Revised Edition

A Textbook of Refrigeration and Air Conditioning

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A TEXTBOOK OF

REFRIGERATION AND AIR CONDITIONING

[For the Students of B.E.; UPSC (Engg. Services); UPSC (Civil Services); Section 'B' of AMIE (India) and Diploma Courses]

(S.I. UNITS)

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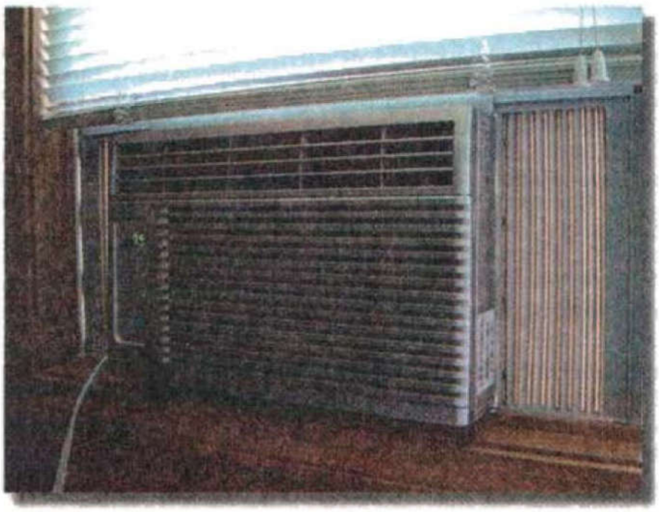

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CHAPTER

2

Air Refrigeration Cycles

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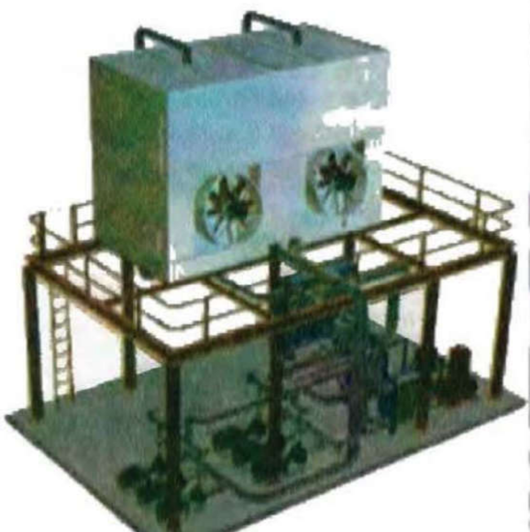
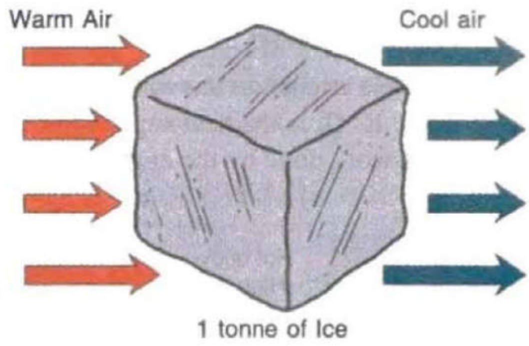


2.1 Introduction

In an air refrigeration cycle, the air is used as a refrigerant. In olden days, air was widely used in commercial applications because of its availability at free of cost. Since air does not change its phase *i.e.* remains gaseous throughout the cycle, therefore the heat carrying capacity per kg of air is very small as compared to vapour absorbing systems. The air-cycle refrigeration systems, as originally designed and installed, are now practically obsolete because of their low coefficient of performance and high power requirements. However, this system continues to be favoured for air refrigeration because of the low weight and volume of the equipment. The basic elements of an air cycle refrigeration system are the compressor, the cooler or heat exchanger, the expander and the refrigerator.

Before discussing the air refrigeration cycles, we should first know about the unit of refrigeration, coefficient of performance of a refrigerator and the difference between the heat engine, a refrigerator and a heat pump.



One tonne (1000 kg) of ice requires 335 kJ/kg to melt. When this is accomplished in 24 hours, it is known as a heat transfer rate of 1 tonne of refrigeration (1TR).

2.2 Units of Refrigeration

The practical unit of refrigeration is expressed in terms of 'tonne of refrigeration' (briefly written as TR). A *tonne of refrigeration* is defined as the amount of refrigeration effect produced by the uniform melting of one tonne (1000 kg) of ice from and at 0°C in 24 hours.

Since the latent heat of ice is 335 kJ/kg, therefore one tonne of refrigeration,

$$1\text{TR} = 1000 \times 335 \text{ kJ in 24 hours}$$

$$= \frac{1000 \times 335}{24 \times 60} = 232.6 \text{ kJ/min}$$

In actual practice, one tonne of refrigeration is taken as equivalent to 210 kJ/min or 3.5 kW (i.e. 3.5 kJ/s).

2.3 Coefficient of Performance of a Refrigerator

The coefficient of performance (briefly written as C.O.P.) is the ratio of heat extracted in the refrigerator to the work done on the refrigerant. It is also known as theoretical coefficient of performance. Mathematically,

$$\text{Theoretical C.O.P.} = \frac{Q}{W}$$

where Q = Amount of heat extracted in the refrigerator (or the amount of refrigeration produced, or the capacity of a refrigerator), and

W = Amount of work done.

Notes: 1. For per unit mass, $\text{C.O.P.} = \frac{q}{w}$

2. The coefficient of performance is the reciprocal of the efficiency (*i.e.* $1/\eta$) of a heat engine. It is thus obvious, that the value of C.O.P is always greater than unity.

3. The ratio of the actual C.O.P to the theoretical C.O.P. is known as *relative coefficient of performance*. Mathematically,

$$\text{Relative C.O.P.} = \frac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$$

Example 2.1. *Find the C.O.P. of a refrigeration system if the work input is 80 kJ/kg and refrigeration effect produced is 160 kJ/kg of refrigerant flowing.*

Solution. Given : $w = 80$ kJ/kg ; $q = 160$ kJ/kg

We know that C.O.P. of a refrigeration system

$$= \frac{q}{w} = \frac{160}{80} = 2 \textbf{ Ans.}$$

2.4 Difference Between a Heat Engine, Refrigerator and Heat Pump

In a heat engine, as shown in Fig. 2.1 (*a*), the heat supplied to the engine is converted into useful work. If Q_2 is the heat supplied to the engine and Q_1 is the heat rejected from the engine, then the net work done by the engine is given by

$$W_E = Q_2 - Q_1$$

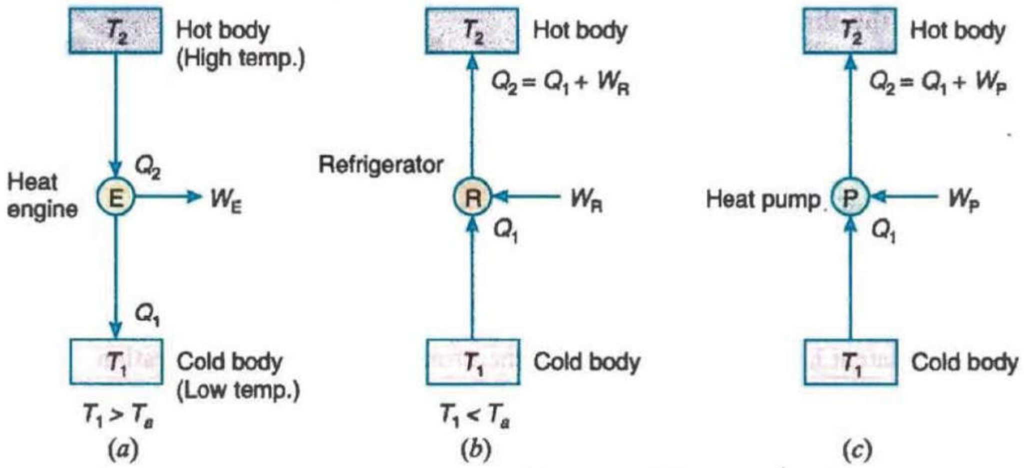


Fig. 2.1. Difference between a heat engine, refrigerator and heat pump.

The performance of a heat engine is expressed by its efficiency. We know that the efficiency or coefficient of performance of an engine,

$$\eta_E \text{ or } (\text{C.O.P.})_E = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W_E}{Q_2} = \frac{Q_2 - Q_1}{Q_2}$$

A refrigerator as shown in Fig. 2.1 (*b*), is a reversed heat engine which either cool or maintain the temperature of a body (T_1) lower than the atmospheric temperature (T_a). This is done by extracting the heat (Q_1) from a cold body and delivering it to a hot body (Q_2). In doing so, work W_R is required to be done on the system. According to First Law of Thermodynamics,

$$W_R = Q_2 - Q_1$$

The performance of a refrigerator is expressed by the ratio of amount of heat taken from the cold body (Q_1) to the amount of work required to be done on the system (W_R). This ratio is called coefficient of performance. Mathematically, coefficient of performance of a refrigerator,

$$(\text{C.O.P.})_R = \frac{Q_1}{W_R} = \frac{Q_1}{Q_2 - Q_1}$$

Any refrigerating system is a heat pump as shown in Fig. 2.1 (*c*), which extracts heat (Q_1) from a cold body and delivers it to a hot body. Thus there is no difference between the cycle of operations of a heat pump and a refrigerator. The main difference between the two is in their operating temperatures. A refrigerator works between the cold body temperature (T_1) and the atmospheric temperature (T_a) whereas the heat pump operates between the hot body temperature (T_2) and the atmospheric temperature (T_a). A refrigerator used for cooling in summer can be used as a heat pump for heating in winter.

In the similar way, as discussed for refrigerator, we have

$$W_P = Q_2 - Q_1$$

The performance of a heat pump is expressed by the ratio of the amount of heat delivered to the hot body (Q_2) to the amount of work required to be done on the system (W_P). This ratio is called coefficient of performance or energy performance ratio (E.P.R.) of a heat pump. Mathematically, coefficient of performance or energy performance ratio of a heat pump,

$$(\text{C.O.P.})_P \text{ or E.P.R.} = \frac{Q_2}{W_P} = \frac{Q_2}{Q_2 - Q_1}$$

$$= \frac{Q_1}{Q_2 - Q_1} + 1 = (\text{C.O.P.})_R + 1$$

From above we see that the C.O.P. may be less than one or greater than one depending on the type of refrigeration system used. But the C.O.P. of a heat pump is always greater than one.

2.5 Open Air Refrigeration Cycle

In an open air refrigeration cycle, the air is directly led to the space to be cooled (*i.e.* a refrigerator), allowed to circulate through the cooler and then returned to the compressor to start another cycle. Since the air is supplied to the refrigerator at atmospheric pressure, therefore, volume of air handled by the compressor and expander is large. Thus the size of compressor and expander should be large. Another disadvantage of the open cycle system is that the moisture is regularly carried away by the air circulated through the cooled space. This leads to the formation of frost at the end of expansion process and clog the line. Thus in an open cycle system, a drier should be used.

2.6 Closed or Dense Air Refrigeration Cycle

In a closed or dense air refrigeration cycle, the air is passed through the pipes and component parts of the system at all times. The air, in this system, is used for absorbing heat from the other fluid (say brine) and this cooled brine is circulated into the space to be cooled. The air in the closed system does not come in contact directly with the space to be cooled.

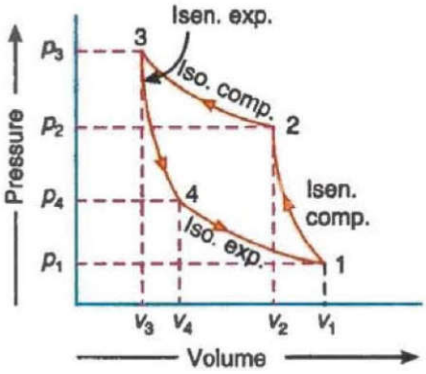
The closed air refrigeration cycle has the following thermodynamic advantages :

1. Since it can work at a suction pressure higher than that of atmospheric pressure, therefore the volume of air handled by the compressor and expander are smaller as compared to an open air refrigeration cycle system.
2. The operating pressure ratio can be reduced, which results in higher coefficient of performance.

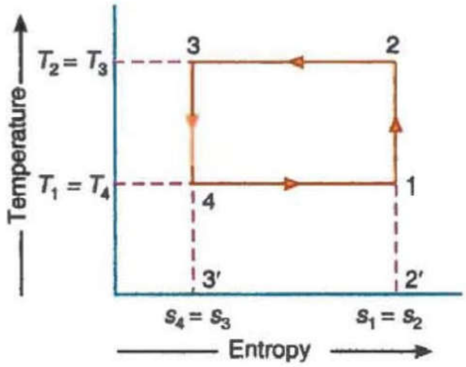
2.7 Air Refrigerator Working on Reversed Carnot Cycle

In refrigerating systems, the Carnot cycle considered is the reversed Carnot cycle. We know that a heat engine working on Carnot cycle has the highest possible efficiency. Similarly, a refrigerating system working on the reversed Carnot cycle, will have the maximum possible coefficient of performance. We also know that it is not possible to make an engine working on the Carnot cycle. Similarly, it is also not possible to make a refrigerating machine working on the reversed Carnot cycle. However, it is used as the ultimate standard of comparison.

A reversed Carnot cycle, using air as working medium (or refrigerant) is shown on *p-v* and *T-s* diagrams in Fig. 2.2(*a*) and (*b*) respectively. At point 1, let p_1, v_1, T_1 be the pressure, volume and temperature of air respectively.



(a) p-v diagram.



(b) T-s diagram.

Fig. 2.2. Reversed Carnot cycle.

The four processes of the cycle are as follows :

1. *Isentropic compression process.* The air is compressed isentropically as shown by the curve 1-2 on *p-v* and *T-s* diagrams. During this process, the pressure of air increases from p_1 to p_2, specific volume decreases from v_1 to v_2 and temperature increases from T_1 to T_2. We know that during isentropic compression, no heat is absorbed or rejected by the air.

2. *Isothermal compression process.* The air is now compressed isothermally (*i.e.* at constant temperature, $T_2 = T_3$) as shown by the curve 2-3 on *p-v* and *T-s* diagrams. During this process, the pressure of air increases from p_2 to p_3 and specific volume decreases from v_2 to v_3. We know that the heat rejected by the air during isothermal compression per kg of air,

$$q_R = q_{2-3} = \text{Area } 2\text{–}3\text{–}3'\text{–}2'$$
$$= T_3(s_2 - s_3) = T_2(s_2 - s_3)$$

3. *Isentropic expansion process.* The air is now expanded isentropically as shown by the curve 3-4 on *p-v* and *T-s* diagrams. The pressure of air decreases from p_3 to p_4, specific volume increases from v_3 to v_4 and the temperature decreases from T_3 to T_4. We know that during isentropic expansion, no heat is absorbed or rejected by the air.

4. *Isothermal expansion process.* The air is now expanded isothermally (*i.e.* at constant temperature, $T_4 = T_1$) as shown by the curve 4-1 on *p-v* and *T-s* diagrams. The pressure of air decreases from p_4 to p_1, and specific volume increases from v_4 to v_1. We know that the heat absorbed by the air (or heat extracted from the cold body) during isothermal expansion per kg of air,

$$q_A = q_{4-1} = \text{Area } 4\text{–}1\text{–}2'\text{–}3'$$
$$= T_4(s_1 - s_4) = T_4(s_2 - s_3) = T_1(s_2 - s_3)$$

We know that work done during the cycle per kg of air

$$w_R = {}^*\text{Heat rejected} - \text{Heat absorbed} = q_R - q_A = q_{2-3} - q_{4-1}$$
$$= T_2(s_2 - s_3) - T_1(s_2 - s_3) = (T_2 - T_1)(s_2 - s_3)$$

∴ Coefficient of performance of the refrigeration system working on reversed Carnot cycle,

$$(\text{C.O.P.})_R = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{q_R - q_A} = \frac{q_{4-1}}{q_{2-3} - q_{4-1}}$$

* In a refrigerating machine, heat rejected is more than heat absorbed.

$$= \frac{T_1(s_2 - s_3)}{(T_2 - T_1)(s_2 - s_3)} = \frac{T_1}{T_2 - T_1}$$

Though the reversed Carnot cycle is the most efficient between the fixed temperature limits, yet no refrigerator has been made using this cycle. This is due to the reason that the isentropic processes of the cycle require high speed while the isothermal processes require an extremely low speed. This variation in speed of air is not practicable.

Note : We have already discussed that C.O.P. of a heat pump,

$$(\text{C.O.P.})_P = (\text{C.O.P.})_R + 1 = \frac{T_1}{T_2 - T_1} + 1 = \frac{T_2}{T_2 - T_1}$$

and C.O.P. or efficiency of a heat engine,

$$(\text{C.O.P.})_E = \frac{w_R}{q_R} = \frac{(T_2 - T_1)(s_2 - s_3)}{T_2(s_2 - s_3)} = \frac{T_2 - T_1}{T_2} = \frac{1}{(\text{C.O.P.})_P}$$

2.8 Temperature Limitations for Reversed Carnot Cycle

We have seen in the previous article that the C.O.P. of the refrigeration system working on reversed Carnot cycle is given by

$$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1}$$

where T_1 = Lower temperature, and

T_2 = Higher temperature.

The C.O.P. of the reversed Carnot cycle may be improved by

1. decreasing the higher temperature (*i.e.* temperature of hot body, T_2), or
2. increasing the lower temperature (*i.e.* temperature of cold body, T_1).

This applies to all refrigerating machines, both theoretical and practical. It may be noted that temperatures T_1 and T_2 cannot be varied at will, due to certain functional limitations. It should be kept in mind that the higher temperature (T_2) is the temperature of cooling water or air available for rejection of heat and the lower temperature (T_1) is the temperature to be maintained in the refrigerator. The heat transfer will take place in the right direction only when the higher temperature is more than the temperature of cooling water or air to which heat is to be rejected, while the lower temperature must be less than the temperature of substance to be cooled.

Domestic Air Conditioner.

Thus, if the temperature of cooling water or air (*i.e.* T_2) available for heat rejection is low, the C.O.P. of the Carnot refrigerator will be high. Since T_2 in winter is less than T_2 in summer, therefore, C.O.P. in winter will be higher than C.O.P. in summer. In other words, the Carnot refrigerators work more efficiently in winter than in summer. Similarly, if the lower temperature fixed by the refrigeration application is high, the C.O.P. of the Carnot refrigerator will be high. Thus a Carnot refrigerator used for making ice at 0°C (273 K) will have less C.O.P. than a Carnot refrigerator used for air-conditioned plant in summer at 20°C when the atmospheric temperature is 40°C. In other words, we can say that the Carnot C.O.P. of a domestic refrigerator is less than the Carnot C.O.P. of a domestic air-conditioner.

Example 2.2. *A machine working on a Carnot cycle operates between 305 K and 260 K. Determine the C.O.P. when it is operated as: 1. a refrigerating machine; 2. a heat pump; and 3. a heat engine.*

Solution. Given : $T_2 = 305$ K ; $T_1 = 260$ K

1. C.O.P. of a refrigerating machine

We know that C.O.P. of a refrigerating machine,

$$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1} = \frac{260}{305 - 260} = 5.78 \textbf{ Ans.}$$

2. C.O.P. of a heat pump

*We know that C.O.P. of a heat pump,

$$(\text{C.O.P.})_P = \frac{T_2}{T_2 - T_1} = \frac{305}{305 - 260} = 6.78 \textbf{ Ans.}$$

3. C.O.P. of a heat engine

**We know that C.O.P. of a heat engine,

$$(\text{C.O.P.})_E = \frac{T_2 - T_1}{T_2} = \frac{305 - 260}{305} = 0.147 \textbf{ Ans.}$$

Example 2.3. *A Carnot refrigeration cycle absorbs heat at 270 K and rejects it at 300 K.*

1. *Calculate the coefficient of performance of this refrigeration cycle.*
2. *If the cycle is absorbing 1130 kJ/min at 270 K, how many kJ of work is required per second ?*
3. *If the Carnot heat pump operates between the same temperatures as the above refrigeration cycle, what is the coefficient of performance ?*
4. *How many kJ/min will the heat pump deliver at 300 K if it absorbs 1130 kJ/min at 270 K.*

Solution. Given : $T_1 = 270$ K ; $T_2 = 300$ K

1. Coefficient of performance of Carnot refrigeration cycle

We know that coefficient of performance of Carnot refrigeration cycle,

$$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1} = \frac{270}{300 - 270} = 9 \textbf{ Ans.}$$

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* We know that C.O.P. of a heat pump, $(\text{C.O.P.})_P = (\text{C.O.P.})_R + 1 = 5.78 + 1 = 6.78$ **Ans.**

** We know that C.O.P. of a heat engine, $(\text{C.O.P.})_E = \dfrac{1}{(\text{C.O.P.})_P} = \dfrac{1}{6.78} = 0.147$ **Ans.**

2. Work required per second

Let W_R = Work required per second.

Heat absorbed at 270 K (*i.e.* T_1),

$$Q_1 = 1130 \text{ kJ/min} = 18.83 \text{ kJ/s} \qquad \text{...(Given)}$$

We know that $$(\text{C.O.P.})_R = \frac{Q_1}{W_R} \quad \text{or} \quad 9 = \frac{18.83}{W_R}$$

$$\therefore \quad W_R = 2.1 \text{ kJ/s} \textbf{ Ans.}$$

3. Coefficient of performance of Carnot heat pump

We know that coefficient of performance of a Carnot heat pump,

$$(\text{C.O.P.})_P = \frac{T_2}{T_2 - T_1} = \frac{300}{300 - 270} = 10 \textbf{ Ans.}$$

4. Heat delivered by heat pump at 300 K

Let Q_2 = Heat delivered by heat pump at 300 K.

Heat absorbed at 270 K (*i.e.* T_1),

$$Q_1 = 1130 \text{ kJ/min} \qquad \text{... (Given)}$$

We know that

$$(\text{C.O.P.})_P = \frac{Q_2}{Q_2 - Q_1} \quad \text{or} \quad 10 = \frac{Q_2}{Q_2 - 1130}$$

$$\therefore \quad 10Q_2 - 11\,300 = Q_2 \quad \text{or} \quad Q_2 = 1256 \text{ kJ/min} \textbf{ Ans.}$$

Example 2.4. *A cold storage is to be maintained at –5°C while the surroundings are at 35°C. The heat leakage from the surroundings into the cold storage is estimated to be 29 kW. The actual C.O.P. of the refrigeration plant is one-third of an ideal plant working between the same temperatures. Find the power required to drive the plant.*

Solution. Given : $T_1 = -5°C = -5 + 273 = 268$ K; $T_2 = 35°C = 35 + 273 = 308$ K ; $Q_1 = 29$ kW ;

$$(\text{C.O.P.})_{actual} = \frac{1}{3}(\text{C.O.P.})_{ideal}$$

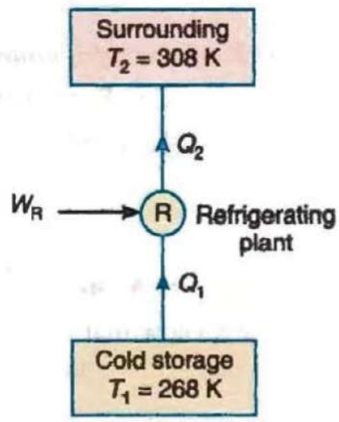


Fig. 2.3

The refrigerating plant operating between the temperatures T_1 and T_2 is shown in Fig. 2.3.

Let W_R = Work or power required to drive the plant.

We know that the coefficient of performance of an ideal refrigeration plant,

$$(\text{C.O.P.})_{ideal} = \frac{T_1}{T_2 - T_1} = \frac{268}{308 - 268} = 6.7$$

$\therefore$ Actual coefficient of performance,

$$(\text{C.O.P.})_{actual} = \frac{1}{3}(\text{C.O.P.})_{ideal} = \frac{1}{3} \times 6.7 = 2.233$$

We also know that $$(\text{C.O.P.})_{actual} = \frac{Q_1}{W_R}$$

$$\therefore \quad W_R = \frac{Q_1}{(\text{C.O.P.})_{actual}} = \frac{29}{2.233} = 12.987 \text{ kW} \textbf{ Ans.}$$

Example 2.5. *Two refrigerators A and B operate in series. The refrigerator A absorbs energy at the rate of 1kJ/s from a body at temperature 300 K and rejects energy as heat to a body at temperature T. The refrigerator B absorbs the same quantity of energy which is rejected by the refrigerator A from the body at temperature T, and rejects energy as heat to a body at temperature 1000 K. If both the refrigerators have the same C.O.P., calculate:*

1. The temperature T of the body;

2. The C.O.P. of the refrigerators; and

3. The rate at which energy is rejected as heat to the body at 1000 K.

Solution. Given : $Q_1 = 1$ kJ/s ; $T_1 = 300$ K ; $T_2 = T$; $T_3 = 1000$ K

The arrangement of the refrigerators *A* and *B* is shown in Fig. 2.4.

