

H. W for 30th April 2024

2022-23

1. Test the convergence of the series

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{x}{4 \cdot 5 \cdot 6} + \frac{x^2}{7 \cdot 8 \cdot 9} + \dots$$

Ans:- convergent if $x \leq 1$, divergent $x > 1$.

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2. Find a fourier series to represent $f(x) = x - x^2$, $-\pi \leq x \leq \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$$

Ans:- $a_0 = -\frac{2}{3}\pi^2$, $a_n = -\frac{4}{n^2}(-1)^n$, $b_n = -\frac{2}{n}(-1)^n$

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3. Find the half range cosine series for the function $f(x) = (x-1)^2$ in $(0, 1)$. Hence prove that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Ans:- $f(x) = \frac{1}{3} + \frac{4}{\pi^2} \left(\cos \pi x + \frac{\cos 2\pi x}{2^2} + \frac{\cos 3\pi x}{3^2} + \dots \right)$

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4. Solve $(D^3 - 1)y = 3x^4 - 2x^3$

Ans:- $y = c_1 e^x + e^{-x/2} \left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x \right) - \{ 3x^4 - 2x^3 + 72x - 12 \}$

5. Solve $\frac{d^2 y}{dx^2} - 4y = x \sinh x$

Ans:- $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \left(\frac{e^x - e^{-x}}{2} \right) - \frac{2}{9} \left(\frac{e^x + e^{-x}}{2} \right)$
or, $y = c_1 e^{2x} + c_2 e^{-2x} - \frac{x}{3} \sinh x - \frac{2}{9} \cosh x$