ABES ENGINEERING COLLEGE, GHAZIABAD (032)

B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-4 (MULTIVARIABLE CALCULUS-I)

QUESTION BANK

1. Evaluate $\iint y\,dxdy$ over the part of the plane bounded by the lines $y = x$ and $y = 4x - x^2$.
2. Evaluate $\iint xy\,dxdy$ over the region in the positive quadrant for which $x + y \le 1$.
3. Let D be the region in the first quadrant bounded by the curves $xy = 16$, $x = y$, $y = 0$ *and* $x = 8$ .Sketch the region of integration of the following integral $\iint_D x^2\,dxdy$ and evaluate it by expressing it as an appropriate repeated integral .
4. Show that $\iint_R r^2 \sin\theta\,drd\theta = \frac{2a^2}{3}$, where R is the region bounded by the semi-circle $r = 2a\cos\theta$, above the initial line .

   Answer is 2a^3/3
5. Evaluate $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta\,drd\theta$.
6. Evaluate the following integral by changing the order of integration: $\int_0^1\int_{y^2}^{2-y} xy\,dx\,dy$.
7. Evaluate the following integral by changing the order of integration: $\int_0^{\infty}\int_0^{y} ye^{-\frac{y^2}{x}}\,dx\,dy$.

   Answer is -1/2
8. Evaluate the following integral by changing the order of integration: $\int_0^2\int_{\frac{x^2}{4}}^{3-x} xy\,dy\,dx$
9. Evaluate the following integral by changing the order of integration: $\int_0^3\int_0^{\frac{6}{x}} x^2\,dy\,dx$
10. Evaluate $\iiint_R (x + y + z)dxdydz$, where R is the region determined by $0 \le x \le 1, 0 \le y \le x^2, 0 \le z \le x + y$.

    Answer: 87/140
11. Let D be the region in the first quadrant bounded by $x = 0, y = 0$ *and* $x + y = 1$. Change the variable x, y *to* u, v *where* $x + y = u, y = uv$ and evaluate $\iint_D xy(1 - x - y)^{1/2}\,dxdy$.
12. Using the transformation $x - y = u, x + y = v$, show that $\iint_R \sin\left(\frac{x-y}{x+y}\right)dxdy = 0$, where R is bounded by the coordinate axes and $x + y = 1$ in the first quadrant.

    since, no region is bounded in the 1st quadrant, so area=0
13. Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dxdy$ by changing into polar co-ordinates.

    Answer: 2a^4 pie
14. Prove the relation between Beta and Gamma function.
15. Using the beta and gamma functions, evaluate $\int_0^1 \left(\frac{x^3}{1-x^3}\right)^{1/2} dx$.

16. Compute $\iiint_V x^2 dxdydz$ over volume of tetrahedron bounded by $x=0, y=0, z=0$ and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

ANSWERS:

SECTION – B

1: 54/5
2. 1/24
3. 448
5. $\frac{5}{8}\pi a^3$
6. 3/8
7. 1/2
8. 8/3
9. 27
10. 9/2
11. 16/945
12. $\frac{3\pi a^4}{4}$
13. $a^3bc/60$

16. $\frac{b^6}{48}$