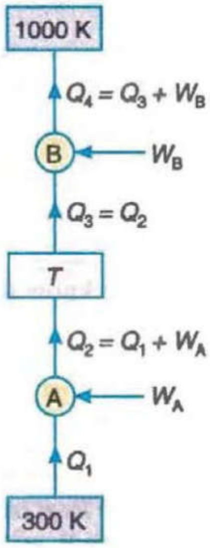


Fig. 2.4

1. *Temperature T of the body*

We know that C.O.P. for refrigerator *A*,

$$(\text{C.O.P.})_A = \frac{T_1}{T_2 - T_1} = \frac{300}{T - 300} \qquad ...(i)$$

and C.O.P. for refrigerator *B*,

$$(\text{C.O.P.})_B = \frac{T_2}{T_3 - T_2} = \frac{T}{1000 - T} \qquad ...(ii)$$

Since C.O.P. of both the refrigerators is same, therefore equating equations (*i*) and (*ii*),

$$\frac{300}{T - 300} = \frac{T}{1000 - T}$$

or
$$300 \times 1000 - 300\,T = T^2 - 300\,T$$

$$\therefore \qquad T = \sqrt{300 \times 1000} = 547.7 \text{ K} \quad \textbf{Ans.}$$

2. *C.O.P. of the refrigerators*

Since C.O.P. of both the refrigerators is same, therefore substituting the value of *T* in equation (*i*) or equation (*ii*),

$$(\text{C.O.P.})_A = (\text{C.O.P.})_B = \frac{300}{547.7 - 300} = 1.21 \quad \textbf{Ans.}$$

3. *Rate at which energy is rejected as heat to the body at 1000 K*

We know that work done by refrigerator *A*,

$$W_A = \frac{Q_1}{(\text{C.O.P.})_A} = \frac{1}{1.21} = 0.826 \text{ kJ/s}$$

and heat rejected by refrigerator *A*,

$$Q_2 = Q_1 + W_A = 1 + 0.826 = 1.826 \text{ kJ/s}$$

Now workdone by refrigerator *B*,

$$W_B = \frac{Q_3}{(\text{C.O.P.})_B} = \frac{1.826}{1.21} = 1.51 \text{ kJ/s} \qquad ... (\because Q_3 = Q_2)$$

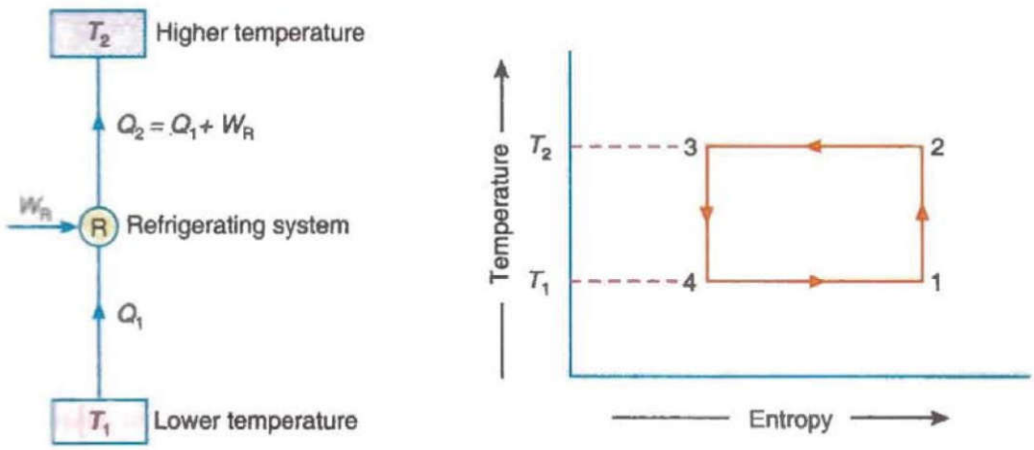
$\therefore$ Heat rejected to the body at 1000 K,

$$Q_4 = Q_3 + W_B = 1.826 + 1.51 = 3.336 \text{ kJ/s} \quad \textbf{Ans.}$$

Example 2.6. *A refrigerating system operates on the reversed Carnot cycle. The higher temperature of the refrigerant in the system is 35°C and the lower temperature is – 15°C. The capacity is to be 12 tonnes. Determine : 1. C.O.P. ; 2. Heat rejected from the system per hour ; and 3. Power required.*

Solution. Given : $T_2 = 35°C = 35 + 273 = 308$ K ; $T_1 = -15°C = -15 + 273 = 258$ K ; $Q_1 = 12$ TR $= 12 \times 210 = 2520$ kJ/min

The refrigerating system operating on the reversed Carnot cycle is shown in Fig. 2.5.



(*a*) (*b*)

Fig. 2.5

1. C.O.P.

We know that

$$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1} = \frac{258}{308 - 258} = 5.16 \textbf{ Ans.}$$

2. Heat rejected from the system per hour

Let W_R = Work or power required to drive the system.

We know that $(\text{C.O.P.})_R = \dfrac{Q_1}{W_R}$

$$\therefore \quad W_R = \frac{Q_1}{(\text{C.O.P.})_R} = \frac{2520}{5.16} = 488.37 \text{ kJ/min}$$

and heat rejected from the system,

$$Q_2 = Q_1 + W_R = 2520 + 488.37 = 3008.37 \text{ kJ/min}$$
$$= 3008.37 \times 60 = 180\,502.2 \text{ kJ/h} \textbf{ Ans.}$$

3. Power required

We know that work or power required,

$$W_R = 488.37 \text{ kJ/min} = \frac{488.37}{60} = 8.14 \text{ kJ/s or kW} \textbf{ Ans.}$$

Example 2.7. *1.5 kW per tonne of refrigeration is required to maintain the temperature of – 40°C in the refrigerator. If the refrigeration cycle works on Carnot cycle, determine the following:*

1. C.O.P. of the cycle ; 2. Temperature of the sink ; 3. Heat rejected to the sink per tonne of refrigeration ; and 4. Heat supplied and E.P.R., if the cycle is used as a heat pump.

Solution. Given : $W_R = 1.5$ kW ; $Q_1 = 1$ TR ; $T_1 = -40°C = -40 + 273 = 233$ K

1. C.O.P. of the cycle

The refrigeration cycle working on Carnot cycle is shown in Fig 2.6.

Since 1.5 kW per tonne of refrigeration is required to maintain the temperature in the refrigerator, therefore amount of work required to be done,

$$W_R = 1.5 \text{ kW} = 1.5 \text{ kJ/s} = 1.5 \times 60 = 90 \text{ kJ/min}$$

and heat extracted from the cold body,

$$Q_1 = 1 \text{ TR} = 210 \text{ kJ/min}$$

We know that $$(\text{C.O.P.})_R = \frac{Q_1}{W_R} = \frac{210}{90} = 2.33 \text{ **Ans.**}$$

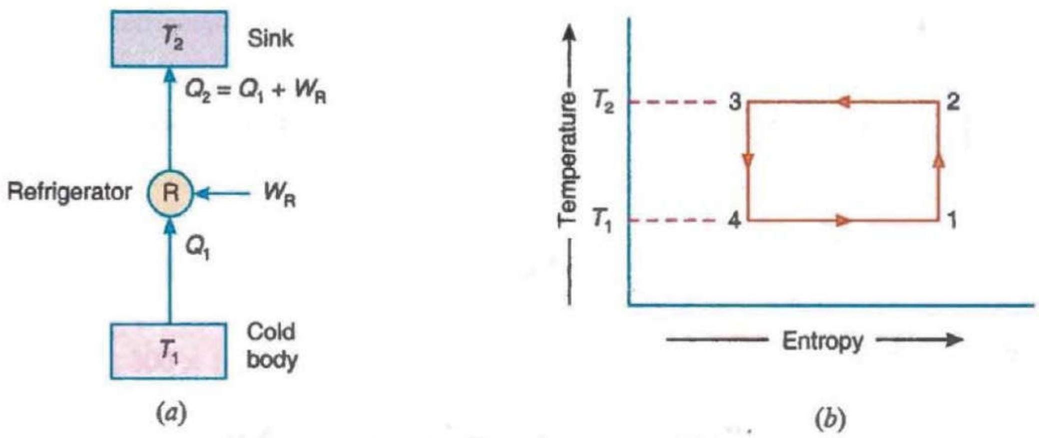


Fig. 2.6

2. Temperature of the sink

Let T_2 = Temperature of the sink.

We know that $$(\text{C.O.P.})_R = \frac{T_1}{T_2 - T_1} \quad \text{or} \quad 2.33 = \frac{233}{T_2 - 233}$$

$$\therefore \quad T_2 = \frac{233}{2.33} + 233 = 333 \text{ K} = 333 - 273 = 60°\text{C} \text{ **Ans.**}$$

3. Heat rejected to the sink per tonne of refrigeration

We know that heat rejected to the sink,

$$Q_2 = Q_1 + W_R = 210 + 90 = 300 \text{ kJ/min} \text{ **Ans.**}$$

4. Heat supplied and E.P.R., if the cycle is used as a heat pump

We know that heat supplied when the cycle is used as a heat pump is

$$Q_2 = 300 \text{ kJ/min} \text{ **Ans.**}$$

and $$\text{E.P.R.} = (\text{C.O.P.})_R + 1 = 2.33 + 1 = 3.33 \text{ **Ans.**}$$

Example 2.8. *The capacity of a refrigerator is 200 TR when working between – 6°C and 25°C. Determine the mass of ice produced per day from water at 25°C. Also find the power required to drive the unit. Assume that the cycle operates on reversed Carnot cycle and latent heat of ice is 335 kJ/kg.*

Solution. Given : $Q = 200$ TR ; $T_1 = -6°C = -6 + 273 = 267$ K ; $T_2 = 25°C = 25 + 273 = 298$ K ; $t_w = 25°C$; $h_{fg(ice)} = 335$ kJ/kg

Mass of ice produced per day

We know that heat extraction capacity of the refrigerator

$$= 200 \times 210 = 42\,000 \text{ kJ/min} \qquad \ldots (\because 1\text{TR} = 210 \text{ kJ/min})$$

and heat removed from 1 kg of water at 25°C to form ice at 0°C

$$= \text{Mass} \times \text{Sp. heat} \times \text{Rise in temperature} + h_{fg(ice)}$$

$$= 1 \times 4.187(25 - 0) + 335 = 439.7 \text{ kJ / kg}$$

∴ Mass of ice produced per min

$$= \frac{42\,000}{439.7} = 95.52 \text{ kg / min}$$

and mass of ice produced per day $= 95.52 \times 60 \times 24 = 137\,550$ kg $= 137.55$ tonnes **Ans.**

Power required to drive the unit

We know that C.O.P. of the reversed Carnot cycle

$$= \frac{T_1}{T_2 - T_1} = \frac{267}{298 - 267} = 8.6$$

Also
$$\text{C.O.P.} = \frac{\text{Heat extraction capacity}}{\text{Work done per min}}$$

∴
$$8.6 = \frac{42\,000}{\text{Work done per min}}$$

or
$$\text{Work done per min} = 42\,000 / 8.6 = 4884 \text{ kJ/min}$$

∴ Power required to drive the unit

$$= 4884 / 60 = 81.4 \text{ kW}$$ **Ans.**

Example 2.9. *Five hundred kgs of fruits are supplied to a cold storage at 20°C. The cold storage is maintained at –5°C and the fruits get cooled to the storage temperature in 10 hours. The latent heat of freezing is 105 kJ/kg and specific heat of fruit is 1.256 kJ/kg K. Find the refrigeration capacity of the plant.*

Solution. Given : $m = 500$ kg ; $T_2 = 20°C = 20 + 273 = 293$ K; $T_1 = -5°C = -5 + 273 = 268$ K ; $h_{fg} = 105$ kJ/kg ; $c_F = 1.256$ kJ/kg K

We know that heat removed from the fruits in 10 hours,

$$Q_1 = m\, c_F (T_2 - T_1)$$
$$= 500 \times 1.256\,(293 - 268) = 15\,700 \text{ kJ}$$

and total latent heat of freezing,

$$Q_2 = m \times h_{fg} = 500 \times 105 = 52\,500 \text{ kJ}$$

∴ Total heat removed in 10 hours,

$$Q = Q_1 + Q_2 = 15\,700 + 52\,500 = 68\,200 \text{ kJ}$$

and total heat removed in one minute

$$= 68\,200/10 \times 60 = 113.7 \text{ kJ/min}$$

∴ Refrigeration capacity of the plant

$$= 113.7/210 = 0.541 \text{ TR } \textbf{Ans.} \qquad \ldots(\because 1\text{TR} = 210 \text{ kJ/min})$$

Example 2.10. *A cold storage plant is required to store 20 tonnes of fish. The fish is supplied at a temperature of 30°C. The specific heat of fish above freezing point is 2.93 kJ/kg K. The specific heat of fish below freezing point is 1.26 kJ/kg K. The fish is stored in cold storage which is maintained at – 8°C. The freezing point of fish is – 4°C. The latent heat of fish is 235 kJ/kg. If the plant requires 75 kW to drive it, find :*

1. The capacity of the plant, and 2. Time taken to achieve cooling.

Assume actual C.O.P. of the plant as 0.3 of the Carnot C.O.P.

Solution. Given : $m = 20 \text{ t} = 20\,000$ kg ; $T_2 = 30°C = 30 + 273 = 303$ K ; $c_{AF} = 2.93$ kJ/kg K ; $c_{BF} = 1.26$ kJ/kgK ; $T_1 = -8°C = -8 + 273 = 265$ K ; $T_3 = -4°C = -4 + 273 = 269$ K; $h_{fg(Fish)} = 235$ kJ/kg ; $P = 75$ kW $= 75$ kJ/s

1. *Capacity of the plant*

We know that Carnot C.O.P.

$$= \frac{T_1}{T_2 - T_1} = \frac{265}{303 - 265} = 6.97$$

∴ Actual C.O.P. $= 0.3 \times 6.97 = 2.091$

and heat removed by the plant $=$ Actual C.O.P. $\times$ Work required

$$= 2.091 \times 75 = 156.8 \text{ kJ/s}$$

$$= 156.8 \times 60 \text{ kJ/min} = 9408 \text{ kJ/min}$$

∴ Capacity of the plant

$$= 9408 / 210 = 44.8 \text{ TR } \textbf{Ans.} \qquad \ldots (\because 1\text{TR} = 210 \text{ kJ/min})$$

2. *Time taken to achieve cooling*

We know that heat removed from the fish above freezing point,

$$Q_1 = m \times c_{AF}(T_2 - T_3)$$

$$= 20\,000 \times 2.93\,(303 - 269) = 1.992 \times 10^6 \text{ kJ}$$

Similarly, heat removed from the fish below freezing point,

$$Q_2 = m \times c_{BF}(T_3 - T_1)$$

$$= 20\,000 \times 1.26\,(269 - 265) = 0.101 \times 10^6 \text{ kJ}$$

and total latent heat of fish,

$$Q_3 = m \times h_{fg(Fish)} = 20\,000 \times 235 = 4.7 \times 10^6 \text{ kJ}$$

∴ Total heat removed by the plant

$$= Q_1 + Q_2 + Q_3$$

$$= 1.992 \times 10^6 + 0.101 \times 10^6 + 4.7 \times 10^6 = 6.793 \times 10^6 \text{ kJ}$$

and time taken to achieve cooling

$$= \frac{\text{Total heat removed by the plant}}{\text{Heat removed by the plant per min}}$$

$$= \frac{6.793 \times 10^6}{9408} = 722 \text{ min} = 12.03 \text{ h } \textbf{Ans.}$$

2.9 Air Refrigerator Working on a Bell-Coleman Cycle (or Reversed Brayton or Joule Cycle)

A Bell-Coleman air refrigeration machine was developed by Bell-Coleman and Light Foot by reversing the Joule's air cycle. It was one of the earliest types of refrigerators used in ships carrying frozen meat. Fig. 2.7 shows a schematic diagram of such a machine which consists of a compressor, a cooler, an expander and a refrigerator.

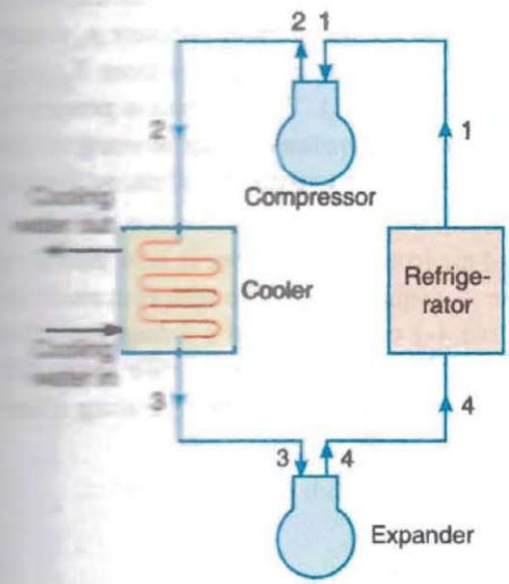


Fig. 2.7. Open cycle air Bell-Coleman Refrigerator.

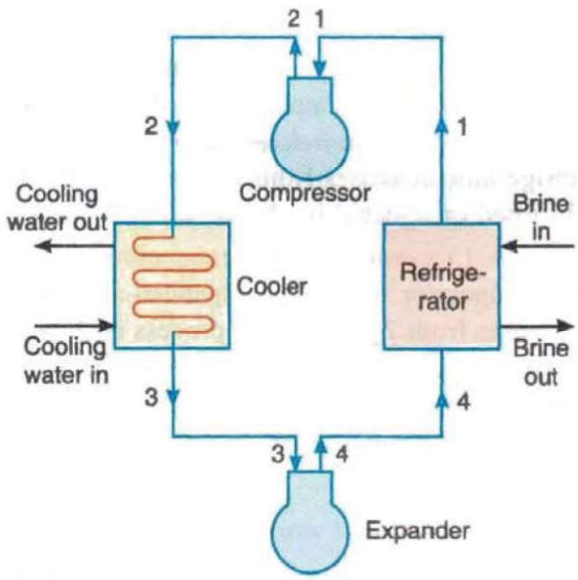
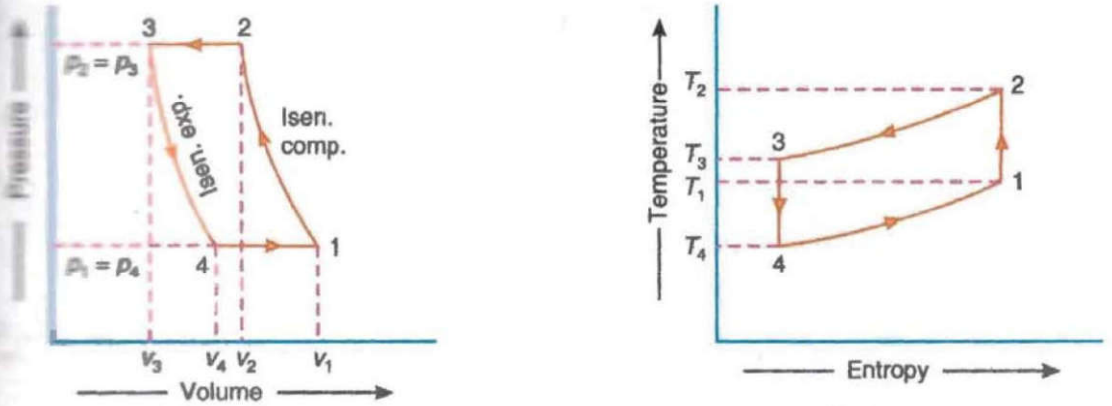


Fig. 2.8. Closed cycle or dense air Bell-Coleman Refrigerator.

The Bell-Coleman cycle (also known as reversed Brayton or Joule cycle) is a modification of reversed Carnot cycle. The cycle is shown on *p-v* and *T-s* diagrams in Fig. 2.9 (*a*) and (*b*). At point 1, let p_1, v_1 and T_1 be the pressure, volume and temperature of air respectively. The four processes of the cycle are as follows :

1. ***Isentropic compression process.*** The cold air from the refrigerator is drawn into the compressor cylinder where it is compressed isentropically in the compressor as shown by the curve 1-2 on *p-v* and *T-s* diagrams. During the compression stroke, both the pressure and temperature increases and the specific volume of air at delivery from compressor reduces from v_1 to v_2. We know that during isentropic compression process, no heat is absorbed or rejected by the air.



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 2.9. Bell-Coleman cycle.

2. *Constant pressure cooling process.* The warm air from the compressor is now passed into the cooler where it is cooled at constant pressure p_3 (equal to p_2), reducing the temperature from T_2 to T_3 (the temperature of cooling water) as shown by the curve 2-3 on *p-v* and *T-s* diagrams. The specific volume also reduces from v_2 to v_3. We know that heat rejected by the air during constant pressure per kg of air,

$$q_R = Q_{2-3} = c_p (T_2 - T_3)$$

3. *Isentropic expansion process.* The air from the cooler is now drawn into the expander cylinder where it is expanded isentropically from pressure p_3 to the refrigerator pressure p_4 which is equal to the atmospheric pressure. The temperature of air during expansion falls from T_3 to T_4 (*i.e.* the temperature much below the temperature of cooling water, T_3). The expansion process is shown by the curve 3-4 on the *p-v* and *T-s* diagrams. The specific volume of air at entry to the refrigerator increases from v_3 to v_4. We know that during isentropic expansion of air, no heat is absorbed or rejected by the air.

4. *Constant pressure expansion process.* The cold air from the expander is now passed to the refrigerator where it is expanded at constant pressure p_4 (equal to p_1). The temperature of air increases from T_4 to T_1. This process is shown by the curve 4-1 on the *p-v* and *T-s* diagrams. Due to heat from the refrigerator, the specific volume of the air changes from v_4 to v_1. We know that the heat absorbed by the air (or heat extracted from the refrigerator or the refrigerating effect produced) during constant pressure expansion per kg of air is

$$q_A = q_{4-1} = c_p (T_1 - T_4)$$

We know that work done during the cycle per kg of air

$$= \text{Heat rejected} - \text{Heat absorbed} = q_R - q_A$$

$$= c_p (T_2 - T_3) - c_p (T_1 - T_4)$$

∴ Coefficient of performance,

$$\text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{q_R - q_A} = \frac{c_p(T_1 - T_4)}{c_p(T_2 - T_3) - c_p(T_1 - T_4)}$$

$$= \frac{(T_1 - T_4)}{(T_2 - T_3) - (T_1 - T_4)}$$

$$= \frac{T_4\left(\frac{T_1}{T_4} - 1\right)}{T_3\left(\frac{T_2}{T_3} - 1\right) - T_4\left(\frac{T_1}{T_4} - 1\right)} \quad \text{... (i)}$$

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} \quad \text{... (ii)}$$

Similarly, for isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma - 1}{\gamma}} \quad \text{... (iii)}$$

Since, $p_2 = p_3$ and $p_1 = p_4$, therefore from equations (*ii*) and (*iii*),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad \frac{T_2}{T_3} = \frac{T_1}{T_4} \quad \text{... (iv)}$$

Now substituting these values in equation (*i*), we get

$$\text{C.O.P.} = \frac{T_4}{T_3 - T_4} = \frac{1}{\frac{T_3}{T_4} - 1}$$

$$= \frac{1}{\left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1} = \boxed{\frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}} \quad \dots (v)$$

where

$$\boxed{r_p = \text{Compression or Expansion ratio} = \frac{p_2}{p_1} = \frac{p_3}{p_4}}$$

Sometimes, the compression and expansion processes take place according to the law pv^n = Constant. In such a case, the C.O.P. is obtained from the fundamentals as discussed below :

We know that work done by the compressor during the process 1-2 per kg of air,

$$w_C = \frac{n}{n-1}(p_2 v_2 - p_1 v_1) = \frac{n}{n-1}(RT_2 - RT_1) \quad \dots (\because pv = RT)$$

and work done by the expander during the process 3-4 per kg of air,

$$w_E = \frac{n}{n-1}(p_3 v_3 - p_4 v_4) = \frac{n}{n-1}(RT_3 - RT_4)$$

$\therefore$ Net work done during the cycle per kg of air,

$$w = w_C - w_E = \frac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]$$

We also know that heat absorbed during constant pressure process 4-1,

$$= c_p (T_1 - T_4)$$

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w} = \frac{c_p (T_1 - T_4)}{\frac{n}{n-1} \times R\left[(T_2 - T_1) - (T_3 - T_4)\right]} \quad \dots (vi)$$

We know that $\quad R = c_p - c_v = c_v(\gamma - 1)$

Substituting the value of R in equation (*vi*),

$$\text{C.O.P.} = \frac{c_p (T_1 - T_4)}{\frac{n}{n-1} \times c_v (\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]}$$

$$= \frac{\gamma (T_1 - T_4)}{\frac{n}{n-1} \times (\gamma - 1)\left[(T_2 - T_1) - (T_3 - T_4)\right]} \quad \dots \left[\because \frac{c_p}{c_v} = \gamma\right]$$

$$= \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{(\gamma - 1)}{\gamma}\left[(T_2 - T_1) - (T_3 - T_4)\right]} \quad \dots (vii)$$

Notes : 1. In this case, the values of T_2 and T_4 are to be obtained from the following relations:

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \quad \text{and} \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}}$$

2. For isentropic compression or expansion, $n = \gamma$. Therefore, the equation (*vii*) may be written as

$$\text{C.O.P.} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} \quad \text{... (same as before)}$$

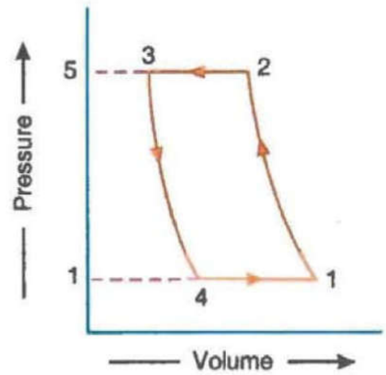
3. We have already discussed that the main drawback of the open cycle air refrigerator is freezing of the moisture in the air during expansion stroke which is liable to choke up the valves. Due to this reason, a closed cycle or dense air Bell-Coleman refrigerator as shown in Fig. 2.8 is preferred. In this case, the cold air does not come in direct contact of the refrigerator. The cold air is passed through the pipes and it is used for absorbing heat from the brine and this cooled brine is circulated in the refrigerated space. The term 'dense air system' is derived from the fact that the suction to the compressor is at higher pressure than the open cycle system (which is atmospheric).

Example 2.11. *In a refrigeration plant working on Bell Coleman cycle, air is compressed to 5 bar from 1 bar. Its initial temperature is 10°C. After compression, the air is cooled up to 20°C in a cooler before expanding back to a pressure of 1 bar. Determine the theoretical C.O.P. of the plant and net refrigerating effect. Take* c_p = 1.005 *kJ/kg K* and c_v = *0.718 kJ/kg K.*

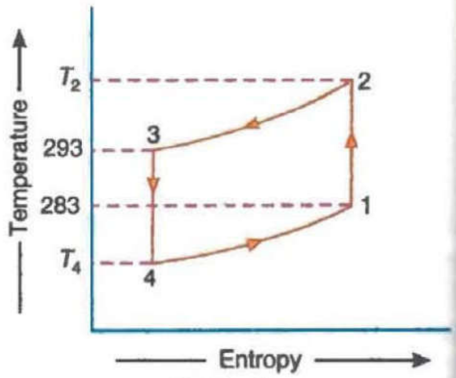
Solution. Given : $p_2 = p_3 = 5$ bar ; $p_1 = p_4 = 1$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 20°C = 20 + 273 = 293$ K ; $c_p = 1.005$ kJ/kg K ; $c_v = 0.718$ kJ/kg K

The *p-v* and *T-s* diagrams for a refrigeration plant working on Bell-Coleman cycle, is shown in Fig. 2.10 (*a*) and (*b*) respectively.

