... $\sin x) + (-\,.1 \cos 2x - .06 \sin 2x) + .03 \cos 3x + \ldots\ldots$

10. -0.0731.

ASSIGNMENT-III

(2 Marks Questions for Section–A)

1. Find the constant term if the function $f(x) = x + x^2$ is expanded in Fourier series defined in $(-1, 1)$.
(G.B.T.U. 2012)
2. Find the constant term if $f(x) = x^2$ is expanded in Fourier series in $(-\pi, \pi)$.
(U.P.T.U. 2015)
3. If $f(x) = |\ x\ |$ is expanded in Fourier series defined in $(-1, 1)$ then find the constant term.
(U.P.T.U. 2013)
4. Find the constant term when $f(x) = |\ x\ |$ is expanded in Fourier series in the interval $(-2, 2)$.
(A.K.T.U. 2017).
5. Define periodic functions and find the period of $f(x) = \cos x + \frac{1}{2}\cos 2x + \frac{1}{3}\cos 3x$.
[G.B.T.U. (AG) 2012]

6. For an even function defined in the interval $(0, 2\pi)$, write down the Fourier series. [G.B.T.U. (AG) 2012]

7. State Dirichlet's conditions for the expansion of $f(x)$ in Fourier series. (U.P.T.U. 2015, 17)

8. If $f(x) = 1$ is expanded in half range sine series in $(0, \pi)$, then find the value of b_n. (A.K.T.U. 2022)

9. If $f(x) = 1$, $0 < x < \pi$ is expanded in half range cosine series then find the value of a_0. (U.P.T.U. 2014)

10. Find the value of the Fourier coefficient a_0 for the function

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$ (A.K.T.U. 2016)

11. If $F(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$, then find F(0). (U.P.T.U. 2014)

12. Find the period of $\sin nx$.

13. Find Fourier half-range sine series for the function $f(x) = x$, $0 < x < 2$. [G.B.T.U. (SUM) 2010]

14. What does the Fourier coefficient a_0 in Fourier series expansion of a function represent? (G.B.T.U. 2010)

15. If $F(x) = x \sin x$ in $(-\pi, \pi)$ then find b_n.

16. What is the smallest period of the function $f(x) = \sin\left(\frac{2n\pi x}{k}\right)$.

17. If $F(x) = x^2$ in $-2 < x < 2$ and $F(x + 4) = F(x)$, then find the value of a_n.

18. What is the product of two odd functions. (G.B.T.U. 2011)

19. If $F(x) = x$ is expanded in a Fourier sine series in $(0, \pi)$ then find b_n. [M.T.U. (SUM) 2011]

20. If $F(x) = x \cos x$ is expanded in a Fourier series in $(-\pi, \pi)$ then find a_0. [G.B.T.U. (AG) 2011]

21. Find the Fourier coefficient for the function $f(x) = x^2$, $0 < x < 2\pi$. (A.K.T.U. 2017)

22. Discuss the convergence of sequence $\{1, 2^1, 2^2, 2^3, 2^4, ...\}$. (A.K.T.U. 2022)

23. Discuss the convergence of sequence $a_n = \frac{2n}{n^2 + 1}$. (A.K.T.U. 2019)

24. Find the constant term when $f(x) = 1 + |x|$ is expanded in Fourier series in the interval $(-3, 3)$. (A.K.T.U. 2022)

25. Find the Fourier constant a_n for $f(x) = x \cos x$ in the interval $(-\pi, \pi)$. (A.K.T.U. 2022)

26. Find the Fourier constant a_1 of $f(x) = x^2$, $-\pi \le x \le \pi$. (A.K.T.U. 2019)

Answers

1. $\frac{2}{3}$

2. $\frac{2}{3}\pi^2$

3. 1

4. $\frac{a_0}{2} = 1$

5. 2π

6. $F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$

8. $\frac{2}{n\pi}[1 - (-1)^n]$

9. 2

10. $\frac{\pi}{2}$

11. 0

12. $\frac{2\pi}{n}$

13. $x = -\frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{2}$

14. Mean value of the function

15. 0

16. $\frac{k}{n}$

17. $\int_0^2 x^2 \cos\frac{n\pi x}{2}\,dx$

18. Even function

19. $-\frac{2}{n}(-1)^n$

20. 0

21. $a_0 = \frac{8}{3}\pi^2$, $a_n = \frac{4}{n^2}$, $b_n = -\frac{4\pi}{n}$

22. Divervent

23. Convergent

24. $a_0 = 5$

25. $a_n = 0$

26. –4.

