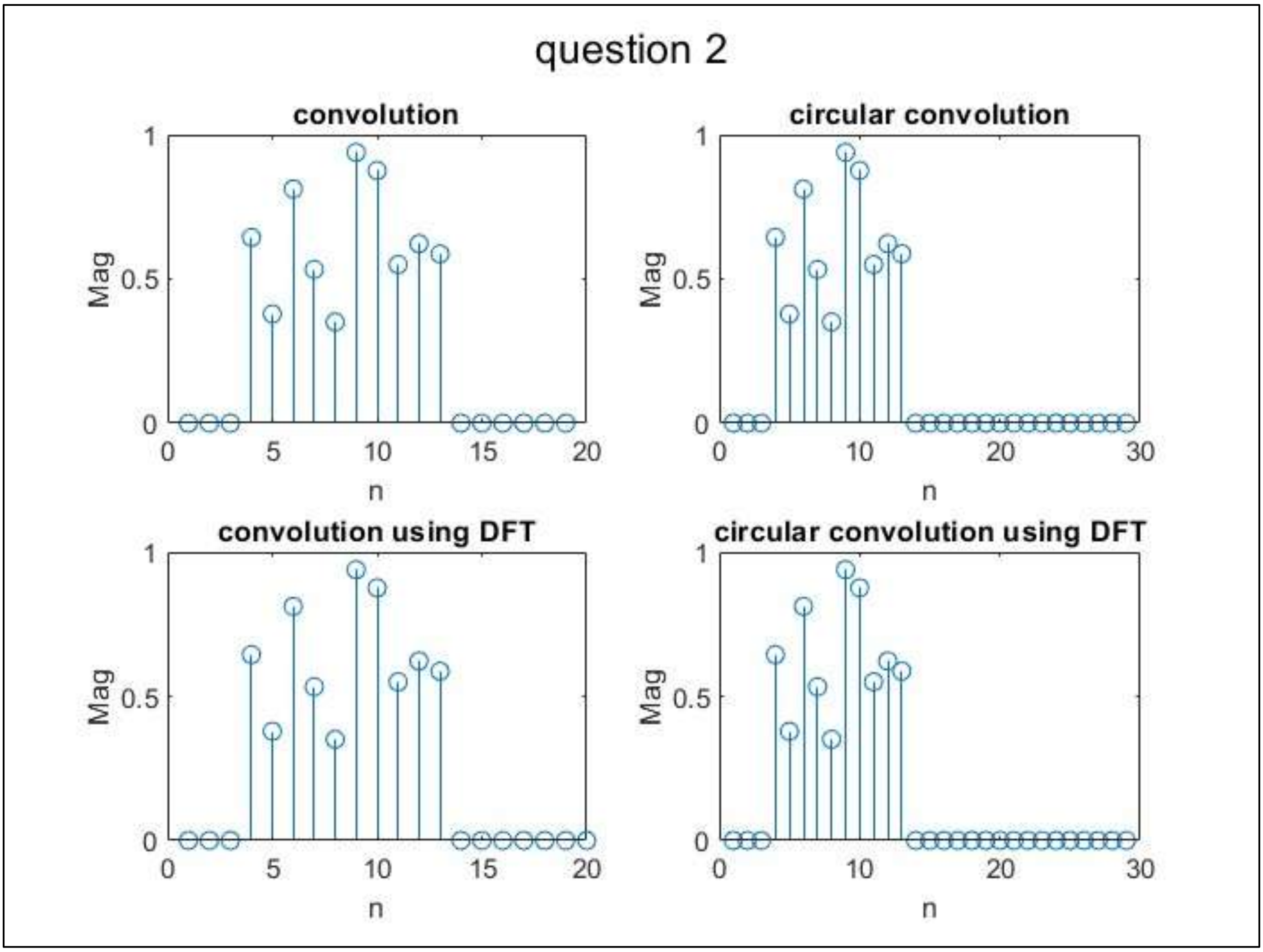
Using a signal of our own, we will be using the DFT to analyse it :

The plot of the DFT :



The high frequencies are 2085.87 Hz and -2089.96 Hz. The second highest magnitude corresponds to 2073.59 Hz and -2077.68 Hz. The third highest magnitude corresponds to 2098.15 Hz and -2102.24 Hz.