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 2.10

We know that isentropic index for compression and expansion process,

$$\gamma = c_p / c_v = 1.005/0.718 = 1.4$$

For isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

and for isentropic expansion process 3-4

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.584$$

$$\therefore \quad T_4 = T_3 / 1.584 = 293/1.584 = 185 \text{ K}$$

Theoretical C.O.P. of the plant

We know that *theoretical C.O.P. of the plant,

$$= \frac{T_4}{T_3 - T_4} = \frac{185}{293 - 185} = 1.713 \text{ Ans.}$$

Net refrigerating effect

We know that net refrigerating effect (*i.e.* heat absorbed during constant pressure process 4–1)

$$= c_p (T_1 - T_4) = 1.005 (283 - 185) = 98.5 \text{ kJ/kg} \text{ Ans.}$$

Example 2.12. *A refrigerator working on Bell-Coleman cycle operates between pressure limits of 1.05 bar and 8.5 bar. Air is drawn from the cold chamber at 10°C, compressed and then it is cooled to 30°C before entering the expansion cylinder. The expansion and compression follows the law $pv^{1.3}$ = constant. Determine the theoretical C.O.P. of the system.*

Solution. Given : $p_1 = p_4 = 1.05$ bar ; $p_2 = p_3 = 8.5$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 30°C = 30 + 273 = 303$ K ; $n = 1.3$

The *p-v* and *T-s* diagrams for a refrigerator working on the Bell-Coleman cycle is shown in Fig. 2.11 (*a*) and (*b*) respectively.

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.

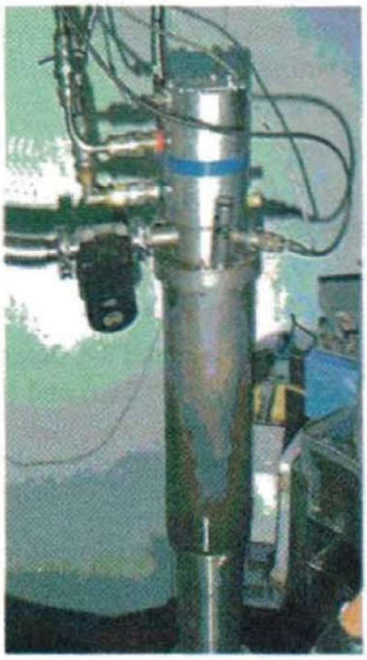
Since the compression and expansion follows the law $pv^{1.3} = C$, therefore

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = (8.1)^{0.231} = 1.62$$

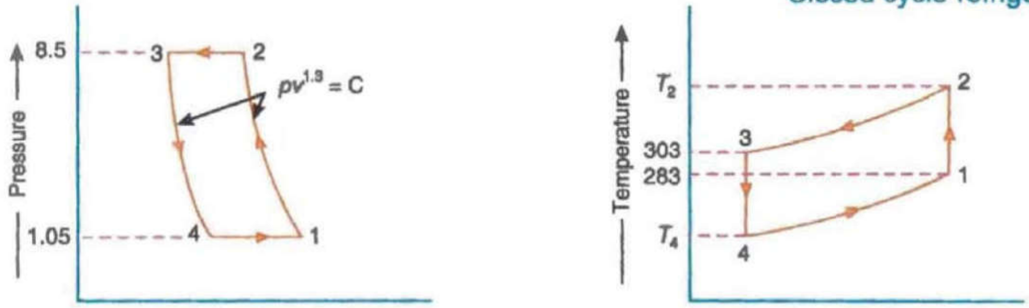
$$\therefore \quad T_2 = T_1 \times 1.62 = 283 \times 1.62 = 458.5 \text{ K}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{8.5}{1.05}\right)^{\frac{1.3-1}{1.3}} = 1.62$$

$$\therefore \quad T_4 = T_3 / 1.62 = 303/1.62 = 187 \text{ K}$$



Closed cycle refrigerator.



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 2.11

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*The theoretical C.O.P. of the plant may also be obtained as follows:

We know that compression or expansion ratio,

$$r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} = \frac{5}{1} = 5 \quad \text{and} \quad \text{C.O.P.} = \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}} - 1} = \frac{1}{(5)^{\frac{1.4-1}{1.4}} - 1} = \frac{1}{1.584 - 1} = 1.712 \text{ Ans.}$$

We know that theoretical coefficient of performance,

$$\text{C.O.P.} = \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{(\gamma - 1)}{\gamma}\left[(T_2 - T_3) - (T_1 - T_4)\right]}$$

$$= \frac{(283 - 187)}{\frac{1.3}{1.3-1} \times \frac{(1.4-1)}{1.4}\left[(458.5 - 303) - (283 - 187)\right]} \quad \text{...(Taking } \gamma = 1.4)$$

$$= \frac{96}{1.24 \times 59.5} = 1.3 \textbf{ Ans.}$$

Example 2.13. *The atmospheric air at pressure 1 bar and temperature –5°C is drawn in the cylinder of the compressor of a Bell-Coleman refrigerating machine. It is compressed isentropically to a pressure of 5 bar. In the cooler, the compressed air is cooled to 15°C, pressure remaining the same. It is then expanded to a pressure of 1 bar in an expansion cylinder, from where it is passed to the cold chamber. Find : 1. the work done per kg of air, and 2. C.O.P. of the plant.*

For air assume law for expansion, $pv^{1.2}$ = constant ; law for compression, $pv^{1.4}$ = constant and specific heat of air at constant pressure = 1 kJ/kg K.

Solution. Given : $p_1 = p_4 = 1$ bar ; $T_1 = -5°C = -5 + 273 = 268$ K ; $p_2 = p_3 = 5$ bar ; $T_3 = 15°C = 15 + 273 = 288$ K ; $n = 1.2$; $\gamma = 1.4$; $c_p = 1$ kJ/kg K

The *p-v* and *T-s* diagrams for a refrigerating machine working on Bell-Coleman cycle is shown in Fig. 2.12 (*a*) and (*b*) respectively.

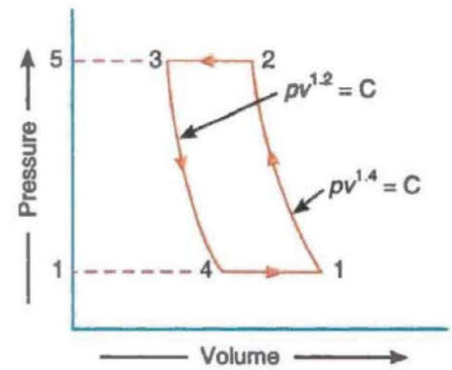
1. *Work done per kg of air*

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

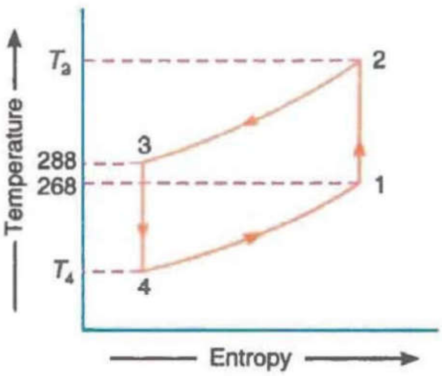
The compression process 1-2 is isentropic and follows the law $pv^{1.4}$ = constant.

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5}{1}\right)^{\frac{1.4-1}{1.4}} = (5)^{0.286} = 1.585$$

or $$T_2 = T_1 \times 1.585 = 268 \times 1.585 = 424.8 \text{ K}$$



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 2.12

Bell-Coleman Refrigeration Machine

The expansion process 3-4 follows the law $pv^{1.2}$ = constant.

$$\therefore \quad \frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.2-1}{1.2}} = (5)^{0.167} = 1.31$$

$$T_4 = T_3/1.31 = 288/1.31 = 220 \text{ K}$$

We know that workdone by the compressor during the isentropic process 1-2 per kg of air,

$$w_C = w_{1-2} = \frac{\gamma}{\gamma-1} \times R\,(T_2 - T_1)$$

$$= \frac{1.4}{1.4-1} \times 0.287(424.8-268) = 159 \text{ kJ/kg}$$

... (Taking R for air = 0.287 kJ/kg K)

and workdone by the expander during the process 3-4 per kg of air,

$$w_E = w_{3-4} = \frac{n}{n-1} \times R\,(T_3 - T_4)$$

$$= \frac{1.2}{1.2-1} \times 0.287(288-220) = 118.3 \text{ kJ/kg}$$

$\therefore$ Net work done per kg of air,

$$w = w_C - w_E = 159 - 118.3 = 40.7 \text{ kJ/kg}$$ **Ans.**

2. C.O.P. of the plant

We know that heat absorbed during constant pressure process 4-1 per kg of air,

$$q_A = c_p\,(T_1 - T_4) = 1(268-220) = 48 \text{ kJ/kg}$$

$$\therefore \quad \text{C.O.P. of the plant} = \frac{\text{Heat absorbed}}{\text{Work done}} = \frac{q_A}{w} = \frac{48}{40.7} = 1.18$$ **Ans.**

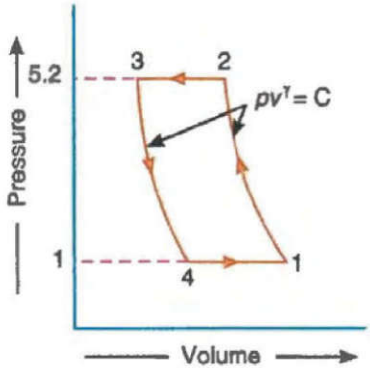
Example 2.14. *A refrigerating machine of 6 tonnes capacity working on Bell-Coleman cycle has an upper limit of pressure of 5.2 bar. The pressure and temperature at the start of compression are 1 bar and 16°C respectively. The compressed air is cooled at constant pressure to a temperature of 41°C, enters the expansion cylinder. Assuming both expansion and compression processes to be isentropic with γ = 1.4, Calculate :*

1. *Coefficient of performance;*
2. *Quantity of air in circulation per minute;*
3. *Piston displacement of compressor and expander;*
4. *Bore of compressor and expansion cylinders. The unit runs at 240 r.p.m. and is double acting. Stroke length is 200 mm ; and*
5. *Power required to drive the unit.*

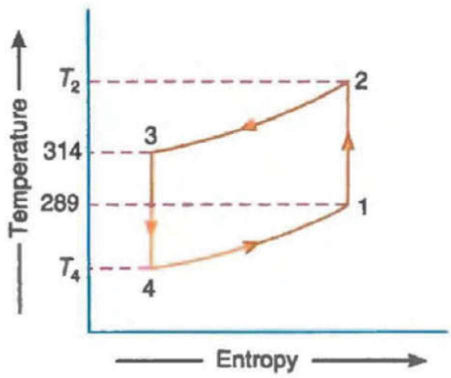
For air, take γ = 1.4, and c_p = 1.003 kJ/kg K.

Solution. Given : $Q = 6$ TR $= 6 \times 210 = 1260$ kJ/min ; $p_2 = p_3 = 5.2$ bar ; $p_1 = p_4 = 1$ bar $= 1 \times 10^5$ N/m² ; $T_1 = 16°C = 16 + 273 = 289$ K ; $T_3 = 41°C = 41 + 273 = 314$ K ; $\gamma = 1.4$

The Bell-Coleman cycle on *p-v* and *T-s* diagrams is shown in Fig. 2.13 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 2.13

1. *Coefficient of performance*

Let T_2 and T_4 = Temperature at the end of compression and expansion respectively.

The compression and expansion are isentropic (*i.e.* $pv^\gamma = C$) and γ for air = 1.4.

We know that
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5.2}{1}\right)^{\frac{1.4-1}{1.4}} = (5.2)^{0.286} = 1.6$$

∴
$$T_2 = T_1 \times 1.6 = 289 \times 1.6 = 462.4 \text{ K}$$

Similarly
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{5.2}{1}\right)^{\frac{1.4-1}{1.4}} = (5.2)^{0.286} = 1.6$$

∴
$$T_4 = T_3 / 1.6 = 314/1.6 = 196.25 \text{ K}$$

We know that coefficient of performance,

$$\text{C.O.P.} = \frac{T_4}{T_3 - T_4} = \frac{196.25}{314 - 196.25} = 1.674 \text{ **Ans.**}$$

Quantity of air in circulation per minute

Let m_a = Mass of air in circulation in kg per minute.

We know that heat extracted from the refrigerating machine (or refrigerating effect produced) per kg of air

$$= c_p\,(T_1 - T_4) = 1.003\,(289 - 196.25) = 93 \text{ kJ/kg}$$

and refrigerating capacity of the machine

$$= 6 \text{ TR } = 6 \times 210 = 1260 \text{ kJ/min}$$

∴ Mass of air in circulation,

$$m_a = 1260 / 93 = 13.548 \text{ kg/min} \quad \textbf{Ans.}$$

Piston displacement of compressor and expander

Let v_1 and v_4 = Piston displacement per minute of compressor and expander respectively.

We know that characteristic gas constant,

$$^*R_a = c_p\left(\frac{\gamma-1}{\gamma}\right) = 1.003\left(\frac{1.4-1}{1.4}\right) = 0.287 \text{ kJ/kg K} = 287 \text{ J/kg K}$$

We also know that $p_1 v_1 = m_a R_a T_1$

$$\therefore \quad v_1 = \frac{m_a R_a T_1}{p_1} = \frac{13.548 \times 287 \times 289}{1 \times 10^5} = 11.237 \text{ m}^3\text{/min} \quad \textbf{Ans.}$$

For constant pressure process 4–1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

$$\therefore \quad v_4 = v_1 \times \frac{T_4}{T_1} = 11.237 \times \frac{196.25}{289} = 7.63 \text{ m}^3\text{/min} \quad \textbf{Ans.}$$

Bore of compressor and expansion cylinders

Let D and d = Bore of compressor and expansion cylinder in metres, respectively.

N = Speed of the unit = 240 r.p.m. ...(Given)

L = Length of stroke = 200 mm = 0.2 m ...(Given)

We know that piston displacement of compressor cylinder,

$$v_1 = \left[\frac{\pi}{4} \times D^2 \times L \times 2\right] N \qquad \ldots (\because \text{ of double acting})$$

---

* We know that $c_p - c_v = R_a$

Dividing by c_p throughout,

$$1 - \frac{1}{\gamma} = \frac{R_a}{c_p} \qquad \ldots \left(\because \frac{c_v}{c_p} = \frac{1}{\gamma}\right)$$

$$\therefore \quad R_a = c_p\left(\frac{\gamma-1}{\gamma}\right)$$

$$11.237 = \left[\frac{\pi}{4}\times D^2\times 0.2\times 2\right]240 = 75.4 D^2$$

$\therefore$ $D^2 = 11.237/75.4 = 0.149$ or $D = 0.386$ m = 386 mm **Ans.**

Similarly piston displacement of expansion cylinder,

$$v_4 = \left[\frac{\pi}{4}\times d^2\times 2\times 2\right]N$$

$$7.63 = \left[\frac{\pi}{4}\times d^2\times 0.2\times 2\right]240 = 75.4\, d^2$$

$\therefore$ $d^2 = 7.63/75.4 = 0.1012$ or $d = 0.318$ m = 318 mm **Ans.**

5. Power required to drive the unit

We know that heat absorbed during the constant pressure process 2–3

$$= m_a c_p (T_1 - T_4) = 13.548\times 1.003\,(289 - 196.25) = 1260 \text{ kJ/min}$$

$\therefore$ Workdone per minute $= \frac{\text{Heat absorbed}}{\text{C.O.P.}} = \frac{1260}{1.674} = 752.7$ kJ/min

and power required to drive the unit

$= 752.7/60 = 12.54$ kJ/s or kW **Ans.**

Note: The power required to drive the unit may also be calculated from the following relation:

We know that $\text{C.O.P.} = \frac{\text{Refrigerating capacity or heat absorbed } (Q)}{\text{Workdone}}$

$$1.674 = \frac{6\times 210}{\text{Workdone}}$$

$\therefore$ Workdone = 6 × 210/1.674 = 752.7 kJ/min

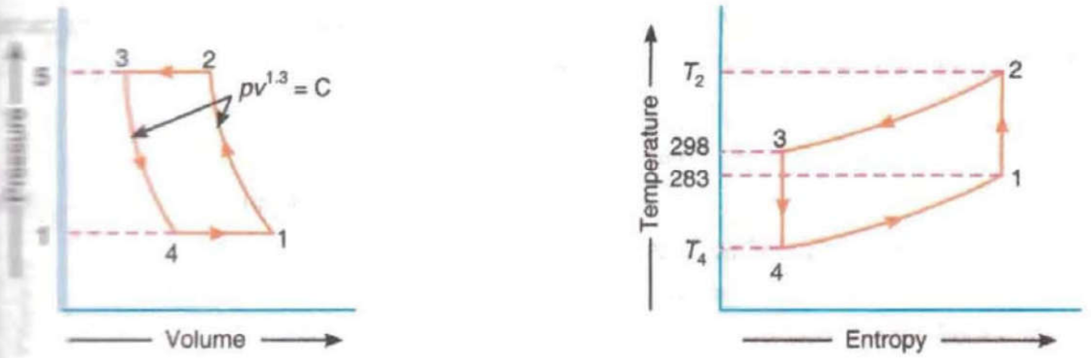
and power required = 752.7/60 = 12.54 kJ/s or kW **Ans.**

Example 2.15. *An air refrigerator works between the pressure limits of 1 bar and 5 bar. The temperature of the air entering the compressor and expansion cylinder are 10°C and 25°C respectively. The expansion and compression follow the law $pv^{1.3}$ = constant. Find the following :*

1. *The theoretical C.O.P. of the refrigerating cycle ;*
2. *If the load on the refrigerating machine is 10 TR, find the amount of air circulated per minute through the system assuming that the actual C.O.P. is 50% of the theoretical C.O.P.*
3. *The stroke length and piston diameter of single acting compressor if the compressor runs at 300 r.p.m. and the volumetric efficiency is 85 %.*
   Take $L/d = 1.5$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.71$ kJ/ kg K.

Solution. Given : $p_1 = p_4 = 1$ bar ; $p_2 = p_3 = 5$ bar ; $T_1 = 10°C = 10 + 273 = 283$ K ; $T_3 = 25°C = 25 + 273 = 298$ K ; $n = 1.3$; $Q = 10$ TR ; Actual C.O.P. = 50% Theoretical C.O.P. ; $N = 300$ r.p.m. ; $\eta_v = 85\% = 085$; $L/d = 1.5$; $c_p = 1.005$ kJ/kg K ; $c_v = 0.71$ kJ/kg K

The p-v and T-s diagrams of the refrigerating cycle are shown in Fig. 2.14 (a) and (b) respectively.



(a) p-v diagram.

(b) T-s diagram.

Fig. 2.14

2. Theoretical C.O.P. of the refrigerating cycle

Let T_2 and T_4 = Temperature at the end of compression and expansion respectively.

We know that
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = (5)^{0.23} = 1.45$$

$$\therefore \quad T_2 = T_1 \times 1.45 = 283 \times 1.45 = 410.3 \text{ K}$$

Similarly
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{5}{1}\right)^{\frac{1.3-1}{1.3}} = (5)^{0.23} = 1.45$$

$$\therefore \quad T_4 = T_3 / 1.45 = 298 / 1.45 = 205.5 \text{ K}$$

We know that heat extracted from the refrigerating system per kg of air,

$$q_A = c_p (T_1 - T_4) = 1.005\,(283 - 205.5) = 78 \text{ kJ/kg}$$

and characteristic gas constant,

$$R = c_p - c_v = 1.005 - 0.71 = 0.295 \text{ kJ/kg K}$$

Workdone during compression process 1-2 per kg of air,

$$w_C = w_{1\text{-}2} = \frac{n}{n-1} \times R\,(T_2 - T_1) = \frac{1.3}{1.3-1} \times 0.295\,(410.3 - 283) \text{ kJ/kg}$$
$$= 162.7 \text{ kJ/kg}$$

and workdone during expansion process 3-4 per kg of air,

$$w_E = w_{3\text{-}4} = \frac{n}{n-1} \times R\,(T_3 - T_4) = \frac{1.3}{1.3-1} \times 0.295\,(298 - 205.5) \text{ kJ/kg}$$

$$= 118.2 \text{ kJ/kg}$$

$\therefore$ Net workdone per kg of air supplied,

$$w = w_C - w_E = w_{1\text{-}2} - w_{3\text{-}4} = 162.7 - 118.2 = 44.5 \text{ kJ/kg}$$

We know that theoretical C.O.P. of the refrigerating cycle

$$= \frac{\text{Heat extracted}}{\text{Workdone}} = \frac{q_A}{w} = \frac{78}{44.5} = 1.75 \text{ Ans.}$$

2. Amount of air circulated per minute

Let m_a = Mass of air circulated per minute.

Since the actual C.O.P. is 50% of the theoretical C.O.P., therefore actual heat extracted or refrigerating capacity of the system per kg of air

$$= 78 \times 0.5 = 39 \text{ kJ/kg}$$

We know that refrigerating capacity of the system

$$= 10 \text{ TR} = 10 \times 210 = 2100 \text{ kJ/min} \qquad \text{...(Given)}$$

∴ Mass of air circulated per minute,

$$m_a = 2100/39 = 53.8 \text{ kg / min} \textbf{ Ans.}$$

3. Stroke length and piston diameter of the compressor

Let L = Stroke length, and

d = Piston diameter.

Since the mass of air supplied to the compressor at point 1 is $m_a = 53.8$ kg / min, therefore its volume,

$$v_1 = \frac{m_a R T_1}{p_1} = \frac{53.8 \times 295 \times 283}{1 \times 10^5} = 45 \text{ m}^3\text{/min}$$

... (R is taken in J/kg K and p is in N/m²)

We also know that volume (v_1),

$$45 = \left(\frac{\pi}{4} \times d^2 \times L\right) N \times \eta_v = \left[\frac{\pi}{4} \times d^2 \times 1.5\, d\right] 300 \times 0.85$$

$$= 300\, d^3 \qquad \text{... } (\because L/d = 1.5)$$

∴ $d^3 = 45 / 300 = 0.15$ or $d = 0.53$ m = 530 mm **Ans.**

and $L = 1.5\, d = 1.5 \times 530 = 795$ mm **Ans.**

Example 2.16. *A dense closed cycle refrigeration system working between 4 bar and 16 bar extracts 126 MJ of heat per hour. The air enters the compressor at 5°C and into the expander at 20°C. Assuming the unit runs at 300 r.p.m., find out 1. Power required to run the unit; 2. Bore of compressor ; and 3. Refrigerating capacity in tonnes of ice at 0°C per day. Take the following :*

The compressor and expander are double acting and stroke for compressor and expander is 300 mm. The mechanical efficiency of compressor is 80%. The mechanical efficiency of expander is 85%. Assume the compression and expansion are isentropic.

Solution. Given : $p_1 = p_4 = 4$ bar $= 4 \times 10^5$ N/m² ; $p_2 = p_3 = 16$ bar $= 16 \times 10^5$ N/m² ; $Q = 126$ MJ/h = 2100 kJ/min ; $T_1 = 5°\text{C} = 5 + 273 = 278$ K ; $T_3 = 20°\text{C} = 20 + 273 = 293$ K ; $N = 300$ r.p.m ; $L = 300$ mm = 0.3 m; $\eta_C = 80\% = 0.8$; $\eta_E = 85\% = 0.85$

The cycle on *p-v* and *T-s* diagrams is shown in Fig. 2.15 (*a*) and (*b*) respectively.

1. Power required to run the unit

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

The compression and expansion are isentropic (*i.e.* $pv^\gamma = C$) and γ for air = 1.4. We know that

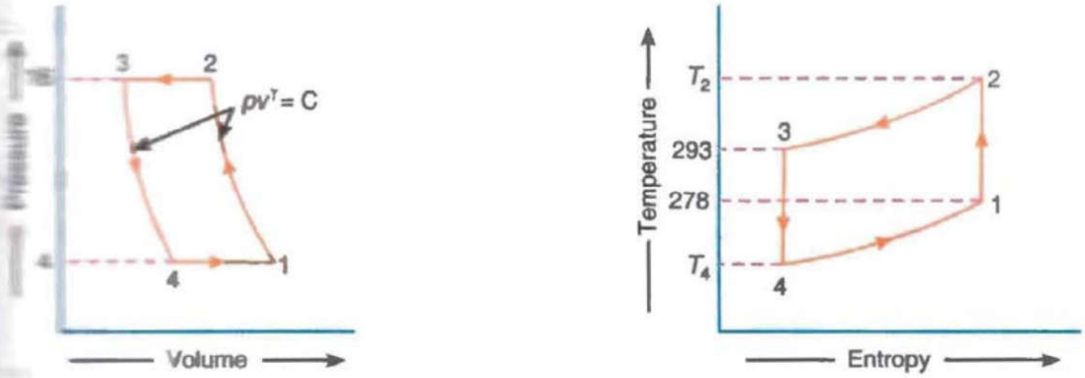
$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

$$\therefore \qquad T_2 = T_1 \times 1.486 = 278 \times 1.486 = 413 \text{ K}$$

Similarly

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

$$\therefore \qquad T_4 = T_3 / 1.486 = 293 / 1.486 = 197 \text{ K}$$



(*a*) *p-v* diagram.

(*b*) *T-s* diagram.

Fig. 2.15

We know that heat extracted from the refrigeration system per kg of air,

$$q_A = c_p (T_1 - T_4) = 1(278 - 197) = 81 \text{ kJ/kg}$$

... (Taking c_p for air = 1 kJ/kg K)

∴ Mass of air circulated,

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{2100}{81} = 25.9 \text{ kg / min}$$

We know that workdone during compression process 1-2 per kg of air,

$$w_C = w_{1-2} = \frac{\gamma}{\gamma - 1} \times R\,(T_2 - T_1) \frac{1}{\eta_C}$$

$$= \frac{1.4}{1.4 - 1} \times 0.287(413 - 278) \frac{1}{0.8} = 169.5 \text{ kJ/kg}$$

and workdone during expansion process 3-4 per kg of air,

$$w_E = w_{3-4} = \frac{\gamma}{\gamma - 1} \times R\,(T_3 - T_4) \eta_E$$

$$= \frac{1.4}{1.4 - 1} \times 0.287(293 - 197)0.85 = 82 \text{ kJ/kg}$$

∴ Net workdone per kg of air supplied in the system,

$$w = w_C - w_E = w_{1-2} - w_{3-4} = 169.5 - 82 = 87.5 \text{ kJ/kg}$$

and power required to run the system

$$= \frac{m_a \times w}{60} = \frac{25.9 \times 87.5}{60} = 37.8 \text{ kW } \textbf{Ans.}$$

2. Bore of compressor

Let D = Bore of compressor in metres.