ASSIGNMENT-IV

1. Define analytic function and state the necessary and sufficient condition for function to be analytic.
(M.T.U. 2012

2. If $f(z) = u + iv$ is analytic, then show that the family of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ are mutually orthogonal.
(M.T.U. 2012

3. Using the Cauchy-Riemann equations, show that $f(z) = |z|^2$ is not analytic at any point.
(M.T.U. 2013

4. Find the constants a, b and c such that the function $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + y^2)$ is analytic. *(M.T.U. 2013)*
5. Define analytic function with an example. *(A.K.T.U. 2018)*
6. Find the values of a and b for which the function $f(z) = \cos x\,(\cosh y + a \sinh y) + i \sin x\,(\cosh y + b \sinh y)$ is analytic.
7. If $f(z) = u + iv$ is an analytic function and $u = x^2 - y^2 - y$, then find its conjugate harmonic function $v(x, y)$. *(A.K.T.U. 2016)*
8. If $f(z) = u + iv$ is an analytic function and $v = y^2 - x^2$, then find its conjugate harmonic function $u(x, y)$.
9. If $u = \dfrac{x^2 - y^2}{(x^2 + y^2)^2}$ is the real part of analytic function $f(z) = u + iv$, then find $f(z)$ in terms of z.
10. Let $u(x, y) = 2x(1 - y)$ for all real x and y. Find a function $v(x, y)$ so that $f(z) = u + iv$ is analytic.
11. If $u(x, y) = x^3y - xy^3$ is the real part of analytic function $f(z) = u(x, y) + (x, y)$, then find its conjugate harmonic function $v(x, y)$. *(M.T.U. 2014)* *(A.K.T.U. 2022)*
12. Define Harmonic function. *(A.K.T.U. 2016)*
13. If $u(x, y) = x^2 - y^2$, prove that u satisfies Laplace equation.
14. Let $u(x, y) = x^3 + ax^2y + bxy^2 + 2y^3$ be a harmonic function and $v(x, y)$ its harmonic conjugate. If $u(0, 0) = 1$, then find $|a + b + v(1, 1)|$. *(A.K.T.U. 2017)*
15. Prove that $\sinh z$ is analytic.
16. Find the image of the straight line $2x - y + 3 = 0$ in the w-plane under the transformation $w = z - 2$. *(A.K.T.U. 2022)*
17. Find the points of invariant of the transformation $w = \dfrac{2z + 3}{z + 2}$.
18. Show that an analytic function with constant real part is constant. *(A.K.T.U. 2016)*
19. If $u + iv$ is analytic, show that $v - iu$ and $-v + iu$ are also analytic. *(A.K.T.U. 2019)*
20. Write the Cauchy's Reimann conditions in polar coordinates system. *(A.K.T.U. 2019)*
21. Show that complex function $f(z) = z^3$ is analytic.
22. Define conformal mapping.

Answers

4. $a = -\dfrac{1}{2}, b = -2, c = \dfrac{1}{2}$

6. $a = -1, b = -1$

7. $2xy + x + c$

8. $2xy + c$

9. $\dfrac{1}{z^2} + c$

10. $x^2 - (y - 1)^2$

11. $x^4 + y^4 - 6x^2y^2 + c$

14. 10

16. $2u - v + 7 = 0$

17. $z = \pm\sqrt{3}$

20. $\dfrac{\partial u}{\partial r} = \dfrac{1}{r}\dfrac{\partial v}{\partial \theta}$ and $\dfrac{\partial v}{\partial r} = -\dfrac{1}{r}\dfrac{\partial u}{\partial \theta}$

ASSIGNMENT- V

1. Evaluate $\int_0^{1+i} z^2 \, dz$.

2. Evaluate the integral $\int_C \frac{e^{iz}}{z^3} dz$ where C : $|z| = 1$. *(M.T.U. 2013)*

3. (*i*) Define isolated and non-isolated singular points. *(M.T.U. 2012)*
   (*ii*) Define removable and essential singular points with example. *(M.T.U. 2012)*

4. (*i*) Define singular point of an analytic function. Find nature and location of the singularity of
$$f(z) = \frac{z - \sin z}{z^2}.$$ *(M.T.U. 2013)*

   (*ii*) Find the nature of singularity of $f(z) = \frac{z - \sin z}{z^3}$ at $z = 0$.

