ABES ENGINEERING COLLEGE, GHAZIABAD

Subject: Fundamentals of Mechanical Engineering

Unit 1

Topic: Introduction to Mechanics (Stress and Strain)

Lecture Notes

INDEX

Introduction to Mechanics (Stress and Strain)

Normal and shear Stress, strain, Hookes' law, Poisson's ratio, elastic constants and their relationship, stress-strain diagram for ductile and brittle materials, factor of safety.

1.0 STRESS:

When an external force is applied on a body, internal resistance is developed within the body, internal resistance is developed within the body to balance the effect of externally applied forces. The resistive force per unit area against deformation is called as stress. It is represented by σ .

Mathematically it can represented as

$$\sigma = \frac{\mathrm{Re}\,sistive \cdot Force}{Area} = \frac{P}{A},$$

In general there are two type of stresses

(a)Normal Stress

(b)Shear Stress

(a)**NORMAL STRESS:**

The intensity of force perpendicular or normal to the section is called normal stress. It is denoted by σ .Mathematically it can be represented as

$\sigma = \lim\limits_{A \to 0}\left(\frac{F_n}{A}\right)$, Where F_n is the force acting normal to the section and A is its corresponding area.

(b)**SHEAR STRESS:**

When a body is subjected to load P consisting of two equal and opposite parallel forces not in same line, it tends to shear off across the resisting section. The stress induced in the body is called shear stress and corresponding strain is called shear strain.

The most common occurrences of pure shear are in riveted and cotter joints .

In the resisting cross-section area parallel to load P is A , then the average shear stress is

$$\tau = \frac{P}{A},$$

Consider of length L fixed at the bottom face AB, with unit width. Let the force P be applied at face DC tangentially to face AB. As a result of force P , the cuboid distorts from ABCD to ABC_1D_1 through an angle ϕ.

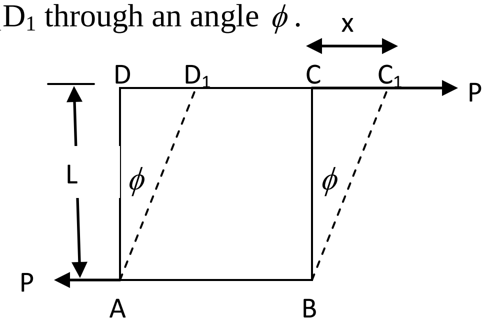

Fig 1

Shear stress $\tau = \dfrac{P}{AB \times 1}$,

1.01Strain:

When a single force (or a system of forces) acts on a body, it undergoes some deformation. This deformation per unit length is known as strain. It is denoted by $\in$. Mathematically it can be represented as

Strain $\in = \dfrac{\delta l}{l}$, where δl is the change of length of body and l is the original length of the body.

The strains can also be classified into two categories.

(1)**Normal or Longitudinal Strain**

(2)Shear Strain

(1)NORMAL STRAIN:

Normal strain can be defined as change in length per unit of length. Mathematically it can be represented as

$$\in = \frac{\delta l}{l}$$

Depending upon the type of force of longitudinal strain can be divided into two categories:

(a)**Tensile strain**

(b)Compressive strain

(a)TENSILE STRAIN:

When a member is subjected to equal and opposite axial tensile forces at its end, it creates elongation in its length. The ratio of elongation in the length to the original length of the member is termed as tensile strain.

(b)COMPRESSIVE STRAIN:

When a member is subjected to equal and opposite axial compressive forces at its length. The ratio of this reduced length to the original length of the member is termed as compressive strain.

(2)SHEAR STRAIN:

When a body is subjected to two equal and opposite parallel forces not in same line it tends to shear off across the resisting section. As a result the cuboid(member) distort as shown in fig from ABCD to ABC_1D_1 through an angle ϕ.Then the ratio of angular deformation to original length along the force is termed as shear strain.

