Steady-State Analysis of Single Phase AC Circuit

ALTERNATING QUANTITY

An alternating quantity is that which acts in alternate directions and whose magnitude undergoes a definite cycle of changes in definite intervals of time. When a simple loop revolves in a magnetic field, an alternating emf is induced in the loop. If the loop revolves with an uniform angular velocity the induced alternating emf is sinusoidal in nature. The important alternating quantities, that will be discussed in the chapter are current and voltage.

ALTERNATING VOLTAGE

Alternating voltage may be generated by rotating a coil in a stationary magnetic field or rotating a magnetic field across a stationary coil. The value of the voltage generated in each case depends on:

(i) The number of turns in the coils.

(ii) The strength of the field.

(iii) The speed at which the coil or magnetic field rotates

- **Period:** – This is the length of time in seconds that the waveform takes to repeat itself from start to finish. This value can also be called the Periodic Time, (T) of the waveform for sine waves, or the Pulse Width for square waves.
- **Cycle**: – One complete set of all the possible values of a wave forms its one complete cycle.
- **Frequency**: – This is the number of times the waveform repeats itself within a one second time period. Frequency is the reciprocal of the time period, (f = 1/T) with the standard unit of frequency being the Hertz, (Hz).
- **Amplitude**: – This is the magnitude or intensity of the signal waveform measured in volts or amps.
- **Phase**: – Phase of a wave at a point is defined as the fraction of time period or the angle covered by the wave in reaching at that point from the position where the wave had crossed its reference position last time.

ADVANTAGES OF SINE WAVE

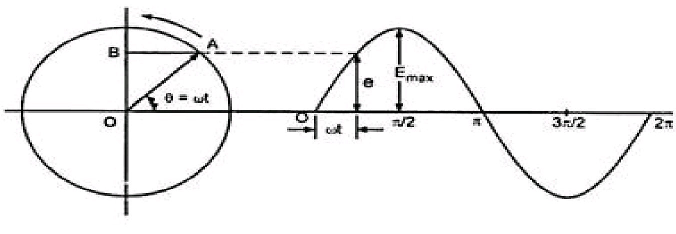
- Sine wave can be expressed in a simple mathematical form.
- The resultant of two or more quantities varying sinusoidally at the same frequency is another sinusoidal quantity of same frequency.
- Rate of change of any sinusoidal quantity is also sinusoidal

REPERSENTATION OF ALTERNATING QUANTITY BY PHASOR

An alternating quantity can be represented by rotating vectors or phasors provided they satisfy the following conditions

- Length of vector should represent the amplitude of alternating quantity at a suitable scale.
- The vector should be in horizontal position when the quantity is zero and increasing positively
- The angular velocity of rotating vector should be such that it should complete its one revolution in same time in which the alternating quantity completes its one cycle

So if a vector fulfills the above conditions than it can represent the given alternating quantity and the instantaneous value of quantity at any instant can be obtained by projecting the vector on y-axis at same instant. In this representation the anticlockwise rotation of vector is considered positive and clockwise rotation is considered negative. Fig. Shows a vector OA representing an alternating voltage v = E_{max} Sin wt .

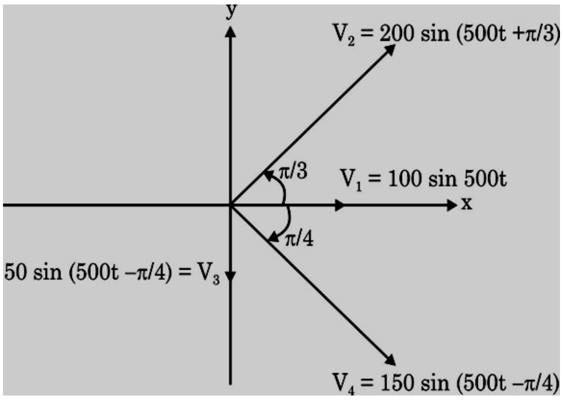


Phasor Representation of An Alternating Quantity

1.Draw a phasor diagram showing the following voltage $\left.\begin{array}{l}\mathbf{V_1 = 100sin500t\ ,\ \ V_2 = 200sin(500\ t + \pi/3)} \\ \mathbf{V_3 = -50cos500t,\ \ V_4 = 150sin(500\ t - \pi/4)}\end{array}\right]$ **Find RMS value of result voltage**

Ans. $V_1 = 100\sin 500t$, $V_2 = 200\sin\left(500t + \frac{\pi}{3}\right)$, $V_3 = -50\cos 500t$ Or $V_3 = 50\sin\left(500t - \frac{\pi}{2}\right)$

$V_4 = 150\sin\left(500t - \frac{\pi}{4}\right)$



Finding $V_{\max} \Rightarrow V_x = 100 + 200\cos\frac{\pi}{3} + 150\cos\frac{\pi}{4} + 50\cos\frac{\pi}{2} = 100 + 100 + 75\sqrt{2} = 306.067$ V

$V_y = 100\sin 0° + 200\sin\frac{\pi}{3} - 150\sin\frac{\pi}{2} - 50 = 0 + 173.2 - 106.67 - 50 = 17.133$

So, $V_{\max} = \sqrt{V_x^2 + V_y^2} = 306.55$ Volts And $V_{rms} = \frac{V_{\max}}{\sqrt{2}} = 216.76$ Volts.

AVERAGE AND RMS VALUE, FORM FACTOR AND PEAK FACTOR

AVERAGE VALUE: The average of all the instantaneous values of an alternating quantity over one complete cycle is called Average Value**.** If we consider symmetrical waves the positive half cycle will be exactly equal to negative half cycle. Therefore, the average over a complete cycle will be zero. In such cases only positive half cycle is considered to determine the average value. For sine waves avg. value is 0.636 times the maximum value.

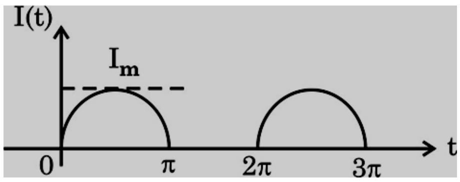
RMS VALUE: It is defined as that steady current which, when flows through a given resistor produces same quantity of heat as produced by the alternating current when flows through the same resistor for the same period of time. It is also called effective value or true value. For sine wave its value is 0.707 times the maximum value. Root Mean Square is the actual value of an alternating quantity which tells us an energy transfer capability of an AC source.

PEAK FACTOR: Peak Factor is also known as Crest Factor or Amplitude Factor. It is the ratio of maximum value to rms value of an alternating wave. For a sinusoidal alternating voltage its value is 1.4142. This factor plays an important role in determination of dielectric strength of insulations for a given AC circuit.

FORM FACTOR: it is defined as the ratio of rms value to the average value of a wave. For sine wave the value of form factor is 1.11. This factor is used for conversion from rms to average value and vice-versa.

Q. Find Avg value, RMS value and form factor of half wave rectified alternating current.

Ans.



$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} i^2 d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_{\max}^2 \sin^2 d\theta} = \frac{I_{\max}}{\sqrt{2\pi}}\sqrt{\int_0^{\pi}\sin^2\theta d\theta} = \frac{I_{\max}}{\sqrt{4\pi}}\sqrt{\int_0^{\pi}(1-\cos 2\theta d\theta)}$$

$$= \frac{I_{\max}}{\sqrt{4\pi}}\sqrt{\left(\theta - \frac{1}{2}\sin 2\theta\right)_0^{\pi}} = \frac{I_{\max}}{\sqrt{4\pi}} \times \sqrt{\pi} = \frac{I_{\max}}{2}$$

$$I_{avg} = \frac{1}{2\pi}\int_0^{\pi} i\, d\theta = \frac{1}{2\pi}\int_0^{\pi} I_{\max}\sin\theta d\theta = \frac{I_{\max}}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{I_{\max}}{2\pi}\left[-\cos\pi + \cos 0\right] = \frac{I_m}{2\pi}\left[1+1\right]$$

$$= \frac{I_{\max}}{\pi}$$

Form Factor $= \frac{I_{rms}}{I_{avg}} = \frac{I_{\max}/2}{I_{\max}/\pi} = \frac{\pi}{2} = 1.57$

Q. An alternating voltage is given by v = 141.4 sin 314t. Find :

- **(i) Frequency**
- **(ii) r.m.s value**
- **(iii) Average value**
- **(iv) The instantaneous value of voltage when 't' is 3 m sec.**

(v) The time taken for the voltage to reach 100V for the first time after passing through zero value.

Ans. Given $v = 141.4 \sin 314t$

Compare above equation by $v = V_m \sin wt$

We have $V_m = 141.4V$ and $w = 314$ rad/sec.

(i) $w = 2\pi f$ or $f = \frac{w}{2\pi} = \frac{314}{2\pi} = 50Hz$

(ii) $V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{141.4}{\sqrt{2}} = 100V$

(iii) $V_{av} = \frac{2V_m}{\pi} = \frac{2(141.4)}{\pi} = 90.07V$

(iv) Instantaneous voltage at 3m sec.

$$= 141.4 \sin 314\left(3 \times 10^{-3}\right)$$
$$= 141.4 \sin(0.942 \text{ rad})$$
$$= 141.4(0.8087) = 114.395V$$

(v) $100 = 141.4 \sin 314\, t \Rightarrow 314\, t = 0.7855$

Or $t = \frac{1}{314}(0.7855) = 2.5 \times 10^{-3}$ sec

Q. The equation of an alternating current is $i = 42.42 \sin 628t$ Determine: (i)its maximum value (ii)frequency (iii)rms value (iv)average value (v)form factor

Ans. Given $i = 42.42 \sin 628t$ Compare with $i = I_{\max} \sin \omega t$

$I_{\max} = 42.42$, $\omega = 628$

(i) Maximum value of current $I_{\max} = 42.42$Amp.

