Fundamentals of Mechanical Engineering

BME-101 (3 Credit Course)

NPTEL → Moocs Platform

Unit 1: Introduction to Mechanics

⇓

Branch of Physical Science in which we study motion or rest.

1) Mechanics is a branch of Physical Science which deals with state of rest or motion of a body.

* It is classified as :-

(1) Statics (body at rest)
(2) Dynamics (body at motion)
in

Dynamics (Body in motion)

↓ Kinetics

↓ Kinematics

Kinetics → cause of motion is not studied

Kinematics → cause of motion is studied

2) <u>State of Body</u> :-

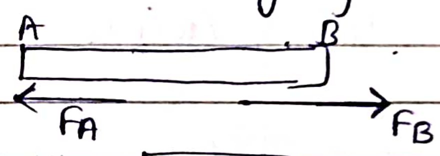
Body

↓ Solid Mech. (Rigid & Non Rigid Bodies)

↓ Fluid Mech. (Liquid & gas)

3) <u>Force</u> : Force is defined as an agent which produces or tends to produce or destroys or tends to destroy motion of a body.

Tends to destroy Motion $\xrightarrow{\text{Example}}$ Tug of war



$\boxed{F_A = F_B}$

<u>OR</u>

→ According to Newton Second Law of Motion 1 N is the force require to produce an acceleration of $1 m/s^2$ in a body of unit mass

<u>Ex</u>:- Horse applies force to pull the cart. (produces motion)
(1)

2) Force applied on giant wall (tends to produce motion)

3) Game of Tug of war, when rope is balance (destroys motion)

but one party (A) gets weaker other party (B) pulls off. (A parties best effort to destroy the motion) (tends to destroy the motion)

(4) Specification of force

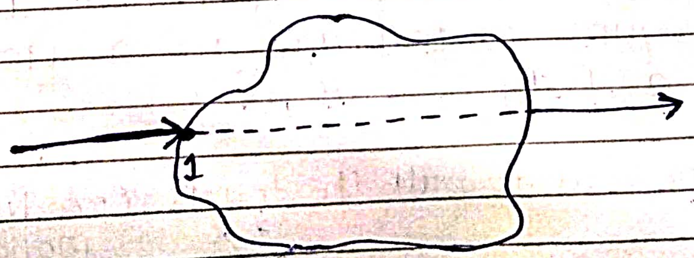
(i) Magnitude (for ex:- 100 N, 10 Kg N, 5 Mega Newton)

(ii) Point of Application (The point at which or through which force is applied)

(iii) Line of Action

(iv) Direction

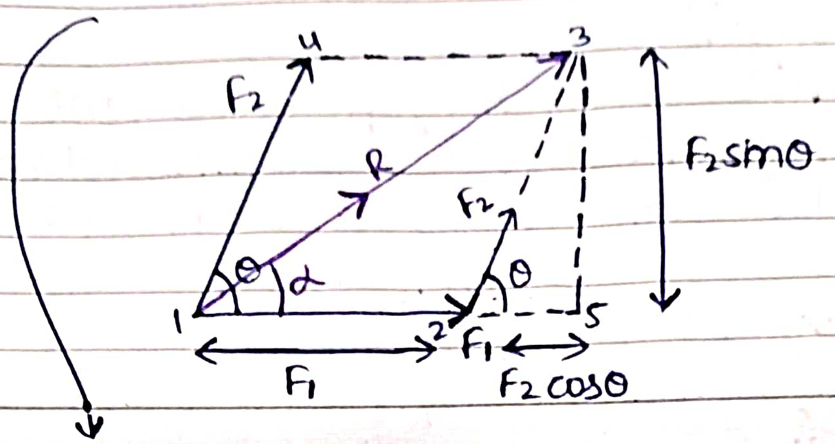
(5) Transmissibility of forces :- (for short note)

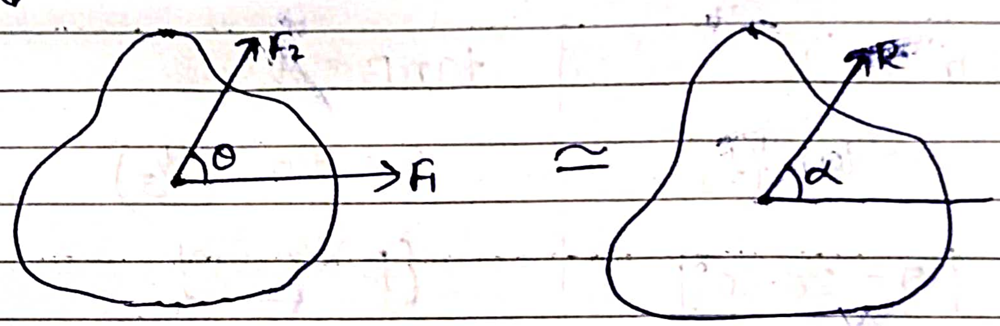


* same :-

1. Magnitude
2. Direction
3. Line of action

(6) Law of parallelogram of forces : (Short Note + Numerical)





In $\Delta 153$

$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Q) Two forces equal to P & 2P resp. act on a particle when the first force is inc. by 120N & second force is doubled the dir. of resultant remains same. Determine the value of force P

Sol) $\tan\alpha = \frac{F_2\sin\theta}{F_1+F_2\cos\theta} = \frac{2P\sin\theta}{P+2P\cos\theta}$

$\tan\alpha = \frac{4P\sin\theta}{(P+120)+4P\cos\theta}$

$\frac{2P\sin\theta}{P+2P\cos\theta} = \frac{\overset{2}{\cancel{4}}P\sin\theta}{P+120+4P\cos\theta}$

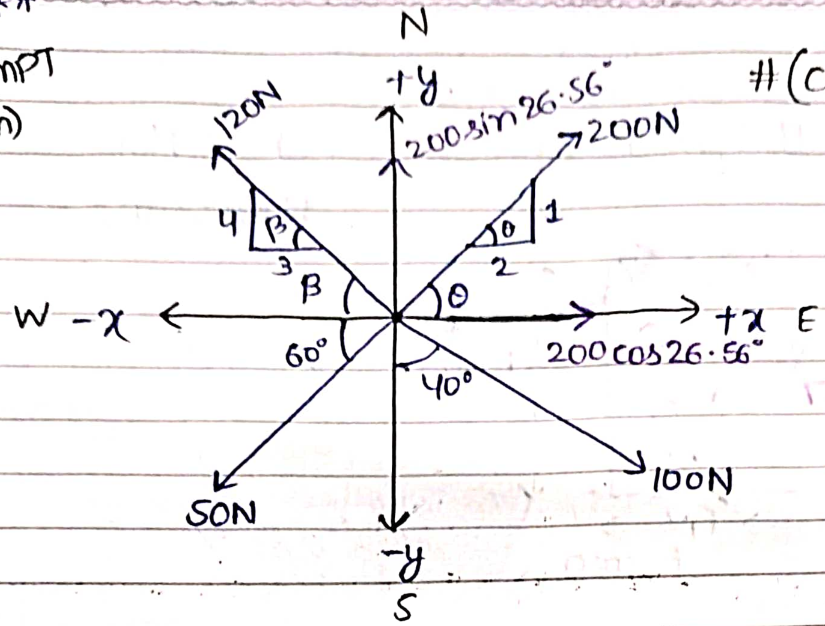
$P+120+4\cancel{P}\cos\theta = 2P+4\cancel{P}\cos\theta$

$\boxed{120 = P}$

$\boxed{P = 120\,N}$ ans

Q) ***
V.V. IMPT
(Exam)

(concurrent force system)



Resultant = ?
& its inclination from + x-axis

Sol)

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

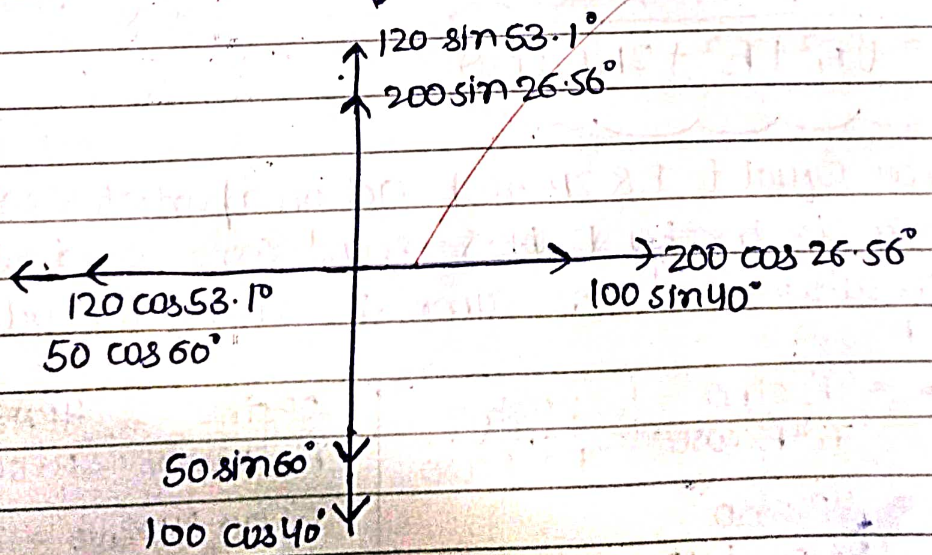
$\boxed{\theta = 26.56^\circ}$

$\tan\beta = \frac{4}{3}$

$\beta = \tan^{-1}\left(\frac{4}{3}\right)$

$\boxed{\beta = 53.1^\circ}$

FBD



$$\Sigma F_x = 200\cos 26.56^\circ = 178.89$$
$$100\sin 40^\circ = 64.27 \quad = 243.16$$

(for $+x$ axis)

$$= 120\cos 53.1^\circ = 72.05$$
$$= 50\cos 60^\circ = 25 \quad = 97.05$$

(for $-x$ axis)

$$\therefore \Sigma F_x = 243.16 - 97.05 = 146.11$$

$$\Sigma F_y = 120\sin 53.1^\circ = 95.96 \quad = 185.38 \text{ \{for +y-axis}$$
$$= 200\sin 26.56^\circ = 89.42$$

$$= 50\sin 60^\circ = 43.30 \quad = 119.9 \text{ \{for -y-axis)}$$
$$= 100\cos 40^\circ = 76.60$$

$$\therefore F_y = 185.38 - 119.9 = 65.48$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$R = \sqrt{(146.11)^2 + (65.48)^2}$$

$$R = \sqrt{21348.13 + 4287.63}$$

$$R = \sqrt{25698.13}$$

$$\boxed{R = 160.3\ N}$$

angle of R $\rightarrow$

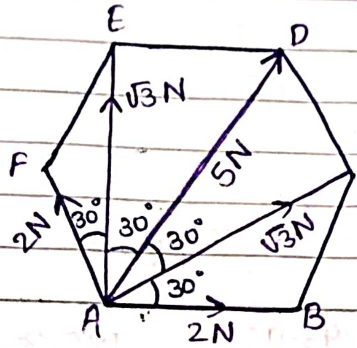
$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x}$$

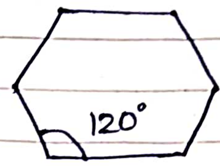
$$\tan\alpha = \frac{65.48}{146.11}$$

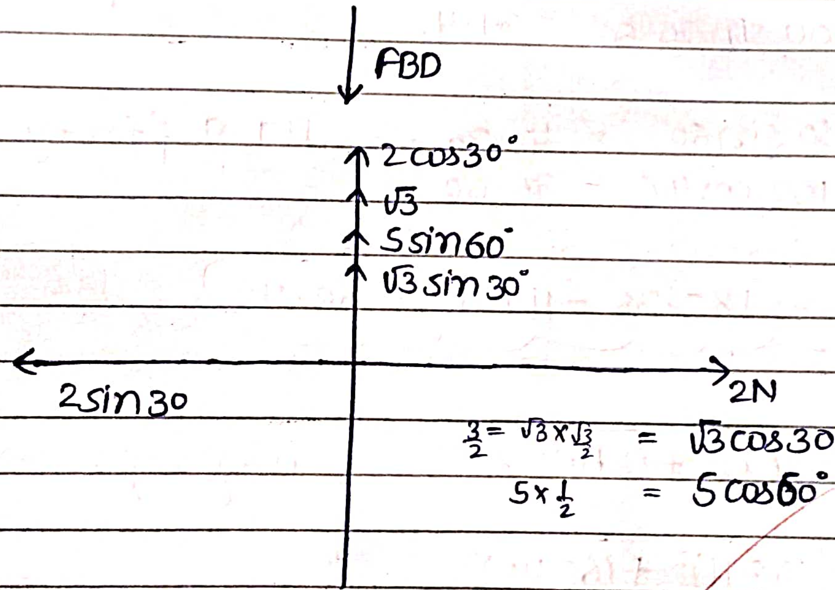
$$\boxed{\tan\alpha = 0.448}$$

Q) Find the resultant of forces 2, $\sqrt{3}$, 5, $\sqrt{3}$, 2 that act at a angular point of a regular hexagon towards the other angular point taken in order.

Sol)







$\frac{3}{2} = \sqrt{3} \times \frac{\sqrt{3}}{2} = \sqrt{3}\cos 30$

$5 \times \frac{1}{2} = 5\cos 60°$ { 60 isliye $30° + 30°$ from + x axis)

$\Sigma F_x = 2 + \frac{3}{2} + \frac{5}{2} = 6$ (+ x-axis)

$= 2 \times \frac{1}{2} = 1$ (− x-axis)

$\Sigma F_x = 5N$

$\Sigma F_y = 5\sqrt{3}\,N$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\boxed{R = 10N}$

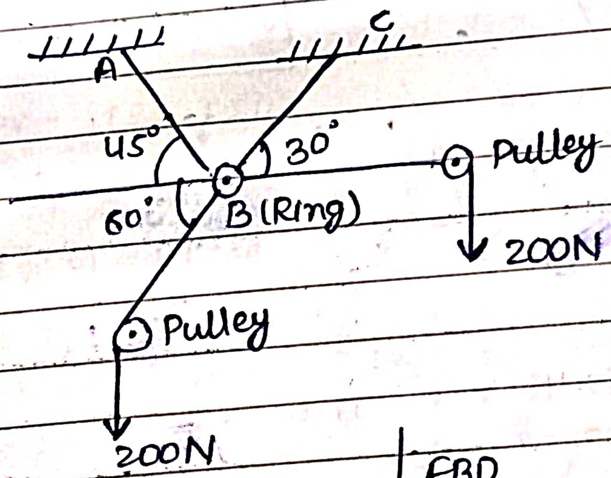
$\tan\alpha = \dfrac{\Sigma F_y}{\Sigma F_x}$

$= \dfrac{5\sqrt{3}}{5}$

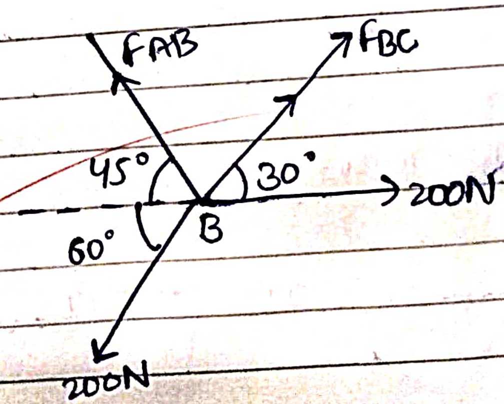
$\tan\alpha = \sqrt{3}$

$\boxed{\alpha = 60^\circ}$

Q) Calculate the tensile force in the cables AB & BC as shown in figure assume pullies are frictionless.

