ABES ENGINEERING COLLEGE, GHAZIABAD

Subject: Fundamentals of Mechanical Engineering (BME101)

Unit 1

Topic: Introduction to Mechanics (Force system)

Lecture Notes

INDEX

1.1 Force system

When several forces act on a body, then they are called a force system. Force systems are also known as system of forces. If all the forces in a system do not lie in a single plane they constitute the system of forces in space. A force system may be co planar or non-coplanar. If in a system all forces lie in the same plane then force system is called as coplanar force system. But if in a system all the forces lie in a different plane, then the force system is known as non co planar force system.

Depending upon the orientation of the forces and position of the line of action of forces, system of forces can be classified as shown below

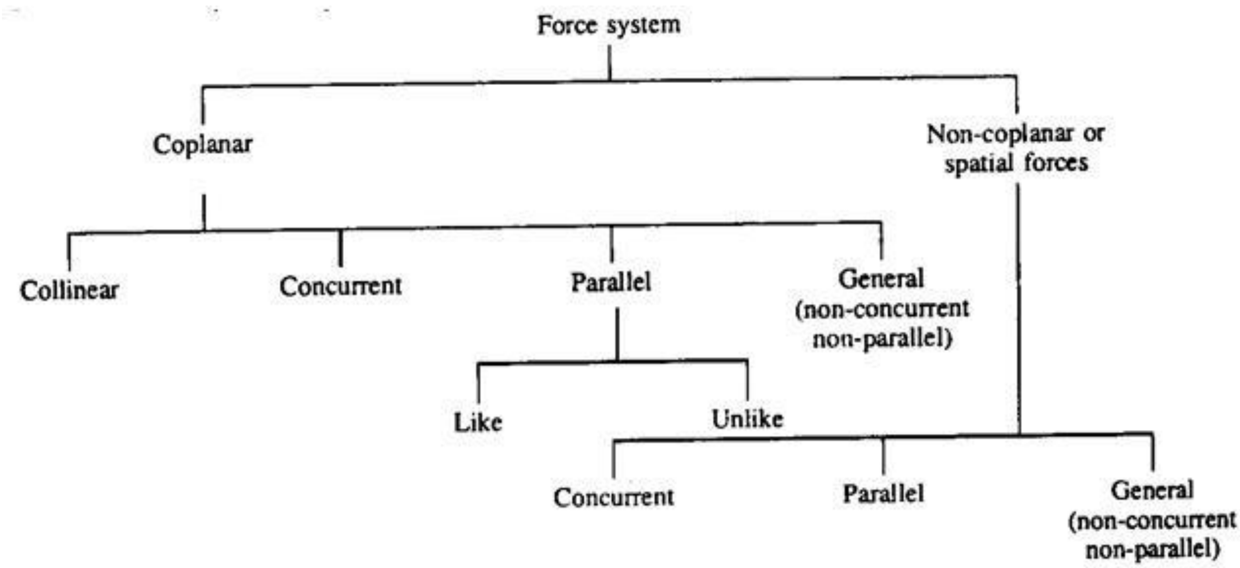


Fig. 1.01 Classification of Force System

Coplanar forces: If all the forces in a system lie in a single plane, it is called a coplanar force

Non coplanar forces: if all the forces in a system lie in different planes , it is called a non-coplanar force

Concurrent forces: If the lines of action of all the forces in a system pass through a single point it is called a concurrent force

Non concurrent forces: If the line of action of all the forces in a system do not meet at a single point,it is called non concurrent forces

Co linear forces: The lines of action of all forces are acting along the same straight line

Coplanar concurrent forces: all forces lie in the same plane, having different directions but their lines of action act at a single point

Coplanar non concurrent forces: All the forces lie in the same plane but their lines of action do not pass through a single point

Non coplanar concurrent forces: All the forces do not lie in the same plane but their line of action pass through a single point

Non coplanar & non concurrent forces: All the forces do not lie in the same plane and their line of action do not pass through a single point

Parallel forces: If the lines of action of all the forces are parallel to each other. The force system is called parallel force system. If the line of action is in the same direction, such types of forces are called like parallel forces. If the lines of action of forces are in different direction, such type of forces is called unlike parallel forces

1.1.1 Force: Force is an external agent ,either a push or pull, which when acting on a body changes or tends to change ,the state of rest or of uniform motion of the body.
The rate of change of momentum is called force. It is denoted by F.
According to newton's second law of motion
Force= mass x acceleration

$$F = ma = m\frac{dv}{dt} = \frac{d}{dt}(mv),$$

$$F = \frac{dP}{dt}.........(1)$$

Where P is the momentum. From the above equation(1), It can be concluded that force is the rate of change of momentum. The concept of force is essential as an agency which changes or tends to change condition of rest or of uniform motion of the body.
In general there are two types of forces :
(i) tensile force (ii) compressive force

(i)Tensile force: It is an axial force which is acting outward and coincides with the neutral axis of the component. Tensile force tries to elongate the component.

(ii)Compressive force: It is an axial force which is acting towards and coincides with the neutral axis of the component. Compressive force tries to contract the component. It means compressive force decreases the length of the component

1.1.2 Characteristics of Force :

A force is a vector quantity and it is completely defined by the following parameters
(i) Magnitude (ii) Point of application (iii) line of action (iv) Direction

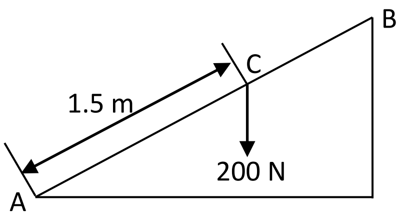


Fig. 1.02 Ladder AB

With reference to the Fig. 1.02, ladder AB is at rest against a wall and a person of weight 200N stands at point c on the ladder. The force applied by the person on the has the following characteristics
(a) Magnitude is 200N
(b) the point of application is atc which is 1.5m from the floor along the ladder
(c) the line of action of force is vertical
(d) the direction is downward

1.1.3 Effects of force:

The force acting on the system having following effect

a) Change its state of rest or motion
b) Change its shape or size
c) It may bring the body at rest
d) It may give rise to internal stress in the body

1.1.4 Moment of a force:

Moment of a force about a point is the product of force and the perpendicular distance of the point from the line of action of the force. let a force P act on a body which is hinged at O. then moment of force P about the point O in the body $= P \times OA$,

Where OA is the perpendicular distance of point from O from the line of action of force P, as shown in Fig. 1.03.

Point O is called Moment Centre and perpendicular distance OA is the moment arm or arm of the force.

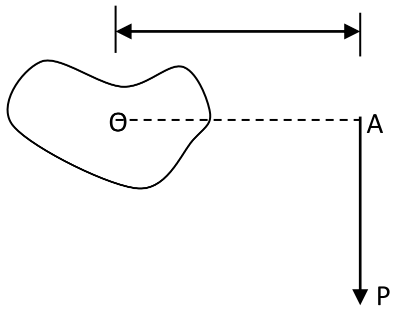


Fig.1.03

When a force acts on a body, it causes or tends to cause a change of state of rest or of uniform motion of the body. The action of the moment tends to cause a rotational motion of the body. Moment is the turning effect produced by the force on which it acts. The tendency of rotation or turning of the body due to moment of force may be clockwise or anticlockwise. In general, clockwise moment is taken as positive .

