ABES Engg. college, GZB

B.Tech ; Even sem (2023-24)

Sessional Test - I (BAS-203)

Section-A

1(a) $u_n = \sqrt{n^4+1} - \sqrt{n^4-1}$

$$= \frac{\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)\left(\sqrt{n^4+1}+\sqrt{n^4-1}\right)}{\left(\sqrt{n^4+1}+\sqrt{n^4-1}\right)}$$

$$= \frac{2}{\left(\sqrt{n^4+1}+\sqrt{n^4-1}\right)} \quad \text{— 1}$$

1

$$V_n = \frac{1}{n^2}$$

2

$$\lim_{n\to\infty} \frac{u_n}{V_n} = 2 \text{ (Fixed, finite non zero quantity)}$$

by P-test, $P = 2 (>1)$ Hence the series is convergent. 1

1(b) $u_n = \dfrac{x^{n-1}}{(3n-2)(3n-1)3n}$

$$u_{n+1} = \frac{x^n}{(3n+1)(3n+2)(3n+3)} \quad 1$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \frac{1}{x}$$

Hence by Ratio test, Σu_n is

convergent if $\frac{1}{x} > 1$ or $x < 1$

divergent if $\frac{1}{x} < 1$ or $x > 1$

test fail if $x = 1$ — 2

when $x = 1$

$$u_n = \frac{1}{(3n-2)(3n-1)3n}$$

$$V_n = \frac{1}{n^3}$$

$$\lim_{n\to\infty} \frac{u_n}{V_n} = \frac{1}{27} \text{ (Fixed, finite non-zero quantity)}$$

P-test, $P = 3 (>1)$ Hence the given series is convergent

Σu_n is convergent if $x \le 1$

divergent if $x > 1$ 2

2(a) $f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{1}\right)$, $l = 1$

$$b_n = \frac{2}{1}\int_0^1 (2x-1)\sin n\pi x \, dx$$

$$\boxed{b_n = \frac{-2}{n\pi}\left[(-1)^n + 1\right]} \quad \text{— 4}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2}{n\pi}\left[(-1)^n+1\right] \sin n\pi x \quad 1$$

$$= \frac{-2}{\pi}\left(\sin 2\pi x + \frac{1}{2}\sin 4\pi x + \cdots\right)$$

2(b) $\because f(x)$ is an even funct^n

hence $\boxed{b_n = 0}$ — 1

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{1}{2}\int_{-2}^{2} f(x)\,dx$$

$$= \frac{2}{2}\int_0^2 x\,dx = \left(\frac{x^2}{2}\right)_0^2 = 2 \quad 1$$

(2)

$$a_n = \frac{1}{2}\int_{-2}^{2} f(x)\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{2}{2}\int_0^2 x\cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{4}{n^2\pi^2}\left((-1)^n - 1\right) \quad (2)$$

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2}\left((-1)^n - 1\right)\cos\left(\frac{n\pi x}{2}\right)$$

(1)

3(a)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 dx \quad (2)$$

$$= \frac{\pi^2}{6}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \cos nx\, dx \quad (2.5)$$

$$= \frac{1}{n^2}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\left(\frac{\pi - x}{2}\right)^2 \sin nx\, dx \quad (2.5)$$

$$= 0$$

$$f(x) = \frac{\pi^2}{12} + \sum_{n=1}^{\infty} \frac{\cos nx}{n^2} \quad (3)$$

3(b)

$$u_n = \frac{1\cdot 3\cdot 5 \cdots (2n-1)}{2\cdot 4\cdot 6 \cdots (2n)} \cdot \frac{x^{2n+1}}{2n+1} \quad (1)$$

$$u_{n+1} = \frac{1\cdot 3\cdot 5 \cdots (2n-1)(2n+1)}{2\cdot 4\cdot 6 \cdots (2n)(2n+2)} \cdot \frac{x^{2n+3}}{2n+3} \quad (1)$$

$$\lim_{n\to\infty} \frac{u_n}{u_{n+1}} = \frac{1}{x^2} \quad (2)$$

Hence by Ratio test, $\sum u_n$ is convergent if $\frac{1}{x^2} > 1$ or $x^2 < 1$

divergent if $\frac{1}{x^2} < 1$ or $x^2 > 1$ (2)

test fail if $x^2 = 1$.