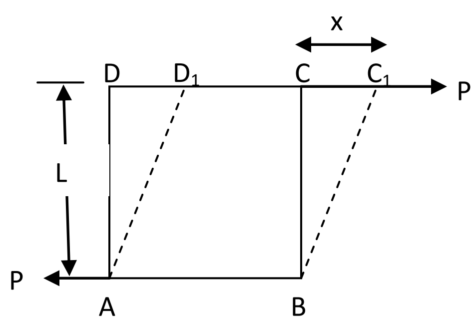

Fig 2

Shear Strain = $\dfrac{Angular \cdot Deformation}{Original \cdot Length} = \dfrac{CC_1}{L} = \phi$(since ϕ is small, so $\tan\phi \approx \phi$)

Other types of strain:

(1)**Volumetric Strain**

(2)Lateral Strain

(3)Longitudinal Strain

(1)VOLUMETRIC STRAIN:

It is defined as the ratio of change in volume of the specimen to the original volume of specimen. It is denoted by $\in_V$. $\in_V = \dfrac{\delta V}{V}$

(2)LATERAL STRAIN:

When a circular bar of diameter d and length l is subjected to normal tensile stress, it produces tensile strain in the direction of normal stress, and the body elongates in the direction of stress. There is a contraction in cross-sectional area of the body. As the length l increases to $l + \delta l$, the diameter d decreases to $d - \delta d$ .The contraction produces a lateral strain.

Lateral strain $\in_d = -\frac{\delta d}{d}$,

(3)LONGITUDINAL STRAIN :

Longitudinal strain is defined as the ratio of change in linear dimension to original dimension. It is denoted by $\in_L$. Mathematically

$$\in_L = \frac{\delta l}{l},$$

1.2 HOOKES' LAW :

When a material is loaded within elastic limit, stress is directly proportional to strain. Mathematically

$$Stress \propto Strain; \sigma \propto \in,$$
$$Stress = Cons\tan t \cdot of \cdot proportionality \times Strain$$
$$or \sigma = E \cdot \in; E = \frac{\sigma}{\in},$$
$$E = \frac{Stress}{Strain}$$

Where the constant of proportionality E is called Young's modulus or modulus of elasticity.

Stress $\sigma = \frac{P}{A}$,Strain $\in = \frac{\delta l}{l}$,According to hook's Law

$$E = \frac{\frac{P}{A}}{\frac{\delta l}{l}} = \frac{Pl}{A\delta l}; \delta l = \frac{PL}{AE}$$

1.3 POISSON'S RATIO:

It is defined as the ratio of lateral strain to longitudinal strain. It is denoted by µ. Mathematically

$$\mu = \frac{Laterial \cdot Strain}{Longitudinal \cdot Strain}$$

1.4 ELASTIC CONSTANT:

(a)**Modulus of elasticity**

(b)Modulus of rigidity

(c)Bulk modulus

(a)Modulus of elasticity:

It is defined as the ratio of stress to strain within the proportionality limit. It is denoted by E.

Modulus of elasticity = $\frac{Stress}{Strain}, E = \frac{\sigma}{\in}$,

(b)Modulus of rigidity:

Modulus of rigidity is defined as the ratio of shear stress to shear strain. It is denoted by G.

Modulus of rigidity = $\frac{Shear \cdot Stress}{Shear \cdot Strain}, G = \frac{\tau}{\phi}$

(c)Bulk Modulus:

When a body is subjected to the mutually perpendicular like and equal direct stresses, then the ratio of direct stress to corresponding volumetric strain is called bulk modulus. It is denoted by K.

Bulk modulus $K = \frac{\sigma}{\in_V} = \frac{\sigma}{\frac{\delta V}{V}}$

RELATION BETWEEN DIFFERENT ELASTIC CONSTANT:

(i)Relation between young's modulus and bulk modulus:

Bulk modulus $K = \frac{Hydrostatic \cdot Pessure}{Volumetric \cdot Strain} = \frac{\sigma}{\in_V} = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)} = \frac{E}{3\sigma(1-2\mu)}$,

$E = 3K(1-2\mu).........(1)$,

(ii)Relation between modulus of elasticity and modulus of rigidity:

Consider a square block ABCD of side L and of thickness unity perpendicular to the plane of drawing as shown in fig 3.

Let the block be subjected to shear stress of intensity τ as shown in Fig 3. Due to these stresses the square is subjected to some distortion, such that the diagonal AC will get elongated and diagonal BD will be shortened.