(ii) $\omega = 2\pi f$ so, $f = \frac{\omega}{2\pi}$ $\Rightarrow f = \frac{628}{2 \times 3.14} = 100Hz$

(iii) Rms value of current $I_{rms} = \frac{I_{\max}}{\sqrt{2}} = \frac{42.42}{\sqrt{2}} = 30Amp.$

(iv) Average value of current $I_{av} = \frac{2I_{\max}}{\pi}$ $\Rightarrow I_{av} = \frac{2 \times 42.42}{\pi} = 27$ Amp.

(v) Form factor $= \frac{I_{rms}}{I_{av}} = \frac{30}{27} = 1.11$.

ANALYSIS OF PURE RESISTIVE, INDUCTIVE AND CAPACTIVE CIRCUIT

CIRCUIT WITH PURE RESISTANCE ONLY

A pure resistance is that in which there is only a ohmic voltage drop. Consider a circuit having a pure resistance R as shown is Fig below. Let the instantaneous value of the alternating voltage applied be, v = V_m Sin ωt The instantaneous value of current,

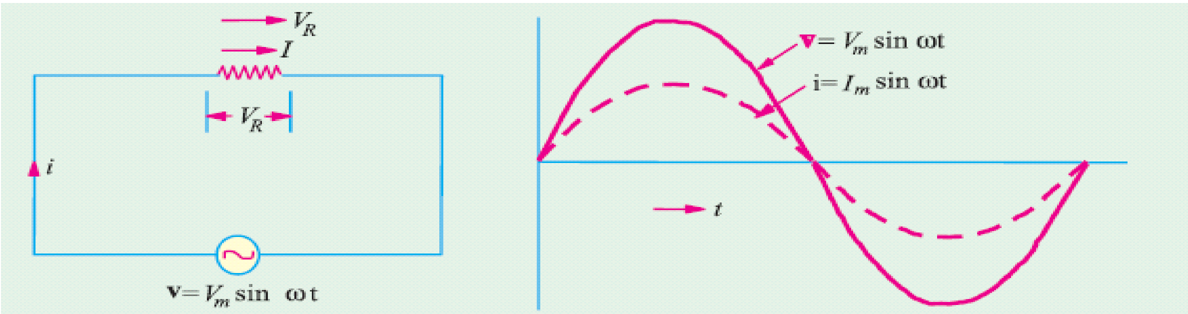
$$\mathbf{i = e/R = (V_m \sin \omega t)/R = I_m \sin \omega t \qquad \text{where } I_m = \frac{V_m}{R} \text{ A.}}$$

we find that in pure resistive circuit the voltage and current are in same phase.

Instantaneous power $\mathbf{p = V_m \sin \omega t * I_m \sin \omega t = V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t}$.

Instantaneous power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$ of frequency double that of supply. On taking average over complete cycle the fluctuating part reduces to zero and we get average power

$$\mathbf{P = \frac{1}{2\pi} \int_0^{2\pi} vi\, d(\omega t) = \frac{V_m I_m}{2} \text{ or } V_{RMS} * I_{RMS} \text{ watt}}$$



CIRCUIT WITH PURE INDUCTANCE ONLY

A pure inductive circuit has only inductor with no resistance as shown below. When an alternating voltage is applied to it, a back emf is induced it and the magnitude of this induced emf is equal and opposite to supply voltage at all instants. Let voltage applied be $v = V_m \sin \omega t$ and instantaneous value of self-induced emf is e given by $e = -L\frac{dI}{dt}$.

So at any instant $\quad v = -(e) \quad$ or $\quad V_m \sin \omega t = -\left(-L\frac{di}{dt}\right) \quad$ or $\quad V_m \sin \omega t = L\frac{dI}{dt}$

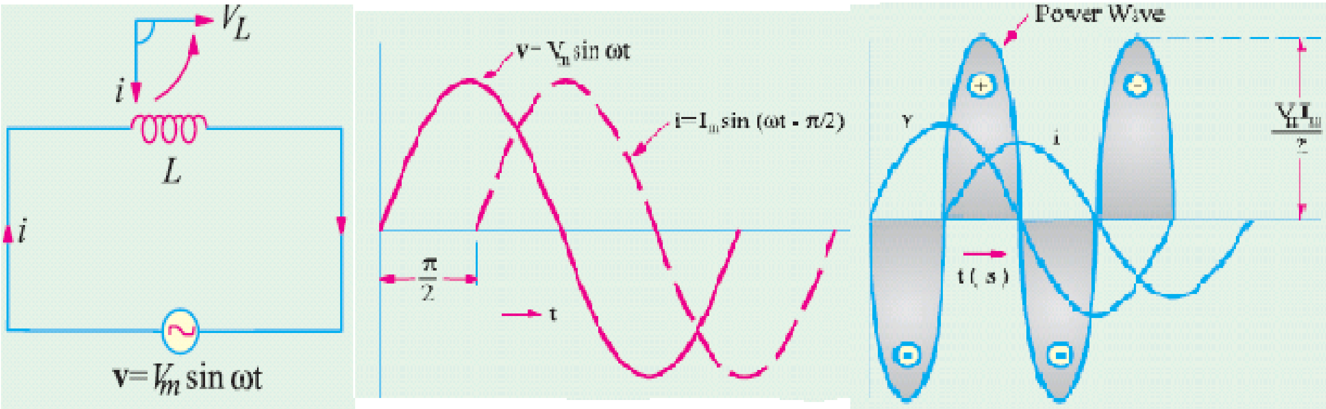
$$\text{or} \quad di = \frac{V_m}{L} \sin \omega t \; .$$

On integrating $\quad \int di = \int \frac{V_m}{L} \sin \omega t \quad$ or $\quad i = \frac{V_m}{\omega L}(-\cos \omega t) \quad$ or $\quad i = \frac{V_m}{\omega L}(\sin(\omega t - \pi/2))$

$$\text{or} \quad i = I_m (\sin(\omega t - \pi/2)) \qquad \text{where } I_m = \frac{V_m}{X_L} \text{ A.}$$

we find that the current lags the applied voltage by 90° or $\pi/2$ radian. The quantity X_L is called inductive reactance and its units is ohm.

$$X_L = \omega L \;\; \Omega$$



Instantaneous power $\quad p = vi = V_m \sin \omega t * I_m (\sin(\omega t - \pi/2)) = -\frac{V_m I_m}{2} Sin2\omega t.$

The instantaneous power consists only a fluctuating part with frequency double of supply frequency. On taking average over complete cycle the fluctuating part reduces to zero and we get average power

$$P = \frac{1}{2\pi}\int_0^{2\pi} vi \, d(\omega t) = \frac{1}{2\pi}\int V_m \sin \omega t * I_m(\sin \omega t - \pi/2) \, d(\omega t) = -\frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\omega t}{2} \, d(\omega t) = 0 \,.$$

CIRCUIT WITH PURE CAPACTANCE ONLY

When an alternating voltage is applied to the plates of a capacitor (C), the capacitor is charged first in one direction and then in the opposite direction. Let applied voltage be $v = V_m \sin \omega t$, and q is the charge on the plates of capacitor then

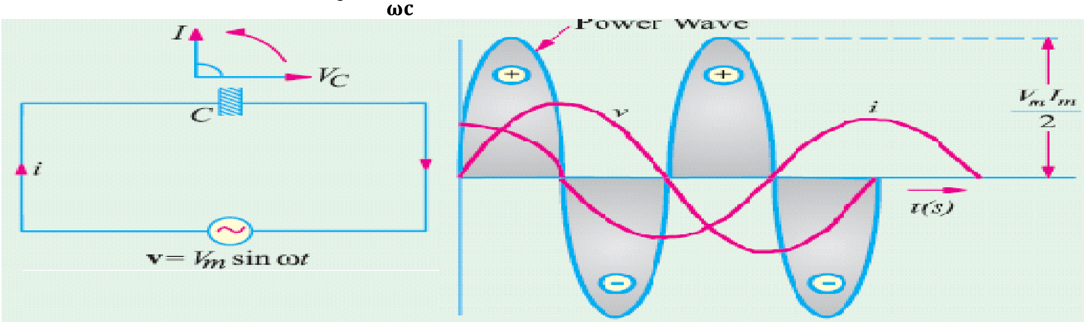
$$q = Cv = C.\, V_m \sin \omega t$$

As current is given by rate of flow of charge so instantaneous current is given by

$$i = \frac{dq}{dt} = \frac{d(Cv)}{dt} = C.\, V_m \cos \omega t.\ \Omega \quad \text{or}$$

$$i = \frac{V_m}{\frac{1}{\omega c}} \sin(\omega t + \pi/2) = I_m (\sin(\omega t + \pi/2)). \qquad \text{Where } I_m = \frac{V_m}{X_c} \text{ A}$$

we find that the current leads the applied voltage by 90° or $\pi/2$ radian. The quantity X_C is called capacitive reactance and its units is ohm $\quad X_c = \frac{1}{\omega c} \;\; \Omega.$



Instantaneous power $\quad p = vi = V_m \sin \omega t * I_m (\sin(\omega t + \pi/2)) = \frac{V_m I_m}{2} Sin2\omega t.$

