# Model Question Paper

BAS-103

Engineering Mathematics - I

## Section-A

(2×7=14) marks

Q.1 Attempt All.

a). If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 3 & 4 & 5 \end{bmatrix}$ find eigen values of $A^2 - 2A + 3I$

b) find the value of 'b' for which rank of matrix A is 2.

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$$

or

b) show that matrix A is unitary, $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

c) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$

or

c) If $f(x, y, z, w) = 0$ then find $\frac{\partial x}{\partial y} \times \frac{\partial y}{\partial z} \times \frac{\partial z}{\partial w} \times \frac{\partial w}{\partial x}$.

d) If $f(x,y) = 5x^2 + 10y^2 + 12xy - 4x - 6y + 1$, find the stationary points?

e) If $pv^2 = k$, ~~find~~ and the relative errors in $p$ and $v$ are 0.05, 0.025 respectively, show that error in $k$ is 10%

f) find $\Gamma\left(-\frac{1}{2}\right)$ or $\Gamma\left(\frac{1}{2}\right)$ or $\Gamma\left(-\frac{5}{2}\right)$.

or

f) find area bounded by curve $y^2 = x$, and $x^2 = y$.

g) find $\nabla(\log r)$ or find curl $\left(\frac{\vec{r}}{r^3}\right)$

h) find constants a and b such that $\vec{A}$ is irrotational

$$\vec{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} + (3xy + 2byz)\hat{k}.$$

or

h) using green's Theorem find area of the region in I quadrant bounded by the curves

$$y = x, \quad y = \frac{1}{x}, \quad y = \frac{x}{4}.$$

## Section-B.

Q2 Attempt any three (7×3=21)

a) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

Hence compute $A^{-1}$. Also evaluate $A^6 - 2A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$.

b) If $y = e^{m\cos^{-1}x}$, show that

$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$. also

calculate $y_n(0)$.

c) If $u, v, w$ are roots of cubic $(\lambda-x)^3 + (\lambda-y)^3 + (\lambda-z)^3 = 0$ in $\lambda$, then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

or

c) find maximum and minimum distance of point $(1,2,-1)$ to the sphere $x^2+y^2+z^2=24$.

d) find the volume and mass contained in the solid region in the first octant of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, if density at any point is $\rho(x,y,z) = Kxyz$.

e) Verify Gauss Divergence theorem for $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ taken over the cube bounded by the planes $x=0, x=1, y=0, y=1, z=0, z=1$

## Section-C

Attempt all. Attempt any one part from each question [3 to 7) [5x7=35]

Q.3 Attempt any one part. [7x1=7]

a) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ determine two non-singular matrices P and Q such that PAQ is in normal form. Hence find $A^{-1}$

b) Investigate for what values of $\lambda$ and $\mu$ do the system of eq$^n$s
$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have

i) no solution ii) unique sol$^n$ iii) infinite sol$^n$.

or

b) find eigen values and eigen vectors of

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Q4. Attempt any one part (7x1=7)

a) Trace the curve $y^2(2a-x) = x^3$

or

a) If $x^x y^y z^z = c$, show that at $x=y=z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x\log ex)^{-1}$

b) If $u = f(y-z, z-x, x-y)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

Q.5 Attempt any one part (7x1=7)

~~a) expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of x, y.~~

a) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in the neighbourhood of (1,1) upto and inclusive second degree terms. Hence compute $f(1.1, 0.9)$ approximately.

b) Show that functions
$u = x+y+z$
$v = x^2+y^2+z^2-2xy-2yz-2zx$
$w = x^3+y^3+z^3-3xyz$ are functionally

related. Find relation between them.

Q6 Attempt any one part (7×1=7)

a) Change the order of integration & evaluate

$$\int_0^1\int_{y^2}^{2-y} xy\,dx\,dy.$$

b) Evaluate $\iint_R (x-y)^4 e^{x+y}\,dx\,dy$ where R is the square with vertices at (1,0), (2,1) (1,2) and (0,1).

or

b) Show that $\iiint \dfrac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}} = \dfrac{\pi^2}{8}$, by changing into spherical co-ordinates

or change in to Polar co-ordinates & evaluate

b) $$\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$$ Hence show $\int_0^\infty e^{-x^2}dx = \dfrac{\sqrt{\pi}}{2}$

Q7 a) find the directional derivative of $\phi = (x^2+y^2+z^2)^{-1/2}$ at the point P(3,1,2) in the direction of vector $yz\hat{i} + zx\hat{j} + xy\hat{k}$.

b) Verify Stoke's theorem, for $\vec{F} = (x^2-y^2)\hat{i} + 2xy\hat{j}$ integrated round the rectangle in the plane z=0 and bounded by lines x=0, y=0, x=a, y=b.

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Q8 Show that $\vec{A} = (6xy+z^3)\hat{i} + (3x^2-z)\hat{j} + (3xz^2-y)\hat{k}$ is irrotational. find the velocity potential $\phi$ such that $\vec{A} = \nabla\phi$.

or final

Extra Questions

1. The plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ meets the axes in A, B, C. Apply Dirichlet's integral to find volume of tetrahedron OABC. Also find mass if density at any point is kxyz

2. Verify Gauss divergence theorem $\vec{F} = 2(x-z)\hat{i} + x^2y\hat{j} - xz^2\hat{k}$. taken over the region bounded by x=0, x=1, y=0, y=1, z=0, z=1.

3. find directional derivative of $\nabla(\nabla f)$ at the point (1,-2,1) in the direction of normal to the surface $xy^2z = 3x+z^2$ where $f = 2x^3y^2z^4$.

4. Show that vector field $\vec{F} = \dfrac{\vec{r}}{r^3}$ is irrotational as well as solenoidal. Find the scalar potential.

5. If $u = \operatorname{cosec}^{-1}\left[\dfrac{x^{1/2}+y^{1/2}}{y^{1/3}+x^{1/3}}\right]^{1/2}$ prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13+\tan^2 u)$$