4.1)

a) generalized_policy_iteration.m

```
% generalized_policy_iteration: Function solving the given MDP using the
%                               Generalized Policy Iteration algorithm
%
% Inputs:
%       world:                  A structure defining the MDP to be solved
%       precision_pi:           Maximum value function change before
%                               terminating Policy Improvement step
%       max_ite_pi:             Maximum number of iterations for Policy
%                               Improvement loop
%       precision_pe:           Maximum value function change before
%                               terminating Policy Evaluation step
%       max_ite_pe:             Maximum number of iterations for Policy
%                               Evaluation loop
%
% Outputs:
%       V:                      An array containing the value at each state
%       policy_index:           An array summarizing the index of the
%                               optimal action index at each state
%
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%
% This script is adapted from the course on Optimal & Learning Control for
% Autonomous Robots at the Swiss Federal Institute of Technology in Zurich
% (ETH Zurich). Course Instructor: Jonas Buchli. Course Webpage:
% http://www.adrlab.org/doku.php/adrl:education:lecture:fs2015
%
% --
% Revision history
% [20.03.07, SZ]    first version

function [V, policy_index] = generalized_policy_iteration(world, precision_pi,
precision_pe, max_ite_pi, max_ite_pe)
    %% Initialization
    % MDP
    mdp = world.mdp;
    T = mdp.T;
    R = mdp.R;
    gamma = mdp.gamma;
    iteration_pi = 0;
    % Dimensions
    num_actions = length(T);
```
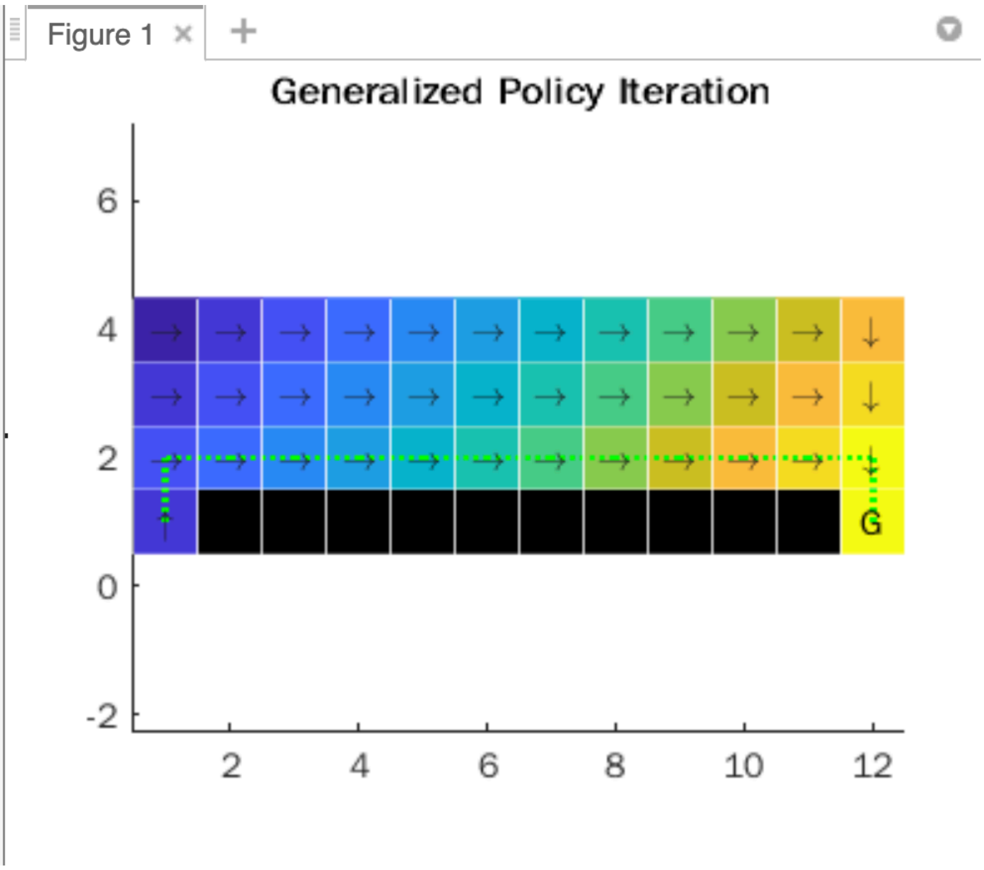

```
    num_states = size(T{1}, 1);

    % Intialize value function
    V = zeros(num_states, 1);

    % Initialize policy
    % Note: Policy here encodes the action to be executed at state s. We
    %       use deterministic policy here (e.g., [0,1,0,0] means take
    %       action indexed 2)
    random_act_index = randi(num_actions, [num_states, 1]);
    policy = zeros(num_states, num_actions);
    for s = 1:1:num_states
        selected_action = random_act_index(s);
        policy(s, selected_action) = 1;
    end

    while true
        %% [TODO] policy Evaluation (PE) (Section 2.6 of [1])
       iterations_pe = 0
       iteration_pi = iteration_pi + 1
       while iterations_pe <= max_ite_pe
        delta = 0
        iterations_pe = iterations_pe + 1
        for s = 1:1:num_states %loop for each state

            v = V(s,1); %initialize v value
            cur_state_index = s
            action_index = find(policy(s,:))
            noise_alpha = 0
            [next_state_index, next_state_noisy_index, reward] = ...
            one_step_gw_model(world, cur_state_index, action_index, noise_alpha)
%compute next state, reward of transition when applying a = pi(s)
            V(s,1) = reward + gamma*V(next_state_index,1) %compute value of
policy
            abs_diff = abs(v-V(s,1));
            delta = max(abs_diff,delta);
        end

        if delta < precision_pe

        break

        end

       end

        % V = ...;

        %% [TODO] Policy Improvment (PI) (Section 2.7 of [1])
        policyISstable = true;
        for s = 1:1:num_states

            b = policy(s,:);

            %compute argmax of cumulative reward function:
            cur_state_index = s;
            noise_alpha = 0;

            for a = 1:4
            [next_state_index, next_state_noisy_index, reward] =
one_step_gw_model(world, cur_state_index, a, noise_alpha)
            V_temp = reward + gamma*V(next_state_index,1);

            if a == 1
            temp = V_temp
```

```
            end

            if V_temp >= temp

                temp = V_temp;
                argmax_a = a;   %computing which action maximizes cumulative
reward

            end

            end

            policy(s,:) = zeros(1,4);   %updating policy
            policy(s,argmax_a) = 1;

         if find(b) ~= find(policy(s,:))
            fprintf('policy not stable')
            policyISstable = false;
         end

        end

        if policyISstable == true

            break
        end
        % policy = ...;

        % Check algorithm convergence
        % if ...
        %       break
        % end
    end
    iteration_pi
    % Return deterministic policy for plotting
    [~, policy_index] = max(policy, [], 2);
end
```

Heat Map and Results:



Difference between VI and PI:

In value iteration, the policy is evaluated only once and then improved.
Value iteration converges faster than PI.