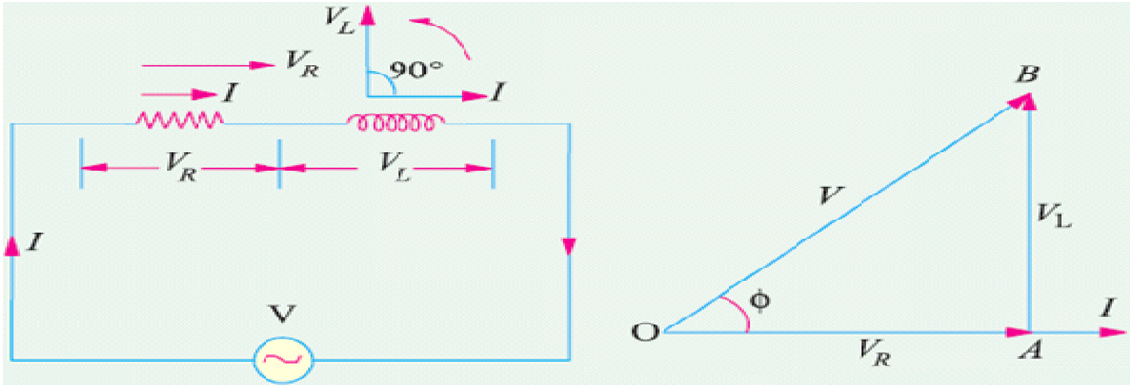
The instantaneous power consists only a fluctuating part with frequency double of supply frequency. On taking average over complete cycle the fluctuating part reduces to zero and we get average power

$$P = \frac{1}{2\pi}\int_0^{2\pi} vi\, d(\omega t) \quad = \frac{1}{2\pi}\int V_m \sin \omega t * I_m(\sin \omega t + \pi/2)\, d(\omega t) \quad = \quad \frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\omega t}{2}\, d(\omega t) \quad = 0 .$$

ANALYSIS OF SERIES RL AND RC CIRCUIT

CIRCUIT WITH RESISTANCE AND INDUCTANCE IN SERIES

A pure resistance R and a pure inductive coil of inductance L are shown connected in series. Let V = RMS value of the applied voltage, I = RMS value of current, V_R = IR voltage drop across R (in phase with I), V_L = I.X_L voltage drop across coil (ahead of I by 90°)

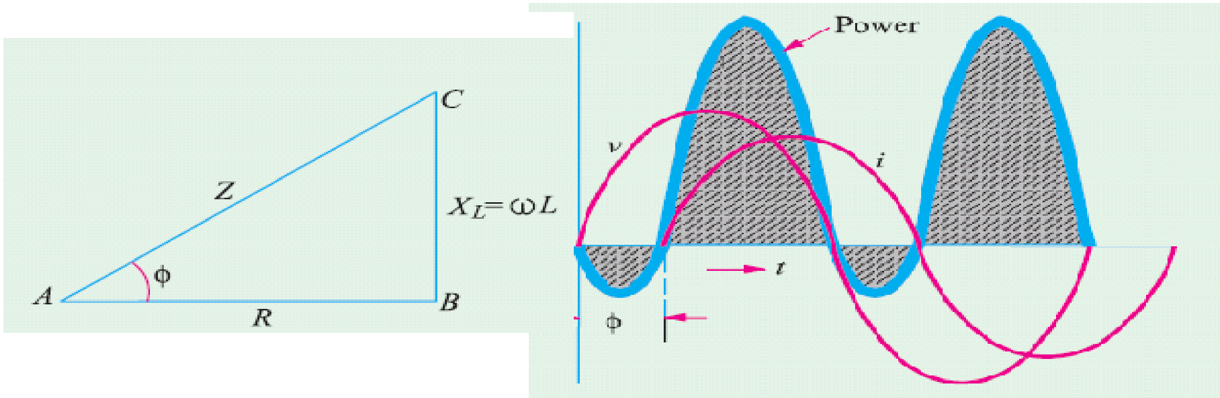


These voltage drops are shown in voltage triangle OAB in Fig. Vector OA represents ohmic drop V_R and AB represents inductive drop V_L. The applied voltage V is the vector sum of the two i.e.

$$V = \sqrt{V_R{}^2 + V_L{}^2} = \sqrt{(IR)^2 + (IX_L)^2} \;=\; I\sqrt{(R)^2 + (X_L)^2} = I * Z$$

Where Z is known as impedance of the circuit given by $Z = \sqrt{(R)^2 + (X_L)^2}\ \Omega$.

The current I lags voltage V by an angle Φ such that $\tan\Phi = \frac{V_L}{V_R} = \frac{X_L}{R}$ or $\Phi = \tan^{-1}(\frac{V_L}{V_R}) = \tan^{-1}(\frac{X_L}{R})$ **lagging**.



If the voltage is given by **$v = V_m \sin \omega t$**, then circuit current is given by **$i = I_m (\sin \omega t - \Phi)$** **Where $I_m = V_m/Z$ A**

The instantaneous power of the circuit can be calculated as **$p = V_m \sin \omega t * I_m (\sin \omega t - \Phi)$**

$$\text{or} \quad p \;=\; \frac{V_m I_m}{2}\, 2 \sin \omega t * \sin(\omega t - \Phi) \;=\; \frac{V_m I_m}{2}[\cos\Phi - \cos(2\omega t - \Phi)].$$

The instantaneous power can be divided in two components as

(i) $\frac{V_m I_m}{2}\cos\Phi$ is a constant value component independent of time and

(ii) $\frac{V_m I_m}{2}\cos(2\omega t - \Phi)$ is fluctuating part with double of supply frequency.

On taking average over complete cycle the fluctuating part reduces to zero and we get the average power over complete cycle

$$P = \frac{V_m I_m}{2}\cos\Phi \qquad \text{or} \qquad P = V_{RMS}\, I_{RMS} \cos\Phi \qquad \text{watt}$$

ACTIVE, REACTIVE AND APPARENT POWER

In an ac circuit the power is classified in three categories. The three powers drawn by the circuit are as follows

(i) Apparent Power (S): It is given by the product of RMS values of applied voltage and RMS value of circuit current

$$S = VI = (IZ).I = I^2Z \qquad \text{VA or KVA}$$

(ii) Active Power (P): It is the power which is actually dissipated in the circuit resistance. It is given by product of RMS value of voltage, RMS value of circuit current and cosine of angle between them

$$P = VI\cos\Phi = I^2R \qquad \text{Watt or Kwatt}$$

(iii)Reactive Power(Q): It is the power developed in the inductive reactance of the circuit. It is given by product of RMS value of voltage, RMS value of circuit current and sine of angle between them

$$Q = VI \sin\Phi = I^2X_L \qquad \text{VAR or KVAR}$$

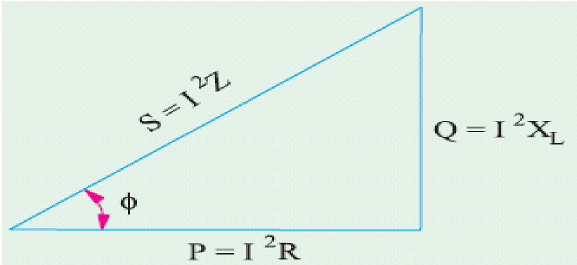


Fig. shows Power triangle for a RL Series circuit. From this triangle it is clear that

$$S = \sqrt{P^2 + Q^2} \quad \text{and} \quad P = S\cos\Phi; \quad Q = S\sin\Phi$$

CIRCUIT WITH RESISTANCE AND CAPACITOR IN SERIES

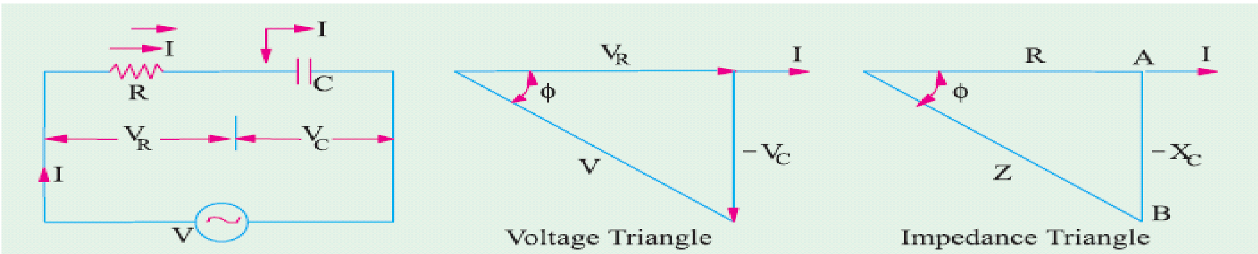
A pure resistance R and a capacitor of capacitance C are connected in series as shown Fig. Let V = RMS value of the applied voltage, I = RMS value of current, V_R = IR voltage drop across R (in phase with I), V_C = I.X_C voltage drop across capacitor (lags behind of I by 90°)

As capacitive reactance X_C is taken negative, V_C is shown along negative direction of Y-axis in the voltage triangle. Now the applied voltage can be represented as

$$V = \sqrt{V_R^2 + (-V_C^2)} = \sqrt{(IR)^2 + (IX_C)^2} = I\sqrt{(R)^2 + (X_C)^2} = I * Z$$

Where Z is known as impedance of the circuit given by $Z = \sqrt{(R)^2 + (X_C)^2}\ \Omega.$

The current leads voltage V by an angle Φ such that $\tan\Phi = -\frac{V_C}{V_R} = -\frac{X_C}{R}$ or $\Phi = -\tan^{-1}(\frac{V_C}{V_R}) = -\tan^{-1}(\frac{X_L}{R})$



If the voltage is given by $v = V_m \sin\omega t$, then circuit current is given by $i = I_m(\sin\omega t+\Phi)$ Where $I_m = V_m/Z$ A

The instantaneous power of the circuit can be calculated as $p = V_m \sin\omega t * I_m(\sin\omega t+\Phi)$

$$\text{or} \quad p = \frac{V_m I_m}{2} 2\sin\omega t * \sin(\omega t+\Phi) = \frac{V_m I_m}{2}[\cos\Phi - \cos(2\omega t+\Phi)]$$

The instantaneous power can be divided in two components as

(i) $\frac{V_m I_m}{2}\cos\Phi$ is a constant value component independent of time and

(ii) $\frac{V_m I_m}{2}\cos(2\omega t+\Phi)$ is fluctuating part with double of supply frequency.

