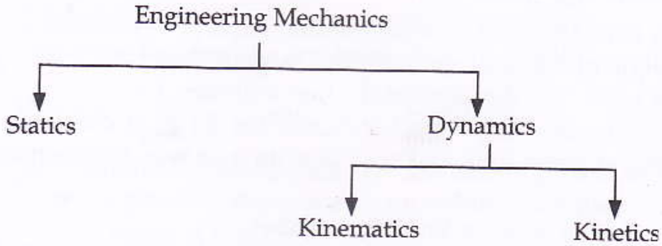
Chapter
1

Engineering Mechanics

Engineering mechanics is a basic subject which describes and predicts the effects of forces on rigid bodies. It essentially deals with the study of bodies at rest or in motion when subjected to external mechanical disturbances. A knowledge of its basic concepts and principles is a must for an engineer engaged in the design and construction of various types of machines and structures.



The subject of mechanics of a rigid body is divided into Statics and Dynamics. *Statics* deals with forces in terms of their distribution and effect on a body in equilibrium, *i.e.*, at absolute or relative rest. *Dynamics* deals with the study of bodies in motion. Dynamics is further divided into Kinematics and Kinetics. *Kinematics* is concerned with the description of motion of objects independent of causes of motion. Here study is made of motion interrelationship among position, velocity, acceleration and time without taking into account of the forces causing motion. In *Kinetics,* both the motion and its causes are considered. Kinetics relates the action of forces and the resulting motion.

1.1. MATTER, PARTICLE AND BODY

Matter is anything that occupies space, possesses mass and offers resistance to any external force. Iron, stone, wood salt, water and air etc represent matter.

A *particle* is an object that has infinitely small volume (occupies negligible space) but has a mass which can be considered to be concentrated at a point.

A *body* has a definite shape and consists of numbers of particles.

A body in which the distance between any two particles remains constant, *i.e.*, the size and shape of the body do not change, is called a *rigid* body. A rigid body exhibits no relative deformation between two parts when it is acted upon by a force system.

An *elastic* body undergoes deformation but regains its original shape after removal of the external forces.

1.2. NEWTON'S LAWS OF MOTION

First law: Every body continues in its state of rest or of uniform motion in a straight line if there is no unbalanced force acting upon it.

Newton's first law of motion gives the concept of inertia. Inertia is the property of matter by which a resistance is offered for change in the state of rest or uniform motion.

Second law: The rate of change of linear momentum is directly proportional to the impressed force and it takes place in the direction of the impressed force.

Newton's second law of motion connects rate of change of momentum and external force.

Third law: To every action, there is equal and opposite reaction.

1.3. SCALAR AND VECTOR QUANTITIES

A quantity is said to be *scalar* if it is completely defined by its magnitude alone. Mass, length, volume, temperature and time etc are scalar quantities.

Any quantity which possesses magnitude as well as direction is called a *vector* quantity. A vector quantity needs both magnitude and direction for its complete specification. Some examples of vector quantities are displacement, velocity, acceleration, momentum and force.

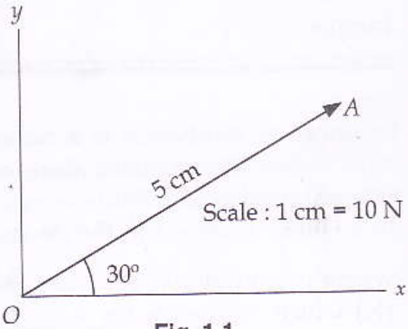


Fig. 1.1

A vector quantity is represented by a directed segment of a straight line. The length of the line represents the magnitude of the vector on some scale. The direction of the line indicates the direction of the vector, and arrowhead indicates the sense of direction. For example a force of 50 N acting in the x-y plane and making 30° with x-axis is represented by segment OA as shown in Fig. 1.1.

A vector is denoted by a letter of English alphabet with an arrow at its top such as $\vec{a}$, $\vec{A}$, $\vec{X}$ etc. The magnitude of any vector $\vec{A}$ is denoted by $|\vec{A}|$.

In Bow's method of force representation, a force is designated by writing two capital letters one on either side of it. With reference to Fig. 1.4, a force of P_1 = 10 kN is represented by AB and P_2 = 15 kN is represented by CD.

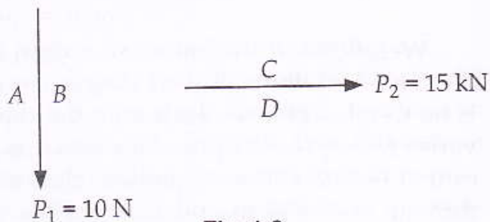


Fig. 1.2

1.4. MASS, FORCE AND WEIGHT

Mass is an indication of the quantity of matter present within a system. The more mass means more matter.

Force is an external agent which tends to change the speed or direction of a system. A force is applied whenever the system needs to be accelerated or decelerated. A force on a rigid body may produce one or both of the following effects:

(*i*) linear displacement

(*ii*) turning or rotating moment.

A force is a vector quantity determined completely by its

– magnitude
– point of application
– line of action, and
– direction.

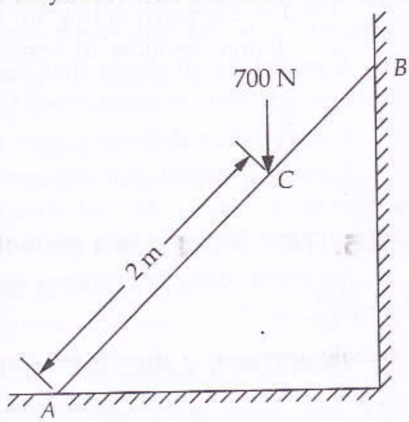


Fig. 1.3

With reference to Fig. 1.3, *AB* is the ladder that rests against a wall and a person weighing 700 N stands at point *C* on the ladder. For complete specification of the force exerted by the person on the ladder, the following characteristics need to be specified:

- magnitude is 700 N
- the point of application is at a distance of 2 m from the floor along the ladder
- the line of action of force is vertical, and
- the direction is downward.

Newton's second law of motion states that force is proportional to rate of change of momentum. That is

$$\begin{aligned}\text{Force} &\propto \text{rate of change of momentum}\\ &\propto \text{rate of change of (mass} \times \text{velocity)}\\ &\propto \text{mass x rate of change of velocity}\\ &\propto \text{mass x acceleration}\end{aligned}$$

Thus $\quad F \;\alpha\; m\,a; \qquad F = \dfrac{m\,a}{g_c}$

where m is mass, a is acceleration and g_c is proportionality constant. The SI unit of force is newton (N) which represents the force required to accelerate 1kg mass with an acceleration of $1\ \text{m/s}^2$

Thus $\quad 1\ \text{N} = \dfrac{1\ \text{kg} \times 1\ \text{m/s}^2}{g_c} \quad \text{and} \quad g_c = \dfrac{1\ \text{kg m}}{\text{Ns}^2}$

The constant of proportionality becomes unity for defining unit of force as newton and accordingly dropped. Such a practice corresponds to writing Newton's second law as

$$\text{Force} = \text{mass x acceleration}; \quad F = m\,a$$

Weight is a force which the system exerts due to gravitational acceleration. Weight is actually the force with which the system is attracted towards the earth. Where there is no gravity, there is no weight and this condition of the system is known as weightlessness. The weight of a system varies from place to place but mass remains constant. The mass of a given collection of matter cannot be altered by a change of location, a deformation of shape, a change of temperature, and through chemical reactions which the matter may undergo.

The weight (W) of system equals the product of mass (m) and local gravitational acceleration. The mass of a system remains constant whereas weight varies with change in value of gravitational acceleration from one place to another. The value of g at sea level is $9.8066\ \text{m/s}^2$ and is generally taken as $9.81\ \text{m/s}^2$.

The force system acting on a body may

- change its state of rest or motion
- accelerate or retard its motion
- change its shape or size
- turn or rotate it, and
- keep it in equilibrium.

1.5. TENSION AND COMPRESSION

A structural member is said to be in tension when it is subjected to two equal and opposite pulls and the member tends to elongate/increase in length. The stress induced in the bar is called tensile stress, and the tensile stress σ_t at any cross-section X-X is given by

$$\sigma_t = \frac{P}{A}$$

where A is the cross-sectional area of the bar.

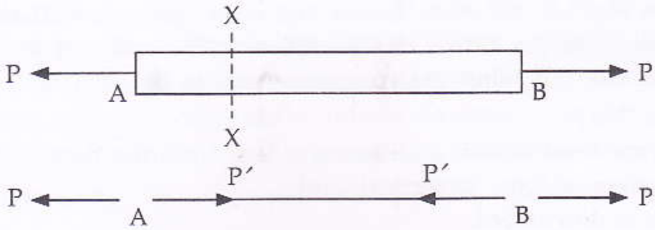


Fig. 1.4. Bar under tension

At every point of bar AB, equal and opposite forces P' (where $P' = P$) act. The force P' acting at point A balances the external force P; hence when considering the equilibrium of A, the arrow showing P' must be directed toward B. Likewise, when considering the equilibrium of B, the arrow points towards A.

The hoisting ropes used is cranes and passenger elevators are the elements subjected to tensile loading. An element under tension is called a *tie.*

If the structural member is subjected to two equal and opposite pushes, and the member tends to shorten/decrease in length, the member is said to be in compression. The stress so produced is called compressive stress, and the compressive stress σ_c at any cross-section X-X is given by

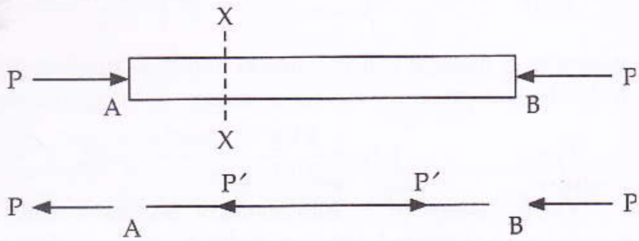


Fig. 1.5. Bar under compression

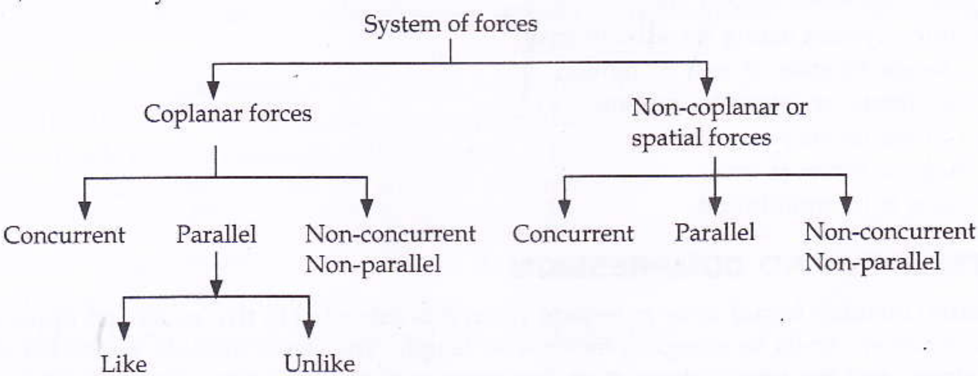
$$\sigma_c = \frac{P}{A}$$

Under equilibrium conditions of the bar, the resistance force P' (where $P' = P$) will be directed as shown in Fig. 1.5. A bar under compression is called a *strut.*

1.6. SYSTEM OF FORCES

When several forces of different magnitude and direction act upon a body, they constitute a force system.

Considering the plane in which forces are applied and depending upon the position of line of action, forces may be classified as shown follows:



- *Collinear forces*: The lines of action of all forces lie along the same straight line.

Fig. 1.6

Example: Force on a rope in a tug of war.

- *Coplanar parallel forces :* The lines of action of all forces are parallel to each other and lie in a single plane.

  Example: System of vertical loads (including reactions) acting on a beam.

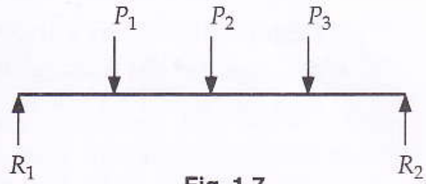


Fig. 1.7

- *Coplanar concurrent forces*: All forces lie in the same plane, have different directions but their lines of action act at one point (pass through a single point). The point where the lines of action of the forces meet is known as the point of concurrency of the force system.

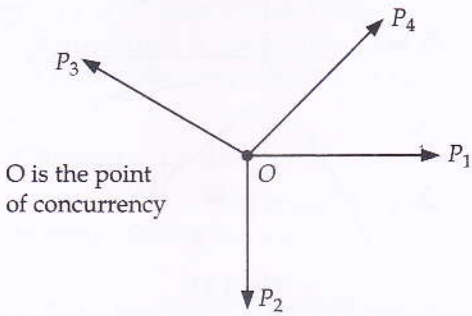


Fig. 1.8

Example: Forces on a rod resting against a wall.

- *Coplanar non-concurrent forces*: All forces lie in the same plane but their lines of action do not pass through a single point.

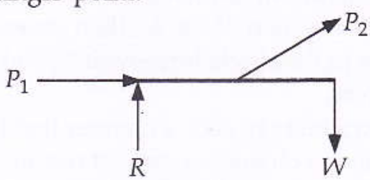


Fig. 1.9

Example: Forces on a ladder resting against wall and a person standing on a rung which is not at its centre of gravity.

- *Non-coplanar concurrent forces*: All forces do not lie in the same plane but their lines of action pass through a single point.

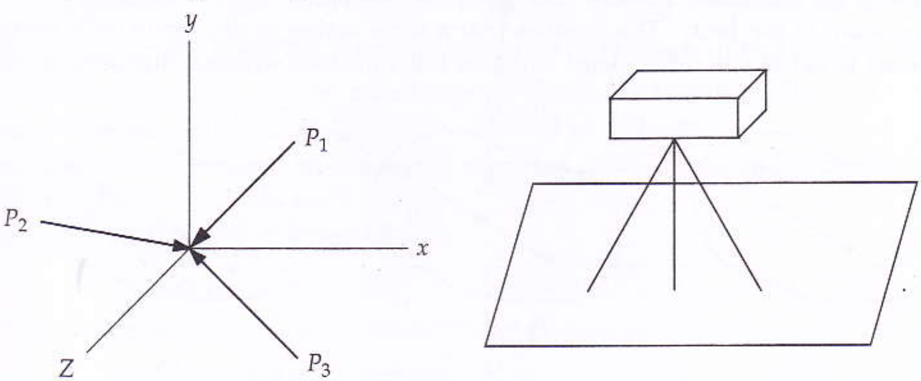


Fig. 1.10

Example: forces on a tripod carrying a camera.

- *Non-coplanar and non-concurrent forces:* All forces do not lie in the same plane, and their lines of action do not meet at a single point.

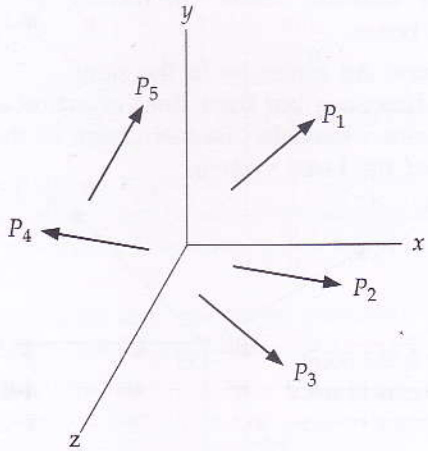


Fig. 1.11

Example: Forces acting on a moving bus.

1.7. EQUILIBRIUM, RESULTANT AND EQUILIBRANT

When two or more than two forces act on a body in such a way that the body remains in a state of rest or of uniform motion (no acceleration), then the system of forces is said to be in *equilibrium.*

When a body is acted upon by a system of forces, then vectorial sum of all the forces is known as *resultant*. Hence resultant refers to the single force which produces the same effect as is done by the combined effect of several forces.

A number of forces may act on a body in such a manner that the body is not in equilibrium. The resultant of several forces may cause a change of state of rest or of uniform motion. A single force may have to be applied to the body to bring it in equilibrium state. That single force is known as *equilibrant*. Equilibrant is equal and opposite to the resultant of several forces acting on the body.

1.8. PRINCIPLE OF TRANSMISSIBILITY

When the point of application of a force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there occurs no change in the equilibrium state of the body. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

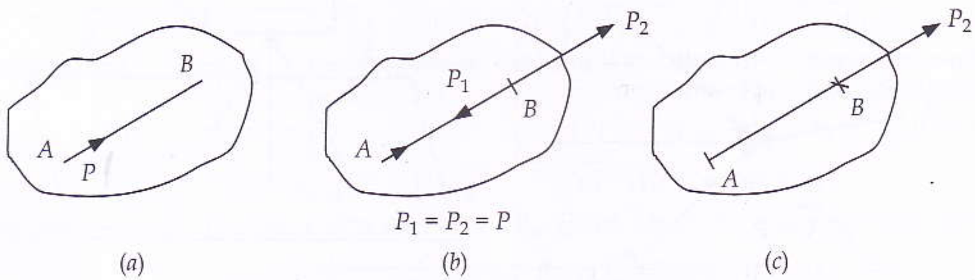


Fig 1.12. Principle of transmissibility of forces

Consider a force P acting at point A on rigid body (Fig 1.14). B is another point on the line of action of force P. At point B, apply two oppositely directed forces (P_1 and P_2) equal to and collinear with P. Such an application will in no way alter the action of given force P. At point A, forces P and P_1 are equal but opposite and accordingly cancel each other. That leaves a force $P_2 = P$ at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing its effect on the body.

1.9. TWO-DIMENSIONAL CONCURRENT FORCE SYSTEMS

When a number of concurrent forces act on a body to keep the body in equilibrium, then resultant of these forces will be the vector sum of all these forces. The process of determination of resultant in known as *composition* of forces. Also from a given force, two forces in two directions can be obtained, which will produce the same effect as produced by single given force. This process is known as *resolution* of a force.

1.9.1 Parallelogram Law of Forces

The parallelogram law of forces is used to determine the resultant of two forces acting at a point in a plane and inclined to each other at an angle.

It states that:

"If two forces, acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."

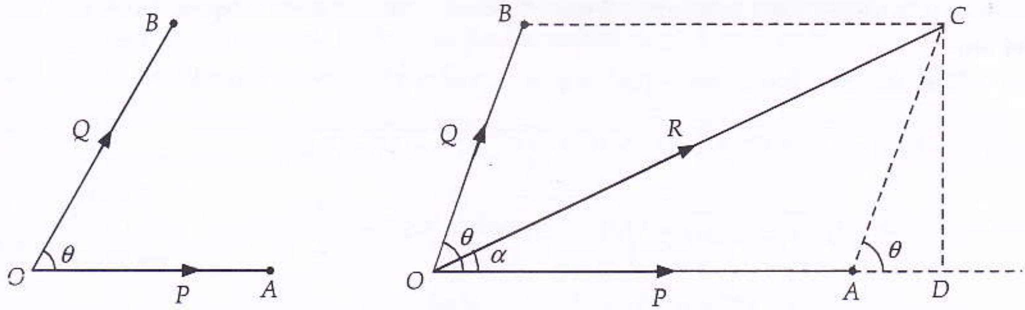


Fig. 1.13.

Consider two forces P and Q acting on a body at O as shown in Fig 1.13. The force P is represented in magnitude and direction by $\overrightarrow{OA}$ whereas the force Q is represented in magnitude and direction by $\overrightarrow{OB}$. Let the angle between the two forces be θ. The resultant of these two forces is obtained by the diagonal $\overrightarrow{OC}$ of the parallelogram $OACB$, as shown in Fig. 1.13.

The relationship between P, Q and R can be derived as follows:

Drop perpendicular from C and let it meet OA extend at point D. In $\Delta\, CAD$, side CA is parallel and equal to OB, *i.e.*, it represents force Q.

The resultant R of P and Q is given by

$$R = OC = \sqrt{OD^2 + CD^2} = \sqrt{(OA + AD)^2 + CD^2}$$

But
$$OA = P$$
$$AD = AC \cos\theta = Q \cos\theta$$
$$CD = AC \sin\theta = Q \sin\theta$$

$$\therefore \quad R = \sqrt{(P + Q\cos\theta)^2 + (Q\sin\theta)^2} = \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$$

$$= \sqrt{P^2 + Q^2(\sin^2\theta + \cos^2\theta) + 2PQ\cos\theta} = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \quad \text{...(1.1)}$$

The inclination of the resultant R to the direction of force P is given by

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\alpha = \tan^{-}\left[\frac{Q\sin\theta}{P + Q\cos\theta}\right] \quad \text{...(1.2)}$$

Note: It is not necessary that for the law of parallelogram for forces to be valid, one of the two forces should be along the x-axis. The forces P and Q may be in any direction as shown in Fig. 1.14.

If the angle between the forces is θ, then

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

The direction of resultant will be

$$\alpha = \tan^{-1}\frac{Q\sin\theta}{P + Q\cos\theta}$$

α is the angle which the resultant makes with the direction of P. If β is the angle which the force P makes with the x-axis, the angle made by the resultant with x-axis will be $(\alpha + \beta)$.

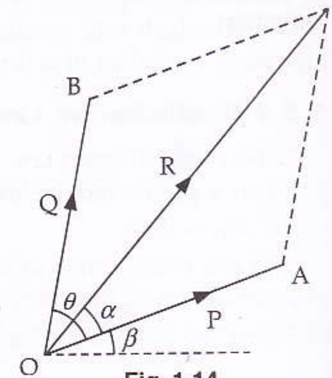


Fig. 1.14

Special cases:

(*i*) When the two forces are equal and θ is the angle between them:

$$R = -\sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{P^2 + P^2 + 2P \times P\cos\theta}$$

$$= \sqrt{2P^2(1 + \cos\theta)} = \sqrt{2P^2 \times 2\cos^2\frac{\theta}{2}} = 2P\cos\frac{\theta}{2} \quad \text{...(1.3)}$$

and $$\alpha = \tan^{-1}\left(\frac{Q\cos\theta}{P + Q\cos\theta}\right) = \tan^{-1}\left(\frac{P\sin\theta}{P + P\cos\theta}\right) = \tan^{-1}\left(\frac{\sin\theta}{1 + \cos\theta}\right)$$

$$= \tan^{-1}\left[\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right] = \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\therefore \quad \alpha = \frac{\theta}{2} \quad \text{...(1.4)}$$

i.e., the resultant bisects the angle between the forces.

(*ii*) When the two forces act at right angles; *i.e*, $\theta = 90°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 90°} = \sqrt{P^2 + Q^2} \quad (\because \cos 90° = 0) \quad \text{...(1.5)}$$

and $$\alpha = \tan^{-1}\frac{Q\sin 90°}{P + Q\cos 90°} = \tan^{-1}\left(\frac{Q}{P}\right) \quad (\because \sin 90° = 1)$$

(*iii*) When the two forces act in the same line and same sense, *i.e.*, $\theta = 0^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 0^\circ} = \sqrt{P^2 + Q^2 + 2PQ} \qquad (\because \cos 0^\circ = 1)$$

$$= P + Q \qquad ...(1.6)$$

Apparently the resultant is maximum when the forces are collinear and act in the same direction.

(*iv*) When the two forces have the same line of action but opposite senses, *i.e.*, $\theta = 180^\circ$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ} = \sqrt{P^2 + Q^2 - 2PQ} \qquad (\because \cos 180^\circ = -1)$$

$$= P - Q \qquad ...(1.7)$$

Obviously the resultant is minimum when the two forces are collinear but act in opposite direction.

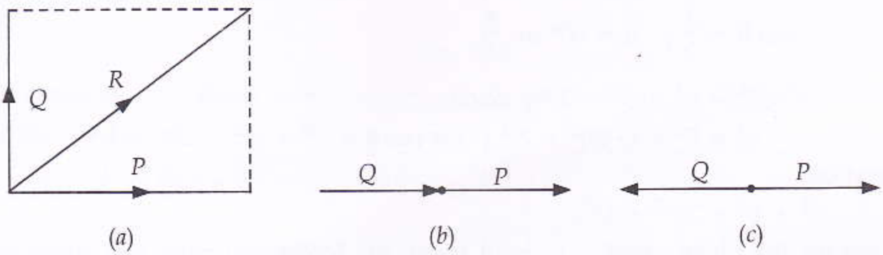


Fig. 1.15

The results obtained vide equations (1.5), (1.6) and (1.7) have been represented in Fig 1.15. These results lead us to conclude that when the forces acting on a body are collinear, their resultant is equal to the algebraic sum of the forces.

EXAMPLE 1.1

(*i*) **Two forces of equal mangnitude P act at an angle θ to each other. What will be their resultant ?**

(*ii*) **Resultant of two equal forces is equal to either of them. Determine the angle between the forces.**

(*iii*) **The resultant of two forces (P + Q) and (P – Q) equals $\sqrt{3P^2 + Q^2}$. Show that the forces are then inclined to each other at an angle of 60°.**

(*iv*) **The resultant of two forces P and Q is R. If one of the forces is reversed in direction, The resultant is S. Then for the identity**

$$\mathbf{R^2 + S^2 = 2\,(P^2 + Q^2)}$$

to hold good, show that the forces can have any angle of inclination between them.

Solution : Solution to these forces stems from the parallelogram law of forces.

(*i*) $$R^2 = P^2 + Q^2 + 2PQ\cos\theta = P^2 + P^2 + 2P^2\cos\theta$$

$$= 4P^2\left(\frac{1+\cos\theta}{2}\right) = 4P^2\cos^2\frac{\theta}{2}$$

$\therefore$ $$R = 2P\cos\frac{\theta}{2}$$

(*ii*) $R = 2P\cos\dfrac{\theta}{2}$ as outlined above.

Since the resultant equals either of the two equal forces,

$$P = 2P\cos\frac{\theta}{2}; \quad \cos\frac{\theta}{2} = \frac{1}{2}$$

$$\frac{\theta}{2} = 60^\circ$$

$$\therefore \quad \theta = 120^\circ \text{ or } \frac{2\pi}{3}$$

(*iii*) $$3P^2 + Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cos\theta$$

$$= 2P^2 + 2Q^2 + 2(P^2 - Q^2)\cos\theta$$

or $$(P^2 - Q^2) = 2(P^2 - Q^2)\cos\theta$$

or $$\cos\theta = \frac{1}{2}; \quad \theta = 60^\circ \text{ or } \frac{\pi}{3}$$

(*iv*) $$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$S^2 = P^2 + (-Q)^2 + 2P(-Q)\cos\theta = P^2 + Q^2 - 2PQ\cos\theta$$

upon summation

$$R^2 + S^2 = 2(P^2 + Q^2)$$

Obviously for the given identify to hold good, the forces can have any angle of inclination between them.

EXAMPLE 1.2

The resultant of two forces P and Q is at right angles to P. Show that the angle between the forces is $\cos^{-1}(-P/Q)$.

Solution : Let θ be the angle between the two given forces P and Q. Then the inclination α which the resultant makes with force P is

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$$

Since $\alpha = 90^\circ$ (given), we get

$$\tan 90^\circ = \frac{Q\sin\theta}{P + Q\cos\theta}$$

$$\infty = \frac{Q\sin\theta}{P + Q\cos\theta}$$

This is possible only when the denominator is zero

$$P + Q\cos\theta = 0; \ \cos\theta = (-P/Q)$$

$$\therefore \quad \theta = \cos^{-1}(-P/Q)$$

EXAMPLE 1.3

Two locomotives on opposite banks of a canal pull a vessel moving parallel to the banks by means of two horizontal ropes. The tensions in these ropes have been measured to be 20 kN and 24 kN while the angle between them is 60°. Find the resultant pull on the vessel and the angle between each of the ropes and the sides of the canal.