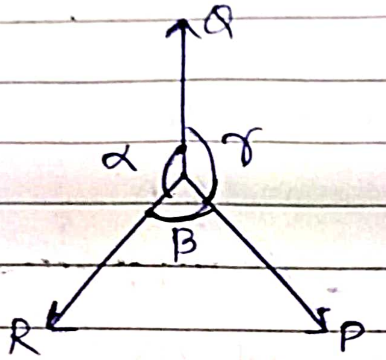


FBD



Practice Sheet -2

(7) Lami's Theorem :- If three co-planar concurrent forces be in equilibrium then each force is directly proportional to the sine angle b/w two other forces.



$P \propto \sin\alpha$ —— ①

$Q \propto \sin\beta$ —— ②

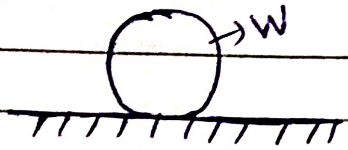
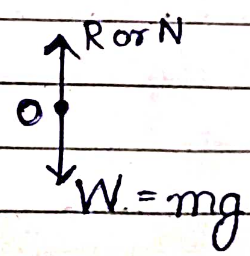
$R \propto \sin\gamma$ —— ③

from ①, ② & ③

$$\boxed{\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}}$$

) Free Body Diagram (F.B.D)

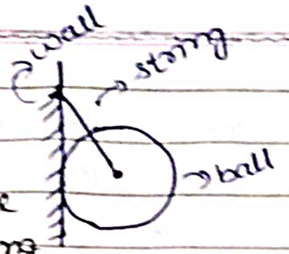
An isolated or separated view of a body with all external forces is called Free Body Diagram.

System	F.B.D
	

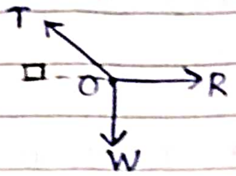
{R = Reaction Force
N = Normal}

FBD

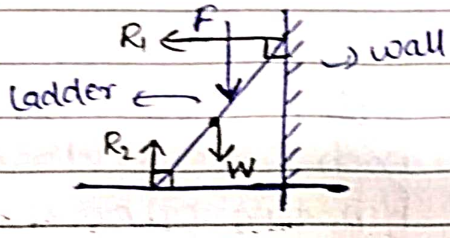
(2)

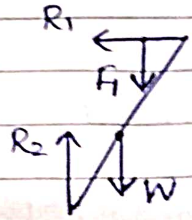


(Tension force hamesha string और wall के POC के dir. की तरफ lagega

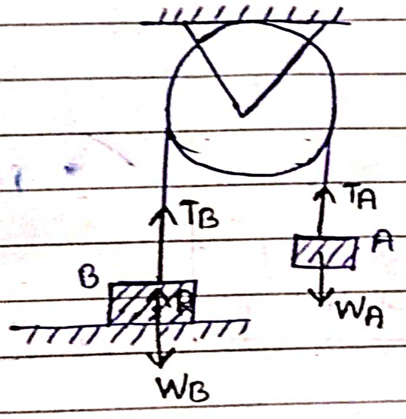


(3)

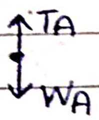




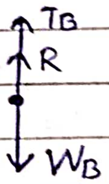
(4)



* for weight A:-



* for weight B:-



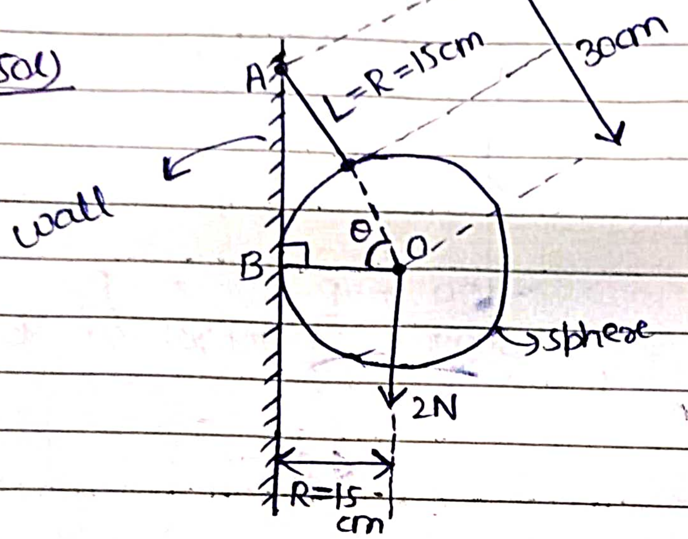
$$\frac{T_A}{T_B} = e^{\mu\theta}$$

{here $\mu = 0$}
coeff. of friction.

$$T_A = T_B$$

Q1) A smooth sphere of radius 15 cm & weight 2N is supported in contact with a smooth vertical ~~ball~~ wall by string whose length is equal to the radius of sphere. The string joins a point on the wall and a point on the surface of sphere, workout inclination and the tension in the string and the reaction on the wall.

Sol)

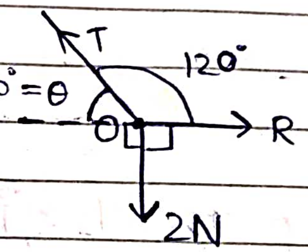


$\boxed{\mu = 0}$

To find :-

θ, T, R

* FBD



In ΔABO

AO = 30 cm

OB = 15 cm

$\cos\theta = \frac{OB}{OA} = \frac{15}{30} = \frac{1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$

$\boxed{\theta = 60°}$

Method 2

Apply Lami's Theorem

$\frac{T}{\sin 90°} = \frac{R}{\sin 150°} = \frac{2}{\sin 120°}$

T = ? R = ?

$\frac{T}{1} = \frac{2}{\sin 120°}$

$T = \frac{2 \times 2}{\sqrt{3}}$

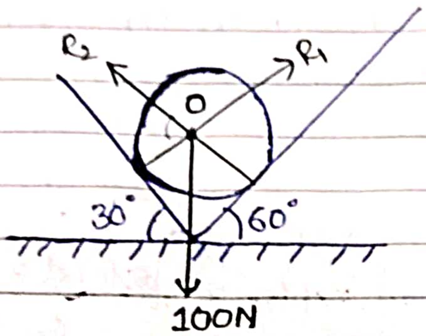
$\boxed{T = \frac{4}{\sqrt{3}}}$

$\boxed{T = 2.31\ N}$

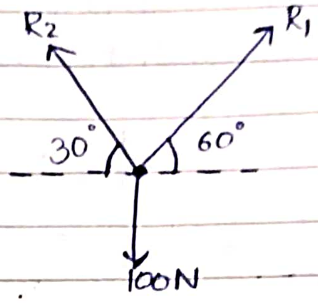
$\boxed{R = 1.15\ N}$

Q) A heavy spherical ball of weight 100N rest in a V shape trough whose sides are inclined at 30° & 60° from horizontal. Determine the pressure exerted on each side neglect friction

Sol)



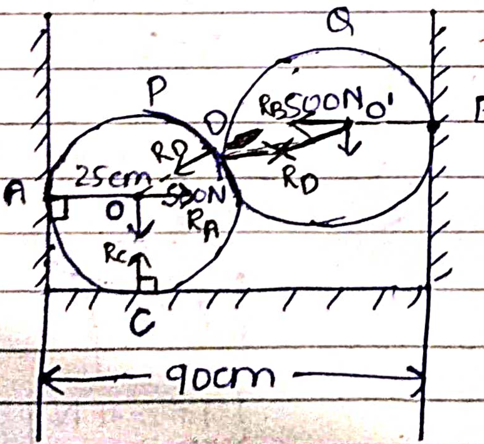
FBD ⇒



$$\frac{100}{\sin 90^\circ} = \frac{R_1}{\sin(90+30)} = \frac{R_2}{\sin(90+60)}$$

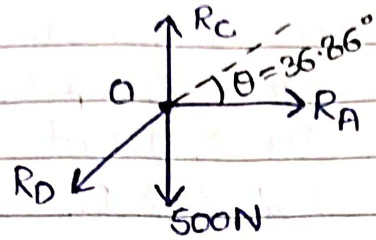
***

Q) Two smooth spheres P & Q each of radius 25 cm and weight → 500N rest in a channel having vertical wall as shown in fig. if the distance b/w the wall is 90cm. Make calculations for the pressure exerted on the wall & the floor at point of contact A, B, C

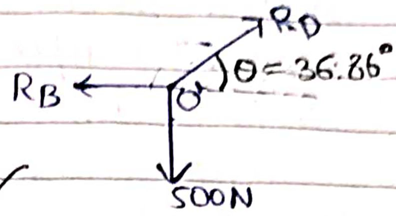


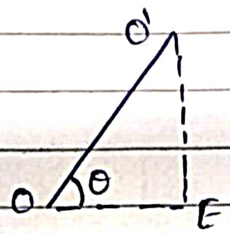
to find R_A, R_B & R_C

Sol) For sphere P



For sphere Q





$\boxed{OO' = 50cm}$

$OE = 90 - (25+25)$

$\boxed{OE = 40cm}$

$\cos\theta = \frac{OE}{OO'}$

$\cos\theta = \frac{40}{50}$

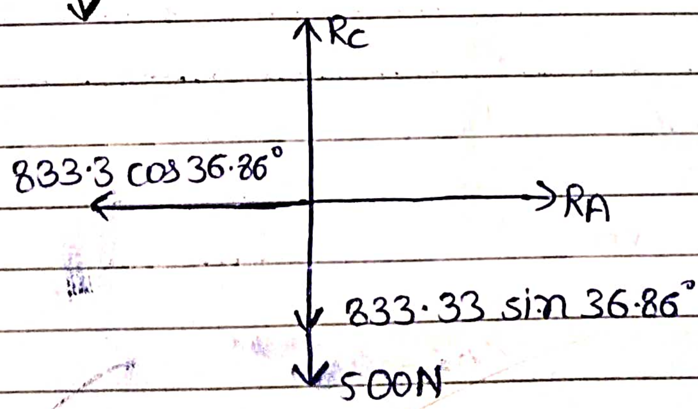
$\boxed{\theta = 36.86°}$

Apply Lami's Theorem

$\frac{500}{\sin 143.14} = \frac{R_B}{\sin(90+36.86)} = \frac{R_D}{\sin 90}$

$R_B = 666.66N$

$R_D = 833.33N$



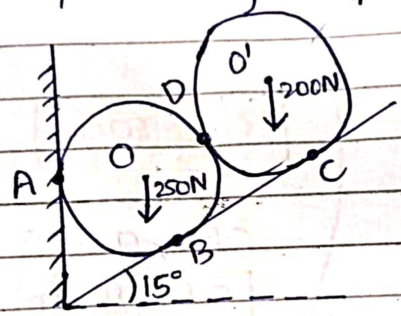
$\Sigma F_x = R_A = 833.33 \cos 36.86°$, $\Sigma F_y = R_C = 500 + 833.33 \sin 36.8$

$\boxed{R_A = 666.67 N}$ $\boxed{R_C = 1000N}$

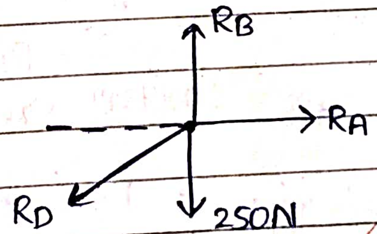
$R_A = 120.61\,N$, $R_B = 272.68\,N$, $R_3 = 193.18\,N$

$R_D = 51.76\,N$

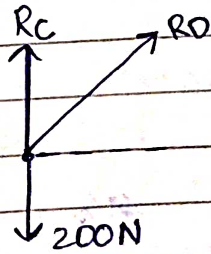
Q) Two rollers of same diameter are supported by an ~~two~~ inclined plane and vertical wall as shown in the fig. The upper and lower roller are 200 N and 250 N respectively in weight. Assuming smooth surface find the reaction induced at the points of support A, B, C & D.



Sol) for sphere 1

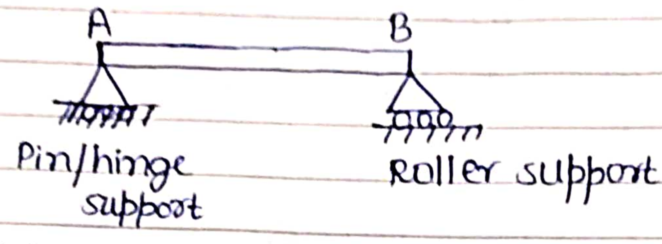


for sphere 2

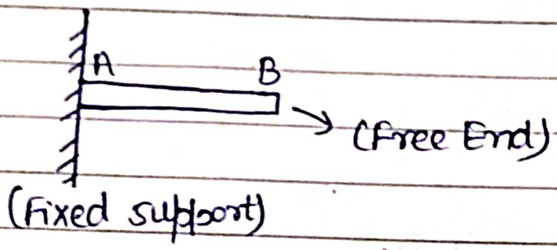


Types of Beam :- There are 5 types of Beams.