On taking average over complete cycle the fluctuating part reduces to zero and we get the average power over complete cycle

$$P = \frac{V_m I_m}{2}\cos\Phi \quad \text{or} \quad P = V_{RMS} I_{RMS}\cos\Phi. \quad \text{watt}$$

Q. An A.C. Voltage e(t) = 141.4 Sin 120t is applied to a series RC circuit. The circuit is obtained as i(t) = 12.25Sin (120t + 30°) Determine : (i)Value of R and C (ii)Power factor (iii)Power delivered

ANS Given **e(t) = 141.4 Sin 120t** ***i(t)*= 12.25 Sin(120+30°)** and **ϕ = 30°**

Impedance (z) $= \frac{141.4\angle 0^\circ}{12.25\angle 30^\circ}$ or $Z = 11.54\angle -30^\circ$

(a) Resistance $(R) = Z\cos\phi$ $(R) = 11.54\cos 30^\circ$ Resistance (R) = 10Ω

Reactance $(X_C) = Z\sin\phi$ = 11.54 sin 30° (X_C)= 5.77Ω

Capacitance $(C) = \frac{1}{2\pi f X_C}$ $C = \frac{1}{120\times 5.77}$ C = 0.00144 *F* or 1.44 *mF*

(b) Power factor ($\cos\phi$) $\cos\phi = \cos 30^\circ$ or $\cos\phi = 0.866$ leading

(c) Power delivered by the source

$$= V_{rms} I_{rms}\cos\phi = \frac{141.4}{\sqrt{2}} \times \frac{12.25}{\sqrt{2}} \times \cos 30^\circ = 750\ watt$$

Q.. A load having impedance of $(1+j1)\Omega$ is connected to an a.c voltage represented as $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$ Volt. Find the current in load, expressed in the form of $i = I_m \sin(\omega t + \varnothing) A$ Find the real power Consumed by the load.

Ans Impedance $Z = (1 + j1)\Omega$ or $Z = \sqrt{2}\angle 45^\circ$ { in polar form}

$V = 20\sqrt{2}\cos(\omega t + 10^\circ)$ or, $V = 20\sqrt{2}\sin(\omega t + 10^\circ + 90^\circ)$ or $V = 20\sqrt{2}\sin(\omega t + 100^\circ)$

Also $V = 20\sqrt{2}\angle 100^\circ$ { in polar form}

(i) Current $(I) = \dfrac{V}{Z}$ or $I = \dfrac{20\sqrt{2}\angle 100^\circ}{\sqrt{2}\angle 45^\circ}$ or $I = \dfrac{20\sqrt{2}}{\sqrt{2}}\angle(100^\circ - 45^\circ)$

$I = 20\angle 55^\circ$ or, $I = 20\sin(\omega t + 55^\circ)$

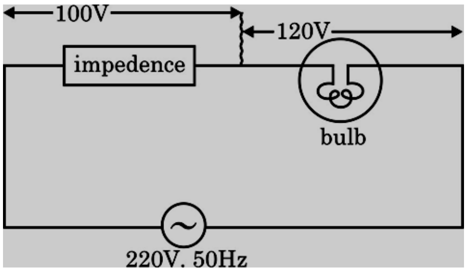
(ii) Real Power $= VI\cos\varnothing$

$$P = \frac{20\sqrt{2}}{\sqrt{2}}\left(\frac{20}{\sqrt{2}}\right).\cos 45^\circ \text{ or } P = 20.\frac{20}{\sqrt{2}}.\cos 45^\circ \text{ or } P = \frac{400}{\sqrt{2}.\sqrt{2}}$$

$P = 200W$

Q. A 120 V, 60 W lamp is to be operated on 220 V, 50 Hz supply mains. Calculate value of : (i)non inductive resistance (ii)pure inductance which are connected in series with lamp in order that lamp should operate on correct voltage.

Ans



Current in bulb $I = \dfrac{P}{V}$

$$I = \frac{60W}{120V} = 0.5 \text{ Amp.}$$

(i) If impedance is non inductive resistance

$\therefore$ Resistance and bulb are connected in series so current will be same in each

so, $I = 0.5$ Amp.

Voltage drop across Resistance = 220 – 120 = 100V

so, $R = \dfrac{V}{I} = \dfrac{100}{0.5} = 200\Omega$

(ii) If impedance is pure inductance then $V_L = 184.4$

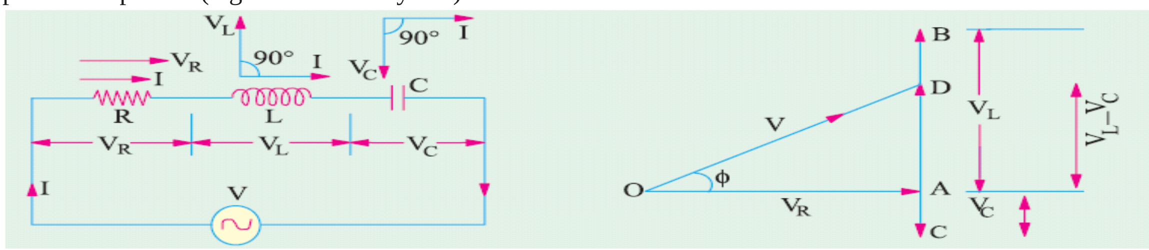
$$X_L = \frac{V}{I} = \frac{184.4}{0.5} = 368.8\Omega$$

$X_L = 2\pi f L$ or $L = \dfrac{X_L}{2\pi f} = \dfrac{368.8}{2\times\pi\times 50} = 1.174$ H.

ANALYSIS OF RLC SERIES CIRCUIT

CIRCUIT WITH RESISTANCE, INDUCTANCE AND CAPACITOR IN SERIES

Let these three elements are joined in series and V = RMS value of the applied voltage, I = RMS value of the resultant current, $V_R = IR$ voltage drop across R (in phase with I), $V_L = I.X_L$ voltage drop across inductor (ahead of I by 90°) and $V_C = I.X_C$ voltage drop across capacitor (lags behind of I by 90°)



In voltage triangle of given Fig. OA represents V_R, AB and AC represent the inductive and capacitive drops respectively. It will be seen that V_L and V_C are 180° out of phase with each other i.e. they are in direct opposition to each other. Subtracting BD (= AC) from AB, we get the net reactive drop AD = ($V_L - V_C$). The applied voltage V is represented by OD which is the vector sum of OA and AD. So

$$\mathbf{OD} = \sqrt{OA^2 + OD^2} \quad \text{or} \quad \mathbf{V} = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = \mathbf{I^* \, Z},$$

where Z is known as impedance of RLC series circuit given by $\mathbf{Z} = \sqrt{(R)^2 + (X_L - X_C)^2}\ \mathbf{\Omega}$.

Here $X_L - X_C = X_{Net}$, known as net reactance of the circuit.

Phase angle of the circuit is given by $\quad \Phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{X_{Net}}{R}\right)$

If the expression of applied voltage is $\mathbf{v = V_m \sin \omega t}$ then the circuit current expression is given by $\mathbf{i = I_m \sin(\omega t \pm \Phi)}$ where the sign of phase angle depends on following cases

Case 1: The + ve sign is to be used when current leads or X_{Net} is negative i.e. $X_C > X_L$.

Case 2: The - ve sign is to be used when current lags or X_{Net} is positive i.e. $X_L > X_C$.

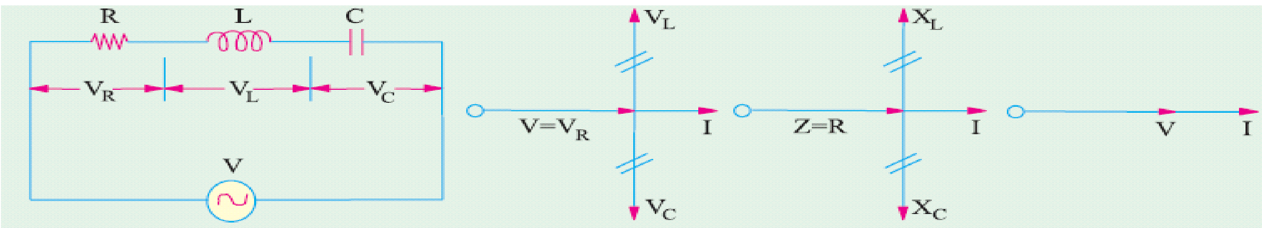
Case 3: The value of phase angle Φ is zero when voltage and current are in same phase or X_{Net} is zero i.e $X_L = X_C$.

SUMMARRY OF RESULTS FOR SERIES AC CIRCUITS

Type of Impedance	*Value of Impedance*	*Phase angle for current*	*Power factor*
Resistance only	R	0°	1
Inductance only	ωL	90° lag	0
Capacitance only	$1/\omega C$	90° lead	0
Resistance and Inductance	$\sqrt{[R^2 + (\omega L)^2]}$	$0 < \phi < 90°$ lag	1 > p.f. > 0 lag
Resistance and Capacitance	$\sqrt{[R^2 + (-1/\omega C)^2]}$	$0 < \phi < 90°$ lead	1 > p.f. > 0 lead
R-L-C	$\sqrt{[R^2 + (\omega L \sim 1/\omega C)^2]}$	between 0° and 90° lag or lead	between 0 and unity lag or lead

RESONANCE IN RLC SERIES CIRCUIT

A RLC series circuit is said to be in electrical resonance when the net reactance of the circuit becomes zero and the frequency at which this happens is known as resonating frequency (f_o) of the circuit. At resonance the voltage drops across inductor ($V_L = IX_L$) and capacitor ($V_C = IX_C$) are equal and opposite to each other so they cancel out each other and net supply voltage is dropped across circuit resistance ($V_R = V$). The two reactance's taken together act as a short-circuit since no voltage develops across them. The phasor diagram of series resonating circuit is shown in figure.