c) Monte Carlo

1) code: monte_carlo.m:

```
% monte_carlo: Function solving the given MDP using the on-policy Monte
%               Carlo method
%
% Inputs:
%       world:                  A structure defining the MDP to be solved
%       epsilon:                A parameter defining the 'sofeness' of the
%                               epsilon-soft policy
%       k_epsilon:              The decay factor of epsilon per iteration
%       omega:                  Learning rate for updating Q
%       training_iterations:    Maximum number of training episodes
%       episode_length:         Maximum number of steps in each training
%                               episodes
%
% Outputs:
```

```matlab
%       Q:                        An array containing the action value for
%                                 each state-action pair
%       policy_index:             An array summarizing the index of the
%                                 optimal action index at each state
%
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% http://www.adrlab.org/doku.php/adrl:education:lecture:fs2015
%
% --
% Revision history
% [20.03.07, SZ]    first version

function [Q, policy_index] = ...
    monte_carlo(world, epsilon, k_epsilon, omega, training_iterations,
episode_length)
    %% Initialization
    % MDP
    mdp = world.mdp;
    gamma = mdp.gamma;

    % States
    STATES = mdp.STATES;
    ACTIONS = mdp.ACTIONS;

    % Dimensionts
    num_states = size(STATES, 2);
    num_actions = size(ACTIONS, 2);

    % Create object for incremental plotting of reward after each episode
    windowSize = 10; %Sets the width of the sliding window fitler used in
plotting
    plotter = RewardPlotter(windowSize);

    % Initialize Q
    Q = zeros(num_states, num_actions);
    terminal_state = 12
    % [TODO] Initialize epsilon-soft policy
    % policy = ...; % size: num_states x num_actions
    policy = zeros(num_states, num_actions);
    policy = initialize_random_policy(epsilon,num_states,num_actions);
    %% On-policy Monte Carlo Algorithm (Section 2.9.3 of [1])
    initial_state = randi([1, num_states]);
    curr_state = initial_state;
```
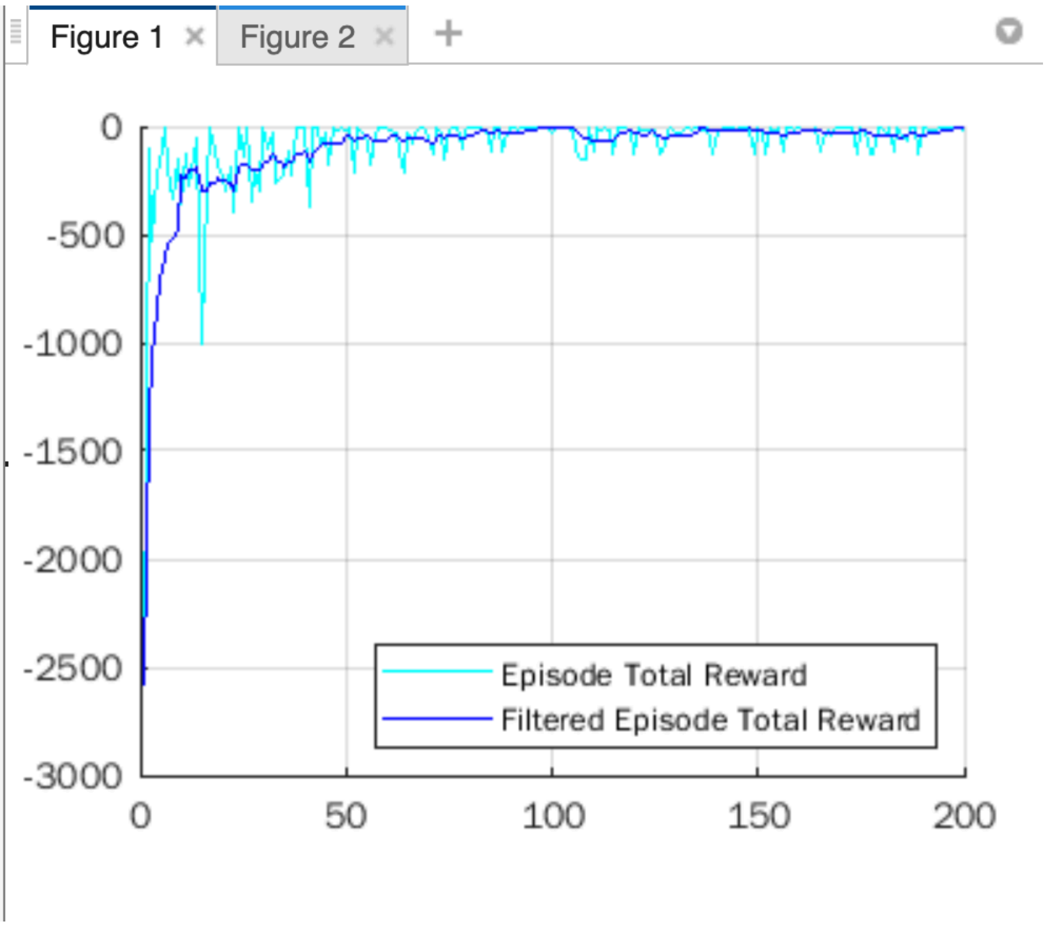
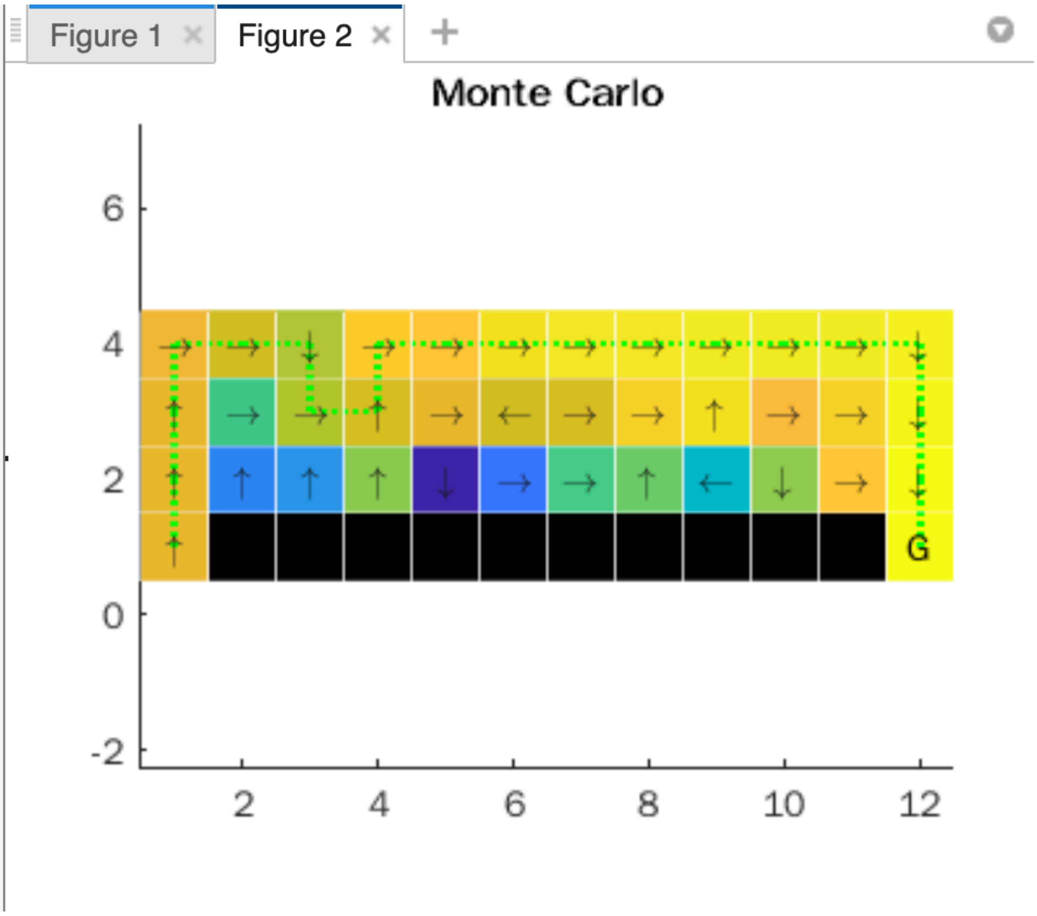

```
    cur_state_index = curr_state;
    state_sequence = [curr_state];
    reward_sequence = [];
    action_sequence = [];

    for train_loop = 1:1:training_iterations
        %% [TODO] Generate a training episode
        initial_state = randi([1, num_states]); %randomly initialize states
        cur_state_index = initial_state;
        R = 0; %Initialize Return
        state_sequence = [curr_state];
        reward_sequence = []; %initialize reward sequence
        action_sequence = []; %initialize state  sequence
        episode_index = 0
         while cur_state_index ~= terminal_state & episode_index <
episode_length % episode termination criteria
            episode_index = episode_index + 1;
            policy_prob = policy(cur_state_index,:)
            % Sample current epsilon-soft policy
            action =sample_from_epsilon_policy(epsilon,policy_prob);

            % Interaction with environment
            [next_state_index, ~, reward] = one_step_gw_model(world,
cur_state_index, action, 1);
            state_sequence = [state_sequence,next_state_index];
            reward_sequence = [reward_sequence, reward];
            action_sequence = [action_sequence, action];


            cur_state_index = next_state_index;
            % Log data for the episode
            % ...
        end
        N = length(state_sequence);
        i = 0;
        reward_sequence
        action_sequence
        state_sequence
        for i = 1:N-1

            s = state_sequence(N-i);
            a = action_sequence(N-i);
            r = reward_sequence(N-i)
            R = gamma*R + r ; % cumulative return
            Q(s,a) = Q(s,a) + omega*(R - Q(s,a));


        end
        R
        % Update Q(s,a)
        % Q = ...;

        %% [TODO] Update policy(s,a)
         for i = 1:N-1

             x = state_sequence(N-i);
             u_optim = arg_max_Q(Q,x);

          for a = 1:4

            if a == u_optim
             policy(x,u_optim) = 1 - 0.75*epsilon;
            else
             policy(x,a) = 0.25*epsilon;
            end
```

```
            end

           end

          % policy = ...;

          %% [TODO] Update the reward plot
          EpisodeTotalReturn = R; % Sum of the reward obtained during the episode
          plotter = UpdatePlot(plotter, EpisodeTotalReturn);
          drawnow;
          pause(0.1);

          %% Decrease the exploration
          % Set k_epsilon = 1 to maintain constant exploration
          epsilon = epsilon * k_epsilon;
      end

      % Return deterministic policy for plotting
      [~, policy_index] = max(policy, [], 2);
end
```

