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B. Tech.
(SEM II) THEORY EXAMINATION 2022-23
ENGINEERING MATHEMATICS-II

Time: 3 Hours *Total Marks: 70*

समय: 03 घण्टे पूर्णांक: 70

Note:

1. Attempt all Sections. If require any missing data; then choose suitably.
2. The question paper may be answered in Hindi Language, English Language or in the mixed language of Hindi and English, as per convenience.

नोट: 1. सभी प्रश्नो का उत्तर दीजिए। किसी प्रश्न में, आवश्यक डेटा का उल्लेख न होने की स्थिति में उपयुक्त डेटा स्वतः मानकर प्रश्न को हल करें।

2. प्रश्नों का उत्तर देने हेतु सुविधानुसार हिन्दी भाषा, अंग्रेजी भाषा अथवा हिंदी एवं अंग्रेजी की मिश्रित भाषा का प्रयोग किया जा सकता है।

SECTION A

1. **Attempt *all* questions in brief.** 2 x 7 = 14

निम्न सभी प्रश्नों का संक्षेप में उत्तर दीजिए।

(a) Solve: $(D^3+2D^2-3D)y=e^x, D=\frac{d}{dx}$.

हल कीजिये:

$(D^3+2D^2-3D)y=e^x, D=\frac{d}{dx}$.

(b) Explain the first shifting property of the Laplace transform with example.
लाप्लास परिवर्तन के प्रथम स्थानांतरण गुण को उदाहरण सहित समझाइये।

(c) Discuss the convergence of sequence $\{u_n\}$, where $u_n=\sin(1/n)$.
अनुक्रम $\{u_n\}$ के अभिसरण पर चर्चा करें, जहां $u_n=\sin(1/n)$.

(d) Show that the function $f(z)=|z|^2$ is not analytic at origin.
दिखाएँ कि फ़ंक्शन $f(z)=|z|^2$ मूल रूप से विश्लेषणात्मक नहीं है।

(e) Classify the singularity of $f(z)=\frac{e^{1/z}}{z}$.

$f(z)=\frac{e^{1/z}}{z}$ की एकलता का वर्गीकरण कीजिए

(f) Find the inverse Laplace transform of $F(s)=\frac{1}{s^2+2s+2}$.

$F(s)=\frac{1}{s^2+2s+2}$ का व्युत्क्रम लाप्लास रूपांतरण ज्ञात कीजिए।

(g) Find the invariant points of the transformation $w=\frac{2z+6}{z+7}$.

ट्रांसफॉर्मेशन $w=\frac{2z+6}{z+7}$ के अपरिवर्तनीय बिंदु ज्ञात कीजिए।

SECTION B

2. **Attempt any *three* of the following:** **7 x 3 = 21**

निम्न में से किसी तीन प्रश्नों का उत्तर दीजिए।

(a) Solve the following differential equation:
निम्नलिखित अवकल समीकरण को हल करें:

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-12y=x^3\log x.$$

(b) Find the Laplace transform of the function $f(x)=x^3\sin x$. Hence, prove that $\int_0^{\infty} e^{-x}x^3\sin x dx=0.$

$f(x)=x^3\sin x$ फ़ंक्शन का लाप्लास रूपांतरण ज्ञात कीजिए। सिद्ध करें कि $\int_0^{\infty} e^{-x}x^3\sin x dx=0$.

(c) Test the convergence of following series:
निम्नलिखित श्रृंखला के अभिसरण का परीक्षण करें:

$\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+\ldots$, Where x is a real number.

$\frac{1}{1.2.3}+\frac{x}{4.5.6}+\frac{x^2}{7.8.9}+\ldots$, जहाँ x एक वास्तविक संख्या है।

(d) Show that the function *f(z)* defined by $f(z)=\frac{x^3y^5(x+iy)}{x^6+y^{10}}, z\neq 0, f(0)=0$ is not analytic at the origin even though it satisfies Cauchy-Riemann equations at the origin.

दिखाएँ कि $f(z)=\frac{x^3y^5(x+iy)}{x^6+y^{10}}, z\neq 0, f(0)=0$ द्वारा परिभाषित फ़ंक्शन *f(z)* मूल बिंदु पर विश्लेषणात्मक नहीं है, यद्यपि यह मूल बिंदु पर कॉची-रीमैन समीकरणों को संतुष्ट करता हो।

(e) Using Cauchy-integral formula, evaluate $\oint_C \frac{\sin 2z}{(z+3)(z+1)^2}dz$, where C is a rectangle with vertices at $3\pm i, -2\pm i$.