Let the shear stress τ cause shear strain ϕ as shown in Fig 3, so that AC becomes AC_1.

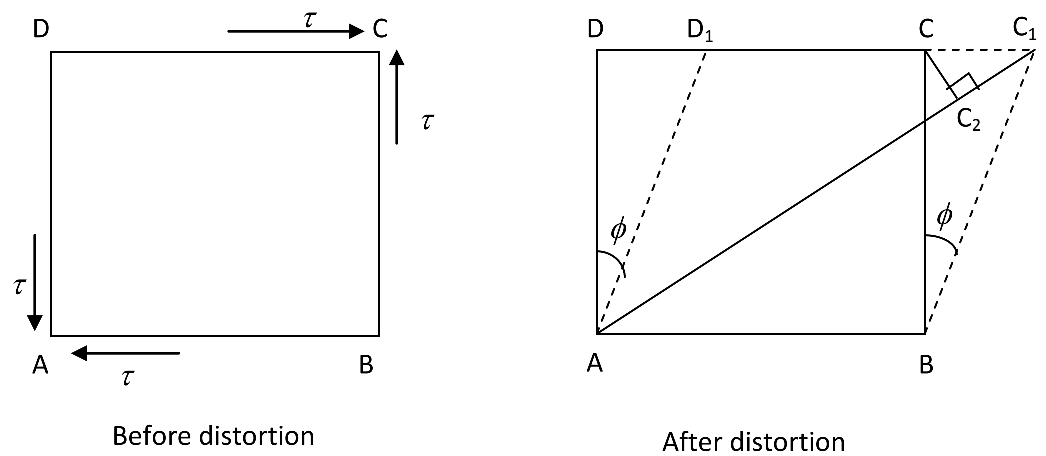

Fig 3

Strain of AC $= \dfrac{AC_1 - AC}{AC} = \dfrac{C_1C_2}{AC} = \dfrac{CC_1 Cos45^0}{AB\sqrt{2}} = \dfrac{CC_1}{2AB} = \dfrac{1}{2}\left(\dfrac{CC_1}{AB}\right) = \dfrac{1}{2}\phi$

Thus, we see that the linear strain of diagonal AC is half of the shear strain and is tensile in nature. Similarly, linear strain of diagonal BD is also equal to half of the strain strain, but is compressive in nature.

Now the linear strain of diagonal AC $= \dfrac{\phi}{2} = \dfrac{\tau}{2G}........(1)$,

Where τ is shear stress and G is modulus of rigidity. The tensile strain of the diagonal AC due to tensile stress on diagonal AC is

Strain on AC $= \dfrac{\tau}{E}\left(\because E = \dfrac{\sigma}{\in}\right)$,

And the tensile strain on the diagonal AC due to compressive stress on diagonal BD

$= \mu \cdot \dfrac{\tau}{E}$,

Combined effect of the above two stresses on diagonal AC is $= \dfrac{\tau}{E} + \mu \cdot \dfrac{\tau}{E} = \dfrac{\tau}{E}(1+\mu)......(2)$,

–

Equating equation (1) and(2), we get

$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu),$$
$$E = 2G(1+\mu).....(3)$$

Where G is modulus of rigidity and E is modulus of elasticity.