Results (Monte Carlo)





2. Impact of varying policy parameter epsilon:

d)

Code: q_learning.m:

```
% q_learning: Function solving the given MDP using the off-policy
%             Q-Learning method
%
% Inputs:
%       world:                  A structure defining the MDP to be solved
%       epsilon:                A parameter defining the 'sofeness' of the
%                               epsilon-soft policy
%       k_epsilon:              The decay factor of epsilon per iteration
%       omega:                  Learning rate for updating Q
%       training_iterations:    Maximum number of training episodes
%       episode_length:         Maximum number of steps in each training
%                               episodes
%       noise_alpha:            A parameter that controls the noisiness of
%                               observation (observation is noise-free when
%                               noise_alpha is set to 1 and is more
%                               corrupted when it is set to values closer
%                               to 0)
%
% Outputs:
%       Q:                      An array containing the action value for
%                               each state-action pair
%       policy_index:           An array summarizing the index of the
%                               optimal action index at each state
%
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% http://www.adrlab.org/doku.php/adrl:education:lecture:fs2015
%
% --
% Revision history
% [20.03.07, SZ]    first version

function [Q, policy_index] = q_learning(world, epsilon, k_epsilon, omega,
training_iterations, episode_length, noise_alpha)
    %% Initialization
    % MDP
    mdp = world.mdp;
    gamma = mdp.gamma;
    terminal_state = 12
```
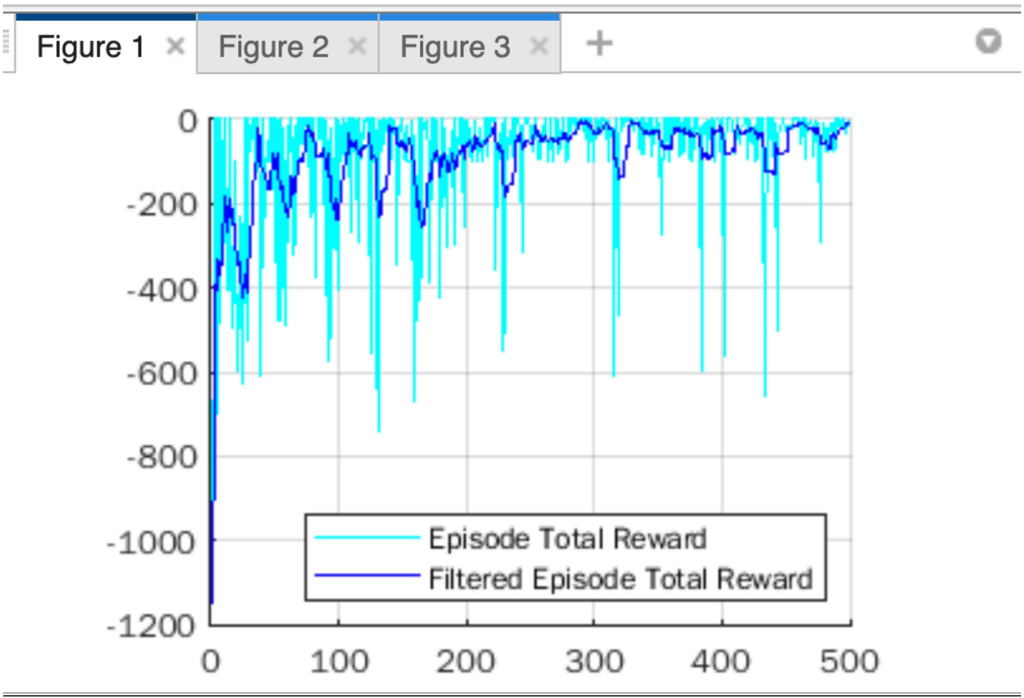
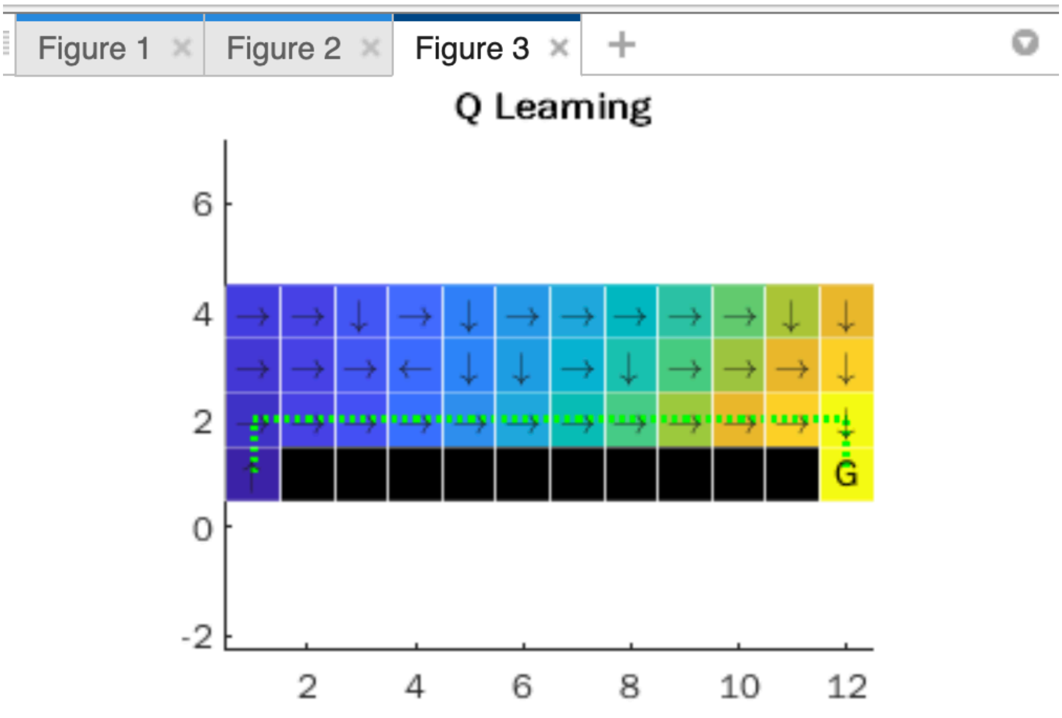

```
    % States
    STATES = mdp.STATES;
    ACTIONS = mdp.ACTIONS;

    % Dimensionts
    num_states = size(STATES, 2);
    num_actions = size(ACTIONS, 2);

    % Create object for incremental plotting of reward after each episode
    windowSize = 10; %Sets the width of the sliding window fitler used in
plotting
    plotter = RewardPlotter(windowSize);

    % Initialize Q
    Q = zeros(num_states, num_actions);

    % [TODO] Initialize epsilon-soft policy
    % policy = ...; % size: num_states x num_actions
    policy = initialize_random_policy(epsilon,num_states,num_actions);
    %% Q-Learning Algorthim (Section 2.9 of [1])
    for train_loop = 1:1:training_iterations
        %% [TODO] Generate a training episode
        initial_state = randi([1, num_states]); %randomly initialize states
%         initial_state = mdp.s_start_index;
        cur_state_index = initial_state;
        reward_sequence = [];
        episode_index = 0;
        R = 0;
        while cur_state_index ~= terminal_state & episode_index <=
episode_length
            % Sample current epsilon-soft policy
            policy_prob = policy(cur_state_index,:);
            % Sample current epsilon-soft policy
            action =sample_from_epsilon_policy(epsilon,policy_prob);
            episode_index = episode_index + 1;
            % Interaction with environment
            % Note: 'next_state_noisy_index' below simulates state
            %       observarions corrupted with noise. Use this for
            %       Q-learning correspondingly for the last part of
            %       Problem 2.2 (d)
            [next_state_index, next_state_noisy_index, reward] = ...
              one_step_gw_model(world, cur_state_index, action, noise_alpha);
            %Q learning update rule::
            argmax_u_prime = arg_max_Q(Q, next_state_index);
            Q(cur_state_index,action) = Q(cur_state_index,action) +
omega*(reward + gamma*Q(next_state_index,argmax_u_prime) -
Q(cur_state_index,action));
            reward_sequence = [reward_sequence, reward];

            % Log data for the episode
            % ...
            x = cur_state_index;
            u_optim = arg_max_Q(Q,x);

          for a = 1:4

            if a == u_optim
             policy(x,u_optim) = 1 - 0.75*epsilon;
            else
             policy(x,a) = 0.25*epsilon;
            end


            % Update Q(s,a)
            % Q = ...;
         end
```

```
        %% [TODO] Update policy(s,a)
        % policy = ...;
           N = length(reward_sequence);
          for i = 1:N-1

            r = reward_sequence(N-i+1);
            R = gamma*R + r ; % cumulative return

          end

          cur_state_index = next_state_index;
        end

        %% [TODO] Update the reward plot
        EpisodeTotalReturn = R %Sum of the reward obtained during the episode
        plotter = UpdatePlot(plotter, EpisodeTotalReturn);
        drawnow;
        pause(0.1);

        %% Decrease the exploration
        k_epsilon = 1 %to maintain constant exploration
        epsilon = epsilon * k_epsilon;

    end

    % Return deterministic policy for plotting
    [~, policy_index] = max(policy, [], 2);
end
```