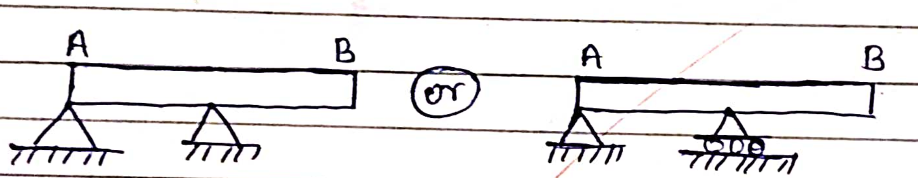
1) Simply Supported Beam



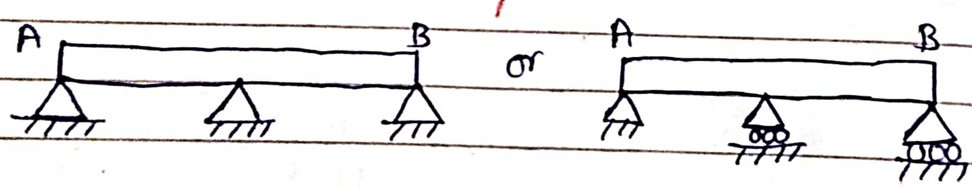
2) Cantilever beam :-



3) Overhanging Beam

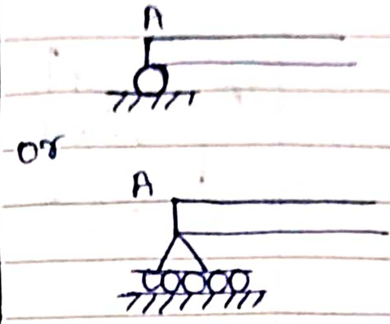
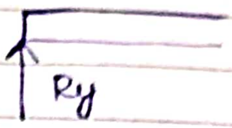
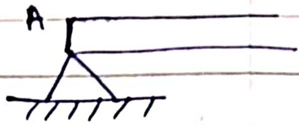
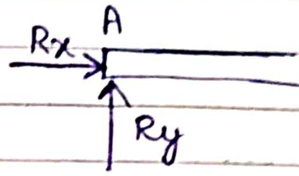
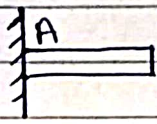
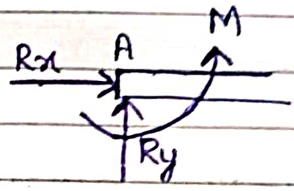


4) Continuous beam :-

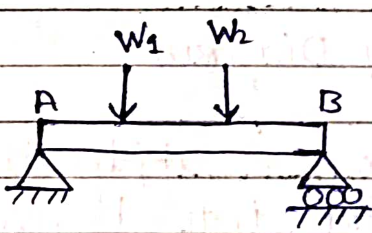
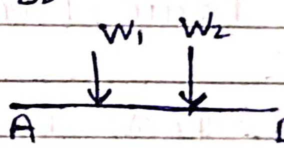
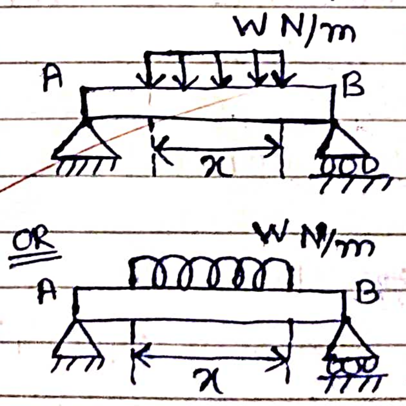
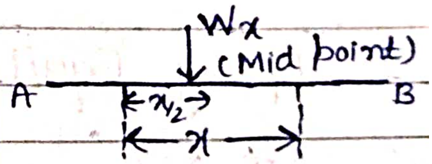


5) Fixed Beam :-

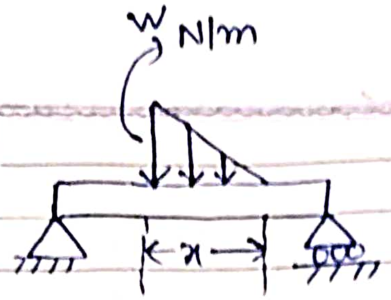
Types of support :-

S.No.	Support	Reactions	
1)	Roller support		
2)	Pin / Hinge support		
3)	Fixed support		 M → moment

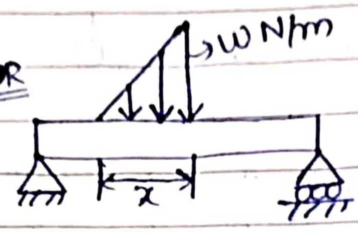
Types of Loads :-

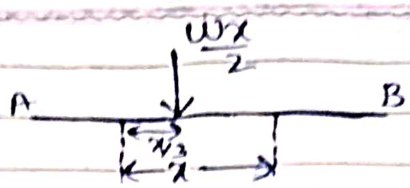
S.No	Load		FBD
1	Point or concentrated Load		
2	Uniformly distributed Load (U.D.L)		 $\boxed{\text{Load} = Wx \text{ N}}$

3) Uniformly Varying Load (U.V.L)

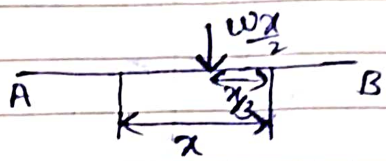


OR





$$\text{Load} = \frac{1}{2} \times x \times W$$
$$= \frac{wx}{2}$$



$$\boxed{\text{Load} = \frac{wx}{2}}$$

Load is always taken from Base which is at a distance of $x/3$

Steps to solve support Reaction

1) Draw Free body Diagram.

a) Remove all supports & apply reactions.
b) Convert all Loads into horizontal and vertical points Loads

2) Apply Equilibrium condition.

(i) $\Sigma F_x = 0$ —— (i)

(Forces along $+x$-axis $\rightarrow$ take '+')
(Forces along $-x$-axis $\rightarrow$ take '−')

(ii) $\Sigma F_y = 0$ — (ii)

(Force along '+' y-axis / upward (↑) → take '+')
(Forces along '-' y-axis / downward (↓) → take '-')

(iii) $\Sigma M = 0$ (about Left most support) — (iii)

↻M (clockwise → +

↺M (anticlockwise → −

3) Solve the equation obtained from equilibrium condition (i), (ii) & (iii) and get support reaction.

P.S-2

Q2) Two forces, one of which is double the other has resultant of 260N. if the dir. of the larger force is reversed and the other remains unaltered, the resultant reduces to 180N. Find the values of forces.

Sol) let the mag. of forces be F & 2F

Resultant = 260N ⇒ $\sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta} = 260$

$$F^2 + 4F^2 + 4F^2\cos\theta = (260)^2$$

$$5F^2 + 4F^2\cos\theta = 67600 \quad \text{— (1)}$$

condition-2

$$\sqrt{F_1^2 - 2F_1F_2\cos\theta + F_2^2} = 180$$

$$F^2 - 2F(2F)\cos\theta + (2F)^2 = 32400$$

$$5F^2 - 4F^2\cos\theta = 32400 \quad \text{— (2)}$$

Adding (1) & (2)

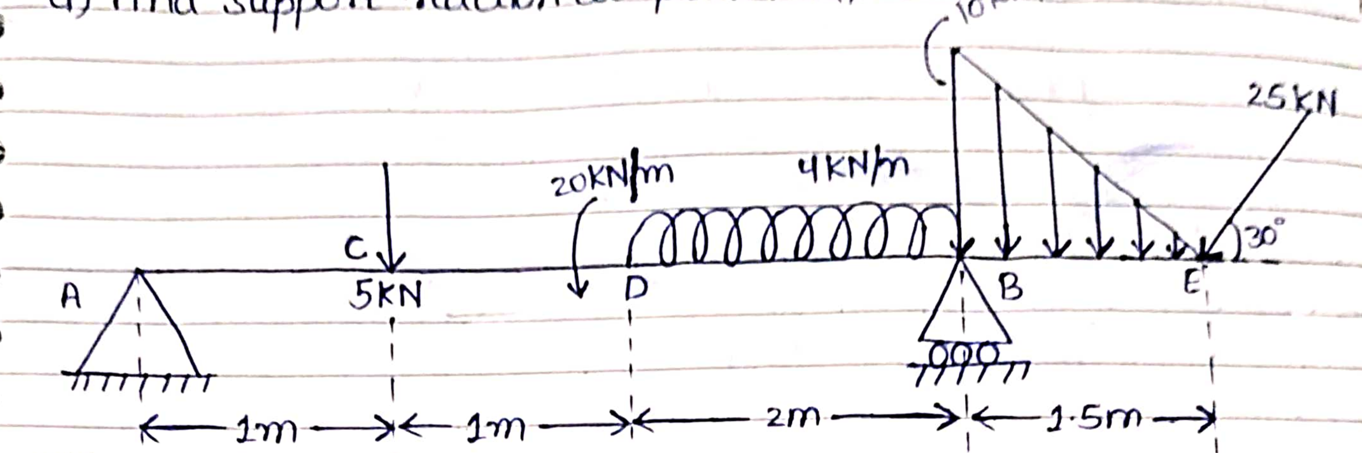
$$1\cancel{0}F^2 = 10000\cancel{0}$$

$$F^2 = 10000$$

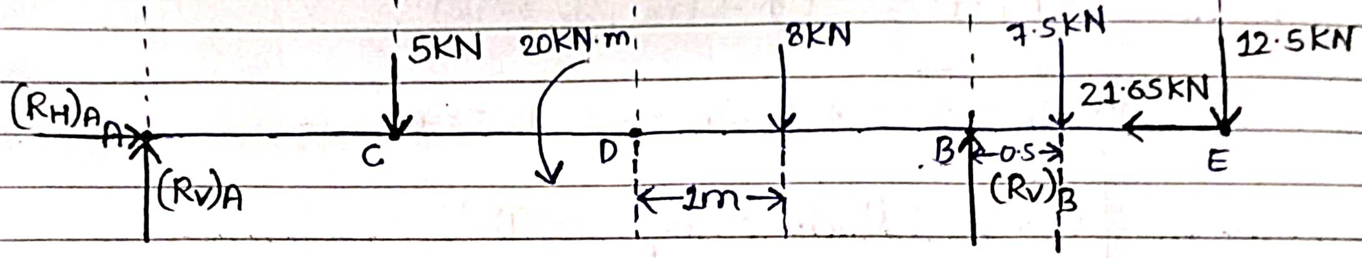
$$\boxed{F = 100N} \quad \boxed{F_1 = 100N}, \boxed{F_2 = 200N}$$

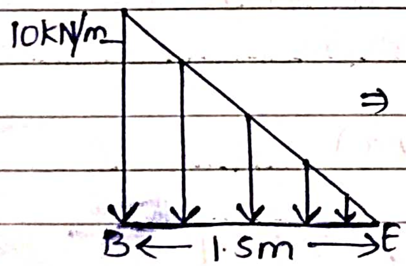
$\frac{1}{2} \times 1.5 \times 10$

Q) Find support reaction at point A & B



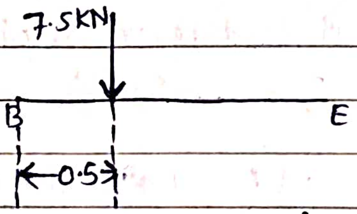
Sol)



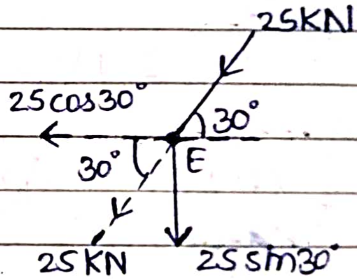


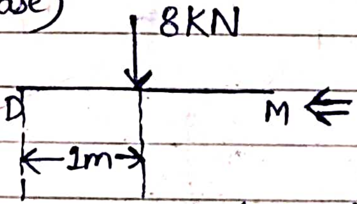
$= \frac{1}{2} \times 1.5 \times 10$

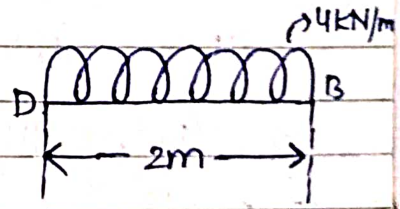
$= 7.5KN$



(0.5 isliye aaya kyuki $\frac{1}{3}$rd of base)







1m → 4KN

2m → 4×2 KN

→ 8KN

4K/m

4×2

* For the beam to be in Equilibrium :-

$\Sigma F_x = 0$ $\quad$ $(+ve\ x \to +)$ $\longrightarrow$ +ve

$(-ve\ x \to -)$ $\longleftarrow$ -ve

$(R_H)_A - 21.65 = 0$

$(R_H)_A = 21.65$ ans

$\Sigma F_y = 0$ $\quad$ $(\uparrow \to +ve, \downarrow \to -ve)$

$(R_V)_A - 5 - 8 + (R_V)_B - 7.5 - 12.5 = 0$

$\boxed{(R_V)_A + (R_V)_B = 33}$ —— ①

$\Sigma M_A = 0$ $\quad$ (about left most support → A)

$(\circlearrowright +, \circlearrowleft -)$

$(M = F \times \perp \text{ distance})$

$\{(R_H)_A \times 0 + (R_V)_A \times 0 + (5\times1) - (20) + 8\times3 - (R_V)_B \times 4 + (7.5\times4.5) + (12.5\times5.5) + (21.65\times0) = 0\}$

$\rightarrow 5 - 20 + 24 - ((R_V)_B \times 4) + 33.75 + 68.75 = 0$

$\rightarrow 111.50 = (R_V)_B \times 4$

$\rightarrow (R_V)_B = \frac{111.5}{4}$

$\rightarrow \boxed{(R_V)_B = 27.875}$

Now,

$(R_V)_A = 33 - 27.875$

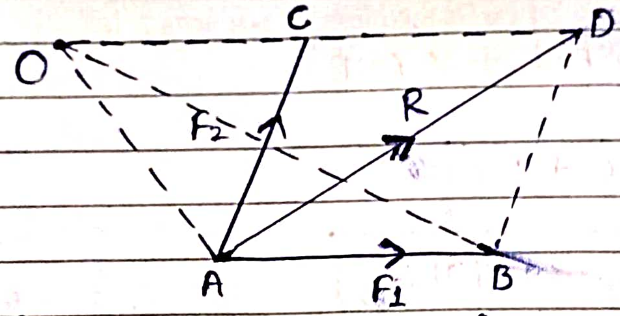
$\boxed{(R_V)_A = 5.125\,kN}$

Varignon's Theorem → The algebraic sum of moment due to individual forces is equal to the moment due to resultant force.

M due to R = M due to F_1 + M due to F_2 + - - - - -

$$M_R = \Sigma M_{F_1} + \Sigma M_{F_2}$$

$$M_R = \sum_{F=F_1}^{F_n} M$$



$$(M \text{ due to } F_1)_O + (M \text{ due to } F_2)_O = (M \text{ due to } R)_O$$

→ There are two forces F_1 (AB) and F_2 (AC). By Parallelogram Law, the resultant of F_1 & F_2 ~~is~~ R (AD). There is a point O in the extendend line CD.

Moment of force F_1 about O is = 2 × ar. of ΔAOB

Similarly

Moment of Force F_2 about O is $= 2 \times$ ar. of ΔAOC

Moment of Resultant force (R) $= 2 \times$ ar. of ΔAOD —①

But,

ar. of ΔAOD = ar. of ΔAOC + ar. of ΔACD

{ar. ΔACD = ar. ΔABD}
= ar. ΔAOB

ar. of ΔAOD = ar. of ΔAOC + ar. of AOB —②

Substitute ② in ①

$$(M_R)_O = 2(\Delta AOC + \Delta AOB)$$

$$(M_R)_O = 2\Delta AOC + 2\Delta AOB$$

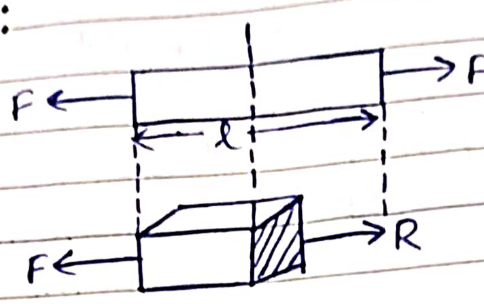
$$\boxed{(M_R)_O = (M_{F_2})_O + (M_{F_1})_O}$$

hence proved

Unit 1 (b)

Strength of Materials (S.O.M.)