Since the mass of air supplied to the compressor at point 1 is m_a = 25.9 kg/min, therefore its **volume,**

$$v_1 = \frac{m_a R T_1}{p_1} = \frac{25.9 \times 287 \times 278}{4 \times 10^5} = 5.17 \text{ m}^3/\text{min}$$

... ($\because$ R for air = 287 J/kg K)

We also know that volume,

$$v_1 = \left(\frac{\pi}{4} \times D^2 \times L \times 2\right) N \qquad \text{... } (\because \text{ of double acting})$$

$$5.17 = \left(\frac{\pi}{4} \times D^2 \times 0.3 \times 2\right) 300 = 141.4\, D^2$$

$\therefore$ D^2 = 5.17/141.4 = 0.037 or D = 0.192 m = 192 mm **Ans.**

3. *Refrigerating capacity in tonnes of ice at 0°C per day*

We know that heat extracted or refrigerating capacity of the system per day

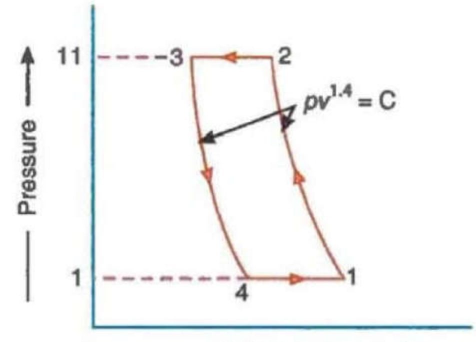
$= 126 \times 24 = 3024$ MJ $= 3024 \times 10^3$ kJ

Since the latent heat of ice is 335 kJ/kg, therefore ice formation capacity of the system per day

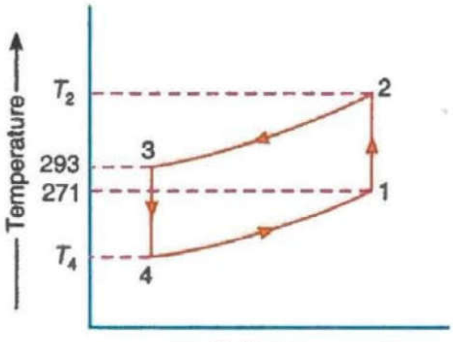
$= 3024 \times 10^3/335 = 9000$ kg = 9 tonnes **Ans.**

Example 2.17. *In an open cycle air refrigeration machine, air is drawn from a cold chamber at – 2°C and 1 bar and compressed to 11 bar. It is then cooled at this pressure, to the cooler temperature of 20°C and then expanded in expansion cylinder and returned to the cold room. The compression and expansion are isentropic, and follows the law* $pv^{1.4}$ = *constant. Sketch the p-v and T-s diagrams of the cycle and for a refrigeration of 15 tonnes, find : 1. theoretical C.O.P; 2. rate of circulation of the air in kg/min ; 3. piston displacement per minute in the compressor and expander ; and 4. theoretical power per tonne of refrigeration.*

Solution. Given : T_1 = –2°C = –2 + 273 = 271 K ; $p_1 = p_4$ = 1 bar = 1 × 10^5 N/m^2; $p_2 = p_3$ = 11 bar ; T_3 = 20°C = 20 + 273 = 293 K ; γ = 1.4 ; Q = 15 TR



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 2.16

The *p-v* and *T-s* diagrams of the cycle are shown in Fig. 2.16 (*a*) and (*b*) respectively.

1. *Theoretical C.O.P.*

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = (11)^{0.286} = 1.985$$

$$\therefore \quad T_2 = T_1 \times 1.985 = 271 \times 1.985 = 538 \text{ K}$$

Similarly
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{11}{1}\right)^{\frac{1.4-1}{1.4}} = (11)^{0.286} = 1.985$$

$$\therefore \quad T_4 = T_3 / 1.985 = 293/1.985 = 147.6 \text{ K}$$

We also know that theoretical C.O.P.

$$= \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} = \frac{271 - 147.6}{(538 - 293) - (271 - 147.6)}$$

$$= \frac{123.4}{121.6} = 1.015 \textbf{ Ans.}$$

2. Rate of circulation of the air in kg/min

Refrigeration capacity $= 15$ TR ...(Given)

$\therefore$ Heat extracted/min $= 15 \times 210 = 3150$ kJ/min ... ($\because$ 1 TR = 210 kJ/min)

We know that heat extracted from cold chamber per kg of air,

$$q_A = c_p (T_1 - T_4) = 1(271 - 147.6) = 123.4 \text{ kJ/kg}$$

... ($\because$ c_p for air = 1kJ/kg K)

$\therefore$ Rate of circulation of air,

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{3150}{123.4} = 25.5 \text{ kg/min} \textbf{ Ans.}$$

3. Piston displacement per minute in the compressor and expander

Let v_1 and v_4 = Piston displacement per minute in the compressor and expander respectively.

We know that
$$p_1 v_1 = m_a R_a T_1$$

$$\therefore \quad v_1 = \frac{m_a R_a T_1}{p_1} = \frac{25.5 \times 287 \times 271}{1 \times 10^5} = 19.8 \text{ m}^3\text{/min} \textbf{ Ans.}$$

...(Taking $R_a = 287$ J/kg K)

For constant pressure process 4-1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

$$\therefore \quad v_4 = v_1 \times \frac{T_4}{T_1} = 19.8 \times \frac{147.6}{271} = 10.8 \text{ m}^3 \textbf{ Ans.}$$

4. Theoretical power per tonne of refrigeration

We know that net workdone on the refrigeration machine per minute

$$= m_a \text{ (Heat rejected – Heat extracted)}$$

$$= m_a c_p [(T_2 - T_3) - (T_1 - T_4)]$$

$$= 25.5 \times 1[(538 - 293) - (271 - 147.6)] = 3100 \text{ kJ/min}$$

$\therefore$ Theoretical power of the refrigerating machine

$$= 3100/60 = 51.67 \text{ kW}$$

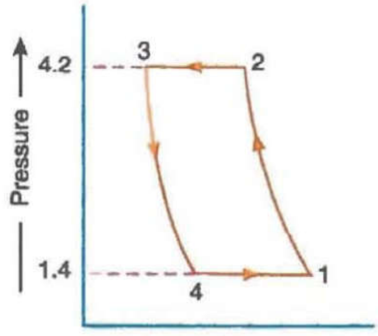
and theoretical power per tonne of refrigeration

$$= 51.67/15 = 3.44 \text{ kW/TR} \textbf{ Ans.}$$

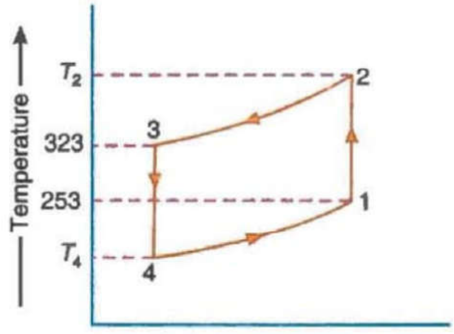
Example 2.18. *A dense air machine operates on reversed Brayton cycle and is required for a capacity of 10 TR. The cooler pressure is 4.2 bar and the refrigerator pressure is 1.4 bar. The air is cooled in the cooler at a temperature of 50°C and the temperature of air at inlet to compressor is –20°C. Determine for the ideal cycle : 1. C.O.P.; 2. mass of air circulated per minute ; 3. theoretical piston displacement of compressor ; 4. theoretical piston displacement of expander ; and 5. net power per tonne of refrigeration. Show the cycle on p-v and T-s planes.*

Solution. Given : Q = 10 TR; $p_2 = p_3$ = 4.2 bar ; $p_1 = p_4$ = 1.4 bar = 1.4×10^5 N/m^2; T_3 = 50°C = 50 + 273 = 323 K ; T_1 = –20°C = –20 + 273 = 253 K

The cycle on *p-v* and *T-s* planes is shown in Fig. 2.17 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram.



(*b*) *T-s* diagram.

Fig. 2.17

1. *Coefficient of performance (C.O.P.)*

Let T_2 and T_4 = Temperatures at the end of compression and expansion respectively.

Let us assume the compression and expansion to be isentropic and γ for air as 1.4. We know that

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.2}{1.4}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

$\therefore$ $$T_2 = T_1 \times 1.369 = 253 \times 1.369 = 346 \text{ K}$$

Similarly $$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{4.2}{1.4}\right)^{\frac{1.4-1}{1.4}} = (3)^{0.286} = 1.369$$

$\therefore$ $$T_4 = T_3 / 1.369 = 323 / 1.369 = 236 \text{ K}$$

We know that $$\text{C.O.P.} = \frac{T_1 - T_4}{(T_2 - T_3) - (T_1 - T_4)} = \frac{253 - 236}{(346 - 323) - (253 - 236)}$$

$$= \frac{17}{6} = 2.83 \textbf{ Ans.}$$

2. *Mass of air circulated per minute*

Since the capacity of the machine is 10 TR, therefore heat extracted per min

= 10 × 210 = 2100 kJ/min ... ($\because$ 1 TR = 210 kJ/min)

We know that heat extracted from the refrigerator per kg of air

= $c_p(T_1 - T_4)$ = 1 (253 – 236) = 17 kJ/kg

∴ Mass of air circulated per minute.

$$m_a = \frac{\text{Heat extracted / min}}{\text{Heat extracted / kg}} = \frac{2100}{17} = 123.5 \text{ kg/min} \textbf{ Ans.}$$

3. Theoretical piston displacement of compressor

Let v_1 = Theoretical piston displacement of compressor per min.

We know that
$$v_1 = \frac{m_a R_a T_1}{p_1} = \frac{123.5 \times 287 \times 253}{1.4 \times 10^5} = 64 \text{ m}^3 \textbf{ Ans.}$$

4. Theoretical displacement of expander

Let v_4 = Theoretical displacement of expander per minute.

We know that for constant pressure process 4-1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

or
$$v_4 = \frac{v_1 \times T_4}{T_1} = 64 \times \frac{236}{253} = 60 \text{ m}^3 \textbf{ Ans.}$$

5. Net power per tonne of refrigeration

We know that net work done on the refrigerating machine per minute

$$= m_a \text{ (Heat rejected – Heat extracted)}$$
$$= m_a c_p [(T_2 - T_3) - (T_1 - T_4)]$$
$$= 123.5 \times 1 [(346 - 323) - (253 - 236)] = 741 \text{ kJ/min}$$

∴ Net power of the refrigerating machine

$$= 741/60 = 12.35 \text{ kW}$$

and net power per tonne of refrigeration

$$= 12.35/10 = 1.235 \text{ kW/TR} \textbf{ Ans.}$$

Example 2.19. *An air refrigeration used for food storage provides 25 TR. The temperature of air entering the compressor is 7°C and the temperature at exit of cooler is 27°C. Find : 1. C.O.P. of the cycle; and 2. power per tonne of refrigeration required by the compressor. The quantity of air circulated in the system is 3000 kg / h. The compression and expansion both follows the law $pv^{1.4}$ = constant and take $\gamma = 1.4$; and $c_p = 1$ kJ/kg K for air.*

Solution. Given : $Q = 25$ TR ; $T_1 = 7°C = 7 + 273 = 280$ K ; $T_3 = 27°C = 27 + 273 = 300$ K ; $m_a = 3000$ kg / h = 50 kg / min

The refrigeration cycle on *p-v* and *T-s* diagram is shown in Fig. 2.18 (*a*) and (*b*) respectively.

1. C.O.P. of the cycle

Let T_2 and T_4 = Temperature of air at the end of compression and expansion respectively.

Since the capacity of the refrigerator is 25 TR, therefore heat extracted from the refrigerator

$$= 25 \times 210 = 5250 \text{ kJ/min} \qquad \dots (i)$$

Also the heat extracted from the refrigerator

$$= m_a c_p (T_1 - T_4) = 50 \times 1 (280 - T_4)$$
$$= 50 (280 - T_4) \text{ kJ/min} \qquad \dots (ii)$$

From equations (*i*) and (*ii*),

$$50\,(280 - T_4) = 5250$$

$$280 - T_4 = 105 \quad \text{or} \quad T_4 = 280 - 105 = 175 \text{ K}$$

We know that

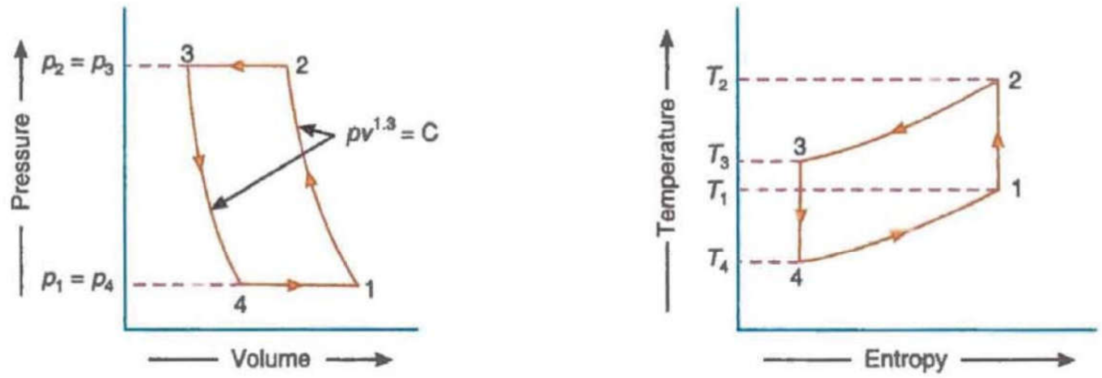
$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \ldots (iii)$$

and

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \qquad \ldots (iv)$$

From equations (*iii*) and (*iv*),

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \quad \text{or} \quad T_2 = \frac{T_1 \times T_3}{T_4} = \frac{280 \times 300}{175} = 480 \text{ K}$$



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 2.18

We know that C.O.P. of the cycle

$$= \frac{T_1 - T_4}{\frac{n}{n-1} \times \frac{\gamma - 1}{\gamma}\left[(T_2 - T_3) - (T_1 - T_4)\right]}$$

$$= \frac{280 - 175}{\frac{1.3}{1.3-1} \times \frac{1.4-1}{1.4}\left[(480 - 300) - (280 - 175)\right]} = 1.13 \textbf{ Ans.}$$

2. Power per tonne of refrigeration

We know that heat absorbed during the constant pressure process 4–1

$$= m_a\, c_p\, (T_1 - T_4) = 50 \times 1\, (280 - 175) = 5250 \text{ kJ/min}$$

$$\therefore \qquad \text{Work done/min} = \frac{\text{Heat absorbed}}{\text{C.O.P.}} = \frac{5250}{1.13} = 4646 \text{ kJ/min}$$

and power per tonne of refrigeration

$$= \frac{4646}{60 \times 25} = 3.1 \text{ kW/ TR } \textbf{Ans.}$$

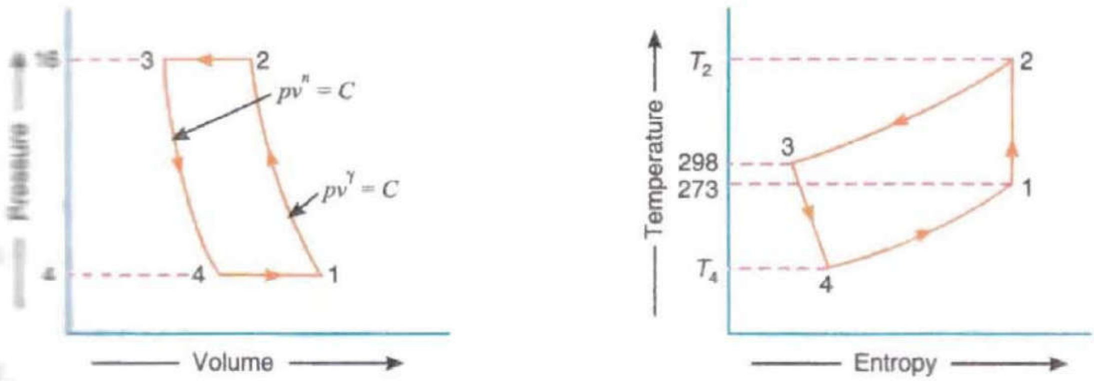
Example 2.20. *A dense air refrigeration system of 10 tonnes capacity works between 4 bar and 16 bar. The air leaves the cold chamber at 0°C and discharges air at 25°C to the expansion cylinder after air cooler. The expansion and compression cylinders are double acting. The mechanical efficiency of compressor and expander are 85% and 80% respectively. The compressor speed is 250 r.p.m. and has a stroke of 250 mm. Determine:*

1. C.O.P. ; 2. Power required; and 3. Bore of compression and expansion cylinders.

Assume isentropic compression and expansion as polytropic with n = 1.25.

Solution. Given : $Q = 10$ TR ; $p_1 = p_4 = 4$ bar ; $p_2 = p_3 = 16$ bar ; $T_1 = 0°C = 273$ K ; $T_3 = 25°C = 25 + 273 = 298$ K ; $\eta_{mc} = 85\% = 0.85$; $\eta_{me} = 80\% = 0.8$; $N = 250$ r.p.m. ; $L = 250$ mm $= 0.25$ m ; $n = 1.25$

The *p-v* and *T-s* diagrams for the cycle are shown in Fig. 2.19 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 2.19

Let

T_2 = Temperature of air at the end of isentropic compression in the compressor, and

T_4 = Temperature of air at the end of polytropic expansion in the turbine.

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

... (Taking γ for air = 1.4]

$$\therefore \quad T_2 = T_1 \times 1.486 = 273 \times 1.486 = 405.7 \text{ K}$$

Similarly, for polytropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{n-1}{n}} = \left(\frac{16}{4}\right)^{\frac{1.25-1}{1.25}} = (4)^{0.2} = 1.32$$

$$\therefore \quad T_4 = T_3 / 1.32 = 298 / 1.32 = 225.7 \text{ K}$$

1. C.O.P.

We know that workdone by the compressor during the isentropic process 1-2 per kg of air,

$$w_C = w_{1-2} = \frac{\gamma}{\gamma-1} \times R\,(T_2 - T_1) \times \frac{1}{\eta_{mc}}$$

$$= \frac{1.4}{1.4-1} \times 0.287(405.7-273) \times \frac{1}{0.85} = 156.8 \text{ kJ/kg}$$

... (Taking R for air = 0.287 kJ/kg K)

and workdone by the expander during the process 3-4 per kg of air,

$$w_E = w_{3-4} = \frac{n}{n-1} \times R\,(T_3 - T_4)\eta_{me}$$

$$= \frac{1.25}{1.25-1} \times 0.287(298-225.7)\,0.8 = 83 \text{ kJ/kg}$$

∴ Net workdone per kg of air,

$$w = w_C - w_E = w_{1-2} - w_{3-4} = 156.8 - 83 = 73.8 \text{ kJ/kg}$$

We know that heat absorbed during constant pressure process 4-1 per kg of air (or refrigerating effect produced per kg of air)

$$q_A = c_p\,(T_1 - T_4) = 1.005\,(273 - 225.7) = 47.3 \text{ kJ/kg}$$

...(Taking c_p = 1.005 kJ/kg K)

∴ $$\text{C.O.P.} = \frac{\text{Heat absorbed}}{\text{Workdone}} = \frac{q_A}{w} = \frac{47.3}{73.8} = 0.641$$ **Ans.**

2. Power required

Let m_a = Mass of air circulated in kg per minute.

We know that refrigeration capacity of the system,

$$Q = 10 \text{ TR} = 10 \times 210 = 2100 \text{ kJ/min}$$

and refrigerating effect per kg of air

$$= c_p\,(T_1 - T_4) = 1.005\,(273 - 225.7) = 47.3 \text{ kJ/kg}$$

∴ Mass of air circulated,

$$m_a = 2100/47.3 = 44.4 \text{ kg/min}$$

and workdone per minute = $m_a \times w = 44.4 \times 73.8 = 3277$ kJ/min

∴ Power required = 3277/60 = 54.6 kJ/s or kW **Ans.**

3. Bore of compression and expansion cylinders

Let D and d = Bore of compression and expansion cylinders respectively, and

v_1 and v_4 = Piston displacement of compressor and expander respectively.

We know that

$$p_1 v_1 = m_a R_a T_1$$

$$\therefore \quad v_1 = \frac{m_a R_a T_1}{p_1} = \frac{44.4 \times 287 \times 273}{4 \times 10^5} = 8.7 \text{ m}^3/\text{min}$$

...($\because$ R_a = 287 J/kg K)

We also know that

$$v_1 = \left[\frac{\pi}{4} \times D^2 \times L \times 2\right] N \qquad \text{...}(\because \text{ of double acting})$$

$$8.7 = \left[\frac{\pi}{4} \times D^2 \times 0.25 \times 2\right] 250 = 98.2\, D^2$$

$$\therefore \quad D^2 = 8.7/98.2 = 0.0886 \quad \text{or} \quad D = 0.2976 \text{ m} = 297.6 \text{ mm}$$ **Ans.**

Now for constant pressure process 4-1,

$$\frac{v_4}{T_4} = \frac{v_1}{T_1}$$

$$\therefore \quad v_4 = v_1 \times \frac{T_4}{T_1} = 8.7 \times \frac{225.7}{273} = 7.2 \text{ m}^3/\text{min}$$

We know that

$$v_4 = \left[\frac{\pi}{4} \times d^2 \times L \times 2\right] N$$

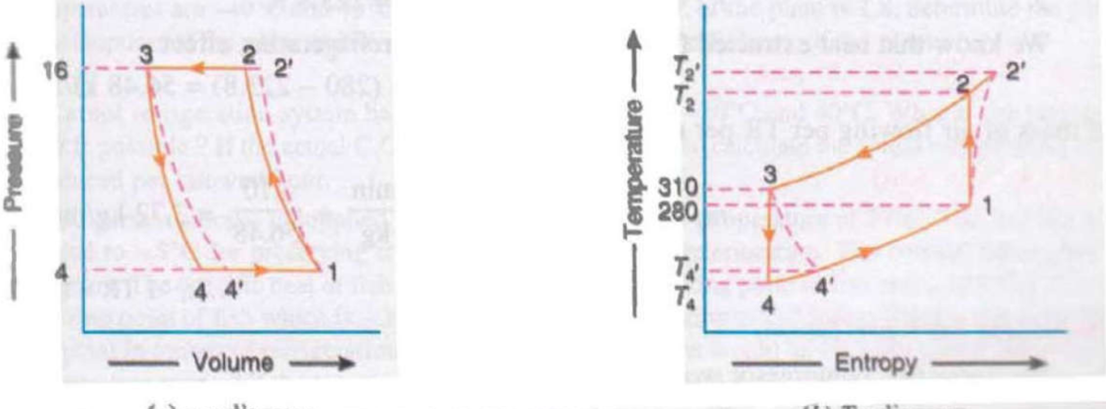
$$7.2 = \left[\frac{\pi}{4} \times d^2 \times 0.25 \times 2\right] 250 = 98.2\, d^2$$

$$\therefore \quad d^2 = 7.2 / 98.2 = 0.0733 \quad \text{or} \quad d = 0.271 \text{ m} = 271 \text{ mm}$$ **Ans.**

Example 2.21. *A dense air refrigeration cycle operates between pressures of 4 bar and 16 bar. The air temperature after heat rejection to surroundings is 37°C and air temperature at exit of refrigerator is 7°C. The isentropic efficiencies of turbine and compressor are 0.85 and 0.8 respectively. Determine compressor and turbine work per TR ; C.O.P.; and power per TR. Take γ = 1.4 and c_p = 1.005 kJ/kg K.*

Solution. Given : $p_1 = p_4$ = 4 bar ; $p_2 = p_3$ = 16 bar ; T_3 = 37°C = 37 + 273 = 310 K ; T_4 = 7°C = 7 + 273 = 280 K ; η_T = 0.85 ; η_C = 0.8 ; γ = 1.4 ; c_p = 1.005 kJ/kg K

The *p-v* and *T-s* diagrams for the cycle are shown in Fig. 2.20 (*a*) and (*b*) respectively.



(*a*) *p-v* diagram. (*b*) *T-s* diagram.

Fig. 2.20

Let T_2 = Temperature of air at the end of isentropic compression in the compressor,

$T_{2'}$ = Actual temperature of air leaving the compressor,

T_4 = Temperature of air at the end of isentropic expansion in the turbine, and

$T_{4'}$ = Actual temperature of air leaving the turbine.

We know that for isentropic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

$$\therefore \quad T_2 = T_1 \times 1.486 = 280 \times 1.486 = 416 \text{ K}$$

Similarly, for isentropic expansion process 3-4,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{16}{4}\right)^{\frac{1.4-1}{1.4}} = (4)^{0.286} = 1.486$$

$$\therefore \quad T_4 = T_3 / 1.486 = 310 / 1.486 = 208.6 \text{ K}$$

We know that isentropic efficiency of the compressor,

$$\eta_C = \frac{\text{Isentropic increase in temperature}}{\text{Actual increase in temperature}} = \frac{T_2 - T_1}{T_{2'} - T_1}$$

$$0.8 = \frac{416 - 280}{T_{2'} - 280} = \frac{136}{T_{2'} - 280}$$

$$\therefore \quad T_{2'} = \frac{136}{0.8} + 280 = 450 \text{ K}$$

and isentropic efficiency of the turbine,

$$\eta_T = \frac{\text{Actual increase in temperature}}{\text{Isentropic increase in temperature}} = \frac{T_3 - T_{4'}}{T_3 - T_4}$$

$$0.85 = \frac{310 - T_{4'}}{310 - 208.6} = \frac{310 - T_{4'}}{101.4}$$

$$\therefore \quad T_{4'} = 310 - 0.85 \times 101.4 = 223.8 \text{ K}$$

We know that heat extracted from the refrigerator or refrigerating effect

$$= c_p (T_1 - T_{4'}) = 1.005\,(280 - 223.8) = 56.48 \text{ kJ/kg}$$

and mass of air flowing per TR per minute,

$$m_a = \frac{\text{Heat extracted per min}}{\text{Heat extracted per kg}} = \frac{210}{56.48} = 3.72 \text{ kg/min}$$

...($\because$ 1 TR = 210 kJ/min)

Compressor work per TR

We know that compressor work,

$$W_C = m_a c_p (T_{2'} - T_1)$$
$$= 3.72 \times 1.005\,(450 - 280) = 635.6 \text{ kJ/min}$$ **Ans.**

Turbine work per TR

We know that turbine work,

$$W_T = m_a c_p (T_3 - T_4')$$
$$= 3.72 \times 1.005\,(310 - 223.8) = 322.3 \text{ kJ/min } \textbf{Ans.}$$

C.O.P.

