Practice Questions

S.No.	Questions	BLT	CO
1.	**Test the analyticity of the functions and find the derivative.** (i) $f(z)=e^x(\cos y+i\sin y)$ (ii) $f(z)=\frac{1}{z}$ (iii) $\log z$ (iv) $\sinh z$	**K3**	4
2.	Show that the function defined by f (z) = $\sqrt{\lvert xy\rvert}$ is not regular at the origin although C-R equations are satisfied there.	**K3**	4
3.	Show that the function $f(z)$ defined by $f(z)=\begin{cases}\frac{x^3y^5(x+iy)}{x^6+y^{10}}, & z\neq 0\\ 0 & z=0\end{cases}$ is not analytic at the origin even though it satisfies Cauchy-Riemann equation at the origin.	**K3**	4
4.	Show that the function $f(z)$ defined by $f(z)=e^{-z^{-4}},(z\neq 0)$ *and* $f(0)=0$ is not analytic, although Cauchy-Riemann equations are satisfied at the point.	**K3**	4
5.	**Construct an analytic function whose real part $u(x, y)$ is:** (i) $e^x(x\cos y - y\sin y)$ (ii) $e^{-x}(x\sin y-y\cos y)$ *and* $f(0)=i$	**K3**	4
6.	**Construct an analytic function whose imaginary part $v(x, y)$ is:** (i) $\log(x^2+y^2)+x-2y$ (ii) $\tan^{-1}(y/x), x\neq 0, y\neq 0$ (iii) $e^{-x}(x\cos y+y\sin y)$	**K3**	4
7.	Determine the analytic functions f(z)= u+iv such that u+v = $e^x(\cos y - \sin y)$	**K3**	4
8.	If $u-v=(x-y)(x^2+4xy+y^2)$, then find an analytic function $f(z) = u + iv$ in terms of z.	**K3**	4
9.	If $u+v=\frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$, then find an analytic function $f(z) = u + iv$ in terms of z.	**K3**	4
10.	Find the Bilinear transformation which maps the points $z=1,-i,1$ to the points $w=i,0,-i$ respectively. Show also that transformation maps the region outside the circle $\lvert z\rvert=1$ into the half-plane $R(w)\geq 0$	**K3**	4
11.	Find the bilinear transformation which maps the points $z=0,-1,i$ onto $w=i,0,\infty$. Also find the image of the unit circle $\lvert z\rvert=1$	**K3**	4
12.	Find the bilinear transformation which maps the points $z=0,1,\infty$ onto $w=i,-1,-i$.	**K3**	4
13.	Obtain the invariant points of the transformation (i) $w=2-\frac{2}{z}$ (ii) $w=\frac{1+z}{1-z}$	**K3**	4
14.	Obtain the fixed points of the transformation (i) $w=\frac{3z-4}{z-1}$ (ii) $w=\frac{2z-5}{z+4}$	**K3**	4
15.	State Cauchy Integral Theorem and hence evaluate: Evaluate $\int_C \frac{z}{(z-3)^4}dz$ where C is $\lvert z\rvert=1$.	**K3**	5
16.	Evaluate $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3}dz$ where C is $\lvert z\rvert=4$.	**K3**	5
17.	Using Cauchy's integral formula, evaluate the following integrals: (i) $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle $\lvert z+1+i\rvert=2$. (ii) $\int_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-2)(z-3)}dz$ where C is $\lvert z\rvert=4$. (iii) $\int_C \frac{z}{(z-1)(z-2)}dz$ where C is the circle $\lvert z-2\rvert=\frac{1}{2}$.	**K3**	5
18.	Evaluate: (i). $\int_C \frac{e^z}{(z^2+\pi^2)^2}dz$ where C is the circle $\lvert z\rvert=4$. (ii) $\int_C \frac{z}{(z-1)(z-2)^2}dz$ where C is the circle $\lvert z\rvert=4$	**K3**	5