Moment of forces has following significant aspects:

(i) Moment is the turning effect produced by a force on which it acts
(ii) The moment of the force is equal to the product of force and the perpendicular distance between point and line of action of force
(iii) Moment of a force is represented by M

M_O =force x perpendicular distance

$= P \times L$

1.1.5 Couple: Two parallel forces equal in magnitude but opposite in direction, and separated by a finite distance are said to form a couple.

The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces.

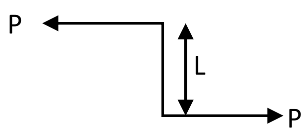


Fig. 1.04

Two equal force of magnitude P is separated by a distance L and acting opposite direction is as shown in Fig. 1.04. These two force form a couple. Mathematically it can be written as

$C = P \times L$

Where C represent the Couple .

1.1.6 Characteristics of a couple:

(i) A couple consists of a pair of equal and opposite forces which are separated by a certain distance

(ii) The translatory effect of a couple on the body is zero

(iii)A couple cannot be balanced by a singular force

(iv)The rotational effect of a couple about any point is a constant and it is equal to the product of the magnitude of the magnitude of forces and the perpendicular distance between the two forces

1.2 Principle of transmissibility:

The conditions of equilibrium or motion of a rigid body remains unchanged if a force acting on a given point of the rigid body is replaced by a force of same magnitude and direction but acting to any other point along the same line of action of the force. This principle is applicable only for rigid body

Let F be the force acting on a rigid body at point (a) as shown in Fig.1.05. According to the law of transmissibility of forces this force has the same effect on the state of the body if force F is applies at point(b). it means the force F has shifted from point a to another to another point b along the same line of action of force, it does not change the condition of equilibrium of body. Any force acting at a point on a rigid body can be transmitted to act at any other point along its line of action without changing its effect on the rigid body

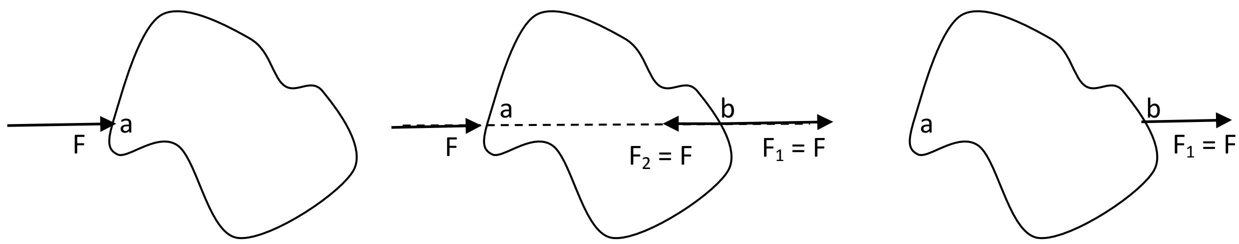


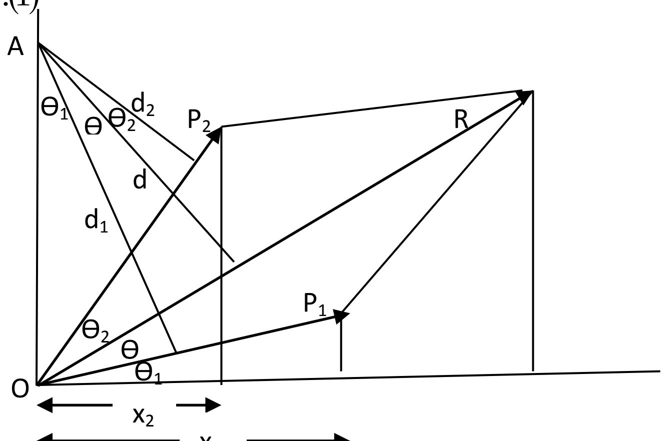
Fig. 1.05

1.3 Varignon's Theorem:

Varignon's theorem states that 'the algebraic sum of moments of two forces about any point in their plane is equal to the moment of their resultant about the same point'.Varignon's theorem is also known as law of moment.

1.3.1 Proof of Varignon's Theoram: Let R be the resultant of forces P_1 and P_2 and O be the moment center as shown in Fig.1.06 . Let d,d_1 and d_2 be the moment arms of the forces R,P_1 and P_2 respectively. According to Varignon's theorem

$Rd = P_1 d_1 + P_2 d_2$...........(1)



Moment of force at A due to $P_1 = P_1 \times d_1 = P_1 \times (OA\cos\theta_1) = OA \times (P_1 \cos\theta_1) = OA \times x_1 .....(2)$,
Moment of force at A due to $P_2 = P_2 \times d_2 = P_2 \times (OA\cos\theta_2) = OA \times (P_2 \cos\theta_2) = OA \times x_2 .....(3)$,
Adding equation (2) and (3), we get
Moment of force at A due to force P_1 and P_2 = $OA \times (x_1 + x_2) .....(4)$,
From geometry, $x = x_1 + x_2$, Putting the value of $x_1 + x_2$ in equation(4), we get
Moment of force at A due to force P_1 and P_2 = $OA \times (x_1 + x_2). = OA \times x....(5)$,
Moment of force at A due to $R = R \times d = R \times (OA\cos\theta) = OA \times (R\cos\theta) = OA \times x.....(6)$,
From equation(5) and (6),
Moment of force at A due to force P_1 and P_2 = Moment of force at A due to R,
$Rd = P_1 d_1 + P_2 d_2$

Varignon's theorem is applicable when two or more than two coplanar forces are acting on the body. If a system of forces consists of more than two forces then according to Varignon's theorem if a number of coplanar forces are acting simultaneously on a particle, the algebraic sum of the moments of all the forces about any point is equal to the moment of their resultant force about the same point

1.4 **Resultant of concurrent coplanar forces:**

All the forces lie in the same plane and line of action of forces passes through the single point is called concurrent co planar forces. We will consider the following cases:

(i) When two forces act at a point
(ii) When more than two forces act at a point

When more than two forces act at a point:

a) **Analytical method:**

The resultant of forces can be calculated by method of resolution as discussed below

i. Resolve all the forces along horizontally and find out the algebraic sum of all the horizontal components i.e. ΣF_x
ii. Resolve all the forces along vertically and find out the algebraic sum of all the vertical components i.e. ΣF_Y
iii. Resultant (R) of the forces will be calculated by

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

iv. The resultant force will be inclined at an angle α

b) **Graphical method:**

The resultant of several forces acting at a point is found graphically with the help of the polygon law of forces. Polygon law states that

"If a number of co planar forces are acting at appoint such that they can be represented in magnitude and direction by the sides of a polygon taken in same order, then their resultant is represented in magnitude and direction by the closing side of the polygon taken in opposite order

Let the three forces F_1, $F_2$and F_3 act at a point . The resultant is calculated graphically by drawing polygon of forces by following steps

(i) Choose a suitable scale to represent the given forces

(ii) Take any point a. from a , draw a vector ab parallel to OF_1
Cut ab= force F_1 to the scale

(iii) From point b, draw line bc parallel to OF_2. Cut bc=Force F_2

(iv) From point c, draw line cd parallel to OF_3. Cut cd=force F_3

(v) Join point a to d. This is closing side of the polygon. Hence ad represents the resultant of forces of same magnitude and direction
Magnitude of resultant R=length ad x suitable scale

The resultant is acting from a to e

1.4.1 Resultant of co planar non concurrent system:

Resultant of a force system is single force. A pure moment or a force and a couple. But they have the same rotational and translatory effect as the given system of forces .