1.5 STRESS-STRAIN DIAGRAM:

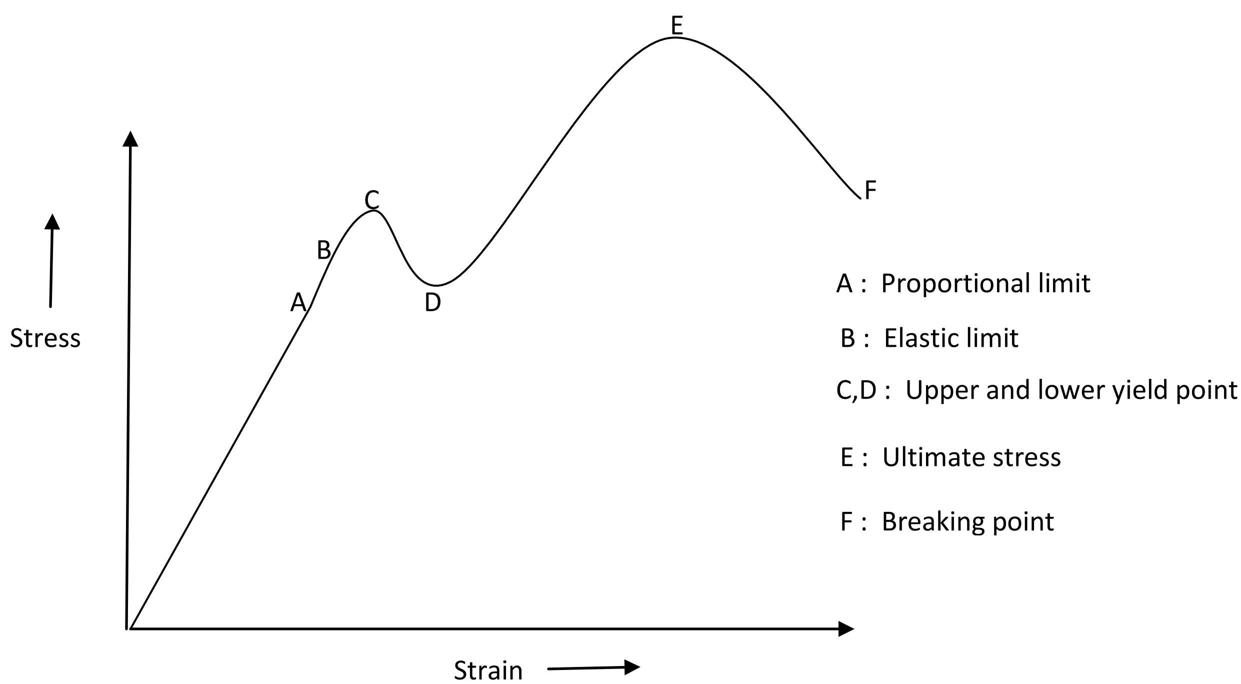

Fig 4 Stress-Strain diagram for ductile

Stress-Strain diagram is a graphical representation of stress versus strain In this diagram, stress is plotted along vertical y axis and strain along x axis .The basis purpose to draw the stress-strain diagram to know the behavior of the material due to load application.

Different significant point of stress-strain diagram for ductile material:

- **Proportional limit:** Stress is a linear function of strain and the material obeys Hook's law. In this limit, stress is proportional to strain. OA represent the proportional limit.
- **Elastic limit:** Beyond proportional limit, stress-strain diagram depart from straight line. Upto elastic limit , after removal of load, it recover its original position. AB represent the elastic limit.

- **Yield point:** Beyond elastic limit, the material shown considerable strain even through there is no increase in load. The behavior of the material is inelastic and the onset of plastic deformation is called yielding of the material. Yielding pertains to the region C-D. The point C is called upper yield point and point D is the lower yield point.

- **Ultimate stress point:** After yielding has taken place, the material becomes strain hardened(strength of the specimen increases) and an increase in load is required to take the material to its maximum stress at the point E.

- **Breaking Point:** In the portion EF, there is falling off the load from maximum until fracture takes place at F. The point F is referred to as the fracture or breaking point and corresponding stress is called the breaking stress.

1.51 STRESS-STRAIN DIAGRAM FOR BRITTLE MATERIAL:

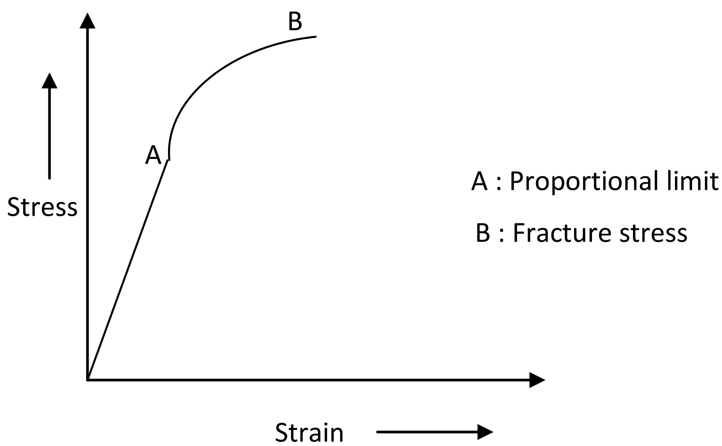

Fig 5 Stress-Strain diagram for Brittle material

Brittle material fails with only little elongation after the proportional limit is exceed. Brittle material fail in tension at relatively low value of strain. For brittle material, like cast iron, no appreciable deformation is obtained and the failure occurs without yielding

1.6 FACTOR OF SAFETY

Factor of safety is defined as the ratio of yield strength or ultimate strength to the working stress (Allowable stress).