* **Stress:**



R → Int. Resistive Force

$$\boxed{F = R}$$

$$\text{Stress} = \frac{\text{Internal Resistive Force}}{\text{cross-section Area}}$$

$$\sigma = \frac{R}{A}$$

but $R = F$

$$\sigma = \frac{F}{A}$$

σ = sigma

→ The internal Resistive Force develop in a body per unit area is termed as stress.

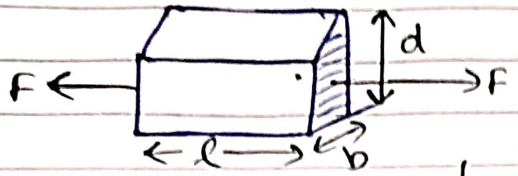
Unit → N/m^2, Pascal (Pa), KPa (kilo), MPa (mega)

$$\left(1\,MPa = \frac{1\,N}{mm^2} \right)$$

→ If tensile load is applied

(compressive load.)

Case 1)



$l\uparrow \quad b\downarrow \quad d\downarrow$

↳ Force kis dimension dirn me apply hua = l

$\sigma = \frac{F}{A}$

$$\boxed{\sigma = \frac{F}{b \times d}}$$

* Due to application of load the dimension of body will also change in such a way that length will increase width (b) and height (d) will decrease.

→ The ratio of change in length and original length is termed as Linear / Longitudinal strain. Whereas,

the ratio of change in width and original width or ratio of change in height and original height is termed as Lateral strain.

Longitudinal strain $(\varepsilon_l) = \frac{\delta l}{l}$
(along F)

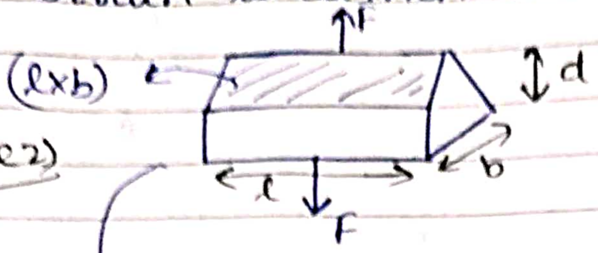
Lateral strain $(\varepsilon_b) = \frac{\delta b}{b}$

Lateral strain $(\varepsilon_d) = \frac{\delta d}{d}$

* Strain is dimension less Quantity (no-unit)

$(l \times b)$

case 2)



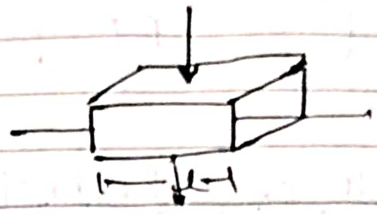
l and b will be same and d will increase

Force d dimension है lagi.

$$\text{Longi. Strain } (\varepsilon_d) = \frac{\delta d}{d}$$

$$\text{lateral Strain } (\varepsilon_l) = \frac{\delta l}{l}$$

$$\text{lateral Strain } (\varepsilon_b) = \frac{\delta b}{b}$$



$$\boxed{\sigma = \frac{F}{A} = \frac{F}{b \times l}}$$

* If compressive load is applied

C·L → $l\downarrow$, $b\uparrow$, $d\uparrow$

* If Tensile load is applied

T·L → $l\uparrow$, $b\downarrow$, $d\downarrow$

Impt. for short notes

Hooke's law :- In the elastic region, stress is directly proportional to the strain.

Stress ∝ Strain

$$\sigma \propto \varepsilon$$

$$\sigma = E\,\varepsilon$$

$$① \text{ — } \boxed{E = \frac{\sigma}{\varepsilon}}$$

σ = sigma

$\varepsilon \Rightarrow$ Epsilon

$\frac{\delta L}{L} = \varepsilon_l$

$\frac{\text{del}\,L}{L} = \varepsilon_l$

where,

E = Young's Modulus

OR

Modulus of Elasticity.

$\sigma = F/A$

$E = \frac{\sigma}{\varepsilon_l}$

$\sigma = \frac{F}{A}$ $\Rightarrow \frac{F\,l}{A\,\delta l}$

$\delta l = \frac{Fl}{AE}$

$\sigma = \frac{F}{A}$

$\varepsilon_l = \frac{\delta l}{l}$

$\boxed{\delta l = \frac{Fl}{AE}}$

$\sigma = \frac{F}{A}$

$\varepsilon_l = \frac{\delta l}{l}$

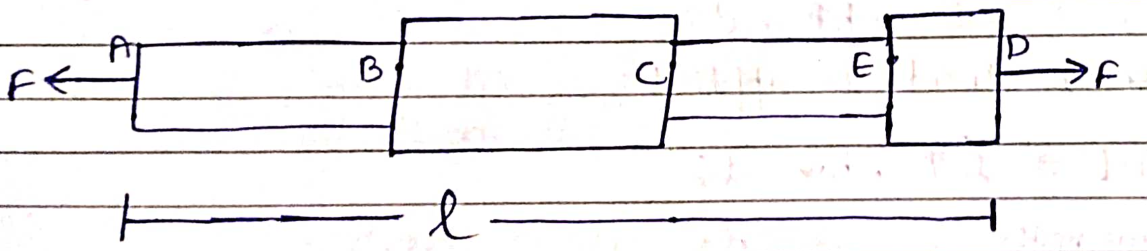
$E = \frac{\sigma}{\varepsilon_l} \Rightarrow \frac{Fl}{A\,\delta l}$

$\boxed{\delta l = \frac{Fl}{AE}}$

Principle of superposition :- The total elongation due to several loads acting on the body is the algebraic sum of elongations caused by individual loads.

(Imp. for Num.)

$\boxed{\delta l = \frac{Fl}{AE}}$



$$(\delta l) = (\delta l)_{AB} + (\delta l)_{BC} + (\delta l)_{CE} + (\delta l)_{ED}$$

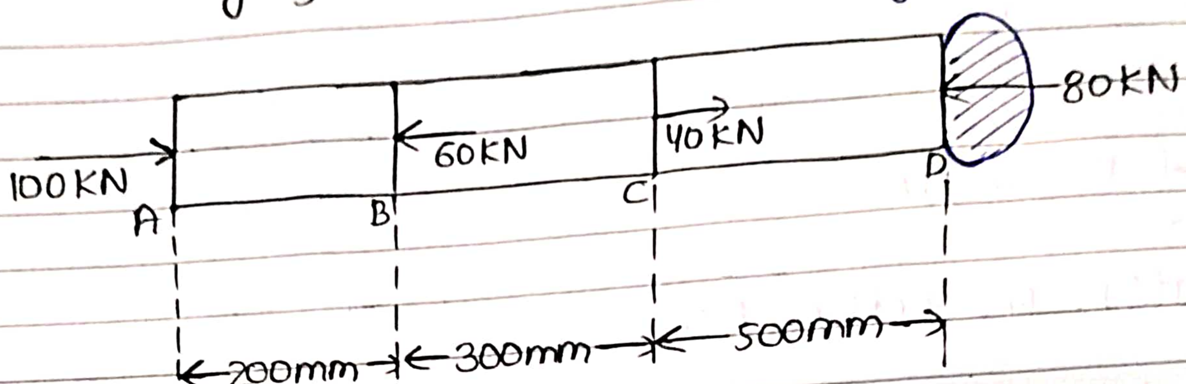
$$\boxed{\delta l = \sum_{A}^{D} \delta l}$$

$1000mm^2 \Rightarrow 10^{-3}$

$1m \Rightarrow 100$

Q1) A member A, B, C & D. of uniform Diameter of 200mm is subjected to point loads as shown in Fig. Determine the net change in the length of bar. Take Modulus of elasticity of the bar as $E = 200\ GN/m^2$

↓ Giga Newton



$r = 2d$

$d = \frac{r}{2}$

Sol Given:-

$E = 200\ GN/m^2$

$= (200 \times 10^9)\ N/m^2$

$1m = 1000mm \Rightarrow 1mm \Rightarrow \frac{1}{1000} m$

$\rightarrow d = 200mm$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (200)^2 = 31400\ mm^2$ ~~0.000314~~ $= 0.0314\ m^2$

$\frac{125600}{4} = 3140$

$1m = 1000mm$

$1m^2 = 1000000$

$1mm^2 = \frac{1}{100000} m$

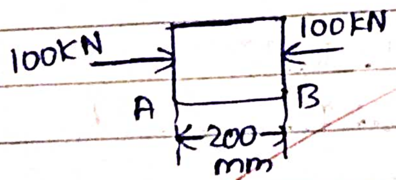
To find $\delta l_{AD} = ?$

As per Principle of superposition.

$\delta l_{AD} = \delta l_{AB} + \delta l_{BC} + \delta l_{CD}$

Tensile → +

compr. → −



$\delta l_{AB} = \frac{F_{AB}\, l_{AB}}{AE}$

$F_{AB} = 100KN = 100 \times 10^3\ N$

$\delta l_{AB} = -\frac{(100 \times 10^3 \times 0.2)}{0.0314 \times 200 \times 10^9} = -\frac{(10^{5-9} \times 0.2)}{0.0314 \times 2 \times 1}$

$= -\frac{(10^{-4-2} \times 0.2)}{0.0314 \times 2}$

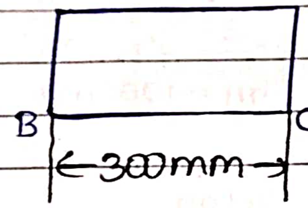
$$\delta l_{AB} = -\frac{(10^{-6} \times 0.2)}{0.0314 \times 2}$$

$$\delta l_{AB} = -\frac{(0.2 \times 10^{-6})}{2 \times 0.0314 \times \cancel{10^{-6}}}$$

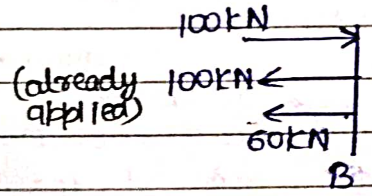
$$\delta l_{AB} = -\frac{(0.2 \times 10^{-6})}{0.0628}$$

$$\boxed{\delta l_{AB} = -(3.18 \times 10^{-6})\, m}$$

- Elongation in section BC

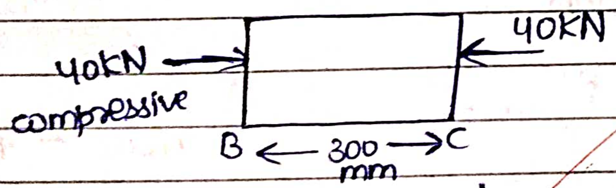


$$(\delta l)_{BC} = \frac{F_{BC}\, l_{BC}}{AE}$$



$$(\delta l)_{BC} = \frac{F_{BC} \times l}{AE}$$

$$= -\frac{(40 \times 10^{3}) \times 0.3}{0.0314 \times 200 \times 10^{9}}$$

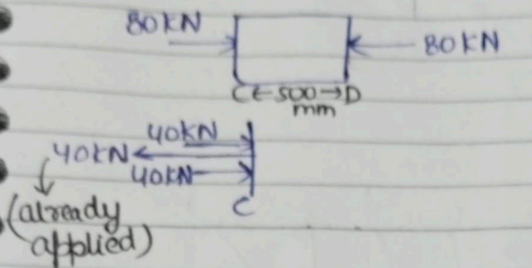


$$= -\frac{(4 \times 10^{4-11}) \times 3 \times 10^{-1}}{0.0314 \times 2}$$

$$= -\frac{4 \times 10^{-7+(-1)}}{0.0628}$$

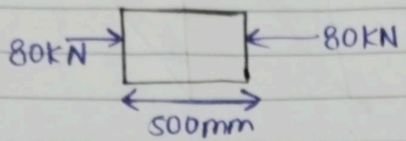
$$= -1.92 \times 10^{-6}\, m$$

· Elongation in section CD



$$\delta l_{CD} = \frac{F_{CD}\, l_{CD}}{AE}$$

$$= -\frac{(80\times10^{3})\times 0.5}{0.0314\times 200\times10^{9}}$$



$$= -\frac{(80\times10^{3})\times 0.5}{0.0314\times200\times10^{9}} \qquad = -40000$$

$$= -(6.36\times10^{-6})\,m$$

$$\delta l_{AD} = -(3.18 + 1.92 + 6.36)\times10^{-6}\,m$$

$$\boxed{\delta l_{AD} = -11.46\times10^{-6}\,m} \quad \text{ans}$$

* <u>Poisson's Ratio</u> (μ):- It is ratio of lateral strain to Longitudenal strain

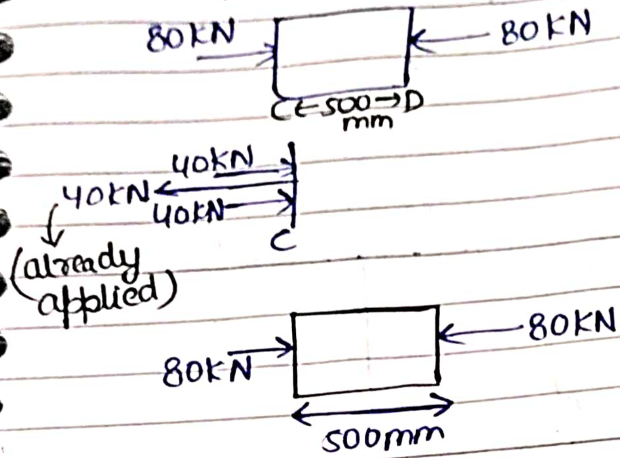
$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

For engg. materials $\longrightarrow$ $\mu = 0.25 - 0.33$

* Shear Stress and Shear Strain :-

Shear stress is the ratio of tangential force per un cross section area

- Elongation in section CD



$$\delta l_{CD} = \frac{F_{CD}\, l_{CD}}{AE}$$

$$= -\frac{(80\times10^3)\times 0.500}{0.0314\times 200\times10^9}$$

$$= -\frac{(80\times10^3)\times 0.500}{0.0314\times 200\times 10^9} \qquad = -40000$$

$$= -(6.36\times10^{-6})\,m$$

$$\delta l_{AD} = -(3.18 + 1.92 + 6.36)\times10^{-6}\, m$$

$$\boxed{\delta l_{AD} = -11.46\times10^{-6}\, m}$$ ans

* **Poisson's Ratio (μ)**:- It is ratio of lateral strain to longitudinal strain

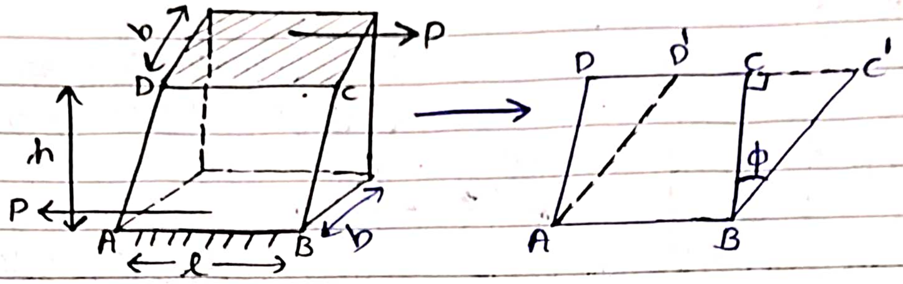
$$\mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

For engg. materials $\longrightarrow \mu = 0.25 - 0.33$

* **Shear Stress and Shear Strain**:-

Shear stress is the ratio of tangential force per unit cross section area.