Question 2



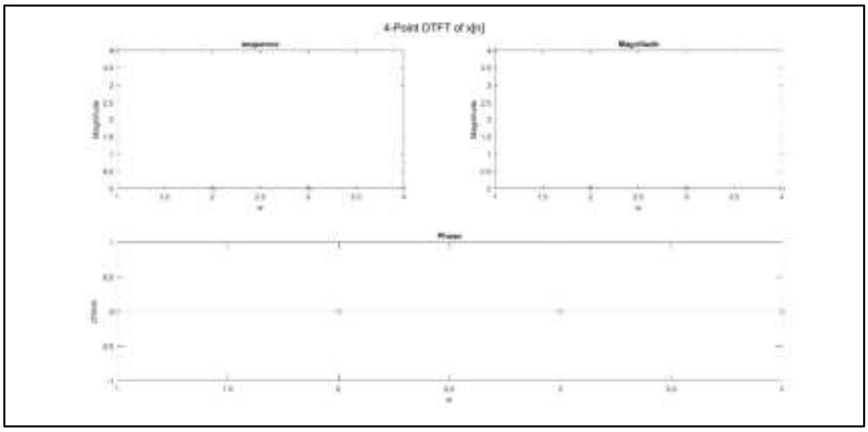
As we can see, the output obtained from both of these methods for convolution and circular convolution are accurate as they have identical outputs.

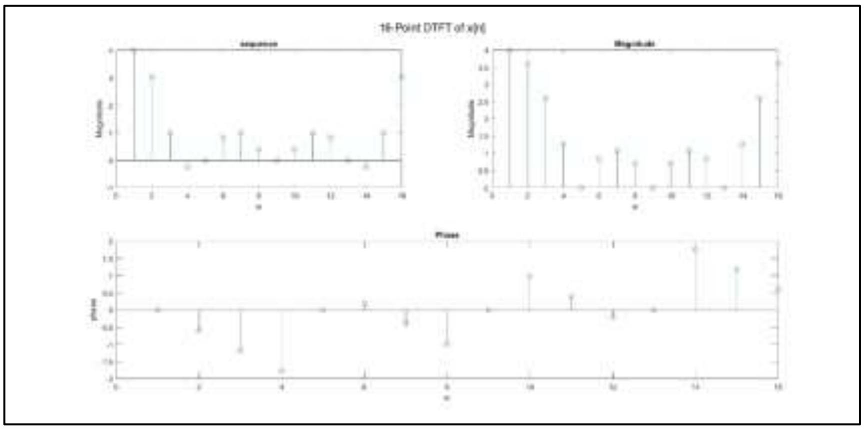
Question – 3

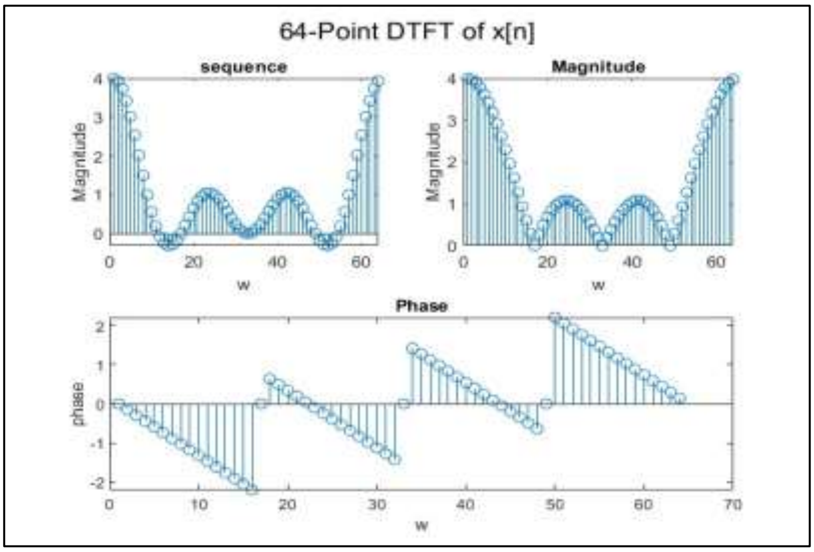
Plotting DFTs of the given signals :

To find the frequencies, we are using the fftshift function and using a scale that represents the frequency scale in Hz. This is done later and is not present in the scripts submitted, the method is similar to that of question 1_i.

a) x[n] = [ones(L,1); zeros(N-L,1)], for fixed L = 4 repeat for N = 4, 16, 64 ;