Solution : The vessel is acted upon at point O by the force system as depicted in the figure given below:

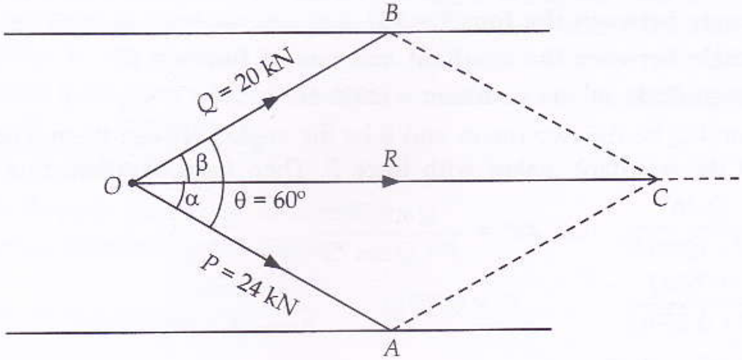


Fig. 1.16

The resultant force exerted on the vessel can be worked out by applying the parallelogram law of forces. That is

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} = \sqrt{24^2 + 20^2 + 2 \times 24 \times 20 \times \cos 60^\circ}$$

$$= \sqrt{576 + 400 + 480} = \sqrt{1456} = \mathbf{38.16\ N}$$

The inclination of resultant with direction of force P is

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{20\sin 60^\circ}{24 + 20\cos 60^\circ} = \frac{20 \times 0.866}{24 + 20 \times 0.5} = 0.5094$$

$$\alpha = \mathbf{27^\circ}$$

$$\therefore \quad \beta = 60 - 27 = \mathbf{33^\circ}$$

EXAMPLE 1.4

The magnitude of two forces is such that when acting at right angles produce a resultant force of and when acting at 60 degree produce a resultant equal to $\sqrt{28}$. Work out the magnitude of the two forces.

Solution : Let P and Q be the magnitude of the two forces. From parallelogram law of forces,

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

where θ is the angle between the two forces.

Then according to the given conditions,

$$20 = P^2 + Q^2 + 2PQ\cos 90^\circ = P^2 + Q^2 \qquad (\because \cos 90^\circ = 0)$$

and

$$28 = P^2 + Q^2 + 2PQ\cos 60^\circ = P^2 + Q^2 + PQ \qquad \left(\because \cos 60^\circ = \frac{1}{2}\right)$$

From the above identities $PQ = 8$; $Q^2 = 64/P^2$

That gives: $20 = P^2 + 64/P^2$

or $P^4 - 20P^2 + 64 = 0$

or $(P^2 - 16)(P^2 - 4) = 0$

$\therefore \quad P = 2$ or 4 units

The corresponding values of force Q will be 4 or 2 units.

EXAMPLE 1.5

Find the magnitude of two concurrent forces which conform to the following data:

angle between the forces = 75°

angle between the resultant and one of forces = 45°

magnitude of the resultant = 2000 N

Solution : Let P and Q be the two forces and θ be the angle between them. Further, let α be the inclination which the resultant makes with force P. Then from parallelogram law of forces

$$(i)\ \tan\alpha = \frac{Q\sin\theta}{P+Q\sin\theta};\quad \tan 45° = \frac{Q\sin 75°}{P+Q\cos 75°}$$

$$1 = \frac{0.996Q}{P+0.259Q} \qquad \therefore \quad P = 0.707Q$$

$$(ii)\ R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$(2000)^2 = (0.707Q)^2 + Q^2 + 2 \times (0.707Q) \times Q\cos 75°$$

$$= 0.5Q^2 + Q^2 + 0.366Q^2 = 1.866Q^2$$

$$\therefore \qquad Q = \sqrt{\frac{(2000)^2}{1.866}} = \mathbf{1464\ N}$$

and $$P = 0.707Q = 0.707 \times 1464 = \mathbf{1035\ N}$$

EXAMPLE 1.6

Two forces, one of which is double the other has resultant of 260 N. If the direction of the larger force is reversed and the other remains unaltered, the resultant reduces to 180 N. Determine the magnitude of the force and the angle between the forces.

Solution : Let P and Q (= $2P$) be the two forces and θ be the angle of inclination between them. From parallelogram law of forces

$$R^2 = P^2 + Q^2 + 2PQ\cos\theta$$

Then according to the given conditions,

$$260^2 = P^2 + (2P)^2 + 2P(2P)\cos\theta$$

$$= P^2 + 4P^2 + 4P^2\cos\theta = 5P^2 + 4P^2\cos\theta \qquad ...(i)$$

and $$180^2 = P^2 + (-2P)^2 + 2P(-2P)\cos\theta$$

$$= P^2 + 4P^2 - 4P^2\cos\theta = 5P^2 - 4P^2\cos\theta \qquad ...(ii)$$

Adding identities (i) and (ii) we get

$$10P^2 = 260^2 + 180^2 = 67600 + 32400 = 100000$$

$$\therefore \qquad P = \sqrt{\frac{100000}{10}} = \mathbf{100\ N}$$

$$Q = 2P = 2 \times 100 = \mathbf{200\ N}$$

Substituting $P = 100$ N in identity (*i*), we get

$$260^2 = 5 \times 100^2 + 4 \times 100^2\cos\theta$$

$$\cos\theta = \frac{260^2 - 5\times 100^2}{4 \times 100^2} = 0.44$$

$$\therefore \qquad \theta = \mathbf{63.896°}$$

EXAMPLE 1.7

Two forces equal to P and 2 P respectively act on a particle. When the first force is increased by 120 N and the second force is doubled, the direction of the resultant remains the same. Determine the value of force P.

Solution : Let α be the inclination which the resultant makes with force P. Then for the Ist case

$$\tan \alpha = \frac{2P\sin\theta}{P+2P\cos\theta} \quad \text{...}(i)$$

where θ is the angle between the two given forces.

Subsequently when the forces get changed

$$\tan \alpha = \frac{4P\sin\theta}{(P+120)+4P\cos\theta} \quad \text{...}(ii)$$

From identities (i) and (ii)

$$\frac{2P\sin\theta}{P+2P\cos\theta} = \frac{4P\sin\theta}{(P+120)+4P\cos\theta)}$$

or
$$\frac{1}{P+2P\cos\theta} = \frac{2}{(P+120)+4P\cos\theta}$$

or
$$(P + 120) + 4P \cos \theta = 2P + 4P \cos \theta$$

That gives: $\quad \boldsymbol{P = 120 \text{ N}}$

EXAMPLE 1.8

The resultant of two forces P and Q acting at a point is R. The resultant R gets doubled when Q is either doubled or its direction is reversed. Show that P, Q and R conform to the ratio

$$P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$$

Solution : From parallelogram law of forces, the resultant of forces$P$ and Q with θ angle of inclination between them is given by

$$R^2 = P^2 + Q^2 + 2PQ \cos \theta \quad \text{...}(i)$$

When Q is doubled

$$(2R)^2 = P^2 + (2Q)^2 + 2P \times (2Q) \cos \theta$$
$$4R^2 = P^2 + 4Q^2 + 4\,PQ \cos \theta \quad \text{...}(ii)$$

When Q is reversed in direction,

$$(2R)^2 = P^2 + (-\,Q)^2 + 2P \times (-\,Q) \cos \theta$$
$$4R^2 = P^2 + Q^2 - 2PQ \cos \theta \quad \text{...}(iii)$$

Adding identities (i) and (iii)

$$5R^2 = 2P^2 + 2Q^2 \quad \text{...}(iv)$$

Multiplying identity (iii) by 2 and adding the result to identity (ii)

$$12\,R^2 = 3P^2 + 6Q^2$$

or
$$4R^2 = P^2 + 2Q^2 \quad \text{...}(v)$$

From identities (iv) and (v)

$$5R^2 = 2P^2 + (4R^2 - P^2)$$

or
$$R^2 = P^2; \qquad R = P$$

Then from identity (v)

$$4P^2 = P^2 + 2Q^2$$

$$3P^2 = 2Q^2 \quad ; \quad Q = \sqrt{3/2}\,P$$

Therefore, $\quad P : Q : R \equiv P : \sqrt{3/2}\,P : P \equiv \sqrt{2} : \sqrt{3} : \sqrt{2}$

1.9.2. Equilibrium of a Particle

A particle will be in equilibrium when resultant of all the forces acting on it is zero

$$\text{Resultant} \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

For a particle to be in equilibrium:

$$R = 0; \quad (\Sigma F_x)^2 + (\Sigma F_y)^2 = 0$$

Now $(\Sigma F_x)^2$ and $(\Sigma F_y)^2$ are positive quantities and their sum cannot be zero unless each of then is zero.

$$\therefore \qquad \Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0 \qquad \text{...(1.8)}$$

Hence, if any number of forces acting at a particle are in equilibrium, then the algebraic sum of their resolved parts in any two perpendicular directions are separately zero.

Equations 1.8 are called the equations of equilibrium of a particle in plane.

A two-force body is subjected to only two forces. For its equilibrium, the resultant force must be zero. Accordingly the two forces must be collinear and must have the same magnitude but opposite direction. These aspects have been shown below in Fig. 1.17 (a) and (b).

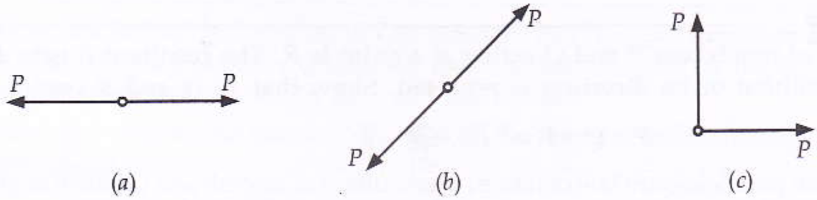


Fig. 1.17

The particle subjected to two forces of equal magnitude (Fig. 1.17 (c)) shall not be in equilibrium because the forces have different lines of action.

A three-force body is subjected to only three forces. For the equilibrium, the acting forces must be concurrent and must form a closed triangle. This aspect has been illustrated in Fig. 1.18.

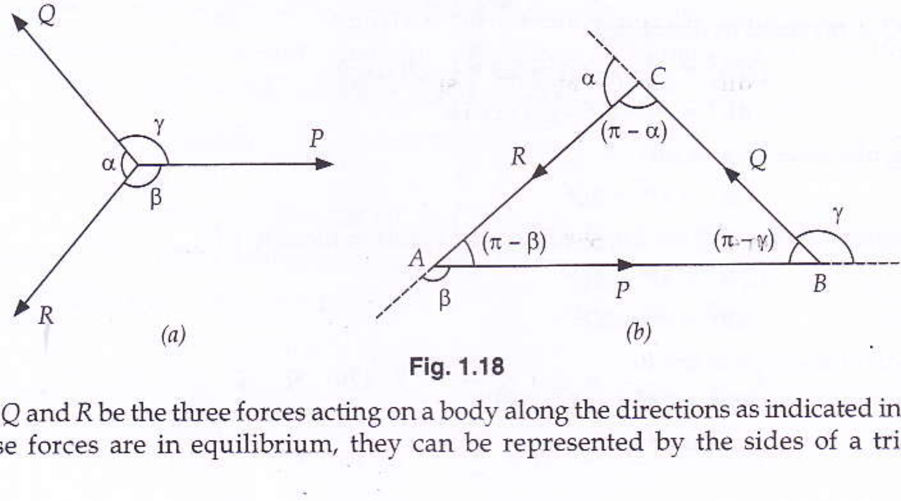


Fig. 1.18

Let P, Q and R be the three forces acting on a body along the directions as indicated in Fig1.18 (a). Since these forces are in equilibrium, they can be represented by the sides of a triangle ABC

[Fig 1.18 (*b*)] which is drawn so as to have its sides respectively parallel to the direction of forces. Thus

$$\vec{AB} = P; \quad \vec{BC} = Q; \quad \text{and} \quad \vec{CA} = R$$

The exterior and interior angles of the triangle ABC of forces will be as shown in Fig 1.18 (*b*). Applying sine rule for the triangle ABC

$$\frac{AB}{\sin(\pi - \alpha)} = \frac{BC}{\sin(\pi - \beta)} = \frac{CA}{\sin(\pi - \gamma)}$$

$$\therefore \qquad \frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

"If a body is in equilibrium under the action of three forces, then each force is proportional to the sine of the angle between the other two forces."

This is referred to as Lami's Theorem.

1.9.3 Triangle Law of Forces

"If two forces acting simultaneously on a body are represented by the sides of a triangle taken in order, their resultant is represented by the closing side of the triangle taken in the opposite order."

Consider two forces P and Q acting on the body as shown in Fig 1.19 (*a*).

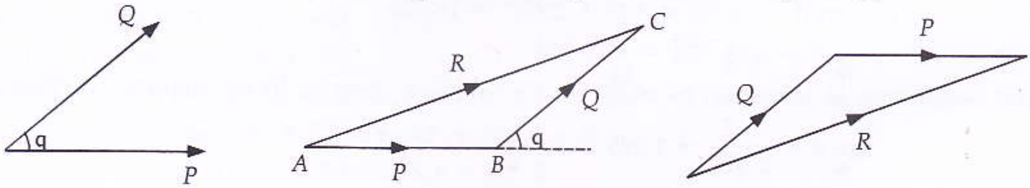


Fig. 1.19

Line AB be drawn to represent force P and BC to represent Q. The triangle ABC is completed by drawing the closing line AC. Line AC represents the resultant in magnitude, line of action and direction.

It is to be noted that addition of forces P and Q in any order gives the same resultant R.

EXAMPLE 1.9.

The resultant of two forces P and Q is 200 N and it makes an angle of 30° with the horizontal as shown in Fig. 1.20. Determine the magnitude of components P and Q.

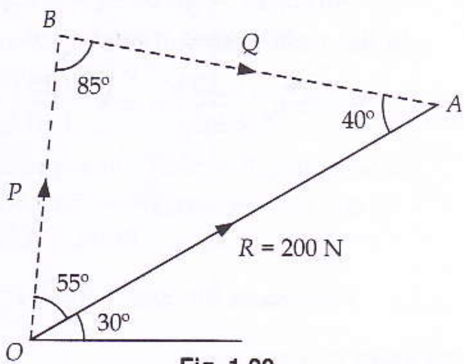


Fig. 1.20

Solution : The triangle OAB constitutes the force triangle with resultant directed along OA, and the components P and Q lying along directions OB and BA respectively.

Angle $OBA = 180 - 55° - 40° = 85°$

Applying sine rule to ΔOAB, we get

$$\frac{OA}{\sin 85°} = \frac{OB}{\sin 40°} = \frac{BA}{\sin 55°}$$

$$\text{or} \quad \frac{200}{\sin 85°} = \frac{P}{\sin 40°} = \frac{Q}{\sin 55°}$$

$$\therefore \quad P = 200 \times \frac{\sin 40°}{\sin 85°} = 200 \times \frac{0.643}{0.996} = \mathbf{129.1\ N} \text{ from } O \text{ to } B$$

$$Q = 200 \times \frac{\sin 55°}{\sin 85°} = 200 \times \frac{0.819}{0.996} = \mathbf{164.4\ N} \text{ from } B \text{ to } A$$

EXAMPLE 1.10.

The force system shown in the adjoining figure has a resultant of 2 kN acting up along the Y-axis. Make calculations for the force F both in magnitude and direction.

Solution : The resultant force $R = 2$ kN acts upwards along O-Y axis. Resolving all the forces in vertical direction, we get

$$F \sin\theta - 2.4 \sin 30^\circ = 2$$

or $$F \sin\theta = 2 + 2.4 \sin 30^\circ = 2 + 2.4 \times \frac{1}{2} = 3.2 \text{ kN} \quad ...(i)$$

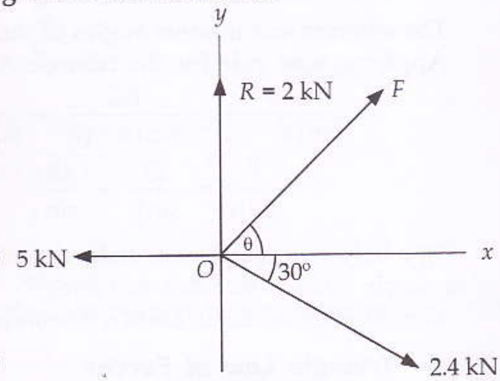


Fig. 1.21

Resolving all the forces in horizontal direction, we get

$$F \cos\theta + 2.4 \cos 30^\circ - 5 = 0$$

$$F \cos\theta = 5 - 2.4 \cos 30^\circ = 5 - 2.4 \times 0.866 = 2.922 \quad ...(ii)$$

Squaring identities (*i*) and (*ii*) and then adding, we get

$$F^2 (\sin^2\theta + \cos^2\theta) = 3.2^2 + 2.922^2 = 18.778$$

$$\therefore \quad F = \sqrt{18.778} = \mathbf{4.33\ kN}$$

The orientation of force can be obtained by dividing identity (*i*) by identity (*ii*). That gives

$$\tan\theta = \frac{3.2}{2.922} = 1.095$$

$$\therefore \quad \theta = \mathbf{47.6^\circ}$$

EXAMPLE 1.11

Two cables which have known tensions of 40 N and 60 N are attached to the top of a tower *AB* as shown in Fig. 1.22. What tension will be induced in the guy wire *AC* if the resultant of the forces exerted at the top *A* by the cables acts vertically downwards?

Solution : Since the resultant of the forces acts vertically downwards, we have $\Sigma F_x = 0$ and that gives

$$40 \cos 15^\circ - 60 \cos 30^\circ + T \cos\theta = 0 \quad ...(i)$$

In the right angled triangle ABC

$$\theta = \tan^{-1}\left(\frac{AB}{BC}\right) = \tan^{-1}\left(\frac{15}{10}\right) = 56.31^\circ$$

Substituting $\theta = 56.31^\circ$ in expression (*i*), we get

$$40 \cos 15^\circ - 60 \cos 30^\circ + T \cos 56.31^\circ = 0$$

or $$38.64 - 51.96 + 0.555\, T = 0$$

$$\therefore \quad \text{Tension in the guy wire } T = \frac{51.96 - 38.64}{0.555} = \mathbf{24\ N}$$

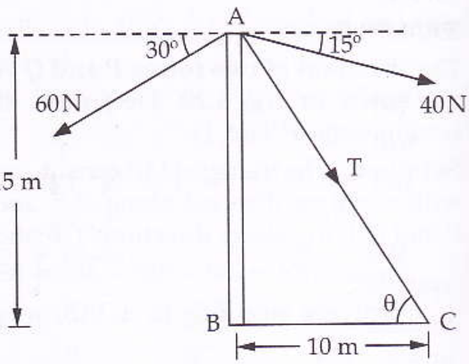


Fig. 1.22

EXAMPLE 1.12

A block of weight 300 N is acted upon by a horizontal force $F = 500$ N and a pressure P exerted by the inclined as shown in Fig. 1.23. If the resultant of the force system lies parallel to the plane, work out the magnitude of pressure P and the resultant force.

Solution : Let the co-ordinates be chosen parallel and perpendicular to the plane.

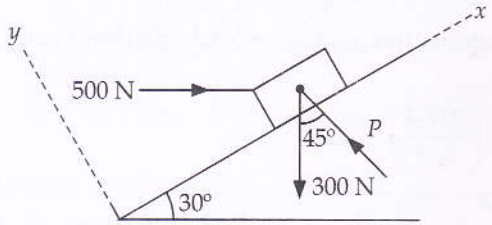


Fig. 1.23

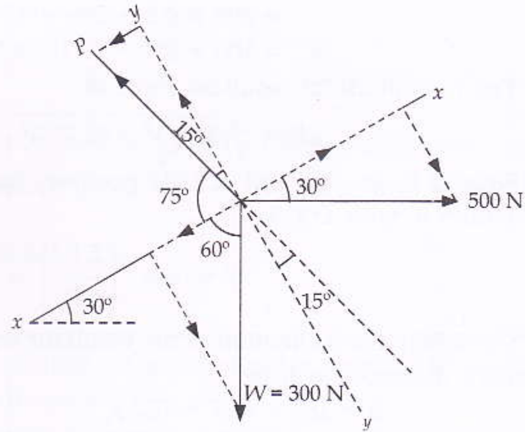


Fig. 1.24

As the resultant R lies along the plane, its component perpendicular to the plane is zero. Then with reference to Fig. 1.24.

$$R = -P \sin 15^\circ + 500 \cos 30^\circ - 300 \cos 60^\circ$$
$$= -0.259P + 433 - 150$$
$$= -0.259P + 283$$

Also
$$P \cos 15^\circ - 500 \sin 30^\circ - 300 \cos 60^\circ = 0$$
$$P \cos 15^\circ = 500 \sin 30^\circ + 300 \sin 60^\circ$$
$$P \times 0.966 = 250 + 259.8 = 509.8$$
$$P = \frac{509.8}{0.966} = 527.74 \text{ N}$$
$$\therefore \quad R = -0.259 \times 527.74 + 283 = \mathbf{146.31 \text{ N}}$$

EXAMPLE 1.13

Determine the magnitude and direction of the resultant of the following set of forces acting on a body

(*i*) **200 N inclined 30° with east towards north,**

(*ii*) **250 N towards the north,**

(*iii*) **300 N towards north west, and**

(*iv*) **350 N inclined at 40° with west towards south.**

What will be the equilibrant of the given force system ?

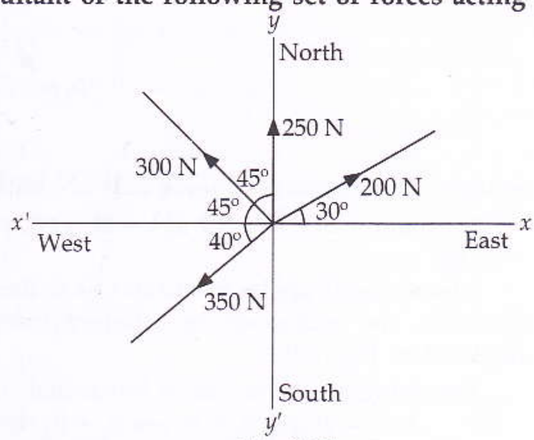


Fig. 1.25

Solution : The given force system has been depicted in Fig. 1.25.

Resolving all the forces along X-direction

$$\Sigma F_x = 200 \cos 30^\circ + 250 \cos 90^\circ + 300 \cos 135^\circ + 350 \cos 220^\circ$$
$$= 200 \times 0.866 + 250 \times 0 + 300 \times (-0.707) + 350 \times (-0.766)$$
$$= 173.2 + 0 - 212.1 - 268.1 = -307 \text{ N (along } OX')$$

Resolving all the forces along Y-direction

$$\Sigma F_y = 200 \sin 30^\circ + 250 \sin 90^\circ + 300 \sin 135^\circ + 350 \sin 220^\circ$$

$$= 200 \times 0.5 + 250 \times 1 + 300 \times 0.707 + 350 \times (-0.642)$$
$$= 100 + 250 + 212.1 - 224.7 = 337.4 \text{ N (along OY)}$$

The magnitude of resultant force is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-307)^2 + (337.4)^2} = \mathbf{456\ N}$$

Since ΣF_x is –ve and ΣF_y is positive, the resultant lies in the second quadrant and its inclination θ' with OX' is

$$\theta' = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{337.4}{307}\right) = 47.7°$$

Obviously the inclination of the resultant with OX (positive direction) will be

$$\theta = 180 - 47.7 = 132.3°$$

An equal and opposite force will be required to h old the body in position. The equilibrant will thus have a magnitude of 456 N, inclined at 47.7° to the x-axis (fourth quadrant) and is opposite to the resultant. Both the resultant and equilibrant are shown in Fig. 1.26.

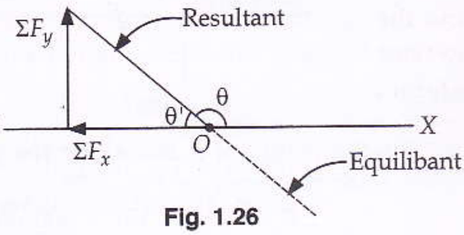


Fig. 1.26

EXAMPLE 1.14

Determine the resultant, both in magnitude and direction, of the four forces acting on the body as shown in the figure given below:

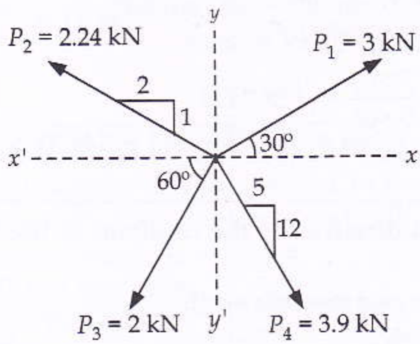


Fig. 1.27

Solution : Inclination of force 2.24 kN with OX′ = $\tan^{-1}(1/2) = 26.56°$

Inclination of force 3.9 kN with OY′ = $\tan^{-1}(5/12) = 22.62°$

Measuring all angles from axis O-X in the anticlockwise direction, the inclination of different forces will be as depicted in Fig. 1.28.

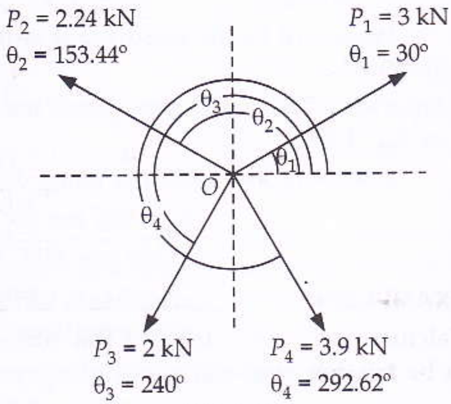


Fig. 1.28

Resolving all the forces in horizontal direction,

$$\Sigma F_x = P_1 \cos\theta_1 + P_2 \cos\theta_2 + P_3 \cos\theta_3 + P_4 \cos\theta_4$$
$$= 3\cos 30° + 2.24 \cos 153.44° + 2\cos 240° + 3.9 \cos 292.62°$$
$$= 2.598 - 2.004 - 1.0 + 1.50 = 1.094 \text{ kN}$$

Resolving all the forces in vertical direction,

$$\Sigma F_y = P_1 \sin\theta_1 + P_2 \sin\theta_2 + P_3 \sin\theta_3 + P_4 \sin\theta_4$$

$$= 3 \sin 30^\circ + 2.24 \sin 153.44^\circ + 2 \sin 240^\circ + 3.9 \sin 292.62^\circ$$
$$= 1.50 + 1.00 - 1.73 - 3.60 = -2.83 \text{ kN}$$

The magnitude of the resultant force is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(1.094)^2 + (-2.83)^2} = \sqrt{1.197 + 8.009} = \mathbf{3.034\ kN}$$

and the inclination of resultant with the horizontal is

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{-2.83}{1.094}\right) = \mathbf{68.86^\circ}$$

EXAMPLE 1.15

Find the resultant of forces 2, $\sqrt{3}$, 5, $\sqrt{3}$ and 2 N that act at an angular point of a regular hexagon towards the other angular points taken in order.

Solution : Let *ABCDEF* be the regular hexagon with A as the angular point where forces are acting. The forces are:

2 N along *AB* ; $\sqrt{3}$ along *AC* ; 5 N along *AD* ;
$\sqrt{3}$ N along *AE* and 2 N along *AF*

Sum of the interior angles of a regular polygon

$$= (2n - 4) \text{ right angles}$$

where n is the number of sides of the polygon. For a hexagon, $n = 6$ and therefore sum of interior angles

$$= (2 \times 6 - 4) \times 90 = 720^\circ$$

$$\therefore \text{Value of each interior angle} = \frac{720}{6} = 120^\circ$$

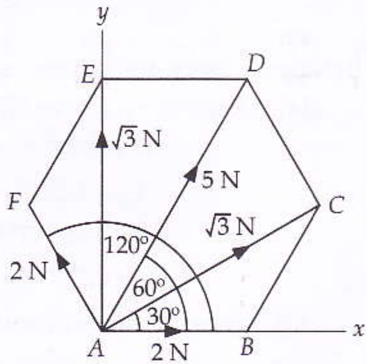


Fig. 1.29

Since included angle of a regular hexagon is 120°, the angular position of these forces will be as indicated in Fig. 1.29.