कॉची-इंटीग्रल सूत्र का उपयोग करके $\oint_C \frac{\sin 2z}{(z+3)(z+1)^2}dz$ का मूल्यांकन करें। जहाँ पर C, $3\pm i, -2\pm i$ शीर्षों वाला एक आयत है।

SECTION C

3. **Attempt any *one* part of the following:** **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Solve the following differential equation by the variation of parameters:
प्राचल परिवर्तन विधि द्वारा निम्नलिखित अवकल समीकरण को हल करें:

$$\frac{d^2y}{dx^2}+y=\operatorname{cosec} x.$$

(b) Solve the differential equation by the changing the independent variable:
स्वतंत्र चर को बदलकर अवकल समीकरण को हल करें:

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin x^2.$$

4. **Attempt any *one* part of the following:** **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) State convolution theorem of the Laplace transforms. Hence, find inverse Laplace transform of $\frac{1}{s^2(s+1)^2}$.

लाप्लास ट्रांसफॉर्म के convolution theorem लिखिए। $\frac{1}{s^2(s+1)^2}$. का व्युत्क्रम लाप्लास रूपांतरण ज्ञात कीजिए।

(b) Using Laplace transform, solve the following differential equation:

लाप्लास ट्रांसफॉर्म का उपयोग करके, निम्नलिखित अवकल समीकरण को हल करें:

$$\frac{d^2y}{dx^2}+y=6\cos 2x,\ y(0)=3\ \&\ y'(0)=1.$$

5. **Attempt any *one* part of the following:** **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Find a Fourier series to represent $f(x)=x-x^2, -\pi\le x\le\pi$. Hence, show that

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}.$$

$f(x)=x-x^2, -\pi\le x\le\pi$. को व्यक्त करने के लिए फूरियर श्रृंखला ज्ञात कीजिये। तथा दर्शाइए कि $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}$.

(b) Find the half range cosine series for the function $f(x)=(x-1)^2$ in the interval $(0,1)$. Hence, prove that

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}.$$

अंतराल (0,1) में फ़ंक्शन $f(x)=(x-1)^2$ के लिए हाफ रेंज कोसाइन श्रृंखला ज्ञात करें। तथा सिद्ध करें कि $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots=\frac{\pi^2}{8}$.

6. **Attempt any *one* part of the following:** **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Determine an analytic function $f(z)=u+iv$ in terms of z whose real part $u(x,y)$ is $e^x(x\cos y-y\sin y)$ and $f(1)=e$.

z के पदों के रूप में एक विश्लेषणात्मक फ़ंक्शन $f(z)=u+iv$ निर्धारित कीजिये जिसका वास्तविक भाग $u(x,y)=e^x(x\cos y-y\sin y)$ है और $f(1)=e$. है।

(b) Find the bilinear transformation which maps the points $z=0,-1,i$ onto $w=i,0,\infty$. Also, find the image of the unit circle $|z|=1$.

ऐसा द्विरेखीय परिवर्तन ज्ञात कीजिये जो बिंदुओं $z=0,-1,i$ को $w=i,0,\infty.$, पर मैप करता है। इकाई वृत्त $|z|=1$ की इमेज भी ज्ञात कीजिये।

7. **Attempt any *one* part of the following:** **7 x 1 = 7**

निम्न में से किसी एक प्रश्न का उत्तर दीजिए।

(a) Expand $f(z) = \dfrac{7z-2}{z^3 - z^2 - 2z}$ in the following regions:

निम्नलिखित क्षेत्रों में $f(z) = \dfrac{7z-2}{z^3 - z^2 - 2z}$ का विस्तार कीजिये।

$(i)\ 0 < |z| < 1 \quad (ii)\ 1 < |z| < 2 \quad (iii)\ |z| > 2.$

(b) Using contour integration, evaluate the real integral $\int_0^{\pi} \dfrac{a\,d\theta}{a^2 + \sin^2\theta}, a > 0.$

contour integration का उपयोग करके, वास्तविक समाकलन $\int_0^{\pi} \dfrac{a\,d\theta}{a^2 + \sin^2\theta}, a > 0.$ का आकलन करें।

B.Tech

(SEM II) Theory Examination 2022-23

Engineering Mathematics-II (BAS 203)

(solution)