Tou $\leftarrow \boxed{\tau = \dfrac{\text{Tangential force}}{\text{surface area}} = \dfrac{P}{l \times b}}$



* Shear strain :-

$$\text{shear strain}(\phi) = \tan\phi$$

$$\boxed{\phi = \frac{CC'}{BC}}$$

under elastic limit

$$\tau \propto \phi$$

$$\tau = G\phi$$

$$\boxed{G = \frac{\tau}{\phi}}$$

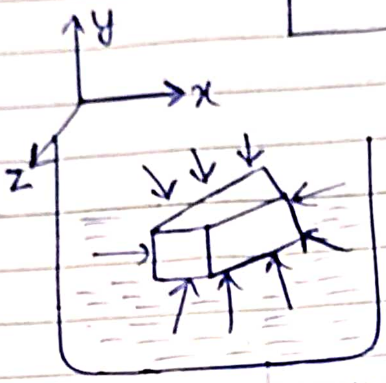
modulus of rigidity

Hydrostatic stress & Volumetric strain :-

Hydrostatic stress causes change in volume of the body and this change in volume per unit

Volume is called as Volumetric Strain

$$\boxed{\varepsilon_v = \frac{\delta V}{V}}$$



Consider cuboid elements of dimensions x, y, z it is fully immersed into the water it is subjected to equal external pressure at all the points of the body, this external pressure is called as hydrostatic stress

The initial volume of element is x, y, z after application of pressure the sides of cuboid changes to $(x+\delta x), (y+\delta y), (z+\delta z)$

$$V_1 = xyz$$
$$V_2 = (x+\delta x)(y+\delta y)(z+\delta z)$$

$$\varepsilon_V = \frac{V_2 - V_1}{V_1}$$

$$\varepsilon_V = \frac{(x+\delta x)(y+\delta y)(z+\delta z) - xyz}{xyz}$$

$$\varepsilon_V = \frac{yz\,dx + xz\,dy + xy\,dz}{xyz}$$

$$\varepsilon_V = \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z}$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

Volumetric strain is the summation of linear normal strain x, y, z direction. Under elastic limit hydrostatic stress is directly proportional to volumetric strain.

Hydro. stress $\propto$ Volumetric strain

$$\sigma_V \propto \epsilon_V$$

$$\sigma_V = K\epsilon_V$$

$$\boxed{K = \frac{\sigma_V}{\epsilon_V}}$$

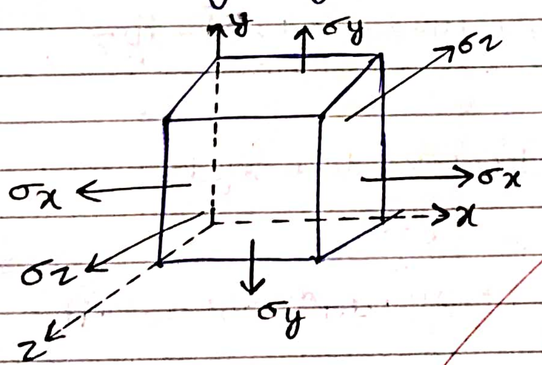
Bulk Modulus

$\boxed{E = \frac{\sigma}{\epsilon}}$, $\boxed{G = \frac{\tau}{\phi}}$, $\boxed{K = \frac{\sigma_V}{\epsilon_V}}$

Relationship b/w elastic constant (E, G & K) (Theory + Numericals)

*(i) Relationship b/w E, K & μ

ST1) Consider a cubic element which is subject to volumetric stress along x, y & z directions.



lateral strain → strain along perp. of axis of force.

$$\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

Longitudinal strain = strain along axis of force.

Strain in x-direction =

→ Strain in x-direction due to σ_x

→ Strain in x-direction due to σ_y (−)

→ Strain in x-direction due to σ_z (−)

= lateral strain

$\sigma v = \varepsilon v$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

where

$$\sigma_x = \sigma_y = \sigma_z = \sigma \qquad \frac{\sigma}{E} - \frac{2\mu\sigma}{E}$$

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) \qquad \frac{\sigma}{E}(1-2\mu)$$

Similarly

$$\varepsilon_y = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_z = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

$$K = \frac{\sigma V}{\varepsilon V} \qquad (\sigma V = \sigma)$$

$$K = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$\boxed{E = 3K(1-2\mu)} \text{ ---- (A)}$$

(ii) Relationship b/w E, G, & μ

(x)

→ Consider a cubic element fixed at bottom. Shear stress (τ) is applied on the top surface in this cube as shown in figure.

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\mu\sigma_y}{E} - \frac{\mu\sigma_z}{E}$$

where

$$\sigma_x = \sigma_y = \sigma_z = \sigma \qquad \frac{\sigma}{E} - \frac{2\mu\sigma}{E}$$

$$\varepsilon_x = \frac{\sigma}{E}(1-2\mu) \qquad \frac{\sigma}{E}(1-2\mu)$$

Similarly

$$\varepsilon_y = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_z = \frac{\sigma}{E}(1-2\mu)$$

$$\varepsilon_V = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$\varepsilon_V = \frac{3\sigma}{E}(1-2\mu)$$

$$K = \frac{\sigma_V}{\varepsilon_V} \qquad (\sigma_V = \sigma)$$

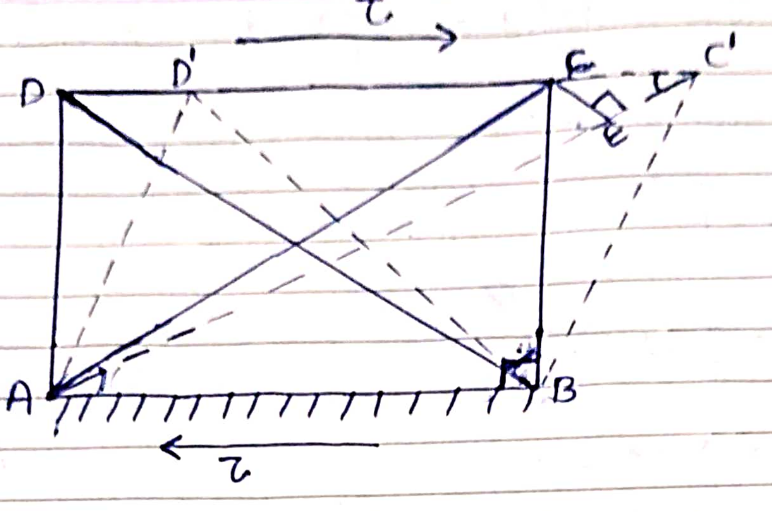
$$K = \frac{\sigma}{\frac{3\sigma}{E}(1-2\mu)}$$

$$\boxed{E = 3K(1-2\mu)} \text{ ---- } (A)$$

(ii) Relationship b/w E, G, & μ

(x)

→ Consider a cubic element fixed at bottom. Shear stress (τ) is applied on the top surface in this cube as shown in figure.



AC' > AC (tensile loading)
BD' < BD (compressive Loading)

Strain in diagonal AC = Strain in due to tensile stress in AC − Strain in due to comp. stress in BD

$$= \frac{\tau}{E} - \left(\frac{-\mu\tau}{E}\right)$$

$$= \frac{\tau}{E}(1+\mu) \quad \text{——— (1)}$$

Also,

Strain in diagonal ~~BD~~ AC $= \dfrac{AC' - AC}{AC}$ $\left(\dfrac{\text{change in dim.}}{\text{original dim.}}\right)$

$$= \frac{AC' - AE}{AC} \quad (\because AC = AE)$$

$$= \frac{EC'}{AC} \quad \text{——— (2)}$$

$\because$ CC' is very small (in $\Delta ECC'$)

$$\angle EC'C = \angle ECC' = 45^\circ$$

$$EC' = CC' \cos 45^\circ$$

$$EC' = \frac{CC'}{\sqrt{2}} \quad \text{——— (3)}$$

In ΔACB

$\angle ABC = 90^\circ$ (since it is cube)

$AB = BC$

$$\frac{BC}{AC} = \cos 45^\circ$$

$$AC = \sqrt{2}BC \quad \text{——— (4)}$$

Substituting EC' and AC from eq^n (3) & (4) in eq^n (2)

$$\text{Strain in diagonal } AC = \frac{CC'}{\sqrt{2}\sqrt{2}BC}$$

$$= \frac{CC'}{2BC} \quad \text{——— (5)}$$

Also,

Strain -

$$\phi = \tan\phi = \frac{CC'}{BC}$$

$$\phi = \frac{CC'}{BC} \quad \text{——— (6)}$$

Substituting value of $\frac{CC'}{BC}$ from eq^n (6) to eq^n (5)

Strain in AC = $\frac{\phi}{2}$ —— (7)

Also,

$$\tau \propto \phi$$
$$\tau = G\phi$$
$$\phi = \frac{\tau}{G} \quad \text{—— (8)}$$

Substitute the value of ϕ from eq^n 8 to eq^n (7)

$$= \frac{\tau}{2G} \quad \text{—— (9)}$$

Equating eq^n (1) & (9)

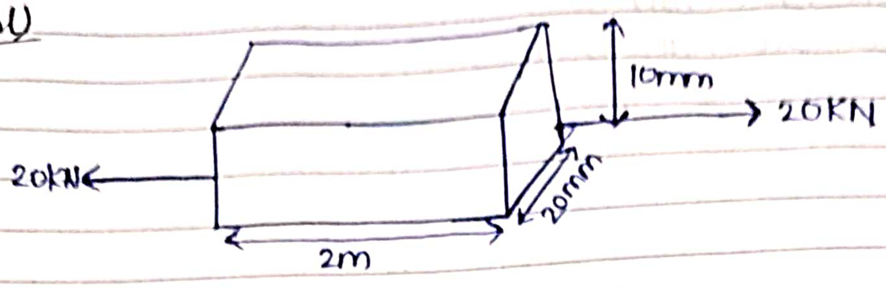
$$\frac{\tau}{2G} = \frac{\tau}{E}(1+\mu)$$

$$\boxed{E = 2G(1+\mu)} \quad \text{—— (B)}$$

21/11/23

Q) A steel bar 2m long and 20mm x 10mm in cross-section is subjected to a tensile load of 20 KN along its longitudinal axis. Make calculations for changes in length, width & thickness of the bar stating whether it is increased or decreased. Take Modulus of ~~ric~~ elasticity as 2×10^5 N/mm^2 and poisson's ratio as 0.3

Sol)



$l = 2m$, $b = 20mm$, $t = 10mm$
$= 2\times10^3 mm$

$P = 20KN$ $\quad E = 2\times10^5 N/mm^2$
$= 20\times10^3 N$ $\quad \mu = 0.3$

$\delta l, \delta b, \delta t = ?$ also Nature

$\sigma = \frac{P}{A}$ $\quad E = \frac{\sigma}{\varepsilon_l}$ $\quad \left(\varepsilon_l = \frac{\delta l}{l}\right)$

$\sigma = \frac{P}{b\times t}$ $\quad E = \frac{\sigma l}{\delta l}$ $\quad \delta l = \frac{PL}{AE}$

$\sigma = \frac{20\times10^3}{20\times10}$ $\quad \delta l = \frac{\sigma l}{E}$

$\sigma = 100 N/mm^2$ $\quad \delta l = \frac{100\times\cancel{2}\times10^3}{\cancel{2}\times10^5} = \frac{10^5}{10^5} = 1mm$ (increasing)

$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ $\quad (\mu < 1)$

Lateral Strain $= \frac{\delta b}{b}$

Longitudinal strain $= \frac{\delta l}{l} = \frac{1}{2\times10^3}$

$0.3 = \frac{\delta b \times 2\times10^{3}}{20}$

Now, $\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$

$$0.3 = \frac{\frac{\delta b}{20}}{\frac{1}{2\times10^{3}}}$$

$$\boxed{\delta b = 3\times10^{-3}\,mm\ (\text{decrease})}$$ ans

Now,

lateral strain $= \frac{\delta t}{t}$

Long. $= \frac{1}{2\times10^{3}}$

$$0.3 = \frac{\frac{\delta t}{10}}{\frac{1}{2\times10^{3}}} \Rightarrow 0.3 = \frac{\delta t \times 2\times10^{3}}{10}$$

$$\Rightarrow \frac{0.3}{200} = \delta t$$

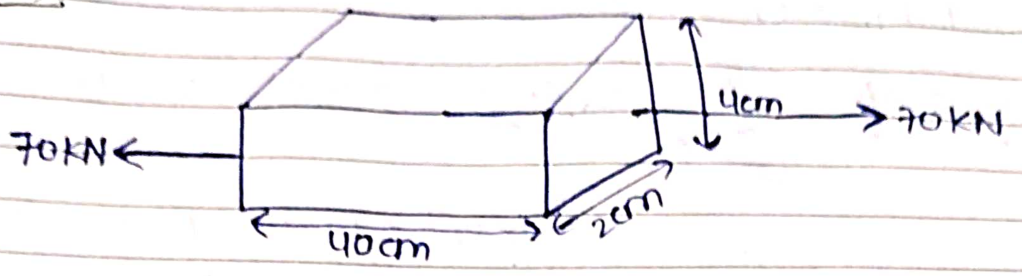
$$\boxed{\delta t = 1.5\times10^{-3}\,mm\ (\text{decrease})}$$ ans

*** IMPT

Q2) A bar 2cm x 4cm in cross section & 40cm Long is subjected to a tensile Load of 70kN. It is found that the length increases by 0.176 mm and lateral dimension of 4cm decreases by 0.0044 mm Find

(i) Young's Modulus (E)
(ii) Poisson's ratio (μ)
(iii) Change in volume of the bar (δV) (e_v) (δ_v)
(iv) Bulk Modulus (K)

Sol)



$l = 40\text{ cm} = 400\text{mm}$
$b = 2\text{ cm} = 20\text{mm}$
$t = 4\text{ cm} = 40\text{mm}$

$E = ?$
$\mu = ?$
$\delta V = ?$
$K = ?$

$P = 70\text{KN}$
$= 70\times10^3\text{N}$

$\delta l = 0.176\text{mm} (\uparrow)$
$\delta t = 0.0044\text{ mm} (\downarrow)$