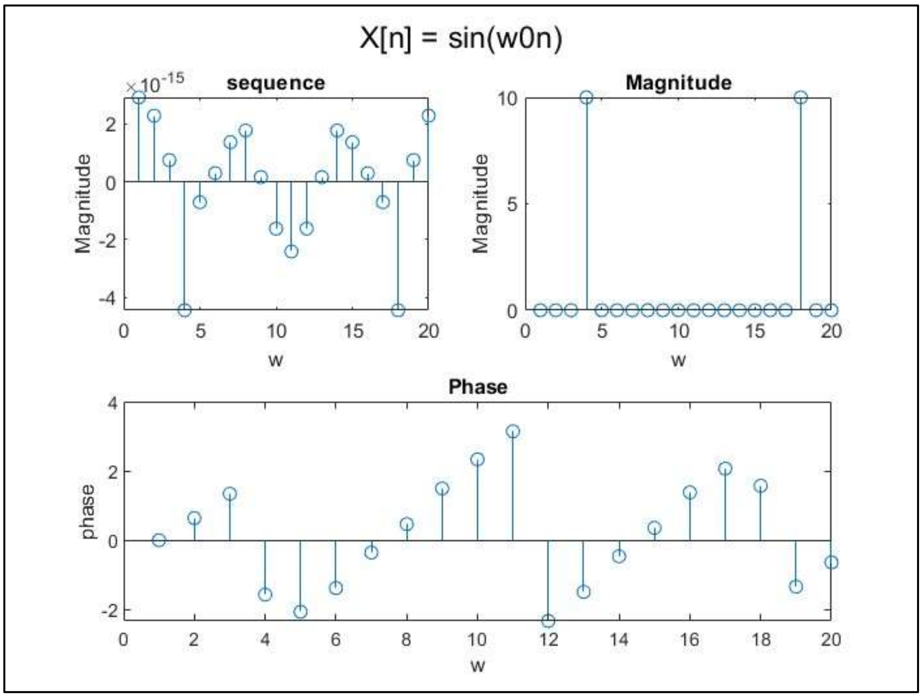






The highest magnitude corresponds to 0Hz and then there are frequencies that have 0 magnitude hence will be our low frequencies.

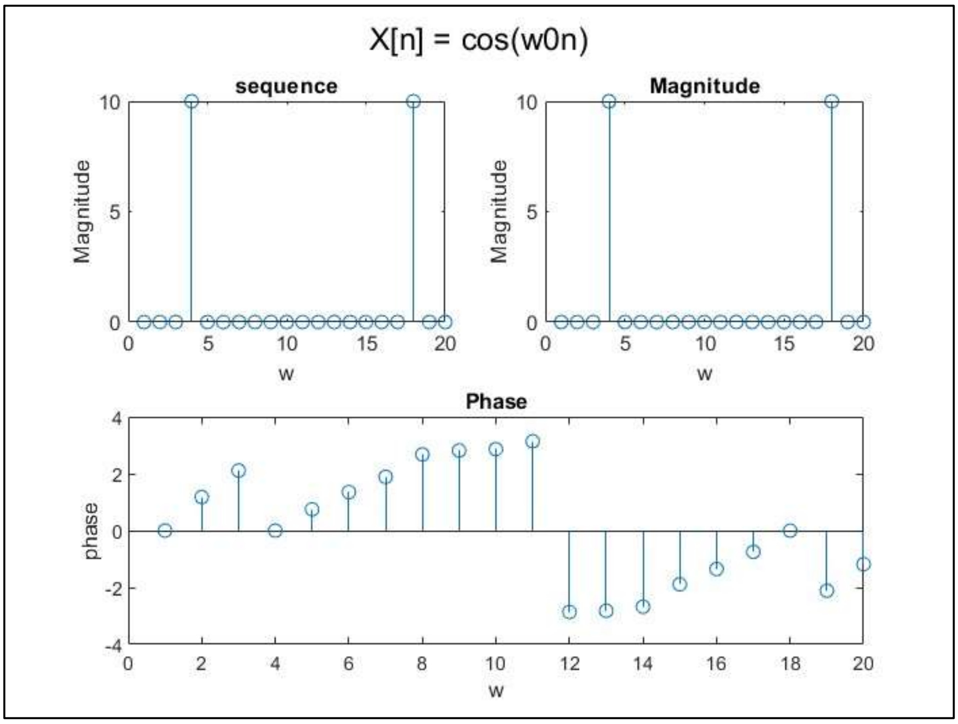
b) $x[n] = \sin(\omega 0 n)$, for $\omega 0 = 3\pi/10$ and N = 20 ;



We can observe that the frequency corresponding to the high magnitude is +3 Hz and – 3 Hz.