Let F_1,F_2 and F_3 constitute a system of forces acting on the body which are the non-concurrent but co planar . Each force can be replaced by a force of the same magnitude and acting in the same direction at point O and a couple of magnitude $M_i = F_i d_i$ where d_i is the perpendicular distance between the line of action of force F_i and point O.

Following steps may be used to find out the resultant of non-concurrent coplanar forces:

(i) To determine the value of ΣF_x and ΣF_y

(ii) To determine the value of resultant by equation(1)

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} .....(1)$$

(iii) To determine the direction of R w.r.t x axis by using equation(2)

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_y} .....(2)$$

(iv) To determine the algebraic sum of the moments of all forces about any given point O i.e. ΣM_O

(v) Mark the position of the resultant such that it produces the same direction of moment about point O

(vi) Applying varignon's theorem to find the position of the resultant i.e.

$$\Sigma M_O = R \times d$$

(vii) To determine the x and y intercepts of the line of action of resultant

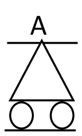
$$x = \frac{d}{\sin\theta}, y = \frac{d}{\cos\theta}$$

1.5 Types of supports :

Various types of support and reactions developed at these ends are following

1. Roller support
2. Hinge or pin support
3. Fixed support

1. Roller support: in roller supported the beam and is supported on a roller. In this case reaction is developed normal to the support end due to rollers are free to roll along the support. In roller support, one vertical reaction is developed w.r.t. to motion of roller so that displacement in vertical direction is prevented . Roller supported at A is shown in Fig. 1.07(a).



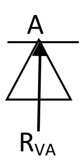


Fig.1.07 (a)

2. Hinge or pin joint: In this case, the position of the end of the beam is fixed but the end is free to rotate. In hinge support, two reactions (one along horizontal and other along vertical)developed in two mutually perpendicular directions. In this types of support, displacements in horizontal and vertical are prevented but do not provide any resistance to moment. Hinge supported at A is shown in Fig. 1.07(b).

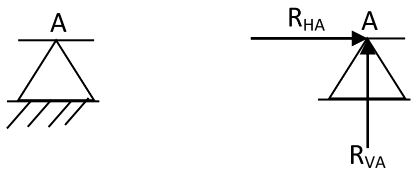


Fig.1.07 (b)

3. Fixed support: At fixed support the end of the beam is neither permitted to move in any direction nor allowed to rotate. Displacements in horizontal, vertical and rotation are prevented
   In fixed support one vertical, one horizontal and one moment is acting at fixed support. Fixed supported at A is shown in Fig. 1.07(c).

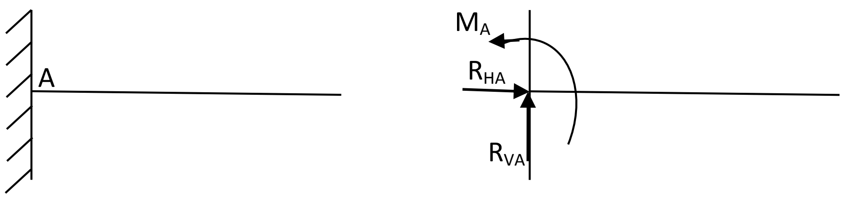


Fig.1.07 (c)

1.6 Types of loads:

In a structure, generally self-load, load due to persons, wind load are acting on the structure. The different types of loads are as following

1. Point or concentrated load
2. Uniformly distributed load(*udl*)
3. Uniformly varying load (*uvl*)

1. Point or concentrated load: if a load is acting on a beam over a very small length compared to the span of the beam, it is approximated as a point load. It is generally represented by an arrow as shown in Fig1.08 (a)

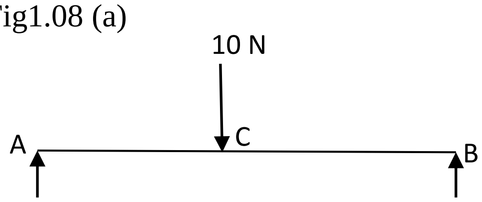


Fig. 1.08 (a)

2. Uniformly distributed load: (udl) : If the intensity of load remain constant over a considerable length, it is called uniformly distributed load. It is shown in Fig.1.08 (b)

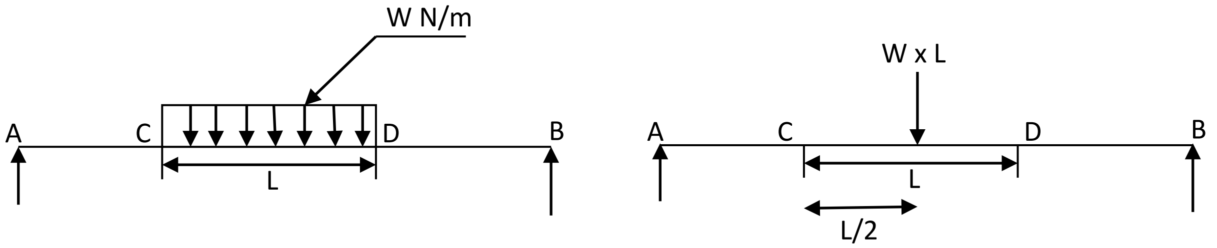


Fig. 1.08 (b)

Equivalent load = Load intensity x distance = W x L,
Equivalent load will be acting at the middle of the distance at which the uniformly distributed load is acting.

3. Uniformly varying load: if the intensity of the load is varying linearly over a considerable length it is called uniformly varying load. This load is also called triangular load. Area of the triangle represents the total load and the centroid of the triangle represents the centre of gravity of the load. It is shown in Fig. 1.08 (c).

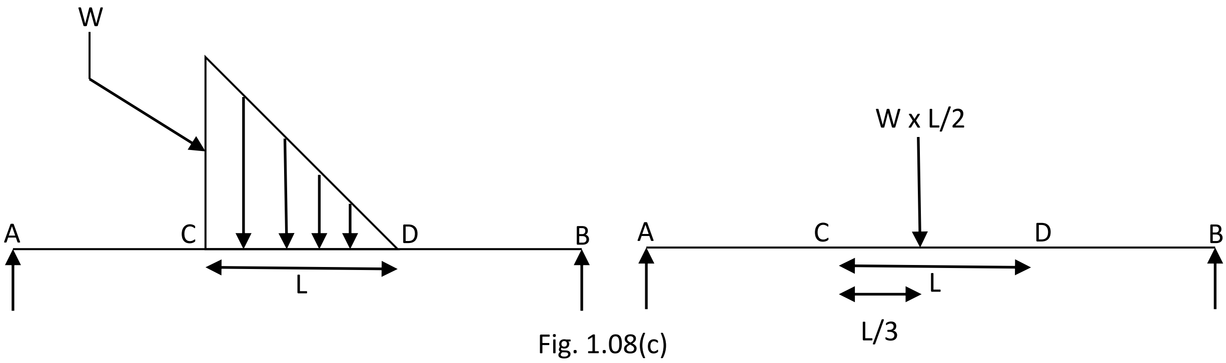


Fig. 1.08(c)

Total equivalent load = Area of triangle = $\frac{W \times L}{2}$

And equivalent load is acting at a distance of 1/3 from the big end and 2/3 from the small end
Ex: water pressure acting on the side wall of the water tank

1.7 Free body diagram:

It is a diagram of the body in which the body under consideration is free from the entire contact surface and all the forces acting on it (both applied and not applied) are drawn. In this diagram, all the supports like walls, floors, hinges, etc. are removed and replaced by the reactions which these supports exert on the body. For the analysis of problem based on equilibrium, It is necessary to isolate the body from all contact surfaces and draw all forces acting on it. While drawing the free body diagrams, It includes, reaction from other contact surface, internal forces and external forces.
Some examples of system with their free body diagram is as shown in Fig.1.09

(i) A sphere resting on a frictionless plane surface

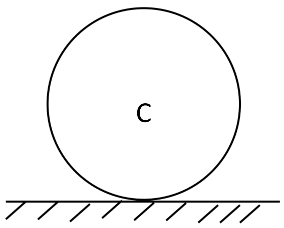


Fig.1.09

Free body diagram of sphere is representation in Fig. 1.09 (a).