Resolving all the forces horizontally, *i.e.*, along *AX*

$$\Sigma F_x = 2 + \sqrt{3} \cos 30^\circ + 5 \cos 60^\circ + \sqrt{3} \cos 90^\circ + 2 \cos 120^\circ$$
$$= 2 + \sqrt{3} \times \frac{\sqrt{3}}{2} + 5 \times \frac{1}{2} + \sqrt{3} \times 0 + 2\left(-\frac{1}{2}\right) = 5 \text{ N}$$

Resolving all the forces vertically, i.e., along *AY*

$$\Sigma F_y = \sqrt{3} \sin 30^\circ + 5 \sin 60^\circ + \sqrt{3} \sin 90^\circ + 2 \sin 120^\circ$$
$$= \sqrt{3} \times \frac{1}{2} + 5 \times \frac{\sqrt{3}}{2} + \sqrt{3} \times 1 + 2 \times \frac{\sqrt{3}}{2} = 5\sqrt{3} \text{ N}$$

The magnitude of the resultant force is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{5^2 + (5\sqrt{3})^2} = \sqrt{100} = \mathbf{10\ N}$$

and the inclination of the resultant with horizontal is

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\frac{5\sqrt{3}}{5} = \tan^{-1}(\sqrt{3}) = \mathbf{60^\circ}$$

EXAMPLE 1.16

Calculate the tensile force in the cables AB and BC as shown in Fig. 1.30. Presume the pulleys to be frictionless.

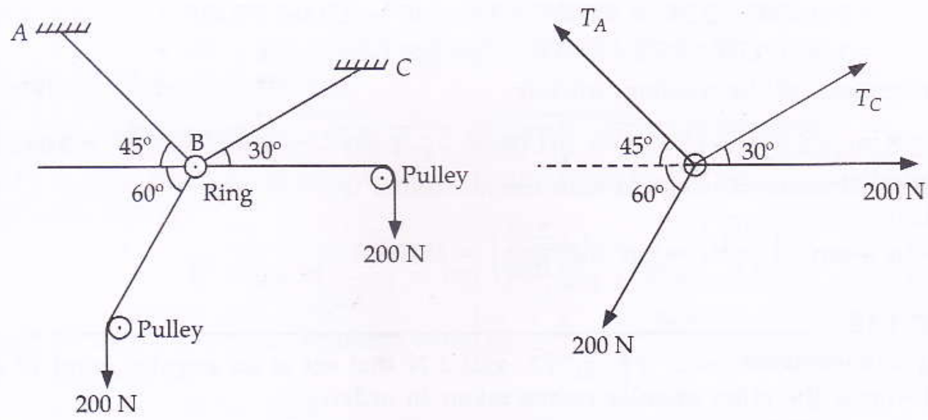


Fig. 1.30

Solution : Since the system is in equilibrium,

$\Sigma F_x = 0 ;\quad 0 = -T_A \cos 45° + T_C \cos 30° + 200 - 200 \cos 60°$

$= -0.707\, T_A + 0.866\, T_C + 200 - 100$

or $\quad -T_A + 1.22\, T_C + 141.0 = 0 \qquad \ldots(i)$

$\Sigma F_y = 0 ;\quad 0 = T_A \sin 45° + T_C \sin 30° - 200 \sin 60° = 0.707\, T_A + 0.5\, T_C - 173$

or $\quad T_A + 0.707\, T_C - 244 = 0 \qquad \ldots(ii)$

Adding expressions (*i*) and (*ii*),

$$1.927\, T_C - 103 = 0 ; \quad T_C = \frac{103}{1.927} = \mathbf{53.45\ N}$$

Then from expression (*ii*)

$$T_A + 0.707 \times 53.45 - 244 = 0$$

$$\therefore \quad T_A = 244 - 0.707 \times 53.45 = 244 - 37.79 = \mathbf{206.21\ N}$$

EXAMPLE 1.17

A right circular roller of weight W rests on a smooth horizontal plane and is held in position by string AC as shown in Fig. 1.31. The roller is pulled by a horizontal force P applied to its centre. Find the reaction at B and the tension of the string.

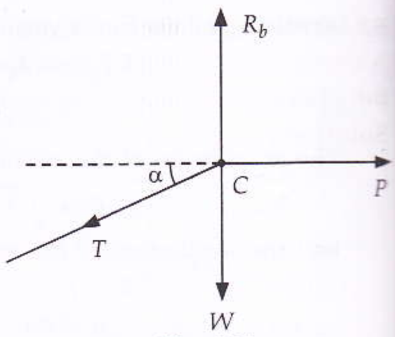


Fig. 1.31

Solution : The roller is in equilibrium under the action of following set of forces:

(*i*) Self weight acting vertically downward

(*ii*) Reaction R_b acting vertically upward

(*iii*) Force P acting horizontally towards right

(*iv*) Tension T of the string acting in the direction from C towards A

For these concurrent forces, the conditions of equilibrium are:

$\Sigma F_x = 0 ;\quad P - T \cos \alpha = 0$

$$\therefore \quad T = \frac{P}{\cos \alpha}$$

$$\Sigma F_y = 0; \qquad R_b = T\sin\alpha - W = 0$$

$$\therefore R_b = T\sin\alpha + W$$

$$= \frac{P}{\cos\alpha}\sin\alpha + W = P\tan\alpha + W$$

EXAMPLE 1.18

A string ABC of length 50 cm is tied to two points A and C at the same level. A smooth ring of weight 500 N which can slide along the string is at B, 30 cm away from A along the string and pulled by a horizontal force P as shown in Fig. 1.32. If point B is 15 cm below the level of AC, determine the magnitude of force P. It may be presumed that tensions in the string on both sides of B are same.

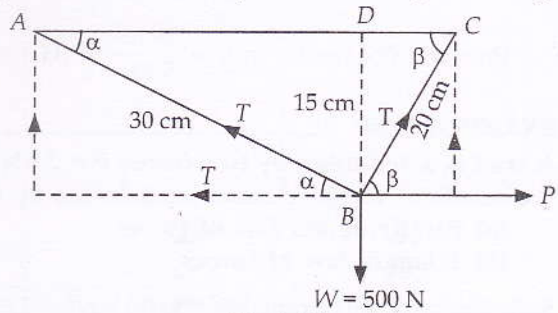


Fig. 1.32

Solution : From the geometrical configuration of the given system

$$\sin\alpha = \frac{15}{30};\ \alpha = 30^\circ$$

$$\sin\beta = \frac{15}{30};\ \beta = 48.59^\circ$$

Since the system of forces is in equilibrium,

$$\Sigma F_x = 0 \text{ and } \Sigma F_y = 0$$

Resolution of forces in the horizontal direction gives,

$$P + T\cos\beta - T\cos\alpha = 0$$

or $\quad P + T\cos 48.59^\circ - T\cos 30^\circ = 0$

or $\quad P = T\cos 30^\circ - T\cos 48.59^\circ = 0.866\,T - 0.661\,T = 0.205\,T$

Resolution of forces in the vertical direction gives,

$$T\sin\beta + T\sin\alpha - 500 = 0$$

or $\quad T\sin 48.59^\circ + T\sin 30^\circ = 500$

or $\quad 0.75\,T + 0.5\,T = 500$

$$\therefore \quad T = \frac{500}{1.25} = \mathbf{400\ N}$$

and $\quad P = 0.205\,T = 0.205 \times 400 = \mathbf{82\ N}$

EXAMPLE 1.19

A uniform ladder weighing 80 N rests against a smooth vertical wall at a height of 12 m above the ground; the foot of ladder being 10 m from the wall. Determine the pressure due to wall.

Solution : The three foces acting in the system are:

(*i*) reaction R_1 due to wall (acts at rights angles to wall as the wall is smooth)

(*ii*) reaction R_2 of the ground

(*iii*) weight W of the ladder

Under equilibrium conditions, the reaction R_2 passes through the point of intersection of R_1 and W. The three forces R_1, R_2 and W are then represented respectively by the sides AN, OA and ON of the triangle OAN. (Fig. 1.33)

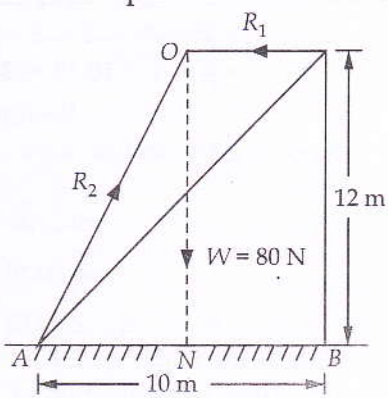


Fig. 1.33.

$$\frac{R_1}{AN} = \frac{R_2}{OA} = \frac{W}{ON};$$

$$AN = 5\text{ m}$$

$$OA = \sqrt{5^2 + 12^2} = 13\text{ m}$$

$$\therefore \quad \frac{R_1}{5} = \frac{R_2}{13} = \frac{80}{12}$$

Pressure R_1 due to wall $= \dfrac{80 \times 5}{12} = \mathbf{33.3\ N}$

EXAMPLE 1.20

A stud is acted upon by two forces $P = 2$ kN and $Q = 3$ kN as shown in Fig. 1.34. Determine the resultant of these concurrent forces by using

(*a*) Parallelogram law of forces

(*b*) Triangle law of forces.

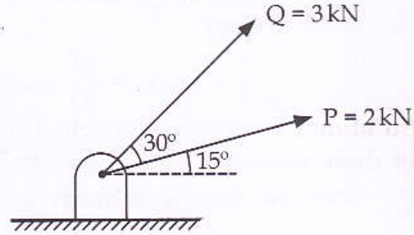


Fig. 1.34

Solution : (*a*) Resultant $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

where θ is the angle between the forces P and Q; $\theta = 30°$

$$R = \sqrt{2^2 + 3^2 + 2 \times 2 \times 3\cos 30°}$$

$$= \sqrt{4 + 9 + 10.39} = \mathbf{4.836\ N}$$

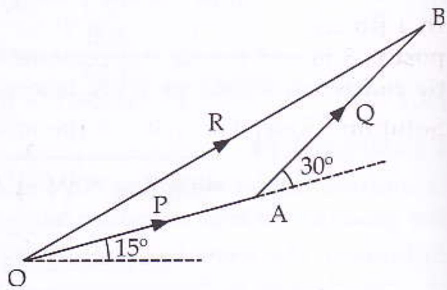
If α is the direction of resultant with respect to direction of force P, then

$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{3\sin 30°}{2 + 3\cos 30°} = \frac{3 \times 0.5}{2 + 3 \times 0.866} = 0.326$$

$$\alpha = \tan^{-1} 0.326 = 18.06°$$

$\therefore$ Angle made by resultant with x-axis $= 18.06 + 15 = \mathbf{33.06°}$

(*b*) The force diagram of these concurrent forces is as shown below:



$$P = OA = 2\text{ kN}$$

$$Q = AB = 3\text{ kN}$$

and the included angle between these forces is

$$\angle OAB = 180° - 30° = 150°$$

Then applying the law of cosines

$$R^2 = P^2 + Q^2 - 2PQ\cos(OAB)$$

$$= 2^2 + 3^2 - 2 \times 2 \times 3\cos 150°$$

$$= 4 + 9 + 10.39 = 23.39$$

$$\therefore \quad \text{Resultant R} = \sqrt{23.39} = \mathbf{4.836\ N}$$

Applying the law of sines

$$\frac{R}{\sin OAB} = \frac{Q}{\sin AOB}$$

$$\sin AOB = \frac{Q}{R}\sin OAB = \frac{3}{4.836}\sin 150 = 0.310$$

$$\therefore \quad \angle AOB = \sin^{-1}(0.31) = 18.06°$$

This is the angle which the resultant makes with direction of force P.

$\therefore$ Angle of inclination of the resultant with the horizontal $= 15 + 18.06 = \mathbf{33.06°}$

EXAMPLE 1.21

A weight of 2000 N is supported by two chains *AC* and *BC* as shown in Fig. 1.35. Determine the tension in each chain.

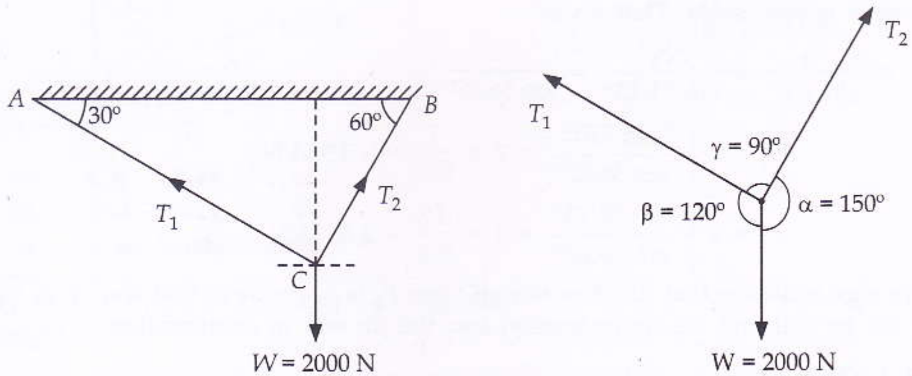


Fig. 1.35

Solution : Let T_1 and T_2 be the tensions in chains AC and BC respectively. Since the lines of action of these tensions and the weight W meet at a point, Lami's theorem can be applied. That is

$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\gamma}$$

or

$$\frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{2000}{\sin 90^\circ}$$

$$\therefore \quad T_1 = 2000 \times \frac{\sin 150^\circ}{\sin 90^\circ} = 2000 \times \frac{0.5}{1} = 1000 \text{ N}$$

$$T_2 = 2000 \times \frac{\sin 120^\circ}{\sin 90^\circ} = 2000 \times \frac{0.866}{1} = 1732 \text{ N}$$

EXAMPLE 1.22

In a jib crane, the jib and the tie rod are 5 m and 4 m long respectively. The height of crane post is 3 m and the tie rod remains horizontal. Determine the forces produced in the jib and tie rod when a load of 2 kN is suspended at the crane head.

Solution : Refer Fig. 1.36 for the arrangement of the system.

$$\sin\theta = \frac{3}{5} = 0.6\,; \quad \theta = 36.87^\circ$$

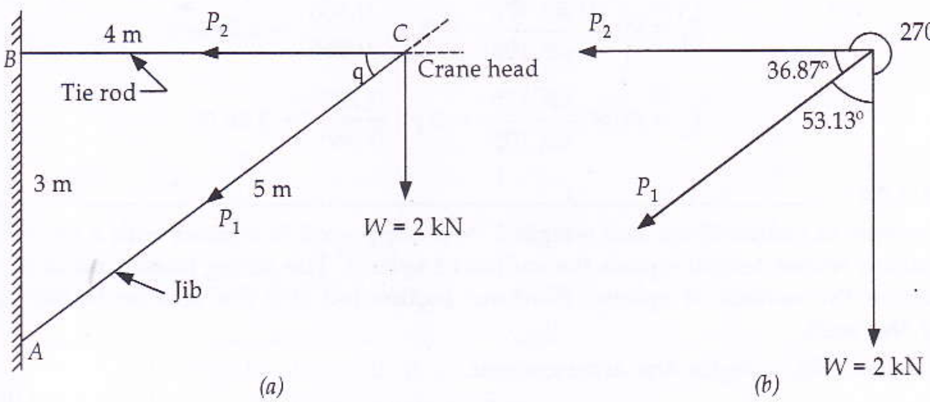


Fig. 1.36

Let P_1 and P_2 be the forces developed in the jib and tie rod respectively. The three forces P_1 P_2 and W are shown in Fig. 1.45 (*b*) with the angles between the forces calculated from the given directions. The lines of action of forces P_1, P_2 and weight W meet at the point C, and therefore Lami's theorem is applicable. That gives

$$\frac{P_1}{\sin 270^\circ} = \frac{P_2}{\sin 53.13^\circ} = \frac{2}{\sin 36.87^\circ}$$

$$\therefore \qquad P_1 = 2 \times \frac{\sin 270^\circ}{\sin 36.87^\circ} = 2 \times \frac{1}{0.6} = \mathbf{-3.33\ kN}$$

$$P_2 = 2 \times \frac{\sin 53.13^\circ}{\sin 36.87^\circ} = 2 \times \frac{0.8}{0.6} = \mathbf{2.667\ kN}$$

The – ve sign indicates that the direction of force P_1 is opposite to that shown in Fig. 1.45 (*a*) Obviously the tie rod will be under tension and the jib will in compression.

EXAMPLE 1.23

A machine weighing 5 kN is supported by two chains attached to some point on the machine One chain goes to the hook in the ceiling and has an indication of 45° with the horizontal The other chain goes to the eye bolt in the wall and is inclined at 30° to the horizontal. Make calculations for the tensions induced in the chain.

Solution : Refer Fig. 1.37 for the arrangement. The machine is acted upon by the following set of forces:

(*i*) weight of machine W = 5 kN acting vertically downwards,

(*ii*) tension T_1 in the chain OA which goes to hook in the ceiling,

(*iii*) tension T_2 in the chain OB which goes to the eye bolt.

These forces are concurrent and meet at point O. Applying Lami's theorem

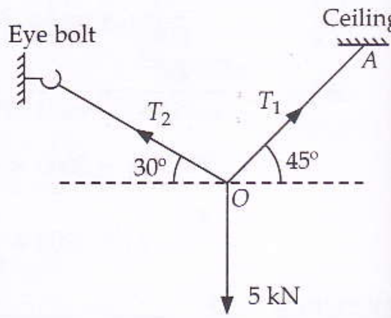


Fig. 1.37.

$$\frac{T_1}{\sin(90+30)} = \frac{T_2}{\sin(90+45)} = \frac{W}{\sin(180-30-45)}$$

or
$$\frac{T_1}{\sin 120^\circ} = \frac{T_2}{\sin 135^\circ} = \frac{W}{\sin 105^\circ}$$

$$\therefore \qquad T_1 = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 5 \times \frac{0.866}{0.966} = \mathbf{4.48\ kN}$$

$$T_2 = W \times \frac{\sin 135}{\sin 105} = 5 \times \frac{0.707}{0.966} = \mathbf{3.66\ N}$$

EXAMPLE 1.24

A smooth sphere of radius 15 cm and weight 2 N is supported in contact with a smooth vertical wall by a string whose length equals the radius of sphere. The string joins a point on the wall and a point on the surface of sphere. Workout inclination and the tension in the string and reaction of the wall.

Solution : Refer. Fig. 1.38 for the arrangement.

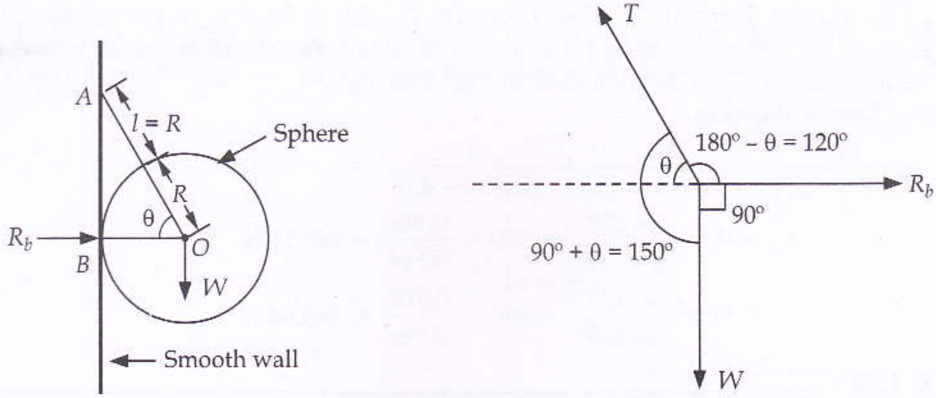


Fig. 1.38

In triangle AOB,

$$\cos\theta = \frac{OB}{OA} = \frac{R}{2R} = \frac{1}{2}; \quad \theta = \mathbf{60^\circ}$$

The sphere is in equilibrium under the action of following forces:

(*i*) Weight $W = 2$ N of the sphere which acts vertically downwards through the centre O.

(*ii*) Tension T in the string

(*iii*) Reaction R_b of the wall at the point of contact B. Since the wall is smooth, this reaction acts perpendicular to the wall.

These three forces are concurrent, *i.e.*, meet at point O and as such the Lami's theorem is applicable.

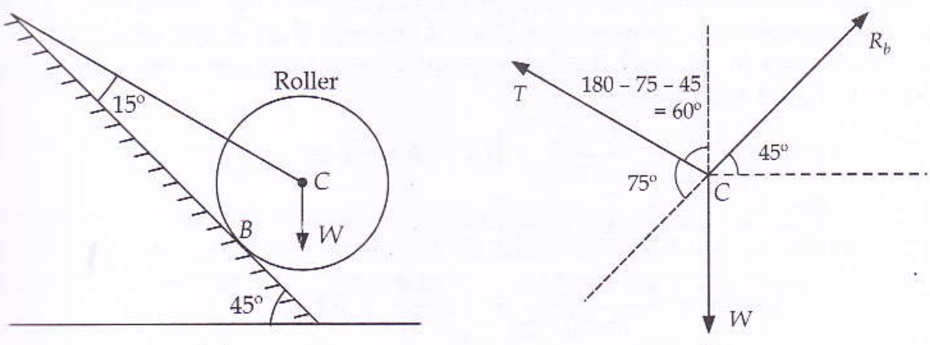
$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin 120^\circ} = \frac{R_b}{\sin 150^\circ}$$

$$\therefore \quad T = W\,\frac{\sin 90^\circ}{\sin 120^\circ} = 2 \times \frac{1}{0.866} = \mathbf{2.31\ N}$$

$$R_b = W\,\frac{\sin 150^\circ}{\sin 120^\circ} = 2 \times \frac{0.5}{0.866} = \mathbf{1.15\ N}$$

EXAMPLE 1.25

A roller of weight 500 N rests on a smooth inclined plane and is kept free from rolling down by a string as shown in Fig. 1.39. Work out tension in the string and reaction at the point of contact *B*.



(*a*) (*b*)

Fig. 1.39

Solution : The lines of action for tension T, weight W and reaction R_b at the contact point meet at C (the centre of the roller) and as such Lami's theorem is applicable. The angles between various segments around point C are as indicated in Fig. 1.48 (b).

Invoking Lami's theorem,

$$\frac{W}{\sin(60+45)} = \frac{R_b}{\sin(75+45)} = \frac{T}{\sin(90+45)}$$

$$\therefore \quad R_b = W \times \frac{\sin 120^\circ}{\sin 105^\circ} = 500 \times \frac{0.866}{0.966} = \mathbf{448.24\ N}$$

$$T = W \times \frac{\sin 135^\circ}{\sin 105^\circ} = 500 \times \frac{0.707}{0.966} = \mathbf{365.94\ N}$$

EXAMPLE 1.26

A uniform wheel of 50 cm diameter and 1 kN weight rests against a rigid rectangular block of thickness 20 cm (Fig. 1.40). Considering all surfaces smooth, determine

(a) least pull to be applied through the centre of wheel to just turn it over the corner of the block,

(b) reaction of the block

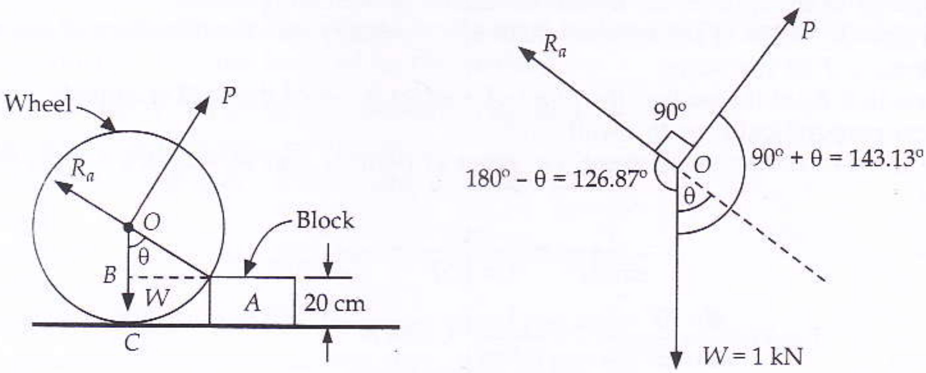


Fig. 1.40

Solution : The wheel is acted upon by the following forces when it is just about to turn over the block.

(i) weight $W = 1$ kN of the wheel acting vertically downwards through centre of the sphere

(ii) Reaction R_a of the block

(iii) Pull P which must be applied normal to OA if it is to be minimum.

When the sphere is just about to turn over the block, it will lose contact with the floor and apparently, the reaction at the contact point C would be zero. Further the wheel is in equilibrium and as such the forces W, R_a and P meet at point O and the Lami's theorem applies.

In the right angled triangle AOB,

$$\cos\theta = \frac{OB}{OA} = \frac{50-20}{50} = 0.6\ ; \quad \theta = 53.13^\circ$$

$$\frac{W}{\sin 90^\circ} = \frac{P}{\sin 126.87^\circ} = \frac{R_a}{143.13^\circ}$$

$$\therefore \quad P = W \times \frac{\sin 126.87^\circ}{\sin 90^\circ} = 1 \times \frac{0.8}{1} = \mathbf{0.8\ kN}$$

$$R_a = W \times \frac{\sin 143.13^\circ}{\sin 90^\circ} = 1 \times \frac{0.6}{1} = \mathbf{0.6\ kN}$$

EXAMPLE 1.27

Figure 1.41 shows a weight W tied to the end of a cord of length l. Determine the magnitude of force F to pull the weight at an angle α as indicated in the figure. Proceed to find the tension in the cord.

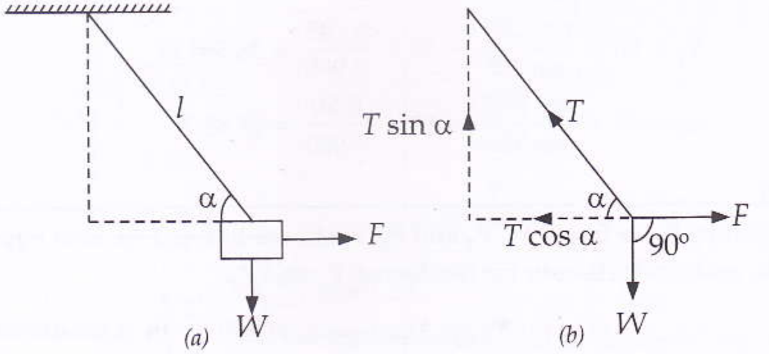


Fig. 1.41

Solution : The three forces F, T and W meet at a point and as such the Lami's theorem is applicable. Therefore

$$\frac{T}{\sin 90^\circ} = \frac{W}{\sin (180^\circ - \alpha)} = \frac{F}{\sin (90^\circ + \alpha)}$$

$$\therefore \quad F = W \times \frac{\sin (90 + \alpha)}{\sin (180 - \alpha)} = W \times \frac{\sin 90^\circ \cos \alpha + \cos 90^\circ \sin \alpha}{\sin \alpha}$$

$$= W \times \frac{\cos \alpha}{\sin \alpha} = \mathbf{W \cot \alpha}$$

$$T = W \times \frac{\sin 90^\circ}{\sin (180^\circ - \alpha)} = W \times \frac{1}{\sin \alpha} = \mathbf{W \operatorname{cosec} \alpha}$$

EXAMPLE 1.28

An electric light fixture weighing 50 N hangs from point C by two strings AC and BC as shown in Fig. 1.42. The string AC is inclined at 60° to the horizontal and string BC is 45° to the vertical. Using Lami's theorem or otherwise determine the forces in the strings AC and BC.