$\sigma = \frac{P}{A}$

$= \frac{70\times10^3}{20\times40}$

$= \frac{7\times10^2}{8}$

$\boxed{\sigma = 87.5}$

$E = \frac{\sigma}{\varepsilon l}$ $\left(\varepsilon l = \frac{\delta l}{l}\right)$

$E = \frac{\sigma l}{\delta l}$

$\cancel{\delta l = \frac{\sigma l}{E}}$

$E = \frac{87.5\times400}{0.176} =$ ~~19 800 N~~/mm²
$= 198863.63$

$\mu = \frac{\text{lateral Strain}}{\text{Longitudinal strain}}$

lateral strain $= \frac{\delta b}{b}$ or $\frac{\delta t}{t}$ ✓

X

$\therefore \mu = \frac{\frac{\delta t}{t}}{\frac{\delta l}{l}} = \frac{\frac{0.0044}{40}}{\frac{0.176}{400}} = 0.25$ ans

$\varepsilon_v = \varepsilon_l + \varepsilon_b + \varepsilon_t$

$= \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta t}{t}$ $= \frac{0.176}{400} + \frac{2.2\times10^{-3}}{20} + \frac{0.0044}{40}$

$= 4.4\times10^{-4} + 1.1\times10^{-4} + 1.1\times10^{-4}$

$= (4.4 + 1.1 + 1.1)\times10^{-4}$

$\boxed{\varepsilon_v = 6.6\times10^{-4}}$

$\mu = \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}}$

$0.25 = \frac{\frac{\delta b}{20}}{\frac{0.176}{400}}$

$0.25 = \frac{400\times\delta b}{20\times0.176}$

$\frac{0.88}{400} = \delta b$

$\boxed{2.2\times10^{-3} = \delta b}$

Now,

$\varepsilon_v = \frac{\delta V}{V}$

$\delta V = \varepsilon_v \times V$

$= 6.6\times10^{-4} \times (l\times b\times t)$

$= 6.6\times10^{-4}\times(400\times20\times40)$

$= 2112000\times10^{-4}$ mm³

$= 211.2$ mm³

$\boxed{\delta V = 0.2112\,cm^3}$ ans

1 cm = 10 mm

1 mm = $\frac{1}{10}$ cm

1 cm² = 100 mm²

1 cm³ = 1000 mm³

$\frac{2112000}{10000}$

Now, for K

$E = 3K(1-2\mu)$

$19.866 = 3K(1-2\times0.25)$

$19.866 = 3K(1-0.5)$

$19.866 = 3K(0.5)$

$\frac{19.866}{1.5} = K$

$13.244 = K$

$\boxed{K = 13.257\times10^4\ N/mm^2}$ ans

* **Important**

$$E = 3K(1-2\mu) \quad \text{——— (A)}$$

$$E = 2G(1+\mu) \quad \text{——— (B)}$$

Eliminate μ from eq^n A & B

$$\mu = \frac{1}{2}\left(1-\frac{E}{3K}\right) \quad \text{——— (1)}$$

$$\mu = \left(\frac{E}{2G} - 1\right) \quad \text{——— (2)}$$

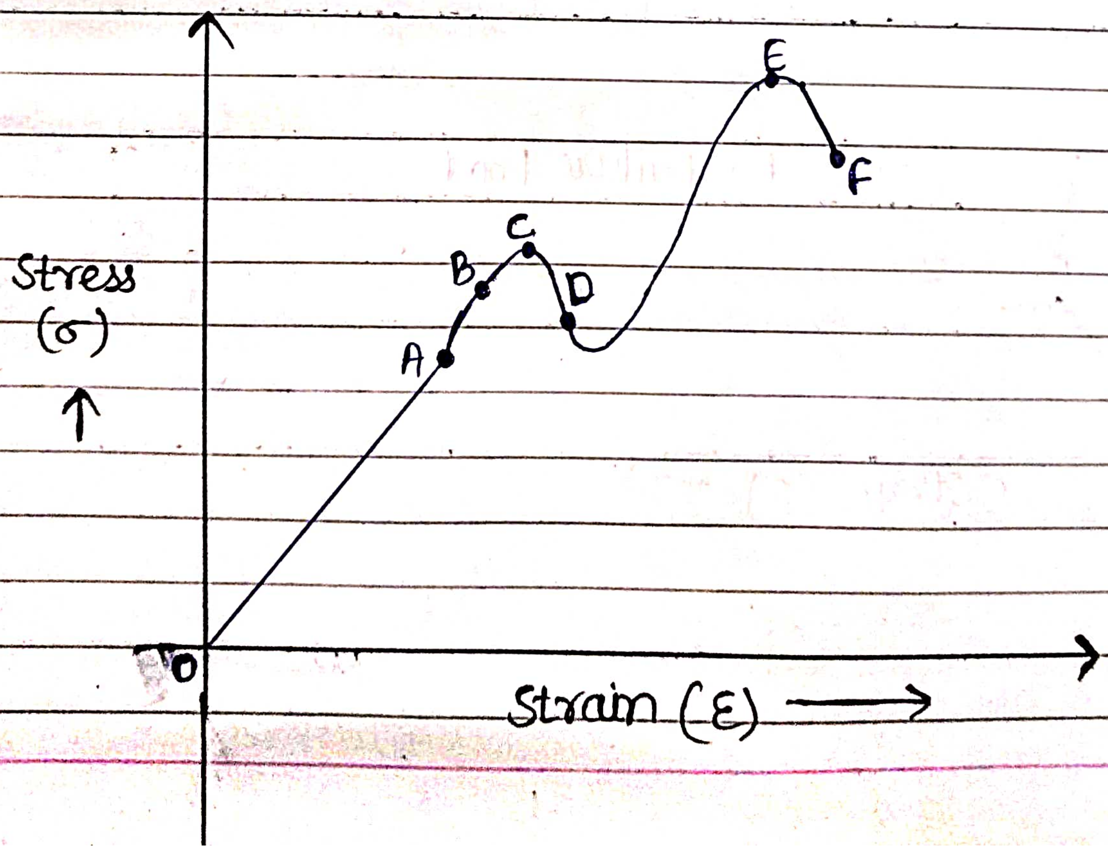
equating (1) & (2)

$$\frac{1}{2}\left(1-\frac{E}{3K}\right) = \frac{E}{2G} - 1$$

$$\boxed{E = \frac{9GK}{(G+3K)}}$$ hence proved

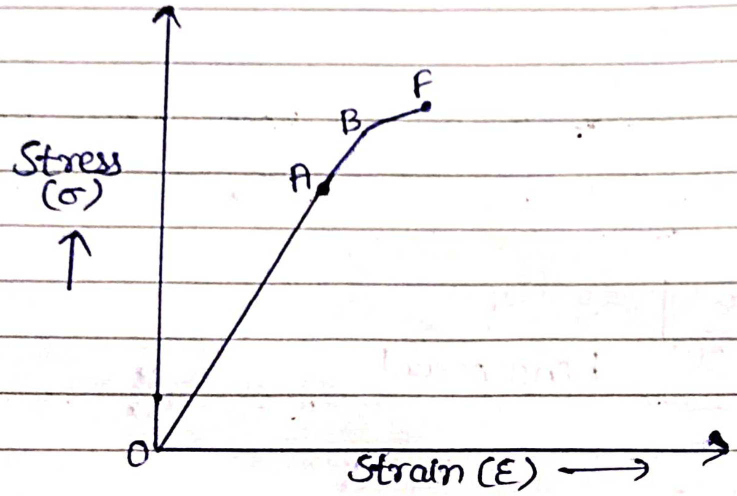
Stress- Strain diagram :-

(1) For mild steel

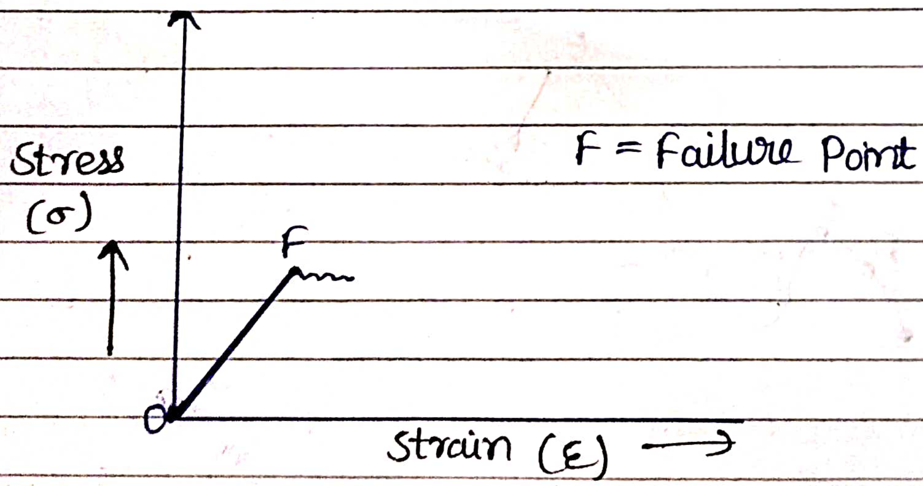


OA = Proportionality limit
OB = Elastic limit
C = Upper yield Point
D = Lower yield Point
E = Ultimate stress Point
F = Failure / Breaking Point

2) For Ductile Materials (Al, Cu etc)



3) For Brittle materials (cast. Iron etc)



Practice Sheet-1

Q) Write short note on Principle of transmissibility of forces.

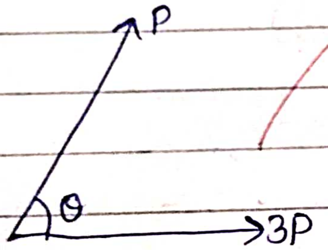
→ The principle of transmissibility states that the point of application of a force can be moved anywhere along its line of action without changing the external reaction forces on a rigid body.

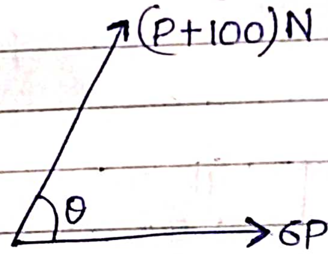
* Conditions to be same:-

(i) Magnitude (ii) Direction (iii) Line of Action.

Q2) Two forces equal to P & 3P respectively act on a particle. When the first force is increased by 100N & the second force is doubled, the dir. of resultant remains the same. Determine the value of P.

Sol)





θ = same {both the cases}

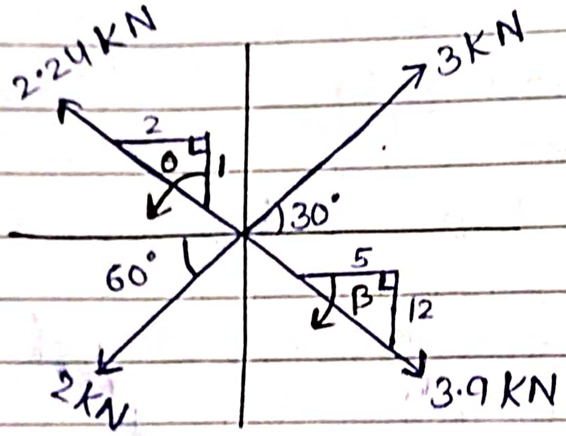
$$\tan\theta = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\Rightarrow \frac{3P\sin\theta}{P + 3P\cos\theta} = \frac{6P\sin\theta}{(P+100) + 6P\cos\theta}$$

$$\Rightarrow P + 100 + \cancel{6P\cos\theta} = 2P + \cancel{6P\cos\theta}$$

$$\Rightarrow \boxed{P = 100N}$$

Q3) A system of 4 forces acting on a body as shown in fig. Determine the magnitude resultant & its inclination with + x-axis.



$\tan\theta = \frac{2}{1}$

$\theta = \tan^{-1}(2)$

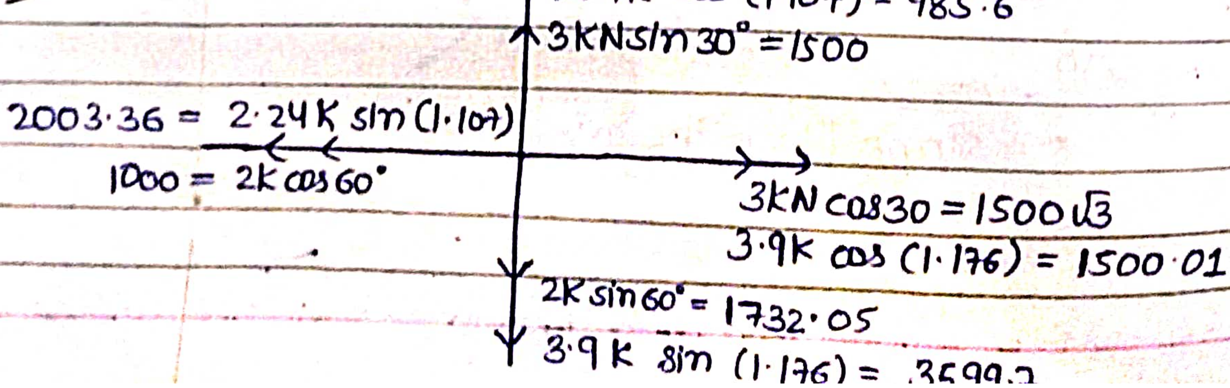
$= 1.107$

$\tan\beta = \frac{12}{5}$

$\beta = \tan^{-1}(12/5)$

$= 1.176$

FBD



$\Sigma F_x = 1500\sqrt{3} + 1500\cdot01 - 2003\cdot36 - 1000$
$= 1094\cdot65$ N

$\Sigma F_y = 985\cdot6 + 1500 - 1732\cdot05 - 3599\cdot7$
$= -2846\cdot15$ N

Practice sheet -2

Q1) Write short notes on Lami's Theorem

Ans) If 3 co-planar concurrent forces acting on a point equilibrium then each force is directly proportional to the sine angle b/w two other forces

Q2) Two forces, one of which is double the other has resultant of 260N. If the dir. of the larger force is reversed and the other force remains unaltered, the resultant reduces to 180N. Find the value of forces.

