ABES Engineering College, Ghaziabad
MODEL PAPER (Odd Semester) 2023-24
B.TECH. [Branch/Section:All] **SEM: I.**

Subject Name: Engineering Mathematics-I Subject Code : BAS103	**Max. Marks : 70**
Name: _____	**Time : 3 Hours**

Instructions :
1. *Attempt the questions as per the instructions given*
2. *Assume missing data suitably*

CO1	**Statement of CO1**: Understand the concept of complex matrices, Eigen values, Eigen vectors and apply the concept of rank to evaluate linear simultaneous equations
CO2	**Statement of CO2**: Remember the concept of differentiation to find successive differentiation, Leibnitz Theorem, and create curve tracing, and find partial and total derivatives
CO3	**Statement of CO3:** Identify the application of partial differentiation and apply for evaluating maxima, minima, series and Jacobians.
CO4	**Statement of CO4**: Remember the concept of Beta and Gamma function; analyze area and volume and Dirichlet's theorem in multiple integral
CO5	**Statement of CO5**: Apply the concept of Vector Calculus to analyze and evaluate directional derivative, line, surface and volume integrals

Section – A

Q.1	Attempt **all** the parts **(2 x 7=14)**			
(a)	Find characteristic equation of the matrix whose eigen values are 1, 2.	K2	CO1	2
(b)	About which axes the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is symmetrical?	K2	CO1	2
(c)	Find the nth derivative of $y=x^{n-1}\log x$	K2	CO2	2
(d)	The time T of a complete oscillation of a simple pendulum of length L is governed by the equation $= 2\pi\sqrt{\frac{L}{g}}$, where g is a constant. Find the approximate error in the calculate value T corresponding to an error of *2%* in the value of L.	K2	CO3	2
(e)	Evaluate $\int_0^1\int_0^1 \frac{dxdy}{\sqrt{1-x^2}\sqrt{1-y^2}}$	K2	CO4	2
(f)	Find the constant 'b' such that the vector $(bx+4y^2z)\hat{i}+(x^3\sin z-3y)\hat{j}-(e^x+4\cos x^2y)\hat{k}$ is solenoidal vector.	K2	CO5	2
(g)	Find the unit normal vector to the surface $(x-1)^2+y^2+(z+2)^2=9$ at the point $(3,1,-4)$.	K2	CO5	2

Section – B

Q.2	Attempt **any three** parts of the following **(7 x 3 = 21)**			
(a)	Find the eigen values and eigen vectors of the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$	K4	CO1	7
(b)	If $y=e^{m\cos^{-1}x}$, find the value of y_n at $x=0$.	K4	CO2	7
(c)	Find the maximum and minimum distances of the point $(3,4,12)$ from the sphere $x^2+y^2+z^2=1$.	K4	CO3	7
(d)	Find the volume of solid surrounded by the surface $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}+\left(\frac{z}{c}\right)^{2/3}=1$.	K4	CO4	7
(e)	Verify the divergence theorem for $\vec{F}=\left(x^3-yz\right)\hat{i}+\left(y^3-zx\right)j+\left(z^3-xy\right)k$, taken over the cube bounded by planes $x=0, y=0, z=0, x=1, y=1, z=1$.	K4	CO5	7

Section – C

Q.3	Attempt **any one** part of the following **(7x 1 = 7)**			

(a)	Find the rank of the following matrix by reducing to normal form: $\begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$.	K3	CO1	7
(b)	Determine the values of λ, μ ,such that the following system of equations has (i) Unique solution, (ii) no solution, (iii) Infinite no. of solutions. $x+y+z=6,\ x+2y+5z=10,\ 2x+3y+\lambda z=\mu$	K4	CO1	7
Q.4	Attempt **any one** part of the following **(7x 1 = 7)**			
(a)	If $u=\cos^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=-2\cot u$. Also evaluate $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}$	K4	CO2	7
(b)	Trace the curve $y^2(2a-x)=x^3$	K4	CO2	7
Q.5	Attempt **any one** part of the following **(7x 1 = 7)**			
(a)	Expand x^y in power of (*x-1*) and (*y-1*) up to the third-degree term and hence evaluate $(1.1)^{1.02}$.	K4	CO3	7
(b)	If u, v, w be the roots of the equation$(x-a)^3+(x-b)^3+(x-c)^3=0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$	K4	CO3	7
Q.6	Attempt **any one** part of the following **(7x 1 = 7)**			
(a)	Change the order of integration in $\int_0^2\int_{\frac{x^2}{4}}^{3-x} xydydx$ and hence evaluate the same.	K4	CO4	7
(b)	Evaluate by changing the variable $\iint_R (x+y)^2\,dx\,dy$, where *R* is the region bounded by the parallelogram $x+y=0, x+y=2, 3x-2y=0, 3x-2y=3$.	K4	CO4	7
Q.7	Attempt **any one** part of the following **(7x 1 = 7)**			
(a)	Find the directional derivative of $\phi=x^2yz+4xz^2$ at the point $(1,-2,1)$ in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$. Find also the greatest rate of increase of $\emptyset$.	K4	CO5	7
(b)	State and Verify Green's theorem by evaluating $\int_C[(x^3-xy^3)dx+(y^2-2xy)dy]$, where C is the square having the vertices at the point $(0,0),(2,0),(2,2)\&(0,2)$.	K4	CO5	7