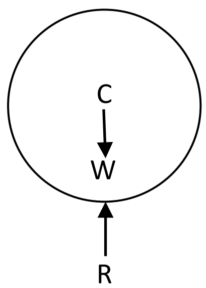


Fig.1.09 (a)

1.8 Equilibrium of coplanar force system:

When external forces are acting on a stationary body, the body may start moving or may start rotating about any point. But if the body does not start moving and also does not start rotating about any point, then the body is said to be in equilibrium

There are three conditions for equilibrium when system of force is subjected to rigid body

1. Algebraic sum of all the horizontal forces acting upon the body must be zero
2. Algebraic sum of all the vertical forces acting upon the body must be zero
3. Algebraic sum of moments acting on the body must be zero

Mathematically the condition for equilibrium is:

$\Sigma F_x = 0, \Sigma F_y = 0. \Sigma M = 0$

Equations of equilibrium:

(i)A non-concurrent coplanar force system will be in equilibrium, if the resultant of all the forces and moment is zero

Equation of equilibrium for non-concurrent forces are

$\Sigma F_x = 0, \Sigma F_y = 0. \Sigma M = 0$

(ii)For concurrent force systems the lines of action of all forces meet at a point and hence the moment of those force is equal to zero

Equation of equilibrium for concurrent forces are

$\Sigma F_x = 0, \Sigma F_y = 0.$

Types I. Problem based on resultant of coplanar con-current forces

1. Five forces are acting on a regular hexagon at an angular point as shown in Fig.1.10. Calculate their resultant in magnitude and direction.

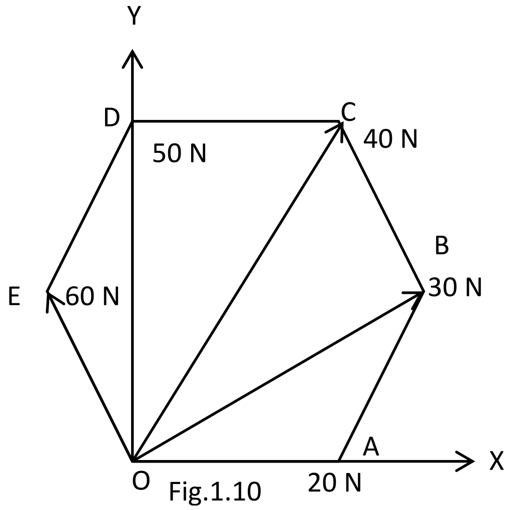


Fig.1.10

Solution:

Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis.

Included angle for regular polygon of n sides = $\frac{2n-4}{n} \times 90^0$

For regular hexagon n= 6, then included angle between two adjacent sides =(8/6) x 90^0 = $120^{0,}$

i.e Angle between OA and AB =120^0,

$\angle AOB = \angle ABO = \angle BOC = \angle COD = \angle DOE = 30^0,$

Resolving all the inclined forces along X and Y Axis,

$\Sigma F_X = 20 + 30Cos30^0 + 40Cos60^0 + 60Cos120^0 = 35.98N,$

$\Sigma F_Y = 30Sin30^0 + 40Sin60^0 + 50 + 60Sin120^0 = 151.60N,$

$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(35.98)^2 + (151.60)^2} = 155.8N,$

$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{151.60}{35.98} = 4.213, \alpha = Tan^{-1}(4.213) = 76.648^0$

2. Three forces act as shown in Fig.1.11. Determine magnitude and direction ϴ of F_1 so that resultant is directed along axis A and has magnitude of 1KN.

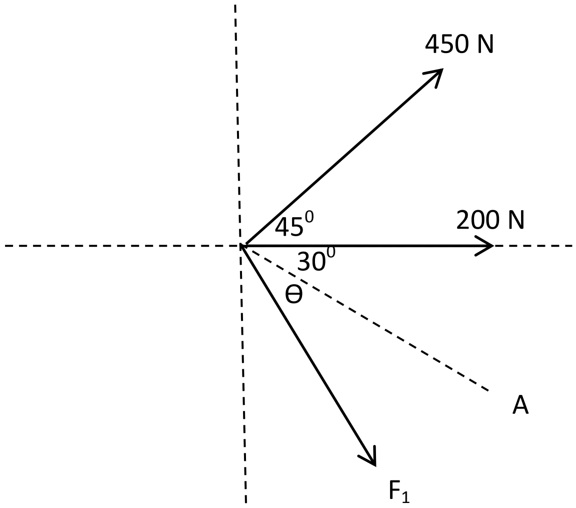


Fig. 1.11

Solution: Since resultant of 1000 N is acting along A direction, then

X component of Resultant (R) = $\sum F_X$ and Y component of Resultant(R) = $\sum F_Y$

$$1000Cos60^0 = 200 + 450Cos45^0 + F_1Cos(\theta + 30^0),$$
$$\therefore F_1Cos(\theta + 30^0) = 347.83................(1),$$
$$-1000Sin30^0 = 450Sin45^0 - F_1Sin(\theta + 30^0),$$
$$\therefore F_1Sin(\theta + 30^0) = 818.2....................(2),$$

Dividing equation (2) by equation (1) , we get

$$\frac{F_1Sin(\theta + 30^0)}{F_1Cos(\theta + 30^0)} = \frac{818.2}{347.83}, Tan(\theta + 30^0) = 2.352, \theta = 36.97^0, F_1 = 889.06N$$

Type II. Problem based on Moment of Forces

3. Four forces of magnitude 20 N , 40 N , 60 N and 80 N are acting respectively along the four sides of square ABCD as shown in Fig.1.12. Determine the moment about point A. It is given that each side of square is 2 m.