Ans) Let the force be F_1 & $2F_1$

$$R = \sqrt{(F_1)^2 + (2F_1)^2 + 2F_1F_2\cos\theta}$$

$$260^2 = F_1^2 + 4F_1^2 + 4F_1^2\cos\theta$$

$$260^2 = 5F_1^2 + 4F_1^2\cos\theta$$

$$67600 = 5F_1^2 + 4F_1^2\cos\theta \quad \text{--- (1)}$$

Now,

$$180 = \sqrt{F_1^2 + F_2^2 - 2F_1F_2\cos\theta}$$

$$32400 = F_1^2 + 4F_1^2 - 4F_1^2\cos\theta$$

$$32400 = 5F_1^2 - 4F_1^2\cos\theta \quad \text{--- (2)}$$

(1) + (2)

$$10F_1^2 = 100000$$

$$F_1^2 = 10000$$

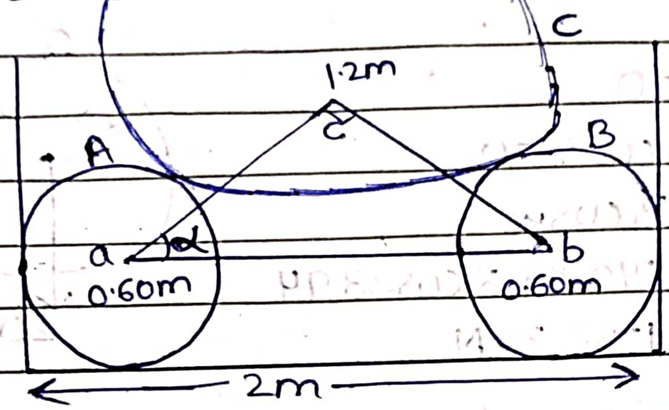
$$\boxed{F_1 = 100N}, \quad \boxed{F_2 = 200N}$$

$2K \sin 60° = 1732.05$

$3.9k \sin(1.176) = 3599.7$

Q3) Refer to the system of cylinder arranged in fig. The cylinder A & B weight 1000N each and weight of cylinder C is 2000N. Determine the forces exerted at the contact points.

Sol)



a, b, c are centre of spheres

$$ab = 2 - \frac{0.6}{2} - \frac{0.6}{2} = 1.4m$$

$$ac = 0.3 + 0.6 = 0.9m$$

$$\cos\alpha = \frac{1.4/2}{0.9} = 0.777 \;\#\; ; \alpha = 38.94°$$

Applying Lami's Theorem to the forces acting on sphere C,

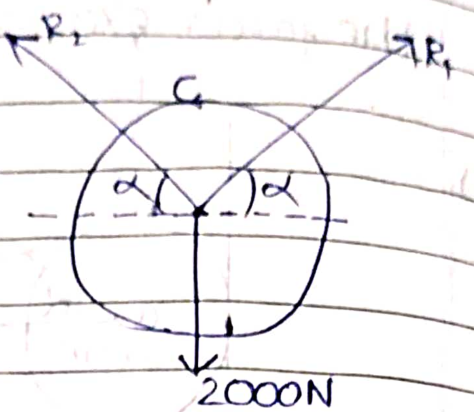
$$\frac{R_1}{\sin(90+\alpha)} = \frac{R_2}{\sin(90+\alpha)} = \frac{2000}{\sin(180-2\alpha)}$$

$$R_1 = R_2 = 2000 \times \frac{\sin(90+\alpha)}{\sin(180-2\alpha)}$$

$$= 2000 \times \frac{\sin(90+38.94)}{\sin(180-2\times 38.94)}$$

$= 2000 \times \frac{0.7777}{0.9777}$

$= 1590.87\,N$



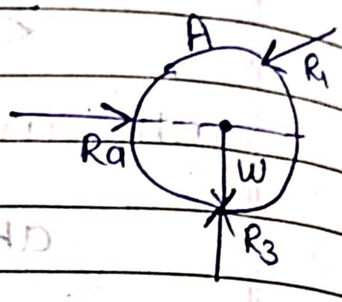
$\Sigma F_x = 0$

$R_a - R_1 \cos\alpha = 0$

$R_a = R_1 \cos\alpha$

$= 1590.87 \times \cos 38.94$

$= 1237.38\,N$



$\Sigma F_y = 0$

$R_1 \sin\alpha + W - R_3 = 0$

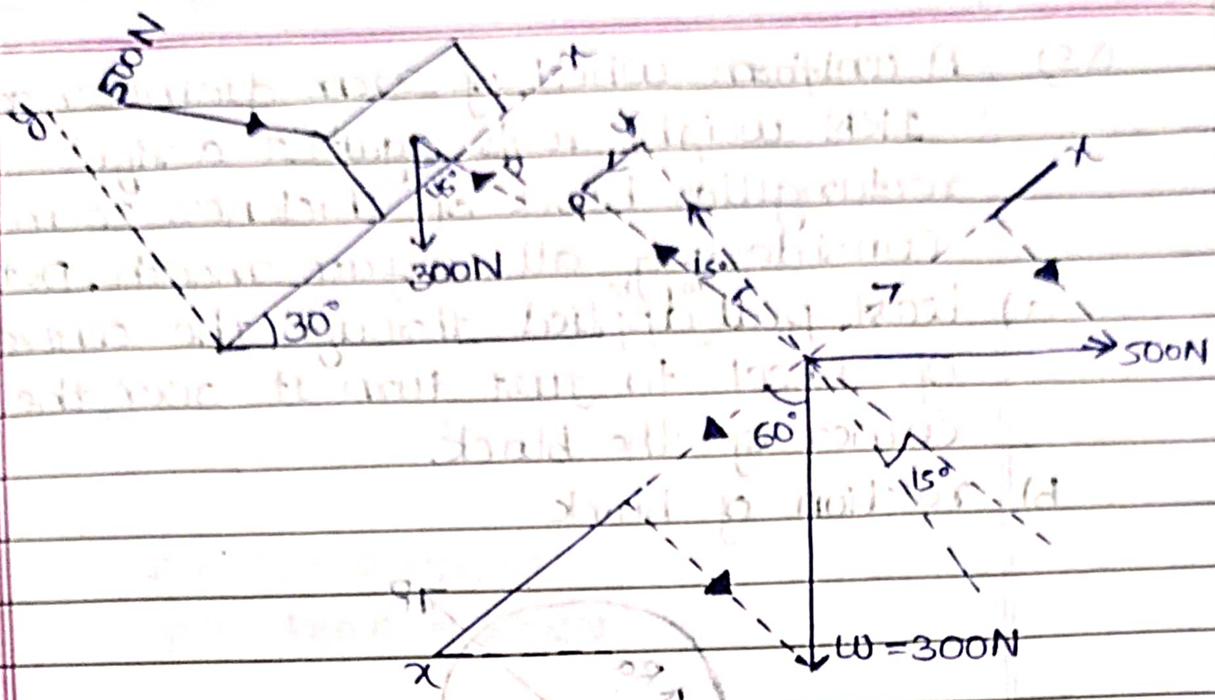
$R_3 = R_1 \sin\alpha + W$

$= 1590.87 \sin 38.94 + 1000$

$= 1999.87\,N$

Q4) A block of weight 300N is acted upon by a horizontal force F = 500N and a pressure exerted by the inclined as shown in fig. If the resultant of the force system lies parallel to the plane, work out the magnitude of pressure P & the resultant force.

→ let the co-ordinates be choosen parallel & perpendicular to the plane.



As the resultant R lies along the plane, its component perpendicular to the plane is 0.

$R = -P\sin 15° + 500\cos 30° - 300\cos 60°$

$= -0.259P + 433 - 150$

$= -0.259P + 283$

Also,

$P\cos 15° - 500\sin 30° - 300\cos 60° = 0$

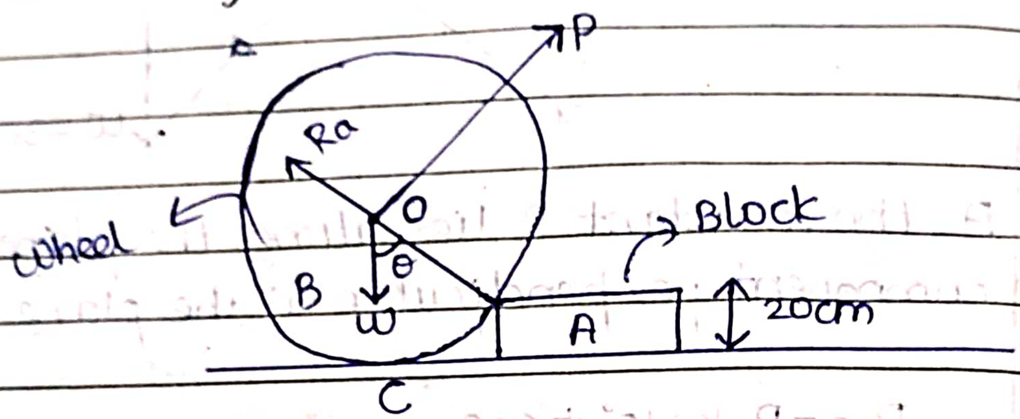
$P \times 0.966 = 500\sin 30° + 300\cos 60°$

$P \times 0.966 = 250 + 259.8 = 509.8$

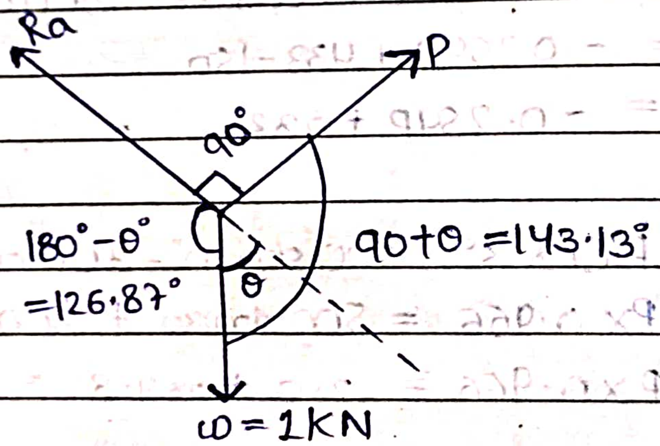
$P = \frac{509.8}{0.966} = 527.74N$

$\therefore R = -0.259 \times 527.74 + 283 = 146.31N$

Q5) A uniform wheel of 50cm diameter and 1KN weight rests against a rigid rectangular block of thickness 20cm. Considering all surface smooth. Determine
a) least pull to be applied through the centre of wheel to just turn it over the corner of the block.
b) reaction of block



Sol)



$$\cos\theta = \frac{OB}{OA} = \frac{50-20}{50} = 0.6 \; ; \; \theta = 53.13^\circ$$

When sphere is just about to turn, then it leaves contact with floor and reaction at point C will be 0

Wheel is in equilibrium,

Applying Lami's Theorem

$$\frac{W}{\sin 90^\circ} = \frac{P}{\sin 126.87^\circ} = \frac{R_a}{\sin 143.13^\circ}$$

$\therefore P = W \times \sin 126.87^\circ$

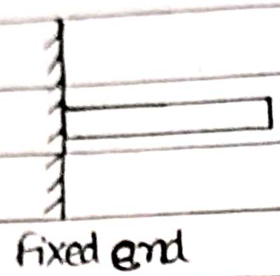
$P = 1 \times 0.8 = 0.8 KN$

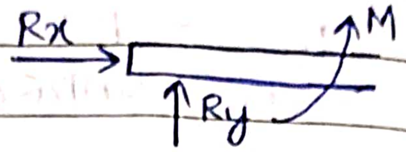
$$R_a = \frac{W \times \sin 143.13^\circ}{\sin 90^\circ} = 1 \times 0.6 = 0.6 KN$$

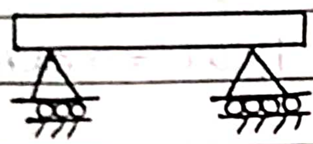
Practice Sheet-3

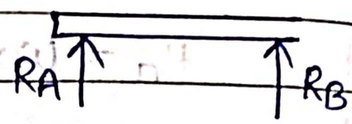
Q1) Draw the support reactions for given beams:-

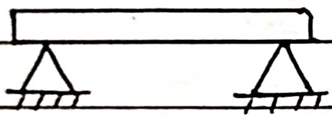
Reactions

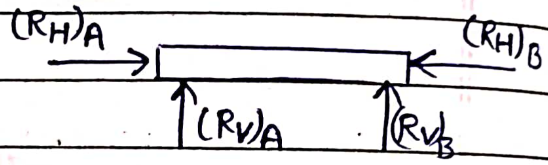
1) 

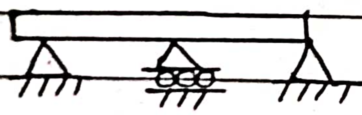


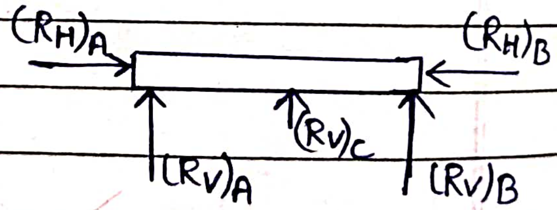
2) 

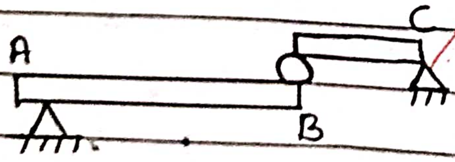


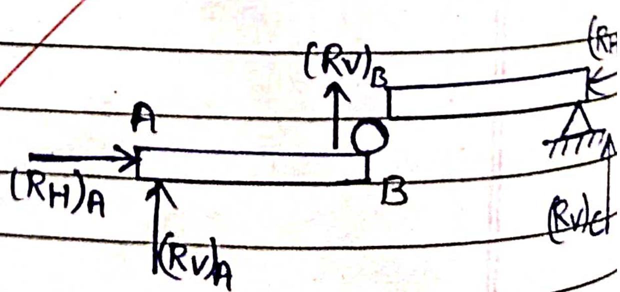
3) 



4) 

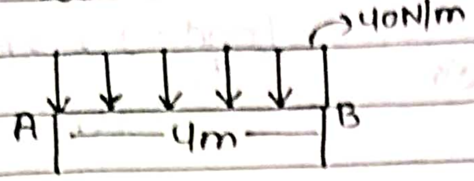


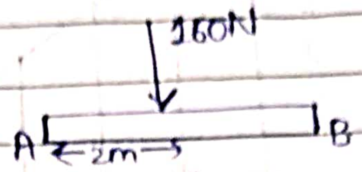
5) 



Q2) Convert these load into equivalent point loads:-

(1)



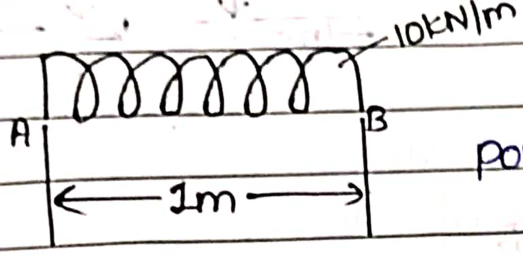


given 40 N/m

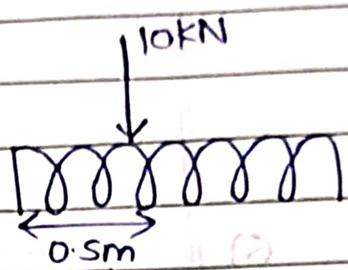
→ In 4m ⇒ 40×4

⇒ 160N

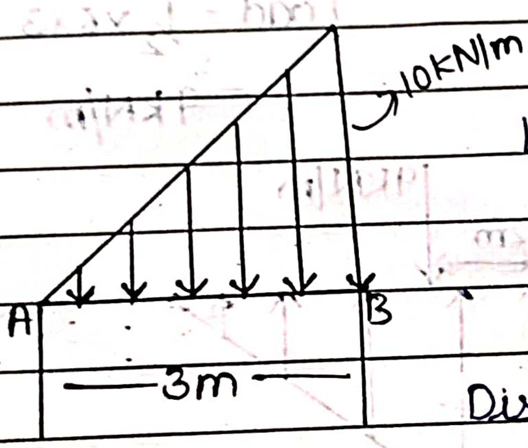
(2)