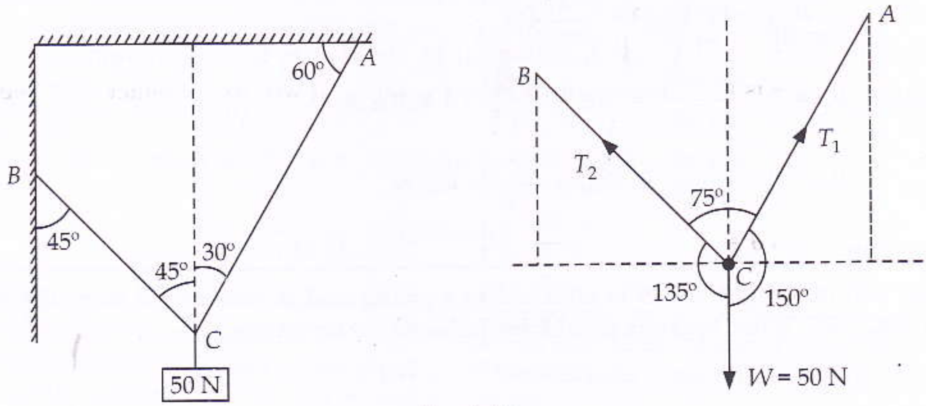


Fig. 1.42

Solution : Let T_1 and T_2 be the tensions in strings AC and BC respectively. The line of action of tensions in AC and BC and weight W of the light fixture meet at point C, and therefore Lami's theorem is applicable. That gives

$$\frac{T_1}{\sin 135^\circ} = \frac{T_2}{\sin 150^\circ} = \frac{50}{\sin 75^\circ}$$

$$\therefore \quad T_1 = 50 \times \frac{\sin 135^\circ}{\sin 75^\circ} = 50 \times \frac{0.707}{0.966} = \mathbf{36.594\ N}$$

$$T_2 = 50 \times \frac{\sin 150^\circ}{\sin 75^\circ} = 50 \times \frac{0.50}{0.966} = \mathbf{25.88\ N}$$

EXAMPLE 1.29

A body acted upon by three forces P_1, P_2 and P_3, as shown in Fig. 1.43, is in equilibrium. If P_2 = 300 N, make calculations for the forces P_1 and P_3.

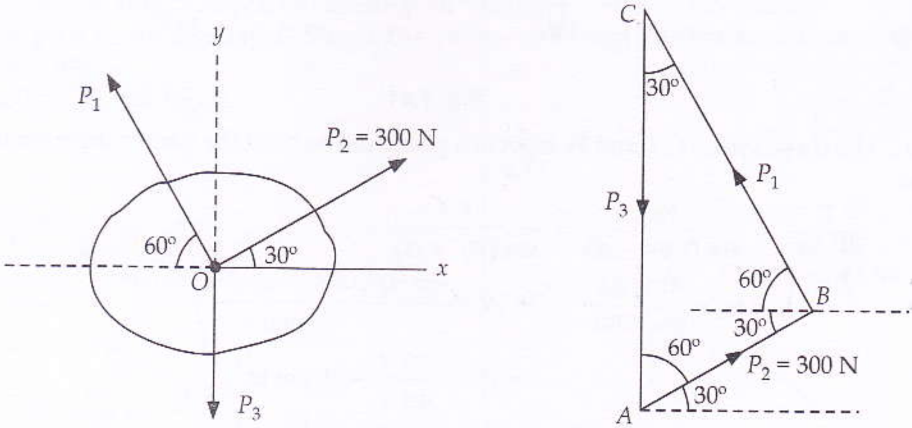


Fig. 1.43

Solution : The given forces can be represented by the sides AB, BC and CA of the triangle ABC (Fig. 1.43)

$$AB = P_2 = 300\text{ N}; \quad BC = P_1 \text{ and } CA = P_3$$

Then applying the law of sines,

$$\frac{P_1}{\sin 60^\circ} = \frac{P_2}{\sin 30^\circ} = \frac{P_3}{\sin 90^\circ}$$

That gives: $$P_1 = P_2 \frac{\sin 60^\circ}{\sin 30^\circ} = 300 \times \frac{0.866}{0.5} = \mathbf{519.6\ N}$$

$$P_3 = P_2 \frac{\sin 90^\circ}{\sin 30^\circ} = 300 \times \frac{1}{0.5} = \mathbf{600\ N}$$

EXAMPLE 1.30

A spherical ball of weight 100 N is attached to a string and is suspended from the ceiling as shown in Fig. 1.44. What tension would be induced in the string ?

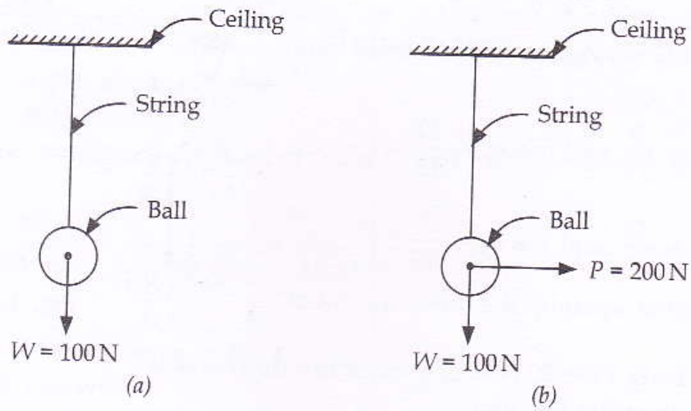


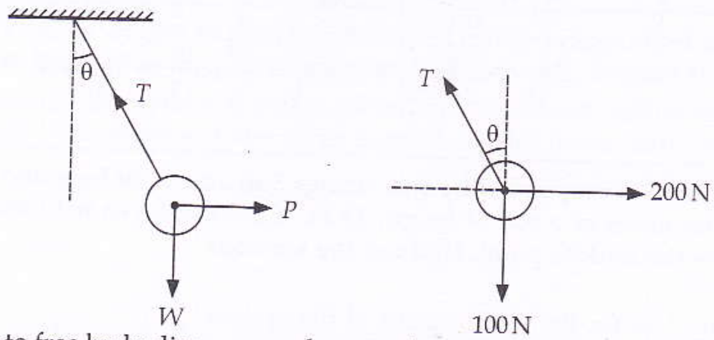
Fig. 1.44

Subsequently a horizontal force of 200 N is applied to the ball as shown in Fig. 1.53. Calculate the resultant tension in the string and the angle which the string makes with the vertical.

Solution : The ball is in equilibrium under the action of two forces namely the weight W of the ball and the tension T induced in the string

$$\therefore \quad T = W = 80 \text{ N}$$

(*ii*) When the horizontal force P is applied the ball, it will be in equilibrium under the action of three forces namely weight of the ball, horizontal force applied and the tension induced in the string.



With reference to free body diagram, we have under equilibrium state

$$\Sigma F_x = 0: \quad -T\sin\theta + 200 = 0 \qquad \text{...(i)}$$

$$\Sigma F_y = 0: \quad T\cos\theta - 100 = 0 \qquad \text{...(ii)}$$

From expressions (*i*) and (*ii*)

$$\tan\theta = \frac{200}{100} = 2\;; \quad \theta = 63.55°$$

$$T = \frac{200}{\sin 63.55} = 223.46 \text{ N}$$

EXAMPLE 1.31

A body of weight 20 N is suspended by two strings 5 m and 12 m long and other ends being fastened to the extremities of a rod of length 13 m. If the rod is to remain horizontal, determine the tensions in the strings.

Solution: Refer Fig. 1.45 for the arrangement of the system. In triangle ABC,

$$AC = 5 \text{ m};\ BC = 12 \text{ m and } AB = 13 \text{ m}$$

$$5^2 + 12^2 = 13^2; \quad AC^2 + BC^2 = AB^2$$

and accordingly ΔABC is a right angled triangle with $\angle ACB = 90°$

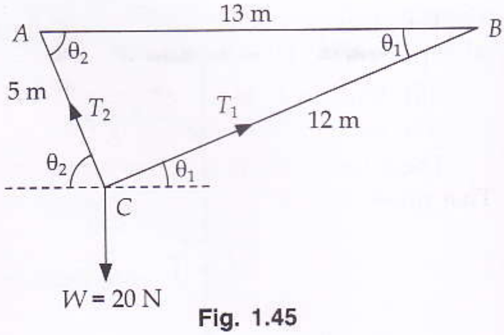


Fig. 1.45

Further: $\sin\theta_1 = \dfrac{5}{13}$ and $\cos\theta_1 = \dfrac{12}{13}$

$\sin\theta_2 = \dfrac{12}{13}$ and $\cos\theta_2 = \dfrac{5}{13}$

The weight is acted upon by the following set of forces :

(*i*) weight of body $W = 20$ N acting vertically downwards,

(*ii*) tension T_1 in string *CB*, and

(*iii*) tension T_2 in string *CA*

Since the lines of action of the tensions and weight meet at point *C*, Lami's theorem holds good. That is

$$\frac{T_1}{\sin(90+\theta_2)} = \frac{T_2}{\sin(90+\theta_1)} = \frac{W}{\sin\angle ACB}$$

or
$$\frac{T_1}{\cos\theta_2} = \frac{T_2}{\cos\theta_1} = \frac{W}{\sin 90°}$$

$\therefore$
$$T_1 = \frac{W}{\sin 90°} \times \cos\theta_2 = \frac{20}{1}\times\frac{5}{12} = \mathbf{7.69\ N}$$

$$T_2 = \frac{W}{\sin 90°} \times \cos\theta_1 = \frac{20}{1}\times\frac{12}{13} = \mathbf{18.46\ N}$$

EXAMPLE 1.32

A body of weight 20 N is suspended by two strings 5 m and 12 m long and other ends being fastened to the extremities of a rod of length 13 m. If the rod be so held that the body hangs immediately below the middle point, find out the tensions in the strings.

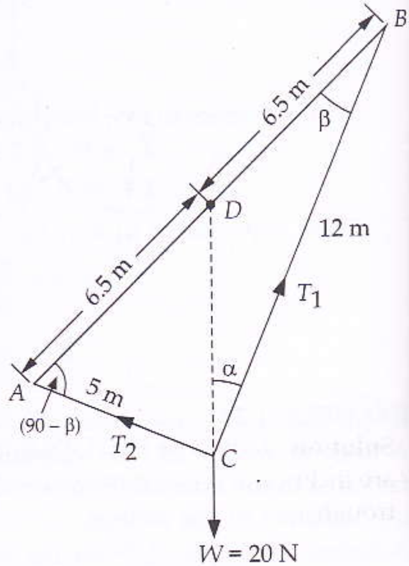


Fig. 1.46

Solution: Refer Fig. 1.46 for the arrangement of the system.

In triangle *ABC*,

$$AC = 5\text{ m};\ BC = 12\text{ m and } AB = 13\text{ m}$$

$$5^2 + 12^2 = 13^2; \quad (AC)^2 + (BC)^2 = AB^2$$

and accordingly ΔABC is a right angled triangle with $\angle ACB = 90°$

Applying sine rule to triangles *ACD* and *BCD*,

$$\frac{6.5}{\sin(90-\alpha)} = \frac{CD}{\sin(90-\beta)}; \quad \frac{6.5}{\cos\alpha} = \frac{CD}{\cos\beta} \quad ...(i)$$

and
$$\frac{6.5}{\sin\alpha} = \frac{CD}{\sin\beta} \quad ...(ii)$$

From expressions (*i*) and (*ii*),

$$\frac{6.5}{\cos\alpha}\times\cos\beta = \frac{6.5}{\sin\alpha}\times\sin\beta$$

or $\tan\alpha = \tan\beta \quad \therefore \quad \alpha = \beta$

The system is acted upon by the following set of forces :

(*i*) weight $W = 20$ N of the body acting vertically downwards

(*ii*) tension T_1 in the string *CB*, and

(*iii*) tension T_2 in the string *CA*

These forces are concurrent and meet at point *C*, and therefore Lami's theorem is applicable. That gives

$$\frac{W}{\sin 90^\circ} = \frac{T_1}{\sin(90+\alpha)} = \frac{T_2}{\sin(180-\alpha)}$$

or $$\frac{20}{1} = \frac{T_1}{\cos\alpha} = \frac{T_2}{\sin\alpha}$$

Since $\alpha = \beta$, we can write

$$20 = \frac{T_1}{\cos\beta} = \frac{T_2}{\sin\beta}$$

From the given geometry $\cos\beta = \frac{12}{13}$ and $\sin\beta = \frac{5}{13}$. Therefore

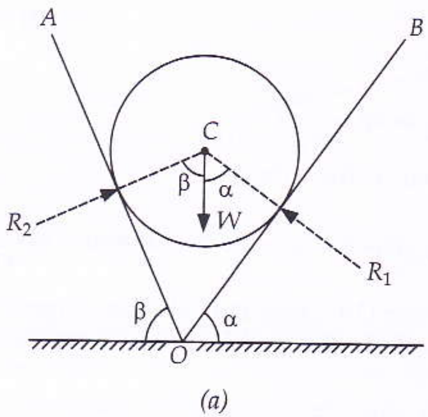
$$T_1 = 20\cos\beta = 20 \times \frac{12}{13} = \mathbf{18.46\ N}$$

and $$T_2 = 20\sin\beta = 20 \times \frac{5}{13} = \mathbf{7.7\ N}$$

EXAMPLE 1.33

A heavy spherical ball of weight *W* rests in a *V*-shaped trough whose sides are inclined at α and β to the horizontal. Determine the pressure exterted on each side. Neglect friction.

Subsequently a similar spherical ball is placed on the side of inclination α and it is made to rest on the first ball. Work out the force exerted by the lower ball on the side inclined at β.



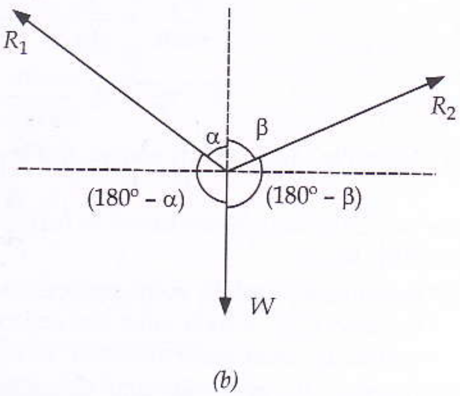


Fig. 1.47 (a)

Solution : Reference Fig. 1.47(a), the trough is formed by two planes *OA* and *OB* and these planes are inclined at α and β respectively with the horizontal. The spherical ball with centre *C* lies in this trough.

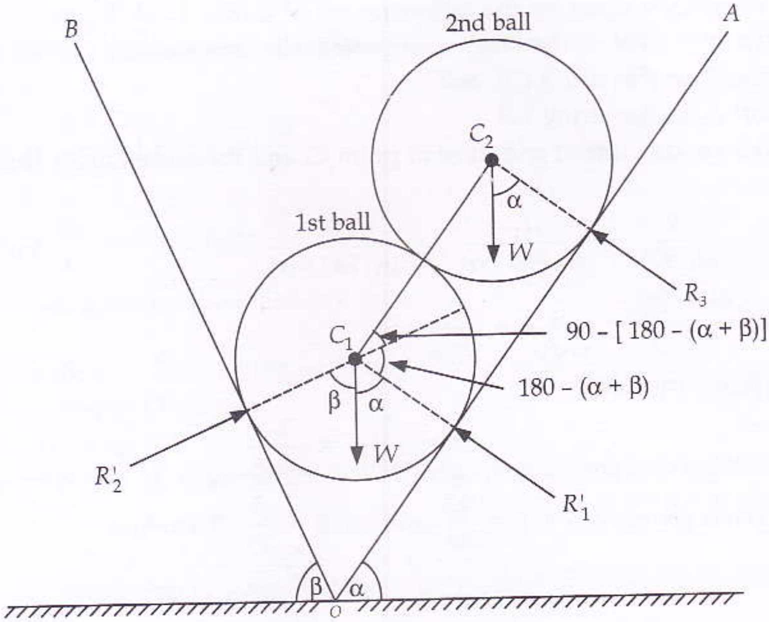


Fig. 1.47 (b)

Let R_1 and R_2 be the reactions at surfaces OA and OB respectively. Since there is no friction, these reactions will be normal to the respective surfaces.

Since the system is in equilibrium condition, the forces R_1, R_2 and W meet at point C, the centre of the ball. The system of forces is as shown in Fig. 2.47 (*b*).

Applying Lami's theorem at point C,

$$\frac{R_1}{\sin(180-\beta)} = \frac{R_2}{\sin(180-\alpha)} = \frac{W}{\sin\{180-(\alpha+\beta)\}}$$

or
$$\frac{R_1}{\sin\beta} = \frac{R_2}{\sin\alpha} = \frac{W}{\sin(\alpha+\beta)}$$

$$\therefore \qquad R_1 = W\frac{\sin\beta}{\sin(\alpha+\beta)} \text{ and } R_2 = W\frac{\sin\alpha}{\sin(\alpha+\beta)}$$

(*b*) Refer Fig. 1.57 which shows the two balls placed in the trough.

The two balls may be assumed to form a single body which is in equilibrium under the action of following forces :

(*i*) reaction R_1' and R_3 acting perpendicular to surface OA. Since the balls are of equal size, the line C_1C_2 which joins the centres of the two balls is parallel to OA. These reactions would be then perpendicular to C_1C_2 also.

(*ii*) reaction R_2' perpendicular to surface OB.

(*iii*) weights of the two balls.

Resolving these forces along C_1C_2, we get

$$R_2' \cos\left[90-\{180-(\alpha+\beta\}\right] - W\sin\alpha - W\sin\alpha$$

It is to be noted that components of R_1' and R_3 along C_1C_2 are zero.

$$R_2' \sin\{180-(\alpha+\beta\} = 2W\sin\alpha$$

or $\qquad R_2' \sin(\alpha + \beta) = 2\,W \sin\alpha$

$$\therefore \qquad R_2' = \frac{2W \sin\theta}{\sin(\alpha+\beta)}$$

EXAMPLE 1.34

Two rollers of the same diameter are supported by an inclined plane and a vertical wall as shown in Fig. 1.48. The upper and the lower rollers are respectively 200 N and 250 N in weight. Assuming smooth surfaces, find the reactions induced at the points of support *A*, *B*, *C* and *D*.

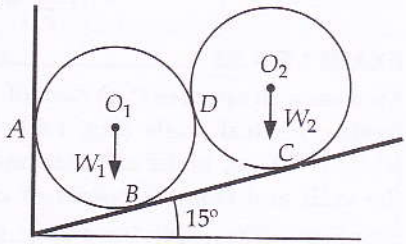


Fig. 1.48

Solution : The upper cylinder is kept in equilibrium by the following set of concurrent forces:

(*i*) weight 200 N acting vertically downward through its centre O_2

(*ii*) reaction R_c acting perpendicular to the inclined plane

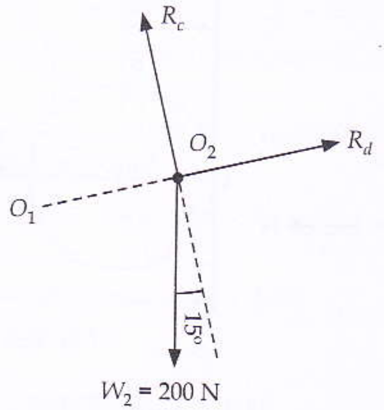
(*iii*) pressure R_d from the lower cylinder in the direction O_1O_2

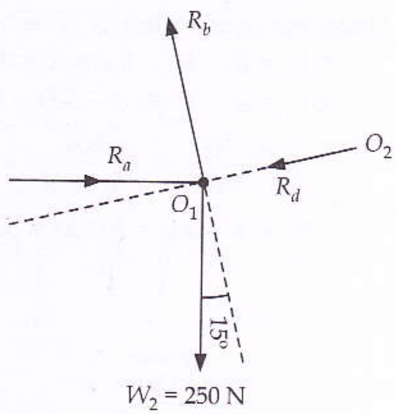
Since these three forces are concurrent, the Lami's theorem applies. That gives

$$\frac{R_d}{\sin(180-15)} = \frac{R_c}{\sin(90+15)} = \frac{W}{\sin 90}$$

$$\therefore \quad R_d = W \times \frac{\sin 165^\circ}{\sin 90^\circ} = 200 \times \frac{0.2588}{1} = \mathbf{51.76\ N}$$

$$R_c = W \times \frac{\sin 105^\circ}{\sin 90^\circ} = 200 \times \frac{0.9659}{1} = \mathbf{193.18\ N}$$





The following set of forces keep the lower cylinder in equilibrium :

(*i*) weight 250 N acting vertically downward through its centre O_1

(*ii*) reaction R_a acting perpendicular to the vertical wall

(*iii*) reaction R_b acting perpendicular to the inclined plane

(*iv*) pressure R_d from the upper cylinder in the direction O_2O_1

Resolving the forces along O_1O_2, we get

$$R_a \cos 15^\circ - W_1 \sin 15^\circ - R_d = 0$$

$$R_a \times 0.966 - 250 \times 0.259 - 51.76 = 0$$

$$\therefore \quad R_a = \frac{(250 \times 0.259) + 51.76}{0.966} = \frac{116.51}{0.966} = \mathbf{120.61\ N}$$

Resolving the forces perpendicular to O_1O_2, we get

$$R_b - R_a \sin 15° - W_1 \cos 15° = 0$$

$$R_b = R_a \sin 15° + W_1 \cos 15°$$

$$= (120.61 \times 0.2588) + (250 \times 0.9659)$$

$$= 31.21 + 241.47 = \mathbf{272.68\ N}$$

EXAMPLE 1.35

Two smooth spheres P, Q each of radius 25 cm and weighing 500 N, rest in a horizontal channel having vertical walls (Fig. 1.49). If the distance between the walls is 90 cm, make calculations for the pressure exerted on the wall and floor at points of contact A, B and C.

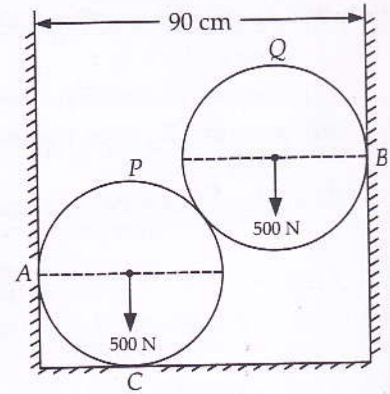


Fig. 1.49

Solution : The following points need consideration

(*i*) the spheres are smooth and as such the pressures at various points of contact would be normal to the surface.

(*ii*) at the point of contact between the two spheres, the reactions would act along the line joining their centres.

With reference to Fig. 1.50, the line C_1C_2 makes an angle a with the horizontal line passing through centre C_1 of sphere P.

$$\cos \alpha = \frac{b - r - r}{2r} = \frac{90 - 25 - 25}{50} = \frac{40}{50}$$

$$\therefore \quad \alpha = 36.87°$$

Considering the equilibrium of sphere Q

$$\Sigma F_x = 0; \quad R_b - R \cos \alpha = 0$$

$$\Sigma F_y = 0; \quad R \sin \alpha - 500 = 0$$

$$\therefore \quad R = \frac{500}{\sin \alpha} = \frac{500}{\sin 36.87} = \mathbf{833.33\ N}$$

$$R_b = R \cos \alpha = 833.33 \times \cos 36.87 = \mathbf{666.66\ N}$$

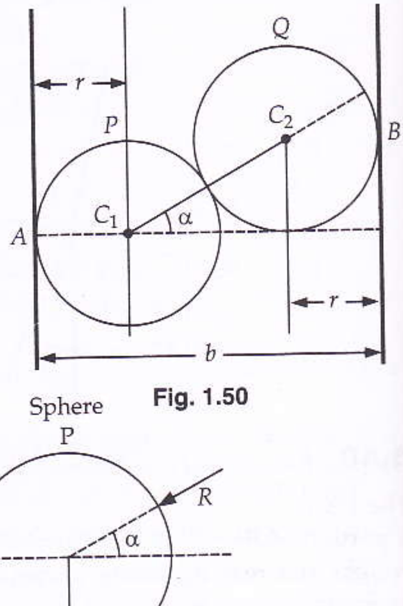
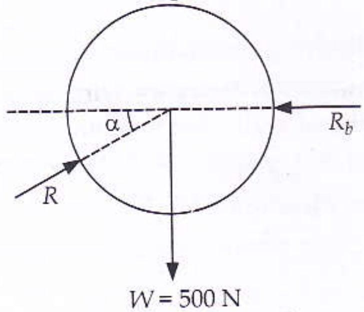
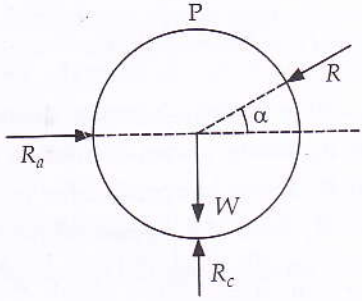


Fig. 1.50





Considering equilibrium of sphere P,

$$\Sigma F_x = 0\,; \quad R_a - R\cos\alpha = 0$$

$$R_a = R\cos\alpha = 833.33 \times \cos 36.87 = \mathbf{666.67\ N}$$

$$\Sigma F_y = 0\,; \quad R_c - W - R\sin\alpha = 0$$

$$R_c = W + R\sin\alpha = 500 + 833.33 \times \sin 36.87 = \mathbf{1000\ N}$$

EXAMPLE 1.36

Refer to the system of cylinders arranged as depicted in Fig. 1.51. The cylinders A and B weigh 1000 N each and the weight of cylinder C is 2000 N. Determine the forces exerted at the contact points.

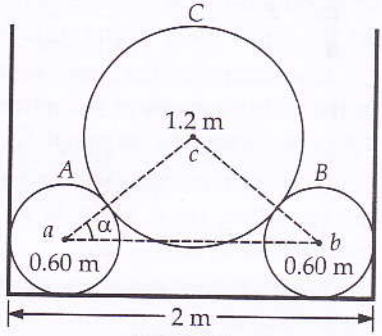


Fig. 1.51

Solution : a, b and c are centres of spheres.

$$ab = 2 - \frac{0.6}{2} - \frac{0.6}{2} = 1.4\ \text{m}$$

$$ac = 0.3 + 0.6 = 0.9\ \text{m}$$

$$\cos\alpha = \frac{1.4/2}{0.9} = 0.7777\,; \quad \alpha = 38.94^\circ$$

Applying Lami's theorem to the forces acting on sphere C,

$$\frac{R_1}{\sin(90+\alpha)} = \frac{R_2}{\sin(90+\alpha)} = \frac{2000}{\sin(180-2\alpha)}$$

$$R_1 = R_2 = 2000 \times \frac{\sin(90+\alpha)}{\sin(180-2\alpha)} = 2000 \times \frac{\sin(90+38.94)}{\sin(180-2\times 38.94)}$$

$$= 2000 \times \frac{0.7777}{0.9777} = \mathbf{1590.87\ N}$$

Considering the free body diagram of cylinder A,

$$\Sigma F_x = 0; \quad R_a - R_1\cos\alpha = 0$$

$$R_a = R_1\cos\alpha = 1590.87 \times \cos 38.94$$

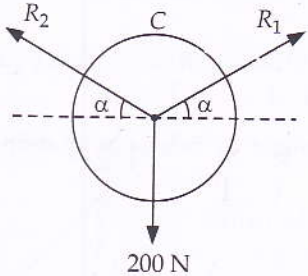
$$= \mathbf{1237.38\ N}$$

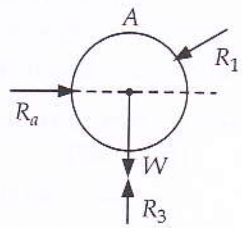
$$\Sigma F_y = 0; \quad R_1\sin\alpha + W - R_3 = 0$$

$$R_3 = R_1\sin\alpha + W$$

$$= 1590.87\sin 38.94 + 1000$$

$$= \mathbf{1999.87\ N}$$





1.10. FREE BODY DIAGRAM

The force analysis of a structure is made in a simplified way by considering the equilibrium of a portion of the structure. For that, the portion is drawn separately showing applied forces, self weight and reactions at the point of contact with other bodies. The resulting diagram is known as *free body diagram* (FBD). In a free body diagram, all the supports (like walls, floors, hinges etc) are removed and replaced by the reactions which these supports exert on the body.