The frequency at which the net reactance of the series circuit is zero is called the resonant frequency (f_o). At resonance

$$\mathbf{X_{Net} = 0 \quad or \quad X_L - X_C = 0}$$

$$\mathbf{i.e \quad \omega_o L = 1/(\omega_o C)}$$

$$\omega_o = \sqrt{LC}\ \text{rad/sec} \qquad \text{or} \qquad f_o = \frac{1}{2\pi\sqrt{LC}}\ \text{Hz}$$

The expression for f_o shows that circuit resistance has no effect on the value of resonating frequency of the circuit. When a series *R-L-C* circuit is in resonance, it possesses minimum impedance $\boldsymbol{Z = R}$. Hence, circuit current is maximum, it being limited by value of *R* alone. The current at resonance is given by

$$\mathbf{I_o} = \frac{V}{Z_0} = \frac{V}{R}\ \mathbf{A} \qquad \text{in phase with the applied voltage.}$$

Since circuit current is maximum, it produces large voltage drops across *L* and *C*. Magnitudes of these voltage drops (V_L or V_C) is much larger than supply voltage and that's why series resonance is also known as voltage resonance. But these drops being equal and opposite, cancel each other when consider together. *L* and *C* from a part of circuit across which there is no voltage drop, however, large current is flowing. A series resonant circuit is sometimes called **acceptor** circuit because they offer minimum impedance to the frequency signal at which they resonates.

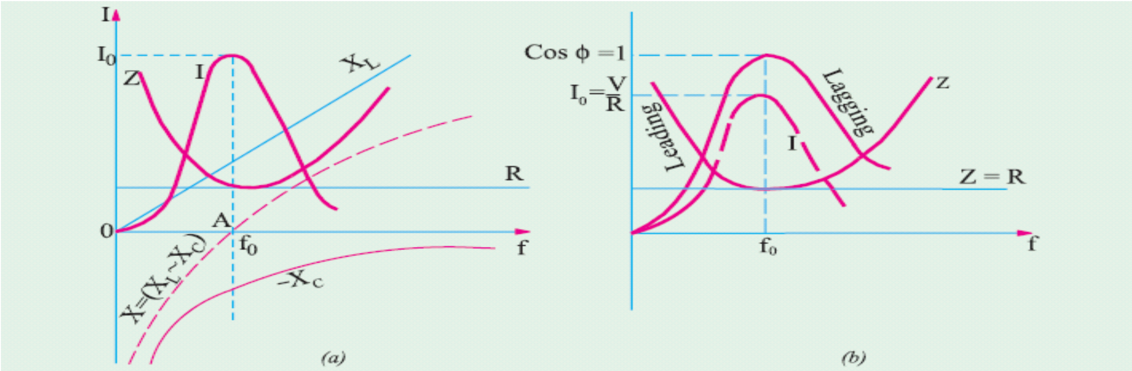
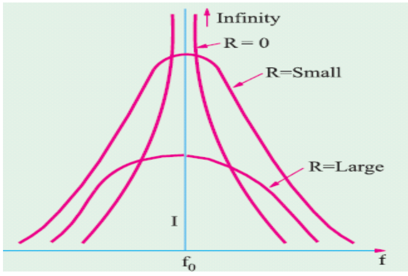
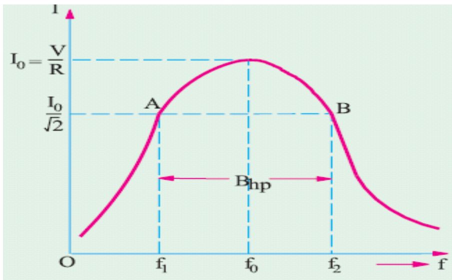


Figure shows the graphical representation of various circuit parameter of RLC Series circuit with variable supply frequency by keeping all other factors constant. The figure gives an idea that how various circuit parameters are changing above and below resonating frequency.





RESONANCE CURVE The curve, between circuit current and the frequency of the applied voltage, is known as resonance curve. The shapes of resonance curve, for different values of R is shown in fig. For smaller values of R, the resonance curve is sharply peaked and such a circuit is said to be sharply resonant or highly selective. However, for larger values of R, resonance curve is flat and is said to have poor selectivity. The ability of a resonant circuit to discriminate one particular frequency from the band of applied frequencies is called its selectivity. The selectivity or sharpness of resonance curve is inversely proportional to circuit resistance.

BANDWIDTH (Δf) It is defined as the band of frequencies which lies on either side of resonating frequency where the circuit current reduces to $1/\sqrt{2}$ times its value at resonance. Narrower is the bandwidth, higher the selectivity of the circuit and vice versa. The bandwidth is also known as half power bandwidth as at the corner points the power consumption of circuit becomes half of its value at resonance. At the corner points of a band width the net reactance of circuit is equal to the circuit resistance. We can derive the expression of corner frequencies as follows:
At lower corner frequency f_1 the nature of circuit is capacitive. Also net reactance is equal to circuit resistance, so at corner frequency f_1 we get

$$X_C - X_L = R \quad \text{or} \quad \frac{1}{\omega_1 C} - \omega_1 L = R$$

$$\text{or} \quad \omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0$$ which is a quadratic equation.

So writing its roots $$\omega_1 = \frac{-\frac{R}{L} \pm \sqrt{(\frac{R}{L})^2 + \frac{4}{LC}}}{2} \quad \text{or} \quad \omega_1 = -\frac{R}{2L} \pm \sqrt{(\frac{R}{2L})^2 + \frac{1}{LC}} \ .$$

Now put $\alpha = \frac{R}{2L}$ & $\omega_0 = \frac{1}{\sqrt{LC}}$ and considering only positive root we get

$$\omega_1 = -\alpha + \sqrt{\alpha^2 + \omega_0{}^2} \ \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)} .$$

Similarly at upper corner frequency f_2 we get

$$X_L - X_C = R \quad \text{or} \quad \omega_2 L - \frac{1}{\omega_2 C} = R$$

or $$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$ which is a quadratic equation.

On solving we get $$\omega_2 = \alpha + \sqrt{\alpha^2 + \omega_0{}^2} \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

Now in equation (i) and (ii) neglecting the term α^2 as its magnitude is very small so we get

$$\omega_1 = \omega_0 - \alpha \quad \text{or} \quad \omega_1 = \omega_0 - \frac{R}{2L} \ \text{rad/sec} \ ;$$

Similarly we get $$\omega_2 = \omega_0 + \frac{R}{2L} \ \text{rad/sec}$$

and $$\Delta\omega = \omega_2 - \omega_1 = \frac{R}{L} \ \text{rad/sec}$$

Now corner frequencies can be written as $$f_1 = f_0 - \frac{R}{4\pi L} \ \text{Hz} \quad \& \quad f_2 = f_0 + \frac{R}{4\pi L} \ \text{Hz}$$

The bandwidth can be obtained as $\Delta f = f_2 - f_1 = \frac{R}{2\pi L}$ Hz

On multiplication of equation (i) and (ii) we get

$$\omega_1 * \omega_2 = (-\alpha + \sqrt{\alpha^2 + \omega_0^2})(\alpha + \sqrt{\alpha^2 + \omega_0^2}) \quad \text{or} \quad \omega_0 = \sqrt{\omega_1 \omega_2} \quad \text{or} \quad f_0 = \sqrt{f_1 f_2}$$

Thus we can show that resonating frequency is geometric mean of corner frequencies.

QUALITY FACTOR (Q- FACTOR)

Quality factor for a coil is defined as the ratio of reactance to resistance of the coil. It is a unit less quantity. For a pure inductive coil the value of quality factor is infinite. As the resistance of coil increases the value of quality factor of coil decreases. It is given by

$$Q = \frac{X_L}{R} = \frac{\omega_0 L}{R} = \tan \phi$$ where ϕ is the power factor angle of the coil.

The *Q*-factor of a series resonating circuit can be defined in the following different ways.

(i) it is given by the voltage magnification produced by the circuit at resonance.

$$\text{Voltage magnification ratio} = \frac{V_L}{V_{Supply}} = \frac{V_L}{V_R} = = \frac{I\,X_L}{I\,R} = \frac{X_L}{R} = \tan \phi$$

where ϕ is the power factor angle of the coil formed by series combination of resistance and inductance present in the circuit.

(ii) The *Q*-factor may also be defined as the energy gain of the circuit. it is given by

$$\text{Q- factor} = 2\pi \frac{\textit{Maximum energy stored}}{\textit{Energy dissipated per cycle}} = 2\pi \frac{\frac{1}{2} L\, I_m^2}{I^2 R\, T_0} = 2\pi f_0 \frac{\frac{1}{2} L(\sqrt{2} I)^2}{I^2 R}$$

$$= 2\pi f_0 \frac{L}{R} = \frac{\omega_0 L}{R} = \tan \phi \; .$$

(iii) As we know that $\omega_0 = \frac{1}{\sqrt{LC}}$ so using this we get Q- factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$. This relation is used to find the value of Q factor in terms of component of RLC series circuit.

(iv) In the case of series resonance, higher *Q*-factor means not only higher voltage magnification but also higher selectivity of the tuning coil. The Q factor can also be written as

$$\text{Q- factor} = \frac{\omega_0}{\frac{R}{L}} = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{f_0}{\Delta f} = \frac{f_0}{f_2 - f_1}$$

Q . A series R-L-C circuit has R=10Ω, L = 0.1 H and C = 8 μF. Determine.