We know that net workdone per TR,

$$W_{net} = W_C - W_T = 635.6 - 322.3 = 313.3 \text{ kJ/min}$$

$$\therefore \quad \text{C.O.P.} = \frac{\text{Heat extracted per min}}{\text{Net workdone}} = \frac{210}{313.3} = 0.67 \textbf{ Ans.}$$

Power per TR

We know that net workdone per TR,

$$W_{net} = 313.3 \text{ kJ/min}$$

$$\therefore \quad \text{Power per TR} = 313.3/60 = 5.22 \text{ kW } \textbf{Ans.}$$

EXERCISES

1. A Carnot cycle machine operates between the temperature limits of 47°C and –30°C. Determine the C.O.P. when it operates as 1. a refrigerating machine ; 2. a heat pump ; and 3. a heat engine. [**Ans.** 3.16 ; 4.16 : 0.24]

2. A heat pump is used for heating the interior of a house in a cold climate. The ambient temperature is –5°C and the desired interior temperature is 25°C . The compressor of the heat pump is to be driven by a heat engine working between 1000°C and 25°C. Treating both cycles as reversible, calculate the ratio in which the heat pump and the heat engine share the heating load. [**Ans.** 7]

3. A refrigerating plant is required to produce 2.5 tonnes of ice per day at – 4°C from water at 20°C. If the temperature range in the compressor is between 25°C and – 6°C, calculate power required to drive the compressor. Latent heat of ice = 335 kJ/kg and specific heat of ice = 2.1 kJ/kg K. [**Ans.** 1.437 kW]

4. A refrigerator using Carnot cycle requires 1.25 kW per tonne of refrigeration to maintain a temperature of – 30°C. Find : 1. C.O.P. of the Carnot refrigerator; 2. Temperature at which heat is rejected; and 3. Heat rejected per tonne of refrigeration. [**Ans.** 2.8 ; 55.4°C ; 284 kJ/min]

5. Ten tonnes of fish is frozen to –30°C per day. The fish enters the freezing chamber at 30°C and freezing occurs at –3°C. The frozen fish is cooled to –30°C. The specific heats of fresh and frozen fish are 3.77 kJ/kg K and 1.67 kJ/kg K respectively while latent heat of freezing is 251.2 kJ/kg K. Find the tonnage of the plant which runs for 18 hours per day. The evaporator and condensor temperatures are –40°C and 45°C respectively. If the C.O.P. of the plant is 1.8, determine the power consumption of the plant in kW. Also find the refrigerating efficiency of the plant. [**Ans.** 18.6 TR ; 36.1 kW ; 65.7%]

6. A Carnot refrigeration system has working temperature of –30°C and 40°C. What is the maximum C.O.P. possible ? If the actual C.O.P. is 75% of the maximum, calculate the actual refrigerating effect produced per kilowatt hour. [**Ans.** 3.47 ; 0.743 TR]

7. A refrigerator storage is supplied with 30 tonnes of fish at a temperature of 27°C. The fish has to be cooled to – 9°C for preserving it for long period without deterioration. The cooling takes place in 10 hours. The specific heat of fish is 2.93 kJ/kg K above freezing point of fish and 1.26 kJ/kg K below freezing point of fish which is – 3°C. The latent heat of freezing is 232 kJ/kg. What is the capacity of the plant in tonnes of refrigeration for cooling the fish ? What would be the ideal C.O.P. between this temperature range ? If the actual C.O.P. is 40% of the ideal, find the power required to run the cooling plant. [**Ans.** 78 TR ; 7.33 ; 93.3 kW]

8. A refrigerating system working on Bell-Coleman cycle receives air from cold chamber at – 5°C and compresses it from 1 bar to 4.5 bar. The compressed air is then cooled to a temperature of 37°C before it is expanded in the expander. Calculate the C.O.P. of the system when compression and expansion are (*i*) isentropic ; and (*ii*) follow the law $pv^{1.25}$ = constant. **[Ans.** 1.86 ; 1.98**]**

9. A Bell-Coleman refrigerator works between 4 bar and 1 bar pressure limits. After compression, the cooling water reduces the air temperature to 17°C. What is the lowest temperature produced by the ideal machine ? Compare the coefficient of performance of this machine with that of the ideal Carnot cycle machine working between the same pressure limits, the temperature at the beginning of compression being –13°C. **[Ans.** –78°C ; 2.07, 1.02**]**

10. An air refrigerator working on Bell-Coleman cycle takes air into the compressor at 1 bar and 268 K. It is compressed in the compressor to 5 bar and cooled to 298 K at the same pressure. It is further expanded in the expander to 1 bar and discharged to take the cooling load. The isentropic efficiencies of the compressor and expander are 85% and 90% respectively. Determine ; 1. refrigeration capacity of the system if the air circulated is 40 kg / min ; 2. power required for the compressor ; and 3. C.O.P. of the system. **[Ans.** 13.14 TR ; 46 kW; 0.812**]**

11. An air refrigeration system having pressure ratio of 5 takes air at 0°C. It is compressed and then cooled to 19°C at constant pressure. If the efficiency of the compressor is 95% and that of expander is 75%, determine: 1. the refrigeration capacity of the system, if the flow of air is 75 kg/min ; 2. the power of the compressor ; and 3. C.O.P. of the system. Assume compression and expansion processes to be isentropic. Take $\gamma = 1.4$; $c_p = 1$ kJ/kg K ; and $c_v = 0.72$ kJ/kg K. **[Ans.** 31.68 TR; 106.6 kW; 1.71**]**

12. A 5 tonne refrigerating machine operating on Bell Coleman cycle has an upper limit of pressure of 12 bar. The pressure and temperature at the start of compression are 1 bar and 17°C respectively. The compressed air cooled at constant pressure to a temperature of 40°C enters the expansion cylinder. Assuming both the expansion and compression processes to be isentropic with $\gamma = 1.4$; Determine : 1. C.O.P.; 2. quantity of air in circulation per minute; 3. piston displacement of compressor and expander; 4. bore of compressor and expansion cylinders. The unit runs at 250 r.p.m. and is double acting. Stroke length is 200 mm ; and 5. power required to drive the unit.Take $c_p = 1$ kJ/kg K ; $c_v = 0.71$ kJ/kg K ; $R = 0.287$ kJ/kg K.
[Ans. 0.952 ; 7.65 kg/min ; 6.37 m³/min, 3.35 m³/min ; 284 mm ; 18.4 kW**]**

13. An air refrigerator used for food storage, provides 50 TR. The temperature of air entering the compressor is 7°C and the temperature before entering into the expander is 27°C. Assuming a 70% mechanical efficiency, find : 1. actual C.O.P; and 2. the power required to run the compressor.

    The quantity of air circulated in the system is 100 kg/min. The compression and expansion follow the law $pv^{1.3}$ = constant.

    Take $\gamma = 1.4$; $c_p = 1$ kJ/kg K for air. **[Ans.** 1.13 ; 110.6 kW**]**

14. A dense air refrigerating system operating between pressures of 17.5 bar and 3.5 bar is to produce 10 tonnes of refrigeration. Air leaves the refrigerating coils at –7°C and it leaves the air cooler at 15.5°C. Neglecting losses and clearance, calculate the net work done per minute and the coefficient of performance. For air $c_p = 1.005$ kJ/kg K and $\gamma = 1.4$. **[Ans.** 1237 kJ/min ; 1.7**]**

15. A dense air refrigeration machine operating on Bell-Coleman cycle operates between 3.4 bar and 17 bar. The temperature of air after the cooler is 15°C and after the refrigerator is 6°C. For a refrigeration capacity of 6 tonnes, find : 1. Temperature after compression and expansion; 2. Air circulation required in the cycle per minute; 3. Work of compressor and expander; 4. Theoretical C.O.P.; and 5. Rate of water circulation required in the cooler in kg/min, if the rise in temperature is limited to 30°C.
[Ans. 169°C, –91.2°C; 12.9 kg/min; 2112 kJ/min; 1377 kJ/min; 1.72; 199.6 kg/min**]**

QUESTIONS

1. How is the effectiveness of a refrigeration system measured ?
2. Explain the term "tonne of refrigeration".

3. Discuss the advantages of the dense air refrigerating system over an open air refrigeration system.
4. What is the difference between a refrigerator and a heat pump ? Derive an expression for the performance factor for both if they are running on reversed Carnot cycle.
5. Prove that the performance factor of a Bell-Coleman cycle refrigeration system is given by

$$\text{C.O.P.} = \frac{T_2}{T_3 - T_2}$$

where T_2 and T_3 are the temperatures of air at the inlet and discharge of compressor respectively. Explain, with a neat sketch, the working of this cycle.

OBJECTIVE TYPE QUESTIONS

1. The heat removing capacity of one tonne refrigerator is equal to
   (a) 21 kJ/min (b) 210 kJ/min (c) 420 kJ/min (d) 620 kJ/min
2. One tonne refrigerating machine means that
   (a) one tonne is the total mass of the machine
   (b) one tonne of refrigerant is used
   (c) one tonne of water can be converted into ice
   (d) one tonne of ice when melts from and at 0°C in 24 hours, the refrigeration effect produced is equivalent to 210 kJ/min
3. The coefficient of performance is always .... one.
   (a) equal to (b) less than (c) greater than
4. The ratio of heat extracted in the refrigerator to the workdone on the refrigerant is called
   (a) coefficient of performance of refrigeration (b) coefficient of performance of heat pump
   (c) relative coefficient of performance (d) refrigerating efficiency
5. The relative coefficient of performance is equal to
   (a) $\dfrac{\text{Theoretical C.O.P.}}{\text{Actual C.O.P.}}$ (b) $\dfrac{\text{Actual C.O.P.}}{\text{Theoretical C.O.P.}}$
   (c) Actual C.O.P. × Theoretical C.O.P.
6. In a refrigerating machine, if the lower temperature is fixed, then the C.O.P. of the machine can be increased by
   (a) increasing the higher temperature (b) decreasing the higher temperature
   (c) operating the machine at a lower speed (d) operating the machine at a higher speed
7. If the condenser and evaporator temperatures are 312 K and 273 K respectively, then reversed Carnot C.O.P. is
   (a) 5 (b) 7 (c) 9 (d) 10
8. The C.O.P. of a reversed Carnot cycle is most strongly depend upon
   (a) evaporator temperature (b) condenser temperature
   (c) specific heat (d) refrigerant
9. The efficiency of Carnot heat engine is 80%. The C.O.P. of a refrigerator operating on the reversed Carnot cycle is equal to
   (a) 0.25 (b) 0.40 (c) 0.60 (d) 0.80
10. The C.O.P. for a reversed Carnot refrigerator is 4. The ratio of its highest temperature to the lowest temperature will be
    (a) 1 (b) 1.25 (c) 1.75 (d) 2
11. In a closed or dense air refrigeration cycle, the operating pressure ratio can be reduced, which results in .... coefficient of performance.
    (a) lower (b) higher
12. Air refrigeration cycle is used in
    (a) commercial refrigerators (b) domestic refrigerators
    (c) air-conditioning (d) gas liquefaction

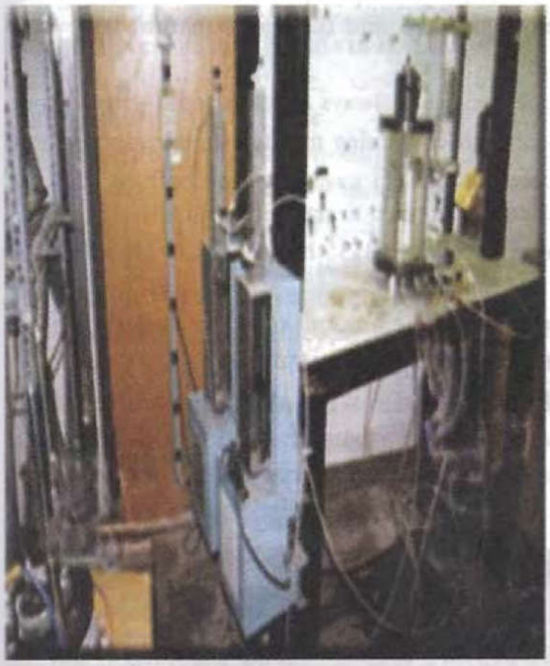
13. In a refrigerating machine, heat rejected is .... heat absorbed.
    (*a*) equal to (*b*) less than (*c*) greater than
14. Air refrigerator works on
    (*a*) Carnot cycle (*b*) Rankine cycle
    (*c*) reversed Carnot cycle (*d*) Bell-Coleman cycle
15. In air-conditioning of aeroplanes, using air as a refrigerant, the cycle used is
    (*a*) reversed Carnot cycle (*b*) reversed Joule cycle
    (*c*) reversed Brayton cycle (*d*) reversed Otto cycle

ANSWERS

1. (*b*)	2. (*d*)	3. (*c*)	4. (*a*)	5. (*b*)
6. (*b*)	7. (*b*)	8. (*a*)	9. (*a*)	10. (*b*)
11. (*b*)	12. (*d*)	13. (*c*)	14. (*c*), (*d*)	15. (*c*)

CHAPTER

16 Psychrometry



16.1 Introduction

The psychrometry is that branch of engineering science which deals with the study of moist air *i.e.* dry air mixed with water vapour or humidity. It also includes the study of behaviour of dry air and water vapour mixture under various sets of conditions. Though the earth's atmosphere is a mixture of gases including nitrogen (N_2), oxygen (O_2), argon (Ar) and carbon dioxide (CO_2), yet for the purpose of psychrometry, it is considered to be a mixture of dry air and water vapour only.

16.2 Psychrometric Terms

Though there are many psychrometric terms, yet the following are important from the subject point of view :

1. *Dry air.* The pure dry air is a mixture of a number of gases such as nitrogen, oxygen, carbon dioxide, hydrogen, argon, neon, helium etc. But the nitrogen and oxygen have the major portion of the combination.

The dry air is considered to have the composition as given in the following table :

Table 16.1. Composition of dry air.

S.No.	*Constituent*	*By volume*	*By mass*	*Molecular mass*
1.	Nitrogen (N_2)	78.03%	75.47%	28
2.	Oxygen (O_2)	20.99%	23.19%	32
3.	Argon (Ar)	0.94%	1.29%	40
4.	Carbon-dioxide (CO_2)	0.03%	0.05%	44
5.	Hydrogen (H_2)	0.01%	—	2

The molecular mass of dry air is taken as 28.966 and the gas constant of air (R_a) is equal to 0.287 kJ/kg K or 287 J/kg K.

The molecular mass of water vapour is taken as 18.016 and the gas constant for water vapour (R_v) is equal to 0.461 kJ/kg K or 461 J/kg K.

Notes : (*a*) The pure dry air does not ordinarily exist in nature because it always contains some water vapour.

(*b*) The term air, wherever used in this text, means dry air containing moisture in the vapour form.

(*c*) Both dry air and water vapour can be considered as perfect gases because both exist in the atmosphere at low pressure. Thus all the perfect gas terms can be applied to them individually.

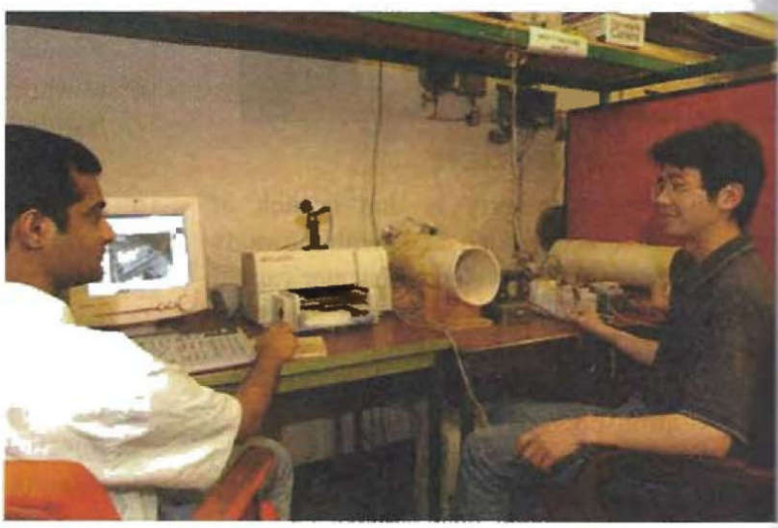
(*d*) The density of dry air is taken as 1.293 kg/m^3 at pressure 1.0135 bar or 101.35 kN/m^2 and at temperature 0°C (273 K).

2. *Moist air.* It is a mixture of dry air and water vapour. The amount of water vapour present in the air depends upon the absolute pressure and temperature of the mixture.

3. *Saturated air.* It is a mixture of dry air and water vapour, when the air has diffused the maximum amount of water vapour into it. The water vapours, usually, occur in the form of superheated steam as an invisible gas. However, when the saturated air is cooled, the water vapour in the air starts condensing, and the same may be visible in the form of moist, fog or condensation on cold surfaces.

4. *Degree of saturation.* It is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature.

5. *Humidity.* It is the mass of water vapour present in 1 kg of dry air, and is generally expressed in terms of gram per kg of dry air (g / kg of dry air). It is also called ***specific humidity*** or ***humidity ratio.***



Psychrometric properties of air.

6. *Absolute humidity.* It is the mass of water vapour present in 1 m^3 of dry air, and is generally expressed in terms of gram per cubic metre of dry air (g/m^3 of dry air). It is also expressed in terms of grains per cubic metre of dry air. Mathematically, one kg of water vapour is equal to 15 430 grains.

7. *Relative humidity.* It is the ratio of actual mass of water vapour in a given volume of moist air to the mass of water vapour in the same volume of saturated air at the same temperature and pressure. It is briefly written as RH.

8. *Dry bulb temperature.* It is the temperature of air recorded by a thermometer, when it is not affected by the moisture present in the air. The dry bulb temperature (briefly written as DBT) is generally denoted by t_d or t_{db}.

9. *Wet bulb temperature.* It is the temperature of air recorded by a thermometer, when its bulb is surrounded by a wet cloth exposed to the air. Such a thermometer is called *wet bulb thermometer. The wet bulb temperature (briefly written as WBT) is generally denoted by t_w or t_{wb}.

10. *Wet bulb depression.* It is the difference between dry bulb temperature and wet bulb temperature at any point. The wet bulb depression indicates relative humidity of the air.

11. *Dew point temperature.* It is the temperature of air recorded by a thermometer, when the moisture (water vapour) present in it begins to condense. In other words, the dew point temperature is the saturation temperature (t_{sat}) corresponding to the partial pressure of water vapour (p_v). It is, usually, denoted by t_{dp}. Since p_v is very small, therefore the saturation temperature by water vapour at p_v is also low (less than the atmospheric or dry bulb temperature). Thus the water vapour in air exists in the superheated state and the moist air containing moisture in such a form (*i.e.* superheated state) is said to be *unsaturated air*. This condition is shown by point *A* on temperature-entropy (*T-s*) diagram as shown in Fig. 16.1. When the partial pressure of water vapour (p_v) is equal to the saturation pressure (p_s), the water vapour is in dry condition and the air will be *saturated air*.

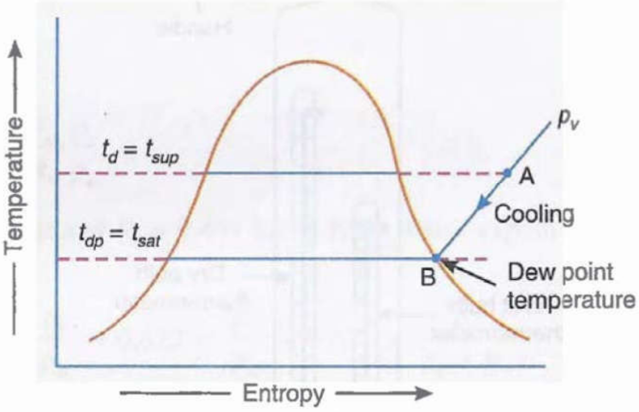


Fig. 16.1. *T-s* diagram.

If a sample of unsaturated air, containing superheated water vapour, is cooled at constant pressure, the partial pressure (p_v) of each constituent remains constant until the water vapour reaches the saturated state as shown by point *B* in Fig. 16.1. At this point *B*, the first drop of dew will be formed and hence the temperature at point *B* is called *dew point temperature*. Further cooling will cause condensation of water vapour.

From the above we see that the dew point temperature is the temperature at which the water vapour begins to condense.

Note : For saturated air, the dry bulb temperature, wet bulb temperature and dew point temperature is same.

* A wet bulb thermometer has its bulb covered with a piece of soft cloth (or silk wick) which is exposed to the air. The lower part of this cloth is dipped in a small basin of water. The water from the basin rises up in the cloth by the capillary action, and then gets evaporated. It may be noted that if relative humidity of air is high (*i.e.* the air contains more water vapour), there will be little evaporation and thus there will be a small cooling effect. On the other hand, if relative humidity of air is low (*i.e.* the air contains less water vapour), there will be more evaporation, and thus there will be more cooling effect.

12. ***Dew point depression.*** It is the difference between the dry bulb temperature and dew point temperature of air.

13. ***Psychrometer.*** There are many types of psychrometers, but the sling psychrometer, as shown in Fig. 16.2, is widely used. It consists of a dry bulb thermometer and a wet bulb thermometer mounted side by side in a protective case that is attached to a handle by a swivel connection so that the case can be easily rotated. The dry bulb thermometer is directly exposed to air and measures the actual temperature of the air. The bulb of the wet bulb thermometer is covered by a wick thoroughly wetted by distilled water. The temperature measured by this wick covered bulb of a thermometer is the temperature of liquid water in the wick and is called wet bulb temperature.

The sling psychrometer is rotated in the air for approximately one minute after which the readings from both the thermometers are taken. This process is repeated several times to assure that the lowest possible wet bulb temperature is recorded.

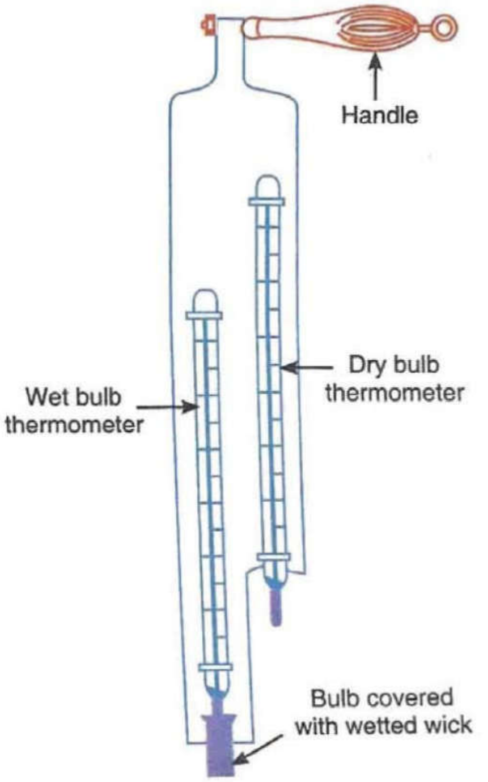
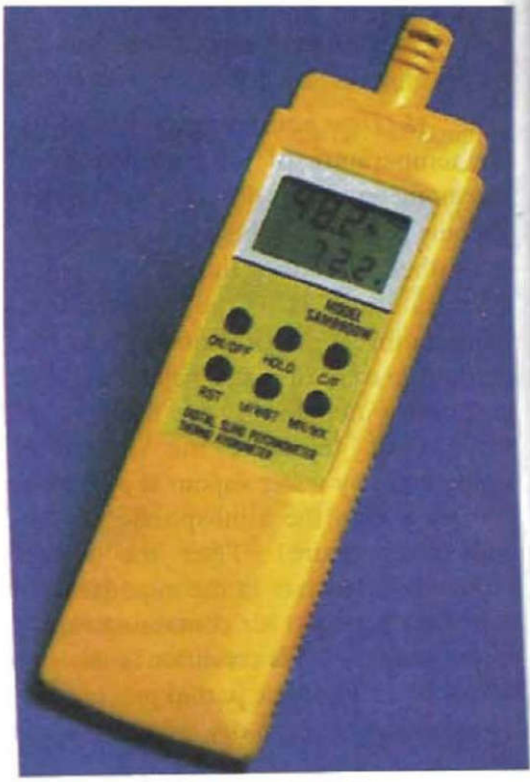


Fig. 16.2. Sling psychrometer.



Digital psychrometer.

16.3 Dalton's Law of Partial Pressures

It states, "*The total pressure exerted by the mixture of air and water vapour is equal to the sum of the pressures, which each constituent would exert, if it occupied the same space by itself.*" In other words, the total pressure exerted by air and water vapour mixture is equal to the barometric pressure. Mathematically, barometric pressure of the mixture,

$$p_b = p_a + p_v$$

where p_a = Partial pressure of dry air, and

p_v = Partial pressure of water vapour.

16.4 Psychrometric Relations

We have already discussed some psychrometric terms in Art. 16.2. These terms have some relations between one another. The following psychrometric relations are important from the subject point of view :

1. *Specific humidity, humidity ratio or moisture content.* It is the mass of water vapour present in 1 kg of dry air (in the air-vapour mixture) and is generally expressed in g / kg of dry air. It may also be defined as the ratio of mass of water vapour to the mass of dry air in a given volume of the air-vapour mixture.

Let p_a, v_a, T_a, m_a and R_a = Pressure, volume, absolute temperature, mass and gas constant respectively for dry air, and

p_v, v_v, T_v, m_v and R_v = Corresponding values for the water vapour.

Assuming that the dry air and water vapour behave as perfect gases, we have for dry air,

$$p_a v_a = m_a R_a T_a \quad \ldots (i)$$

and for water vapour,

$$p_v v_v = m_v R_v T_v \quad \ldots (ii)$$

Also

$$v_a = v_v$$

and

$$T_a = T_v = T_d \quad \ldots \text{(where } T_d \text{ is dry bulb temperature)}$$

From equations (*i*) and (*ii*), we have

$$\frac{p_v}{p_a} = \frac{m_v R_v}{m_a R_a}$$

∴ Humidity ratio,

$$W = \frac{m_v}{m_a} = \frac{R_a p_v}{R_v p_a}$$

Substituting $R_a = 0.287$ kJ/kg K for dry air and $R_v = 0.461$ kJ/kg K for water vapour in the above equation, we have

$$W = \frac{0.287 \times p_v}{0.461 \times p_a} = 0.622 \times \frac{p_v}{p_a} = 0.622 \times \frac{p_v}{p_b - p_v}$$

$$\ldots (\because\ p_b = p_a + p_v)$$

Consider unsaturated air containing superheated vapour at dry bulb temperature t_d and partial pressure p_v as shown by point *A* on the *T-s* diagram in Fig. 16.3. If water is added into this unsaturated air, the water will evaporate which will increase the moisture content (specific humidity) of the air and the partial pressure p_v increases. This will continue until the water vapour becomes saturated at that temperature, as shown by point *C* in Fig. 16.3, and there will be more evaporation of water. The partial pressure p_v increases to the saturation pressure p_s and it is maximum partial pressure of water vapour at temperature t_d. The air containing moisture in such a state (point *C*) is called *saturated air.*

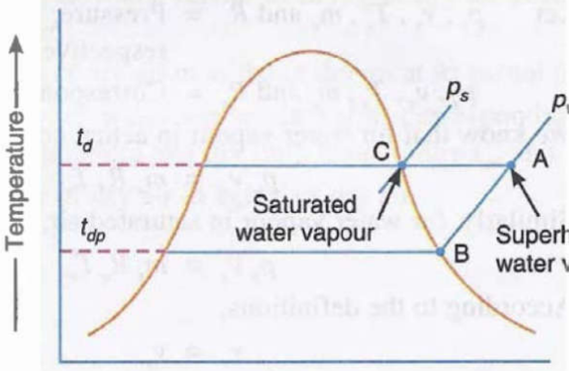


Fig. 16.3. *T-s* diagram.