Section A

Q-1 → a) Solve $(D^3+2D^2-3D)y=e^x$

Solu^n → $D(D^2+2D-3)y=e^x$

Solu^n → $y=CF+PI$

Find CF → A.E $m(m^2+2m-3)=0$

$m_1=0$ $\quad (m-1)(m+3)=0 \Rightarrow m_2=1, \; m_3=-3$

$$CF = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}$$

$$CF = c_1 e^{0x} + c_2 e^{x} + c_3 e^{-3x}$$

$$PI = \frac{1}{(D^3+2D^2-3D)} e^x, \text{ put } D=a=1$$

$$= \frac{1}{(1+2-3)} e^x = \frac{1}{0} e^x \text{ case of failure}$$

$$PI = x \frac{1}{3D^2+4D-3} e^x, \text{ put } D=a=1$$

$$= x \frac{1}{(3+4-3)} e^x = \frac{x}{4} e^x$$

$$y = CF + PI = c_1 + c_2 e^x + c_3 e^{-3x}$$

Q-b) Solution → First shifting Property →

If $L\{F(t)\}=f(p)$ then $L\{e^{at}F(t)\}=f(p-a)$

& $L\{e^{-at}F(t)\}=f(p+a)$

Ex $\quad L\{e^{3t}\sin 2t\} = \dfrac{2}{(p-3)^2+4}$

c) $u_n = \sin\left(\frac{1}{n}\right)$, $\lim_{n\to\infty} \sin\frac{1}{n} = 0$ (finite)

Sequence is convergent

d) $f(z) = |z|^2$, $f(z) = |x+iy|^2 = \left(\sqrt{x^2+y^2}\right)^2 = \underset{u}{(x^2+y^2)} + \underset{v}{0i}$

$\frac{\partial u}{\partial x} = 2x$, $\frac{\partial u}{\partial y} = 2y$, $\frac{\partial v}{\partial x} = 0$, $\frac{\partial v}{\partial y} = 0$, at origin

Hence C-R equ^n ~~does not~~ are satisfied but $f(z)$ is not analytic.

$\because$ $f(z) = |z|^2$ is differentiable only at $z=0$ and nowhere else therefore $f(z)$ is differentiable at $z=0$ but not analytic anywhere.

e) $f(z) = \frac{e^{1/z}}{z}$, Isolated essential singularity $z=0$.

f) $F(s) = \frac{1}{s^2+2s+2} = \frac{1}{(s^2+2s+1)+1} = \frac{1}{(s+1)^2+1}$

$$L^{-1}\left\{\frac{1}{(s+1)^2+1}\right\} = e^{-t}\sin t$$

g) $w = \frac{2z+6}{z+7}$, To find invariant points put $w=z$

$z = \frac{2z+6}{z+7}$ or $(z-1)(z+6)=0$ or $z=1,-6$

Section-B

2a) solve $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 12y = x^3\log x$

Solution $\rightarrow$ $(D(D-1)+2D-12)y = e^{3z}z$

$(D^2+D-12)y = e^{3z}z$, Ans $\rightarrow$ $y = CF + PI$ (in terms of z)

Find CF $\rightarrow$ A.E $\rightarrow$ $m^2+m-12=0$, $m=-4,3$

$CF = c_1e^{-4z} + c_2e^{3z}$

Find PI $\rightarrow$ $PI = \frac{1}{D^2+D-12}e^{3z}z$

replace $D \rightarrow D+3$ $\quad PI = \frac{e^{3z}\times 1}{(D+3)^2+(D+3)-12}z$

$PI = e^{3z}\left[\frac{1}{D^2+7D}z\right]$

$$PI = \frac{e^{3z}}{(D^2+7D)}(z) = e^{3z}\left[\frac{1}{D^2+7D} z\right] = \frac{e^{3z}}{7D}\left(1+\frac{D}{7}\right)^{-1} z$$

$$= \frac{e^{3z}}{7D}\left(1-\frac{D}{7}+\frac{D^2}{49} - - -\right) z = \frac{e^{3z}}{7D}\left(z - \frac{D}{7} z\right)$$

$$= \frac{e^{3z}}{7D}\left(z - \frac{1}{7}\right) = \frac{e^{3z}}{7}\int\left(z-\frac{1}{7}\right)dz = \frac{e^{3z}}{7}\left(\frac{z^2}{2} - \frac{1}{7}z\right)$$

