QUESTION BANK

Module 1: Ordinary Differential Equation of Higher Order [10]

Linear differential equation of nth order with constant coefficients, Simultaneous linear differential equations, Cauchy-Euler equation. Second order linear differential equations with variable coefficients, Solution by Changing independent variable, Method of variation of parameters Application of Differential Equation

CO1: Remember the concept of differentiation to evaluate LDE of nth order with constant coefficient and LDE with variable coefficient of 2nd order

Q No.	Question	BLT	CO
1.	Solve the following ordinary linear differential equations: (a) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 12y = 0$ (b) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ (c) $(D^4 - n^4)y = 0, where\ D = \frac{d}{dx}$ (d) $(D^4 - 2D^2 + 4)^2 y = 0, where\ D = \frac{d}{dx}$ (e) $(D^4 + m^4)y = 0, where\ D = \frac{d}{dx}$ (f) $\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0, where\ R^2C = 4L\ \ and\ \ R, C, L\ are constants$ (g) $\frac{d^2y}{dx^2} + y = 0, given that\ y(0) = 2\ and\ \ y\left(\frac{\pi}{2}\right) = -2$ (h) $\frac{d^5y}{dx^5} - 14\frac{d^3y}{dx^3} = 0$	**K3**	**1**
2.	(a) Form a differential equation if its general solution is $y = Ae^x + Be^{-x}$ (b) Prove that the functions {1, x, x^2} are linearly independent. Hence, form a differential equation whose roots are {1, x, $x^{2.}$}. (c) Form a differential equation whose set of independent solutions is{e^x, $x\ e^x$, $x^2\ e^{x.}$}. (d) For what values of ***a*** the characteristic equation of the differential equation $\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + y = 0$ has equal number of roots?	**K3**	**1**
3.	Solve the following ordinary linear differential equations: (i) $(D\ \ +1)^3 y = e^{-x}$ (j) $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1 + e^{-x})^2$ (k) $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + (k^2 + l^2)y = 2e^{2x}$ (l) $(D\ \ -1)^2(D+2)y = e^{-2x} + 2\cosh x$ (m) $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{d\ \ y}{dx} - y = e^x + 2$	**K3**	**1**
4.	**(i)** Solve: $(D^2 + 4)\ \ y = \sin 3x + \cos 2x$ **(ii)** Solve: $(D^3 + 4)\ \ y = \sin(2x + 1)$ **(iii)** Solve: $(D^2 - 4D + 1)\ \ y = \cos x \cos 2x + \sin^2 x$ **(iv)** Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y + 37\sin 3x = 0$ and find the value of y when x= 1/2 given that $\frac{dy}{dx} = 0, y = 3$ when x = 0. **(v)** Solve: $(D^2 + 4)\ \ y = \sin 3x\ \sin^3 x$	**K3**	**1**

	(vi) Solve $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$ **(vii)** $\frac{d^4y}{dx^4} - m^4 y = \cos mx$		
5..	**(i) Solve:** $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2 + 2x + 1$ **(ii) Solve :** $(D^3 - D^2 - 6D)y = 1 + x^2$ **(iii) Solve:** $(y'' - 6y' + 9y) = 2x^2 - x + 3$ **(iv)** Find the solution of the equation $(D^2 - 1)\ y = 1$ which vanishes when $x = 0$ and tends to a finite limit as $x \to -\infty$ and D stands for d/dx	K3	1
6.	Find the complete solution of the following ordinary linear differential equations: **(i)** $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12y = (x-1)e^{2x}$ **(ii)** $(D^2 - 3D + 2)y = 2e^x \cos\left(\frac{x}{2}\right)$ **(iii)** $(D^4 - 1)y = \cos x \cosh x$ **(iv)** $(D^2 - 2D + 1)y = xe^x \cos x \ \ or\ xe^x \sin x$ **(v)** $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = \frac{e^{-2x}}{(x+1)}$ **(vi)** $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x \cos x + sinx \cos 3x$ **(vii)** $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = xe^{3x} + \sin 2x$	K3	1
7.	Find the complete solution of the following ordinary linear differential equations: **(i)** $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x \quad or\ \ x\sin x$ **(ii)** $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 8x^2 e^{-2x} \sin 2x$	K3	1
8.	Find the complete solution of the following differential equations **(i)** $(D^2 + a^2)y = sec\ ax$ **(ii)** $(D^2 + 1)y = tan\ x$ **(iii)** $(D^2 + 1)y = x - \cot x$ **(iv)** $(D^2 + 2D + 2)y = e^{-x}\sec^3 x$ **(v)** $(D^2 + 3D + 2)y = e^{e^x} = \exp\{\exp(x)\}$	K3	1
9.	Solve the following differential equations: **(i)** $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$ **(ii)** $(x+1)^2\frac{d^2y}{dx^2} + (x+1)\frac{dy}{dx} = (2x+3)(2x+4)$ **(iii)** $(2x+3)^2\frac{d^2y}{dx^2} - 2(2x+3)\frac{dy}{dx} - 12y = 6x$ **(iv)** $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = (\log x)\sin(\log x)$ **(v)** $x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10(x + \frac{1}{x})$ **(vi)** $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ **(vii)** $\left(\frac{d}{dx} + \frac{1}{x}\right)^2 y = x^{-4}$	K3	1

	(viii) $\frac{d^2y}{dx^2}+\frac{1}{x}\frac{dy}{dx}=\frac{12\log x}{x^2}$		
10.	**(i)** Solve the simultaneous equations: $\frac{dx}{dt}+5x-2y=t$, $\frac{dy}{dt}+2x+y=0$, given that $x=0, y=0$ when $t=0$. **(ii)** Solve the simultaneous equations: $\frac{d^2x}{dt^2}+y=sint$, $\frac{d^2y}{dt^2}+x=cost$. **(iii)** $\frac{d^2y}{dt^2}-4\frac{dx}{dt}+3y=\sin 2t : \& \frac{d^2x}{dt^2}+\frac{dy}{dt}+3x=e^{-t}$	**K3**	**1**
11.	Solve the following ODE by the method of variation of parameters **(i)** $\frac{d^2y}{dx^2}+a^2y=\sec ax$ **(ii)** $\frac{d^2y}{dx^2}-y=\frac{1}{\sqrt{(1-e^{-2x})}}$ **(iii)** $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$ **(iv)** $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=e^{-x}\log x$ **(v)** $\frac{d^2y}{dx^2}-6\frac{dy}{dx}+9y=\frac{e^{3x}}{x^2}$ **(vi)** $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$ **(vii)** $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$	**K3**	**1**
12.	Solve the following ODE by changing the independent variable **(i)** $x\frac{d^2y}{dx^2}-\frac{d\ y}{dx}-4x^3y=8x^3\sin x^2$ **(ii)** $x\frac{d^2y}{dx^2}+(4x^2-1)\frac{d\ y}{dx}+4x^3y=2x^3$ **(iii)** $\frac{d^2y}{dx^2}-\cot x\frac{d\ y}{dx}-\sin^2 xy=\cos x-\cos^3 x$ **(iv)** $\cos x\frac{d^2y}{dx^2}+\sin x\frac{d\ y}{dx}-2y\cos^3 x=2\cos^5 x$	**K3**	**1**
13.	An inductance L of **2.0*H*** and a resistance R of 20 ohm are connected in series with an e.m.f. E volt. If the current is zero when t=0, find the current ***i*** at the end of 0.01 second if E = 100 V, using the following differential equation $L\frac{di}{dt}+Ri=E(t)$	**K3**	**1**