Engineering Mathematics-II (BAS-203)
Unit 2 Laplace Transform
Tutorial 6

Que1. Solve the following Differential equation by using Laplace Transform

$$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0\ , \quad y = \frac{dy}{dt} = 0 \text{ and } \frac{d^2y}{dt^2} = 6 \ \ when\ t = 0$$

Que2. A particle moves in a line so that its displacement x from a fixed point O at any time t, is given by $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80 \sin 5t$

Using Laplace Transform,find its displacement at any time t, if intially particle is at rest at $x = 0$

Que3. Solve the intial value problem by using Laplace Transform

$$y'' + y' - 2y = 1 - 2x \ \text{ given that } y = 0, \quad y' = 4 \ \ when\ x = 0$$

Que4. Solve the following Differential equation by using Laplace Transform

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x}\sin x\ , \ \text{ where } \ y(0) = 0, \quad y'(0) = 1$$

Que5. Apply Laplace Transform to solve the equation

$$\frac{d^2y}{dt^2} + y = t\cos 2t\ , \text{ given that } y = \frac{dy}{dt} = 0 \text{ for } t = 0$$

Que6. By using Laplace Transform, find the solution of intial value problem

$$y'' + 9y = 9\,u(t-3) \ \text{ given that } \ y(0) = y'(0) = 0 \text{ where } u(t-3) \text{ is unit step function}$$

Que7. Use Laplace Transform , solve the following differential equation

$$\frac{d^2y}{dx^2} + y = 6\cos 2x\ , \text{ given that } y(0) = 3\ \&\ y'(0) = 1$$ **[2022-23]**

Que8. Solve the following simultaneous differential equations by Laplace Transform

$$3\ \frac{dx}{dt} - y = 2t\ \ , \frac{dx}{dt} + \frac{dy}{dt} - y = 0, \text{ given that } x = y = 0 \text{ when } t = 0$$

Que9. Use Laplace Transform to solve

$$\frac{dx}{dt} + y = \sin t\ \ , \frac{dy}{dt} + x = \cos t, \text{ given that } x = 2, y = 0 \text{ when } t = 0$$

Que10. Solve the following simultaneous differential equations by Laplace Transform

$$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0\ \ , \frac{dx}{dt} + 2y = e^t, \text{ with condition } x = y = 0 \text{ when } t = 0$$

Que11. Use Laplace Transform to solve

$$\frac{dx}{dt} - y = e^t\ , \frac{dy}{dt} + x = \sin t, \text{ given that } x = 1, y = 0 \text{ when } t = 0$$

Answers

Ans1. $y = e^{t} - 3e^{-t} + 2e^{-2t}$

Ans2. $x = 2e^{-2t}(\cos t + \sin t) - 2(\cos 5t + \sin 5t)$

Ans3. $y = e^{x} - e^{-2x} + x$

Ans4. $y = \frac{1}{3}e^{-x}(\sin x + \sin 2x)$

Ans5. $y = -\frac{5}{9}\sin t + \frac{4}{9}\sin 2t - \frac{t}{3}\cos 2t$

Ans6. $y = [1 - \cos 3(t-3)]\, u(t-3)$

Ans7. $y = 5\cos x + \sin x - 2\cos 2x$

Ans8. $y = t + \frac{3}{2} - \frac{3}{2}e^{\frac{2t}{3}}$, $x = \frac{t^2}{2} + \frac{t}{2} - \frac{3}{4}e^{\frac{2t}{3}} + \frac{3}{4}$

Ans9. $x = e^{-t} + e^{t}$, $y = \sin t + e^{-t} - e^{t}$

Ans10. $x = \frac{1}{3} - \frac{5}{7}e^{-t} + \frac{8}{21}e^{\frac{3}{4}t}$, $y = \frac{1}{7}\left(e^{-t} - e^{\frac{3}{4}t}\right)$

Ans11. $x = \frac{1}{2}(e^{t} + \cos t + 2\sin t - t\cos t)$, $y = \frac{1}{2}(t\sin t - e^{t} + \cos t - \sin t)$