Simulation Results:





Impact of changing decay parameter k_epsilon

Difference between monte carlo and q learning algorithm:

Q learning with noise:

4.2)

4.2.a)

1.1 Code - build_stochastic_mdp_nn.m

```
% build_stochastic_mdp_nn: Function implementing the Nearest Neighbour
%                           approach for creating a stochastic MDP
%
% Inputs:
%       world:                  A structure containing basic parameters for
%                               the mountain car problem
%       T:                      Transition model with elements initialized
%                               to zero
%       R:                      Expected reward model with elements
%                               initialized to zero
%       num_samples:            Number of samples to use for creating the
%                               stochastic model
%
% Outputs:
%       T:                      Transition model with elements T{a}(s,s')
%                               being the probability of transition to
%                               state s' from state s taking action a
%       R:                      Expected reward model with elements
%                               R{a}(s,s') being the expected reward on
```

```
%                                transition from s to s' under action a
%
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%
% --
% Revision history
% [20.03.07, SZ]    first version

function [T, R] = build_stochastic_mdp_nn(world, T, R, num_samples)
    % Extract states and actions
    STATES = world.mdp.STATES;
    ACTIONS = world.mdp.ACTIONS;

    % Dimensions
    num_states = size(STATES, 2);
    num_actions = size(ACTIONS, 2);

    % Loop through all possible states
    for state_index = 1:1:num_states
        cur_state = STATES(:, state_index);
        fprintf('building model... state %d\n', state_index);

        % Apply each possible action
        for action_index = 1:1:num_actions
            action = ACTIONS(:, action_index);
%             p_k = cur_state(1);
%             v_k = cur_state(2);
%             v_k_next = v_k + 0.001*action_index


            % [TODO] Build a stochastic MDP based on Nearest Neighbour
            % Note: The function 'nearest_state_index_lookup' can be used
            % to find the nearest node to a countinuous state
            for samples = 1:1:num_samples

                [next_state,reward,is_goal_state] = one_step_mc_model(world,
cur_state, action)
                next_state(1) = next_state(1) + normrnd(0,0.001);
                next_state(2) = next_state(2) + normrnd(0,0.005);
                next_state_nearest = nearest_state_index_lookup(STATES,
next_state);
                T{action_index}(state_index,next_state_nearest) =
T{action_index}(state_index,next_state_nearest) + 1/num_samples;
                % Update transition and reward models
                % T{action_index}(state_index, next_state_index) = ...;
                R{action_index}(state_index, next_state_nearest) = reward;
            end
```

```
        end
    end
end
```