   (*iii*) Discuss the singularity of $\sin\left(\frac{1}{z - a}\right)$. *(A.K.T.U. 2022)*

5. Evaluate $\oint_C \frac{dz}{z - 2}$ around the circle $|z - 2| = 4$. *(A.K.T.U. 2016, 2022)*

6. (*i*) State Cauchy's integral theorem.

   (*ii*) Evaluate $\oint_C (5z^4 - z^3 + 2) \, dz$ around the unit circle $|z| = 1$.

7. If $F(\alpha) = \oint_C \frac{5z^2 - 4z + 3}{z - \alpha} dz$ which C is the ellipse $16x^2 + 9y^2 = 144$, then find F(2).

8. Evaluate $\oint_C \frac{dz}{z^2 + 9}$ where C is $|z - 3i| = 4$.

9. (*i*) Find residue of $f(z) = \left(\frac{z+1}{z-1}\right)^3$ at $z = 1$. *(A.K.T.U. 2019)*

   (*ii*) Find residue of $f(z) = \frac{\cos z}{z(z+5)}$ at $z = 0$. *(A.K.T.U. 2017)*

10. Find the residue of $f(z) = \cot z$ at its pole.

11. (*i*) Find residue of $f(z) = \frac{z^2}{(z^2 + 3z + 2)^2}$ at the pole $z = -1$. *(U.P.T.U. 2014)*

    (*ii*) Find residue of $f(z) = \frac{z^2}{z^2 + 3z + 2}$ at the pole -1. *(M.T.U. 2014)*

    (*iii*) Find residue of $f(z) = \frac{2z + 1}{z^2 - z - 2}$ at the pole $z = -1$.

12. Evaluate $\oint_C \frac{4 - 3z}{z^2 - z} dz$, where C is any simple closed path such that $1 \in C, 0 \notin C$. *(A.K.T.U. 2016)*

13. Write the statement of generalized Cauchy's integral formula for n^{th} derivative of an analytic function at the point $z = z_0$.

14. Evaluate $\oint_C \frac{z - 3}{z^2 + 2z + 5} dz$ when $C \equiv |z| = 1$.

15. Let $I = \int_C \frac{f(z)}{(z-1)(z-2)} dz$ where $f(z) = \sin \frac{\pi z}{2} + \cos \frac{\pi z}{2}$ and C is the curve $|z| = 3$ oriented anti-clockwise. Find the value of I.

16. Let $\sum_{n=-\infty}^{\infty} b_n z^n$ be the Laurent's series expansion of the function $\frac{1}{z \sinh z}$, $0 < |z| < \pi$, then find b_{-2}, b_0 and b_2.

17. Let $f(z) = \sum_{n=0}^{15} z^n$ for $z \in \mathbb{C}$. If $C : |z - i| = 2$, then evaluate $\oint_C \frac{f(z)}{(z-i)^{15}} dz$.

18. Let $u(x, y)$ be the real part of an entire function $f(z) = u(x, y) + i\, v(x, y)$ for $z = x + iy \in$ C. If C is the positively oriented boundary of a rectangular region R in R^2 then evaluate $\oint_C \left(\frac{\partial u}{\partial y} dx - \frac{\partial u}{\partial x} dy \right)$.

19. Consider the function $f(z) = \frac{e^{iz}}{z(z^2+1)}$. Find the residue of f at the isolated singular point in the upper half plane $\{z = x + iy \in \mathbb{C} : y > 0\}$.