A free body diagram can be drawn for any single body of a system, for any subsystem or for the entire system irrespective of whether the system is in equilibrium, *i.e.,* at rest or in uniform motion or in a dynamic state of motion.

Further, while drawing the free diagrams, one must have consideration of internal forces and external forces.

Internal forces: the forces which hold togather the particles of the body and help it to be rigid, *i.e.*, not deform. If more than one body is involved, internal forces hold the bodies together.

Imagine a bar being pulled by two equal and opposite forces applied at the ends. The internal forces will come into play to keep the bar undeformed. These forces cause stresses and strains distributed throughout the material of the body.

External forces: the forces which act on a body or system of bodies externally, *i.e.*, applied from outside. The forces essentially denote the action of other bodies (walls, floors, hinges etc.) on the rigid body being analysed.

Consider a sphere suspended by a string. The weight of the sphere and tension induced in the string represent the external forces. The reactions developed at the contact points, if any, also constitute the external forces.

A complete isolation and systematic representation of all external (applied and reactive) forces acting on a body is an important and effective tool for the solution of problems in mechanical systems.

Given below are a few examples of systems and mechanisms together with their free body diagrams:

(*i*) A sphere resting on a frictionless plane surface (Fig. 1.52)

The forces acting on the sphere when isolated from the surface are:

(*a*) Force W equal to the weight of the sphere. This weight acts downward through the centroid of the sphere.

(*b*) Reaction R at the point of contact with the surface. This reaction acts upwards normal to the surface as it is frictionless.

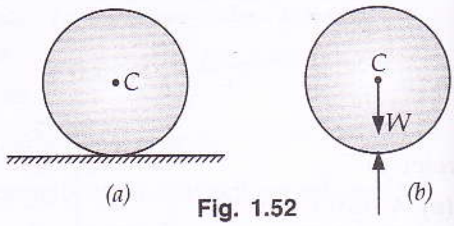


Fig. 1.52

As the sphere is in equilibrium, R and W are equal and collinear, and the free body diagram will be as shown in Fig. 1.62

(*ii*) A circular roller of weight W hangs by a string and rests against a smooth vertical wall (Fig. 1.53)

The force acting on the roller when isolated from the supports are:

(*a*) Force W equal to weight of the roller

(*b*) Wall reaction R_c at the point of contact C with the wall. The reaction will be normal to the wall as it is smooth

(*c*) Tension T in the string along BA.

As the roller is in equilibrium, all the forces will be concurrent and the free body diagram will be as shown in Fig. 1.53

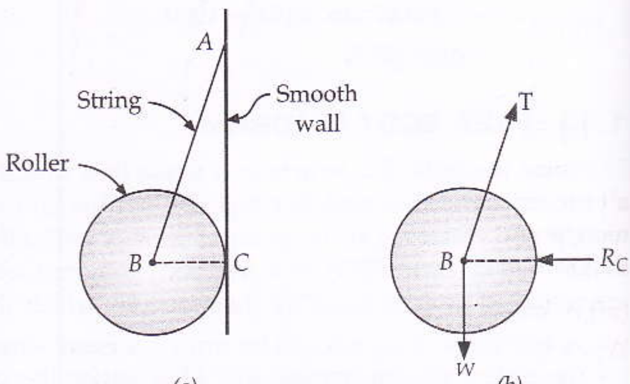


Fig. 1.53

(iii) A sphere resting in a V-shaped groove (Fig. 1.54)

The sphere is isolated from the inclined surface forming the groove, and is considered to be acted upon by the following set of forces:

(a) Weight of the sphere acting vertically down-ward through its centre C

(b) Reaction R_a acting nor-mal to the in-clined plane at the contact point A.

(c) Reaction R_b acting normal to the in-clined plane at the contact point B.

Since the sphere is in equilibrium, all the forces meet at point C and the free body diagram will be as shown in Fig. 1.54

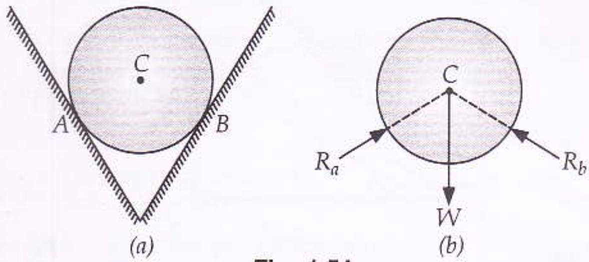


Fig. 1.54

(iv) A drum being rolled along the horizontal comes across a stepped obstacle (Fig. 1.65)

At the instant of obstacle being overcome, the forces acting on the drum are:

(a) Weight W of the drum acting vertically downwards through its centre C

(b) Pull P required to be applied at the position shown to overcome the obstacle

(c) Reaction at A that passes through point B, the intersection of P and W.

For free body diagram of the drum, refer to Fig. 1.55.

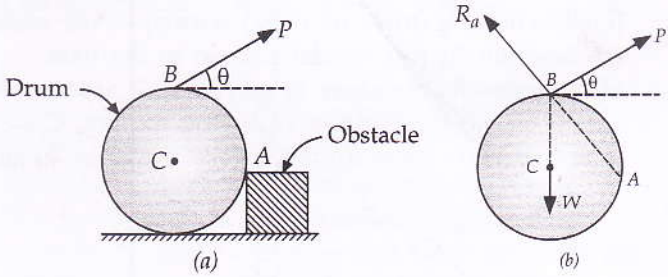


Fig. 1.55

(v) A uniform ladder of weight W leans a against a smooth wall and rests on a rough floor (Fig. 1.56)

The various forces acting on the ladder when it is isolated from the wall and floor are:

(a) Force W equal to the weight of the ladder and acting vertically downward from the mid of the ladder.

(b) Reaction R_b of the wall. This acts right angles to the wall as the wall is smooth.

(c) Reaction R_a at the ground.

Since the ladder is in equilibrium, all the forces must pass though a common point. This aspect then fixes the unknown direction of reaction R_a, and the free body diagram of the ladder will be as shown in Fig. 1.56.

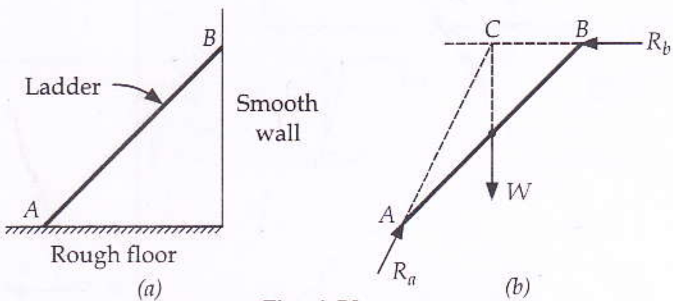


Fig. 1.56.

(vi) A beam loaded and supported (Fig. 1.57)

When the beam is detached from the supports, it subjected to following set of forces:

(a) Weight W acting vertically downwards through mass centre of the beam

(b) Reaction R_b normal to the beam at its smooth contact with the corner

(c) Horizontal applied force P and the couple M

(d) vertical and horizontal reactions (R_{av} and R_{ah}) exerted at the pin connection at B.

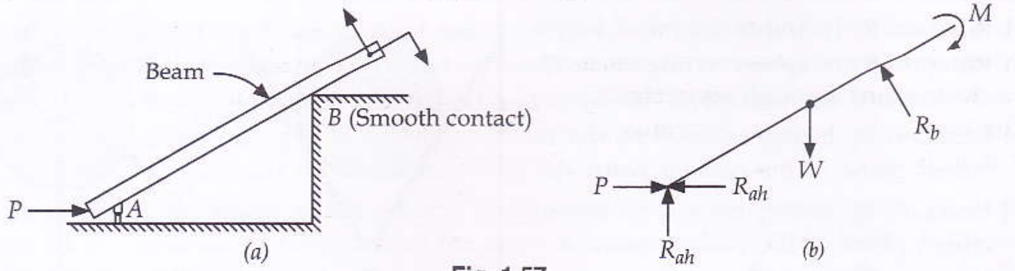


Fig. 1.57

The beam will be in equilibrium under the action of five forces and one couple, and its free body diagram will be as shown in Fig. 1.57.

(*vii*) Two spheres *P* and *Q* placed in a vessel (Fig. 1.58)

Forces acting on sphere P

(a) Weight W_1 of sphere acting downwards through its mass centre C_1

(b) Reaction R_a (towards right) normal to the vertical wall surface

(c) Reaction R_b (up wards) normal to the base

(d) Reaction R_d of sphere Q on sphere P at the point of contact D. This acts in a direction normal to the surface, *i.e.*, along the line C_1C_2 joining the mass centres of the spheres

The free body diagram for sphere P will be as depicted in Fig. 1.58 (b)

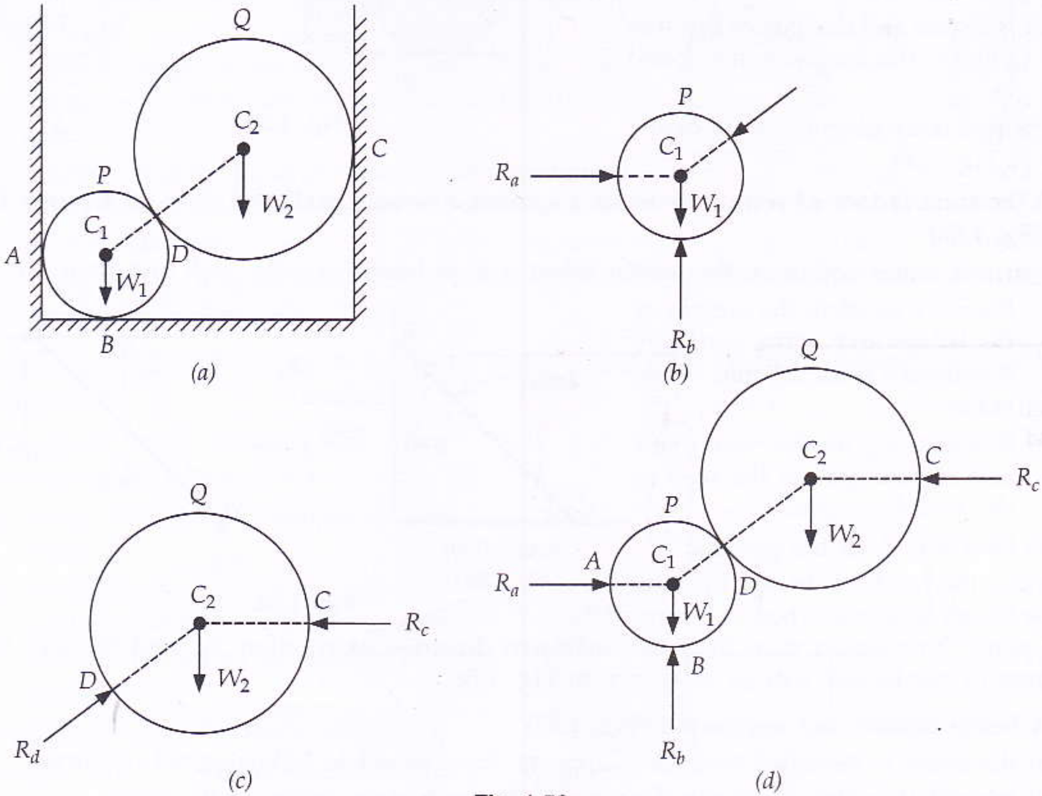


Fig. 1.58

Forces acting on sphere Q

(*a*) Weight of sphere acting downwards through its mass centre C_2

(*b*) Reaction R_c (towards left) and normal to the vertical wall surface.

(*c*) Reaction R_d of sphere P on sphere Q at the point of contact D. This acts in a direction normal to the surface, *i.e.*, along the line C_1C_2 joining the mass centres of the spheres.

The free body diagram for roller Q will be as shown in Fig. 1.68 (*c*).

It is to be noted that

$$\begin{Bmatrix}\text{reaction of sphere } P\\ \text{on sphere } Q\end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix}\text{reaction of sphere } Q\\ \text{on sphere } P\end{Bmatrix}$$

are equal in magnitude, opposite in direction and are collinear. Obviously when both the spheres are taken together, these reactions cancel and do not appear in the free body diagram (Fig. 1.58 (*d*)).

1.11 BEAM: TYPES, LOADS AND SUPPORTS

A beam is a structural member whose longitudinal dimension is large compared to the transverse dimension. The beam is supported along its length and is acted upon by a system of loads at right angles to its axis.

Beams are generally classified as:

- Cantilever beam (Fig. 1.59a): A beam having its one end fixed or built-in and the other end free to deflect. There is no deflection or rotation at the fixed end.
- Fixed beam (Fig. 1.59b): A beam having both of its ends fixed or built-in.
- Simply supported beam (Fig. 1.59c): A beam made to freely rest on supports which may be knife edges or rollers. The term 'freely supported' implies that these supports exert only forces but no moments on the beam. The horizontal distance between the supports is called span.
- Overhanging beam (Fig. 1.59d): A beam having one or both ends extended over the supports. The end portion or portions extend in the form of cantilever beyond the support/supports.
- Continuous beam (Fig. 1.59e): A beam provided with more than two supports. Further such a beam may or may not have overhang.

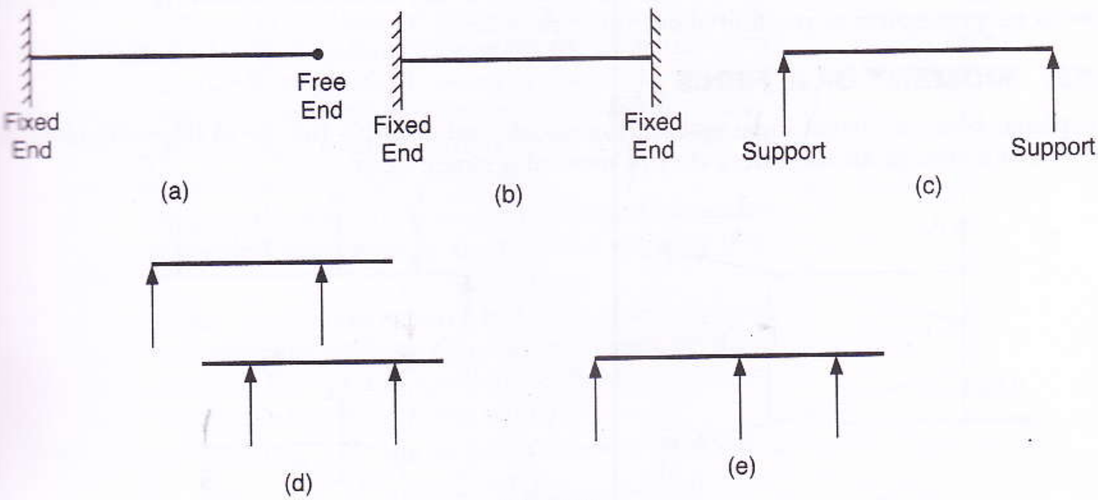


Fig. 1.59

The different types of loads acting on a beam are:

- **Concentrated load:** The load acts at a point on the beam. This point load is applied through a knife edge.
- **Uniformly distributed load:** The load is evenly distributed over a part or the entire length of the beam. The total udl is assumed to act at the centre of gravity of the load. The udl is expressed as N/m length of beam.

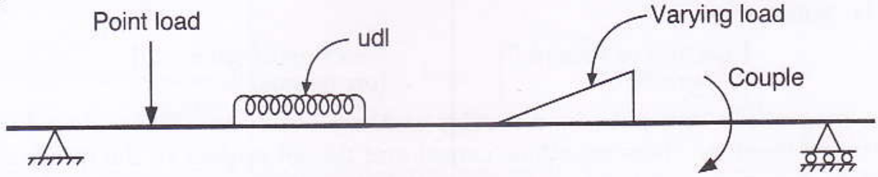


Fig. 1.60.

- **Uniformly varying load:** The load whose intensity varies linearly along the length of beam over which it is applied.
- A beam may be loaded by a couple whose magnitude is expressed as Nm.

A beam may carry any one of the above load systems or combination of two or more loads at a time.

Generally for frames and beams, there are two types of supports: one hinged and the after on rollers.

Roller supports (Fig. 1.61 *a*) are frictionless and provide a reaction at right angles to the roller base. At a hinged support (Fig. 1.61 *b*), the line of action of the reaction depends upon the load system on the frame. This reaction can be split into two components acting in the horizontal and vertical directions.

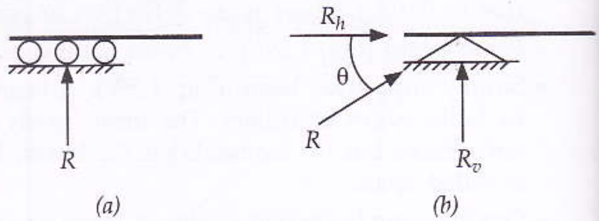


Fig. 1.61

$$R_v = R \sin\theta \text{ and } R_h = R\cos\theta$$

A roller support prevents motion only in the direction perpendicular to its seat. With pin (hinged) support, the beam has no translatory motion in any direction. In both types of supports, there is no prevention of rotation about the connections.

1.12. MOMENT OF A FORCE

A coplanar non-concurrent force system consists of a set of forces that lie in the same plane but the line of action of all the forces do not meet at a single point.

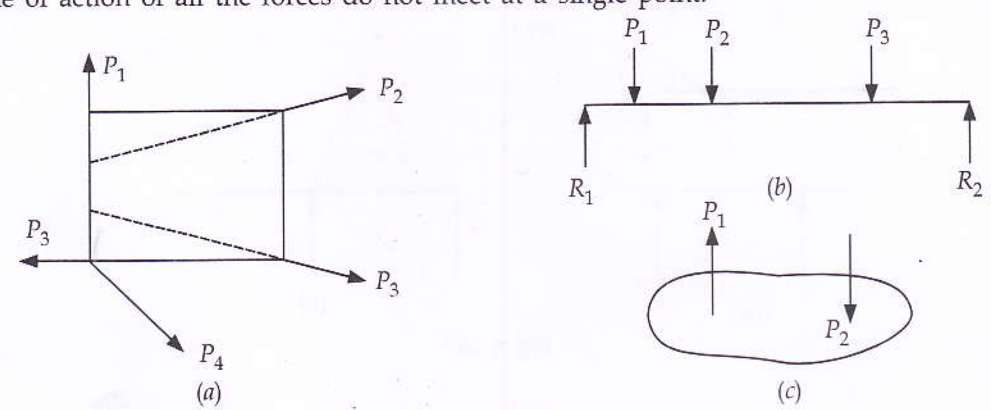


Fig. 1.62

Moment of force about a point is defined as the turning tendency of a force about that point. It is measured by the product of force and the perpendicular distance of the line of action of the force from that point.

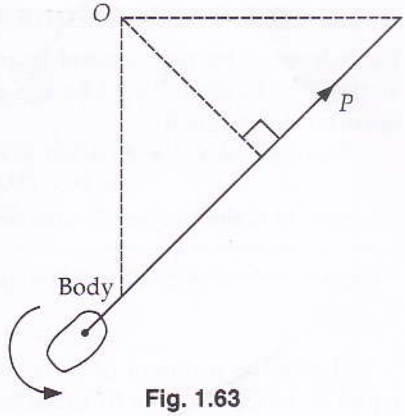


Fig. 1.63

With reference to Fig. 1.63,

P = force acting on a body

l = perpendicular distance between the point O and line of action of force P

Then moment of force P about point O

$$= P \times l$$

The point O is called the *moment centre* and the distance l is the *moment arm* or *arm of the force*.

- The moment of force about a point is a vector which is directed perpendicular to the plane containing the moment centre and the force.
- If force is measured in newton (N) and length in metres (m), then the units of force will be Nm.
- When a force acts on a body, it causes or tends to cause a change of state of rest or of uniform rectilinear motion of the body. The action of moment tends to cause a rotational motion to the body.

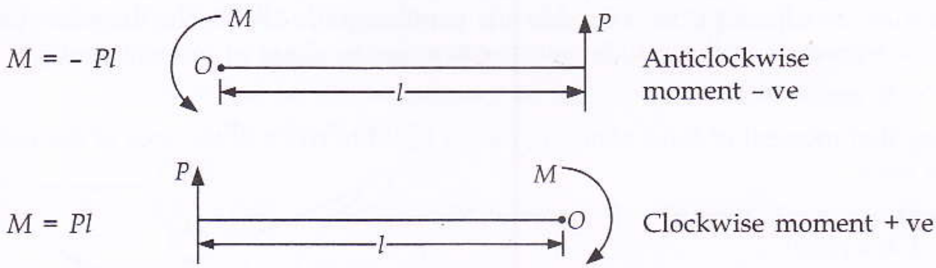


Fig. 1.64

- The tendency of rotation or turning of the body due to moment of force may be clockwise (in the same direction in which hands of the clock move) or anticlockwise (in a direction opposite to the movement of hands of a clock). The corresponding moments are referred to as clockwise moment and anticlockwise moment.

  The general convention is to take the clockwise moments as positive and anticlockwise moments as negative (Fig. 1.64).

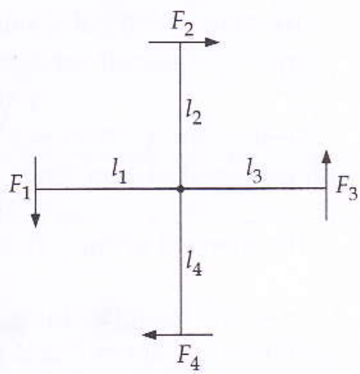


Fig. 1.65

- If a number of moments act on a body (Fig. 1.65), then the resultant moment will be the algebraic sum of these moments, and the effect of resultant moment on the body will be governed by its turning tendency.

With reference to Fig. 1.75, the resultant moment M about the point O is given by

$$M = -F_1 l_1 + F_2 l_2 - F_3 l_3 + F_4 l_4$$

1.13. GRAPHICAL REPRESENTATION OF MOMENT

Let a force P be represented in magnitude and direction by the vector AB. Further, let O be the point about which the moment is to be determined.

Moment of force P about point O,

$$= P \times OM$$

where OM is the perpendicular dropped from point O on line AB

$$= AB \times OM = 2\left[\frac{1}{2} \times AB \times OM\right]$$

$$= 2 \text{ (area of triangle } OAB).$$

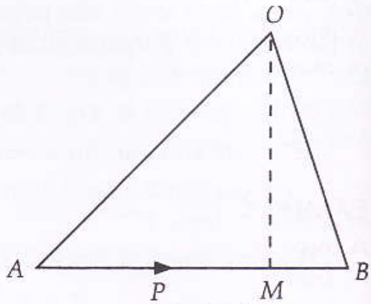


Fig. 1.66

Hence the moment of force about any point is geometrically equal to twice the area of the triangle whose base is the line that represents the force and whose vertex is the point about which the moment is required to be found out.

1.14. VARIGNON'S THEOREM : LAW OF MOMENTS

"Moment of a resultant of two forces, about a point lying in the plane of the forces, is equal to the algebraic sum of moments of these two forces about the same point."

Consider two concurrent forces P and Q represented in magnitude and direction by AB and AC respectively. Let O be the point about which moment is to be taken. Through O, draw a line parallel to the direction of force P and let this line meet the line of action of force Q at point C. With AB and AC as adjacent sides, complete the parallelogram $ABCD$. The diagram AD of this parallelogram represents in magnitude and direction the resultant of forces P and Q.

Join O with points A and B.

Recalling that moment of force about a point is equal to twice of the area of the triangle so formed, we have

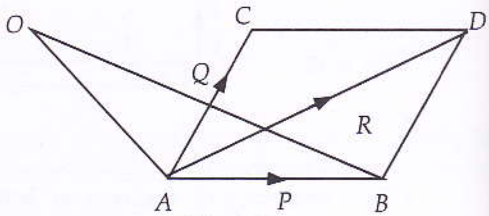


Fig. 1.67

Moment of force P about O = 2 × area of triangle AOB = $2 \times \Delta AOB$ ...(*i*)

Likewise,

moment of force Q about $O = 2 \times \Delta AOC$ ...(*ii*)

moment of force R about $O = 2 \times \Delta AOD$

From geometrical configuration of Fig. 1.67,

$$\Delta AOD = \Delta AOC + \Delta ACD = \Delta AOC + \Delta ABD$$

Further, the Δ's AOB and ABD are on the same base AB and between the same lines and as such are equal in area. Then

$$\Delta AOD = \Delta AOC + \Delta AOB$$

The moment of force R about O may then be re-written as

$$= 2 \times (\Delta AOC + \Delta AOB) \qquad ...(iii)$$

From the identities (*i*), (*ii*) and (*iii*), it is evident that

Moment of forces P and Q about point O

= moment of resultant R about O

This principle can be extended for any number of forces and the generalised law of moments may be stated as

"moment of resultant of a number of forces about a point lying in the plane of forces is equal to the algebraic sum of the moments of these forces about the some point."

1.15. PRINCIPLE OF MOMENTS

A body acted upon by a number of coplanar forces will be in equilibrium, if the algebraic sum of moments of all the forces about a point lying in the same plane is zero. Mathematically

$$\Sigma M = 0$$

i.e., clockwise moments = anticlockwise moments

EXAMPLE 1.37

A force of 200 N is acting at a point *B* as shown in the adjoining figure. Determine the moment of this force about *O*.

Solution : Moment of force about point O

= force × (perpendicular distance between point O and the line of action of force)

$= P \times OM = P \times OB \cos 60°$

$= 200 \times (3 \times 0.866)$

= **519.6 Nm** (clockwise)

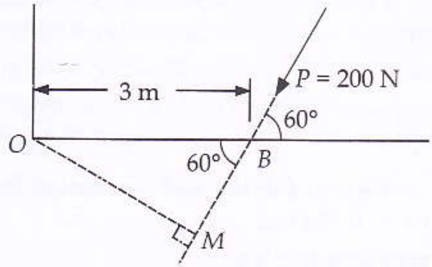


Fig. 1.68

EXAMPLE 1.38

A force of 200 N is acting at point *B* of a lever *AB* which is hinged at its lower end as shown in Fig. 1.69. Find the moment of force about the hinged end.

Solution : Moment of force about point A

= force × (perpendicular distance between point A and the line of action of force)

$= P \times AM$

Now, $AB = \sqrt{0.4^2 + 0.3^2} = 0.5$ m

$$\alpha = \tan^{-1}\left(\frac{0.4}{0.3}\right) = 53.13°$$

$$\gamma = 90 - 53.13 = 36.87°$$

$$\beta = 36.87 - 30 = 6.87°$$

$$AM = AB \sin 6.87° = 0.5 \times 0.1196 = 0.0598 \text{ M}$$

$\therefore$ $M_a = 200 \times 0.0598 =$ **11.96 Nm** (anticlockwise)

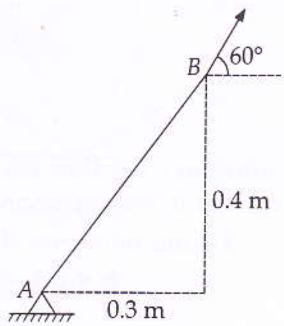


Fig. 1.69

EXAMPLE 1.39

A uniform wooden plank AB of length 3 m has a weight of 40 N. It is supported at end A and at a point D which is 1m from the other end B. Determine the maximum weight W that can be placed at end B so that the plank does not topple.

Solution : Refer to Fig. 1.70 for the given force system. The weight of the plank acts at its centre C.

When the plank is at the state of just being toppled, the reaction R_a at point A is zero.

Taking moments about point D,

$$W \times 1 = 40 \times 0.5$$

$$\therefore \quad W = \mathbf{20\ N}$$

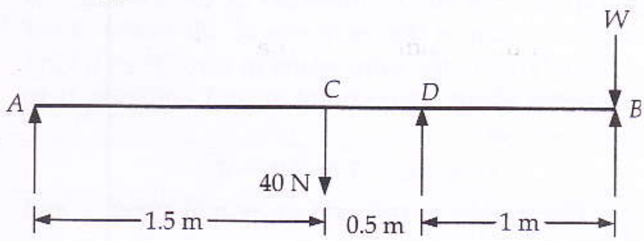


Fig. 1.70

EXAMPLE 1.40

A uniform rodof 10 m length has a self weight of 5 N. The rod carries a weight of 30 N hung from one of its ends. From what point the rod be suspended so that it remains horizontal ?