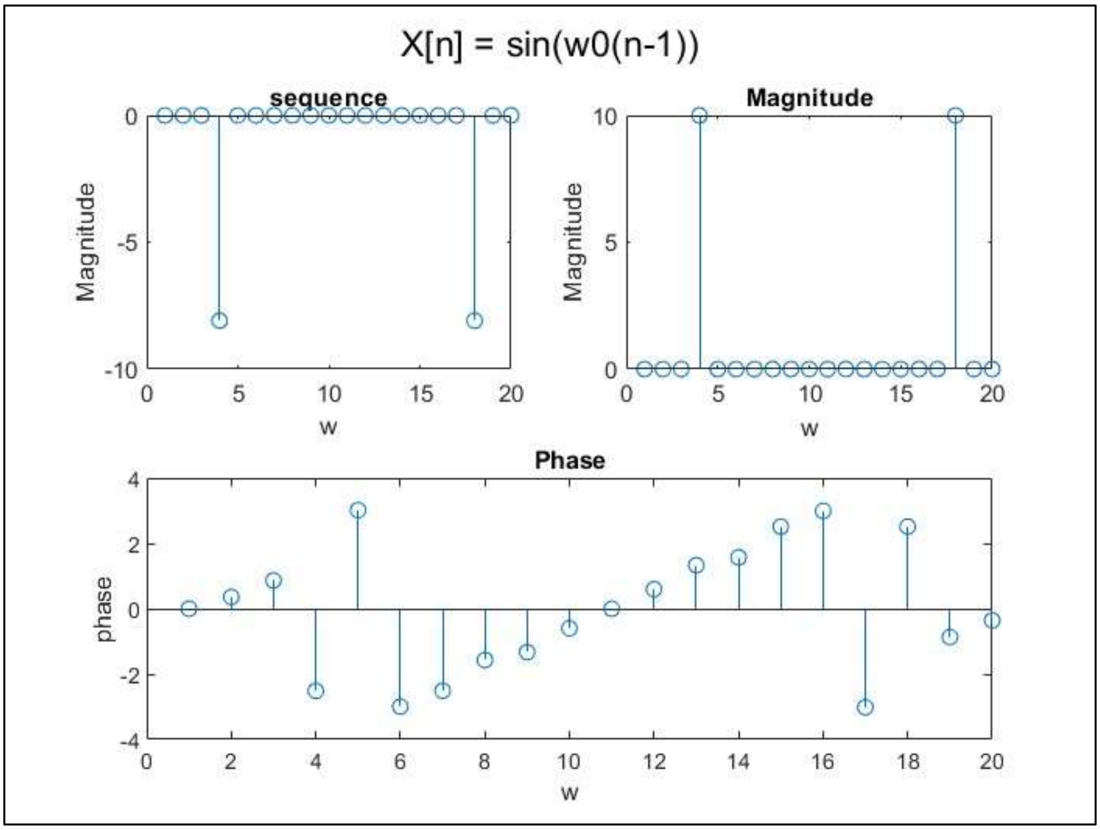
The rest have 0 magnitude and hence will be the low frequencies.

c) $x[n] = \cos(\omega 0 n)$, for $\omega 0 = 3\pi/10$ and N = 20 ;