1.2 What is the stochastic element in the modelling process and what is its significance ?

1.3 What modelling parameters would have the most impact on the quality of the solution ?

2.1) main_p2_mc_rl.m

```
% main_p2_mc_rl: Main script for Problem 4.2 mountain car (RL approach)
%
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% Adam Hall
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%
% --
% Revision history
% [20.03.07, SZ]    first version

clear all;
close all;
clc;

%% General
% Add path
addpath(genpath(pwd));

% Result and plot directory
save_dir = './results/';
% mkdir(save_dir);

%% Problem 4.2 (a)-(b) Create stochastic MDPs for the mountain car problem
% [TODO] Load mountain car model
% change model name correspondingly:
%     (a) 'mountain_car_nn' for the nearest neighbour method
%     (b) 'mountain_car_li' for the linear interpolation approach
load('mountain_car_model/mountain_car_nn');

%% Generalized policy iteration
% Algorithm parameters
precision_pi = 0.1;
precision_pe = 0.01;
max_ite_pi = 100;
```

```
max_ite_pe = 10;

% Solve MDP
[v_gpi, policy_gpi] = generalized_policy_iteration_mc(world, precision_pi, ...
    precision_pe, max_ite_pi, max_ite_pe);

% Visualization
plot_value = true;
plot_flowfield = true;
plot_visualize = true;
plot_title = 'Generalized Policy Iteration';
hdl_gpi = visualize_mc_solution(world, v_gpi, policy_gpi, plot_value, ...
    plot_flowfield, plot_visualize, plot_title, save_dir);

% Save results
save(strcat(save_dir, 'gpi_results.mat'), 'v_gpi', 'policy_gpi');
```

generalized_policy_iteration_mc.m

```
% generalized_policy_iteration: Function solving the given MDP using the
%                               Generalized Policy Iteration algorithm
%
% Inputs:
%       world:                  A structure defining the MDP to be solved
%       precision_pi:           Maximum value function change before
%                               terminating Policy Improvement step
%       max_ite_pi:             Maximum number of iterations for Policy
%                               Improvement loop
%       precision_pe:           Maximum value function change before
%                               terminating Policy Evaluation step
%       max_ite_pe:             Maximum number of iterations for Policy
%                               Evaluation loop
%
% Outputs:
%       V:                      An array containing the value at each state
%       policy_index:           An array summarizing the index of the
%                               optimal action index at each state
%
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% http://www.adrlab.org/doku.php/adrl:education:lecture:fs2015
%
```
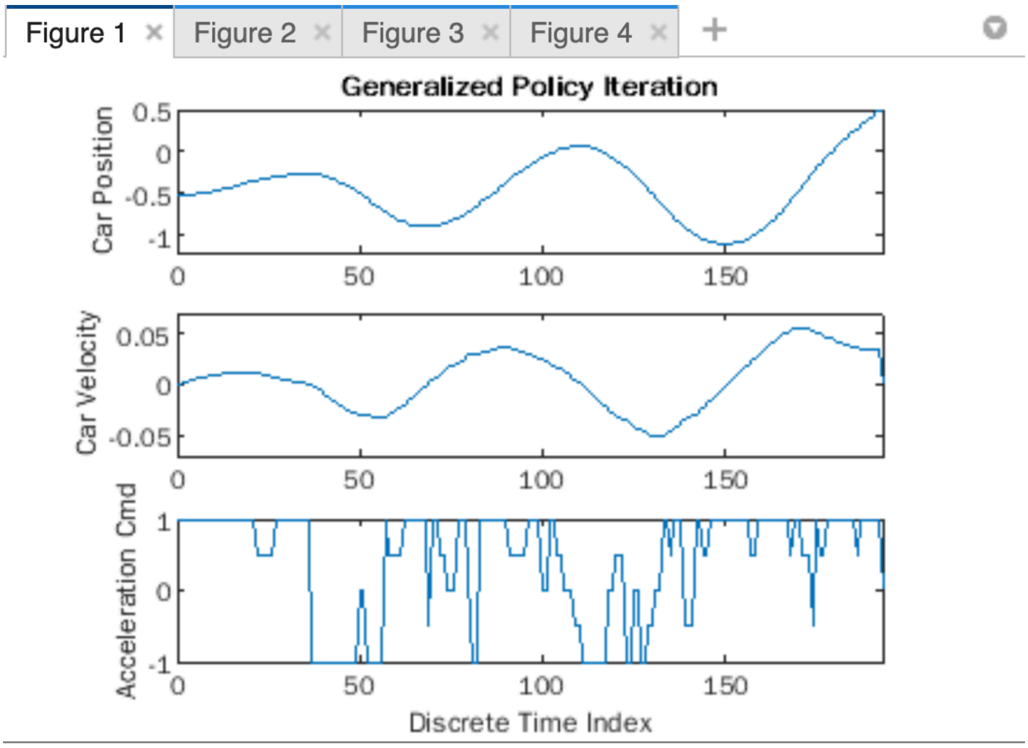

```matlab
% --
% Revision history
% [20.03.07, SZ]    first version

function [V, policy_index] = generalized_policy_iteration_mc(world,
precision_pi, precision_pe, max_ite_pi, max_ite_pe)
    %% Initialization
    % MDP
    mdp = world.mdp;
    T = mdp.T;
    R = mdp.R;
    gamma = mdp.gamma;

    % Discrete states
    POS = world.mdp.POS;
    VEL = world.mdp.VEL;

    % Dimensions
    num_actions = length(T);
    num_states = size(T{1}, 1);

    % Intialize value function
    V = zeros(num_states, 1);

    % Initialize policy
    % Note: Policy here encodes the action to be executed at state s. We
    %       use deterministic policy here (e.g., [0,1,0,0,0] means take
    %       action indexed 2)
    random_act_index = randi(num_actions, [num_states, 1]);
    policy = zeros(num_states, num_actions);
    for s = 1:1:num_states
        selected_action = random_act_index(s);
        policy(s, selected_action) = 1;
    end
    iterations_pi = 0;
    while iterations_pi <= max_ite_pi
        iterations_pe = 0;
        iterations_pi = iterations_pi + 1;
        %% [TODO] policy Evaluation (PE) (Section 2.6 of [1])
        while iterations_pe <= max_ite_pe
            delta = 0;
            iterations_pe = iterations_pe + 1;
            for s = 1:1:num_states
                v = V(s,1);

                %%%%%%Computation of V%%%%%%%%
                temp_v = 0; %temporary variable for value function computation

                for a = 1:num_actions
                    temp_R = 0; %temporary variable for expected return
computation
                    for s_prime = 1:num_states
                       temp_R = temp_R +  T{a}(s,s_prime)*(R{a}(s,s_prime) +
gamma*V(s_prime,1));
                    end
                 temp_v = temp_v + policy(s,a)*temp_R;

                end
                %finished computation of value function
                 V(s,1) = temp_v;
                abs_diff = abs(v-V(s,1));
                delta = max(abs_diff,delta);
                if delta< precision_pe
                    break
                end
            end
```

