ABES ENGINEERING COLLEGE, GHAZIABAD (032)

B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-2: Differential Calculus-I

Question Bank

1. If $u = f(r), where\ r^2 = x^2 + y^2$, $prove\ that\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

2. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$.

3. If $x^2 = au + bv$, $y^2 = au - bv$, $prove\ that\ \left(\frac{\partial u}{\partial x}\right)_y \cdot \left(\frac{\partial x}{\partial u}\right)_v = \frac{1}{2} = \left(\frac{\partial v}{\partial y}\right)_x \left(\frac{\partial y}{\partial v}\right)_u$.

4. If $u = \sin^{-1}\left(\frac{x^{\frac{1}{4}} + y^{\frac{1}{4}}}{x^{\frac{1}{6}} + y^{\frac{1}{6}}}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{1}{144} tan\, u\, (sec^2 u - 12)$

5. If $u = sin^{-1}\left(\frac{x^3+y^3+z^3}{ax+by+cz}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 2\tan u$.

6. If $u = sin^{-1}\left(\frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} + 3\tan u = 0$.

7. If $u = x^4y^2 sin^{-1}\left(\frac{x}{y}\right) + \log x - \log y$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 6x^4y^2 sin^{-1}\left(\frac{x}{y}\right)$.

8. If $u = f(2x - 3y, 3y - 4z, 4z - 2x)$,prove that $\frac{1}{2}\frac{\partial u}{\partial x} + \frac{1}{3}\frac{\partial u}{\partial y} + \frac{1}{4}\frac{\partial u}{\partial z} = 0$.

9. If $w = \sqrt{x^2 + y^2 + z^2}\ and\ x = \cos v, y = u \sin v, z = uv$ then prove that
$u\frac{\partial w}{\partial u} - v\frac{\partial w}{\partial v} = \frac{u}{\sqrt{1+v^2}}$.

10. If $u = x \log xy$, where $x^3 + y^3 + 3xy = 1$, find $\frac{du}{dx}$.

11. If $u = x^2 - y^2 + \sin yz$, where $y = e^x$ and $z = \log x$, find $\frac{du}{dx}$.

12. If $x + y = 2e^\theta \cos \emptyset$ and $x - y = 2ie^\theta \sin \emptyset$, show that $\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial \emptyset^2} = 4xy\frac{\partial^2 V}{\partial x \partial y}$.

13. Trace the curve $y^2(2a - x) = x^3$

14. Trace the curve $y^2(a + x) = x^2(3a - x)$

UNIT-3: Differential Calculus-II

Question Bank

1. If $x = r\sin\theta\cos\emptyset, y = r\sin\theta\, sin\,\emptyset\ , z = r\cos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\emptyset)} = r^2\sin\theta$ and find $\frac{\partial(r,\theta,\emptyset)}{\partial(x,y,z)}$
2. If $y_1 = \frac{x_2x_3}{x_1}, y_2 = \frac{x_1x_3}{x_2}, y_3 = \frac{x_1x_2}{x_3}$ then show that $\frac{\partial(y_1,\ y_2,\ y_3)}{\partial(x_1,\ x_2,\ x_3)} = 4.$
3. If $u = xyz, v = x^2 + y^2 + z^2,\ w = x + y + z$, find the Jacobian $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.
4. If $x + y + z = u, y + z = uv, z = uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.
5. If u, v, w be the roots of the equation$(x-a)^3 + (x-b)^3 + (x-c)^3 = 0$, then find $\frac{\partial(u,v,w)}{\partial(a,b,c)}$.
6. Verify the Chain rule for Jacobian if$x = u, y = u\tan v\,, z = w$.
7. If $u^3 + v^3 = x + y$and $u^2 + v^2 = x^3 + y^3$, show that $\frac{\partial(u,v)}{\partial(x,y)} = \frac{(y^2-x^2)}{2uv(u-v)}$
8. If $u = sin^{-1}x + sin^{-1}y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ find $\frac{\partial(u,v)}{\partial(x,y)}$, Is u, v functionally related. If so, find the relationship. Relationship is pending
9. The period T of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$. Find the maximum error in T due to possible errors up to 1% in *l* and *2.5* in g.
10. The time T of a complete oscillation of a simple pendulum of length *L* is governed by the equation $= 2\pi\sqrt{\frac{L}{g}}$, where g is a constant. Find the approximate error in the calculate value *T* corresponding to an error of *2%* in the value of *L* .
11. The indicated horse-power I of an engine is calculated from the formula $= \frac{PLAN}{33000}$, where $A = \frac{\pi}{4}d^2$. Assuming the error of r percent may have been made in measuring *P, L, N* and *d*. Find the greatest possible error in I.
12. In estimating the cost of pile of bricks measured as $6m \times 50m \times 4m$, the tape is stretched 1% beyond the standard length. If the count is 12 bricks in $1m^3$ and bricks cost Rs. 100per 1000, find the approximate error in the cost.
13. Find the possible percentage error in computing the parallel resistance r of three resistances r_1, r_2, r_3 from the formula $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$ if r_1, r_2, r_3 are each in error by 1.2%.
14. Compute an approximate value of $(1.04)^{3.01}$.
15. Find an approximate value of $\left[(0.98)^2 + (2.01)^2 + (1.94)^2\right]^{\frac{1}{2}}$.

