

Engineering Mathematics-II (BAS-203)

Unit 1 ODE of Higher Order

Tutorial 2

Que1. Obtain the general solution of the differential equation $x^2 y'' + xy' - y = x^3 e^x$ [2015-16]

Que2. Solve $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x)$

Que3. Solve the differential equation $x^2 \frac{d^3 y}{dx^3} + 3x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = x^2 \log x$

Que4. Solve $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = (\log x) \sin(\log x)$

Que5. Solve $x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} - 2y = x - \frac{1}{x^3}$

Que6. Solve $(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$ [2021-22]

Que7. Solve $(2+3x)^2 \frac{d^2 y}{dx^2} + 3(2+3x) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$

Que8. Solve the differential equation $x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 12y = x^3 \log x$ [2022-23]

Que9. Solve the simultaneous differential equation $\frac{dx}{dt} = -3y, \frac{dy}{dt} = 3x$ [2018-19]

Que10. Solve the simultaneous differential equation

$$\frac{d^2 x}{dt^2} - 4 \frac{dx}{dt} + 4x = y, \frac{d^2 y}{dt^2} + 4 \frac{dy}{dt} + 4y = 25x + 16 e^t$$
 [2017-18]

Que11. Solve the simultaneous differential equation

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2 \cos t - 7 \sin t, \frac{dx}{dt} - \frac{dy}{dt} + 2x = 4 \cos t - 3 \sin t$$

Que12. Solve $\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$, given that $x = y = 0$ when $t = 0$

Que13. Solve the following differential equations $\frac{d^2 x}{dt^2} + y = \sin t, \frac{d^2 y}{dt^2} + x = \cos t$ [2015-16]

Que14. Solve $\frac{dx}{dt} + 4(x+y) = 0, \frac{dx}{dt} + 4 \frac{dy}{dt} = -4y$, given that $x = 1, y = 0$ when $t = 0$

Que15. Solve the simultaneous differential equation

$$\frac{d^2 x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}, \frac{d^2 y}{dt^2} - 4 \frac{dx}{dt} + 3y = \sin 2t$$
 [2021-22]

Answers

Que1. $y = c_1 x + c_2 \frac{1}{x} + \left(x - 3 + \frac{3}{x}\right) e^x$

Que2. $y = x \left[c_1 \cos(\sqrt{3} \log x) + c_2 \sin(\sqrt{3} \log x) + \frac{3}{13} \cos(\log x) - \frac{2}{13} \sin(\log x) + \frac{x}{2} \sin(\log x) \right]$

Que3. $y = c_1 + c_2 \log x + c_3 (\log x)^2 + \frac{x^3}{27} (\log x - 1)$

Que4. $y = c_1 \cos(\log x) + c_2 \sin(\log x) - \frac{1}{4} (\log x)^2 \cos(\log x) + \frac{1}{4} \log x \sin(\log x)$

Que5. $y = c_1 x^2 + c_2 \cos(\log x) + c_3 \sin(\log x) - \frac{x}{2} - \frac{1}{50 x^3}$

Que6. $y = c_1 \cos(\log(1+x)) + c_2 \sin(\log(1+x)) + 2 \log(1+x) \sin(\log(1+x))$

Que7. $y = c_1 (2+3x)^2 + c_2 \frac{1}{(2+3x)^2} + \frac{1}{108} [(2+3x)^2 \log(2+3x) + 1]$

Que8. $y = c_1 x^3 + c_2 x^{-4} + \frac{x^3}{98} \log x (7 \log x - 2)$

Que9. $x = c_1 \cos 3t + c_2 \sin 3t$, $y = c_1 \sin 3t - c_2 \cos 3t$

Que10. $x = c_1 e^{3t} + c_2 e^{-3t} + c_3 \cos t + c_4 \sin t + 8 e^t$

$$y = c_1 e^{3t} + 25 c_2 e^{-3t} + (3c_3 - 4c_4) \cos t + (4c_3 + 3c_4) \sin t + 8 e^t$$

Que11. $x = c_1 \cosh \sqrt{2} t + c_2 \sinh \sqrt{2} t + 3 \cos t$

$$y = c_3 \cosh \sqrt{2} t + c_4 \sinh \sqrt{2} t + 2 \sin t$$

Que12. $x = -\frac{1}{27} (1 + 6t) e^{-3t} + \frac{1}{9} \left(t + \frac{1}{3} \right)$

$$y = -\frac{2}{27} (2 + 3t) e^{-3t} - \frac{2}{9} t + \frac{4}{27}$$

Que13. $x = c_1 e^t + c_2 e^{-t} + c_3 \cos t + c_4 \sin t + \frac{t}{4} (\sin t - \cos t)$

$$y = -c_1 e^t - c_2 e^{-t} + c_3 \cos t + c_4 \sin t + \frac{t}{4} (\sin t - \cos t) + \frac{1}{2} (\sin t - \cos t)$$

Que14. $x = (1 - 2t) e^{-2t}$, $y = t e^{-2t}$

Que15. $x = c_1 \cos t + c_2 \sin t + c_3 \cos 3t + c_4 \sin 3t + \frac{e^{-t}}{5} + \frac{2}{15} \cos 2t$

$$y = c_1 (\cos(\sqrt{3}t) - 2 \sin t) + c_2 (\sin(\sqrt{3}t) + 2 \cos t) + 2c_3 \sin 3t - 2c_4 \cos 3t + \frac{1}{15} \sin 2t - \frac{e^{-t}}{5}$$