During design of an element, it is to be kept in mind that actual stress developed in the element does not exceed the working stress. Factor of safety reduces the risk of failure of a product. It is the load carrying capacity of a system beyond which the system actually supports. In general the value of factor of safety is greater than one.

1.7 NUMERICAL PROBLEM BASED ON STRESS AND STRAIN

(a) A metallic rectangular rod 1.5 m long, 40 mm wide and 25 mm thick is subjected to an axial tensile load of 120 KN. Elongation of the rod is measured as 0.9 mm. Calculate stress, strain and modulus of elasticity.(UPTU 2004)

Solution:

Given

$l = 1.5m, b = 40mm = 0.04m, t = 25mm = 0.025m, P = 120KN = 1.2\times10^5 N, \delta l = 0.9mm = 9\times10^{-4} m$

Area of metallic rod $= b\times t = 0.04\times0.025 = 1\times10^{-3} m^2$,

Stress $\sigma = \frac{P}{A} = \frac{1.2\times10^5}{10^{-3}} = 1.2\times10^8 \, N/m^2$,

Strain $= \frac{\delta l}{l} = \frac{9\times10^{-4}}{1.5} = 6\times10^{-4}$,

Modulus of elasticity $E = \frac{Stress}{Strain} = \frac{1.2\times10^8}{6\times10^{-4}} = 2\times10^{11} \, N/m^2$

(b) A bar of 25 mm diameter is subjected to a pull of 60 KN. Measured extension over a gauge length of 250 mm is 0.15 mm and change in diameter is 0.004 mm. Calculate modulus of elasticity, modulus of rigidity and Poisson's ratio.(UPTU 2006)

Solution:

Given Diameter of rod $d = 25mm = 2.5\times10^{-2} m$,

Area of bar $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.025)^2 = 4.90625\times10^{-4} m^2$,

P = 60 KN = $6\times10^4 N$

Gauge length $= l = 250mm = 0.250m$,change in length $\delta l = 0.15mm = 1.5\times10^{-4} m$,

Change in diameter $\delta d = 0.004mm = 4\times10^{-6} m$,

stress $\sigma = \frac{P}{A} = \frac{6\times10^4}{4.90625\times10^{-4}} = 1.222\times10^8 \, N/m^2$,Strain $= \frac{\delta l}{l} = \frac{1.5\times10^{-4}}{0.250} = 6\times10^{-4}$,

Modulus of elasticity $E = \frac{Stress}{Strain} = \frac{1.222\times10^8}{6\times10^{-4}} = 2.036\times10^{11} \, N/m^2$, Longitudinal strain $= 6\times10^{-4}$,

Lateral strain $= \dfrac{\delta d}{d} = \dfrac{4\times10^{-6}}{0.025} = 1.6\times10^{-4}$,

Possion's ratio (μ) = $\dfrac{\text{Laterial strain}}{\text{Longitudinal strain}} = \dfrac{1.6\times10^{-4}}{6\times10^{-4}} = 0.266$,

Applying relation between modulus of elasticity(E) and modulus of rigidity(G).

$$E = 2G(\mu+1), G = \frac{E}{2(\mu+1)} = \frac{2.036\times10^{11}}{2\times1.266} = 8.03\times10^{10}\ N/m^2$$

(c) A 2m long rectangular bar of 7.5 cm x 5 cm is subjected to an axial tensile load of 1000KN. Bar get elongated by 2 mm in length and decreases in width by 10 x 10^{-6} m. Determine the modulus of elasticity E and Poisson's ratio 'm' of the material of the bar.(UPTU 2007-2008)

Solution:

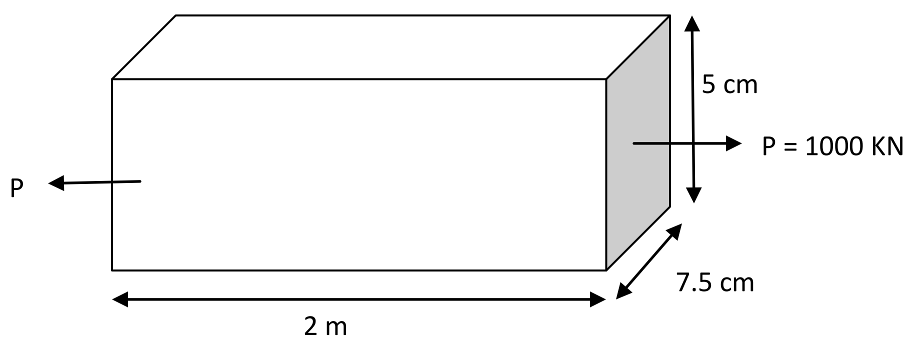

Fig 5.112

Given Length of bar $= l = 2m, \delta l = 0.002m, b = 7.5cm = 0.075m, \delta b = 10\times10^{-6}\,m,$

Thickness $t = 5cm = 0.05m, P = 1000KN = 10^6\,N$,

Stress $\sigma = \dfrac{P}{A} = \dfrac{10^6}{0.075\times0.05} = 2.667\times10^8\ N/m^2$, Linear strain $\in = \frac{\delta l}{l} = \frac{0.002}{2} = 10^{-3}$,From hooke's law

$$E = \frac{\sigma}{\in} = \frac{2.667\times10^{8}}{10^{-3}} = 2.667\times10^{11}\ N/m^{2}\ ,$$

Lateral strain $\in_l = \frac{\delta b}{b} = \frac{10^{-5}}{0.075} = 1.33\times10^{-4}$,

Poisson ratio $\mu = \frac{\in_l}{\in} = \frac{1.33\times10^{-4}}{10^{-3}} = 0.133$

(d) Young's modulus and bulk modulus of steel are 2.1 x 10^{11} Pa and 8.4 x 10^{10} pa respectively. Determine the value of Poisson ratio. (UPTU 2012-2013)

Solution:

Young's modulus = E = 2.1 x 10^{11}Pa , Bulk modulus = 8.4 x 10^{10}Pa,

Applying elastic constant, we get

$$E = 3K(1-2\mu), 2.1\times10^{11} = 3\times8.4\times10^{10}(1-2\mu),$$
$$\mu = 0.079$$

e) A tensile test was conducted on a mild steel bar, the following data was obtained from test:

1) Diameter of the steel bar	**= 3 cm**
2) Gauge length of the bar	**= 20 cm**
3) Load at elastic limit	**=250 KN**
4) Extension at load of 150 KN	**=0.21 mm**
5) Maximum load	**= 380 KN**
6) Total Extension	**= 60 mm**
7) Diameter of the rod at the failure	**= 2.25 cm**

Determine :1) the young's modulus 2) The stress at the elastic limit

3)The percentage elongation 4) The percentage decrease in area

Solution:

1)Area of the rod A $=\frac{\pi}{4}D^2 = \frac{\pi}{4}(0.03)^2 = 7.0685\times10^{-4}\,m^2$

Stress $=\sigma = \frac{Load}{Area} = \frac{150\times10^3}{7.0685\times10^{-4}} = 2.122\times10^8\,N/m^2$,

Strain= $\in= \frac{\delta l}{l} = \frac{0.21\times10^{-3}}{20\times10^{-2}} = 1.05\times10^{-3}$,

Young's modulus $E = \frac{Stress}{Strain} = \frac{2.122\times10^8}{1.05\times10^{-3}} = 2.02\times10^{11}\,N/m^2$,

2)The stress at the elastic limit is given by

Stress $\sigma = \frac{(Load)_{ElasticLimit}}{Area} = \frac{250\times10^3}{7.0685\times10^{-4}} = 3.5368\times10^8\,N/m^2$