Solution : The self weight of the rod 5 N acts at the mid point C of the rod.

Let the rod be suspended at point F which is at distance x from its end B where a load of 30 N is hung.

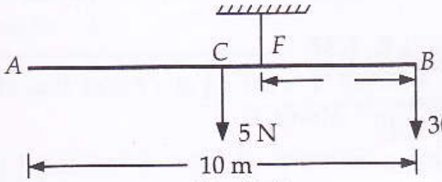


Fig. 1.71

Taking moments about point F, we get

$$5 \times CF - 30x = 0$$

$$5(BC - x) = 30x$$

or $$5(5 - x) = 30x$$

or $$25 - 5x = 30x$$

$$\therefore \quad x = \frac{25}{35} = \mathbf{0.7143\ m}$$

Hence for the rod to remain horizontal, it needs to be suspended from point F such that $BF = 0.7143$ m.

EXAMPLE 1.41

A uniform rod *ABCD* weighs 8 N and is supported at *B* and *C* as shown in the figure given below. Determine the minimum force at *A* which will just overturn the rod.

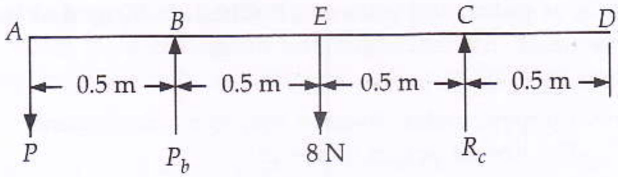


Fig. 1.72

Solution : Let P be the minimum force to be applied at end A which will just overturn the rod. At that instant reaction R_c will be zero.

Taking moments about point B,

$$8 \times 0.5 - P \times 0.5 = 0$$

$$\therefore \quad P = \frac{8 \times 0.5}{0.5} = \mathbf{8\ N}$$

EXAMPLE 1.42

A uniform rod *AB* of weight 100 N is hinged at *A* so as to be able to rotate about *A* in a vertical plane. It is held in horizontal position by a string attached to *B* and inclined at 60° to the horizontal as shown in Fig. 1.73. Find the tension induced in the string.

Solution : For the equilibrium of the system, the algebraic sum of the moments of all forces about a point lying in the same plane in zero. Then taking moments about hinge point A and equating it to zero, we get

$$W \times AC - T \times AD = 0$$

The weight of rod acts at its mid point C and therefore $AC = 1$ m.

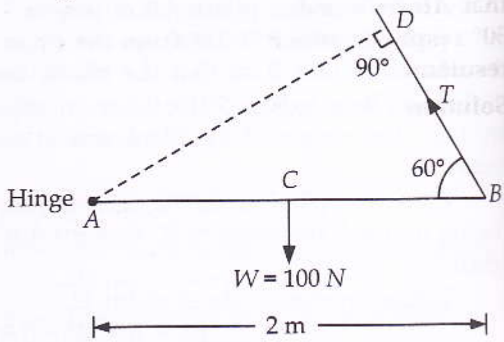


Fig. 1.73

AD = perpendicular distance between hinge point A and line of action of tension T

$= AB \sin 60° = 2 \times 0.866 = 1.732$ m

$\therefore \quad 100 \times 1 - T \times 1.732 = 0$

$$T = \frac{100}{1.732} = \mathbf{57.74\ N}$$

EXAMPLE 1.43

A regular hexagon *ABCDEF* of side 2 m in length is subjected to forces 1, 2, 3, 4, 5 and 6 N along sides *AB, CB, DC, DE, EF* and *FA* respectively as shown in the Fig. 1.74 given below. Make calculations for the sum of moments at point *A*.

Solution : Starting from force represented by side AB, we take moments about point A and get

$$M_a = -1 \times 0 + 2 \times AN + 4 \times AC - 3 \times AE - 5 \times AM - 6 \times 0$$

The clockwise moments have been taken positive and anticlockwise moments as negative. From the geometrical configuration of the system,

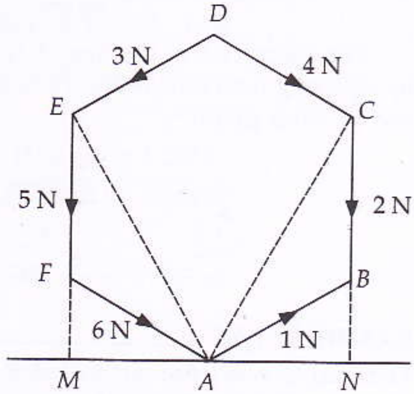


Fig. 1.74

$$\angle BAN = \angle FAM = 30°$$
$$\angle ACB = \angle BAC = 30°$$
$$AN = AM = AB \cos 30°$$
$$= 2 \times \cos 30°$$
$$= 1.732 \text{ m}$$

Also $\quad AN = AC \cos 60°$

$$AC = \frac{1.732}{\cos 60°}$$
$$= 3.464 \text{ m} = AE$$

$$\therefore \quad M_a = -(1 \times 0) + (2 \times 1.732) + (4 \times 3.464) - (3 \times 3.464) - (5 \times 1.732) - (6 \times 0)$$
$$= 0 + 3.464 + 13.586 - 10.392 - 8.66 - 0 = \mathbf{-1.728\ Nm}$$

The negative sign suggests that the resultant moment is anticlockwise.

EXAMPLE 1.44

A horizontal line *ABCD* measuring 9 m is acted upon by forces of magnitude 400, 600, 400 and 200 N at points *A, B, C* and *D* respectively with downward direction. These points are so located that *AB = BC = CD* = 3 m. The lines of action of the forces are inclined at 90°, 60°, 45° and 30° respectively with *AB*. Make calculations for the magnitude, position and direction of the resultant.

Solution : Refer to Fig. 1.75 for the given force system.

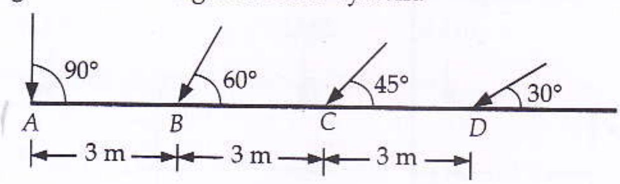


Fig. 1.75

Resolving all the forces horizontally,

$$\Sigma F_x = 200 \cos 30^\circ + 400 \cos 45^\circ + 600 \cos 60^\circ + 400 \cos 90^\circ$$
$$= 173.20 + 282.8 + 300 + 0 = 756 \text{ N}$$

Resolving all the forces vertically,

$$\Sigma F_y = 200 \sin 30^\circ + 400 \sin 45^\circ + 600 \sin 60^\circ + 400 \sin 90^\circ$$
$$= 100 + 282.8 + 519.6 + 400 = 1302.4 \text{ N}$$

Magnitude of the resultant force,

$$F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{756^2 + 1302.4^2} = \mathbf{1505.9\ N}$$

Let α be the angle which the resultant makes with the horizontal direction, *i.e.*, with line *AD*. Then

$$\tan \alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{1302.4}{756} = 1.723$$

$\therefore \quad \alpha = \mathbf{59.87^\circ}$

The distance x of the line of application of the resultant force from end A can be worked out by equating moments of the vertical components of the given forces and the resultant force about end A. That gives

$$1302.4 \times x = (600 \sin 60^\circ) \times 3 + (400 \sin 45^\circ \times 6 + (200 \sin 30^\circ \times 9)$$
$$= 1558.84 + 1697.06 + 900 = 4155.9$$

$$\therefore \quad x = \frac{4155.9}{1302.4} = \mathbf{3.191\ m}$$

EXAMPLE 1.45

Determine reactions at *A* and *B* (Fig. 1.76)

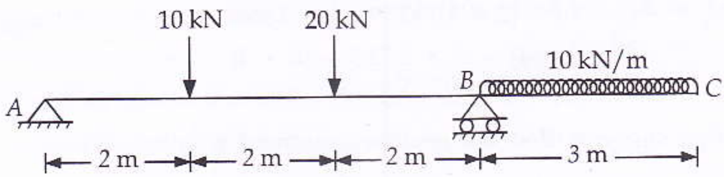


Fig. 1.76

Solution : At the hinged support, the reaction R_a can be in any direction. R_{av} and R_{ah} are its components in the vertical and horizontal directions. At the roller support, the reaction R_b will be in vertical direction only

The uniformly distributed load has a total magnitude of 10 × 3 = 30 kN, and acts at the middle of the 3 m length.

The equivalent free body diagram for the beam will then be as shown below:

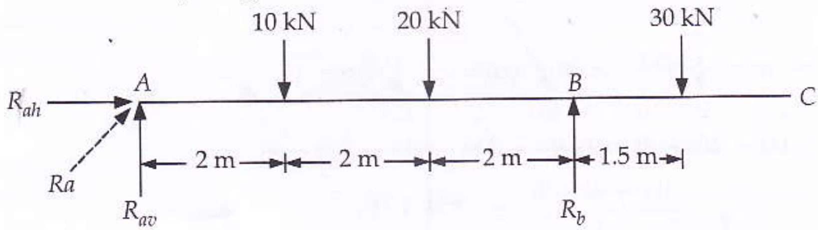


Fig. 1.77

The reactions R_{ah}, R_{av} and R_b can be evaluated by applying the conditions of equilibrium. That is

$$\Sigma M = 0; \quad \Sigma F_x = 0; \quad \Sigma F_y = 0$$

(*i*) Taking moments about A and equating it to zero;

$$(10 \times 2) + (20 \times 4) - (R_b \times 6) + (30 \times 7.5) = 0$$

$$20 + 80 - 6\,R_b + 225 = 0$$

$$\therefore \quad R_b = \frac{225 + 20 + 80}{6} = \frac{325}{6} = \mathbf{54.17\ kN}$$

(*ii*) $\Sigma F_x = 0$ for the whole system.

$$\therefore \quad R_{ah} = 0$$

This is because no other horizontal force, active or reactive, is acting on the beam.

Further, $\Sigma F_y = 0$. That gives $R_{av} - 10 - 20 + R_b - 30 = 0$

$$\therefore \quad R_{av} = 10 + 20 - R_b + 30 = 10 + 20 - 54.17 + 30 = \mathbf{5.83\ kN}$$

EXAMPLE 1.46

Determine reactions at A and B for the beam loaded as shown in Fig. 1.78.

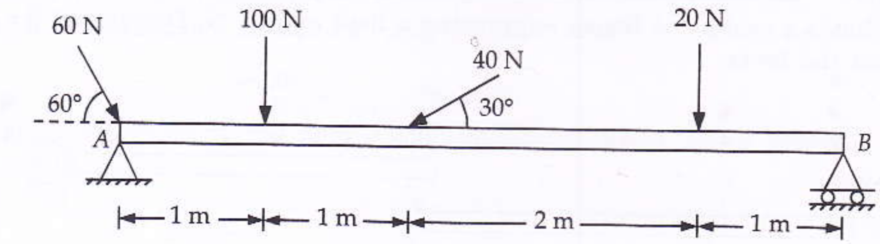


Fig. 1.78

Solution : At the hinged support the reaction R_a can be in any direction. R_{av} and R_{ah} are its components in the vertical and horizontal directions. At the roller support, the reaction R_b will be in vertical direction only.

The equivalent free body diagram for the beam will be then as shown below.

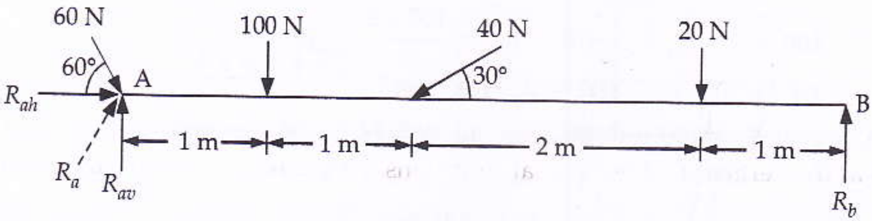


Fig. 1.79

The reactions R_{ah}, R_{av} and R_b can be evaluated by applying the conditions of equilibrium. That is

$$\Sigma M = 0; \quad \Sigma F_x = 0; \quad \Sigma F_y = 0$$

(*i*) Taking moments about A and equating it to zero,

$$(100 \times 1) + (40 \sin 30° \times 2) + (20 \times 4) - (R_b \times 5) = 0$$

$$100 + 40 + 80 - 5\,R_b = 0$$

$$\therefore \quad R_b = \frac{100 + 40 + 80}{5} = \frac{220}{5} = \mathbf{44\ N}$$

(*ii*) $\Sigma F_x = 0$ for the whole system.

$$R_{ah} + 60 \cos 60 - 40 \cos 30° = 0.$$

$$\therefore \quad R_{ah} = 40 \cos 30° - 60 \cos 60° = 34.64 - 30 = \mathbf{4.64\ N}$$

(*iii*) Further, $\Sigma F_y = 0$. That gives

$$R_{av} - 60 \sin 60° - 100 - 40 \sin 30° - 20 + Rb = 0$$

$$R_{av} = 60 \sin 60° + 100 + 40 \sin 30° + 20 - R_b$$

$$= 51.96 + 100 + 20 + 20 - 44 = \mathbf{147.96\ N}$$

Total reaction at hinge support,

$$R_a = \sqrt{R_{ah}^2 + R_{av}^2} = \sqrt{4.64^2 + 147.96^2} = 148\ \text{N}$$

$$\theta = \tan^{-1} \frac{R_{av}}{R_{ah}} = \tan^{-1} \frac{147.96}{4.64} = 88.20°$$

where θ is the angle which the reaction at A makes with the x-axis.

EXAMPLE 1.47

Figure 1.80 shows a system of levers supporting a load of 100 N. Determine the reactions at *A* and *B* on the lever.

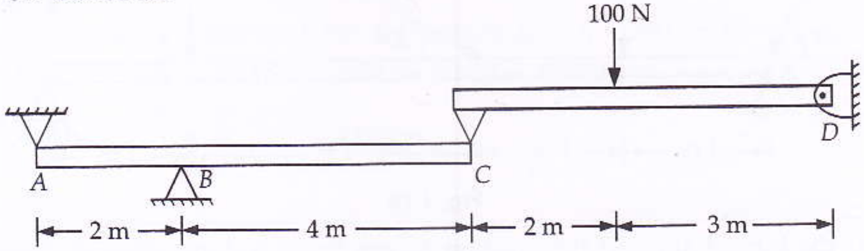


Fig. 1.80

Solution : Refer Fig. 1.81 for the free body diagrams for the upper and lower beams.

Taking moments about end C and equating it to zero,

$$100 \times 2 - R_d \times 5 = 0; \quad R_d = \frac{100 \times 2}{5} = 40\ \text{N}$$

$$\Sigma F_y = 0; \quad R_c - 100 + R_d = 0$$

$$R_c = 100 - R_d = 100 - 40 = 60\ \text{N}$$

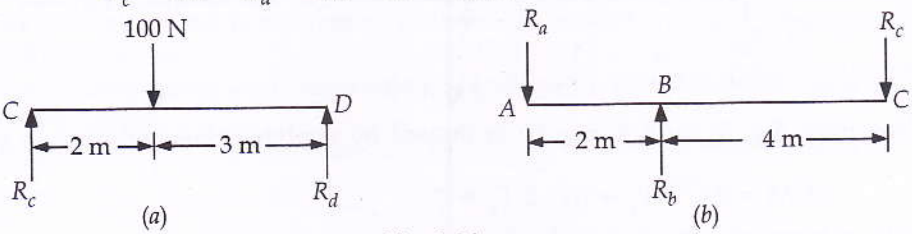


Fig. 1.81

From Fig. 1.81 (*b*),

Taking moments about end A and equating it to zero,

$$R_c \times 6 - R_b \times 2 = 0; \quad R_b = \frac{6R_c}{2} = \frac{6 \times 60}{2} = 180\ \text{N}$$

$$\Sigma F_y = 0; \quad R_a - R_b + R_c = 0$$
$$R_a = R_b - R_c$$
$$= 180 - 60 = 120 \text{ N}$$

Thus the reactions at points A and B on the lever are:

$$R_b = 180 \text{ N (upwards)}$$
$$R_a = 120 \text{ N (downwards)}$$

EXAMPLE 1.48

A beam has been loaded and supported as shown in Fig. 1.82 given below. Determine the reactions at the support points A and B.

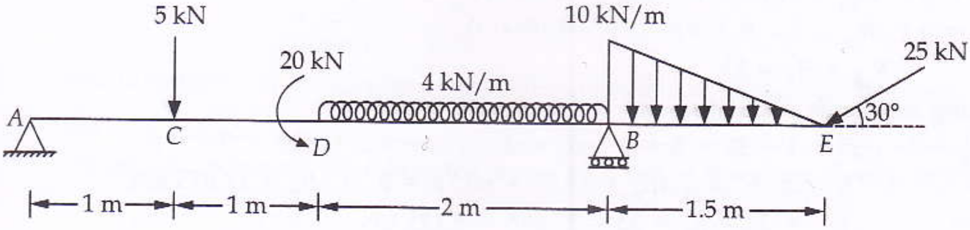
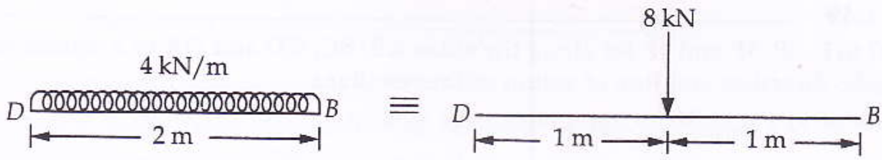
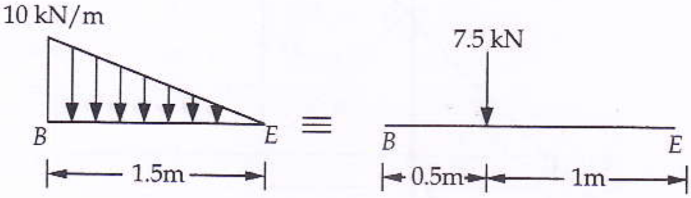


Fig. 1.82

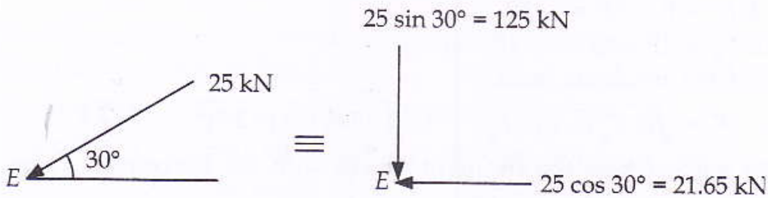
Solution : Equivalent loads and their position of acting are as shown below :



The total load is equal to 4 × 2 = 8 kN and it acts at the centre.



The total load is $\frac{1}{2} \times 1.5 \times 10 = 7.5$ kN and it acts at $\frac{1}{3} \times 1.5 = 0.5$ m from the base, *i.e.*, from point B.



These equivalent loads have been indicated in the diagram of the beam drawn below.

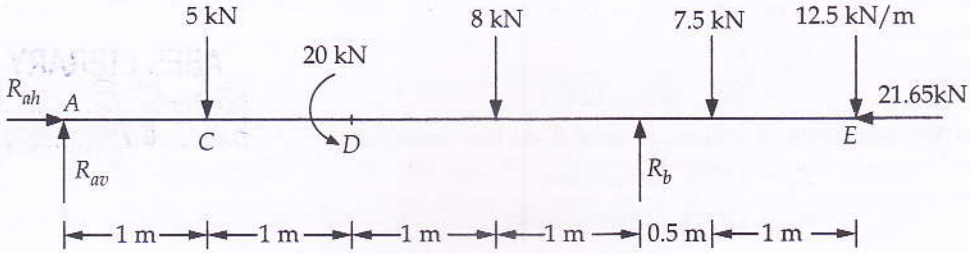


Fig. 1.83

For equilibrium of the beam : $\Sigma F_x = 0$; $\Sigma F_y = 0$ and $\Sigma M = 0$

$\Sigma F_x = 0$; $R_{ah} - 21.65 = 0$; $R_{ah} = 21.65$ kN

$\Sigma F_y = 0$; $R_{av} - 5 - 8 + R_b - 7.5 - 12.5 = 0$

$\therefore \quad R_{av} + R_b = 33$

Taking moments about the end A (clockwise positive)

$\Sigma M_a = 0$; $5 \times 1 - 20 + 8 \times 3 - R_b \times 4 + 7.5 \times 4.5 + 12.5 \times 5.5 = 0$

$5 - 20 + 24 - 4R_b + 33.75 + 68.75 = 0$; $R_b = \mathbf{27.875\ kN}$

Then $\quad R_{av} = 33 - R_b = 33 - 27.875 = \mathbf{5.125\ kN}$

Thus the reactions at support points A and B are

$$R_{ah} = 21.65 \text{ kN} \quad ; \quad R_{av} = 5.125 \text{ and } R_b = 27.875 \text{ kN}$$

EXAMPLE 1.49

Forces equal to P, 2P, 3P and 4P act along the sides AB, BC, CD and DA of a square ABCD. Find the magnitude, direction and line of action of the resultant.

Solution : Refer to Fig. 1.84 for the given force system

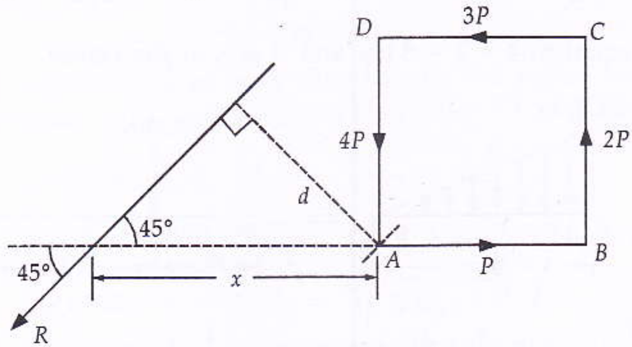


Fig. 1.84

Resolving all the forces horizontally and vertically,

$$\Sigma F_x = P - 3P = -2P$$
$$\Sigma F_y = 2P - 4P = -2P$$

Magnitude of the resultant force,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-2P)^2 + (-2P)^2} = \mathbf{2\sqrt{2}\ P}$$

Let α be the angle which the resultant makes with the horizontal. Then

$$\tan \alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{-2P}{-2P} = 1; \qquad \alpha = 45°$$

Since both ΣF_x and ΣF_y are negative, resultant lies in the 3rd quadrant making an angle of 45° with horizontal as shown in Fig. 1.101.

Taking moments of all forces about point A (clockwise + ve), we get

$$\Sigma M_a = -2P \times a - 3P \times a = -5Pa$$

The resultant R should then lie as shown in Fig. 1.43 so that it can produce anti-clockwise (negative) moment about point A.

If d is the perpendicular distance of resultant from A, then

$$R \times d = \Sigma M_a$$

$$\therefore \quad d = \frac{\Sigma M_a}{R} = \frac{5Pa}{2\sqrt{2}\,P} = \frac{5a}{2\sqrt{2}}$$

The horizontal distance x of the line of action of resultant from point A is

$$x = \frac{d}{\sin\alpha} = \frac{5a}{2\sqrt{2}} \div \sin 45^\circ = 2.5a$$

In the above identities a is a measure of the side of each side of the square block.

EXAMPLE 1.50

A horizontal beam AD of length 12 m is acted upon by a set of forces as shown in Fig. 1.85.

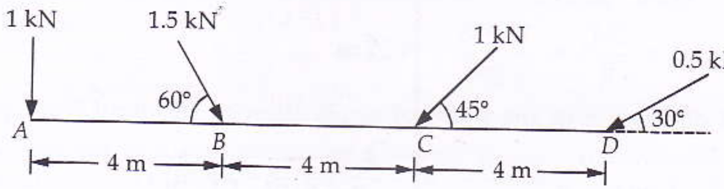


Fig. 1.85

Determine the magnitude, direction and position of the resultant.

Solution : Resolving the forces along x-and y-directions, we get

$$\Sigma F_x = 0 + 1.5\cos 60^\circ - 1\cos 45^\circ - 0.5\cos 30^\circ$$
$$= +0.75 - 0.707 - 0.433 = -0.39 \text{ kN}$$
$$\Sigma F_y = -1 - 1.5\sin 60^\circ - 1\sin 45^\circ - 0.5\sin 30^\circ$$
$$= -1 - 1.299 - 0.707 - 0.25 = -3.256 \text{ kN}$$

Resultant force $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$$= \sqrt{(-0.39)^2 + (3.256)^2} = \sqrt{10.7536} = \mathbf{3.279} \text{ kN}$$

Inclination of resultant with the horizontal,

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{3.256}{0.39} = 8.333;\ \alpha = 83.17^\circ$$

Since both ΣF_x and ΣF_y are negative, the resultant lies in the 3rd quadrant making an angle of 83.17° with horizontal as shown in Fig. 1.86

Taking moments of all forces about point A (clockwise $+ve$), we get

$$\Sigma M_a = (1.5\sin 60^\circ) \times 4 + (1\sin 45^\circ) \times 8 + (0.5\sin 30^\circ) \times 12$$
$$= 5.196 + 5.657 + 3 = 13.853 \text{ kNm}$$

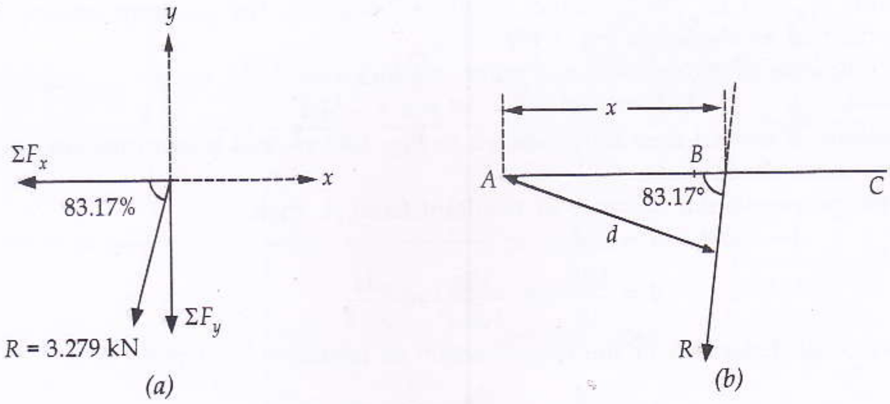


Fig. 1.86

The resultant R should then lie as shown in Fig. 1.104 so that it can produce clockwise (positive) moment about point A.

If d is the perpendicular distance of resultant from A, then

$$R \times d = \Sigma M_a$$

$$\therefore \quad d = \frac{\Sigma M_a}{R} = \frac{13.853}{3.279} = \mathbf{4.22\ m}$$

The horizontal distance x of the point of application of resultant from point A is

$$x = \frac{d}{\sin\alpha} = \frac{4.22}{\sin 83.17^\circ} = \mathbf{4.25\ m}$$

EXAMPLE 1.51

Determine the resultant of the four forces acting tangentially to a circle of radius 3 m as shown in Fig. 1.87. What will be the location of the resultant with respect to centre of the circle?

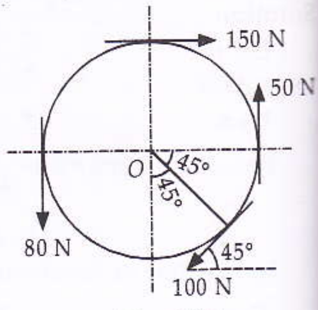


Fig. 1.87

Solution : For these coplanar and non-current forces

$$\Sigma F_x = 150 - 100\cos 45^\circ = 79.29\text{ N}$$

$$\Sigma F_y = 50 - 80 - 100\sin 45^\circ = -100.71\text{ N}$$

$$\text{Resultant } R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(79.29)^2 + (-100.71)^2} = 128.18\text{ N}$$

Let α be the angle which the resultant makes with the horizontal direction as shown in Fig. 1.106. Then

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{100.71}{79.29}\right) = 51.8^\circ$$

Since ΣF_x is positive and ΣF_y is negative, the resultant lies in the fourth quadrant.