(i) resonant frequency

(ii) Q-factor of the circuit at resonance

(iii) the half power frequencies

Ans Given R = 10Ω, L = 0.1 H and C = 8μF

Resonation Frequency $f_r = \frac{1}{2\pi\sqrt{0.1 \times 8 \times 10^{-6}}}$ $\qquad f_r = 177.94 Hz$

(ii) Q factor $Q_r = \frac{X_L}{R} = \frac{\omega_r L}{R} = \frac{2\pi f_r \times L}{R}$

$$Q_r = \frac{2 \times \pi \times 177.94 \times 0.1}{10} = 11.18$$

(iii) Lower half power frequency $f_1 = f_r - \frac{R}{4\pi L} = 177.94 - 7.95 = 169.99 Hz$

Higher half power frequency $f_2 = f_r + \frac{R}{4\pi L} = 177.94 + 7.95 = 185.89 Hz$

ANALYSIS OF PARALLEL CIRCUIT

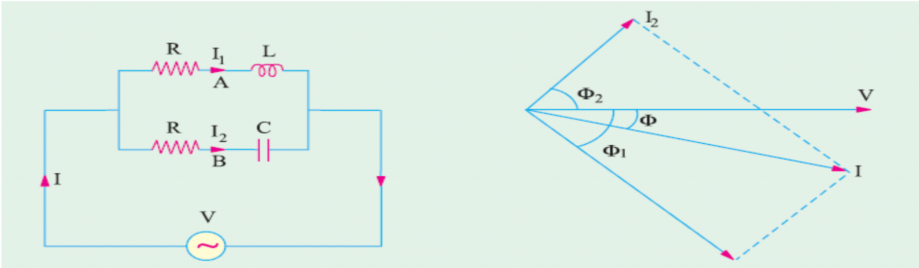
The following methods can be used to solve parallel circuits

VECTOR OR PHASOR METHOD

Consider the circuits shown in Fig. Here, two impedances *A* and *B* have been joined in parallel across an r.m.s. supply of *V* volts. The voltage across two parallel branches *A* and *B* is the same, but currents through them are different.

For Branch A : $\quad Z_1 = \sqrt{R_1^2 + X_L^2}\;;\; I_1 = \frac{V}{Z_1}\;;\; \cos\phi_1 = \frac{R_1}{Z_1}$

Also $\quad \phi_1 = \cos^{-1}\left(\frac{R_1}{Z_1}\right)$ $\qquad$ Current I_1 lags behind the applied voltage V by angle ϕ_1



For Branch B : $Z_2 = \sqrt{R_2^2 + X_C{}^2}$; $I_2 = \frac{V}{Z_2}$; $\cos\phi_2 = \frac{R_2}{Z_2}$

Also $\phi_2 = \cos^{-1}\left(\frac{R_2}{Z_2}\right)$ Current I_2 leads the applied voltage V by angle ϕ_2

The resultant circuit current I is the vector sum of the branch currents I_1 and I_2 and can be found by using parallelogram law of vectors. If I is the resultant current and ϕ its phase, then

$$\mathbf{I\cos\phi = (I_1\cos\phi_1 + I_2\cos\phi_2) \quad and \quad I\sin\phi = (I_2\sin\phi_2 - I_1\cos\phi_1).}$$

The total circuit current is given by $\mathbf{I} = \sqrt{(\mathbf{I\cos\phi})^2 + (\mathbf{I\sin\phi})^2}$ and

The phase angle of this current is given by $\boldsymbol{\phi} = \mathbf{\tan^{-1}}\left(\frac{I\sin\phi}{I\cos\phi}\right)$

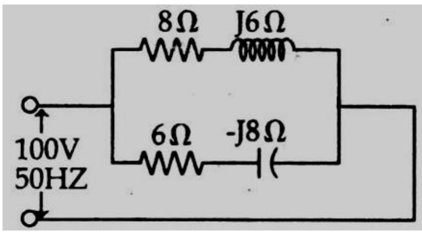
Q. The parallel circuit shown in figure is connected across a single phase 100V, 50Hz ac supply. Calculate

(i) The branch currents

(ii) The total current

(iii) The supply power factor

(iv) The active and reactive power supplied by the supply.



Ans. Let the current and impedance for branches (i) and (ii) are I_1 , Z_1 and I_2 , Z_2 respectively. Also total current is I_T

(i) For branch (i) :

$Z_1 = (8 + j6)\Omega = 10\angle 37°\Omega$ And $I_1 = \frac{100}{10\angle 37°} = 10\angle -37° A$

For branch (ii) : $Z_2 = (6 + j8)\Omega = 10\angle -53°\Omega$ And $I_2 = \frac{100}{10\angle 37°} = 10\angle -53° A$

(ii) Total current, $I_T = I_1 + I_2 = 10\angle -37° + 10\angle 53° = 14.14\angle 8.13° A$

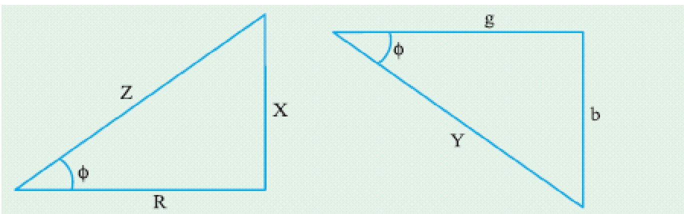
(iii) Supply power factor; $\cos\phi = \cos(8.13°) = 0.989$

(iv) Active power applied; $VI\cos\phi = 100(14.14)\cos(8.13°) = 1.397\, KW$

Reactive power supplied; $VI\sin\phi = 100(14.14)\sin(8.13°) = 199.97\ VAR$

ADMITTANCE METHOD

We defined admittance as the reciprocal of impedance. It is represented by Y where Y= $\frac{1}{Z}$ Ω^{-1} or mho. As the impedance Z of a circuit has two components X and R similarly, admittance Y also has two components as shown in Fig. The X-component is known as conductance (G) and Y-component as susceptance (B). Fig. shows the impedance triangle for a RL circuit along with its admittance triangle.



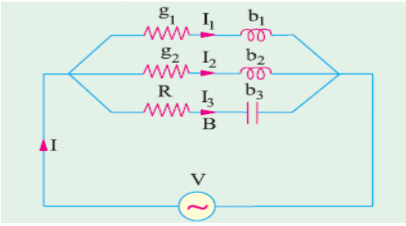
For a RL circuit the impedance is written as Z= $R + jX_L\ \Omega$. The corresponding value of admittance can be calculated as

$$\mathbf{Y} = \frac{1}{Z} = \frac{1}{R+jX_L}\ \Omega^{-1} \text{ or } = \frac{1}{R+jX_L} * \frac{(R-JX_L)}{(R-JX_L)} = \frac{R}{R^2+X_L^2} - j\frac{X_L}{R^2+X_L^2} = \mathbf{G - jB};$$

where G is conductance and B is susceptance of the circuit.

Here $\mathbf{G} = \frac{R}{R^2+X_L^2} = \frac{1}{Z} * \frac{R}{Z} = \boldsymbol{Y}\ \mathbf{cos}\ \phi$. So we define conductance as the real component of admittance whose value is equal to the reciprocal of circuit resistance provided circuit reactance should be zero.

Also $\mathbf{B} = \frac{X_L}{R^2+X_L^2} = = \frac{1}{Z} * \frac{X_L}{Z} = \boldsymbol{Y}\ \mathbf{Sin}\ \phi$. So we define susceptance as the reactive component of admittance whose value is equal to the reciprocal of circuit reactance provided circuit resistance is zero. Here one more thing to consider that opposite to the reactance the inductive susceptance is considered negative and capacitive susceptance is considered to be positive. The advantage with admittance is that they can be added when they are connected in parallel.



Consider the 3-branched circuit of Fig. Total conductance is found by merely adding the conductances of three branches. Similarly, total susceptance is found by algebraically adding the individual susceptances of different branches.

$$\mathbf{G} = \boldsymbol{g_1} + \boldsymbol{g_2} + \boldsymbol{g_3}\ ; \qquad \textbf{and} \qquad \mathbf{B} = (-\boldsymbol{b_1}) + (-\boldsymbol{b_2}) + (\boldsymbol{b_3})$$

Total admittance $\mathbf{Y} = \sqrt{\boldsymbol{G^2} + \boldsymbol{B^2}}\ \boldsymbol{\Omega}^{-1}$, Total current $\mathbf{I = VY}$; Power factor $\mathbf{cos}\ \phi = \mathbf{G/Y}$.

This method is more useful in case of large number of parallel branches.

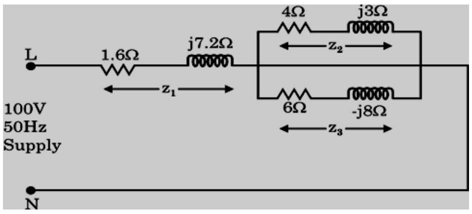
Q. The following figure shows a series parallel circuit find.

(i) Admittance of each parallel branch

(ii) Total circuit impedance

(iii) Supply current and power factor

(ii) Total power supplied by the source



Ans. **(i)** $Z_2 = 4 + j3$ $\quad Z_2 = 5\angle 36.86°$

$$\text{Admittance } (Y_2) = \frac{1}{Z_2} = \frac{1}{5\angle 36.86^\circ} \text{ or } Y_2 = 0.2\angle -36.86\ \mho$$

$Z_3 = 6 - j8 \qquad Z_3 = 10\angle -53.13\Omega$

$$\text{Admittance } (Y_3) = \frac{1}{Z_3} = \frac{1}{10\angle -53.13^\circ} \text{ or } Y_3 = 0.1\angle 53.13°\mho$$

$$\textbf{(ii)}\ \text{Total Impedance (Z)} = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$

$Z = 6 + j8$

$Z = 10\angle 53.13°\ \Omega$

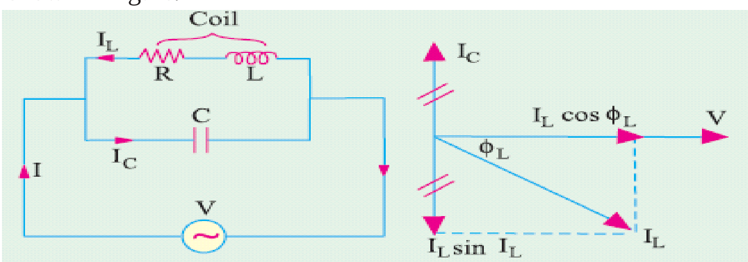
$$\textbf{(iii)}\ \text{Supply current (I)} = \frac{V}{Z} = \frac{100}{10} = 10A$$

Power factor (cosϕ) = cos 53.13 = 0.6 P.F.

$$\textbf{(iv)}\ \text{Power Supplied} = I^2 R = \left(\frac{100}{10}\right)^2 \times 6 = 600 \text{ watt.}$$

RESONANCE IN PARALLEL CIRCUIT

Consider a coil in parallel shown in Fig. Such a circuit is said to be in electrical resonance when the reactive (or wattless) component of line current becomes zero and the frequency at which this happens is known as resonant frequency. The phasor diagram for this circuit is also shown in figure.



