ABES ENGINEERING COLLEGE, GHAZIABAD (032)

B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-1: MATRICES

Question Bank

1. Find the inverse employing elementary transformation $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$
2. Find the inverse of the matrix by using elementary operations: $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$
3. Find the value of 'b' for which the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ is 2
4. If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, determine to non-singular matrices P and Q such that PAQ=I . Hence find A^{-1} .
5. Find the rank of the matrix by reducing it to canonical form $\begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \end{bmatrix}$
6. Find the rank of the matrix by reducing it to normal form : $A = \begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 3 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$
7. Find the rank of the matrix by reducing it to normal form: $A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{bmatrix}$
8. Test the consistency for the following system of equations:
   $2x - y + 3z = 8, -x + 2y + z = 4, 3x + y - 4z = 0$
9. Determine the values of λ and μ such that the system
   $2x - 5y + 2z = 8, 2x + 4y + 6z = 5, x + 2y + \lambda z = \mu$ has (i) no solution (ii) a unique solution (iii) infinite number of solutions.
10. Test the consistency for the following system of equations and if system is consistent, solve them: $x + y + z = 6, x + 2y + 3z = 14, x + 4y + 7z = 30$

11. Determine 'k' such that the system of homogeneous equations $x+y+3z=0$, $4x+3y+kz=0$, $2x+y+2z=0$ have a non-trivial solution.
12. Determine 'b' such that the system of homogeneous equations $2x+y+2z=0$, $x+y+3z=0$, $4x+3y+bz=0$ has (i) trivial solution (ii) non-trivial solution.
13. If the vectors $(0,1,a)$, $(1,a,1)$, $(a,1,0)$ are linearly dependent, then find the value of a.
14. If A is a skew-Hermitian matrix , then show that iA is Hermitian.
15. Define unitary matrix . show that the following matrix is unitary matrix: $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$.
16. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$
17. Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$
18. Find the characteristic equation of the matrix $A=\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence, compute A^{-1}. Also Find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$.
19. Show that the matrix $\begin{bmatrix} 3 & 10 & 5 \\ -2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$ has less than three linearly independent Eigen vectors. Also find them.
20. State and verify Cayley Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$ and also find A^{-1}.

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B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-2: Differential Calculus-I

Question Bank

1. Find the n^{th} derivative of $sinx.sin2x.sin3x$.
2. Find the n^{th} derivative of $\frac{2x+1}{(2x-1)(2x+3)}$.
3. Find the n^{th} derivative of $\dfrac{x}{2x^2+3x+1}$.
4. If $y = xlog\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$
5. Find the n^{th} derivative of $tan^{-1}\left(\frac{1+x}{1-x}\right)$.
6. If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.
7. Apply Leibnitz Theorem to find $y_n \quad if \;\; y = x^{n-1}logx.$
8. If $y = e^{\tan^{-1}x}$, Prove that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$
9. If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$, prove that $(1-x^2)y_n - [2(n-1)x+1]y_{n-1} - (n-1)(n-2)y_{n-2} = 0.$
10. If $x = \tan(\log y)$, Prove that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$
11. If $y = \sin(a\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-a^2)y_n = 0$ and hence find the value of y_n when $x = 0$.
12. If $y = \left[log\left(x+\sqrt{1+x^2}\right)\right]^2$, find $y_n(0)$.
13. If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ and hence find the value of y_n when $x = 0$.
14. If $u = f(r), where\ r^2 = x^2 + y^2\ ,\ prove\ that\ \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r}f'(r)$.
15. If $u = x^2\tan^{-1}\dfrac{y}{x} - y^2\tan^{-1}\dfrac{x}{y}$, then prove that $\dfrac{\partial^2 u}{\partial x\partial y} = \dfrac{x^2-y^2}{x^2+y^2}$.
16. If $z = f(x+ct) + \phi(x-ct)$, show that $\dfrac{\partial^2 z}{\partial t^2} = c^2\dfrac{\partial^2 z}{\partial x^2}$.
17. If $x^2 = au + bv\ ,\ y^2 = au - bv\ ,\ prove\ that\ \left(\dfrac{\partial u}{\partial x}\right)_y . \left(\dfrac{\partial x}{\partial u}\right)_v = \dfrac{1}{2} = \left(\dfrac{\partial v}{\partial y}\right)_x \left(\dfrac{\partial y}{\partial v}\right)_u$.
18. If $u = e^{xyz}$ show that $\dfrac{\partial^3 u}{\partial x\partial y\partial z} = (1+3xyz+x^2y^2z^2)u$.