For saturated air (*i.e.* when the air is holding maximum amount of water vapour), the humidity ratio or maximum specific humidity,

$$W_s = W_{max} = 0.622 \times \frac{p_s}{p_b - p_s}$$

where p_s = Partial pressure of air corresponding to saturation temperature (*i.e.* dry bulb temperature t_d).

2. *Degree of saturation or percentage humidity.* We have already discussed that the degree of saturation is the ratio of actual mass of water vapour in a unit mass of dry air to the mass of water vapour in the same mass of dry air when it is saturated at the same temperature (dry bulb temperature). In other words, it may be defined as the ratio of actual specific humidity to the specific humidity of saturated air at the same dry bulb temperature. It is, usually, denoted by μ. Mathematically, degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{\dfrac{0.622\, p_v}{p_b - p_v}}{\dfrac{0.622\, p_s}{p_b - p_s}} = \frac{p_v}{p_s}\left(\frac{p_b - p_s}{p_b - p_v}\right) = \frac{p_v}{p_s}\left[\frac{1 - \dfrac{p_s}{p_b}}{1 - \dfrac{p_v}{p_b}}\right]$$

Notes : (*a*) The partial pressure of saturated air (p_s) is obtained from the steam tables corresponding to dry bulb temperature t_d.

(*b*) If the relative humidity, $\phi = p_v / p_s$ is equal to zero, then the humidity ratio, $W = 0$, *i.e.* for dry air, $\mu = 0$.

(*c*) If the relative humidity, $\phi = p_v / p_s$ is equal to 1, then $W = W_s$ and $\mu = 1$. Thus μ varies between 0 and 1.

3. *Relative humidity.* We have already discussed that the relative humidity is the ratio of actual mass of water vapour (m_v) in a given volume of moist air to the mass of water vapour (m_s) in the same volume of saturated air at the same temperature and pressure. It is usually denoted by ϕ. Mathematically, relative humidity,

$$\phi = \frac{m_v}{m_s}$$

Let p_v, v_v, T_v, m_v and R_v = Pressure, volume, temperature, mass and gas constant respectively for water vapour in actual conditions, and

p_s, v_s, T_s, m_s and R_s = Corresponding values for water vapour in saturated air.

We know that for water vapour in actual conditions,

$$p_v\, v_v = m_v\, R_v\, T_v \qquad \dots (i)$$

Similarly, for water vapour in saturated air,

$$p_s\, v_s = m_s\, R_s\, T_s \qquad \dots (ii)$$

According to the definitions,

$$v_v = v_s$$

and

$$T_v = T_s$$

Also

$$R_v = R_s = 0.461 \text{ kJ/kg K}$$

$\therefore$ From equations (*i*) and (*ii*), relative humidity,

$$\phi = \frac{m_v}{m_s} = \frac{p_v}{p_s}$$

Thus, the relative humidity may also be defined as the ratio of actual partial pressure of water vapour in moist air at a given temperature (dry bulb temperature) to the saturation pressure of water vapour (or partial pressure of water vapour in saturated air) at the same temperature.

The relative humidity may also be obtained as discussed below :

We know that degree of saturation,

$$\mu = \frac{p_v}{p_s}\left[\frac{1-\frac{p_s}{p_b}}{1-\frac{p_v}{p_b}}\right] = \phi\left[\frac{1-\frac{p_s}{p_b}}{1-\phi\times\frac{p_s}{p_b}}\right] \qquad \ldots \left(\because \phi = \frac{p_v}{p_s}\right)$$

$$\therefore \qquad \phi = \frac{\mu}{1-(1-\mu)\frac{p_s}{p_b}}$$

Note : For saturated air, the relative humidity is 100%.

4. *Pressure of water vapour*. According to Carrier's equation, the partial pressure of water vapour,

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\, t_w}$$

where p_w = Saturation pressure corresponding to wet bulb temperature (from steam tables),

p_b = Barometric pressure,

t_d = Dry bulb temperature, and

t_w = Wet bulb temperature.

5. *Vapour density or absolute humidity*. We have already discussed that the vapour density or absolute humidity is the mass of water vapour present in 1 m^3 of dry air.

Let v_v = Volume of water vapour in m^3/kg of dry air at its partial pressure,

v_a = Volume of dry air in m^3/kg of dry air at its partial pressure,

ρ_v = Density of water vapour in kg/m^3 corresponding to its partial pressure and dry bulb temperature t_d, and

ρ_a = Density of dry air in kg/m^3 of dry air.

We know that mass of water vapour,

$$m_v = v_v\, \rho_v \qquad \ldots (i)$$

and mass of dry air,

$$m_a = v_a\, \rho_a \qquad \ldots (ii)$$

Dividing equation (*i*) by equation (*ii*),

$$\frac{m_v}{m_a} = \frac{v_v\, \rho_v}{v_a\, \rho_a}$$

Since $v_a = v_v$, therefore humidity ratio,

$$W = \frac{m_v}{m_a} = \frac{\rho_v}{\rho_a} \quad \text{or} \quad \rho_v = W\, \rho_a \qquad \ldots (iii)$$

We know that $p_a\, v_a = m_a\, R_a\, T_d$

Since $v_a = \dfrac{1}{\rho_a}$ and $m_a = 1$ kg, therefore substituting these values in the above expression, we get

$$p_a \times \frac{1}{\rho_a} = R_a\, T_d \quad \text{or} \quad \rho_a = \frac{p_a}{R_a\, T_d}$$

Substituting the value of ρ_a in equation (*iii*), we have

$$\rho_v = \frac{W\, p_a}{R_a\, T_d} = \frac{W\,(p_b - p_v)}{R_a\, T_d} \qquad \ldots (\because\ p_b = p_a + p_v)$$

where p_a = Pressure of air in kN/m^2,

R_a = Gas constant for air = 0.287 kJ/ kg K, and

T_d = Dry bulb temperature in K.

16.5 Enthalpy (Total Heat) of Moist Air

The enthalpy of moist air is numerically equal to the enthalpy of dry air plus the enthalpy of water vapour associated with dry air. Let us consider one kg of dry air. We know that enthalpy of 1 kg of dry air,

$$h_a = c_{pa}\, t_d \qquad \ldots (i)$$

where c_{pa} = Specific heat of dry air which is normally taken as 1.005 kJ / kg K, and

t_d = Dry bulb temperature.

Enthalpy of water vapour associated with 1 kg of dry air,

$$h_v = W\, h_s \qquad \ldots (ii)$$

where W = Mass of water vapour in 1 kg of dry air (*i.e.* specific humidity), and

h_s = Enthalpy of water vapour per kg of dry air at dew point temperature (t_{dp}).

If the moist air is superheated, then the enthalpy of water vapour

$$= W\, c_{ps}\,(t_d - t_{dp}) \qquad \ldots (iii)$$

where c_{ps} = Specific heat of superheated water vapour which is normally taken as 1.9 kJ/kg K, and

$t_d - t_{dp}$ = Degree of superheat of the water vapour.

∴ Total enthalpy of superheated water vapour,

$$\begin{aligned}
h &= c_{pa}\, t_d + W\, h_s + W\, c_{ps}\,(t_d - t_{dp}) \\
&= c_{pa}\, t_d + W\,[h_{fdp} + h_{fgdp} + c_{ps}\,(t_d - t_{dp})] && \ldots (\because\ h_s = h_{fdp} + h_{fgdp}) \\
&= c_{pa}\, t_d + W\,[4.2\, t_{dp} + h_{fgdp} + c_{ps}\,(t_d - t_{dp})] && \ldots (\because\ h_{fdp} + 4.2\, t_{dp}) \\
&= c_{pa}\, t_d + 4.2\, W\, t_{dp} + W\, h_{fgdp} + W\, c_{ps}\, t_d - W\, c_{ps}\, t_{dp} \\
&= (c_{pa} + W\, c_{ps})\, t_d + W\,[h_{fgdp} + t_{dp}\,(4.2 - c_{ps})] \\
&= (c_{pa} + W\, c_{ps})\, t_d + W\,[h_{fgdp} + t_{dp}\,(4.2 - 1.9)] \\
&= (c_{pa} + W\, c_{ps})\, t_d + W\,[h_{fgdp} + 2.3\, t_{dp}]
\end{aligned}$$

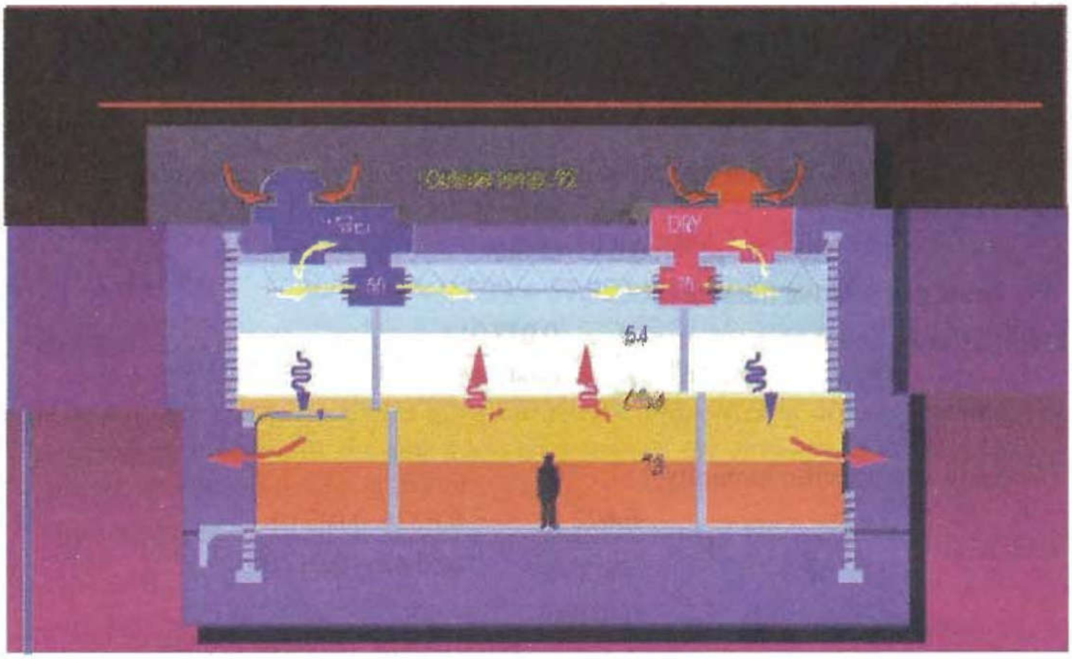
The term $(c_{pa} + W\, c_{ps})$ is called *humid specific heat* (c_{pm}). It is the specific heat or heat capacity of moist air, *i.e.* $(1 + W)$ kg/kg of dry air. At low temperature of air conditioning range, the value of W is very small. The general value of humid specific heat in air conditioning range is taken as 1.022 kJ/kg K.

$$\therefore \qquad h = 1.022\, t_d + W\,(h_{fgdp} + 2.3\, t_{dp}) \text{ kJ/kg}$$

where $\quad h_{fgdp}$ = Latent heat of vaporisation of water corresponding to dew point temperature (from steam tables).

An approximate result may be obtained by the following relation:

$$h = 1.005\, t_d + W\,[2500 + 1.9\, t_d] \text{ kJ/kg}$$



Psychrometric control

Example 16.1. *The readings from a sling psychrometer are as follows :*

Dry bulb temperature = 30° C ; Wet bulb temperature = 20° C ; Barometer reading = 740 mm of Hg.

Using steam tables, determine : 1. Dew point temperature ; 2. Relative humidity ; 3. Specific humidity ; 4. Degree of saturation ; 5. Vapour density ; and 6. Enthalpy of mixture per kg of dry air.

Solution. Given : t_d = 30°C ; t_w = 20°C ; p_b = 740 mm of Hg

1. *Dew point temperature*

First of all, let us find the partial pressure of water vapour (p_v).

From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 20° C is

$$p_w = 0.023\,37 \text{ bar}$$

We know that barometric pressure,

$$p_b = 740 \text{ mm of Hg} \qquad \text{... (Given)}$$

$$= 740 \times 133.3 = 98\,642 \text{ N/m}^2 \qquad \text{... } (\because \; 1 \text{ mm of Hg} = 133.3 \text{ N/m}^2)$$

$$= 0.986\,42 \text{ bar} \qquad \text{... } (\because \; 1 \text{ bar} = 10^5 \text{ N/m}^2)$$

∴ Partial pressure of water vapour,

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\, t_w}$$

$$= 0.023\,37 - \frac{(0.986\,42 - 0.02337)(30 - 20)}{1544 - 1.44 \times 20}$$

$$= 0.023\,37 - 0.006\,36 = 0.017\,01 \text{ bar}$$

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore from steam tables, we find that corresponding to a pressure of 0.017 01 bar, the dew point temperature is

$$t_{dp} = 15° \text{ C} \quad \textbf{Ans.}$$

2. *Relative humidity*

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$p_s = 0.042\,42 \text{ bar}$$

We know that relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{0.01701}{0.04242} = 0.40 \text{ or } 40\% \quad \textbf{Ans.}$$

3. *Specific humidity*

We know that specific humidity,

$$W = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.01701}{0.986\,42 - 0.01701}$$

$$= \frac{0.01058}{0.96941} = 0.010\,914 \text{ kg/kg of dry air}$$

$$= 10.914 \text{ g/kg of dry air} \quad \textbf{Ans.}$$

4. *Degree of saturation*

We know that specific humidity of saturated air,

$$W_s = \frac{0.622\, p_s}{p_b - p_s} = \frac{0.622 \times 0.04242}{0.98642 - 0.04242}$$

$$= \frac{0.02638}{0.944} = 0.027\,945 \text{ kg/kg of dry air}$$

We know that degree of saturation,

$$\mu = \frac{W}{W_s} = \frac{0.010\,914}{0.027\,945} = 0.391 \text{ or } 39.1\% \quad \textbf{Ans.}$$

Note : The degree of saturation (μ) may also be calculated from the following relation :

$$\mu = \frac{p_v}{p_s}\left(\frac{p_b - p_s}{p_b - p_v}\right)$$

$$= \frac{0.01701}{0.04242}\left[\frac{0.98642 - 0.04242}{0.98642 - 0.01701}\right]$$

$$= 0.391 \text{ or } 39.1\% \quad \textbf{Ans.}$$

5. *Vapour density*

We know that vapour density,

$$\rho_v = \frac{W\,(p_b - p_v)}{R_a\,T_d} = \frac{0.010\,914\,(0.98642 - 0.01701)\,10^5}{287\,(273+30)}$$

$$= 0.012\,16\ \text{kg/m}^3 \text{ of dry air} \quad \textbf{Ans.}$$

6. *Enthalpy of mixture per kg of dry air*

From steam tables, we find that the latent heat of vaporisation of water at dew point temperature of 15°C is

$$h_{fgdp} = 2466.1 \text{ kJ/kg}$$

∴ Enthalpy of mixture per kg of dry air,

$$h = 1.022\,t_d + W\,[h_{fgdp} + 2.3\,t_{dp}]$$

$$= 1.022 \times 30 + 0.010\,914\,[2466.1 + 2.3 \times 15]$$

$$= 30.66 + 27.29 = 57.95 \text{ kJ/kg of dry air} \quad \textbf{Ans.}$$

Example 16.2. *On a particular day, the atmospheric air was found to have a dry bulb temperature of 30° C and a wet bulb temperature of 18° C. The barometric pressure was observed to be 756 mm of Hg. Using the tables of psychrometric properties of air, determine the relative humidity, the specific humidity, the dew point temperature, the enthalpy of air per kg of dry air and the volume of mixture per kg of dry air.*

Solution. Given : $t_d = 30$°C ; $t_w = 18$°C ; $p_b = 756$ mm of Hg

Relative humidity

First of all, let us find the partial pressure of water vapour (p_v). From steam tables, we find that the saturation pressure corresponding to wet bulb temperature of 18°C is,

$$p_w = 0.020\,62 \text{ bar} = 0.020\,62 \times 10^5 = 2062 \text{ N/m}^2$$

$$= \frac{2062}{133.3} = 15.47 \text{ mm of Hg} \qquad \ldots (\because 1 \text{ mm of Hg} = 133.3 \text{ N/m}^2)$$

We know that

$$p_v = p_w - \frac{(p_b - p_w)(t_d - t_w)}{1544 - 1.44\,t_w}$$

$$= 15.47 - \frac{(756 - 15.47)(30-18)}{1544 - 1.44 \times 18} \text{ mm of Hg}$$

$$= 15.47 - 5.85 = 9.62 \text{ mm of Hg}$$

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 30°C is

$$p_s = 0.042\,42 \text{ bar} = 0.042\,42 \times 10^5 = 4242 \text{ N/m}^2$$

$$= \frac{4242}{133.3} = 31.8 \text{ mm of Hg}$$

We know that the relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{9.62}{31.8} = 0.3022 \text{ or } 30.22\% \quad \textbf{Ans.}$$

Specific humidity

We know that specific humidity,

$$W = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 9.62}{756 - 9.62} = 0.008 \text{ kg/kg of dry air} \quad \textbf{Ans.}$$

Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore from steam tables, we find that corresponding to 9.62 mm of Hg or 9.62 × 133.3 = 1282.3 N/m² = 0.012 823 bar, the dew point temperature is,

$$t_{dp} = 10.6° \text{ C} \quad \textbf{Ans.}$$

Enthalpy of air per kg of dry air

From steam tables, we also find that latent heat of vaporisation of water at dew point temperature of 10.6°C,

$$h_{fgdp} = 2476.5 \text{ kJ/kg}$$

We know that enthalpy of air per kg of dry air,

$$\begin{aligned} h &= 1.022\, t_d + W\,(h_{fgdp} + 2.3\, t_{dp}) \\ &= 1.022 \times 30 + 0.008\,(2476.5 + 2.3 \times 10.6) \\ &= 30.66 + 20 = 50.66 \text{ kJ/kg of dry air} \quad \textbf{Ans.} \end{aligned}$$

Volume of the mixture per kg of dry air

From psychrometric tables, we find that specific volume of the dry air at 760 mm of Hg and 30°C dry bulb temperature is 0.8585 m³/kg of dry air. We know that one kg of dry air at a partial pressure of (756 – 9.62) mm of Hg occupies the same volume as W = 0.008 kg of vapour at its partial pressure of 9.62 mm of Hg. Moreover, the mixture occupies the same volume but at a total pressure of 756 mm of Hg.

∴ Volume of the mixture (v) at a dry bulb temperature of 30°C and a pressure of 9.62 mm of Hg

= Volume of 1 kg of dry air (v_a) at a pressure of (756 – 9.62) or 746.38 mm of Hg

$$= 0.8585 \times \frac{760}{746.38} = 0.8741 \text{ kg/kg of dry air} \quad \textbf{Ans.}$$

Note : The volume of mixture per kg of dry air may be calculated as discussed below :

We know that $\quad v = v_a = \dfrac{R_a\, T_d}{p_a}$

where

$$\begin{aligned} R_a &= \text{Gas constant for air} = 287 \text{ J/kg K} \\ T_d &= \text{Dry bulb temperature in K} \\ &= 30 + 273 = 303 \text{ K, and} \\ p_a &= \text{Pressure of air in N/m}^2 \\ &= p_b - p_v = 756 - 9.62 = 746.38 \text{ mm of Hg} \\ &= 746.38 \times 133.3 = 994\,92 \text{ N/m}^2 \end{aligned}$$

Substituting the values in the above equation,

$$v = \frac{287 \times 303}{994\,92} = 0.8741 \text{ m}^3\text{/kg of dry air} \quad \textbf{Ans.}$$

Example 16.3. *The humidity ratio of atmospheric air at 28°C dry bulb temperature and 760 mm of mercury is 0.016 kg / kg of dry air. Determine: 1. partial pressure of water vapour ; 2. relative humidity ; 3. dew point temperature ; 4. specific enthalpy; and 5. vapour density.*

Solution. Given : $t_d = 28°C$; $p_b = 760$ mm of Hg ; $W = 0.016$ kg/kg of dry air

1. Partial pressure of water vapour

Let p_v = Partial pressure of water vapour.

We know that humidity ratio (W),

$$0.016 = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622\, p_v}{760 - p_v}$$

$$12.16 - 0.016\, p_v = 0.622\, p_v \quad \text{or} \quad 0.638\, p_v = 12.16$$

$$\therefore \quad p_v = 12.16/0.638 = 19.06 \text{ mm of Hg}$$

$$= 19.06 \times 133.3 = 2540.6 \text{ N/m}^2 \quad \textbf{Ans.}$$

2. Relative humidity

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 28°C is

$$p_s = 0.03778 \text{ bar} = 3778 \text{ N/m}^2$$

∴ Relative humidity,

$$\phi = \frac{p_v}{p_s} = \frac{2540.6}{3778} = 0.672 \text{ or } 67.2\% \quad \textbf{Ans.}$$

3. Dew point temperature

Since the dew point temperature is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore from steam tables, we find that corresponding to a pressure of 2540.6 N/m² (0.025406 bar), the dew point temperature is,

$$t_{dp} = 21.1° \text{ C} \quad \textbf{Ans.}$$

4. Specific enthalpy

From steam tables, latent heat of vaporisation of water corresponding to a dew point temperature of 21.1° C,

$$h_{fgdp} = 2451.76 \text{ kJ/kg}$$

We know that specific enthalpy,

$$h = 1.022\, t_d + W\,(h_{fgdp} + 2.3\, t_{dp})$$

$$= 1.022 \times 28 + 0.016\,(2451.76 + 2.3 \times 21.1)$$

$$= 28.62 + 40 = 68.62 \text{ kJ/kg of dry air} \quad \textbf{Ans.}$$

5. Vapour density

We know that vapour density,

$$\rho_v = \frac{W\,(p_b - p_v)}{R_a\, T_d} = \frac{0.016\,(760 - 19.06)\,133.3}{287\,(273 + 28)}$$

$$= 0.0183 \text{ kg/m}^3 \text{ of dry air} \quad \textbf{Ans.}$$

Example 16.4. *A room 7 m × 4 m × 4 m is occupied by an air-water vapour mixture at 38°C. The atmospheric pressure is 1 bar and the relative humidity is 70%. Determine the humidity ratio, dew point, mass of dry air and mass of water vapour. If the mixture of air-water vapour is further cooled at constant pressure until the temperature is 10°C, find the amount of water vapour condensed.*

Solution. Given : $v = 7 \times 4 \times 4 = 112\ m^3$; $T_d = 38°C = 38 + 273 = 311$ K ; $p_b = 1$ bar ; $\phi = 70\% = 0.7$

Humidity ratio

First of all, let us find the partial pressure of water vapour (p_v). From steam tables, we find that saturation pressure of vapour corresponding to a temperature of 38° C is,

$$p_s = 0.066\ 24 \text{ bar}$$

We know that relative humidity (ϕ),

$$0.7 = \frac{p_v}{p_s}$$

$$\therefore \quad p_v = 0.7\ p_s = 0.7 \times 0.066\ 24 = 0.046\ 368 \text{ bar}$$

We also know that humidity ratio,

$$W = \frac{0.622\ p_v}{p_b - p_v} = \frac{0.622 \times 0.046\ 368}{1 - 0.046\ 368}$$

$$= \frac{0.028\ 841}{0.953\ 632} = 0.0302 \text{ kg/kg of dry air}$$

$$= 30.2 \text{ g/kg of dry air} \quad \textbf{Ans.}$$

Dew point temperature

Since the dew point temperature (t_{dp}) is the saturation temperature corresponding to the partial pressure of water vapour (p_v), therefore, from steam tables, we find that corresponding to a pressure of 0.046 368 bar, the dew point temperature is

$$t_{dp} = 31.56° \text{ C} \quad \textbf{Ans.}$$

Mass of dry air

Let $\quad m_a$ = Mass of dry air, and

p_a = Pressure of dry air = $p_b - p_v$

$= 1 - 0.046\ 368 = 0.953\ 632$ bar

$= 0.953\ 632 \times 10^5 = 95363.2\ N/m^2$ ... ($\because$ 1 bar = $10^5\ N/m^2$)

We know that $\quad p_a\, v = m_a\, R_a\, T_d$

$$\therefore \quad m_a = \frac{p_a\, v}{R_a\, T_d} = \frac{953\,63.2 \times 112}{287 \times 311} = 119.7 \text{ kg} \quad \textbf{Ans.}$$

... (Taking R_a = 287 J/kg K)

Mass of water vapour

Let $\quad m_v$ = Mass of water vapour.

We know that humidity ratio (W),

$$0.0302 = \frac{m_v}{m_a} = \frac{m_v}{119.7} \quad \text{or} \quad m_v = 0.0302 \times 119.7 = 3.61 \text{ kg} \quad \textbf{Ans.}$$

Amount of water vapour condensed

If the temperature is 10° C, the air will be saturated before some water condenses out. From steam tables, we find that saturation pressure of vapour corresponding to 10° C is

$$p_s = p_v = 0.012\ 27 \text{ bar} \qquad \text{... } (\because \text{ Pressure is constant})$$

We know that humidity ratio,

$$W = \frac{0.622\, p_v}{p_b - p_v} = \frac{0.622 \times 0.012\ 27}{1 - 0.012\ 27} = \frac{0.007\ 632}{0.987\ 73}$$

$$= 0.007\ 73 \text{ kg/kg of dry air} = 7.73 \text{ g/kg of dry air}$$

We know that pressure of dry air,

$$p_a = p_b - p_v = 1 - 0.012\ 27 = 0.987\ 73 \text{ bar}$$

$$= 0.987\ 73 \times 10^5 = 987\ 73 \text{ N/m}^2$$

∴ Mass of dry air, $m_a = \dfrac{p_a\, v}{R_a\, T} = \dfrac{987\ 73 \times 112}{287\ (10 + 273)} = 136.2 \text{ kg}$

and mass of water vapour, $m_v = W \times m_a = 0.007\ 73 \times 136.2 = 1.053 \text{ kg}$

∴ Amount of water vapour condensed

$$= 3.61 - 1.053 = 2.557 \text{ kg} \quad \textbf{Ans.}$$

16.6 Thermodynamic Wet Bulb Temperature or Adiabatic Saturation Temperature

The thermodynamic wet bulb temperature or adiabatic saturation temperature is the temperature at which the air can be brought to saturation state, adiabatically, by the evaporation of water into the flowing air.