When $x^2 = 1$

$$\frac{u_n}{u_{n+1}} = \frac{(2n+2)(2n+3)}{(2n+1)^2}$$

$$\lim_{n\to\infty} n\left(\frac{u_n}{u_{n+1}} - 1\right) = \lim_{n\to\infty} n\left(\frac{(2n+2)(2n+3)}{(2n+1)^2} - 1\right)$$

$$= \frac{6}{4} = \frac{3}{2} > 1$$

By Rabbe's test, the series is convergent (4)

Hence, $\sum u_n$ is convergent if $x^2 \le 1$

divergent if $x^2 > 1$

Section-B

4(a) A.E $m^2 + 5m - 6 = 0$

$m = 1, -6$

$$C.F = c_1 e^{x} + c_2 e^{-6x} \quad (2)$$

$$P.I = \frac{1}{D^2+5D-6}\sin 3x + \frac{1}{D^2+5D-6}\cos 2x$$

$$= \frac{1}{-9+5D-6}\sin 3x + \frac{1}{-4+5D-6}\cos 2x$$

$$= \frac{(D-3)\sin 3x}{5(D^2-9)} + \frac{(D+2)\cos 2x}{5(D^2-4)}$$

$$= \frac{3\cos 3x - 3\sin 3x}{5(-9-9)} + \frac{-2\sin 2x + 2\cos 2x}{5(-4-4)}$$

$$= -\frac{\cos 3x - \sin 3x}{30} + \frac{1}{20}(\sin 2x - \cos 2x)$$

$y = C.F + P.I$ (3)

4(b) A.E $m^3-1=0$

$m=+1, \frac{-1\pm\sqrt{3}i}{2}$

$C.F = c_1e^{x} + e^{-x/2}\left(c_2\cos\frac{\sqrt{3}}{2}x + c_3\sin\frac{\sqrt{3}}{2}x\right)$ — 2

$P.I. = \frac{1}{D^3-1}(3x^4-2x^3)$

$= -(1-D^3)^{-1}(3x^4-2x^3)$

$= -[1+(D^3)+(D^3)^2+\dots](3x^4-2x^3)$

$= -[3x^4-2x^3+D^3(3x^4-2x^3)]$

$= -(3x^4-2x^3+72x-12)$ — 3

$y = C.F + P.I$

5(a) $Dx+2y=-\sin t$ — (1)

$-2x+Dy=\cos t$ — (2)

Multiplying (1) by D and (2) by 2

$D^2x+2Dy=-D\sin t$

$-4x+2Dy=2\cos t$

$(D^2+4)x = -D\sin t - 2\cos t$

$(D^2+4)x = -\cos t - 2\cos t$

$(D^2+4)x = -3\cos t$ — 1

A.E $m^2+4=0 \Rightarrow m=\pm 2i$

$C.F = c_1\cos 2t + c_2\sin 2t$ — 1

$P.I = -3\left(\frac{1}{D^2+4}\cos t\right)$

$= -3\left(\frac{1}{-1^2+4}\cos t\right)$

$P.I = -\cos t$ (3)

$x = C.F + P.I$ — 2

$\boxed{x = c_1\cos 2t + c_2\sin 2t - \cos t}$

$\frac{dx}{dt} = -2c_1\sin 2t + 2c_2\cos 2t + \sin t$

$y = -\frac{1}{2}\left[\frac{dx}{dt}+\sin t\right]$

$\boxed{y = c_1\sin 2t - c_2\cos 2t - \sin t}$ — 1

5(b) A.E $m^2-2m+1=0$

$m=1,1$

$C.f = (c_1+c_2x)e^{x}$ — 2

$P.I = \frac{1}{D^2-2D+1}e^{x}x\sin x$

$= e^{x}\left[\frac{1}{(D+1)^2-2(D+1)+1}x\sin x\right]$

$= e^{x}\left[\frac{1}{D^2}x\sin x\right]$

$= e^{x}\cdot\frac{1}{D}(-x\cos x+\sin x)$

$= -e^{x}(x\sin x + 2\cos x)$

$y = C.f + P.I$

$y = (c_1+c_2x)e^{x} - e^{x}(x\sin x + 2\cos x)$ — 3