Subject Name: Engineering Mathematics-I Subject Code : BAS103	**Max. Marks : 70**
Name: _____	**Time : 3 Hours**

Instructions :	1. *Attempt the questions as per the instructions given*
	2. *Assume missing data suitably*

CO1	**Statement of CO1**: Understand the concept of complex matrices, Eigen values, Eigen vectors and apply the concept of rank to evaluate linear simultaneous equations
CO2	**Statement of CO2**: Remember the concept of differentiation to find successive differentiation, Leibnitz Theorem, and create curve tracing, and find partial and total derivatives
CO3	**Statement of CO3**: Identify the application of partial differentiation and apply for evaluating maxima, minima, series and Jacobians.
CO4	**Statement of CO4**: Remember the concept of Beta and Gamma function; analyze area and volume and Dirichlet's theorem in multiple integral
CO5	**Statement of CO5**: Apply the concept of Vector Calculus to analyze and evaluate directional derivative, line, surface and volume integrals

Section – A

Q.1	Attempt **all** the parts		**(2 x 7=14)**		
(a)	Find the value of '**a**' for which the vectors (1, -2, a), (2, -1, 5) and (3, -5, 7a) are linearly dependent.	K2	CO1	2
(b)	Determine the values of λ, μ, for the following system of equations $3x - 2y + z = \mu, 5x - 8y + 9z = 3, 2x + y + \lambda z = -1$ has unique solution.	K2	CO1	2
(c)	Find the nth derivative of the following function: $\dfrac{1}{(1-3x)^2}$	K2	CO2	2
(d)	If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$, prove that $\dfrac{1}{2}\dfrac{\partial u}{\partial x} + \dfrac{1}{3}\dfrac{\partial u}{\partial y} + \dfrac{1}{4}\dfrac{\partial u}{\partial z} = 0.$	K2	CO2	2
(e)	Find the percentage error in the area of an ellipse when an error of +1 % is made in measuring the major and minor axis.	K2	CO3	2
(f)	Prove that $f(mx) = f(x) + (m-1)xf'(x) + \dfrac{(m-1)^2 x^2}{\lfloor 2} f''(x) + \dfrac{(m-1)^3 x^3}{\lfloor 3} f'''(x) + \dots\dots$	K2	CO4	2
(g)	Discuss the value of '**b**' for a Solenoidal vector $\vec{F} = (bx)\hat{i} - (5y)\hat{j} + (2z)\hat{k}.$	K2	CO5	2

Section – B

Q.2	Attempt **any three** parts of the following		**(7 x 3 = 21)**		
(a)	Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$	K4	CO1	7
(b)	If $x = \sin\sqrt{y}$ find the value of y_n at $x = 0.$	K4	CO2	7
(c)	If $u^3 + v^3 + w^3 = x + y + z, u^2 + v^2 + w^2 = x^3 + y^3 + z^3$ and $u + v + w = x^2 + y^2 + z^2,$ then show that $\dfrac{\partial(u, v, w)}{\partial(x, y, z)} = \dfrac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$	K4	CO3	7
(d)	Prove that $\iiint \dfrac{dx\,dy\,dz}{\sqrt{1 - x^2 - y^2 - z^2}} = \dfrac{\pi^2}{8}$, the integral being extended to all positive values of the variables for which the expression is real.	K4	CO4	7
(e)	Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ taken round the rectangle bounded by the lines $x = \pm a, y = 0, y = b.$	K4	CO5	7

Section – C				
Q.3	Attempt **any one** part of the following (**7x 1 = 7**)			
(a)	If A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ determine two non-singular matrices P and Q such that $PAQ=I$. **Hence find A^{-1}.**	K3	CO1	7
(b)	Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence Compute A^{-1}. Also evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.	K4	CO1	7
Q.4	Attempt **any one** part of the following (**7x 1 = 7**)			
(a)	Find the *n*th derivative of $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$	K4	CO2	7
(b)	Trace the curve : $a^2x^2 = y^3(2a - y)$ and also write all necessary steps	K4	CO2	7
Q.5	Attempt **any one** part of the following (**7x 1 = 7**)			
(a)	Compute $f(1.1, 0.9)$ for the function $f(x, y) = \tan^{-1}\left(\dfrac{x}{y}\right)$ by using Taylor's series expansion.	K4	CO3	7
(b)	Find the dimensions of a rectangular box of maximum capacity whose surface area is given when (i) box is open at the top (ii) box is closed.	K4	CO3	7
Q.6	Attempt **any one** part of the following (**7x 1 = 7**)			
(a)	Evaluate the following integral by changing the order of integration : $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$	K4	CO4	7
(b)	Change into polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\,dy$. Hence show that $\int_0^\infty e^{-x^2}dx = \dfrac{\sqrt{\pi}}{2}$.	K4	CO4	7
Q.7	Attempt **any one** part of the following (**7x 1 = 7**)			
(a)	Find the directional derivative of $\varphi(x, y) = 5x^2y - 5y^2z + \dfrac{5}{2}z^2x$ at the point $(1,1,1)$ in the direction of line $\dfrac{x+1}{1} = \dfrac{y}{1} = \dfrac{1-2z}{-2}$.	K4	CO5	7
(b)	A fluid motion is given by $v = (y+z)i + (z+x)j + (x+y)k$, verify that motion is irrational and hence find the velocity potential. Also prove that fluid is incompressible	K4	CO5	7