The equipment used for the adiabatic saturation of air, in its simplest form, consists of an insulated chamber containing adequate quantity of water. There is also an arrangement for extra water (known as make-up water) to flow into the chamber from its top, as shown in Fig. 16.4.

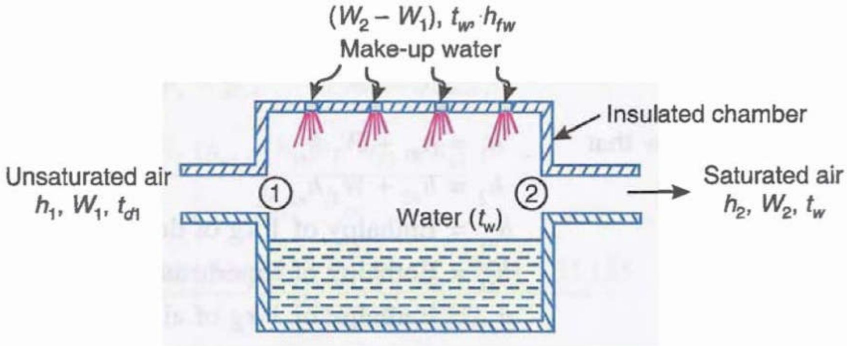


Fig. 16.4. Adiabatic saturation of air.

Let the unsaturated air enters the chamber at section 1. As the air passes through the chamber over a long sheet of water, the water evaporates which is carried with the flowing stream of air, and the specific humidity of the air increases. The make-up water is added to the chamber at this temperature to make the water level constant. Both the air and water are cooled as the evaporation takes place. This process continues until the energy transferred from the air to the water is equal to the energy required to vaporise the water. When steady conditions are reached, the air flowing at section 2 is saturated with water vapour. The temperature of the saturated air at section 2 is known as *thermodynamic wet bulb temperature* or *adiabatic saturation temperature*.

The adiabatic saturation process can be represented on *T-s* diagram as shown by the curve 1-2 in Fig. 16.5.

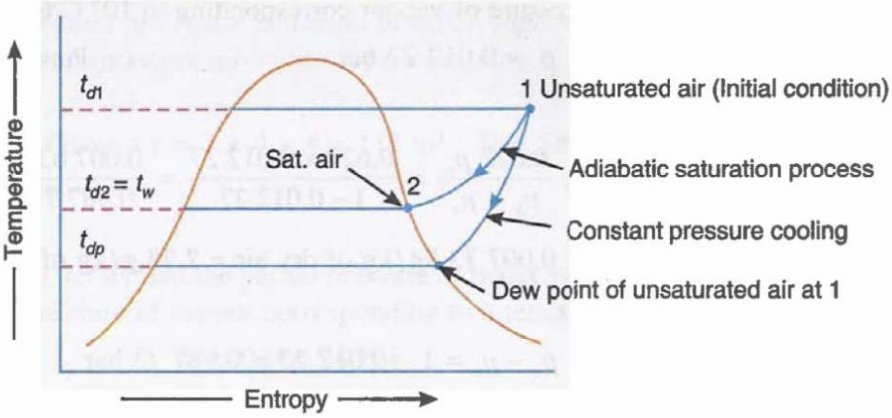


Fig. 16.5. *T-s* diagram for adiabatic saturation process.

During the adiabatic saturation process, the partial pressure of vapour increases, although the total pressure of the air-vapour mixture remains constant. The unsaturated air initially at dry bulb temperature t_{d1} is cooled adiabatically to dry bulb temperature t_{d2} which is equal to the adiabatic saturation temperature t_w. It may be noted that the adiabatic saturation temperature is taken equal to the wet bulb temperature for all practical purposes.

Let h_1 = Enthalpy of unsaturated air at section 1,

W_1 = Specific humidity of air at section 1,

h_2, W_2 = Corresponding values of saturated air at section 2, and

h_{fw} = Sensible heat of water at adiabatic saturation temperature.

Balancing the enthalpies of air at inlet and outlet (*i.e.* at sections 1 and 2),

$$h_1 + (W_2 - W_1)\, h_{fw} = h_2 \quad \ldots (i)$$

or

$$h_1 - W_1\, h_{fw} = h_2 - W_2\, h_{fw} \quad \ldots (ii)$$

The term $(h_2 - W_2\, h_{fw})$ is known as *sigma heat* and remains constant during the adiabatic process.

We know that $h_1 = h_{a1} + W_1\, h_{s1}$

and $h_2 = h_{a2} + W_2\, h_{s2}$

where h_{a1} = Enthalpy of 1 kg of dry air at dry bulb temperature t_{d1},

*h_{s1} = Enthalpy of superheated vapour at t_{d1} per kg of vapour,

h_{a2} = Enthalpy of 1 kg of air at wet bulb temperature t_w, and

h_{s2} = Enthalpy of saturated vapour at wet bulb temperature t_w per kg of vapour.

Now the equation (*ii*) may be written as :

$$(h_{a1} + W_1\, h_{s1}) - W_1\, h_{fw} = (h_{a2} + W_2\, h_{s2}) - W_2\, h_{fw}$$

$$W_1\,(h_{s1} - h_{fw}) = W_2\,(h_{s2} - h_{fw}) + h_{a2} - h_{a1}$$

$$\therefore \quad W_1 = \frac{W_2\,(h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}}$$

* In psychrometry, the enthalpy of superheated vapour at dry bulb temperature t_{d1} is taken equal to the enthalpy of saturated vapour corresponding to dry bulb temperature t_{d1}.

Example 16.5. *Atmospheric air at 0.965 bar enters the adiabatic saturator. The wet bulb temperature is 20° C and dry bulb temperature is 31° C during adiabatic saturation process. Determine : 1. humidity ratio of the entering air ; 2. vapour pressure and relative humidity at 31° C ; and 3. dew point temperature.*

Solution. Given : $p_b = 0.965$ bar ; $t_w = 20°$ C ; $t_d = 31°$ C

1. Humidity ratio of the entering air

Let W_1 = Humidity ratio of the entering air, and

W_2 = Humidity ratio of the saturated air.

First of all, let us find the value of W_2. From psychrometric or steam tables, we find that saturation pressure of vapour at 20° C,

$$p_{v2} = 0.023\ 37 \text{ bar}$$

Enthalpy of saturated vapour at 20° C,

$$h_{s2} = h_{g2} = 2538.2 \text{ kJ/kg}$$

Sensible heat of water at 20° C,

$$h_{fw} = 83.9 \text{ kJ/kg}$$

and enthalpy of saturated vapour at 31° C,

$$h_{s1} = h_{g1} = 2558.2 \text{ kJ/kg}$$

We know that enthalpy of unsaturated air corresponding to dry bulb temperature of 31° C,

$$h_{a1} = m\, c_p\, t_d = 1 \times 1.005 \times 31 = 31.155 \text{ kJ/kg}$$

... (Taking c_p for air = 1.005 kJ/kg°C)

Enthalpy of 1 kg of saturated air corresponding to wet bulb temperature of 20° C,

$$h_{a2} = m\, c_p\, t_w = 1 \times 1.005 \times 20 = 20.1 \text{ kJ/kg}$$

We know that
$$W_2 = \frac{0.622\, p_{v2}}{p_b - p_{v2}} = \frac{0.622 \times 0.023\,37}{0.965 - 0.023\,37} = 0.0154 \text{ kg/kg of dry air}$$

$\therefore$
$$W_1 = \frac{W_2\,(h_{s2} - h_{fw}) + h_{a2} - h_{a1}}{h_{s1} - h_{fw}}$$

$$= \frac{0.0154\,(2538.2 - 83.9) + 20.1 - 31.155}{2558.2 - 83.9}$$

$$= 0.0108 \text{ kg/kg of dry air} \quad \textbf{Ans.}$$

2. Vapour pressure and relative humidity at 31° C

Let p_{v1} = Vapour pressure at 31° C.

We know that humidity ratio of the entering air (W_1),

$$0.0108 = \frac{0.622\, p_{v1}}{p_b - p_{v1}} = \frac{0.622\, p_{v1}}{0.965 - p_{v1}}$$

or
$$0.0104 - 0.0108\, p_{v1} = 0.622\, p_{v1}$$

$\therefore$
$$0.6328\, p_{v1} = 0.0104 \quad \text{or} \quad p_{v1} = 0.0164 \text{ bar} \quad \textbf{Ans.}$$

From psychrometric or steam tables, we find that the saturation pressure corresponding to 31° C is

$$p_s = 0.044\,91 \text{ bar}$$

∴ Relative humidity,

$$\phi = \frac{p_{v1}}{p_s} = \frac{0.0164}{0.044\,91} = 0.365 \text{ or } 36.5\% \text{ Ans.}$$

3. *Dew point temperature*

Since the dew point temperature (t_{dp}) is the saturation temperature corresponding to the partial pressure of water vapour (p_{v1}), therefore from psychrometric or steam tables, we find that corresponding to a pressure of 0.0164 bar, the dew point temperature is

$$t_{dp} = 14.5° \text{ C Ans.}$$

16.7 Psychrometric Chart

It is a graphical representation of the various thermodynamic properties of moist air. The psychrometric chart is very useful for finding out the properties of air (which are required in the field of air conditioning) and eliminate lot of calculations. There is a slight variation in the charts prepared by different air-conditioning manufactures but basically they are all alike. The psychrometric chart is normally drawn for standard atmospheric pressure of 760 mm of Hg (or 1.01325 bar).

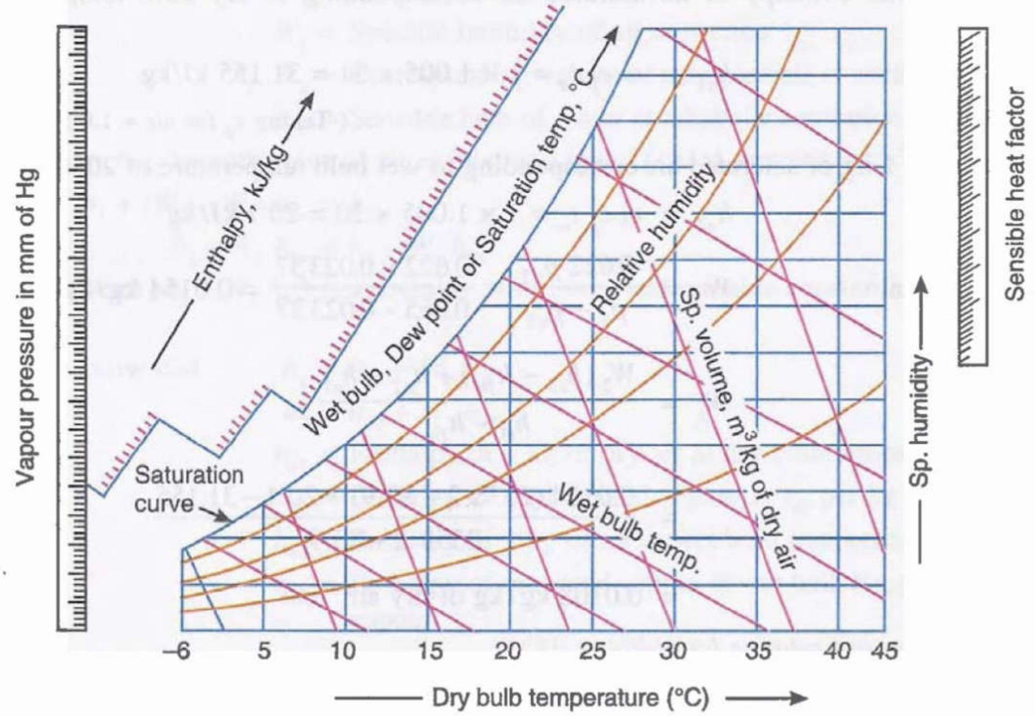


Fig. 16.6. Psychrometric chart.

In a psychrometric chart, dry bulb temperature is taken as abscissa and specific humidity *i.e.* moisture contents as ordinate, as shown in Fig. 16.6. Now the saturation curve is drawn by plotting the various saturation points at corresponding dry bulb temperatures. The saturation curve represents 100% relative humidity at various dry bulb temperatures. It also represents the wet bulb and dew point temperatures.

Though the psychrometric chart has a number of details, yet the following lines are important from the subject point of view :

1. *Dry bulb temperature lines.* The dry bulb temperature lines are vertical *i.e.* parallel to the ordinate and uniformly spaced as shown in Fig. 16.7. Generally the temperature range of these lines on psychrometric chart is from – 6° C to 45° C. The dry bulb temperature lines are drawn with difference of every 5°C and up to the saturation curve as shown in the figure. The values of dry bulb temperatures are also shown on the saturation curve.

2. *Specific humidity or moisture content lines.* The specific humidity (moisture content) lines are horizontal *i.e.* parallel to the abscissa and are also uniformly spaced as shown in Fig. 16.8. Generally, moisture content range of these lines on psychrometric chart is from 0 to 30 g / kg of dry air (or from 0 to 0.030 kg / kg of dry air). The moisture content lines are drawn with a difference of every 1 g (or 0.001 kg) and up to the saturation curve as shown in the figure.

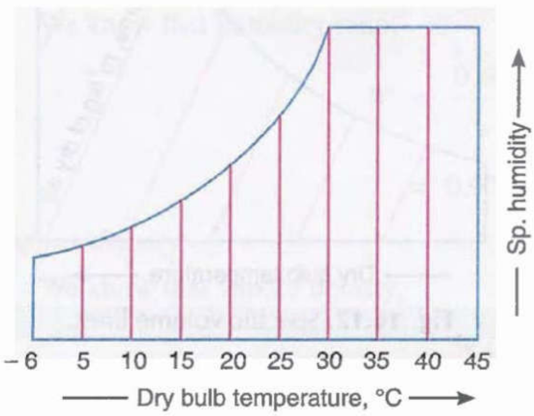


Fig. 16.7. Dry bulb temperature lines.

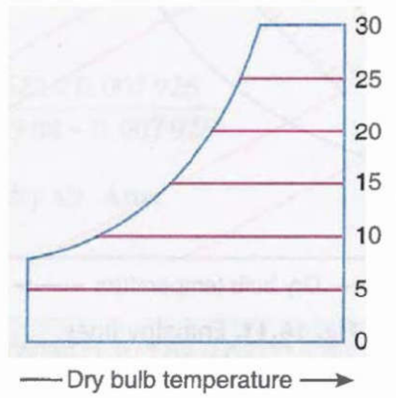


Fig. 16.8. Specific humidity lines.

3. *Dew point temperature lines.* The dew point temperature lines are horizontal *i.e.* parallel to the abscissa and non-uniformly spaced as shown in Fig. 16.9. At any point on the saturation curve, the dry bulb and dew point temperatures are equal.

The values of dew point temperatures are generally given along the saturation curve of the chart as shown in the figure.

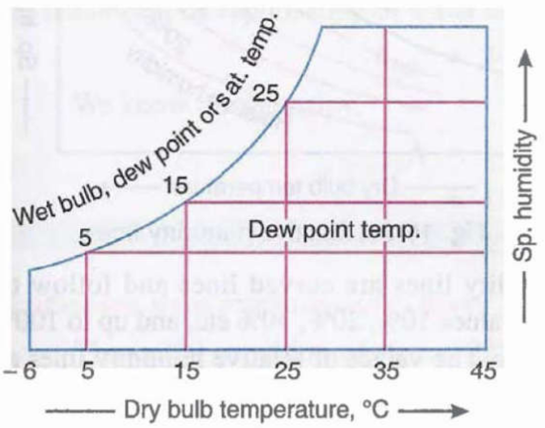


Fig. 16.9. Dew point temperature lines.

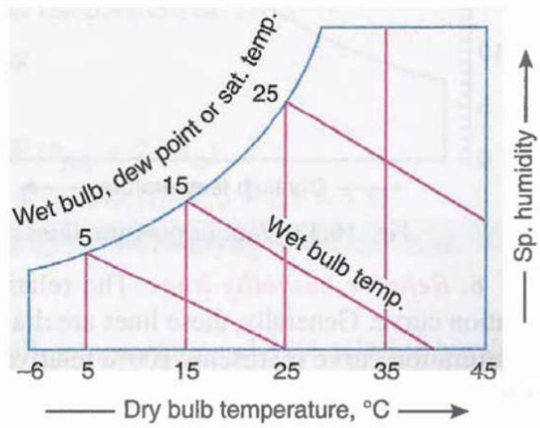


Fig. 16.10. Wet bulb temperature lines.

4. *Wet bulb temperature lines.* The wet bulb temperature lines are inclined straight lines and non-uniformly spaced as shown in Fig. 16.10. At any point on the saturation curve, the dry bulb and wet bulb temperatures are equal.

The values of wet bulb temperatures are generally given along the saturation curve of the chart as shown in the figure.

5. *Enthalpy (total heat) lines.* The enthalpy (or total heat) lines are inclined straight lines and uniformly spaced as shown in Fig. 16.11. These lines are parallel to the wet bulb temperature lines, and are drawn up to the saturation curve. Some of these lines coincide with the wet bulb temperature lines also.

The values of total enthalpy are given on a scale above the saturation curve as shown in the figure.

6. *Specific volume lines.* The specific volume lines are obliquely inclined straight lines and uniformly spaced as shown in Fig. 16.12. These lines are drawn up to the saturation curve.

The values of volume lines are generally given at the base of the chart.

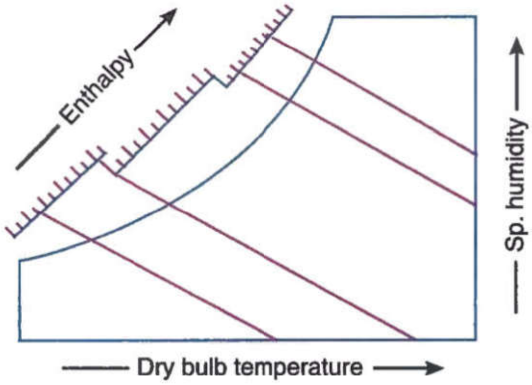


Fig. 16.11. Enthalpy lines.

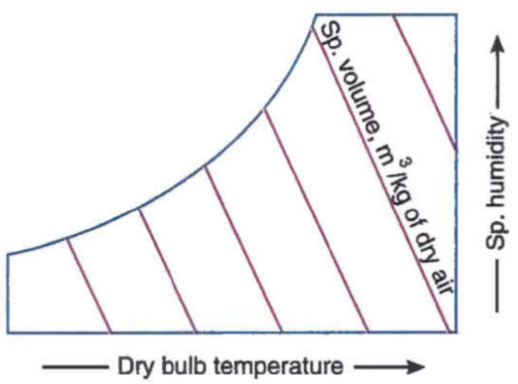


Fig. 16.12. Specific volume lines.

7. *Vapour pressure lines.* The vapour pressure lines are horizontal and uniformly spaced. Generally, the vapour pressure lines are not drawn in the main chart. But a scale showing vapour pressure in mm of Hg is given on the extreme left side of the chart as shown in Fig. 16.13.

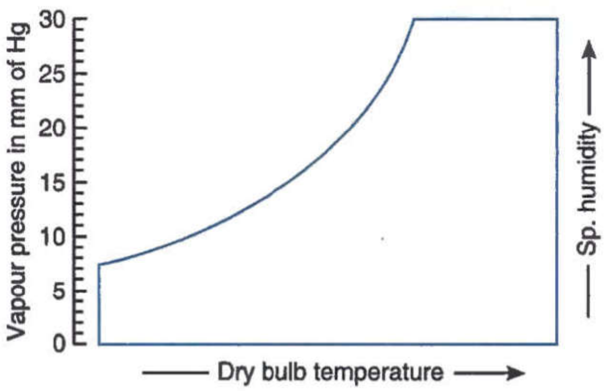


Fig. 16.13. Vapour pressure lines.

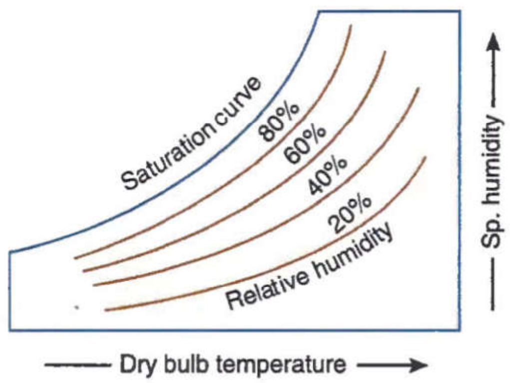


Fig. 16.14. Relative humidity lines.

8. *Relative humidity lines.* The relative humidity lines are curved lines and follow the saturation curve. Generally, these lines are drawn with values 10%, 20%, 30% etc. and up to 100%. The saturation curve represents 100% relative humidity. The values of relative humidity lines are generally given along the lines themselves as shown in Fig. 16.14.

Example 16.6. *For a sample of air having 22°C DBT, relative humidity 30 per cent at barometric pressure of 760 mm of Hg, calculate : 1. Vapour pressure, 2. Humidity ratio, 3. Vapour density, and 4. Enthalpy.*

Verify your results by psychrometric chart.

Solution. Given : t_d = 22° C ; ϕ = 30% = 0.3 ; p_b = 760 mm of Hg = 760 × 133.3 = 101 308 N/m² = 1.01308 bar

1. *Vapour pressure*

Let p_v = Vapour pressure.

From steam tables, we find that the saturation pressure of vapour corresponding to dry bulb temperature of 22° C is

$$p_s = 0.026\,42 \text{ bar}$$

We know that relative humidity (ϕ),

$$0.3 = \frac{p_v}{p_s} = \frac{p_v}{0.026\,42}$$

$$\therefore \quad p_v = 0.3 \times 0.026\,42 = 0.007\,926 \text{ bar} \quad \textbf{Ans.}$$

2. *Humidity ratio*

We know that humidity ratio,

$$W = \frac{0.622\,p_v}{p_b - p_v} = \frac{0.622 \times 0.007\,926}{1.013\,08 - 0.007\,926}$$

$$= 0.0049 \text{ kg/kg of dry air} \quad \textbf{Ans.}$$

3. *Vapour density*

We know that vapour density,

$$\rho_v = \frac{W\,(p_b - p_v)}{R_a\,T_d} = \frac{0.0049\,(1.013\,08 - 0.007\,926)\,10^5}{287\,(273 + 22)}$$

$$= 0.005\,82 \text{ kg/m}^3 \text{ of dry air} \quad \textbf{Ans.}$$

4. *Enthalpy*

From steam tables, we find that saturation temperature or dew point temperature corresponding to a pressure of $p_v = 0.007\,926$ bar is

$$t_{dp} = 3.8° \text{ C}$$

and latent heat of vaporisation of water at dew point temperature of 3.8° C is

$$h_{fgdp} = 2492.6 \text{ kJ/kg}$$

We know that enthalpy,

$$h = 1.022\,t_d + W\,(h_{fgdp} + 2.3\,t_{dp})$$

$$= 1.022 \times 22 + 0.0049\,(2492.6 + 2.3 \times 3.8)$$

$$= 22.484 + 12.256 = 34.74 \text{ kJ/kg of dry air} \quad \textbf{Ans.}$$

Verification from psychrometric chart

The initial condition of air *i.e.* 22° C dry bulb temperature and 30% relative humidity is marked on the psychrometric chart at point *A* as shown in Fig. 16.15.

From point *A*, draw a horizontal line meeting the vapour pressure line at point *B* and humidity ratio line at *C*. From the psychrometric chart, we find that vapour pressure at point *B*,

$$p_v = 5.94 \text{ mm of Hg}$$

$$= 5.94 \times 133.3 = 791.8 \text{ N/m}^2 = 0.007\,918 \text{ bar} \quad \textbf{Ans.}$$

and humidity ratio at point *C*,

$$W = 5 \text{ g/kg of dry air} = 0.005 \text{ kg/kg of dry air} \quad \textbf{Ans.}$$

From the psychrometric chart, we find that enthalpy at point 1,

$$h_1 = 78 \text{ kJ/kg of dry air}$$

Enthalpy at point 2, $h_2 = 39.4$ kJ/kg of dry air

Specific volume at point 1,

$$v_{s1} = 0.889 \text{ m}^3\text{/kg of dry air}$$

Specific volume at point 2,

$$v_{s2} = 0.826 \text{ m}^3\text{/kg of dry air}$$

Let h_3 = Enthalpy of air after mixing at point 3.

We know that mass of outside air at point 1,

$$m_1 = \frac{v_1}{v_{s1}} = \frac{20}{0.889} = 22.5 \text{ kg/min}$$

and mass of saturated air leaving the cooling section,

$$m_2 = \frac{v_2}{v_{s2}} = \frac{50}{0.826} = 60.53 \text{ kg / min}$$

We know that $\dfrac{m_1}{m_2} = \dfrac{h_3 - h_2}{h_1 - h_3}$ or $\dfrac{22.5}{60.53} = \dfrac{h_3 - 39.4}{78 - h_3}$

$$\therefore \quad 0.3717\,(78 - h_3) = h_3 - 39.4$$

or $h_3 = 49.86$ kJ / kg of dry air

Specific humidity, relative humidity, dry bulb temperature

Plot point 3 on the line joining the points 1 and 2 corresponding to enthalpy $h_3 = 49.86$ kJ/kg of dry air, as shown in Fig. 16.56.

From point 3 on the psychrometric chart, we find that specific humidity of the mixture at point 3,

$$W_3 = 0.0122 \text{ kg / kg of dry air} \quad \textbf{Ans.}$$

Relative humidity of the mixture at point 3,

$$\phi_3 = 90\% \quad \textbf{Ans.}$$

and dry bulb temperature of the mixture at point 3,

$$t_{d3} = 19°\text{C} \quad \textbf{Ans.}$$

Volume flow rate of the mixture

From the psychrometric chart, we find that specific volume of the mixture at point 3,

$$v_{s3} = 0.843 \text{ m}^3\text{/kg of dry air}$$

∴ Volume flow rate of the mixture at point 3,

$$v_3 = (m_1 + m_2)\, v_{s3} = (22.5 + 60.53)\, 0.843 = 70 \text{ m}^3\text{/min} \quad \textbf{Ans.}$$

EXERCISES

1. The atmospheric conditions of air are 25°C dry bulb temperature and specific humidity of 0.01 kg per kg of dry air. Find : 1. Partial pressure of vapour ; 2. Relative humidity ; and 3. Dew point temperature. [**Ans.** 0.016 bar ; 50.6% ; 14.1° C]
2. A sling psychrometer reads 40° C dry bulb temperature and 28° C wet bulb temperature. Calculate the following :

1. Specific humidity ; 2. Relative humidity ; 3. Vapour density in air ; 4. Dew point temperature ; 5. Enthalpy of mixture per kg of dry air.

