Curve Tracing :- Finding approximate shapes of the curves from their cartesian or polar eq^ns.

Working process :-

1. Symmetry : 1. If $f(x,-y) = f(x,y)$, curve is symmetrical about x axis.

2. If $f(-x,y) = f(x,y)$, curve is symmetrical about y axis.

3. If $f(y,x) = f(x,y)$, curve is symmetrical about line $y=x$

4. If $f(-y,-x) = f(x,y)$, curve is symmetrical about line $y=-x$

5. If $f(-x,-y) = f(x,y)$, curve is symmetrical in opposite quadrants.

2. Origin :- We check whether curve passes through origin or not. If constant term is missing the curve passes through origin or $f(0,0)=0$

\# If curve passes through origin.

3. Tangent at origin :- Equating lowest degree terms $=0$ we get eq^n of tangent at origin.

4. Asymptotes :- At this step we find asymptotes of the curve.

1) $\parallel$ to x axis.
2) $\parallel$ to y axis
3) oblique asymptotes.

5. Point of Intersection :- put $x=0$ & calculate y
put $y=0$ & calculate x.

6. Nature of curve / Region :- We look for values which are possible for the given eq^n.

7. Construct the Table :- In this step we take random values in the region to plot the curve

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Q.1 Trace the curve $y^2(2a-x) = x^3$ [Cissoid].

Sol. $f(x,y) = 2ay^2 - xy^2 - x^3 = 0$.

Symmetry ① $f(-x,y) \neq f(x,y)$ ∴ curve is not symmetrical on y axis

2. $f(x,-y) = f(x,y) \Rightarrow$ curve is symmetrical about x axis.

3. $f(y,x) \neq f(x,y)$ 4. $f(-y,-x) \neq f(x,y)$

Origin The curve does not contain any constant term. ∴ curve passes through $O(0,0)$

Tangent at origin :- The lowest degree term $2ay^2 = 0$

$y = 0, 0$ is tangent to the given curve. $y=0$ is double point
∴ cusp to the curve.

Asymptotes 1) $\parallel$ to x-axis $\quad -1 \neq 0 \therefore$ No asymptote $\parallel$ to x axis.

2) $\parallel$ to y axis. $\quad 2a - x = 0 \quad x = 2a$ is asymptote.

3) oblique. $\quad \phi_3(m) = -m^2 - 1 = 0 \quad m = \pm i$ No real asymptote.

$\Rightarrow x = 2a$ is the only asymptote for the curve.

Point of Intersection $\quad$ put $y = 0$. $\quad x = 0$.

put $x = 0$, we get $y = 0$.

$(0,0)$ is point of intersection.

Nature of the curve

$$y^2 = \frac{x^3}{2a-x} \qquad = \pm x\sqrt{\frac{x}{2a-x}}.$$

If $x > 2a$ $\quad$ y is imaginary.

$x = 2a$ $\quad$ y does not exist

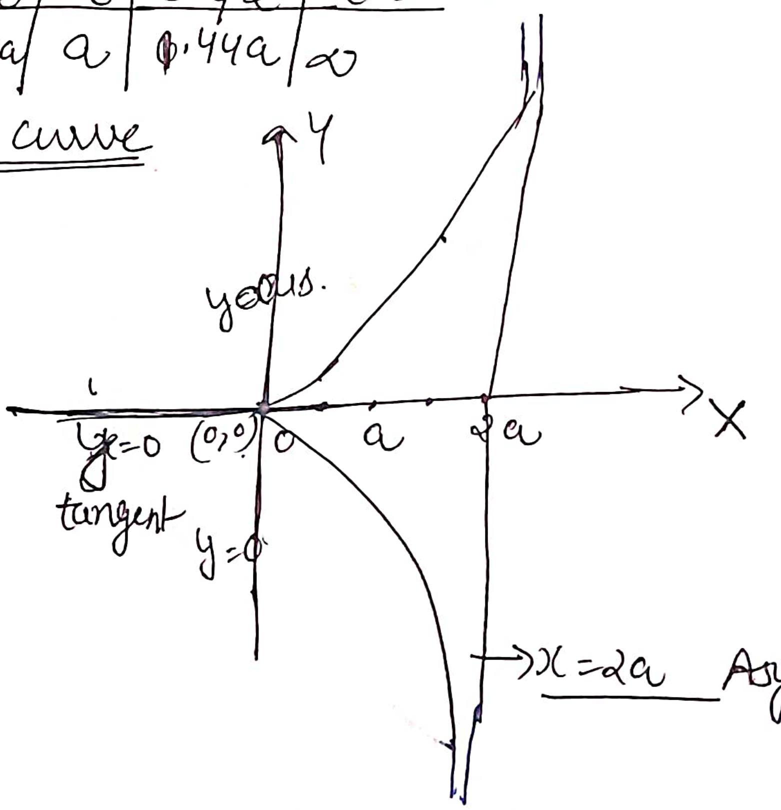
$x < 2a$ $\quad$ y is real.

$\therefore$ x varies from 0 to $2a$.

Construction of Table

x	0	$a/2$	a	$3a/2$	$2a$
y	0	$0.28a$	a	$0.44a$	∞

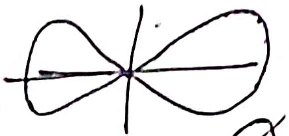
plot the curve



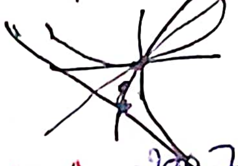
Symmetrical about x-axis

Problems for practice:

1) Trace the curve $y^2 = x^2 - x^4$ →



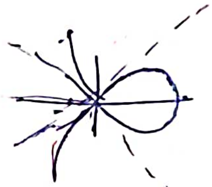
2) Trace the curve $x^3 + y^3 = 3axy$



3) Trace the curve $y^2(a-x) = x^2(a+x)$. 2007

4) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$, [2010-11]
write symmetry

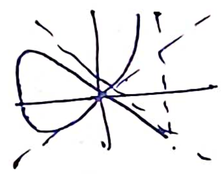
5) $ay^2 = x^2(a-x)$, 2009, 14.



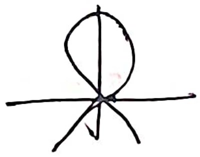
6) $y^2(x^2+y^2) + a^2(x^2-y^2) = 0$ 2010

7) $y = x^3$ 2014, 13.

8) $y^2(2a-x) = x^3$. Cissoid. 2015 Q3



9) $a^2x^2 = y^2(2a-y)$



$x\sqrt{\left(\frac{a-x}{a}\right)}$

$x < 0$

$y = x\sqrt{1-x^2}$