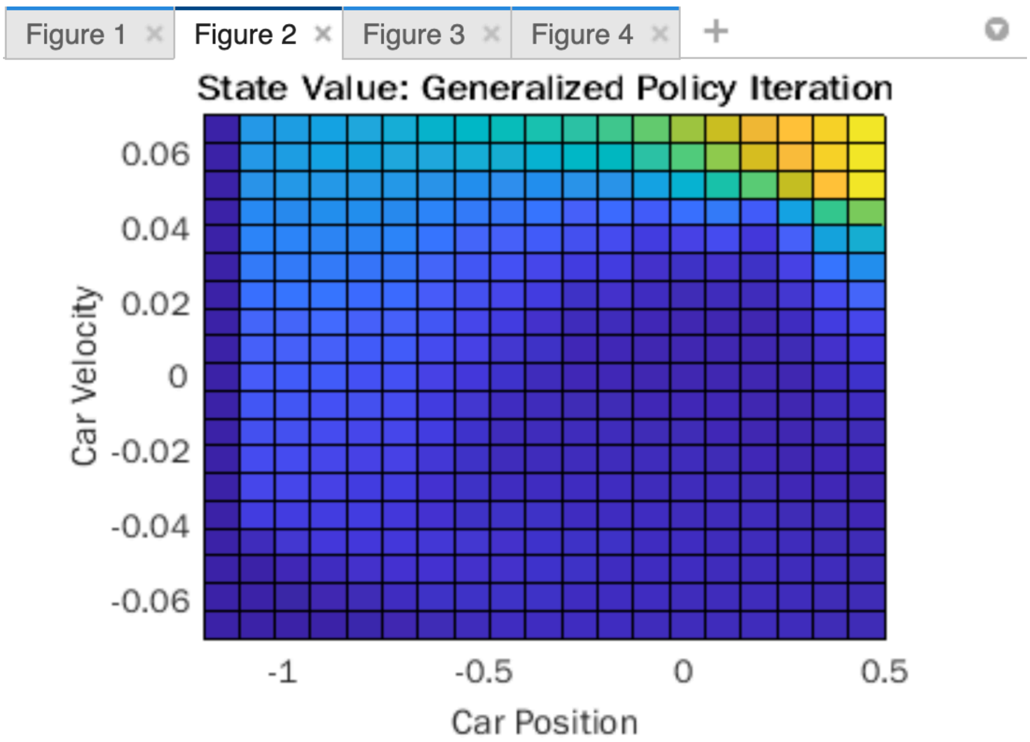
```
        end
        % V = ...;

        %% [TODO] Policy Improvment (PI) (Section 2.7 of [1])


        policy_is_stable = true;

        for s = 1:num_states
            b = policy(s,:);
            %compute argmax of value function
            for a = 1:num_actions
                temp_R = 0; %temporary variable for expected return computation
                for s_prime = 1:num_states
                       temp_R = temp_R +  T{a}(s,s_prime)*( R{a}(s,s_prime) +
gamma*V(s_prime,1));
                end
                if a==1
                    temp = temp_R;
                    arg_max = a;

                end

                if temp_R >= temp

                    temp = temp_R;
                    arg_max = a;
                end

            end
             policy(s,:) = zeros(1,num_actions);   %updating policy
             policy(s,arg_max) = 1;

             if find(b) ~= find(policy(s,:))
                fprintf('policy not stable')
                policyISstable = false;
             end

        end


         if policyISstable == true

            break
        end
        % policy = ...;

        % Check algorithm convergence
        % if ...
        %       break
        % end
    end

      % Return deterministic policy for plotting
      [~, policy_index] = max(policy, [], 2);
end
```

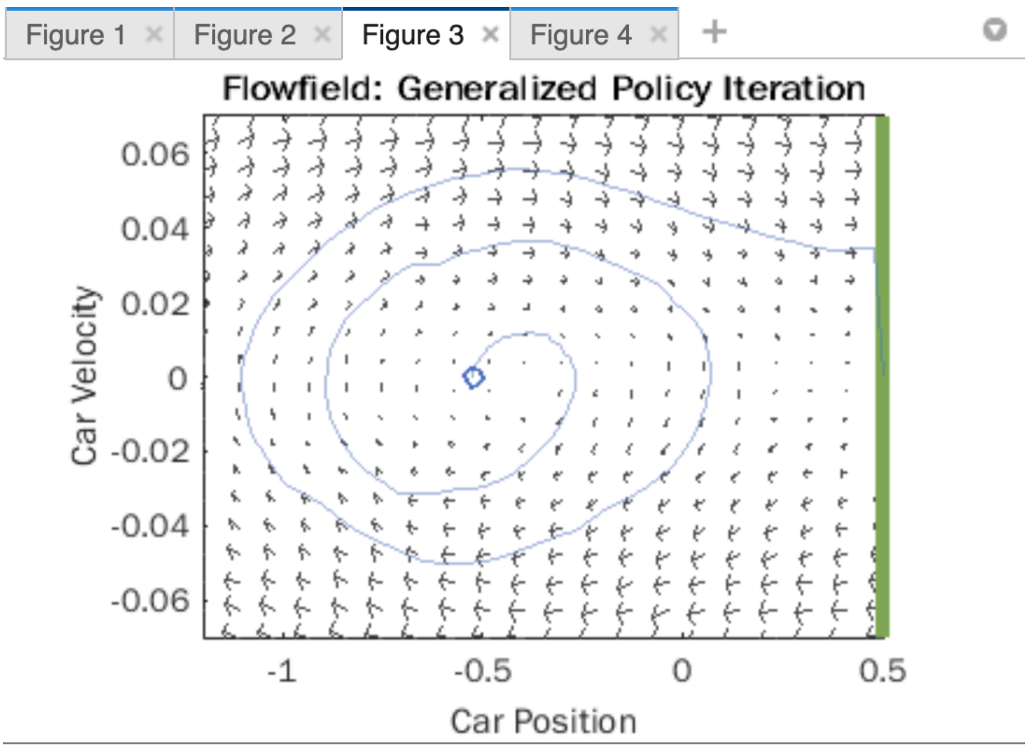
Q: Was the learning algorithm able to find this solution? If not, why do you think that is the case?

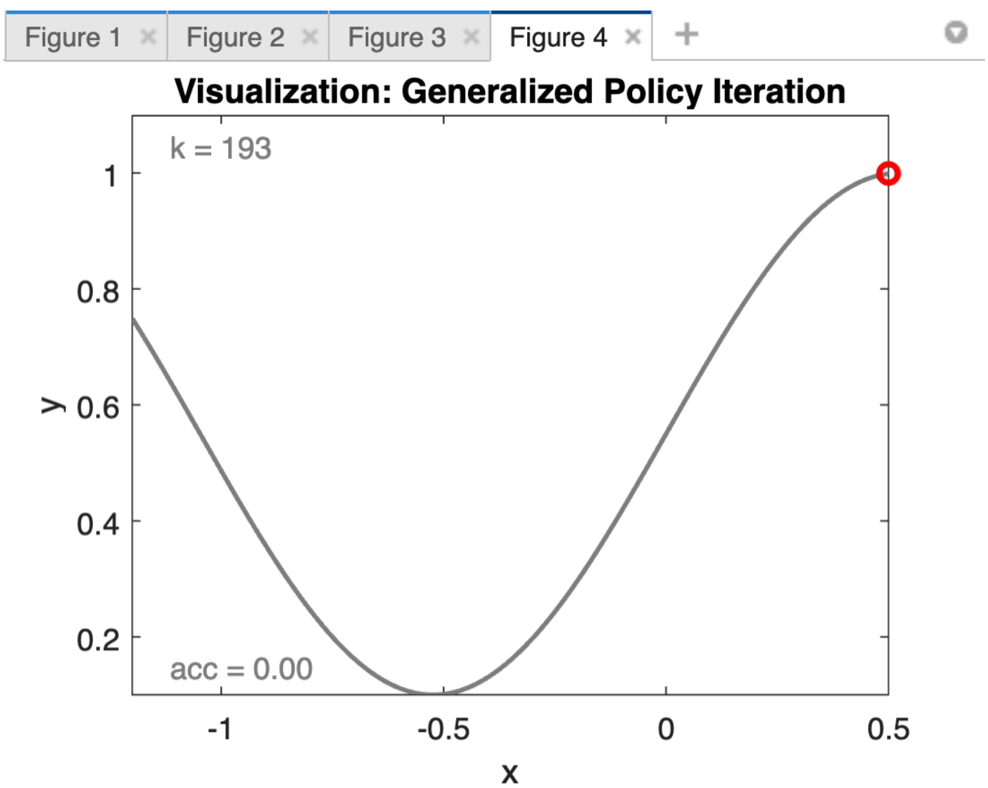
Yes.