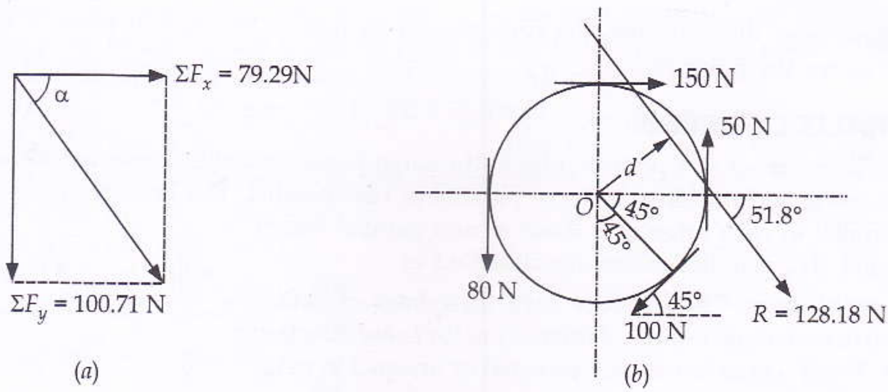


Fig. 1.88

Taking moments of all the forces about the centre (clockwise positive)

$$M_o = 150 \times 3 - 50 \times 3 + 100 \times 3 - 80 \times 3 = 360 \text{ (clockwise)}$$

The resultant R should then lie as shown in Fig. 1.106 (*b*) so that it can produce clockwise (positive) moment about the centre.

If d is the perpendicular distance of resultant from the centre, then

$$R \times d = \Sigma M_o$$

$$\therefore \quad d = \frac{\Sigma M_o}{R} = \frac{360}{128.18} = \mathbf{2.795\ m}$$

The resultant will act at a perpendicular distance of 2.795 m from the centre.

EXAMPLE 1.52

One end of a rope of length l is fixed to a vertical telegraph pole. A man standing on the ground pulls at the other end of the rope with a given force. Find the point at which the rope should be tied to the pole in order that the man will have the best chance of over turning the pole about its end fixed to the ground.

Solution : With reference to Fig. 1.89, AB is the vertical pole grounded at point A.

Let P be the force applied through rope BC (length l) towards C where the man is standing.

Moment of force P about point A,

$$M = P \times AD = P \times AC \sin\theta$$

$$= P \times (BC \cos\theta) \sin\theta$$

$$= Pl \sin\theta \cos\theta = \frac{1}{2} Pl \sin 2\theta$$

The force P and length l are fixed and as such the moment M will be maximum when

$\sin 2\theta = 1$, *i.e.*, when $2\theta = 90°$ and $\theta = 45°$

Obviously the man will have the best chance of overturning the vertical pole when he pulls it through the rope at an angle of 45° with the horizontal.

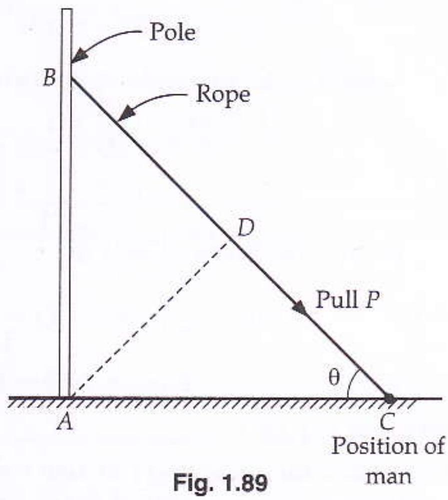


Fig. 1.89

$$AB = BC \sin\theta = l \sin 45° = \frac{l}{\sqrt{2}}$$

Hence to achieve the objective, the rope should be tied to the pole at point B which is at height $l/\sqrt{2}$ above the point A.

1.16. PARALLEL FORCES

Forces which donot meet at a point but lie in the same plane are called coplanar non-concurrent forces. The non-concurrent forces may be parallel or non-parallel. The lines of action of parallel forces are parallel to each other and those of non-parallel forces are not parallel. The parallel forces are classified as

- *Like parallel forces.* These forces have their lines of action parallel to each other and all of them act in the same direction [Fig. 1.90 (*a*)]. These forces may be equal or unequal in magnitude
- *Unlike Parallel forces.* These forces have their lines of action parallel to each other and all of them do not act in the same direction [Fig. 1.90 (*b*)]. These forces may be equal or unequal in magnitude.

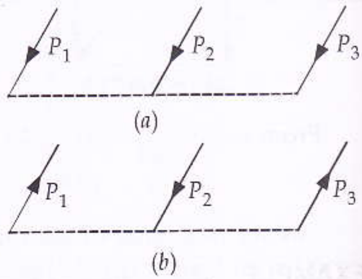


Fig. 1.90

EXAMPLE 1.53

Two like parallel forces are acting at a distance of 200 mm from each other. These forces are equivalent to a single force of 250 N and its line of action is at a distance of 50 mm from one of the forces. Make calculations for the magnitude of the two forces.

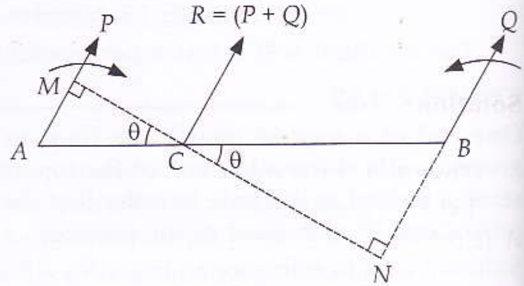


Fig. 1.91

Solution : With reference to Fig. 1.91, let P and Q be the two like parallel forces which act at points A and B. The point of application of the resultant of two like parallel forces lies between the two forces and let it meet AB at C. Then

$$AB = 200 \text{ mm} \; ; \quad AC = 50 \text{ mm} \quad \text{and}$$

$$BC = 200 - 50 = 150 \text{ mm}$$

$$P + Q = R = 250 \text{ N} \qquad \text{...}(i)$$

Applying the principle of moments and taking moments about point C,

$$P \times AC = Q \times BC$$

$$P = Q \times \frac{BC}{AC} = Q \times \frac{150}{50} = 3\,Q \qquad \text{...}(ii)$$

From expressions (*i*) and (*ii*),

$$3Q + Q = 250 \; ; \quad Q = \frac{250}{4} = \mathbf{62.5\ N}$$

$$P = 3 \times 62.5 = \mathbf{187.5\ N}$$

EXAMPLE 1.54

Determine the magnitude of two unlike parallel forces acting at a distance of 400 mm from each other. The resultant of these forces is 80 N and it acts at a distance of 160 mm from the greater of the two forces.

Solution : With reference to Fig. 1.92, let P and Q be the two unlike parallel forces which act at A and B. The point of application of the resultant of two unlike forces lies outside the two forces and near to the larger of the two forces. Let it meet AB at C. Then

$$AB = 400 \text{ mm}; \quad AC = 160 \text{ mm} \quad \text{and} \quad BC = 560 \text{ mm}$$

$$P - Q = R = 80 \text{ N} \qquad \ldots(i)$$

Applying the principle of moments and taking moments about point C,

$$P \times AC = Q \times BC$$

$$P = Q \times \frac{BC}{AC} = Q \times \frac{560}{160} = 3.5\,Q \qquad \ldots(ii)$$

From expressions (i) and (ii),

$$3.5\,Q - Q = 80; \quad Q = \frac{80}{2.5} = \mathbf{32\ N}$$

$$P = 3.5\,Q = 3.5 \times 32 = \mathbf{112\ N}$$

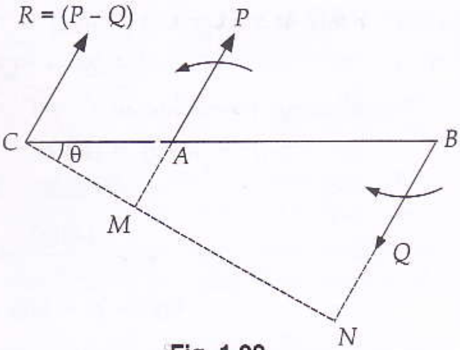


Fig. 1.92

EXAMPLE 1.55

A system of parallel forces acting on a lever is as shown in Fig. 1.93. Determine the magnitude, direction and position of resultant.

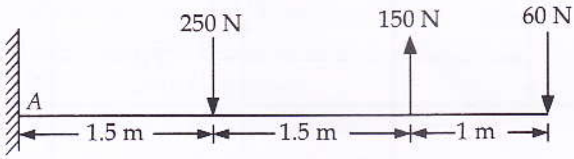


Fig. 1.93

Solution : Taking downward forces as positive, the resultant of the given force system is

$$R = 250 - 150 + 60 = \mathbf{160\ N}$$

Thus the resultant has a magnitude of 160 N and it acts vertically downwards.

Let this resultant act at a distance x from the fixed end A. Taking moments about point A (clockwise moments positive) and applying the principle of moments

moment of resultant = sum of the moments of its components

$$160x = (250 \times 1.5) - (150 \times 3) + (60 \times 4)$$

$$160x = 375 - 450 + 240 = 165$$

$$\therefore \quad x = \frac{165}{160} = \mathbf{1.031\ m}$$

Thus the resultant lies at a distance of 1.031 m from the fixed end A.

EXAMPLE 1.56

For the system of parallel forces depicted in Fig. 1.94, the resultant has a magnitude of 600 N vertically upwards and it acts through a point which is 4 m to the right of force 1500 N. Workout the values of forces *P* and *Q*.

Solution : Taking downward forces as positive, we have

$$-600 = 1500 - P + Q - 3000$$

$$= -P + Q - 1500$$

$$P - Q = -900 \qquad \ldots(i)$$

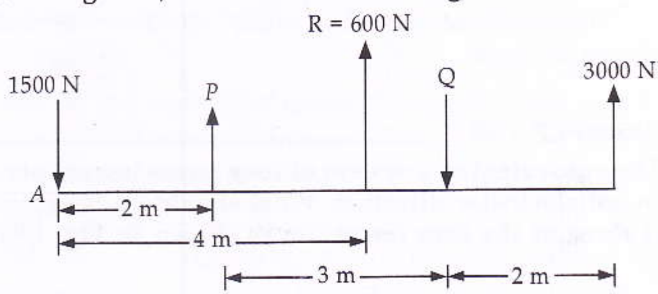


Fig. 1.94

Taking moments about point A (clockwise moments positive) and applying the principle of moments,

Sum of moments of components = moment of resultant

$$-P \times 2 + Q \times 5 - 3000 \times 7 = -600 \times 4$$

$$-2P + 5Q = 21000 - 2400 = 18600 \qquad ...(ii)$$

Substituting the value of $Q = P + 900$ from expression (i)

$$-2P + 5(P + 9800) = 18600$$

$$3P = 18600 - 4500 = 14100$$

$$\therefore \quad P = \frac{14100}{3} = \mathbf{4700\ N}$$

$$900 = P + 900 = 4700 + 900 = \mathbf{5600\ N}$$

EXAMPLE 1.57

A coplanar parallel force system consisting of three forces acts on a rigid bar AB as shown in Fig. 1.95. Determine the simplest equivalent action for the force system.

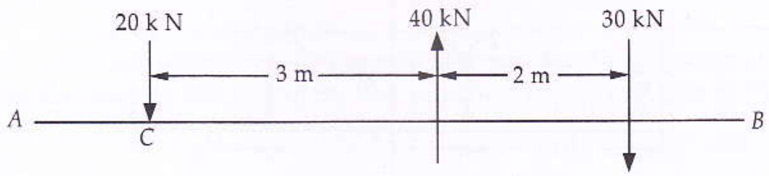


Fig. 1.95

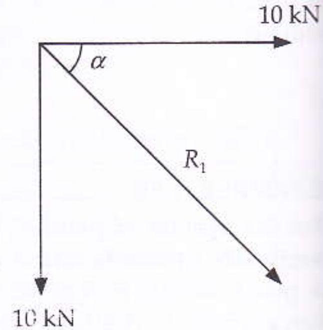
If an additional force of 10 kN acts along the bar from A to B, what would be the simplest equivalent action?

Solution : Let R be the simplest equivalent action. Then

$$R = \Sigma F = -20 + 40 - 30 = -10 \text{ kN} \quad (\downarrow)$$

The location of R can be worked out by taking moments about point C. That gives

$$10 \times x - 40 \times 3 + 30 \times 5 = 0; \quad x = 3 \text{ m}$$



Thus the equivalent action R acts downwards along the line of action of 40 kN force.

(b) The resultant of 10 kN acting downwards and 10 kN force acting horizontally from A to B is

$$R_1 = \sqrt{10^2 + 10^2} = 14.14 \text{ kN}$$

This resultant acts at $\alpha = \tan^{-1}(10/10) = 45°$ below x-axis as shown in figure.

EXAMPLE 1.58

The equivalent of a system of four forces is a couple of moment of magnitude 4 kNm acting in anticlockwise direction. What should be the magnitude and location of the fourth force if three of the four forces are as shown in Fig. 1.96?

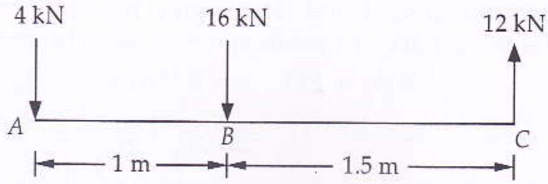


Fig. 1.96

Solution : The equivalent of the given system of forces is a couple and accordingly

(*i*) $\Sigma F_h = 0$, *i.e.*, the algebraic sum of the horizontal components of different forces is zero. Since the given three forces are all vertical, they do not have any horizontal component. Obviously the horizontal component of the fourth force would also be zero. That implies that even the fourth force would be a vertical force.

(*ii*) $\Sigma F_v = 0$, *i.e.*, the algebraic sum of the vertical components of different forces is zero. Let P be the fourth force acting vertically upwards. Then

$$P - 4 - 16 + 12; \quad P = 8 \text{ kN } (\uparrow)$$

Algebraic sum of the moments of given three forces about point A

$$= 12 \times 2.5 - 16 \times 1 = 14 \text{ kNm (anticlockwise)}$$

Moment of couple due to equivalent force system

$$= 4 \text{ kNm (anticlockwise)}$$

$\therefore$ Moment to be developed by the fourth force

$$= 14 - 4 = 10 \text{ kNm}$$

This moment of 10 kNm has to be clockwise so that net moment about point A is equal to 4 kNm (anticlockwise). For providing clockwise moment, the vertical force $P = 8$ kN should lie on left of point A at distance

$$x = \frac{10}{8} = \mathbf{1.25 \text{ m}}$$

EXAMPLE 1.59

A cantilever beam is acted upon by two vertical forces and a couple of moment 1250 Nm as shown in the figure given below:

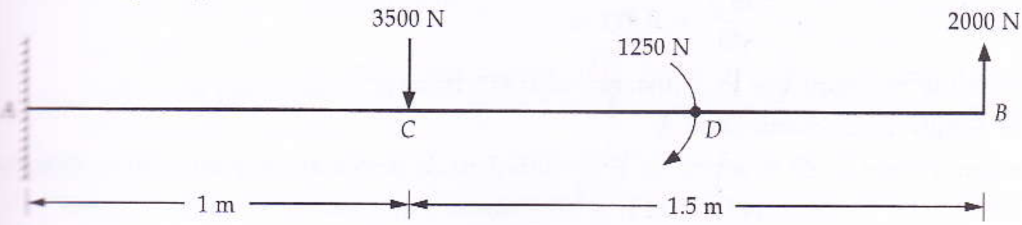


Fig. 1.97

Determine: (*i*) resultant of the system and (*ii*) and equivalent system through the fixed and *A*.

Solution : Resultant force $R = 3500 - 2000 = 1500$ N acting downward

Resultant moment about A (clockwise moment positive)

$$= 3500 \times 1 + 1250 - 2000 \times 2.5$$

$$= 3500 + 1250 - 5000$$

$$= -250 \text{ Nm} \quad \text{(anticlockwise)}$$

The resultant force acts downward and it will give the anti-clockwise moment if it acts towards left of A. Let x be the distance of resultant force (1500 N) from A. Then

$$1500x = 250; \quad x = \mathbf{0.166\ m}$$

Hence the resultant of the system is 1500 N downward and it acts at a distance of 0.166 m left of fixed and A.

(*ii*) This means that we have to determine a single resultant force and a single moment through$A$.

Single resultant force = 1500 N (downward)
Single resultant moment = 250 N (anticlockwise)

EXAMPLE 1.60

A rigid bar is subjected to a system of parallel forces as shown in Fig. 1.98.

Reduce this system to

(*a*) a single force

(*b*) a single force-moment system at *A*

(*c*) a single force moment system at *B*

Solution : (*a*) A single force or resultant

Taking downward forces as positive, the resultant of the given force system is

$$R = -15 + 50 - 10 + 20 = \mathbf{45\ N}$$

Thus the resultant has a magnitude of 45 N, its line of action is parallel to that of given forces and it acts vertically downwards.

Let this resultant act at a distance x from the end A.

Taking moments about A (clockwise moments positive) and applying the principle of moments,

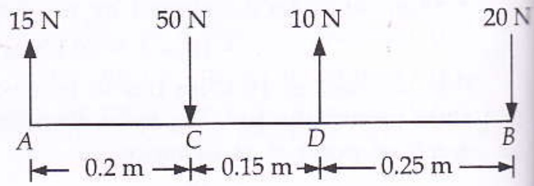


Fig. 1.98

Moment of the resultant = sum of the moments of its components

$$45x = (50 \times 0.2) - (10 \times 0.35) + (20 \times 0.6) = 10 - 3.5 + 12 = 18.5$$

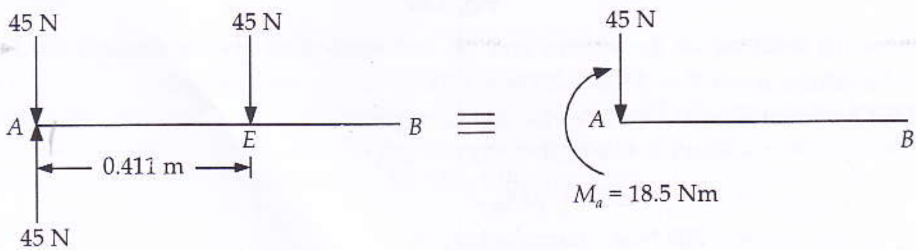
$$\therefore \qquad x = \frac{18.5}{45} = \mathbf{0.411\ m}$$

Thus the resultant lies at a distance of 0.411 from A.

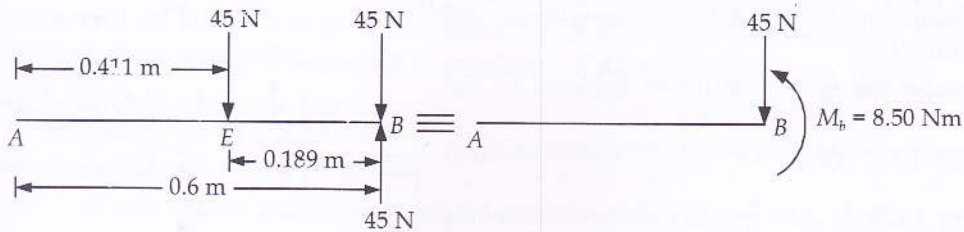
(*b*) **Single force-moment at *A***

When a force of 45 N acting at E is shifted to A, it is a accompanied by a moment

$$M_a = (45 \times 0.411) = \mathbf{18.5\ Nm}$$



(*c*) **Single force - moment at** ***B***



When the force of 45 N acting at E is moved to B, it is accompanied by a anticlock wise moment

$$M_b = -(45 \times 0.189) = \mathbf{-8.50\ Nm}$$

1.17. COUPLE

Two parallel forces equal in magnitude but opposite in direction, and separated by a finite distance are said to form a couple.

The rotational effect of a couple is measured by its moment which is defined as the product of either of the forces and the perpendicular distance between the forces. The perpendicular distance separating the two forces is called *arm of the couple.*

With reference to Fig. 1.99, the moment of couple M is

$$M = P \times l$$

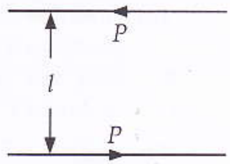


Fig. 1.99

Examples of a couple :

(*i*) Winding of a watch or a clock

(*ii*) Opening or closing a water tap

(*iii*) Unscrewing the cap of an ink bottle

(*iv*) Locking/unlocking of a lock with a key

(*v*) Turn of the cap of a pen

Properties of a couple :

(*i*) If moment is taken about any point lying in the plane of couple, then moment of the couple remains the same.

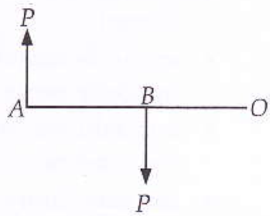


Fig. 1.100

Taking moments about point O, (Fig. 1.100)

$$= P \times AO - P \times BO$$
$$= P\,(AO - BP)$$
$$= P \times AB \text{ clockwise}$$
$$= \text{moment of couple}$$

Obviously a deduction can be made that

"the algebraic sum of the moments of two forces forming a couple about any point in their plane is constant and equal to the moment of the couple."

(*ii*) Two caplanar couples, whose moments are equal and opposite, balance each other.

With reference to Fig. 1.101, there act two couples whose moments are equal and opposite. These couples will balance each other and accordingly there will be no turning effect at point O.

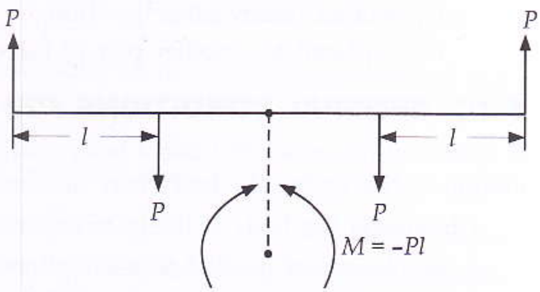


Fig. 1.101

(*iii*) Any two couples will be equivalent if their moments are equal, both in magnitude and direction.

Consider the system of forces depicted in Fig. 1.102

In Fig. 1.102 (*a*) : $M = P \times l = P\,l$ (anticlockwise)

In Fig. 1.120 (*b*) : $M = \dfrac{P}{2} \times 2\,l = P\,l$ (anticlockwise)

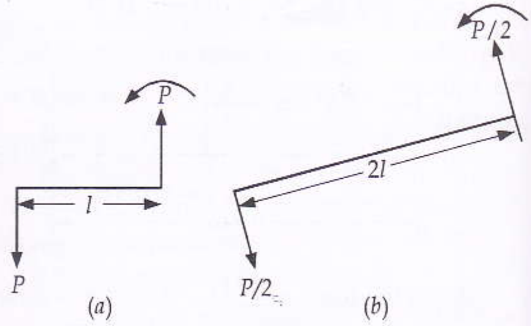


Fig. 1.102

(*iv*) Algebraic sum of moments of a number of couples is equal to the moment of a single couple.

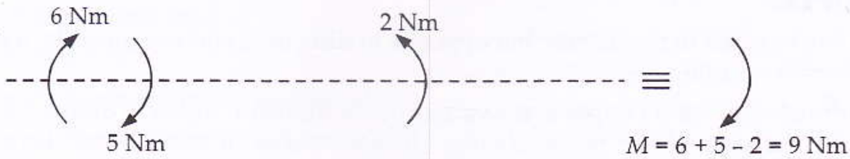


Fig. 1.103

(*v*) A single force P and a couple M acting in the same plans on a body cannot balance each other. However, they are together equivalent to a single force at a distance $e = M/P$ from its original line of action.

The salient aspects of a couple may then be summarised as:

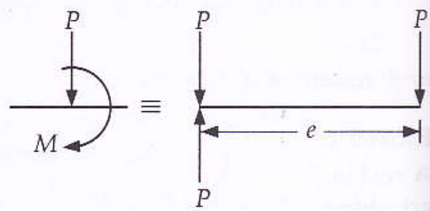


Fig. 1.104

- a couple consists of a pair of equal and opposite forces separated by a definite distance.
- the algebraic sum of the vertical and horizontal components of the forces forming a couple is zero.
- a couple cannot be balanced by a single force, but can be balanced only by a couple of opposite sense.
- any number of coplanar couples can be reduced to a single couple of moment equal to the algebraic sum of the moments of all the couples.
- the translatory effect of a couple on a body is zero.
- the effect of couple on a body remains unchanged if the couple is
  (*a*) rotated through an angle,
  (*b*) shifted to any other position,
  (*c*) replaced by another pair of forces whose rotational effect is same.

1.18. GENERAL CONDITIONS FOR EQUILIBRIUM

A system of forces acting upon a body changes or tends to change the state of rest or of uniform motion of the body. The body may also undergo rotation if a couple also acts on the body.

Obviously, the body is likely to experience

- displacement parallel to x-axis due to component F_x of the resultant force
- displacement parallel to y-axis due to component F_y of the resultant force
- rotation due to resultant couple.

When acted upon by a system of **coplanar concurrent** forces, the body undergoes only displacement and the rotation is zero. The system is then said to be in equilibrium if there is no unbalanced force acting on it, *i.e.*, the resultant of all forces acting on the body is zero. Analytically then

(*i*) the algebraic sum of components of all forces in horizontal direction in their plane is zero, *i.e.*, $\Sigma F_x = 0$.

(*ii*) the algebraic sum of components of all forces in vertical direction in their plane is zero, *i.e.*, $\Sigma F_y = 0$.

Graphically, the force polygon, *i.e.*, the force (vector) diagram must close.

When acted upon by a system of **coplanar non-concurrent** forces, the body undergoes displacement (due to resultant force) as well as rotation due to resultant couple. The system is then said to in equilibrium if the algebraic sum of all the forces and their moments about any point in their plane is zero. Mathematically,

- $\Sigma F_x = 0$, *i.e.*, the algebraic sum of components of all the forces in horizontal direction is zero.
- $\Sigma F_y = 0$, *i.e.*, the algebraic sum of components of all the forces in vertical direction is zero.
- $\Sigma M = 0$, i.e., the algebraic sum of all the moments is zero.

The conditions $\Sigma F_x = 0$ and $\Sigma F_y = 0$ ensure that the system does not reduce to a single force and condition $\Sigma M = 0$ ensures that it does not reduce to a couple.

EXAMPLE 1.61

A rectangle *ABCD* has sides *AB* = *CD* = 80 mm and *BC* = *DA* = 60 mm. Forces of 150 N each act along *AB* and *CD*, and forces of 100 N each act along BC and DA. Make calculations for the resultant moment of the force system.

Solution : Refer to Fig. 1.105 for the arrangement of the force system.

The forces of 150 N each acting along *AB* and *CD* generate a couple of moment

$$= 150 \times 60 = 9000 \text{ Nmm}$$

The forces of 100 N each acting along *BC* and *DA* produce a couple of moment,

$$= 100 \times 80 = 8000 \text{ Nm}$$

$\therefore$ Resultant moment $= 9000 + 8000$

$= \mathbf{17000}$ Nmm (anticlockwise).