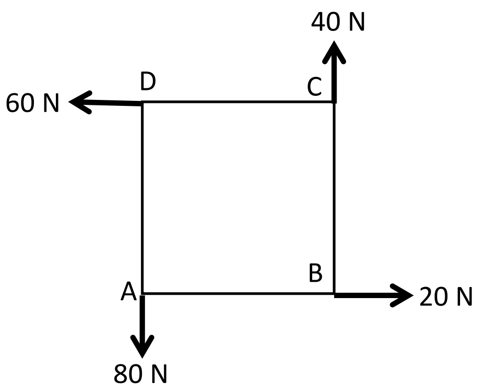


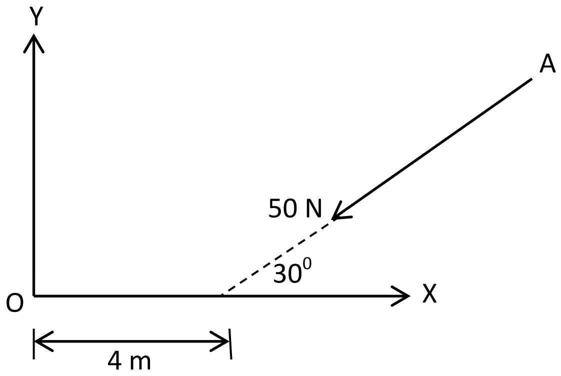
Fig. 1.12

Solution:

Let us assumed that clockwise moment is taken as positive.

$$\Sigma M_A = -40 \times 2 - 60 \times 2 = -200 Nm(Anti - clockwise)$$

4. A force of 50 N is acting at a point A as shown in Fig.1.13. Determine the moment of this force about O.



Solution:

Fig. 1.13

Resolve the 50 N force along X and Y axis. Component of 50 N force along X and Y Axis are 50 $Cos30^0$ and 50 $Sin30^0$ respectively. Moment of Force 50 N about O is

$$\Sigma M_A = 50 Sin 30^0 \times 4 = 100 Nm(Clockwise)$$

5. A force of 200 N is acting at point B of a lever AB which is hinged at its lower end as shown in Fig.1.14. Find the moment of force about the hinged end.

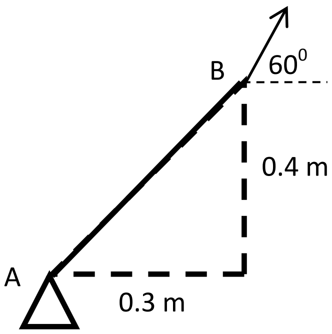


Fig. 1.14

Solution:

The horizontal and vertical components of Force 200 N are

$$F_X = 200 Cos 60^0 = 100 N, F_Y = 200 Sin 60^0 = 173.2 N$$

Moment of Force 200 N about hinged end A =

$$200 Cos 60^0 \times 0.4 + 200 Sin 60^0 \times 0.3 = -11.96 Nm(Anti - clockwise))$$

6. A regular hexagon ABCDEF of side 2 m in length is subjected to forces 1, 2, 3, 4,5 and 6 N along sides AB, CB, CD, DE, EF and FA respectively as shown in Fig.1.15. Determine the sum of moments about point A.

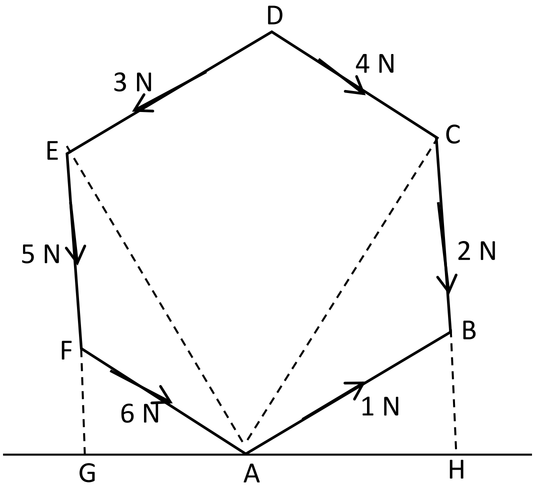


Fig. 1.15

Solution: Clockwise moment is taken as positive.

Moment about point A = $\Sigma M_A = -1\times 0 + 2\times AH + 4\times AC - 3\times AE - 5\times AG - 6\times 0.......(1)$

From the geometric

$\angle BAH = \angle FAG = \angle ACB = \angle BAC = 30^0$,

$AH = AG = ABCos30^0 = 2\times Cos30^0 = 1.732$,

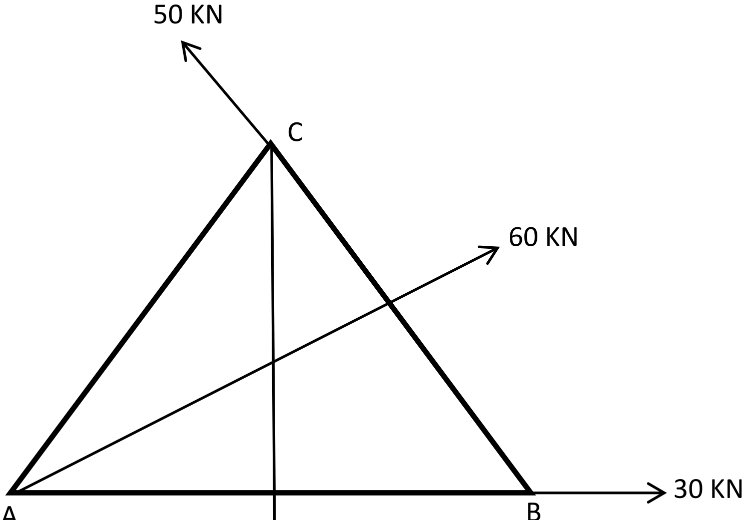
$AH = AC\cos 60^0; AC = \dfrac{1.732}{\cos 60^0} = 3.464, AC = AE = 3.464$,

Putting the value of AH, AC, AE and AG in equation (1), we get

$\Sigma M_A = 2\times 1.732 + 4\times 3.464 - 3\times 3.464 - 5\times 1.732 = -1.728 Nm(Anti-Clockwise)$

Types III. Problem based on resultant of coplanar Non- concurrent forces

7. An equilateral triangular plate of sides 200 mm is acted upon by four forces as shown in Fig.1.16. Determine the magnitude and direction of the resultant of this system of forces and its position from A.(UPTU 2013-2014)



Solution: Resolving all the inclined forces along X and Y Axis is as shown in Fig. 1.16 (a) .

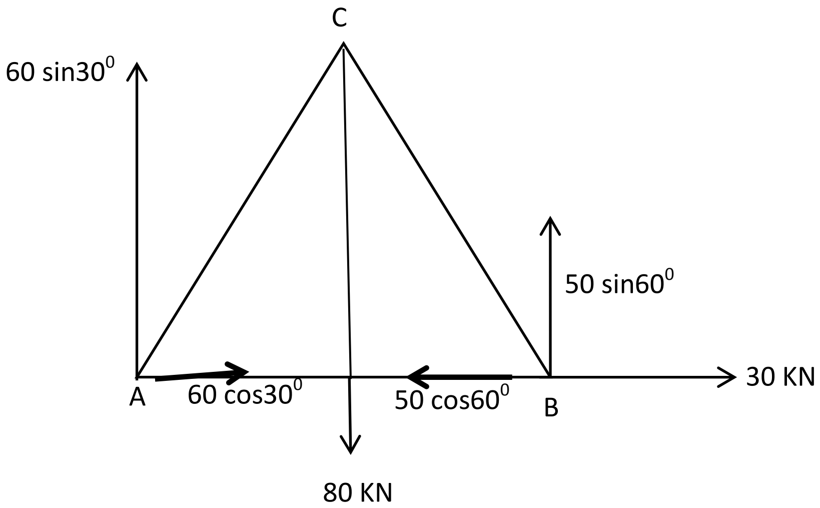


Fig. 1.16 (a)

Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis.

$$\Sigma F_X = 30 + 60\cos 30^0 - 50\cos 60^0 = 30 + 51.96 - 25 = 56.96KN,$$

$$\Sigma F_Y = 60\sin 30^0 - 80 + 50\sin 60^0 = 30 - 80 + 43.30 = -6.698KN,$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(56.96)^2 + (-6.698)^2} = 57.35KN,$$

$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-6.698}{56.96} = -0.1176, \alpha = 6.69^0$$

Let resultant act at a distance of x from point A. For finding the value of line of action of resultant (x), apply Varignon's Theorem. According to this theorem the algebraic sum of moment due to indivisible forces is equal to moment due to resultant force.