Point Load ⇒



(3)

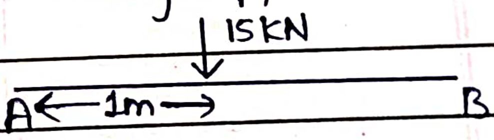


$$\text{Load} = \tfrac{1}{2} \times 3 \times 10000$$

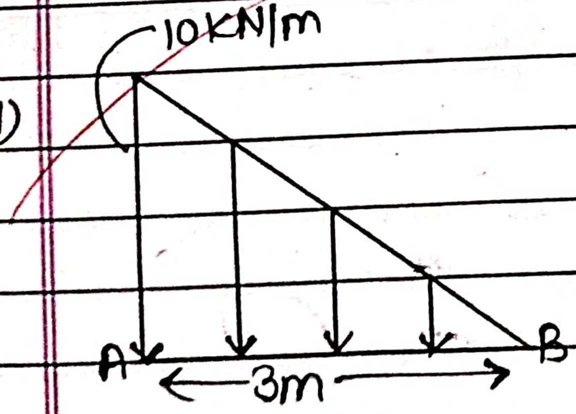
$$= \frac{30000}{2}$$

$$= 15\,kN$$

Distance of application $= 3/3 = 1m$



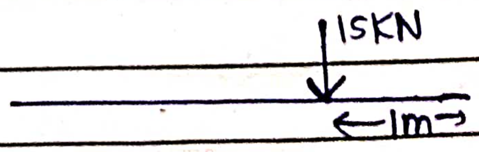
(4)



$$\text{Load} = \tfrac{1}{2} \times 3 \times 10\,kN$$

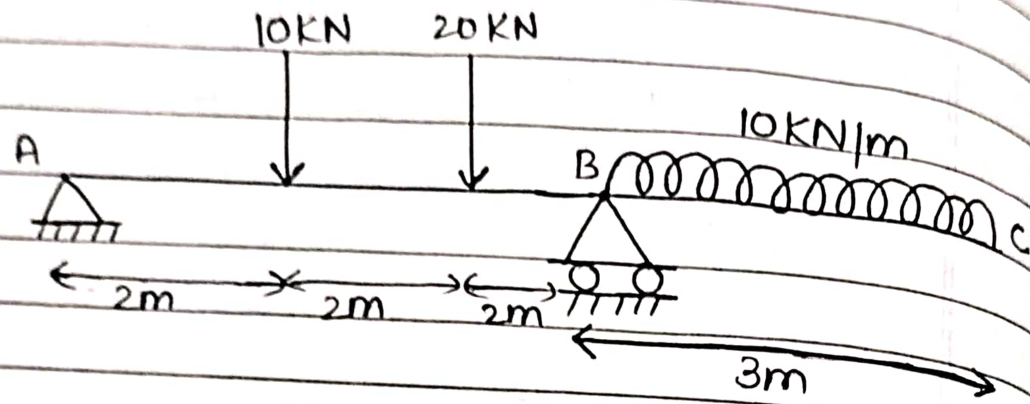
$$= 15\,kN$$

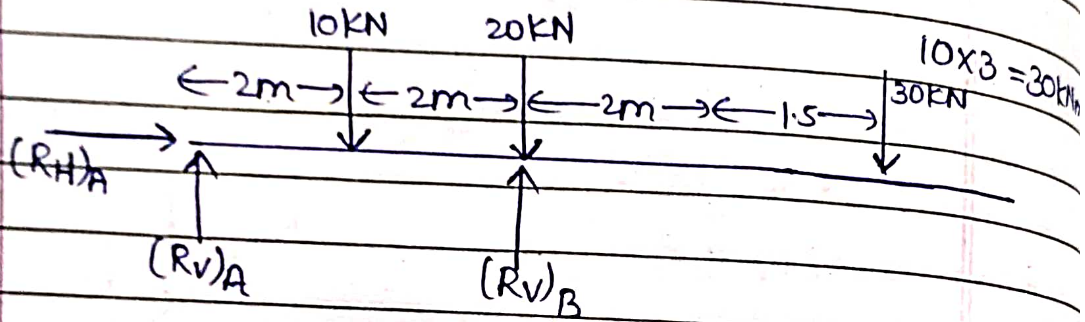
Distance of application = 1m



Practice Sheet - 4

Q1) Find the support reactions for given beam





$\Sigma F_x = 0$

$(R_H)_A = 0$

$\Sigma F_y = 0$

$(R_V)_A - 10KN - 20KN + (R_V)_B - 30KN = 0$

$(R_V)_A + (R_V)_B = 60\ KN$ —— (1)

$M = F \times \perp dist.$

$\Sigma M = 0$

$\Rightarrow (R_H)_A \times 0 + (R_V)_A \times 0 + 10 \times 2 + 20 \times 4 - (R_V)_B \times 6 + 30 \times 7.5 = 0$

$6(R_V)_B = 325$

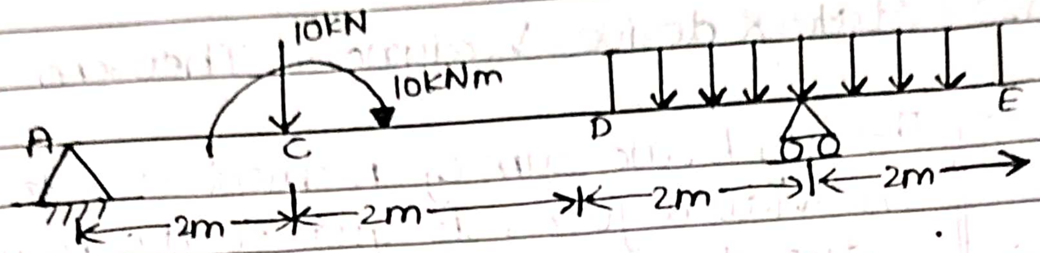
$(R_V)_B = 54.166$ ans

Putting $(R_v)_B$ in ①

$\Rightarrow (R_v)_A + 54.166 = 60$

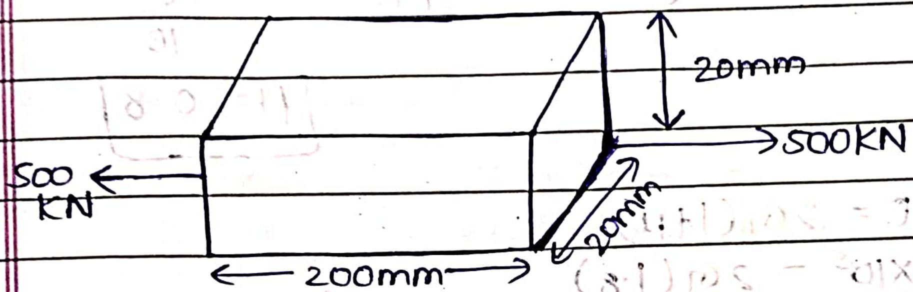
$(R_v)_A = 5.834$ ans

Q2) Find support rxn. for given beam



Q1) A square bar 20mm x 20mm is subjected to a compressive load of 500KN. The contraction over 200mm length is 0.5mm and increase in thickness is 0.04mm. Calculate the value of three elastic constants?

Sol)



$l = 200mm$,
$b = 20mm$
$t = 20mm$

$E, K, G = ?$

$\delta t = 0.04mm$
$\delta l = 0.5$

$P = 500KN$
$P = 500 \times 10^3 N$

$\sigma = \frac{P}{A}$

$= \frac{500 \times 10^3}{20 \times 20}$

$= \frac{5000}{4}$

$\boxed{\sigma = 1250 N/m^2}$

$E = \frac{\sigma}{\varepsilon l}$

$= \frac{\sigma l}{\delta l}$

$E = \frac{1250 \times 200}{0.5}$

$\boxed{E = 500000}$

~~$\mu = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{0.04}{0.5} =$~~

$$\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\frac{\delta t}{t}}{\frac{\delta l}{l}} = \frac{\frac{0.04}{20}}{\frac{0.5}{200}} = \frac{0.04 \times 200}{20 \times 0.5} = \frac{8}{10}$$

$$\boxed{\mu = 0.8}$$

$$E = 2G(1+\mu)$$

$$5\times10^5 = 2G(1.8)$$

$$\frac{5\times10^5}{2\times1.8} = G$$

$$\boxed{G = 1.38\times10^5 \ N/mm^2}$$

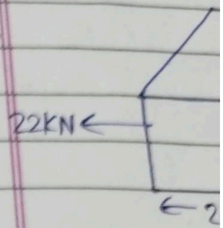
Now $E = 3K(1-2\mu)$

$$500000 = 3K(1-1.6)$$

Q2) At an axial load of 22KN, a 45mm wide by 15mm thick polyamide polymer bar elongates by 3mm while the bar width contracts ~~ee~~ by 0.25mm. The bar is 200mm long at 22KN load ~~less~~ the stress is less than its proportional limit.

Determine m
and chang

Sol)



$l = 200mm$
$b = 45mm$
$t = 15mm$
$P = 22KN$

$\sigma =$

$=$

$=$

$\boxed{\sigma = (}$

$2K \sin 60° = 1732.05$

$3.9K \sin(1.176) = 3599.7$

$$\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\frac{\delta t}{t}}{\frac{\delta l}{l}} = \frac{\frac{0.04}{20}}{\frac{0.5}{200}} = \frac{0.04 \times 200}{20 \times 0.5}$$

$$= \frac{8}{10}$$

$$\boxed{\mu = 0.8}$$

$$E = 2G(1+\mu)$$

$$5 \times 10^5 = 2G(1.8)$$

$$\frac{5 \times 10^5}{2 \times 1.8} = G$$

$$\boxed{G = 1.38 \times 10^5 \ N/mm^2}$$

Now

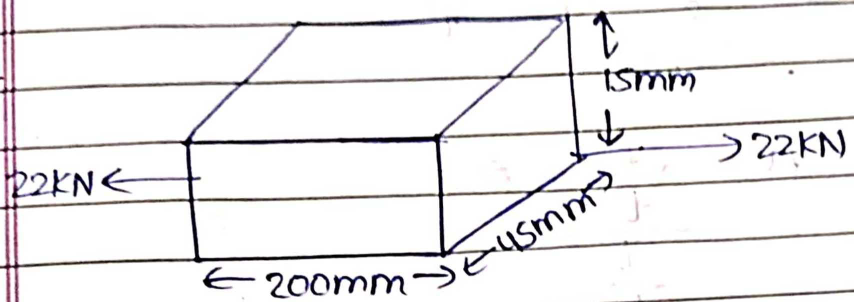
$$E = 3K(1-2\mu)$$

$$500000 = 3K(1-1.6)$$

Q.2) At an axial load of 22KN, a 45mm wide by 15mm thick polyamide polymer bar elongates by 3mm while the bar width contracts ~~as~~ by 0.25mm. The bar is 200mm long at 22KN load ~~less~~ the stress is less than its proportional limit.

Determine modulus of elasticity, poisson's ratio and change in thickness

Sol)



$l = 200mm$ $\quad$ $\delta l = 3mm\ (+)$

$b = 45mm$ $\quad$ $\delta b = 0.25mm\ (-)$

$t = 15mm$

$P = 22KN = 22 \times 10^3 N$

$$\sigma = \frac{P}{A} = \frac{22\times10^3}{45\times15} = \frac{22\times10^3}{675}$$

$$\boxed{\sigma = 0.03\times10^3\ N/mm^2}$$

$$E = \frac{\sigma}{\frac{\delta l}{l}} = \frac{\sigma . l}{\delta l} = \frac{0.03\times10^3\times200}{3}$$

$$\boxed{E = 2\times10^3}$$

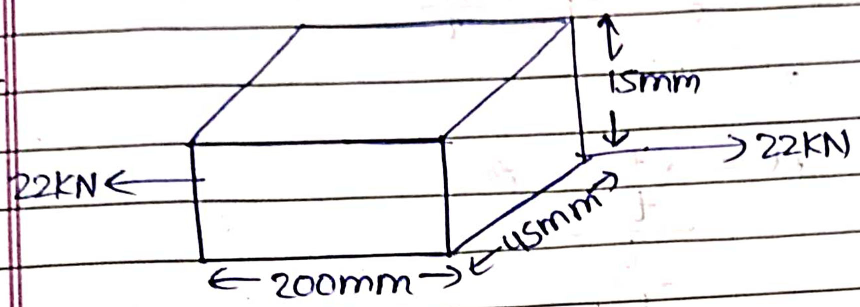
$$\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}} = \frac{\frac{0.25}{45}}{\frac{3}{200}} = \frac{0.25\times200}{45\times3} = \frac{50}{135}$$

$$\boxed{\mu = 0.37}$$

Determine modulus of elasticity, poisson's ratio and change in thickness

Sol



$l = 200mm$ $\quad\quad$ $\delta l = 3mm\ (+)$

$b = 45mm$ $\quad\quad$ $\delta b = 0.25mm\ (-)$

$t = 15mm$

$P = 22KN = 22\times10^3 N$

$$\sigma = \frac{P}{A} = \frac{22\times10^3}{45\times15} = \frac{22\times10^3}{675}$$

$$\boxed{\sigma = 0.03\times10^3\ N/mm^2}$$

$$E = \frac{\sigma}{e_l} = \frac{\sigma \cdot l}{\delta l} = \frac{0.03\times10^3\times200}{3}$$

$$\boxed{E = 2\times10^3}$$

$$\mu = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$$

$$= \frac{\frac{\delta b}{b}}{\frac{\delta l}{l}} = \frac{\frac{0.25}{45}}{\frac{3}{200}} = \frac{0.25\times200}{45\times3} = \frac{50}{135}$$

$$\boxed{\mu = 0.37}$$

$$\cancel{\varepsilon_v = \varepsilon_l + \varepsilon_b + \varepsilon_t}$$

$$\cancel{= \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta t}{t}}$$

$$=$$

$$\mu = \frac{\frac{\delta t}{t}}{\frac{\delta l}{l}}$$

$$0.37 = \frac{\frac{\delta t}{15}}{\frac{3}{200}}$$

$$0.37 = \frac{200 \times \delta t}{45}$$

$$\frac{0.37 \times 45}{200} = \delta t$$

$$\frac{16.65}{200} = \delta t$$

$$\boxed{0.08 = \delta t} \text{ ans}$$