3)Percentage elongation $= \frac{\delta L}{L}\times100 = \frac{60\times10^{-3}}{20\times10^{-2}}\times100 = 30\%$,

4)Percentage decrease in area $= \frac{\delta A}{A}\times100 = \frac{\left(\frac{\pi}{4}\times0.03^2 - \frac{\pi}{4}\times0.0225^2\right)}{\frac{\pi}{4}\times0.03^2}\times100 = 43.75\%$

(f) A steel bar 2 m long and 20 mm x 10 mm in cross-section is subjected to a tensile load of 20 KN along its longitudinal axis. Make calculations for changes in length, width and thickness of the bar stating whether it is increase or decrease. Take modulus of elasticity as 2 x 10 ^{5} N/mm^{2} and Poisson's ratio 0.3.

Solution:

Longitudinal strain

$$\in= \frac{\delta l}{l} = \frac{Stress}{Modulus\cdot of\cdot Elasticity} = \frac{\frac{P}{A}}{E} = \frac{P}{AE} = \frac{20\times10^3}{(0.02\times0.01)\times(2\times10^{11})} = 5\times10^{-4},$$

Change in length δl = Longitudinal strain x original length= $(5\times10^{-4})\times2 = 10^{-3}\,m(Increase)$,

Poisson's ratio $\mu = \frac{\in_{Laterial}}{\in_{Linear}} = \frac{\in_{Laterial}}{5\times10^{-4}}$, $\in_{Laterikal}= 0.3\times5\times10^{-4} = 1.5\times10^{-4}$, The lateral strain equals to $\frac{\delta b}{b}$ *and* $\frac{\delta t}{t}$,

Change in breadth $\delta b = (0.02) \times (1.5 \times 10^{-4}) = 3 \times 10^{-6} m (Decrease)$,

Change in thickness $\delta t = (0.01) \times (1.5 \times 10^{-4}) = 1.5 \times 10^{-6} m (Decrease)$,

(g) A bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on a guage length of 200 mm is 0.1 mm and change in diameter is 0.004 mm. Calculate:

i)Young's modulus **ii) Poisson's ratio**

iii)Bulk modulus

Solution:

Given Diameter of bar $d = 30mm$,

Area of bar $A = \frac{\pi}{4}(0.03)^2 = 2.25\pi \times 10^{-4} m^2$,

Pull $P = 60KN = 6 \times 10^4 N$,Extension $\delta l = 0.1mm = 1 \times 10^{-4} m$,

Change in dia $\delta d = 0.004mm = 4 \times 10^{-6} m$,

i)Young's modulus(E)

Tensile stress $\sigma = \frac{P}{A} = \frac{6 \times 10}{2.25\pi \times 10^{-4}} = 8.487 \times 10^7\ N/mm^2$,

Longitudinal strain, $\in = \frac{\delta l}{l} = \frac{1 \times 10^{-4}}{0.2} = 5 \times 10^{-4}$

Young's modulus $E = \frac{Tensile \cdot Stress}{Longitudinal \cdot Strain} = \frac{8{,}487 \times 10^7}{5 \times 10^{-4}} = 1.6975 \times 10^{11}\ N/m^2$

ii)Poisson's ratio

Poisson ratio $\mu = \frac{\in_{Lateral}}{\in_{Longitudinal}} = \frac{\frac{\delta d}{d}}{5 \times 10^{-4}} = \frac{\frac{4 \times 10^{-6}}{0.03}}{5 \times 10^{-4}} = 0.266$

iii) Bulk modulus(K)

$$K = \frac{E}{3(1-2\mu)} = \frac{1.6975\times10^{11}}{3(1-2\times0.266)} = 1.209\times10^{11}\,N/m^2$$

Problems for exercise

i) A steel bar 2 m long, 40 mm wide and 20 mm thick is subjected to an axial pull of 160 KN in the direction of its length. Find the change in length , width and thickness of the bar. Take E = 200GPa and Poisson's ratio = 0.3

[Ans. 2 mm, 0.012 mm, 0.006 mm]

ii) A steel bar 50 mm x 50 mm in cross-section is 1.2 m long. It is subjected to an axial pull of 200 KN. What are the changes in length, width and volume of the bar , if the value of Poisson's ratio is 0.3? Take E = 200 GPa.