Ans $\rightarrow$ $y = CF + PI$

$$y = c_1 e^{-4z} + c_2 e^{3z} + \frac{e^{3z}}{7}\left(\frac{z^2}{2} - \frac{z}{7}\right) = c_1 e^{-4z} + c_2 e^{3z} + \frac{e^{3z}}{98} z(7z-2)$$

$$y = c_1 \frac{1}{x^4} + c_2 x^3 + \frac{x^3}{98}\left(7\log x - 2\right)\log x$$

2b) $L\{x^3 \sin x\} = -\frac{d^3}{dp^3}\left(\frac{1}{p^2+1}\right)$

$$L\{x^3 \sin x\} = \frac{8p}{(p^2+1)^3} + \frac{16p}{(p^2+1)^3} - \frac{48p^3}{(p^2+1)^4}$$

As per defination of Laplace Transform

$$\int_0^\infty e^{-px}(x^3 \sin x)\,dx = \frac{8p}{(p^2+1)^3} + \frac{16p}{(p^2+1)^3} - \frac{48p^3}{(p^2+1)^4}$$

replace $p \rightarrow 1$

$$\int_0^\infty e^{-x} x^3 \sin x \, dx = \frac{8}{2^3} + \frac{16}{2^3} - \frac{48}{2^4} = 1+2-3 = 0$$

2c) $\frac{1}{1\cdot2\cdot3} + \frac{x}{4\cdot5\cdot6} + \frac{x^2}{7\cdot8\cdot9} + - -$

solu^n $\rightarrow$ $u_n = \frac{x^{n-1}}{(3n-2)(3n-1)3n}$, $u_{n+1} = \frac{x^n}{(3n+1)(3n+2)(3n+3)}$

$$\frac{u_n}{u_{n+1}} = \frac{(3n+1)(3n+2)(3n+3)}{(3n-2)(3n-1)(3n)} \cdot \frac{1}{x}, \quad \lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \frac{1}{x}$$

Hence by ratio test, Σu_n is
convergent if $\frac{1}{x}>1$
divergent if $\frac{1}{x}<1$
and test fails if $\frac{1}{x}=1$ or $x=1$

$$u_n = \frac{1}{(3n-2)(3n-1)3n}, \quad v_n = \frac{1}{n^3}$$

$$\frac{u_n}{v_n} = \frac{n^3}{(3n-2)(3n-1)(3n)}, \quad \lim_{n\to\infty} \frac{u_n}{v_n} = \frac{1}{27} \text{ (fixed, finite, non zero)}$$

Hence Comparison test can be applied.

Now $v_n = \frac{1}{n^3} = \frac{1}{n^p}$, $p = 3 > 1$ by p-test series is convergent at $x=1$.

Finally, Σu_n is convergent if $x \le 1$ and divergent if $x>1$

d) $f(z) = \dfrac{x^3y^5(x+iy)}{x^6+y^{10}}$, $z \neq 0$, $f(0)=0$

Solution $\rightarrow$ $u = \dfrac{x^4y^5}{x^6+y^{10}}$, $v = \dfrac{x^3y^6}{x^6+y^{10}}$

$$\frac{\partial u}{\partial x} = \lim_{x\to 0} \frac{u(x,0)-u(0,0)}{x} = 0, \quad \frac{\partial v}{\partial x} = \lim_{x\to 0} \frac{v(x,0)-v(0,0)}{x} = 0$$

$$\frac{\partial u}{\partial y} = \lim_{y\to 0} \frac{u(0,y)-u(0,0)}{y} = 0, \quad \frac{\partial v}{\partial y} = \lim_{y\to 0} \frac{v(0,y)-v(0,0)}{y} = 0$$

Hence Cauchy Riemann equations are satisfied at the origin

$$f'(0) = \lim_{z\to 0} \frac{f(z)-f(0)}{z} = \lim_{z\to 0} \frac{x^3y^5(x+iy)-0}{(x+iy)\,x^6+y^{10}}$$

$$f'(0) = \lim_{z\to 0} \frac{x^3y^5}{x^6+y^{10}}, \quad \text{at } \boxed{x^3 = my^5}$$

$$f'(0) = \lim_{\substack{x \to 0 \\ y \to 0}} \frac{(my^5)y^5}{m^2y^{10} + y^{10}} = \lim_{y \to 0} \frac{m\cancel{y^{10}}}{\cancel{y^{10}}(m^2+1)}$$

$$f'(0) = \frac{m}{m^2+1}$$

since $f'(0)$ depends on m and m may have any value therefore $f'(0)$ does not exist.
Hence $f(z)$ is not analytic at $z=0$

2e $\quad f(z) = \dfrac{\sin 2z}{(z+3)(z+1)^2}$

Poles of integrand are given by $(z+3)(z+1)^2 = 0$

$z = -3, -1, -1$

Pole $z=-3$ lies outside the rectangle and double pole $z=-1,-1$ lie inside the rectangle.