Moment due to indivisible forces about A =

$$80 \times 200\cos 60^0 - 50\sin 60^0 \times 200 = 8000 - 8660.254 = -660.254KNmm$$

Moment due to resultant force about A = $\Sigma F_Y \times x = 6.698 \times x$,

$$-660.254 = 6.698 \times x, x = \frac{-660.254}{6.698} = -98.57mm$$

Negative value indicate that resultant is acting at a distance of 98.57 mm from point A along negative X Axis(along left from point A).

8. A square ABCD is subjected to forces equal to P, 2P, 3P and 4P along the sides AB, BC, CD and DA. Determine the magnitude, direction and line of action of the resultant.

Solution:

Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis is as shown in Fig. 1.17.

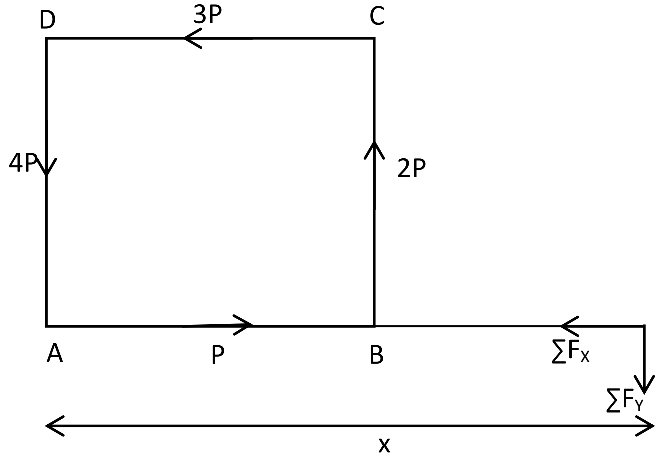


Fig. 1.17

$$\Sigma F_X = P - 3P = -2P, \Sigma F_Y = 2P - 4P = -2P,$$

$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(-2P)^2 + (-2P)^2} = 2\sqrt{2}P,$$

$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-2P}{-2P} = 1, \alpha = 45^0, \left(180^0 + 45^0 = 225^0\right)$$

Since ΣF_x and ΣF_y are both negative, therefore α lies in the 3rd quadrant, so α=225^0

Let resultant act at a distance of x from point A. For finding the value of line of action of resultant(x), apply Varignon's Theorem. According to this theorem the algebraic sum of moment due to indivisible forces is equal to moment due to resultant force. Let a is side of square.

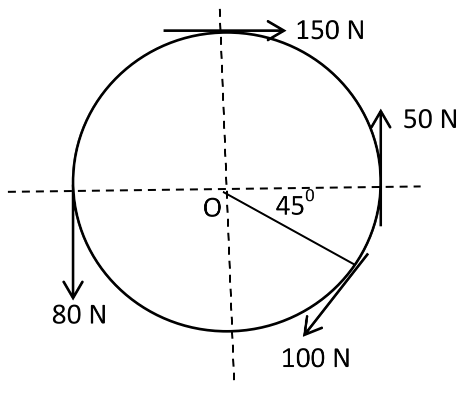
Moment due to indivisible forces about A = $-2P \times a - 3P \times a = -5Pa$

Moment due to resultant force about A = $\Sigma F_Y \times x = 2P \times x$,

$$-5Pa = 2P \times x, x = \frac{-5Pa}{2P} = -2.5a$$

Negative value indicate that resultant is acting at a distance of 2.5a from point A along negative X Axis(along left from point A).

9. Determine the resultant of the four forces acting tangentially to a circle of radius 3m as shown in Fig.1.18. What will be the location of the resultant with respect to centre of the circle?



Solution: Resolving all the inclined forces along X and Y Axis. Let R is the resultant of concurrent forces system and α is the inclination of resultant with X axis.

100 N force is inclined force. Then component of 100 N force along x direction =100 cos45^0 ,

Component of 100 N force along y direction = 100 sin45^0

$$\Sigma F_X = 150 - 100\cos 45^0 = 150 - 70.71 = 79.29N, \Sigma F_Y = 50 - 80 - 100\sin 45^0 = -100.71N,$$
$$R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2} = \sqrt{(79.29)^2 + (-100.71)^2} = 128.18N,$$
$$Tan\alpha = \frac{\Sigma F_Y}{\Sigma F_X} = \frac{-100.71}{79.29} = -1.27, \alpha = 51.8^0$$

Since ΣF_x is positive and ΣF_y is negative, therefore resultant lies in the fourth quadrant.

Moment of all the indivisible forces about the centre = $-50\times3+150\times3-80\times3+100\times3=360Nm$

If d is the perpendicular distance of resultant from the centre, then

$$R\times d = 360, d = \frac{360}{128.18} = 2.795m$$

The resultant will act a perpendicular distance of 2.795 m from the centre.

Types IV. Problem based on condition of equilibrium

10. Five forces of magnitude 2, P, Q, 4 and 3 KN act along AB, CA, AD, AE and FA respectively in a regular hexagon ABCDEF as shown in Fig.1.19. If the system is to be is equilibrium state, determine the value of forces P and Q.

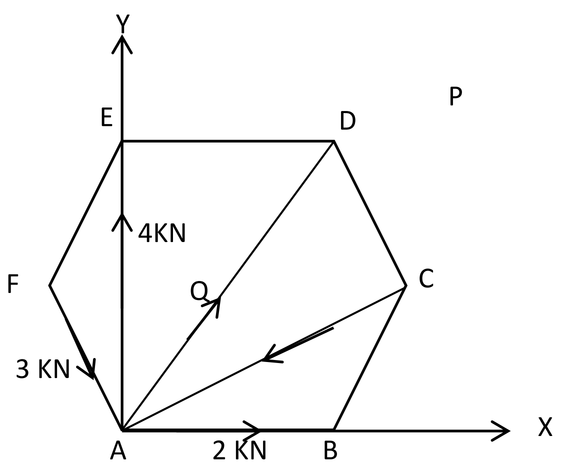


Fig. 1.19

Solution:

The condition of equilibrium for coplanar concurrent forces is

$$\Sigma F_X = 0, \Sigma F_Y = 0,$$

$$\Sigma F_X = 2 - P\cos 30^0 + Q\cos 60^0 - 3\cos 120^0 = 2 - 0.866P + 0.5Q + 1.5$$
$$= 3.5 - 0.866P + 0.5Q = 0 ....(1)$$
$$\Sigma F_Y = -P\sin 30^0 + Q\sin 60^0 + 4 - 3\sin 120^0 = -0.5P + 0.866Q + 4 - 3\times 0.866$$
$$= 1.402 - 0.5P + 0.866Q = 0 ....(2)$$

Solving equation (1) and (2) , we get

$$P = 4.66KN, Q = 1.071KN$$

11. A right circular roller of weight 5000 N rest on a smooth inclined plane and is held in position by a cord AC as shown in Fig. 1.20. Find the tension in the cord if there is a horizontal force of magnitude 1000 N acting at C. (UPTU 2002-2003)