16. Find the extreme value of the function $x^3 + y^3 - 3axy$.

17. Show that the rectangular solid of maximum volume that can be inscribe in a given sphere is a cube.

18. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

19. Find the maximum and minimum distance of the point *(3, 4, 12)* from the sphere $x^2 + y^2 + z^2 = 1$.

20. Find the dimension of a rectangular box of maximum capacity whose surface area is given by when (i) box is open of the top, (ii) box is closed.

21. Expand $e^x \sin y$ in power of *x* and *y* as far as term of the third degree.

22. Expand $tan^{-1} \frac{y}{x}$ in the neighborhood of (1, 1) up to and inclusive of second degree terms. Hence compute $f(1.1, 0.9)$ approximately.

23. Expand x^y in power of (*x-1*) and (*y-1*) up to the third degree term and hence evaluate $(1.1)^{1.02}$.

24. Expand $e^x \cos y$ near the point $\left(1, \frac{\pi}{4}\right)$ by Taylor's theorem.

25. Expand $Sinxy$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ up to second degree terms.

26. Expand $f(x, y) = x^2y + 3y - 2$ in power of (*x-1*) and (*y+2*) by Taylor's theorem.

Answers

UNIT-2: Differential Calculus-I

10. $1 + \log(xy) - \frac{x(x^2+y)}{y(y^2+x)}$.

11. $2(x - e^{2x}) + e^x \cos(e^x logx)(logx + \frac{1}{x})$

UNIT-3: Differential Calculus-II

1. $\frac{1}{r^2 sin\theta}$

3. $-2(x-y)(y-z)(z-x)$

5. $\frac{-2(a-b)(b-c)(c-a)}{(u-v)(v-w)(w-u)}$

8. $\frac{\partial(u,v)}{\partial(x,y)}$=0, relation $v = sinu$

10. 1%

11. 5r%

12. 43.20 Rs

13. 1.2 %

14. 1.12

15. 2.96

16. Maximum value =a^3 , Minimum value = -a^3

18. $\frac{8abc}{3\sqrt{3}}$

19. Maximum distance = 14, Minimum distance = 12

20. (i) $x = y = \sqrt{\frac{S}{3}}$, $z = 1/2\sqrt{\frac{S}{3}}$ (ii) $x = y = z = \sqrt{\frac{S}{6}}$

21. $y + xy + \frac{1}{2} x^2 y - \frac{1}{6} y^3 + \cdots$ ...

22. 0.6857

23. 1.1021

24. $\frac{e}{\sqrt{2}}\{1 + (x-1) - \left(y - \frac{\pi}{4}\right) + \frac{(x-1)^2}{2} - (x-1)\left(y - \frac{\pi}{4}\right) + \cdots.\}$

25. $1 - \frac{\pi^2}{8}\frac{(x-1)^2}{} - \frac{\pi}{2}(x-1)\left(y - \frac{\pi}{2}\right) - \frac{1}{2}(y - \frac{\pi}{2})^2 \ldots.\}$

26. $-10 - 4(x-1) + 4(y+2) - 2(x-1)^2 + 2(x-1)(y+2) + (x-1)^2(y+2)$