Hence
$$\int_C \frac{\sin 2z}{(z+3)(z+1)^2} = \frac{2\pi i}{\lfloor 1} \frac{d}{dz}\left(\frac{\sin 2z}{z+3}\right)_{z\to -1}$$

$$= 2\pi i \left(\frac{(z+3)(2\cos 2z) - \sin 2z}{(z+3)^2}\right)$$

$$= 2\pi i \left[\frac{4\cos(-2) - \sin(-2)}{4}\right]_{z\to -1}$$

$$= \frac{\pi i}{2}(4\cos 2 + \sin 2)$$

Section-C

3(a) $u = \cos x,\ v = \sin x$

$uv_1 - u_1v = 1$

$$A = -\int \frac{Rv}{uv_1 - u_1v}\,dx + c_1$$

$$A = -x + c_1$$

$$B = \int \frac{Ru}{uv_1 - u_1v}\,dx + c_2$$

$$B = \log \sin x + c_2$$

$$y = Au + Bv$$

$$y = (-x + c_1)\cos x + (\log \sin x + c_2)\sin x$$

3(b) $P = -\frac{1}{x},\ Q_1 = -4x^2,\ R = 8x^2 \sin x^2$

$$\left(\frac{dz}{dx}\right)^2 = 4x^2,\ \left(\frac{dz}{dx}\right) = 2x,\ z = x^2$$

$$\frac{d^2z}{dx^2} = 2$$

$$P_1 = \frac{\frac{d^2z}{dx^2} + P\frac{dz}{dx}}{(dz/dx)^2} = 0,\ Q_1 = -1,\ R_1 = 2\sin z$$

$$\frac{d^2y}{dz^2} - y = 2\sin z$$

$$m^2 - 1 = 0 \Rightarrow m = \pm 1$$

$$C.F = c_1e^{z} + c_2e^{-z} = c_1e^{x^2} + c_2e^{-x^2}$$

$$P.I = \frac{1}{D^2 - 1}\,2\sin z = -\sin z$$

$$y = c_1e^{x^2} + c_2e^{-x^2} - \sin x^2$$

4(a) Convolution th^m-

If $L^{-1}\{f(p)\} = F(t)$ and $L^{-1}\{g(p)\} = G(t)$

then $L^{-1}\{f(p)g(p)\} = \int_0^t F(u)\,G(t-u)\,du$

$$L^{-1}\left\{\frac{1}{s^2}\right\} = t = G(t)$$

$$L^{-1}\left\{\frac{1}{(s+1)^2}\right\} = e^{-t}t = f(t)$$

$$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\} = \int_0^t e^{-u}\cdot u\,(t-u)\,du$$

$$= (t+2)e^{-t} + t - 2$$

4(b) $L[y''] + L[y] = 6L(\cos 2x)$

$$p^2\bar{y} - py(0) - y'(0) + \bar{y} = \frac{6p}{p^2+4}$$

$$\bar{y} = \frac{6p}{(p^2+1)(p^2+4)} + \frac{3p+1}{p^2+1}$$

$$y = L^{-1}\left\{\frac{6p}{(p^2+1)(p^2+4)} + \frac{3p+1}{p^2+1}\right\}$$

$$y = 7\cos t - 2\cos 2t + \sin t$$

5(a) $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\,dx = -\frac{2\pi^2}{3}$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\cos nx$$

$$= -4/n^2(-1)^n$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\sin nx = -\frac{2}{n}(-1)^n$$

$$x - x^2 = -\frac{\pi^2}{3} - 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos nx - 2\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin nx$$

$$x - x^2 = -\frac{\pi^2}{3} + 4\left[\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} - \cdots\right] + 2\left[\frac{\sin x}{1} - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \cdots\right]$$

Put $x = 0$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots = \frac{\pi^2}{12}$$

5(b) $f(x)=(x-1)^2$, $(0,1)$

$$a_0=2\int_0^1(x-1)^2dx=\frac{2}{3}$$

$$a_n=2\int_0^1(x-1)^2\cos n\pi x\,dx$$

$$=\frac{4}{n^2\pi^2}$$

$$(x-1)^2=\frac{1}{3}+\frac{4}{\pi^2}\left(\cos\pi x+\frac{\cos 2\pi x}{2^2}+\cdots\right)$$