Net reactive component of circuit current is $I_C - I_L \sin\phi$.

At resonance $\quad I_C - I_L \sin\phi = 0 \quad$ or $\quad I_C = I_L \sin\phi \quad$ or $\quad \frac{V}{X_C} = \frac{V}{Z_{coil}} \frac{X_L}{Z_{coil}}$

$$Z_{coil}^2 = X_L * X_C = \omega_o L * \left(\frac{1}{\omega_o C}\right) = \frac{L}{C} \text{ .... (i)}$$

$$R^2 + X_L^2 = \frac{L}{C} \quad \text{or} \quad (2\pi f_o L)^2 = \frac{L}{C} - R^2$$

$$2\pi f_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{or} \quad f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \text{ .... (ii)}$$

Equation (ii) is the resonant frequency and is given in Hz, R is in ohm, L is the henry and C is the farad. If R is the negligible, then resonating frequency is given by $f_o = \frac{1}{2\pi\sqrt{LC}}$ same as series circuit.

CURRENT AT RESONANCE

As the reactive component of circuit current is zero so the total circuit current is in phase with the applied voltage which is given by $\quad I_o = I_L \cos\phi = \frac{V}{Z_{coil}} * \frac{R}{Z_{coil}} = \frac{V\,R}{Z_{coil}^2} = \frac{VR}{\frac{L}{C}}$ ....**by equation (i)**

So $\quad I_o = \frac{V}{\frac{L}{CR}} = \frac{V}{Z_o} \quad$ **where** $\quad Z_o = \frac{L}{CR}$

known as dynamic impedance of parallel resonating circuit. It should be noted that impedance is 'resistive' only. Since current is minimum at resonance, **L/CR** must represent the maximum impedance of the circuit. In fact, parallel resonance is a condition of maximum impedance or minimum admittance. Current at resonance is minimum, hence in such a circuit (when used in radio work) is also known as rejector circuit because it rejects (or takes minimum current of) the frequency to which it resonates. This resonance is often referred to as current resonance because the current circulating between the two branches is greater than the line current taken from the supply.

Q –FACTOR FOR PARALLEL CIRCUIT

It is defined as the ratio of current circulating between branches to the line current taken from supply. In other words it is defined as the current magnification ratio of the circuit. It is given as $Q_o = \frac{Branch\ current}{Supply\ current} = \frac{I_C}{I_o}$ or

$$Q_o = \frac{\frac{V}{X_C}}{\frac{V}{Z_o}} = \frac{Z_o}{X_C} = \frac{\frac{L}{CR}}{\frac{1}{\omega_o C}} = \frac{\omega_o L}{R} = \tan\phi$$

where ϕ is the power factor angle of coil

COMPARISON OF SERIES AND PARALLEL RESONANCE

item	*series circuit (R–L–C)*	*parallel circuit (R–L and C)*
Impedance at resonance	Minimum	Maximum
Current at resonance	Maximum = V/R	Minimum = $V/(L/CR)$
Effective impedance	R	L/CR
Power factor at resonance	Unity	Unity
Resonant frequency	$1/2\pi\sqrt{(LC)}$	$\frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$
It magnifies	Voltage	Current
Magnification is	$\omega L/R$	$\omega L/R$

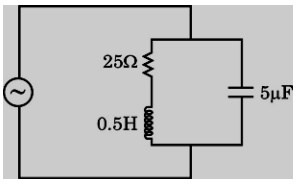
Q.. For the circuit shown below, determine :

(i) Resonant Frequency

(ii) Total impedance of the circuit of resonance.

(iii) Band width

(iv) Quality factor



ANS. **Ans8.** **(i)** Resonance Frequency $(f_r) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$

$$f_r = \frac{1}{2 \times 3.14}\sqrt{\frac{1}{0.5 \times 5 \times 10^{-6}} - \frac{(25)^2}{(0.5)^2}} \qquad f_r = 100.39\ Hz$$

(ii) Total impedance at resonance $Z = \frac{L}{RC}$

$$Z = \frac{0.5}{25 \times 5 \times 10^{-6}} \qquad Z = 4K\Omega$$

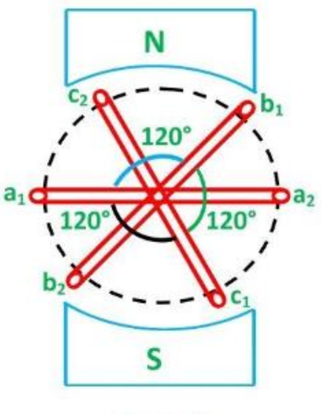
(iii) Band width $= \frac{R}{2\pi L} = \frac{25}{2 \times 3.14 \times 0.5} = 7.96$ Hz.

(iv) Quality factor $(Q) = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{0.5}{5 \times 10^{-6}}} = 12.65$

THREE PHASE AC CIRCUIT

There are two types of system available in electric circuit, single phase and three phase system. In single phase circuit, there will be only one phase, i.e the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. In three phase circuit three phase voltages are send together from the generator to the load. All voltages are displaced from each other by 120°. This type of three phase system can be developed by a set of three identical (having equal no of turns) coils, mounted on a common shaft with a mechanical displacement of 120° from each other and placed in a common magnetic field. When this assembly is rotated by some external means then due to rotation, an emf is induced in all the three coils. The voltages developed in the coils will represent a balanced three phase system in which following conditions are satisfied

1. Amplitude of voltage across all the phases are same
2. Angular velocity of all the phases are same
3. All voltages are displaced from each other by 120 °



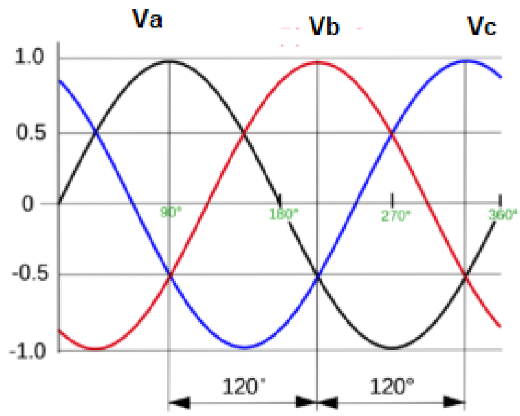


Figure shows how a balanced three phase supply system is generated. Assume the coils are rotating in anticlockwise direction then the three phase voltages can be represented as

$$V_A = V_m \sin \omega t\ ; \quad V_B = V_m \sin(\omega t - 120^\circ)\ ; \quad V_c = V_m \sin(\omega t - 240^\circ) \text{ OR } V_m \sin(\omega t + 120^\circ)$$

ADVANTAGES

Three Phase AC system have following advantages over single phase system

(1) The rating of a machine increase with increase in number of phases or in other words size of a three phase machine is smaller than single phase machine of same rating.

(2) Power factor of a 1-ϕ motor is lower than that of a three-phase motor of same rating.

(3) Three-phase system requires 3/4 th weight of conductor that required by single phase system to transmit the same amount of power at a given voltage and over a given distance.

(4) Three-phase system is more reliable and capable than single phase system.

(5) Three-phase system having higher efficiency compare to single phase system.

(6) The parallel operation of three phase alternator is much easier than single phase alternator.

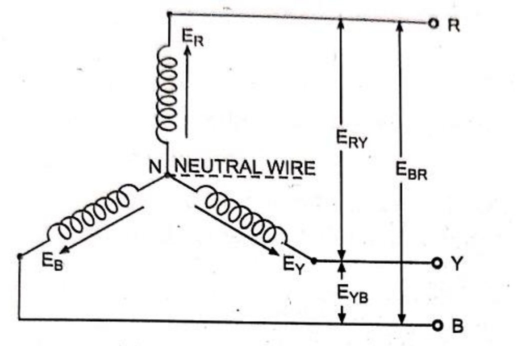
(7) Rotating magnetic fields can be developed with three phase supply which makes three phase motors self-starting.

Different phases of a three phase circuit can be connected as follows

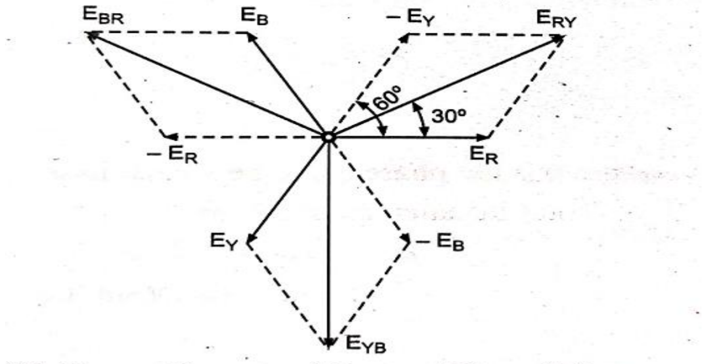
1. STAR (Y) CONNECTION   2. DELTA (Δ) CONNECTION

STAR CONNECTION:

Star connections are formed by joining the similar ends of all the phase coils to form a neutral or star point and the remaining terminals are connected to the supply lines. Some-times an additional wire is provided from neutral point and connections becomes three phase four wire connections. In star connection, Line voltage is the voltage between any two line terminals of three phase circuit and phase voltage is the voltage between any line terminal to the neutral. Let E_R, E_Y and E_B represent the phase voltage in figure and E_{RY}, E_{YB} and E_{BR} be the line voltages with positive phase sequence.



