ABES Engineering College, Ghaziabad
MODEL PAPER (Odd Semester) 2023-24
B.TECH. [Branch/Section:All] **SEM: I.**

Subject Name: Engineering Mathematics-I Subject Code : BAS103 **Name: ____________________**	**Max. Marks : 70** **Time : 3 Hours**

Instructions :	1. *Attempt the questions as per the instructions given* 2. *Assume missing data suitably*

CO1	**Statement of CO1**: Understand the concept of complex matrices, Eigen values, Eigen vectors and apply the concept of rank to evaluate linear simultaneous equations
CO2	**Statement of CO2**: Remember the concept of differentiation to find successive differentiation, Leibnitz Theorem, and create curve tracing, and find partial and total derivatives
CO3	**Statement of CO3:** Identify the application of partial differentiation and apply for evaluating maxima, minima, series and Jacobians.
CO4	**Statement of CO4**: Remember the concept of Beta and Gamma function; analyze area and volume and Dirichlet's theorem in multiple integral
CO5	**Statement of CO5**: Apply the concept of Vector Calculus to analyze and evaluate directional derivative, line, surface and volume integrals

Section – A

Q.1	Attempt **all** the parts **(2 x 7=14)**			
(a)	Find the value of **'a'** for which the vectors (1, -2, a), (2, -1, 5) and (3, -5, 7a) are linearly dependent.	K2	CO1	2
(b)	Determine the values of λ, μ, for the following system of equations $3x-2y+z=\mu, 5x-8y+9z=3, 2x+y+\lambda z=-1$ has unique solution.	K2	CO1	2
(c)	Find the nth derivative of the following function: $\frac{1}{(1-3x)^2}$	K2	CO2	2
(d)	If $\boldsymbol{u = f(2x-3y, 3y-4z, 4z-2x)}$, prove that $\boldsymbol{\frac{1}{2}\frac{\partial u}{\partial x}+\frac{1}{3}\frac{\partial u}{\partial y}+\frac{1}{4}\frac{\partial u}{\partial z}=0}$.	K2	CO2	2
(e)	Find the percentage error in the area of an ellipse when an error of +1 % is made in measuring the major and minor axis.	K2	CO3	2
(f)	Prove that $\boldsymbol{f(mx)=f(x)+(m-1)xf'(x)+\frac{(m-1)^2x^2}{\lfloor 2}f''(x)+\frac{(m-1)^3x^3}{\lfloor 3}f'''(x)+\ldots\ldots}$	K2	CO4	2
(g)	Discuss the value of ***'b'*** for a Solenoidal vector $\vec{F}=(bx)\hat{i}-(5y)\hat{j}+(2z)\hat{k}$.	K2	CO5	2

Section – B

Q.2	Attempt **any three** parts of the following **(7 x 3 = 21)**			
(a)	Find the eigen values and corresponding eigen vectors of the matrix $A=\begin{bmatrix}1&2&2\\0&2&1\\-1&2&2\end{bmatrix}$	K4	CO1	7
(b)	*If* $x=\sin\sqrt{y}$ find the value of y_n at $x=0$.	K4	CO2	7
(c)	*If* $u^3+v^3+w^3=x+y+z, u^2+v^2+w^2=x^3+y^3+z^3$ *and* $u+v+w=x^2+y^2+z^2$, *then show that* $\frac{\partial(u,v,w)}{\partial(x,y,z)}=\frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$	K4	CO3	7
(d)	Prove that $\iiint\frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}=\frac{\pi^2}{8}$, the integral being extended to all positive values of the variables for which the expression is real.	K4	CO4	7
(e)	Verify Stoke's theorem for $\boldsymbol{\vec{F}=(x^2+y^2)\hat{i}-2xy\hat{j}}$ taken round the rectangle bounded by the lines $\boldsymbol{x=\pm a, y=0, y=b}$.	K4	CO5	7

Section – C				
Q.3	Attempt **any one** part of the following (**7x 1 = 7)**			
(a)	If A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ determine two non-singular matrices ***P*** and ***Q*** such that ***PAQ=I.*** **Hence find** $\mathbf{A^{-1}}$**.**	K3	CO1	7
(b)	Verify Cayley-Hamilton theorem for the matrix $\boldsymbol{A}=\begin{bmatrix} \mathbf{2} & \mathbf{-1} & \mathbf{1} \\ \mathbf{-1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{1} & \mathbf{-1} & \mathbf{2} \end{bmatrix}$. Hence Compute $\boldsymbol{A^{-1}}$. Also evaluate $\boldsymbol{A^6-6A^5+9A^4-2A^3-12A^2+23A-9I}$.	K4	CO1	7
Q.4	Attempt **any one** part of the following (**7x 1 = 7)**			
(a)	Find the *n*th derivative of $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$	K4	CO2	7
(b)	Trace the curve : $a^2x^2=y^3(2a-y)$ and also write all necessary steps	K4	CO2	7
Q.5	Attempt **any one** part of the following (**7x 1 = 7)**			
(a)	Compute $f(1.1,0.9)$ for the function $f(x,\ y)\ =\tan^{-1}\left(\frac{x}{y}\right)$ by using Taylor's series expansion.	K4	CO3	7
(b)	Find the dimensions of a rectangular box of maximum capacity whose surface area is given when (i) box is open at the top (ii) box is closed.	K4	CO3	7
Q.6	Attempt **any one** part of the following (**7x 1 = 7)**			
(a)	Evaluate the following integral by changing the order of integration : $\int_0^1\int_{y^2}^{2-x} xy\,dx\,dy$	K4	CO4	7
(b)	Change into polar coordinates and evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$. Hence show that $\int_0^\infty e^{-x^2}\,dx=\frac{\sqrt{\pi}}{2}$.	K4	CO4	7
Q.7	Attempt **any one** part of the following (**7x 1 = 7)**			
(a)	Find the directional derivative of $\boldsymbol{\varphi(x,y)=5x^2y-5y^2z+\frac{5}{2}z^2x}$ at the point $\mathbf{(1,1,1)}$ in the direction of line $\frac{x+1}{1}=\frac{y}{1}=\frac{1-2z}{-2}$.	K4	CO5	7
(b)	A fluid motion is given by $v=(y+z)i+(z+x)j+(x+y)k$, verify that motion is irrational and hence find the velocity potential. Also prove that fluid is incompressible	K4	CO5	7