Converged Heat Map & Results(Nearest Neighbour Case)









build_stochastic_mdp_li.m

```matlab
% build_stochastic_mdp_li: Function implementing the Linear Interpolation
%                          approach for creating a stochastic MDP
%
% Inputs:
%       world:                  A structure containing basic parameters for
%                               the mountain car problem
%       T:                      Transition model with elements initialized
%                               to zero
%       R:                      Expected reward model with elements
%                               initialized to zero
%
% Outputs:
%       T:                      Transition model with elements T{a}(s,s')
%                               being the probability of transition to
%                               state s' from state s taking action a
%       R:                      Expected reward model with elements
%                               R{a}(s,s') being the expected reward on
%                               transition from s to s' under action a
%
% --
% Control for Robotics
% AER1517 Spring 2022
% Assignment 4
%
% --
% University of Toronto Institute for Aerospace Studies
% Dynamic Systems Lab
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%
% --
% Revision history
% [20.03.07, SZ]    first version

function [T, R] = build_stochastic_mdp_li(world, T, R)
      % Extract states and actions
    STATES = world.mdp.STATES;
    ACTIONS = world.mdp.ACTIONS;

    % Number of discrete states and actions
    num_states = size(STATES, 2);
    num_actions = size(ACTIONS, 2);

    % State space dimension
    dim_state = size(STATES, 1);

      % Unique values
    for i = 1:1:dim_state
        unique_states{i} = unique(STATES(i,:));
    end

    % Loop through all possible states
    for state_index = 1:1:num_states
        cur_state = STATES(:, state_index);
        fprintf('building model... state %d\n', state_index);
```

```
        % Apply each possible action
        for action_index = 1:1:num_actions
            action = ACTIONS(:, action_index);

            % Propagate forward
            [next_state, reward, ~] = world.one_step_model(world, ...
                cur_state, action);

            % Find four vertices enclosing next state index
            for i = 1:1:dim_state
                % find cloest discretized values along state dimension i
                node_index_temp = knnsearch(unique_states{i}', next_state(i),
'K', 2);
                node_value_temp = unique_states{i}(node_index_temp);

                % for each state dimension i, store the min-max bounds
                box_min = min(node_value_temp);
                box_max = max(node_value_temp);
                node_value(i,1:2) = [box_min, box_max];

                % normalize next state values
                next_state_normalized(i,1) = ...
                    (next_state(i,1) - box_min) / (box_max - box_min);
            end

            % node values (for two-dim state space)
            node(1:2,1) = [node_value(1,1); node_value(2,1)]; % lower-left
            node(1:2,2) = [node_value(1,2); node_value(2,1)]; % lower-right
            node(1:2,3) = [node_value(1,2); node_value(2,2)]; % upper-right
            node(1:2,4) = [node_value(1,1); node_value(2,2)]; % upper-left

            % [TODO] Assign probability to adjacent nodes (bilinear)
            x = next_state_normalized(1);
            y = next_state_normalized(2);
            prob(1) = (1-x)*(1-y); % min min
            prob(2) = x*(1-y); % max min
            prob(3) = x*y; % max max
            prob(4) = (1-x)*(y); % min max

            % Update probability and reward for each node
            for i = 1:1:4
                node_index = nearest_state_index_lookup(STATES, node(:,i));

                % Update transition and reward models
                T{action_index}(state_index, node_index) = prob(i);
                R{action_index}(state_index, node_index) = reward;
            end

        end
    end
end
```

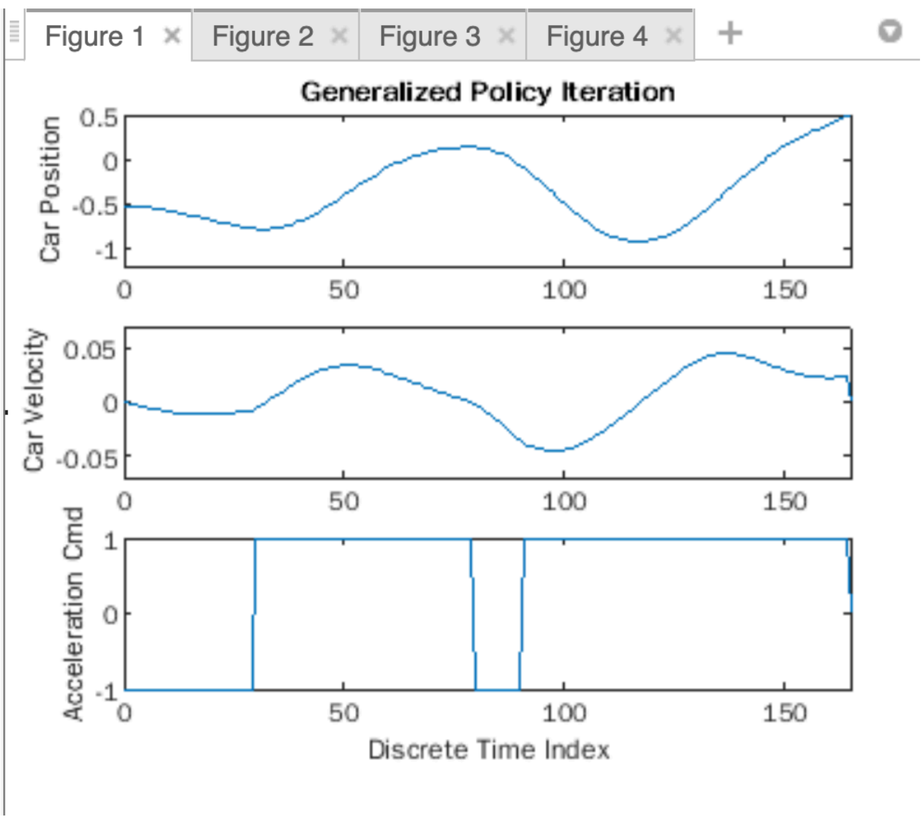


Figure 1
Figure 2
Figure 3
Figure 4
Generalized Policy Iteration
Car Position
0.5
0
-0.5
-1
0
50
100
150
Car Velocity
0.05
0
-0.05
Acceleration Cmd
1
0
-1
Discrete Time Index