20. Let S be the positively oriented circle given by $|z - 3i| = 2$. Then evaluate $\int_S \frac{dz}{z^2+4}$.

21. Let $f(z)$ be an analytic function. Then evaluate $\int_0^{2\pi} f(e^{it}) \cos(t)\, dt$.

22. Let $f(z) = \frac{1}{z^2 - 3z + 2}$, then find the coefficient of $\frac{1}{z^3}$ in the Laurent's series expansion of $f(z)$ for $|z| > 2$.

23. Evaluate: $\int_C \frac{z^2+1}{z^2-1} dz$, where C is the circle $|z| = \frac{3}{2}$. *(A.K.T.U. 2016)*

24. Expand $\frac{1}{(z+1)(z+3)}$ in the region $|z| < 1$. *(A.K.T.U. 2016)*

25. Evaluate: $\int_{|z|=\frac{1}{2}} \frac{e^z}{z^2+1} dz$. *(A.K.T.U. 2017)*

26. Evaluate: $\int_C \frac{e^z}{z+1} dz$, where C is the circle $|z| = 2$. *(A.K.T.U. 2017)*

27. Let $C = \{z \in C;\ |z - i| = 2\}$. Then evaluate: $\frac{1}{2\pi} \oint_C \frac{z^2 - 4}{z^2 + 4} dz$.

28. Let $\gamma = \{z \in C : |z| = 2\}$ be oriented in the counter-clockwise directions. Let $I = \frac{1}{2\pi i} \oint_\gamma z^7 \cos\left(\frac{1}{z^2}\right) dz$, then find the value of I.

29. Find the radius of convergence of the power series

$$\sum_{n=0}^{\infty} 4^{(-1)^n \cdot n} z^{2n}.$$

$\left[\textbf{Hint:} \ a_n = \begin{cases} 4^n, & n = 2k \\ 0, & n = 2k-1 \end{cases} \text{ where } k = 1, 2, 3, \ldots \right]$

30. Consider the power series $\sum_{n=0}^{\infty} a_n z^n$ where $a_n = \begin{cases} \dfrac{1}{3^n}, & \text{if } n \text{ is even} \\ \dfrac{1}{5^n}, & \text{if } n \text{ is odd} \end{cases}$.

What is the radius of convergence of the series?

31. Find the coefficient of $(z - \pi)^2$ in the Taylor's series expansion of

$$f(z) = \begin{Bmatrix} \dfrac{\sin z}{z - \pi}, & \text{if } z \neq \pi \\ -1, & \text{if } z = \pi \end{Bmatrix} \text{ around } \pi.$$

32. If $\sum_{n=-\infty}^{\infty} a_n (z-2)^n$ is the Laurent series of the function $f(z) = \dfrac{z^4 + z^3 + z^2}{(z-2)^3}$ for $z \in \dfrac{C}{\{2\}}$, then find a_{-2}.

Answers

1. $-\frac{2}{3} + \frac{2}{3}i$ **2.** $-\pi i$

4. (*i*) removable singularity at $z = 0$ (*ii*) removable singularity (*iii*) essential singularity

5. $2\pi i$ **6.** (*ii*) 0 **7.** $30\pi i$ **8.** $\frac{\pi}{3}$

9. (*i*) 6 (*ii*) $\frac{1}{5}$ **10.** 1

11. (*i*) – 4 (*ii*) 1 (*iii*) $\frac{1}{3}$

12. $2\pi i$ **13.** $f^n(z_0) = \dfrac{\underline{|n}}{2\pi i} \displaystyle\int_C \frac{f(z)}{(z - z_0)^{n+1}} dz.$ **14.** 0

15. $-4\pi i$ **16.** $b_{-2} = 1, b_0 = -1/6, b_2 = 7/360$ **17.** $2\pi i(1 + 15i)$

18. 0 **19.** $-\frac{1}{2e}$ **20.** $\frac{\pi}{2}$ **21.** $\pi f'(0)$

22. 3 **23.** 0 **24.** $\frac{1}{2}\left[\sum_{n=0}^{\infty} (-1)^n z^n - \frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z}{3}\right)^n\right]$

25. 0 **26.** $\frac{2\pi i}{e}$ **27.** – 2 **28.** $\frac{1}{24}$

29. 0.50 **30.** 3 **31.** $\frac{1}{6}$ **32.** 48.