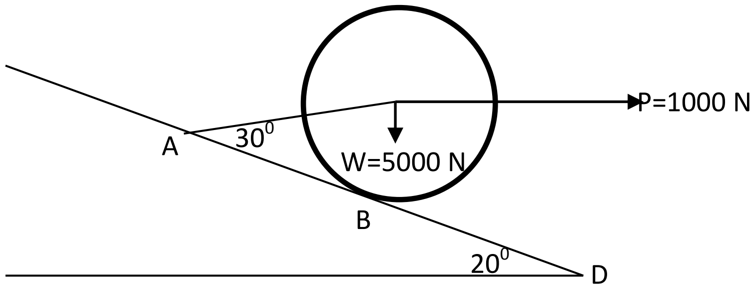


Fig. 1.20

Solution:

Free body diagram (FBD) of circular roller of weight W=5000 N is drawn is as shown in Fig. 1.20 (a)

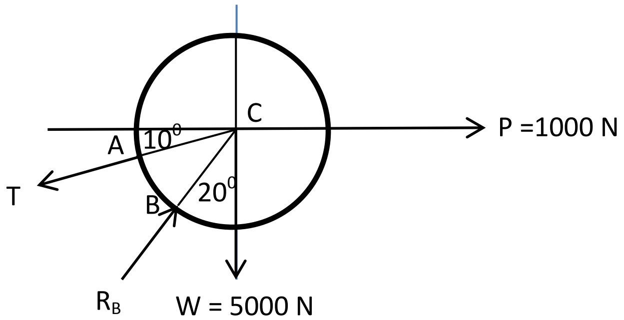


Fig. 1.20 (a)

Resolve all the inclined forces along X and Y axis. Appling condition of equilibrium, we get

$$\Sigma F_X = 0,$$
$$1000 - T\cos 10^0 + R_B \sin 20^0 = 0................(1)$$
$$\Sigma F_Y = 0,$$
$$R_B \cos 20^0 - T\sin 10^0 - 5000 = 0..................(2)$$

Solving equation (1) and (2), we get

$$T = 3059.58N, R_B = 5885.90N$$

Tension in the cord is 3.059 KN.

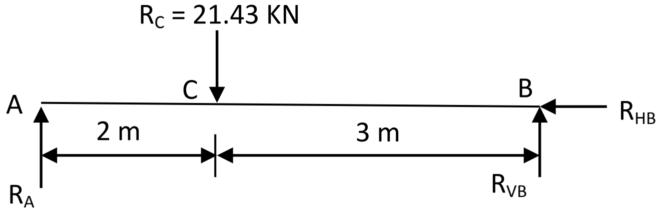


Fig. 1.20 (b)

Applying condition of equilibrium, we get

$$\Sigma F_Y = 0, \Sigma F_X = 0, \Sigma M_A = 0,$$
$$\Sigma F_Y = R_A + R_{VB} - 21.43 = 0, R_A + R_{VB} = 21.43KN......(1)$$
$$\Sigma F_X = R_{HB} = 0$$
$$\Sigma M_A = 21.43 \times 2 - R_{VB} \times 5 = 0, R_{VB} = \frac{42.86}{5} = 8.572KN,$$

Putting the value of R_{VB} in equation (1), we get

$$R_A = 21.43 - 8.572 = 12.85KN$$

Types V : Problem based on support reaction

12. What force and moment is transmitted to the supporting wall at A in the given cantilever beam as shown in Fig.1.21 (UPTU 2002-2003)

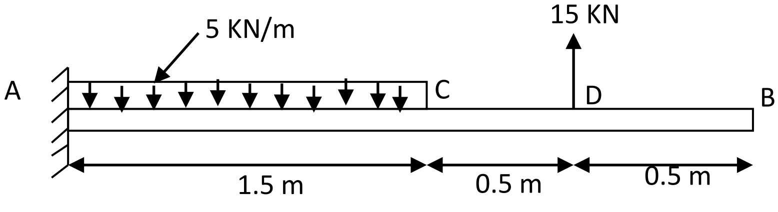


Fig. 1.21

Solution: Free body diagram of Beam AB is as shown in Fig. 1.21 (a) ,

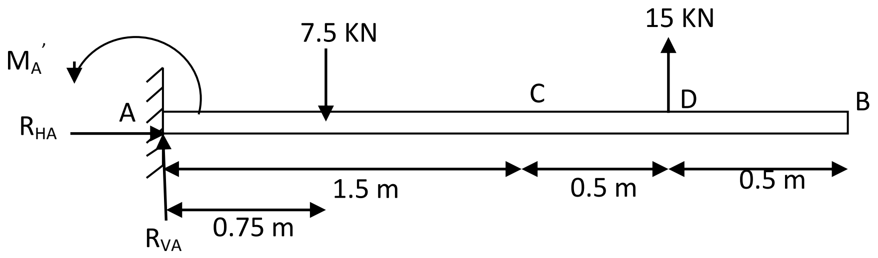


Fig. 1.21 (a)

Applying condition of equilibrium, we get

$$\Sigma F_Y = 0, \Sigma F_X = 0, \Sigma M_A = 0,$$
$$\Sigma F_Y = R_{VA} - 7.5 + 15 = 0, R_{VA} = -7.5KN,$$
$$\Sigma F_X = R_{HA} = 0$$
$$\Sigma M_A = -M_A' + 7.5 \times 0.75 - 15 \times 2 = 0, -M_A' + 5.625 - 30 = 0,$$
$$M_A' = -24.375KNm(Clockwise)$$

Negative sign means, R_{VA} is acting vertically downward (i.e. opposite to the assumed direction).Negative sign means, moment at A will be in opposite direction to assumed direction (i.e. M_A is acting along clockwise direction).

Problems for exercise

1. Find the magnitude and direction of the resultant of the system of coplanar forces as shown in Fig.1.22 (UPTU 200-2001) [Ans. R = 0 N]

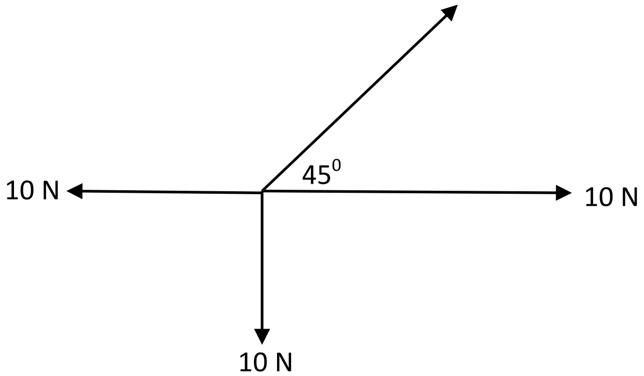


Fig. 1.22

2. The four coplanar forces are acting at a point as shown in Fig.1.23. One of the forces is unknown and its magnitude is as shown by P. The resultant is having a magnitude 500 N and is acting along X- axis. Determine the unknown force P and its inclination with X axis.

