COMPLEX VARIABLES

$f(z)\ e^{z/(z-2)}$

65. Show that $f(z) = z + 2\bar{z}$ is not analytic anywhere in the complex plane.

66. Find the image of $|z - 2i| = 2$ under the mapping $w = \frac{1}{z}$. $1 + 4v = 0$ **(AKTU 2020)**

67. Expand $f(z) = e^{z/(z-2)}$ in a Laurent series about the point $z = 2$. Ans:- $e\left[1 + \frac{2}{z-2} + \frac{4}{(z-2)^2\, 2!} + \cdots\right]$ **(AKTU 2019)**

68. Discuss the nature of singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.
$z = a$ is pole & $z = a$ is non-isolated essential singularity **(AKTU 2020)**

69. If $f(z) = u + iv$ is an analytic function, $f(z)$ in term of z if $u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}$ when $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$. $\cot\frac{z}{2} + \frac{1}{2}(1-i)$ **(AKTU 2020)**

70. Evaluate by contour integration: $\int_0^{2\pi} e^{-\cos\theta}\cos(n\theta + \sin\theta)\,d\theta; n \in I$. $\frac{2\pi}{n!}(-1)^n$ **(AKTU 2019)**

71. Prove that $w = \frac{z}{1-z}$ maps the upper half of the z-plane onto upper half of the w-plane. What is the image of the circle $|z| = 1$ under this tranformation? $2u + 1 = 0$ **(AKTU 2021)**

72. Find a bilinear transformation which maps the point $i, -i, 1$ of the z-plane into $0, 1, \infty$ of the w-plane respectively. $\frac{(i-1)z + (i+1)}{-2z+2}$ **(AKTU 2019)**

73. Evaluate $\oint_C \frac{e^z}{z(1-z)^3}\,dz$, where c is (*i*) $|z| = \frac{1}{2}$ (*ii*) $|z-1| = \frac{1}{2}$ (*iii*) $|z| = 2$ **(AKTU 2021)**
R_1 at $(z=0) = 1$, R_2 at $z = 1 = -\frac{e}{2}$ $2\pi i$ $-\pi i e$ $2\pi i\left(1 - \frac{e}{2}\right)$

74. Find the Taylor's and Laurent's series which reprsent the function $\frac{z^2-1}{(z+2)(z+3)}$ when (*i*) $|z| < 2$ (*ii*) $2 < |z| < 3$ (*iii*) $|z| > 3$. **(AKTU 2021)**
(i) $1 + \frac{3}{2}\sum(-1)^n\left(\frac{z}{2}\right)^n - \frac{8}{3}\sum(-1)^n\left(\frac{z}{3}\right)^n$ Taylor (ii) $1 + \frac{3}{z}\sum(-1)^n\left(\frac{2}{z}\right)^n - \frac{8}{3}\sum(-1)^n\left(\frac{z}{3}\right)^n$ Laurent
(iii) $1 + \frac{3}{z}\sum\left(\frac{2}{z}\right)^n - \frac{8}{z}\sum(-1)^n\left(\frac{3}{z}\right)^n$ Laurent

75. Define harmonic function. **(AKTU 2021)**

76. Find the points of invariant of the transformation $w = \frac{2z+3}{z+2}$, $z = \pm\sqrt{3}$

77. Sate Cauchy integral theorem. **(AKTU 2020)**

78. Discuss the singularity of $\sin\left(\frac{1}{z-a}\right)$. $z = a$ is ~~non~~- isolated essential singularity **(AKTU 2021)**

79. Examine the nature of the function

$$f(z) = \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

in the region including the origin. **(AKTU 2021)**
$f(z)$ is not analytic at origin
$f'(0)$ doesn't exist & C-R eqns are satisfied at origin

80. Evaluate, $I = \frac{1}{2\pi i}\oint_C \frac{z^2 - z + 1}{z - a}\,dz$, where $C \equiv |z-1| = \frac{1}{2}$. If a lie inside then $I = a^2 - a + 1$ and if a lie outside then $I = 0$ **(AKTU 2020)**

81. Define an analytic function. If $f(z) = u + iv$ is an analytic function find $f(z)$ in term of z if $u - v = e^x(\cos y - \sin y)$. Ans:- $e^z + C$ **(AKTU 2017)**

82. Find the image of circle $|z - 1| = 1$ in the complex plane under the mapping $w = 1/z$. **(AKTU 2020)**
$u = \frac{1}{2}$

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$f(z) = \frac{1}{z \cdot 2!} - \frac{1}{4!}z + \frac{1}{6!}z^3 - - - -$

83. Find Laurent series expansion of $\frac{1-\cos z}{z^3}$ about the point $z = 0$ is. *(AKTU 2021)*

84. Find residue at each pole of the function and hence using Cauchy residue theorem evaluate integral $\int \frac{4+3z}{(z-2)(z-3)}dz$, where C: $|z| = 1$. $R_1 = -10$, $R_2 = 13$ $\int f(z)dz = 0$ *(AKTU 2020)*

85. Show that the function defined by $f(z) = \sqrt{|xy|}$ is not regular at the origin, although the Cauchy-Riemann equations are satisfied there. *(AKTU 2018)*

86. Show that complex function $f(z) = z^3$ is analytic. *(AKTU 2018)*

87. Define Conformal mapping. *(AKTU 2018)*

88. Evaluate $\int_0^{1+i}(x^2 - iy)dz$ along the path $y = x$. $\frac{5}{6} - \frac{i}{6}$ *(AKTU 2018)*

89. Find residue of $f(z) = \frac{\cos z}{z(z+5)}$ at $z = 0$. $\frac{1}{5}$ *(AKTU 2018)*

90. Show that $u = x^4 - 6x^2y^2 + y^4$ is harmonic function. Find complex function $f(z)$ whose u is a real part. $f(z) = z^4 + C$ *(AKTU 2018)*

91. Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in regions (i) $-\frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{z}{2}\right)^n - \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$ – Laurent

(i) $1 < |z| < 2$ (ii) $2 < |z|$ $\frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{2}{z}\right)^n - \frac{1}{z}\sum_{n=0}^{\infty}\left(\frac{1}{z}\right)^n$ → Laurent *(AKTU 2018)*

92. Let $f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}}$ when $z \neq 0$, $f(z) = 0$ when $z = 0$. Prove that Cauchy Riemann satisfies at $z = 0$ but function is not differentiable at $z = 0$. $\cancel{w = \frac{3z-5i}{iz-1}}$ *(AKTU 2018)*

93. Find Mobius transformation that maps point $z = 0, -i, 2i$ into the points $w = 5i, \infty, -\frac{i}{3}$ respectively.
$w = \frac{3z-5i}{iz-1}$ *(AKTU 2018)*

94. Using Cauchy Integral formula evaluate $\int_c \frac{\sin z}{(z^2+25)^2}dz$ where c is circle $|z| = 8$.
$\frac{\pi(\sin 5i - 5i\cos 5i)}{125}$ *(AKTU 2018)*

95. Apply residue theorem to evaluate $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)(x^2+4)}$. $= \pi/3$ *(AKTU 2018)*