Put $x=1$

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\cdots=\frac{\pi^2}{12}$$

6(a) $u(x,y)=e^x(x\cos y-y\sin y)$

$$\frac{\partial u}{\partial x}=e^x(x\cos y-y\sin y)+e^x\cos y=\phi_1(x,y)$$

$$\frac{\partial u}{\partial y}=e^x(-x\sin y-y\cos y-\sin y)=\phi_2(x,y)$$

$$\phi_1(z,0)=(z+1)e^z$$

$$\phi_2(z,0)=0$$

$$f(z)=\int\{\phi_1(z,0)-i\phi_2(z,0)\}dz+c$$

$$f(z)=\int(z+1)e^zdz+c=ze^z+c$$

$$f(1)=e+c$$

$$e=e+c$$

$$c=0$$

$$\therefore\ f(z)=ze^z$$

6(b) $$\frac{(w-w_1)(w_2-w_3)}{(w-w_3)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-w_1)\left(\frac{w_2}{w_3}-1\right)}{\left(\frac{w}{w_3}-1\right)(w_2-w_1)}=\frac{(z-z_1)(z_2-z_3)}{(z-z_3)(z_2-z_1)}$$

$$\frac{(w-i)(-1)}{(-1)(0-i)}=\frac{(z-0)(-1-i)}{(z-i)(-1-0)}$$

$$w=\frac{z+1}{z-i}$$

$$z=\frac{iw+1}{w-1}$$

$$|z|=1$$

$$\left|\frac{iw+1}{w-1}\right|=1$$

$$|1+iw|=|w-1|$$

$$|1+i(u+iv)|=|u+iv-1|$$

$$(1-v)^2+u^2=(u-1)^2+v^2$$

$$u-v=0 \text{ or } u=v$$

7(a) $$f(z)=\frac{7z-2}{z^3-z^2-2z}$$

or $$f(z)=\frac{1}{z}-\frac{3}{z+1}+\frac{2}{z-2}$$

(i) $0<|z|<1$

$$f(z)=\frac{1}{z}-\frac{3}{1+z}-\frac{2}{2(1-\frac{z}{2})}$$

$$=\frac{1}{z}-3\sum_{n=0}^{\infty}(-1)^nz^n-\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$$

$\rightarrow$ laurent

(ii) $1<|z|<2$

$$f(z)=\frac{1}{z}-\frac{3}{z(1+\frac{1}{z})}-\frac{2}{2(1-\frac{z}{2})}$$

$$=\frac{1}{z}-\frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z}\right)^n-\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n$$

$\rightarrow$ laurent

(iii) $|z|>2$

$$f(z)=\frac{1}{z}-\frac{3}{z(1+\frac{1}{z})}+\frac{2}{z(1-\frac{2}{z})}$$

$$=\frac{1}{z}-\frac{3}{z}\sum_{n=0}^{\infty}(-1)^n\left(\frac{1}{z}\right)^n-\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^n$$

$\rightarrow$ laurent

7(b) $I = \int_0^{\pi} \frac{a\,d\theta}{a^2 + \frac{(1-\cos 2\theta)}{2}}$

$= 2a \int_0^{\pi} \frac{d\theta}{(2a^2+1) - \cos 2\theta}$ Put $2\theta = \phi$, $d\theta = \frac{d\phi}{2}$

$= a \int_0^{2\pi} \frac{d\phi}{(2a^2+1) - \cos\phi}$

$I = 2a \int_0^{2\pi} \frac{d\phi}{(4a^2+2) - (e^{i\phi} + e^{-i\phi})}$ But $z = e^{i\phi}$

$d\phi = \frac{dz}{zi}$

$I = 2a \int_c \frac{1}{(4a^2+2) - (z + \frac{1}{z})} \frac{dz}{iz}$

$= 2ai \int_c \frac{dz}{z^2 - 2z(1+2a^2)+1} = 2ai \int_c \frac{dz}{(z-\alpha)(z-\beta)}$

only β lies inside c

where
$\alpha = (1+2a^2) + 2a\sqrt{1+a^2}$
$\beta = (1+2a^2) - 2a\sqrt{1+a^2}$

$R(z=\beta) = \frac{-i}{2\sqrt{1+a^2}}$

$I = 2\pi i \left(\frac{-i}{2\sqrt{1+a^2}}\right) = \frac{\pi}{\sqrt{1+a^2}}$