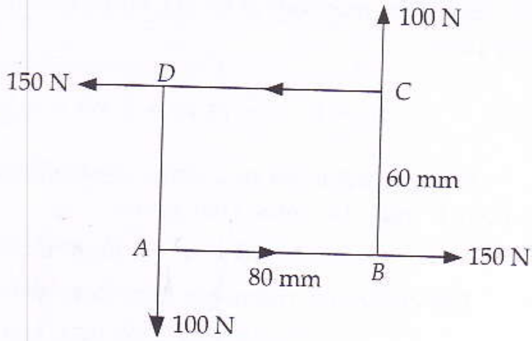


Fig. 1.105

Alternatively, the above result can be obtained by taking moments of all forces about any point, say *A*. That gives

$$M = (100 \times 80) + (150 \times 60) + (100 \times 0) - (150 \times 0)$$
$$= 8000 + 9000 = \mathbf{17000\ Nmm}\ \text{(anticlockwise)}$$

EXAMPLE 1.62

A square block of each side 1.5 m is acted upon by a system of forces along its sides as shown in the adjoining figure. If the system reduces to a couple, determine the magnitude of the forces *P* and *Q*, and the couple

Solution : The system is said to reduce to a couple and as such the algebraic sum of the horizontal and vertical components of different forces is zero.

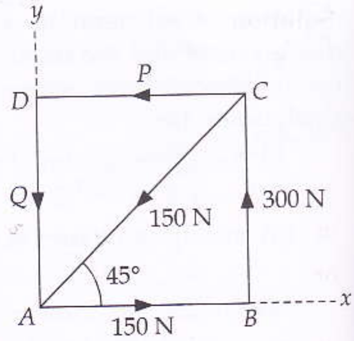


Fig. 1.106

That gives :

$$\Sigma F_x = 150 - 150 \cos 45° - P = 0$$

$$\therefore \quad P = 150 - 150 \times 0.707 = \mathbf{43.95\ N}$$

Also, $\Sigma F_y = 300 - 150 \sin 45° - Q = 0$

$$\therefore \quad Q = 300 - 150 \times 0.707 = \mathbf{193.95\ N}$$

Moment of couple = algebraic sum of the moments of forces about any corner, say A

$$= -300 \times 1.5 - 43.95 \times 1.5 = \mathbf{-515.925\ Nm}$$

The negative sign implies that the moment is anticlockwise.

EXAMPLE 1.63

A 30 cm long rod rests on two pegs whose distance apart is 15 cm. Weights of 3W and 5W respectively are suspended from its ends. Determine the position of pegs if its reactions are to be equal.

Solution : Let the pegs be so located at points E and F such that

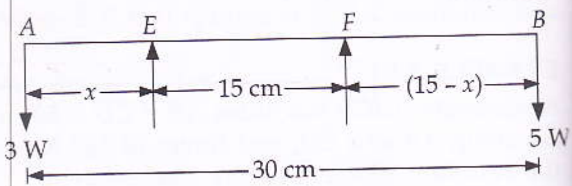


Fig. 1.107

$$AE = x \text{ cm}$$

and $\quad BF = (15 - x)$ cm

Since the reactions at ends E and F are equal, we have

$$R_e = R_f = \frac{1}{2}(3\,W + 5\,W) = 4\,W$$

For the rod to be in a state of equilibrium, the algebraic sum of the moments of the forces about E must be zero. That gives:

$$-3\,Wx - 4\,W \times 15 + 5\,W(30 - x) = 0$$

The clockwise moments have been taken positive and anticlock moments negative

$$-3\,Wx - 60\,W + 150\,W - 5\,Wx = 0$$

Solving: $\quad x = \dfrac{90}{8} = \mathbf{11.25\ cm}$

$$\therefore \quad AE = 11.25 \text{ cm}$$

and $\quad BF = 15 - 11.25 = \mathbf{3.75\ cm}$

EXAMPLE 1.64

A weight of 800 N is hanging from a wooden beam which is carried by two persons as indicated in the adjoining figure. Neglecting weight of the beam, determine the load shared by each person.

Solution : Considering equilibrium of the beam,

$$\Sigma F_y = 0; \qquad P + Q - 800 = 0$$

$$P + Q = 800$$

Taking moments about lower end A of the beam,

$$Q \times AE - W \times AD = 0$$

or $$Q \times AB \cos 60^\circ - W \times AC \cos 60^\circ = 0$$

or $$Q \times (1 \times 0.5) - 800 \times (0.4 \times 0.5) = 0$$

or $$0.5\, Q = 160\ ; \quad Q = \frac{160}{0.5} = \mathbf{320\ N}$$

$$P = 800 - Q$$

$$= 800 - 320 = \mathbf{480\ N}$$

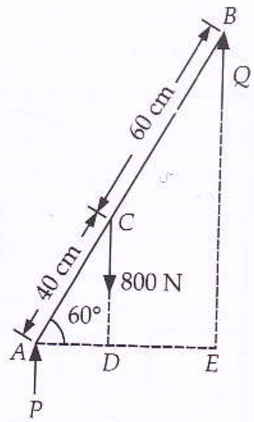
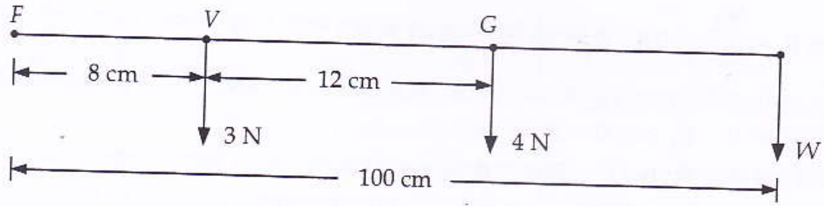


Fig. 1.108

EXAMPLE 1.65

The lever of a lever safety valve is 100 cm long, weighs 4 N and this weight acts at its CG which is 12 cm away from the valve. The valve weighs 3 N, is 10 cm in diameter and is located at a distance of 8 cm from the fulcrum. Make calculations for the weight to be suspended at the end of the lever which will just release the steam at a pressure of 1×10^5 N/m^2.

Solution : Refer Fig. 1.108



F = fulcrum; V = valve; G is position of CG

Fig. 1.109

Force on the valve, P = steam pressure × area of valve

$$= 10^5 \times \left\{\frac{\pi}{4} \times (0.1)^2\right\} = 785\ \text{N}$$

Taking moments about the fulcrum,

$$(W \times 100) + (4 \times 20) + (3 \times 8) - (785 \times 8) = 0$$

or $$100\, W = (785 \times 8) - (4 \times 20) - (3 \times 8)$$

$$= 6280 - 80 - 24 = 6176$$

$\therefore$ $$W = \frac{6176}{100} = \mathbf{61.76\ N}$$

EXAMPLE 1.66

Two smooth spheres, each of radius 20 cm and weight 200 N, rest in a horizontal channel having vertical walls, the distance between which is 72 cm as shown in Fig. 1.110. Find the pressures at contact points E, F and G.

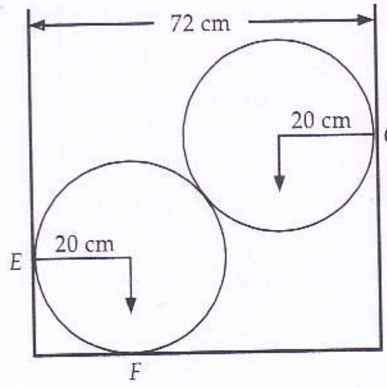


Fig. 1.110

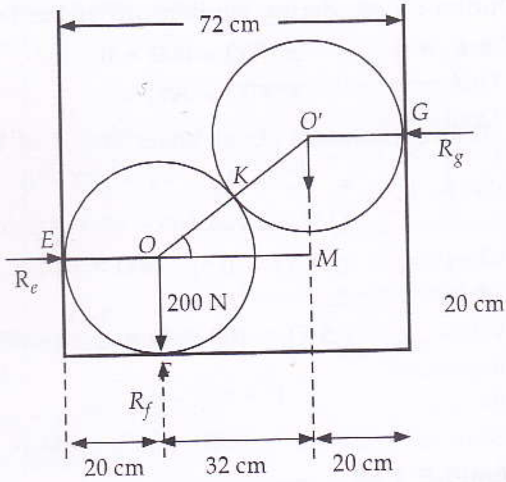


Fig. 1.111

Solution : Let R_e, R_f and R_g be the reactions at surface points E, F and G respectively.

From the geometrical configuration (Fig. 1.111)

$$OO' = 20 + 20 = 40 \text{ cm}$$

$$OM = 32 \text{ cm}$$

$$\cos\theta = \frac{32}{40} = 0.8 \text{ ; } \theta = 36.87^\circ \text{ ; } \sin\theta = 0.6$$

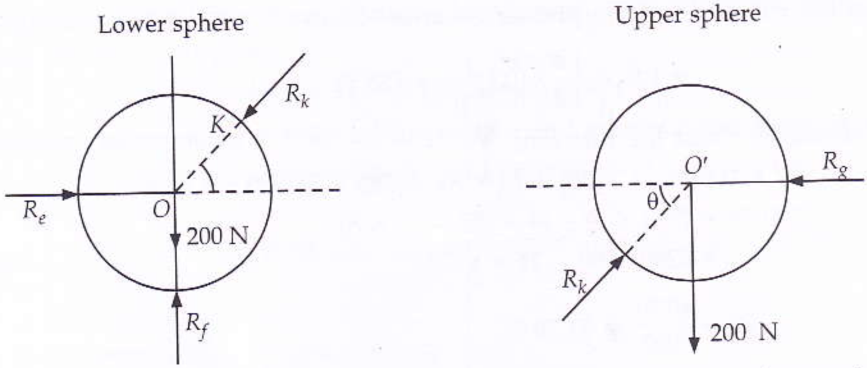
The upper sphere is in equilibrium under the action of forces R_g, R_k and its weight: For that

$$\Sigma F_x = 0; \quad R_k \cos\theta - R_g = 0$$

$$\Sigma F_y = 0; \quad R_k \sin\theta - 200 = 0$$

$$\therefore \quad R_k = \frac{200}{\sin\theta} = \frac{200}{0.6} = \mathbf{333.33\ N}$$

$$R_g = R_k \cos\theta = 333.33 \times 0.8 = \mathbf{266.66\ N}$$



The lower sphere is in equilibrium under the action of R_e, R_f, R_k and its weight. For that

$$\Sigma F_x = 0; \quad R_k \cos\theta - R_e = 0$$

$$\Sigma F_y = 0; \quad R_k \sin\theta + 200 - R_f = 0$$

$$\therefore \quad R_e = R_k \cos\theta = 333.33 \times 0.8 = \mathbf{266.66\ N}$$

$$R_f = R_k \sin\theta + 200 = 333.33 \times 0.6 + 200 = 200 + 200 = \mathbf{400\ N}$$

REVIEW QUESTIONS

A. Conceptual and Conventional questions:

1. Differentiate between statics, kinetics and kinematics.
2. Define matter, particle and body. How does a rigid body differ from an elastic body?
3. Differentiate between scalar and vector quantities. How a vector quantity is represented?
4. Given here is a list of some of the quantities used in mechanics: mass, displacement, momentum, area, volume, velocity, weight, speed, temperature and force. Classify these into scalar and vector quantities.
5. Make a clear distinction between mass, force and weight.
6. What are the two ways for the representation of forces ?
7. Represent graphically a force of 50 N acting upwards at 60° to the horizontal and towards the right.
8. State the effects which a force may produce when it acts on a body.
9. Enumerate the characteristics of a force.
10. What is a force system ? What is its point of concurrency ?
11. How forces are specified depending upon the position of their line of action?
12. Sketch a non-concurrent-non parallel-coplanar force system.
13. State the difference between equilibrium, resultant and equilibrant.
14. Explain the principle of transmissibility of forces.
15. Differentiate between resolution and composition of forces.
16. State : (*a*) parallelogram law of forces, (*b*) triangle law of forces (*c*) Lami's theorem.
17. Is it possible for a force to have a component larger in magnitude than the force itself?
18. Comment on the validity of following statements:
    (*i*) Rectangular components of a force *P* are forces acting at right angles to the force *P*.
    (*ii*) Lines of action of any two components of a force intersect on the line of action of the force.
    (*iii*) There are many possible solutions to the determination of the components of a given force.
19. Define the term free body diagram and state the importance of drawing such a diagram.
20. Define moment and moment of force.
21. List some engineering applications of moment.
22. State the possible displacements of a body subjected to a general force system.
23. State and prove Varignon's theorem.
24. What is a couple? Enumerate its various characteristics.
25. Can a couple be balanced by a single force applied to a body?
26. Is it true that the line of action of the resultant of the parallel forces always passes between the lines of action of these forces?
27. State the equilibrium conditions for bodies acted upon by a system of (*i*) coplanar concurrent forces, (*ii*) coplanar non-concurrent forces.
28. The resultant of two equal forces acting at a point also equals to *P*. Determine the angle between the two forces.
29. Refer Fig. 1.112 for a system of concurrent coplanar forces. If the resultant is zero, find the magnitude and direction of force *P*.

30. A 1 kN force has been resolved into components along AB and AC directions in the x-y plane specified by the angles α and β as shown in Fig. 1.113. If the component along AC is 2 kN and the component along AB is 1.6 N, determine the angles α and β.
31. A body is free to slide on a smooth vertical circular wire and is connected by a string, equal in length to the radius of the circle, to the highest point of the circle. Find the tension developed in the string, and the reaction of the circle.

(Each is equal to weight of the body)

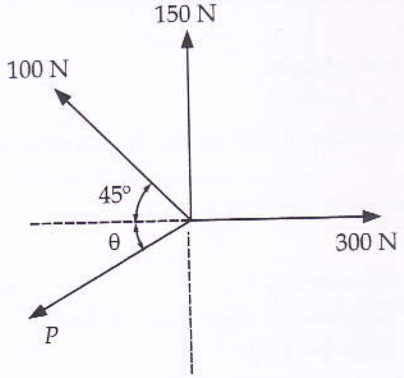


Fig. 1.112

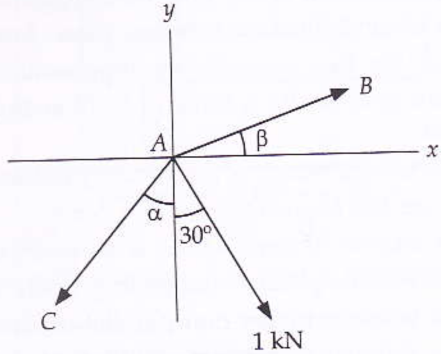


Fig. 1.113

32. A particle is acted upon by the force system shown in Fig. 1.114. Determine the magnitude and direction of the resultant.
33. Three forces $2P$, $3P$ and $4P$ act along three sides of an equilateral triangle taken in order. Find the magnitude and line of action of the resultant force. **(1.732 P, 210°)**
34. A small block of weight 100 N is placed on an inclined plane which makes an angle $\theta = 30°$ with the horizontal. What is the component of this weight (i) parallel to the inclined plane and (ii) perpendicular to the inclined plane. **(8.66 N, 50 N)**

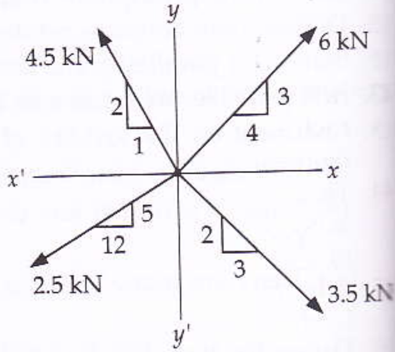


Fig. 1.114

35. A lamp weighing 5 N is suspended from the ceiling by a chain. It is pulled aside by a horizontal chord until the chain makes an angle of 60° with the ceiling. Determine the tensions in the chain and the chord. **(5.77 N, 2.89 N)**
36. A weight W is supported by the two strings at right angles to one another and attached to two points in the same horizontal line. Prove that the tensions induced in the strings are inversely proportional to their lengths.
37. The top of an electric pole is connected with a stay wire which makes an angle of 60° with the horizontal. If a horizontal force of 100 N is essential for keeping the pole vertical determine tension induced in the stay wire. Also workout vertical component of this tension and state the purpose served by it. **(200 N, 173 N, resistance against lifting)**
38. A body of 50 N weight is suspended by two strings 5 m and 12 m long. The other ends of the strings are fastened to the extremities of a rod which is 13 m long. The rod is so held that the body hangs vertically below its mid point. Workout the tensions induced in the strings for the given arrangement. **(46.15 N, 19.23 N)**

39. Find the magnitude of the two like parallel forces acting at a distance of 24 cm, whose resultant is 200 N and its line of action is at a distance of 6 cm from one of the forces.

(50 N, 150 N)

40. Three like parallel force 20 N, 40 N and 60 N are acting at points *A*, *B* and *C* respectively on a straight line *ABC*. The distances are *AB* = 3 m, *BC* = 4 m. Find the resultant and also the distance of the resultant from point *A* on line *ABC*.

(120 N, 4.5 m)

41. Four parallel forces of magnitudes 100 N, 200 N, 50 N and 400 N are shown in Fig. 1.115. Determine the magnitude of the resultant and also the distance of the resultant from point *A*.

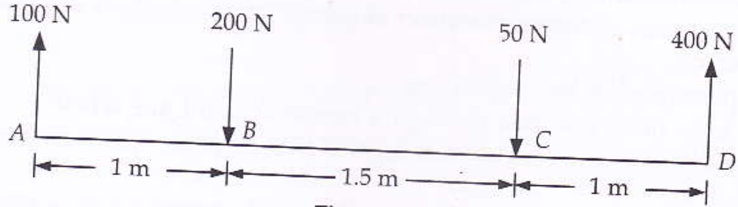


Fig. 1.115

(R = 250 N & 4.3 m)

42. Two like parallel forces *P* and *Q* act on a rigid body at *A* and *B* respectively. If *P* and *Q* be inter changed in position, show that the point of application of the resultant will be displaced through a distance *d* along *AB*, where

$$d = \frac{P-Q}{P+Q} \times AB$$

43. *ABCD* is a rectangle in which *AB* = *CD* = 100 mm and *BC* = *DA* = 80 mm. Forces of 100 N each act along *AB* and *CD* and forces of 50 N each act along *BC* and *DA*. Find the resultant moment of two couples.

(13 Nmm)

44. Three forces, acting on a rigid body, are represented in magnitude, direction and line of action by the three sides of a triangle taken in order. Prove that the forces are equivalent to a couple whose moment is equal to twice the area of the triangles.

45. A beam *AB* of span 8 m is hinged at *A* and is on rollers at *B*. It carries a uniformly distributed load, a concentrated load and an externally applied moment as shown in Fig 1.116. Determine the reactions at *A* and *B* for the loading shown.

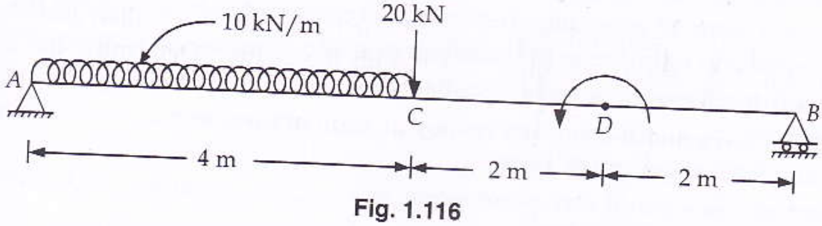


Fig. 1.116

(R_b = 10 kN down and, R_a = 70 kN)

46. Figure 1.137 shows a horizontal beam *AD* of a jib-crane supported horizontally at the point *A* and by a tie rod *BC*. Calculate the tension in the tie rod when a load of 1 kN is acting at point *D* on the beam.

47. A post *BC* of weight 18 N is prevented from sliding with the help of cable *AB* as shown in Fig. 1.105. Assuming all surfaces to be smooth, work out the tension developed in the cable.

(5.19 N)

B. Complete the following statements with most appropriate word/words :

1. Mechanics is a science which describes and predicts the state of .... or .... under the action of forces.
2. .... relates to bodies in motion without any reference to forces.
3. A body which does not deform under the action of applied forces is called the ....
4. A .... is a body of infinitely small volume and is considered to be concentrated at a point
5. A .... quantity is one that has magnitude as well as direction
6. Magnitude, direction, sense and .... are the characteristics of a force
7. According to .... of forces, the effect of a force upon a body is the same at every point in its line of action
8. Fundamental principles of mechanics are ....
9. Forces whose lines of action pass through a common point are called .... forces.
10. A .... is a single force which can replace two or more forces and produce the same effect as the forces.
11. Equlibrant is equal and opposite to the .... of several forces acting or a body.
12. .... is a method of designating a force by writing two capital letters one or either side of the force.
13. Forces whose lines of action pass through a common point are called ...... forces
14. A ...... is a single force which can replace two or more forces and produce the same effect as the forces.
15. The splitting of a force into two perpendicular direction without changing its effect is called ....
16. Two forces P and Q have angle of inclination θ between them. If α is the angle which their resultant makes with the direction of P, then $\tan \alpha$ = ...
17. The resultant of two forces acting on a particle is equal to either of them. The angle of inclination between the forces is then equal to ....
18. "If three forces acting at a point are in equilibrium, then each force will be proportional to the sine of the angle between the other two forces". This is the statement of ....
19. If a system of coplanar concurrent forces is in equilibrium then the vector diagram drawn with these forces must be a .... figure.
20. Equations of equilibrium of a particle subjected to coplanar force system are ....
21. A body isolated from all other members which are connected to it is called the .... body.
22. .... is equal and opposite to the resultant of several forces acting on the body.
23. The rotational tendency of a force is called ....
24. The moment of force about any point is the .... of force and .... distance between the point and line of action of force.
25. The moment of force about any pivot point is a .... quantity and it varies .... with its distance from the point.
26. "The algebraic sum of the moments of a system of forces about any point in their plane is equal to the moment of their resultant about that point."
    This is the generalised statement of ....
27. Force causes .... displacement while moment causes .... displacement.
28. Principle of moment is based on .... theorem.
29. Moment of force about an axis indicates .... of the body about that axis.
30. A couple is formed by two .... and .... parallel forces.

31. Two equal and opposite parallel forces produce a .... whose moment is equal to either force multiplied by their .... distance.
32. The moment of a couple is known as ....
33. If a body is in equilibrium under the action of three forces, then the forces must be .... or ....
34. The conditions of equilibrium for coplanar and non-concurrent forces are ....

Answers :

1. rest, motion 2. kinematics 3. rigid body 4. particle 5. vector 6. point of application 7. principle of transmissibility 8. Newton's of motion parallelogram law of forces, principle of transmissibility etc. 9. concurrent 10. resultant 11. resultant 12. Bow's notation 13. concurrent 14. resultant 15. resolution of forces 16. $\dfrac{Q\sin\theta}{P+Q\cos\theta}$ 17. 120° 18. Lami's theorem 19. closed 20. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ 21. free body 22. equilibriant 23. moment 24. product, perpendicular 25. vector, directly 26. Varignon's theorem 27. linear, angular 28. Varignon's 29. rotational tendency 30. equal, opposite 31. couple, perpendicular 32. torque 33. concurrent, parallel 34. $\Sigma F_x = 0, \Sigma F_y = 0$ and $\Sigma M = 0$

C. Multiple choice questions :

1. For analysis purposes, a body can be modelled as rigid if
   (a) it has continuous distribution of mass
   (b) it is characterized by some preferred configuration of its own
   (c) relative motion between its parts is negligible
   (d) its dimensions are small compared with the co-ordinates describing its motion
2. Which of the following quantities is not a vector quantity?
   (a) energy (b) momentum (c) acceleration (d) force
3. Newtons first law of motion gives the concept of
   (a) work (b) force (c) inertia (d) energy
4. Newton's second law of motion connects
   (a) change of momentum and velocity
   (b) momentum and acceleration
   (c) momentum and rate of change of force
   (d) rate of change of momentum and external force
5. A force which combines with two or more forces to produce equilibrium is called
   (a) resultant (b) equilibrant (c) couple (d) moment
6. According to the principle of transmissibility of forces, the effect of force on a body is
   (a) same at every point in its line of action
   (b) different at different points in its line of action
   (c) minimum when it acts at the centre of gravity of the body
   (d) maximum when its acts at the centre of gravity of the body
7. Resultant of two equal forces is equal to either of them. The angle between the forces is
   (a) 0° (b) 60° (c) 90° (d) 120°
8. Four forces P, $2P$, $3P$ and $4P$ act along the sides, taken in order, of a square. The resultant force is
   (a) zero (b) $12\sqrt{2}\,P$ (c) $2P$ (d) $\sqrt{5}\,P$

9. Given that $\vec{P} + \vec{Q} = \vec{R}$ and also $P = Q = R$. Then the angle between forces $\vec{P}$ and $\vec{Q}$ are
   (*a*) 0 (*b*) $\frac{\pi}{3}$ (*c*) $\frac{2\pi}{3}$ (*d*) π
10. The sum of two forces acting at a point is 16 N. If the resultant force is 8 N and its direction is perpendicular to the minimum force, then the forces are :
    (*a*) 6 N and 10 N (*b*) 8 N and 8 N
    (*c*) 4 N and 12 N (*d*) 2 N and 14 N
11. A system of three forces acts on a body and keeps it in equilibrium. The forces need to be
    (*a*) coplanar only (*b*) concurrent only
    (*c*) coplanar as well as concurrent (*d*) coplanat but may or may not be concurrent
12. Two forces can be in equilibrium if they are
    1. equal in magnitude 2. opposite in direction 3. collinear in action
    Which of these are the essential conditions for equilibrium?
    (*a*) 1 and 3 (*b*) 2 and 3 (*c*) 1 and 3 (*d*) 1, 2 and 3
13. A number of forces acting at a point will lie in equilibrium if
    (*a*) sum of the forces is zero (*b*) sum of the resolved parts is zero
    (*c*) algebric sum of the forces is zero
    (*d*) sum of the resolved parts in any two mutually perependicular direction is zero
14. A circular roller of weight *W* hangs by a tie rod and rests against a smooth vertical wall as shown in Fig. 1.117. The tension in the tie rod *AB* will be
    (*a*) less than weight *W*
    (*b*) greter then *W*
    (*c*) equal to *W*
    (*d*) data insufficient for making force analysis

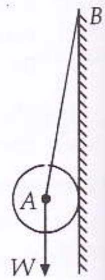


Fig. 1.117

15. The freee body diagram of a body shows the body
    (*a*) with its surroundings and external forces acting on it
    (*b*) isolated from all external forces
    (*c*) isolated from all surroundings
    (*d*) isolated from its surroundings and all external effects acting on it
16. Identify the wrong statement in the context of moment, couple and torque
    (*a*) Two parallel forces equal in magnitude but opposite in direction, and separated by a finite distance are said to form a couple.
    (*b*) Moment of force about a point is defined as the turning tendency of the force about that point.
    (*c*) The moment of a couple is known as torque
    (*d*) The sum of forces forming a couple in any direction has a definite non-zero value.
17. Consider the following statements :
    The effect of couple in a body remains unchanged if the couple is
    1. rotated through an angle 2. shifted to any other position
    3. replaced by another pair of forces whose rotational effect is same
    Which of these statements are correct ?
    (*a*) 1 and 2 (*b*) 1 and 3 (*c*) 2 and 3 (*d*) 1, 2, and 3

18. Consider the following statements
    1. The translatory effect of a couple on a body is zero.
    2. A couple can be balanced by a couple of opposite sense
    3. The moment of a couple about any point is the same.

    Which of these statements is/are correct?

    (*a*) 2 and 3 (*b*) 3 only (*c*) 1 and 2 (*d*) 1, 2, and 3

19. A rectangle *ABCD* has sides $AB = CD = 80$ mm and $BC = DA = 60$ mm. Forces of 150 N each act along *AB* and *CD*, and forces of 100 N each act along *BC* and *DA*. The resultant moment of the force system is

    (*a*) 8000 Nmm (*b*) 9000 Nmm (*c*) 1000 Nmm (*d*) 17000 Nmm

Answers :

1. (*c*)	2. (*a*)	3. (*c*)	4. (*d*)	5. (*b*)	6. (*a*)
7. (*d*)	8. (*b*)	9. (*c*)	10. (*a*)	11. (*c*)	12. (*d*)
13. (*d*)	14. (*b*)	15. (*d*)	16. (*d*)	17. (*d*)	18. (*d*)
19. (*d*)					