(a) Connection Diagram



(b) Phasor Diagram of Line and Phase Voltages

In star connections from circuit diagram it is clear that current flowing in the phase windings and line terminal is same. So for star connections we can say $\mathbf{I_R = I_Y = I_B = I_{PH} = I_L}$

Also assuming load to be balanced so all voltages can be assumed as $\mathbf{E_R = E_Y = E_B = E_{PH}}$

Now from phasor diagram the line voltage E_{RY} is given by

$$\mathbf{E_{RY} = E_R - E_Y = E_R + (-E_Y)}$$

$$\mathbf{E_{RY}} = \sqrt{E_R^2 + E_Y^2 + 2E_R E_Y cos60} = \sqrt{E_{PH}^2 + E_{PH}^2 + 2E_{PH}^2 cos60} = \sqrt{3E_{PH}^2} = \sqrt{3}E_{PH}$$

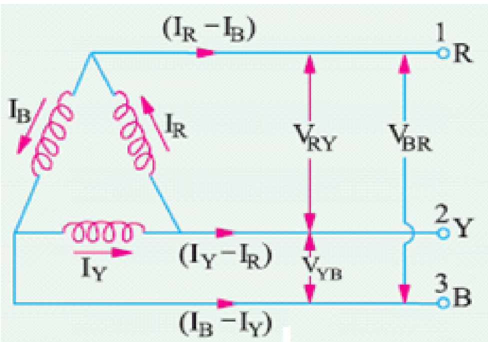
Similarly for other two phases we can prove

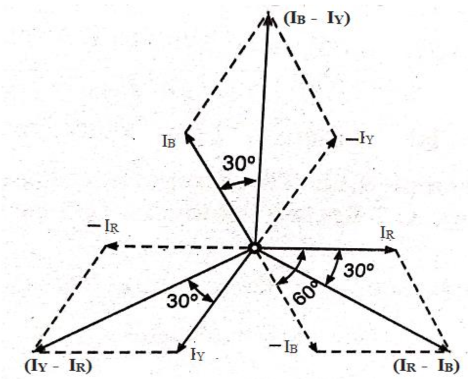
$$E_L = \sqrt{3E_{PH}} \;\; and\; I_L = I_{PH}$$

DELTA CONNECTION:

In this connection the dissimilar ends of the three phase winding are joined together i.e. the 'starting' end of one phase is joined to the 'finishing' end of the other phase and so on a closed loop is obtained. Three leads are taken out from the three junctions and connected to the supply lines. In delta there is no neutral point. The voltage between any two line terminals is equal to the phase voltage. So for a balanced delta connected system

$$\mathbf{E_R = E_Y = E_B = E_{PH} = E_L}$$





Also for balance condition currents can be assumed as $\mathbf{I_R = I_Y = I_B = I_{PH}}$. Now from phasor dia the line current is given by

$$\mathbf{(I_R - I_B) = I_R + (-I_B)} = \sqrt{I_R^2 + I_B^2 + 2I_R I_B cos60}$$

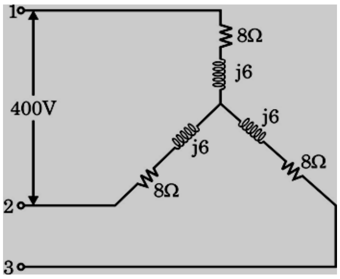
$$= \sqrt{I_{PH}^2 + I_{PH}^2 + 2I_{PH}^2 cos60} = \sqrt{3}I_{PH}$$

Similarly for the other phases we can prove : $I_L = \sqrt{3I_{PH}} \;\; and\; E_L = E_{PH}$

Note: For a given 3 phase load connected across given supply, In delta connections the value of power consumption and line current is three times the value for star connections.

Q. A star connected three phase load has a resistance of 8 ohms and an inductive reactance of 6 ohms in each phase. It is fed from a 400V, three phase balanced apply. Determine line current, power factor, active and reactive powers.

Ans



$$Z_{ph} = 8 + j6 \quad \text{or} \quad Z_{ph} = 10\angle 53.1\Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} \quad V_{ph} = 231V \text{ and } I_{ph} = I_L \quad \{ \text{ star connection}\}$$

$$I_L = \frac{231}{10} \qquad I_L = 23.1\,Amp.$$

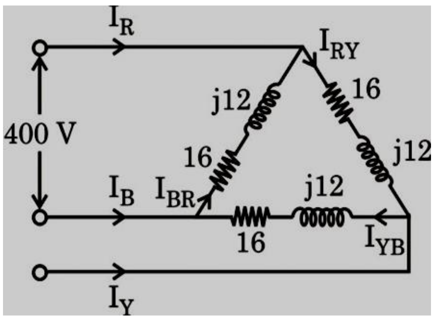
$$cos\phi = \frac{8}{10} = 0.8 \qquad \phi = 36.86^\circ$$

Active power $= \sqrt{3} V_L I_L \cos\phi \quad = \sqrt{3} \times 400 \times 23.1 \times 0.8 \quad = 12.8\ KW$

Reactive Power $= \sqrt{3}\ V_L I_L\ sin\phi \quad = \sqrt{3} \times 400 \times 23.1 \times 0.6 \quad = 9.6\ KVAR$

Q.A balanced delta-connected load of impedance 16 + j12 Ω / phase is connected to a 3-phase 400V supply. Find the phase current, line current, power factor, power, reactive VA and total VA.

Ans



In delta connection,
Phase voltage = Line voltage = 400V

Phase impedance $(Z_P) = \sqrt{R^2 + X^2} = \sqrt{16^2 + 12^2} = 20\Omega$

Phase current $(I_P) = \frac{400}{20} = 20$ Ampere So Line current $(I_L) = \sqrt{3} I_P \quad = \sqrt{3} \times 20 \quad = 34.64$ Ampere.

Power factor $(\cos\phi) = \frac{R_P}{Z_P} = \frac{16}{20} = 0.8$

$\sin\phi = 0.6$

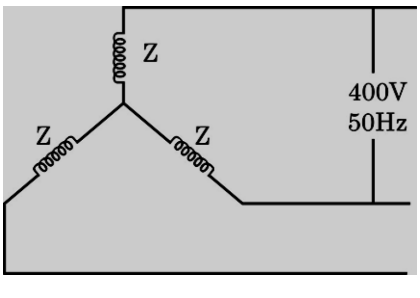
Total power (Apparent Power) $= \sqrt{3}\, V_L I_L = \sqrt{3}(400)(34.64) \quad = 23999.29$ VA

Power (Active) $= \sqrt{3}\, V_L I_L \cos\phi = 19.19943$ kW

Reactive Power $= \sqrt{3} V_L I_L \sin\phi = 14.399\,\text{KVAR}$

Q. Three similar coils each having a resistance of 8 ohm and an inductance of 0.0191 H in series in each phase is connected across a 400V, three phase, 50Hz supply. Calculate the line current, power input, KVA and KVAR taken by the load.

Ans. **If Star Connected Load:**



$$I_L = I_{Ph} = \frac{V_{Ph}}{Z_{Ph}} = \frac{400/\sqrt{3}}{10} = 23.09A$$

$$P = \sqrt{3} V_L I_L \cos\phi$$

$$= \sqrt{3} \times 400 \times 23.09 \times 0.8 \quad = 12797.78W = 12.8KW$$

Reactive Power $Q = \sqrt{3} V_L I_L \sin\phi$

$$= \sqrt{3} \times 400 \times 23.09 \times 0.6 \quad = 9.6KVA$$

Apparent Power $S = \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 23.09 = 16KVA$

$$Z = R + jX_L = (8 + j6)\Omega$$

$$|Z| = 10\Omega$$

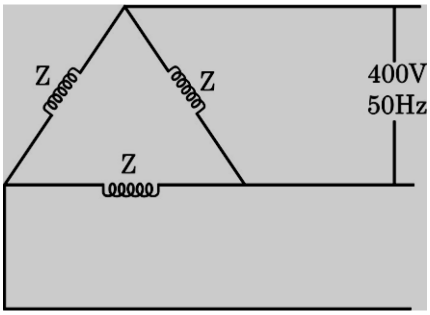
$$\cos\phi = \frac{8}{10} = \frac{R}{Z} = 0.8$$

$$\sin\phi = 0.6$$

$$R = 8\Omega,\ L = 0.0191H$$

$$X_L = 2\pi_f L = 2\pi \times 50 \times 0.061 = 6\Omega$$

If Delta Connected Load:



$$Z = 8 + j6\Omega;\quad |Z| = 10\Omega$$

$$\cos\phi = 0.8;\qquad \sin\phi = 0.6$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{10} = 40A$$

$$I_L = \sqrt{3} I_{ph} = \sqrt{3} \times 40 = 69.28A$$

$$P = \sqrt{3} V_L I_L \cos\phi \quad = \sqrt{3} \times 400 \times 69.28 \times 0.8 \ = 38.4Kw$$

Reactive Power $Q = \sqrt{3} V_L I_L \sin\phi$

$$= \sqrt{3} \times 400 \times 69.28 \times 0.6$$

$$= 28.8KVAr$$

Apparent Power $S = \sqrt{3} V_L I_L = \sqrt{3} \times 400 \times 69.28 = 48KVA$