[Ans. 0.48 mm, 0.006 mm, 480 mm^3]

iii) A steel cube block of 50 mm side is subjected to a force of 6 KN (Tension), 8 KN (Compression) and 4 KN (Tension) along x, y and z direction respectively. Determine the change in volume of the block. Take E= 200 GPa and m=10/3.

[Ans. 0.2 mm^3]

iv) In an experiment, a bar of 30 mm diameter is subjected to a pull of 60 KN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the Poisson's ratio and the values of the three moduli.

[Ans. 0.289 , E = 188.9 GPa, G = 149.2GPa, K = 149.2GPa]

v) The safe stress, for a hollow steel column which carries an axial load of 2200 KN is 120 MN/m^2. If the external diameter of the column is 25 cm, determine the internal diameter.

[Ans. 19.79 cm]

vi) A bar of 20 mm diameter subjected to a pull of 50 KN. The measured extension on gauge length of 250 mm is 0.12 mm and change in diameter is 0.00375 mm. Calculate:

(i)Young's modulus (ii) Poisson's ratio and (iii) Bulk modulus

[Ans. 1.989 x 10^5 N/mm^2 , 0.234 , 1.2465 x $10^5 N/mm^2$]

vii) An axial pull of 40000 N is acting on a bar consisting of three sections of length 30 cm, 25 cm and 20 cm and of diameters 2 cm , 4 cm and 5 cm respectively. If the young's modulus = 2 x 10^5 N/mm^2, determine:

(i)Stress in each section and (ii) Total extension of the bar

[Ans. (i) 127.32, 31.8 , 20.37 N/mm^2,(ii) 0.025 cm]

viii) Determine decrease in the volume of a solid sphere of 250 mm diameter when it is subjected to uniform hydrostatic pressure of 80 N/mm^2. Take E = 2.1 x 10^5 N/mm^2 and μ = 0.3.

[Ans. 3738 mm^3]

ix) Three bars made of copper, zinc and aluminium are of equal length and have cross-sectional areas 500, 750 and 1000 mm^2 respectively. They are rigidly connected at their ends. If this composite system is subjected to a longitudinal pull of 2.5 x 10^5 N, make calculation for the proportion of load carried on each bar and the induced stresses. Take E_C = 1.3 x $10^5 N/mm^2$, E_Z = 1 x 10^5 N/mm^2, E_{al} = 0.8 x $10^5 N/mm^2$.

[Ans. P_c = 0.7378 x 10^5 N , P_z = 0.8522 x 10^5 N ,P_a = 0.9090 x 10^5 N, $\sigma_c = 147.72\, N/mm^2$, $\sigma_z = 113.63\, N/mm^2$, $\sigma_a = 90.9\, N/mm^2$]

x) A brass bar, having cross-section area of 900 mm^2 to axial force as shown in Fig 5.123. Find the total elongation of the bar. Take E = 100GPa. [Ans. -0.111 mm]

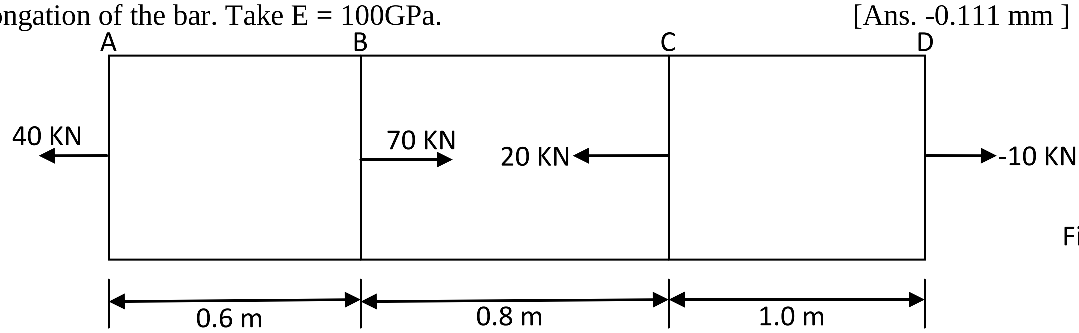

Fig 5.123