[Ans. P = 286.5 N and $\theta = 53^0\ 15'$]

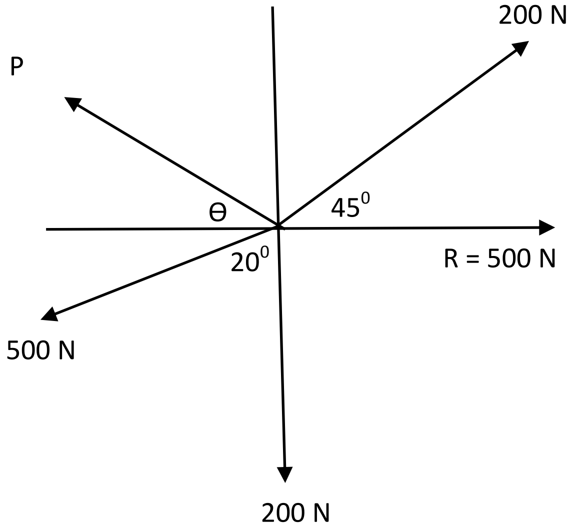


Fig. 1.23

3. A string ABCDE whose extremity A is fixed has weight W_1 and W_2 attached to it at B and C , and passes round a smooth Peg at D carrying a weight of 800 N at the free end E as shown in Fig 1.24. If in a state of equilibrium , BC is horizontal and AB and CD makes angles of 150^0 and 120^0 respectively with BC, make calculations for

(a) The tensions in portions AB, BC,CD and DE of the string

(b) The value of weights W_1 and W_2

(c) The pressure on the peg D

[Ans. T_{AB} = 461.89 N , T_{BC} = 400 N, T_{CD} = 800 N , T_{DE} = 800 N, W_1 = 230.95 N, W_2 = 692.8 N, P_D = 1492.82 N]

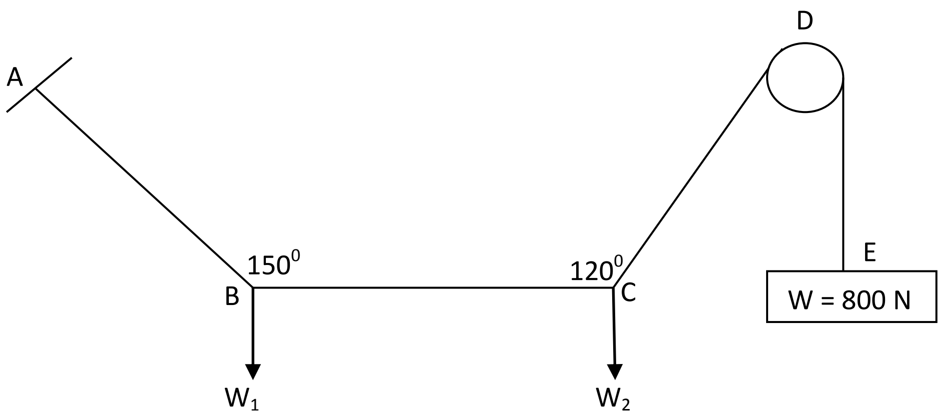


Fig. 1.24

3(a). Determine the resultant of the parallel coplanar force system shown in Fig.1.25.

[Ans. R = 800 N (towards left); d = 627.50 mm]

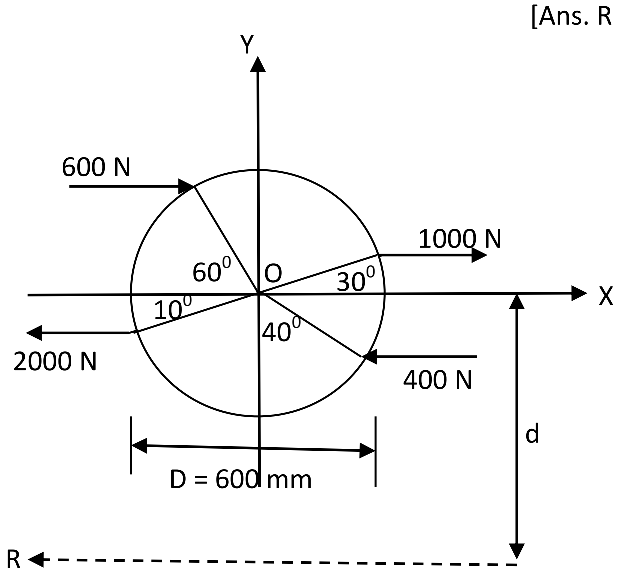


Fig. 1.25

4. The cross-section of a block is an equilateral triangle. It is hinged at A and rests on roller at B . It is pulled by means of a string attached at C. If the weight of the block is M x g and the string is horizontal, determine the force P which should be applied through strings to just lift the block off the roller.

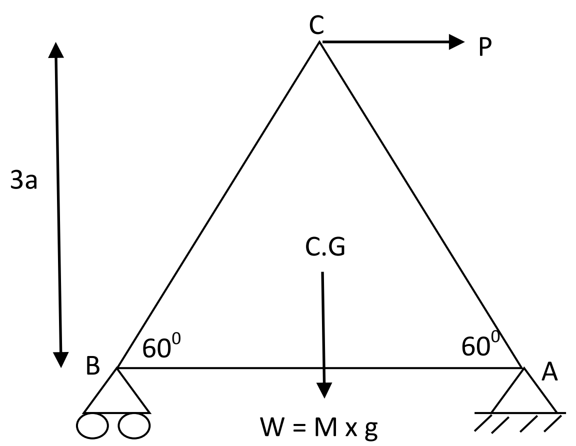
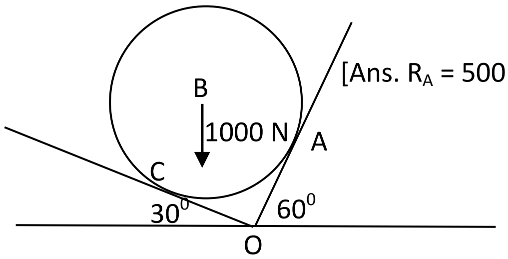


Fig. 1.26

[Ans. $P = \frac{M \times g}{\sqrt{3}}$]

5. A smooth circular cylinder of weight 1000 N and radius 10 cm rests in a right-angled groove whose sides are inclined at an angle of 30^0 and 60^0 to the horizontal as shown in Fig.1.27. Determine the reaction R_A and R_C at the points of contact.



[Ans. R_A = 500 N , R_C = 866.6 N]

Fig. 1.27

6. A beam AB of span 6 m is hinged at A and supported on rollers at the end B and carries load as shown in Fig.1.28. Determine the reactions at A and B.

[Ans. R_{AV} = 5.87 KN , R_{AH} = 3.222 KN ← , R_B = 7.3 KN]

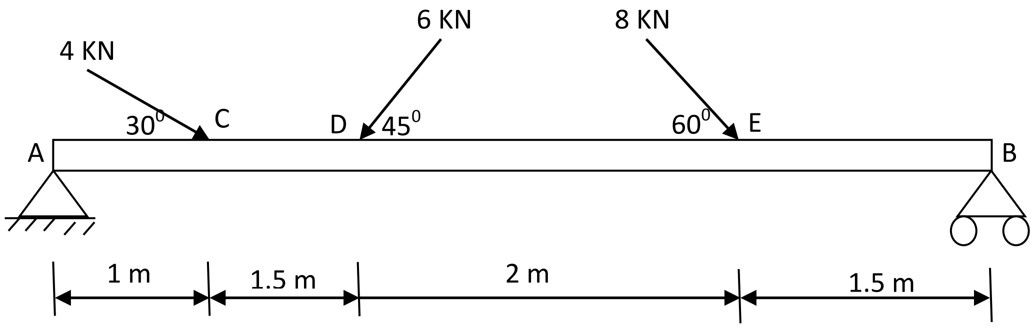


Fig. 1.28

7. Three forces equal to 1 KN, 2KN and 3 KN are respectively acting in order along the three sides of an equilateral triangle. Make calculation for the magnitude, direction and position of their resultant. Assume that a is the side of equilateral triangle.

[Ans. R = 1.732 N , α = 90^0 , d = 3a]