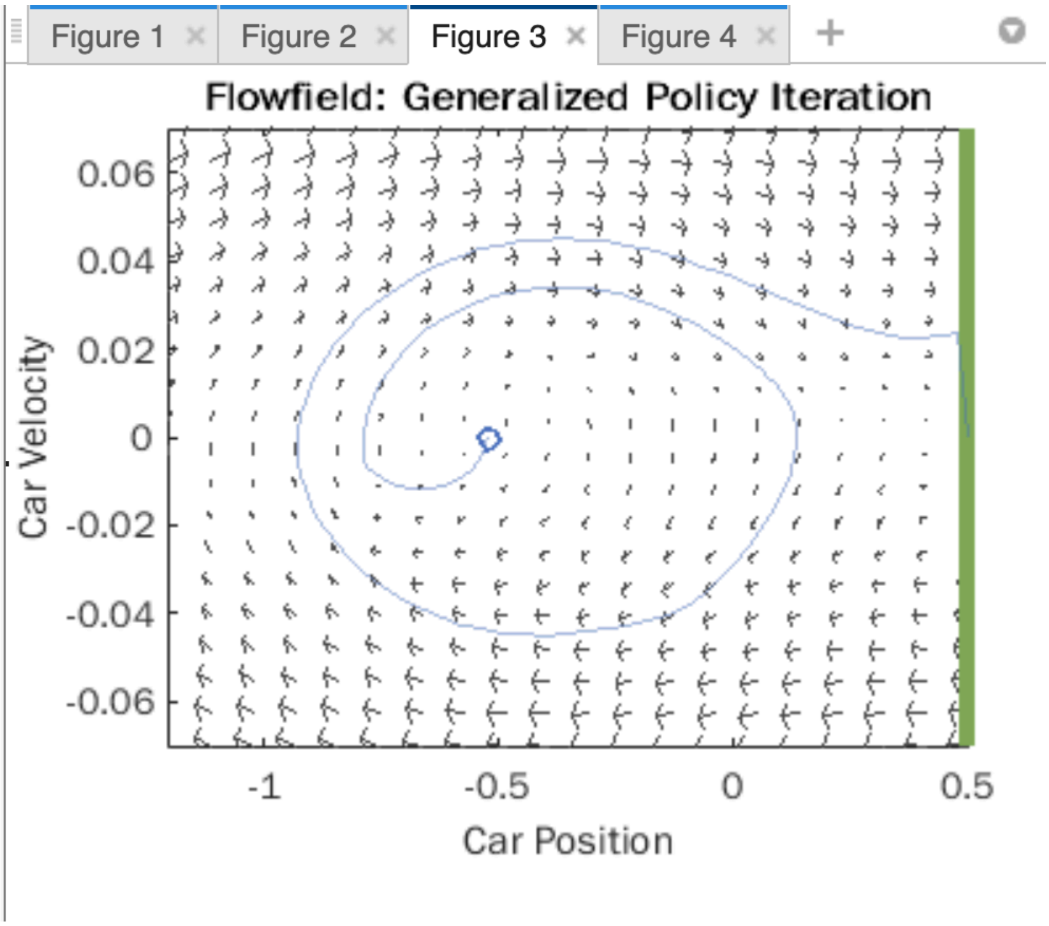


Figure 1
Figure 2
Figure 3
Figure 4
Flowfield: Generalized Policy Iteration
Car Velocity
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-1
-0.5
0
0.5
Car Position

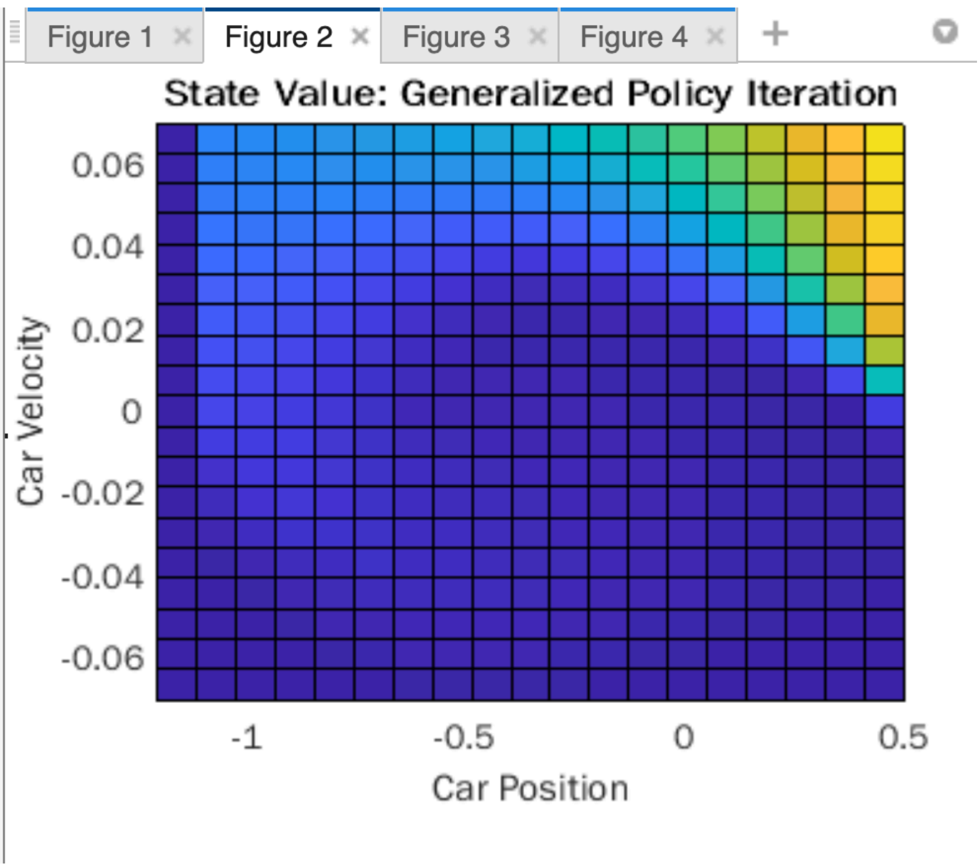
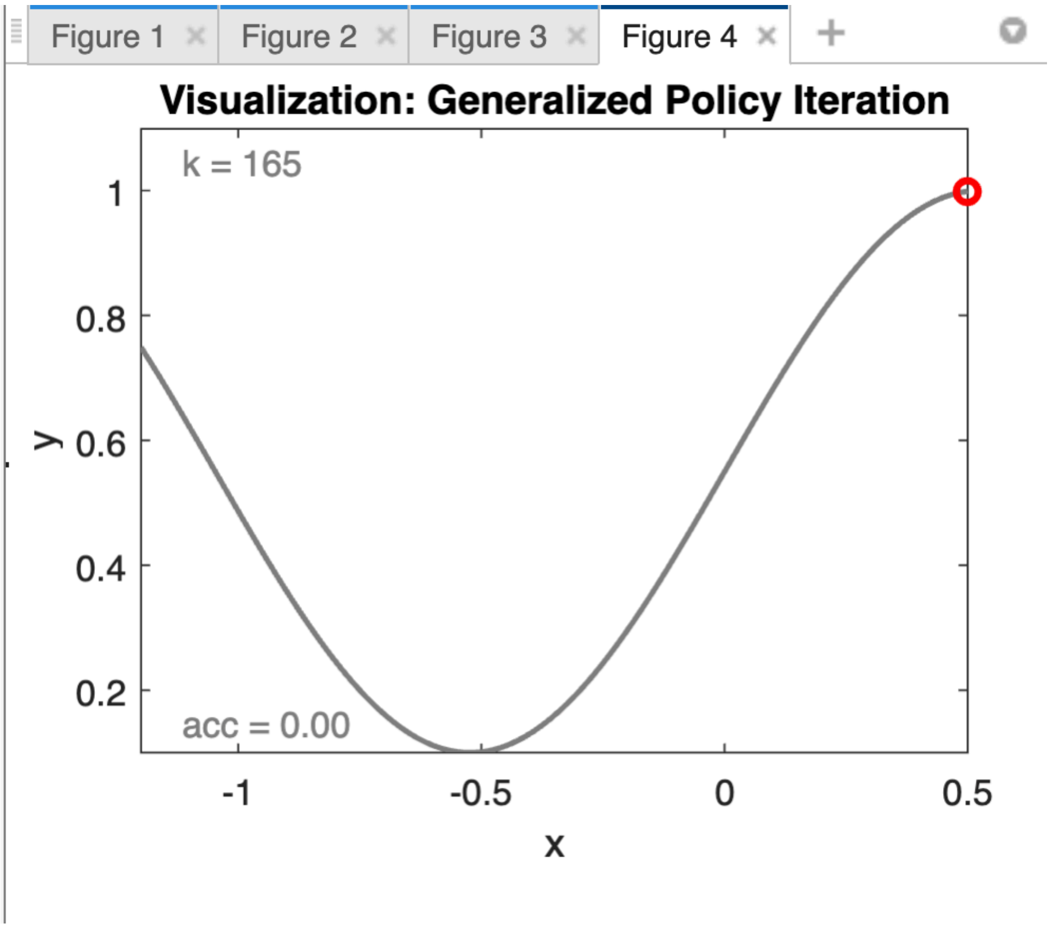


Figure 1
Figure 2
Figure 3
Figure 4
State Value: Generalized Policy Iteration
Car Velocity
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-1
-0.5
0
0.5
Car Position



Effectiveness of LI and NN: