1 (1.) $y''' - 3y'' + 3y' - y = 0$

(2.) $y = (c_1 + x c_2 + x^2 c_3) e^{-x} + \frac{e^{-x}}{3}$

(3) $y = e^{x}(c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x) + \frac{e^{x}\cos x}{2} - \frac{1}{8}\cos 2x + \frac{1}{104}(2\cos 4x - 3\sin 4x)$

(4.) $x = c_1 e^{3t} + c_2 e^{-3t} + c_3\cos t + c_4 \sin t + 8e^{t}$

$y = c_1 e^{3t} + 25 c_2 e^{-3t} + (3c_3 - 4c_4)\cos t + (4c_3 + 3c_4)\sin t + 8e^{t}$.

(5.) $y = c_1 x + \frac{c_2}{x} + \left(1 - \frac{1}{x}\right) e^{x}$

(6.) If $y_1, y_2, \ldots y_n$ are n linearly independent solution of n^{th} order diff. eqn. $a_n \frac{d^n y}{dx^n} + \cdots + a_0 y = 0$ then wronkian

$$W = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ y_1^{n-1} & y_2^{n-1} & \cdots & y_n^{n-1} \end{vmatrix} \neq 0.$$

(7.) $x = c_1\cos t + c_2 \sin t + c_3 \cos 3t + c_4 \sin t + \frac{1}{5} e^{-t} + \frac{2}{15}\cos 2t$

~~y~~ = Find corresponding value of y.

$y = c_5 \cos t + c_6 \sin t + c_7 \sin 3t + c_8 \cos 3t - \frac{1}{5} e^{-t} + \frac{1}{15}\sin 2t$

(8.) $y = c_1 e^{x} + c_2 e^{-x} - e^{-x}\sin^{-1}(e^{-x}) - e^{-x}(e^{2x} - 1)^{\frac{1}{2}}$

(9.) $y = c_1 \cos(\log(1+x)) + c_2(\sin\log(1+x)) + 2\log(1+x)\sin(\log(1+x))$

(10.) by changing the independ var. $y = (c_1 + c_2 x^2) e^{-x^2} + \frac{1}{2}$

12.) Cauchy's equation Let $t = e^z$, $z = \log t$

$$x + Dy = 0 \quad \text{— (1)}$$
$$Dx + y = 0 \quad \text{— (2)}$$

] solve by eliminating either x & y, then find the soln. in terms of z.

Lastly replace z by $\log t$.

Ans:— $x = -c_1 t + \frac{c_2}{t}$, $y = c_1 t + \frac{c_2}{t}$

13. Order - 2, deg. - 2

14. $y = (c_1 + x c_2) e^{2x} + 4 e^{2x}$

16. $x = e^{3t} \left[c_1 e^{(3+\sqrt{10})t} + c_2 e^{(3-\sqrt{10})t} \right]$

Find correspondly value of y. sub. x in first eqn

17. $y = (c_1 + x c_2) e^{-x} + e^{-x} \left[(x+2) \log(x+2) - x \right]$

18. $P.I. = \frac{1}{2!} e^{2x}$

19. $y = 2(\cos x - \sin x)$

20. $x = c_1 \cos \omega t + c_2 \sin \omega t$, $y = c_1 \sin \omega t - c_2 \cos \omega t$

$x^2 + y^2 = c_1^2 + c_2^2$

which is circle with centre $(0,0)$ & radius $\sqrt{c_1^2 + c_2^2}$

21. $y = c_1 \cos x + c_2 \sin x - \cos x \log (\sec x + \tan x)$

22. $y = (c_1 + x c_2) e^{2x} + 4x^2 e^{2x} + \cos 2x + 2x^2 + 4x + 3$

23. $P.I. = \frac{-\cos 2x}{4}$

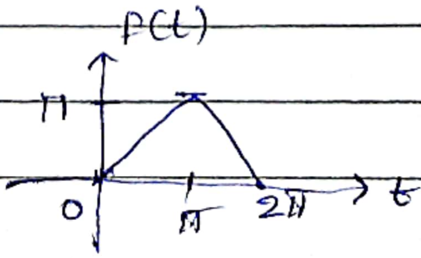
24. $x = c_1 e^{3t} - c_2 e^{-3t}$

$y = c_1 e^{3t} + c_2 e^{-3t}$

26. $e^{-2t} (6 \cos t + 6 \sin t)$

27. $\frac{e^{-3/s+1}}{s+1}$

28. $\frac{1}{s^2} \cdot \frac{1 - e^{-\pi s}}{1 + e^{-\pi s}}$



29. $\dfrac{a\sin at - b\sin bt}{a^2-b^2}$

30. $y = e^t - te^t - \dfrac{t^2}{2}e^t + \dfrac{t^5 e^t}{60}$

31. $\dfrac{\sqrt{\pi}}{(s-1)^{\frac{1}{2}}}$

32. $\frac{1}{\sqrt{2}}\sin(\sqrt{2}t - \sqrt{2}\pi)\, u(t-\pi)$

33. $\frac{1}{3}(\cos t - \cos 2t)$

34. $\dfrac{\omega}{\left(1-e^{\frac{-\pi s}{\omega}}\right)(s^2+\omega^2)}$

35. $\log 2$

36. $\dfrac{3^6}{(s+3)^4}$

37. If $L\{F(t)\} = f(s)$ then $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$

40. $F(t) = 2[u(t) - u(t-\pi)] + \sin t\, [u(t-2\pi)]$

$L\{F(t)\} = 2\left(\dfrac{1-e^{-\pi s}}{s}\right) + \dfrac{e^{-2\pi s}}{s^2+1}$

41. $\frac{1}{a^2}(\cosh at - 1)$

42. Not in syllabus.

43. convergent

44. conv. if $x \leq 1$ & div. if $x > 1$ (Ratio Test)

45. div.

46. conv. if $x < e$ & div. if $x \geq e$ (By Logarithmic test)

47. conv.

48. Conv. if $x \leq 1$ & div. if $x > 1$

49. $F(t) = 1 + \dfrac{8}{\pi^2}\sum_{n=1}^{\infty} \dfrac{1}{n^2}\left(2\cos\dfrac{n\pi}{2} - 1 - \cos n\pi\right)\cos\dfrac{n\pi t}{2}$

50. $f(x) = -\dfrac{\pi}{2} + \dfrac{4}{\pi}\left\{\dfrac{\cos x}{1^2} + \dfrac{\cos 3x}{3^2} + \cdots\right]$ and then put $x=0$.

51. $a_0 = 5$, ~~502~~.

52. $\frac{2}{n\pi}\left[1-(-1)^n\right] = b_n$

53. $a_0 = \pi$, $a_n = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$, $b_n = 0$

$f(x) = \frac{\pi}{2} - \frac{4}{\pi}\left(\frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \cdots\right)$

54. $f(x) = \sum_{n=1}^{\infty} \frac{4}{n^2\pi} \sin \frac{n\pi}{2} \sin nx$

55. $f(x)$ is odd fn., $\therefore a_n = 0$

57. $a_0 = \frac{2\pi^2}{3}$, $a_n = \frac{4}{n^2}(-1)^n$, $b_n = 0$

$x^2 = \frac{\pi^2}{3} - 4\left(\frac{\cos x}{1^2} - \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} \cdots\right)$

Put $x = \pi$.

58. $a_0 = 1$, $a_n = \frac{2}{n\pi} \sin \frac{n\pi}{2}$

$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin \frac{n\pi}{2} \cos nx$

59. $f(x) = \sum_{n=1}^{\infty} \frac{-2}{n}(-1)^n \sin nx$

60. Odd fn., $a_0 = 0$, $a_n = 0$, $b_n = \frac{2}{l}\int_0^l x \cos\left(\frac{\pi x}{l}\right) \sin\left(\frac{n\pi x}{l}\right) dx$

$b_n = \frac{2nl(-1)^n}{n^2-1}$, $n \neq 1$, $b_1 = \frac{-l}{2\pi}$.

61. Even fn. $b_n = 0$, $a_0 = 2$, $a_n = \frac{-2}{n^2-1}$, $a_1 = -\frac{1}{2}$

$f(x) = 1 - \frac{1}{2}\cos x + \sum_{n=2}^{\infty} \frac{-2}{n^2-1} \cos nx \, dx$

62. $a_1 = -4$

~~63. $f(x) = \frac{16}{\pi^2}\left[\frac{-1}{3^2}\sin\frac{3\pi x}{4} + \frac{1}{5^2}\sin\frac{5\pi x}{4} - \frac{1}{7^2}\sin\frac{7\pi x}{4}\right]$~~

63. $b_n = \frac{16}{n^2\pi^2} \sin \frac{n\pi}{2}$

$f(x) = \sum_{n=1}^{\infty} \frac{16}{n^2\pi^2} \sin \frac{n\pi}{2} \sin \frac{n\pi x}{4}$