ABES ENGINEERING COLLEGE, GHAZIABAD (032)

DEPARTMENT OF AS & H

B. TECH SEM-I (2023-24)

Engineering Mathematics - I (BAS103)

Tutorial Sheet:I

Module-I (Matrices)

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1. (a) If A is a Hermitian matrix, then show that iA is Skew-Hermitian matrix.

   (b) Find inverse of the matrix by elementary row transformation $\begin{bmatrix} i & -1 & 2i \\ 2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$

2. (a)Find the ranks of the following matrices $\begin{bmatrix} 1 & 1 & 2 & 3 \\ 1 & 3 & 0 & 3 \\ 1 & -2 & -3 & -3 \\ 1 & 1 & 2 & 3 \end{bmatrix}$

   (b) Find the rank of the following matrix by using normal form

   $$\begin{bmatrix} -2 & -1 & 3 & -1 \\ 1 & 2 & -3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

   (c) Find the value of b such that the rank of the matrix A is 3, $A = \begin{bmatrix} 2 & 4 & 2 \\ 3 & -1 & 2 \\ b & 0 & 1 \end{bmatrix}$

3. Find two non-singular matrices P and Q such that PAQ is in the normal form and hence, find $\rho(A)$, if A is given by $\begin{bmatrix} 2 & 1 & 1 & 3 \\ 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 5 \end{bmatrix}$. Can we find its inverse?

4. (a) Find the value of λ for which the system have a solution: $x + y + z = 1, x + 2y + 4z = \lambda, x + 4y + 10z = \lambda^2$and also find the solution.

   (b) For what values of λ, the system of equations $2x - 2y + z = \lambda x$, $2x - 3y + 2z = \lambda y, -x + 2y + 0z = \lambda z$ possess a non-trivial solution.

5. Express the matrix A = $\begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$ as the sum of Hermitian matrix & skew Hermitian matrix.

6. Examine the following vectors for linear dependence and find the relation between them, if possible:

   $X_1 = (1,1,-1,1), X_2 = (1,-1,-2,-1), X_1 = (3,1,0,1)$

7. Find the eigen values & eigen vectors for the matrix:

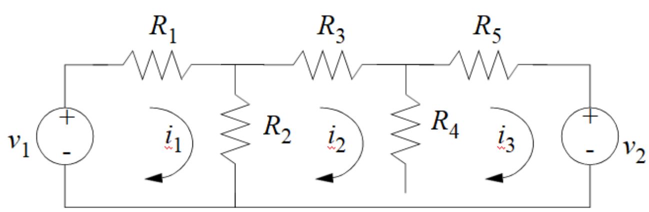
(a) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$

8. (a) Let A be a 3×3 matrix with real entries such that det(A)=6 and the trace of A is 0. If $|I + A| = 0$ where I denotes the identity matrix of order 3 then find the eigen values of A.

   (b) Find the eigen value of the matrix $\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix} 51 \\ 51 \end{bmatrix}$.

9. Find the characteristic equation of the symmetric matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ & hence also find A^{-1} by Cayley – Hamilton theorem. Find the value of $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$

10 (a) A three loop current network with five resistors and two voltage sources is shown in the circuit:



Find the mesh currents i_1, i_2, & i_3 , when the resistance R_1, R_2, R_3, R_4, R_5 are 1ohm each and v_1, v_2 are 5 volts and - 6 volts respectively.

10 (b) *Suppose the message is in encrypted form and is received as*

–4 2 3 10 27 -18 17 27 -25 -15 19 1

along with its encoding matrix*

E = $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$. Decode the message.

<u>**ANS:-**</u> 1- (a) (b) $A^{-1} = \frac{1}{4}\begin{bmatrix} 0 & 1 & -2 \\ -4 & 3i & 2i \\ 0 & 1 & 2 \end{bmatrix}$

2- (a) 3 (b)3, (c) 7/5

3. $P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -2 & 0 \\ -1 & 2 & -3 \end{bmatrix}, Q = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, No

4- (a) $\lambda = 1, 2$ when $\lambda = 1, x = 1 + 2k, y = -3k, z = k$ *and* when $\lambda = 2, x = 2k, y = 1 - 3k, z = k$. 4. (b) $\lambda = 1, -3$

5- $\begin{bmatrix} 0 & 4-2i & 2+3i \\ 4+2i & 0 & 3-2i \\ 2-3i & 3+2i & 2 \end{bmatrix}$ and $\begin{bmatrix} i & -2-i & 2+2i \\ 2-i & 0 & 1-3i \\ -2+2i & -1-3i & i \end{bmatrix}$

6. Linearly dependent; $2X_1 + X_2 = X_3$

7- (a) 0,3,15 $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$, (b) 2,2,8 $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ (c) $1, 2, 2;$ $\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

8- (a) -1, -2, 3 (b) 6

9- $\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$, $A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$, $5A - I$

10 (a) i_1 = 3.8750 amp i_2 = 2.75 amp & i_3 = 4.3750 amp

(b) A B E S I S B E S T