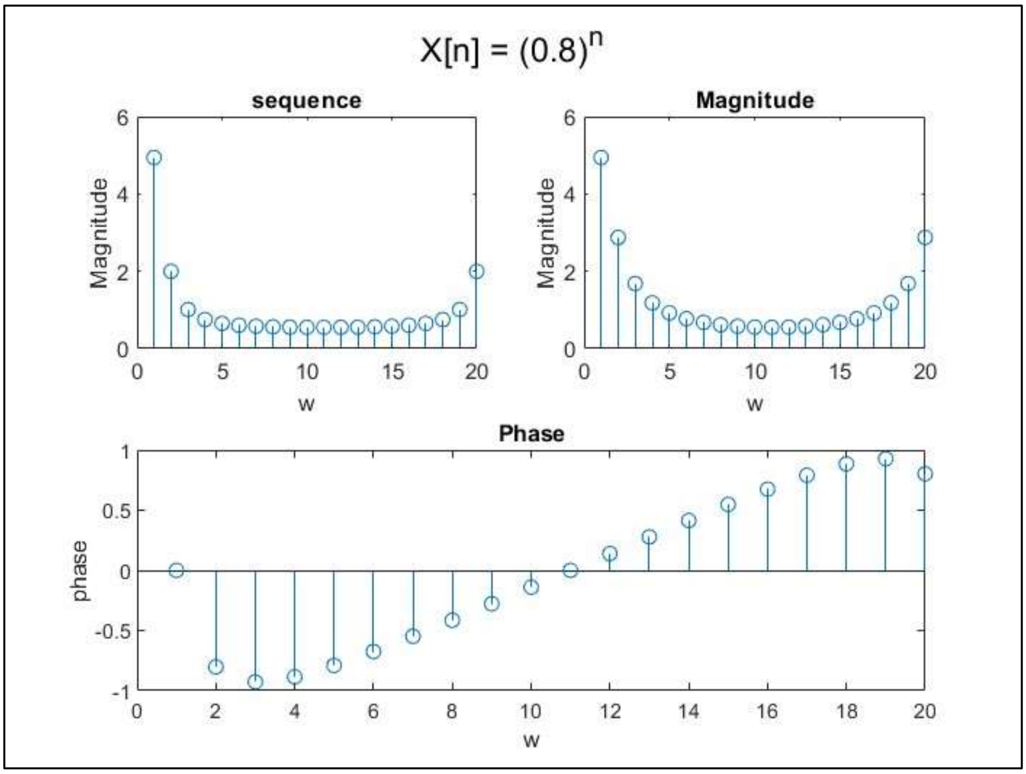
The DFT and magnitude spectrum of this signal can be seen to be similar to that of the earlier one, hence it will have the same high and low frequencies.

d) x[n] = sin(ω0(n − 1)), ω0 = 3π/10 and N = 20 ;



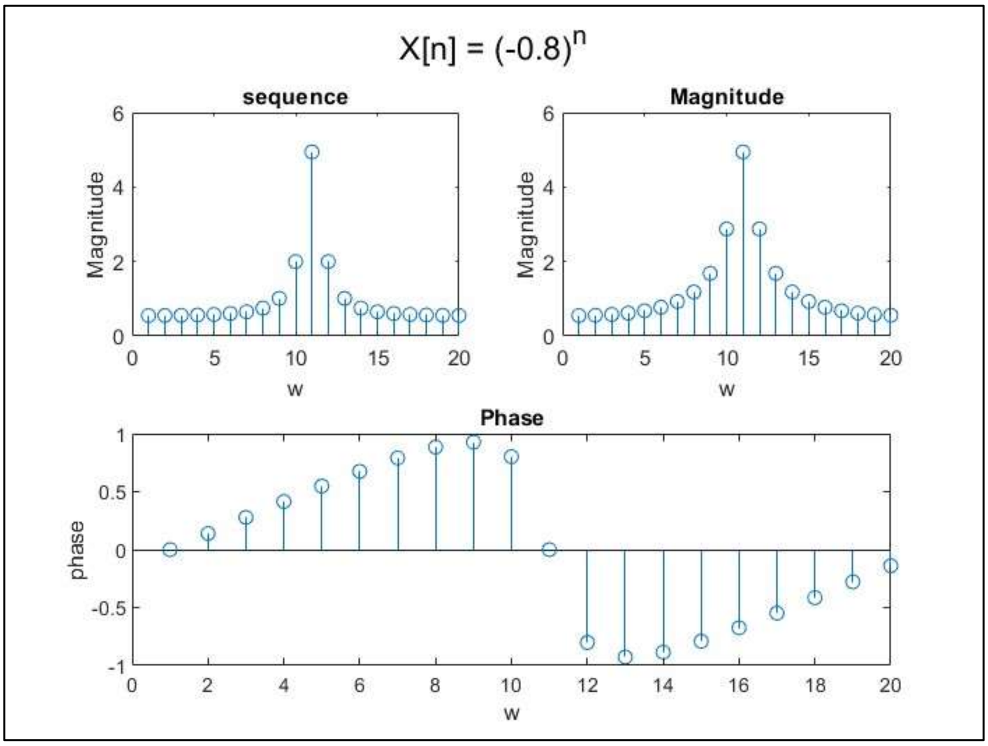
The DFT and magnitude spectrum of this signal can be seen to be similar to that of the earlier one, hence it will have the same high and low frequencies.

e) x[n] = (0.8)n for N = 20 ;



We can observe that the frequency corresponding to the high magnitude is 0 Hz and that of low magnitude are +9Hz and -9Hz.

f) x[n] = $(-0.8)^n$ for N = 20 ;



We can observe that the frequencies corresponding to the high magnitude are + 9Hz and - 9Hz. Frequency of 0 Hz corresponds to the lowest magnitude.