

Fig. 12

Diffraction at N-slit

① Path difference = $(a+b) \sin \theta$.

② Phase difference = $\frac{2\pi}{\lambda} (a+b) \sin \theta = 2\beta$.

③
$$\beta = \frac{\pi}{\lambda} (a+b) \sin \theta$$

③ Resultant amplitude $R' = A \frac{\sin \alpha}{\alpha} \frac{\sin N\beta}{\sin \beta}$

④ Resultant intensity at P.

$$I = R'^2 = A^2 \frac{\sin^2 \alpha}{\alpha^2} \cdot \frac{\sin^2 N\beta}{\sin^2 \beta}$$

⑤ Principal maxima.

$\sin \beta = 0$ $\beta = \pm n\pi$

$$\therefore \lim_{\beta \rightarrow \pm n\pi} \frac{\sin N\beta}{\sin \beta} = \lim_{\beta \rightarrow \pm n\pi} \frac{d}{d\beta} \frac{\sin N\beta}{\sin \beta} = \frac{N \cos N\beta}{\cos \beta} \Big|_{\beta = \pm n\pi}$$

$= N$

$$\therefore I \propto N^2 \text{ for Principal maxima.}$$

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} N^2$$

$$(a+b) \sin \theta = \pm n\lambda$$

⑥ Minima.

$\sin N\beta \neq 0$ | $\sin \beta \neq 0$

$$N\beta = \pm m\pi$$

$$N(a+b) \sin \theta = \pm m\lambda$$

$m \neq 0, N, 2N, 3N, \dots$

$m = 1, 2, 3, \dots, N-1$

$\therefore (N-1)$ minima b/w two successive principal max.

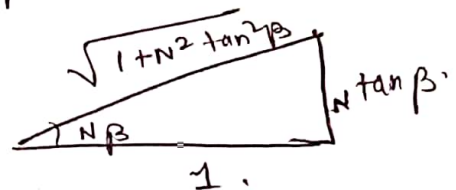
N-slit

⑦ Secondary maxima.

$$\frac{dI}{d\beta} = 0 \Rightarrow \alpha^2 \frac{\sin^2 \alpha}{\alpha^2} \cdot 2 \left[\frac{\sin N\beta}{\sin \beta} \right] \frac{N \cos N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} = 0$$

$$\therefore \tan N\beta = N \tan \beta$$

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{1 + N^2 \tan^2 \beta}}$$



$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{(N^2 \tan^2 \beta)}{(1 + N^2 \tan^2 \beta) \sin^2 \beta}$$

$$= \frac{N^2 \tan^2 \beta}{(1 + N^2 \tan^2 \beta) \sin^2 \beta}$$

$$= \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

$$I'_{SM} = \alpha^2 \frac{\sin^2 \alpha}{\alpha^2} \cdot \frac{N^2}{[1 + (N^2 - 1) \sin^2 \beta]}$$

Ratio \rightarrow

$$\frac{I_{SM}}{I_{PM}} = \frac{1}{1 + (N^2 - 1) \sin^2 \beta}$$

greater the value of N , the weaker are secondary maxima.

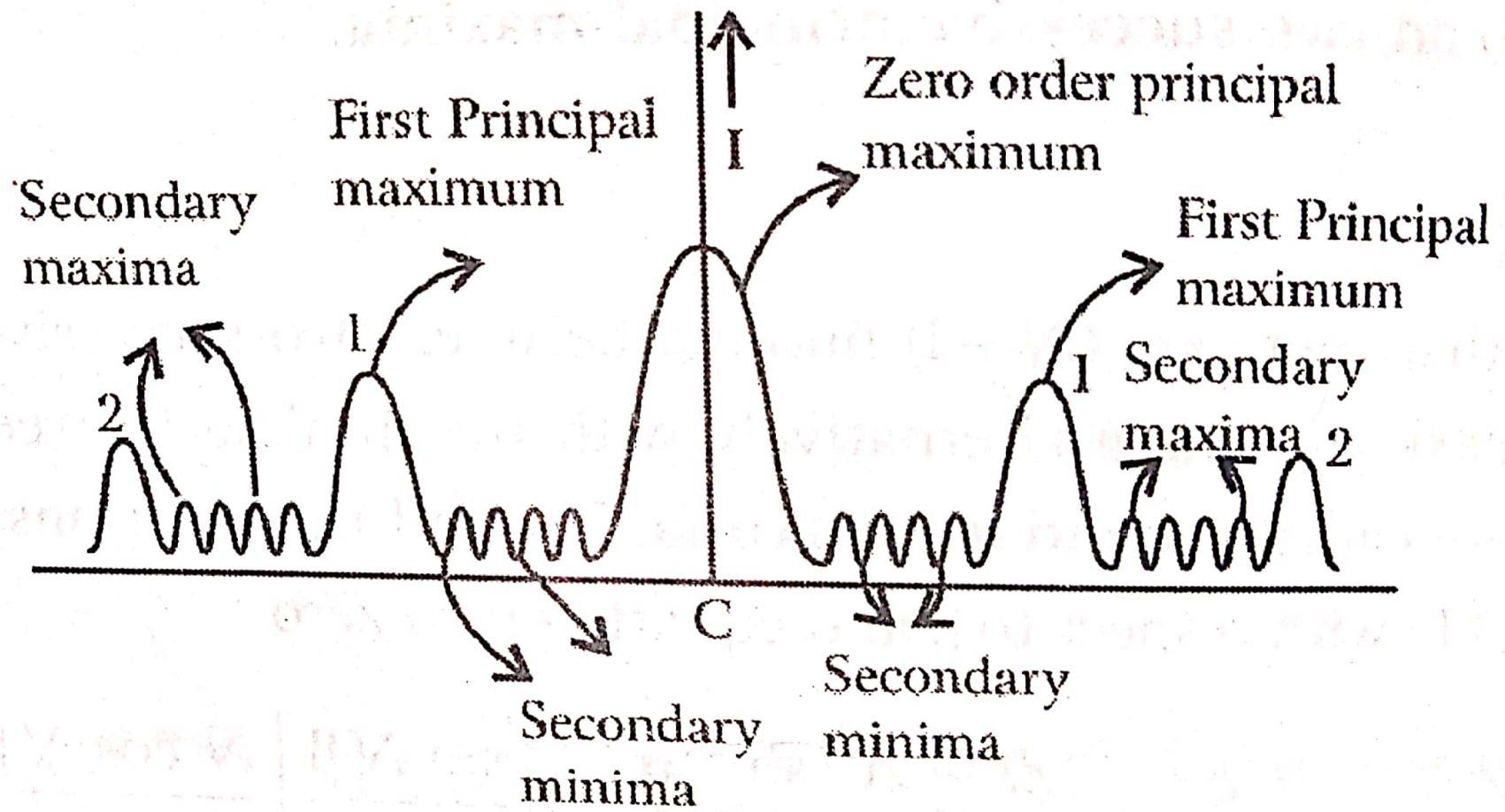


Fig. 14