[**Ans.** 0.019 kg / kg of dry air ; 40.7% ; 0.0208 kg/m^3 of dry air ; 24° C ; 88.38 kJ / kg of dry air]

3. The pressure and temperature of a mixture of dry air and water vapour are 736 mm of Hg and 21°C. The dew point temperature of the mixture is 15°C. Determine the following using steam tables :

1. Partial pressure of water vapour ; 2. Relative humidity ;
3. Specific humidity ; 4. Enthalpy of mixture per kg of dry air ;
5. Specific volume of the mixture per kg of dry air.

[**Ans.** 12.78 mm of Hg ; 68.53% ; 0.011 kg / kg of dry air ; 48.97 kJ/kg of dry air ; 0.875 m^3/kg of dry air]

4. A sample of air is having dry bulb temperature 21°C and relative humidity 30% at barometric pressure of 760 mm of Hg. Find : 1. Partial pressure of vapour ; 2. Specific humidity ; 3. Wet bulb temperature and corresponding saturation pressure ; 4. Precentage humidity or degree of saturation ; 5. Specific volume of dry air ; 6. Dew point temperature ; and 7. Enthalpy of moist air per kg of dry air.

Given : R = 0.287 kJ/kg K ; c_p(dry air) = 1.005 kJ/kg K ; specific heat of superheated vapour = 1.884 kJ/kg K and latent heat of vaporisation at dew point temperature = 2493 kJ/kg. Do not use psychrometric chart. Psychrometric tables can be used.

[**Ans.** 5.595 mm of Hg ; 0.00461 kg / kg of dry air ; 11.5°C, 10.258 mm of Hg ; 0.2945 ; 0.839 m^3/kg of dry air ; 3°C ; 32.812 kJ/kg of dry air]

5. A sample of moist air has a dry bulb temperature of 25° C and a relative humidity of 50 per cent. The barometric pressure is 740 mm of Hg. Calculate : 1. partial pressure of water vapour and dry air ; 2. dew point temperature and specific humidity of air ; 3. enthalpy of air per kg of dry air.

[**Ans.** 0.01583 bar ; 14° C ; 0.0101 kg / kg of dry air ; 50.81 kJ / kg of dry air]

6. The moist air exists at a total pressure of 1.01325 bar and 25° C dry bulb temperature. If the degree of saturation is 50%, determine the following using steam tables :

1. Specific humidity ; 2. Dew point temperature ; and 3. Specific volume of moist air.

[**Ans.** 10.03 g / kg of dry air ; 14° C ; 0.857 m^3 / kg]

7. The atmospheric conditions of air are 35° C dry bulb temperature, 60% relative humidity and 1.01325 bar pressure. If 0.005 kg of moisture per kg of dry air is removed, the temperature becomes 25° C. Determine the final relative humidity and dew point temperature. [**Ans.** 88.6% ; 23° C]

8. The atmospheric air enters the adiabatic saturator at 33° C dry bulb temperature and 23° C wet bulb temperature. The barometric pressure is 740 mm of Hg. Determine the specific humidity and vapour pressure at 33° C. [**Ans.** 0.012 kg / kg of dry air ; 13 mm of Hg]

9. The atmospheric air has 35° C dry bulb temperature and 50% relative humidity. Using psychrometric chart, find (*i*) wet bulb temperature, (*ii*) humidity ratio, (*iii*) dew point temperature, and (*iv*) enthalpy of air per kg of dry air. [**Ans.** 26.2° C ; 0.0178 kg / kg of dry air ; 23° C ; 81 kJ / kg of dry air]

10. The atmospheric air at 750 mm of Hg has 34° C dry bulb temperature and 19° C wet bulb temperature. Using psychrometric chart, find (*a*) partial pressure of vapour, (*b*) saturation pressure corresponding to 34° C, and (*c*) volume of air per kg of dry air.

[**Ans.** 8.5 mm of Hg ; 39.9 mm of Hg ; 0.088 m^3/kg of dry air]

11. The atmospheric air at 15° C dry bulb temperature and 80% relative humidity is supplied to the heating chamber at the rate of 100 m^3/min. The leaving air has a temperature of 22° C without change in its moisture contents. Determine the heat added to the air per min. and final relative humidity of the air. [**Ans.** 843 kJ /min ; 52%]

12. 100 m^3 of air at 35°C and 70% relative humidity is cooled to 20° C and 55% relative humidity by passing it through a cooling coil. Using psychrometric tables only, find : 1. the amount of water vapour removed in kg / h; and 2. the cooling capacity of the coil in tonnes of refrigeration. Assume specific heat of superheated vapour in air as 1.88 kJ/kg K and R for air = 287.14 J/kg K.

[**Ans.** 144 kg / h ; 37 TR]

CHAPTER

Comfort Conditions

17

17.1 Introduction

Strictly speaking, the human comfort depends upon physiological and psychological conditions. Thus it is difficult to define the term 'human comfort'. There are many definitions given for this term by different bodies. But the most accepted definition, from the subject point of view, is given by the American Society of Heating, Refrigeration and Air Conditioning Engineers (ASHRAE) which states : *human comfort is that condition of mind, which expresses satisfaction with the thermal environment.*

17.2 Thermal Exchanges of Body with Environment

The human body works best at a certain temperature, like any other machine, but it cannot tolerate wide range of variations in their environmental temperatures like machines. The human body maintains its thermal equilibrium with the environment by means of three modes of heat transfer *i.e.* evaporation, radiation and convection. The way in which the individual's body maintains itself in comfortable equilibrium will be by its automatic use of one or more of the three modes of heat transfer. A human body feels comfortable when the heat produced by metabolism of human body is equal to the sum of the heat dissipated to the surroundings and the heat stored in human body by raising the temperature of body tissues. This phenomenon may be represented by the following equation :

$$Q_M - W = Q_E \pm Q_R \pm Q_C \pm Q_S$$

where

Q_M = Metabolic heat produced within the body,

W = Useful rate of working,

$Q_M - W$ = Heat to be dissipated to the atmosphere,

Q_E = Heat lost by evaporation,

*Q_R = Heat lost or gained by radiation,

*Q_C = Heat lost or gained by convection, and

**Q_S = Heat stored in the body.

It may be noted that

1. The metabolic heat produced (Q_M) depends upon the rate of food energy consumption in the body. A fasting, weak or sick man, will have less metabolic heat production.

2. The heat loss by evaporation is always positive. It depends upon the vapour pressure difference between the skin surface and the surrounding air. The heat loss of evaporation (Q_E) is given by

$$Q_E = C_d A\,(p_s - p_v)\,h_{fg}\,C_c$$

where

C_d = Diffusion coefficient in kg of water evaporated per unit surface area and pressure difference per hour,

A = Skin surface area = 1.8 m^2 for normal man,

p_s = Saturation vapour pressure corresponding to skin temperature,

p_v = Vapour pressure of surrounding air,

h_{fg} = Latent heat of vaporisation = 2450 kJ / kg,

C_c = Factor which accounts for clothing worn.

The value of Q_E becomes zero when $p_s = p_v$, *i.e.* when the surrounding air temperature is equal to the skin temperature and air is saturated or when it is higher than the skin temperature and the air is nearly saturated.

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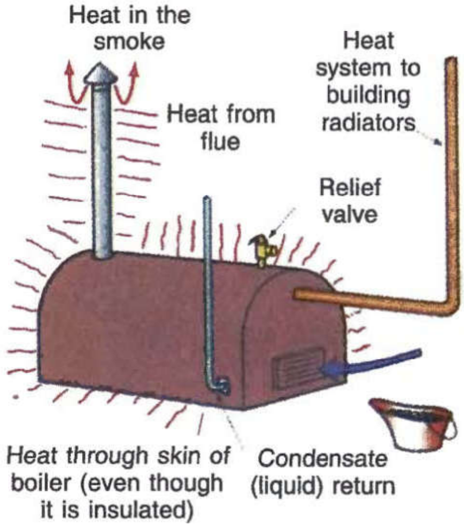
* The *plus* sign is used when heat is lost to the surroundings and *negative* sign is used when heat is gained from surroundings.

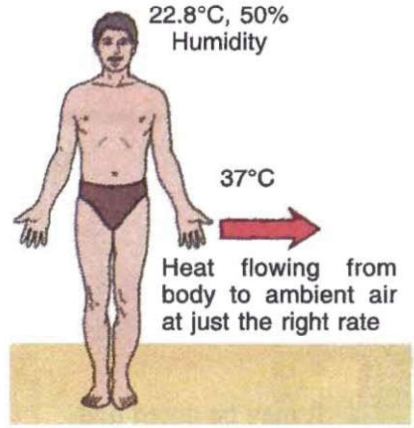
** The *plus* sign is used when the temperature of the body rises and *negative* sign is used when the temperature of the body falls.

The value of Q_E is never negative as when p_s is less than p_v, the skin will not absorb moisture from the surrounding air as it is in saturated state. The only way for equalising the pressure difference is by increasing p_s to p_v by rise of skin temperature from the sensible heat flow from air to skin.

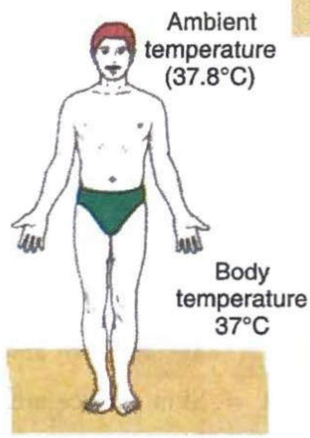
3. The heat loss or gain by radiation (Q_R) from the body to the surroundings depends upon the *mean radiant temperature*. It is the average surface temperature of the surrounding objects when properly weighted, and varies from place to place inside the room. When the mean radiant temperature is lower than the dry bulb temperature of air in the room, Q_R is *positive i.e.* the body will undergo a radiant heat loss. On the other hand, if the mean radiant temperature is higher than the dry bulb temperature of air in the room, Q_R is *negative i.e.* the body will undergo a radiant heat gain.



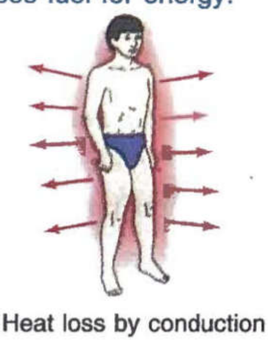
The boiler can be compared to the human body in that it uses fuel for energy.

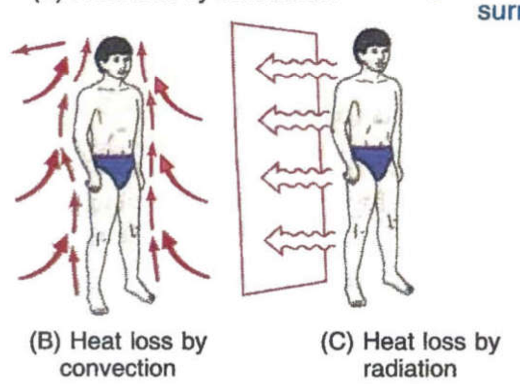


The body's normal temperature is 37°C

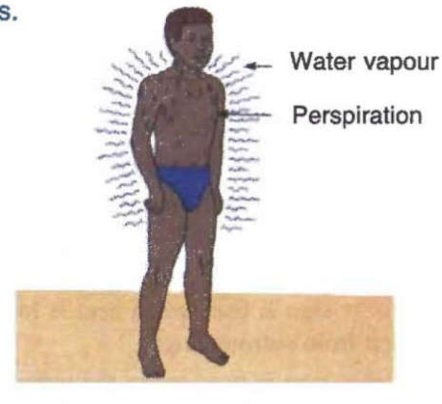


The human body must give off some of its generated heat to the surroundings.





The three direct ways the human body gives off heat (A) conduction (B) convection (C) radiation.



The evaporation of respiration.

4. The heat loss by convection (Q_C) from the body to the surroundings is given by

$$Q_C = UA\,(t_B - t_S)$$

where U = Body film coefficient of heat transfer,

A = Body surface area = 1.8 m^2 for normal man,

t_B = Temperature of the body, and

t_S = Temperature of the surroundings.

When the temperature of the surroundings (t_S) is higher than the temperature of the body (t_B), then Q_C will be *negative*, *i.e.* the heat will be gained by the body. On the other hand, if the temperature of the surroundings (t_S) is lower than the temperature of the body (t_B), then Q_C will be the *positive*, *i.e.* the heat will be lost by the body. Since the body film coefficient of heat transfer increases with the increase in air velocity, therefore higher air velocities will produce uncomfort when t_S is higher than t_B. The higher air velocities are recommended when t_S is lower than t_B.

5. When Q_E, Q_R and Q_C are high and positive and ($Q_E + Q_R + Q_C$) is greater than ($Q_M - W$), then the heat stored in the body (Q_S) will be *negative* *i.e.* the body temperature falls down . Thus the sick, weak, old or a fasting man feels more cold. On the other hand, a man gets fever when high internal body activities increases Q_M to such an extent so that Q_S becomes *positive* for the given Q_E, Q_R and Q_C.

The heat stored in the body has maximum and minimum limits which when exceeded brings death. The usual body temperature, for a normal man (when $Q_S = 0$) is 37°C (98.6°F). The temperature of the body when falls below 36.5°C (98°F) and exceeds 40.5°C (105°F) is dangerous. There is some kind of thermostatic control called *vasomotor control mechanism* in the human body which maintains the temperature of body at the normal level of 37°C, by regulating the blood supply to the skin. When the temperature of the body falls (*i.e.* the heat stored Q_S in the body is *negative*), then the vasomotor control decreases the circulation of blood which decreases conductivity of nerve cells and other tissues between the skin and the inner body cells. This allows skin temperature to fall but allows higher inner temperature of body cells beneath. When the temperature of the body rises (*i.e.* the heat stored Q_S in the body is *positive*) , then the vasomotor control increases blood circulation which increases conductivity of tissues and hence allows less temperature drop between the skin and inner body cells.

The human body feels comfortable when there is no change in the body temperature, *i.e.* when the heat stored in the body Q_S is zero. Any variation in the body temperature acts as a stress to the brain which ultimately results in either perspiration or shivering.

17.3 Physiological Hazards Resulting from Heat

In summer, the temperature of the surroundings is always higher than the temperature of the body. Thus the body will gain heat from the surroundings by means of radiation and convection processes. The body can dissipate heat only through evaporation of sweat. When the heat loss by evaporation is unable to cope with the heat gain, there will be storage of heat in the body and the temperature of body rises. Several physiological hazards exist, the severity of which depends upon the extent and time duration of body temperature rise. Following are some of the physiological hazards which may result due to the rise in body temperature.

1. *Heat exhaustion.* It is due to the failure of normal blood circulation. The symptoms of heat exhaustion include fatigue, headache, dizziness, vomiting and abnormal mental reactions such as irritability. Severe heat exhaustion may cause fainting . It does not cause permanent injury to the body and recovery is usually rapid when the person is removed to a cool place.

2. *Heat cramp.* It results from loss of salt due to an excessive rate of body perspiration. It causes severe pain in the calf and thigh muscles. The heat cramp may be largely avoided by using salt tablets.

3. *Heat stroke.* It is the most serious hazard. When a man is exposed to excessive heat and work, the body temperature may rise rapidly to 40.5°C (105°F) or higher. At such elevated temperatures, sweating ceases and the man may enter a coma, with death imminent. A person experiencing a heat stroke may have permanent damage to the brain. The heat stroke may be avoided by taking sufficient water at frequent intervals. It has been found that a man doing hard work in the sun requires about one litre of water per hour.

17.4 Factors Affecting Human Comfort

In designing winter or summer air conditioning system, the designer should be well conversant with a number of factors which physiologically affect human comfort. The important factors are as follows :

1. Effective temperature, 2. Heat production and regulation in human body, 3. Heat and moisture losses from the human body, 4. Moisture content of air, 5. Quality and quantity of air. 6. Air motion, 7. Hot and cold surfaces, and 8. Air stratification.

These factors are discussed, in detail, in the following articles :

17.5 Effective Temperature

The degree of warmth or cold felt by a human body depends mainly on the following three factors :

1. Dry bulb temperature, 2. Relative humidity, and 3. Air velocity.

In order to evaluate the combined effect of these factors, the term *effective temperature* is employed. It is defined as that index which corelates the combined effects of air temperature, relative humidity and air velocity on the human body. The numerical value of effective temperature is made equal to the temperature of still (*i.e.* 5 to 8 m/min air velocity) saturated air, which produces the same sensation of warmth or coolness as produced under the given conditions.

The practical application of the concept of effective temperature is presented by the *comfort chart*, as shown in Fig. 17.1. This chart is the result of research made on different kinds of people subjected to wide range of environmental temperature, relative humidity and air movement by the American Society of Heating, Refrigeration and Air conditioning Engineers (ASHRAE). It is applicable to reasonably still air (5 to 8 m/min air velocity) to situations where the occupants are seated at rest or doing light work and to spaces whose enclosing surfaces are at a mean temperature equal to the air dry bulb temperature.

In the comfort chart, as shown in Fig. 17.1, the dry bulb temperature is taken as abscissa and the wet bulb temperature as ordinates. The relative humidity lines are replotted from the psychrometric chart. The statistically prepared graphs corresponding to summer and winter season are also superimposed. These graphs have effective temperature scale as abscissa and % of people feeling comfortable as ordinate.

A close study of the chart reveals that the several combinations of wet and dry bulb temperatures with different relative humidities will produce the same *effective temperature.

* From the comfort chart, we see that for a point corresponding to dry bulb temperature of 17.5°C, wet bulb temperature of 12.5°C and relative humidity of 60%, the effective temperature is 16°C. Now for the same feeling of comfort and warmth, there is another point on 100% relative humidity line at which dry bulb temperature and wet bulb temperature are both equal to 16°C. Thus both have an effective temperature of 16°C.

However, all points located on a given effective temperature line do not indicate conditions of equal comfort or discomfort. The extremely high or low relative humidities may produce conditions of discomfort regardless of the existent effective temperature. The moist desirable relative humidity range lies between 30 and 70 per cent. When the relative humidity is much below 30 per cent, the mucous membranes and the skin surface become too dry for comfort and health. On the other hand, if the relative himidity is above 70 per cent, there is a tendency for a clammy or sticky sensation to develop. The curves at the top and bottom, as shown in Fig. 17.1, indicate the percentages of person participating in tests, who found various effective temperatures satisfactory for comfort.

The comfort chart shows the range for both summer and winter condition within which a condition of comfort exists for most people. For summer conditions, the chart indicates that a maximum of 98 percent people felt comfortable for an effective temperature of 21.6°C . For winter conditions, chart indicates that an effective temperature of 20°C was desired by 97.7 percent people. It has been found that for comfort, women require 0.5°C higher effective temperature than men. All men and women above 40 years of age prefer 0.5°C higher effective temperature than the persons below 40 years of age.

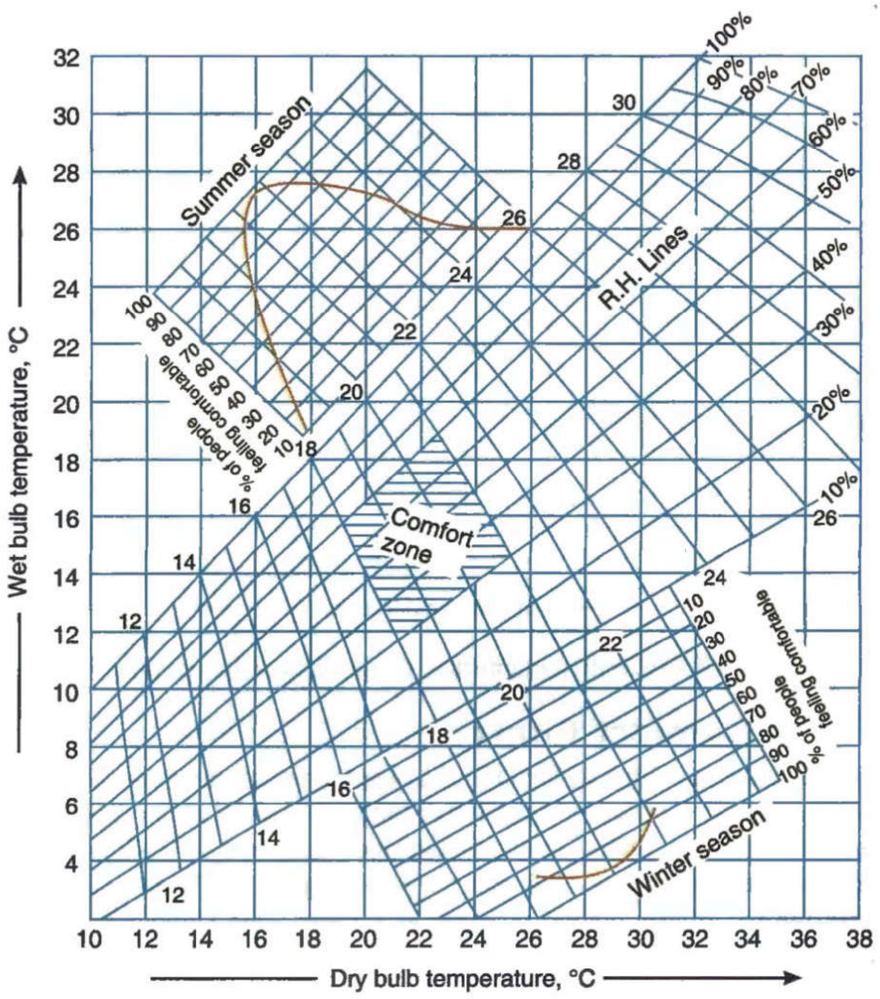


Fig. 17.1. Comfort chart for still air (air velocities from 5 to 8 m/min)

It may be noted that the comfort chart, as shown in Fig. 17.1, does not take into account the variations in comfort conditions when there are wide variations in the mean radiant temperature (MRT). In the range of 26.5°C, a rise of 0.5°C in mean radiant temperature above the room dry bulb temperature raises the effective temperature by 0.5°C. The effect of mean radiant temperature on comfort is less pronounced at high temperatures than at low temperatures.

The comfort conditions for persons at work vary with the rate of work and the amount of clothing worn. In general, the greater the degree of activity, the lower the effective temperature necessary for comfort.

Fig. 17.2 shows the variation in effective temperature with different air velocities. We see that for the atmospheric conditions of 24°C dry bulb temperature and 16°C wet bulb temperature correspond to about 21°C with nominally still air (velocity 6 m / min) and it is about 17°C at an air velocity of 210 m / min. The same effective temperature is observed at higher dry bulb and wet bulb temperatures with higher velocities. The case is reversed after 37.8°C as in that case higher velocities will increase sensible heat flow from air to body and will decrease comfort. The same effective temperature means same feeling of warmth, but it does not mean same comfort.

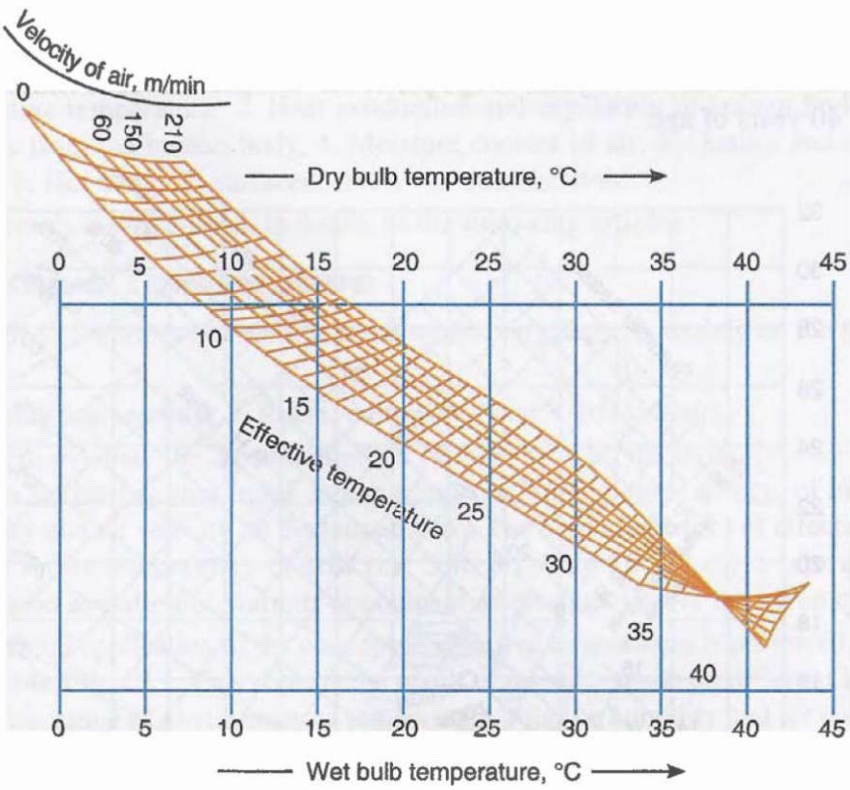


Fig. 17.2. Variation of effective temperature with air velocity.

17.6 Modified Comfort Chart

The comfort chart, as shown in Fig. 17.1,has become obsolete now-a-days due to its short comings of over exaggeration of humidity at lower temperature and under estimation of humidity at heat tolerance level. The modified comfort chart according to ASHRAE is shown in Fig.17.3 and it is commonly used these days. This chart was developed on the basis of research done in 1963 by the institute for environmental research at Kansas State University. The mean radiant temperature was kept equal to dry bulb temperature and air velocity was less than 0.17 m / s.

17.7 Heat Production and Regulation in Human Body

The human body acts like a heat engine which gets its energy from the combustion of food within the body. The process of combustion (called metabolism) produces heat and energy due to the oxidation of products in the body by oxygen obtained from inhaled air. The rate of heat production depends upon the individual's health, his physical activity and his environment. The rate at which the body produces heat is termed as *metabolic rate*. The heat production from a normal healthy person when asleep (called *basal metabolic rate*) is about 60 watts and it is about ten times more for a person carrying out sustained very hard work.

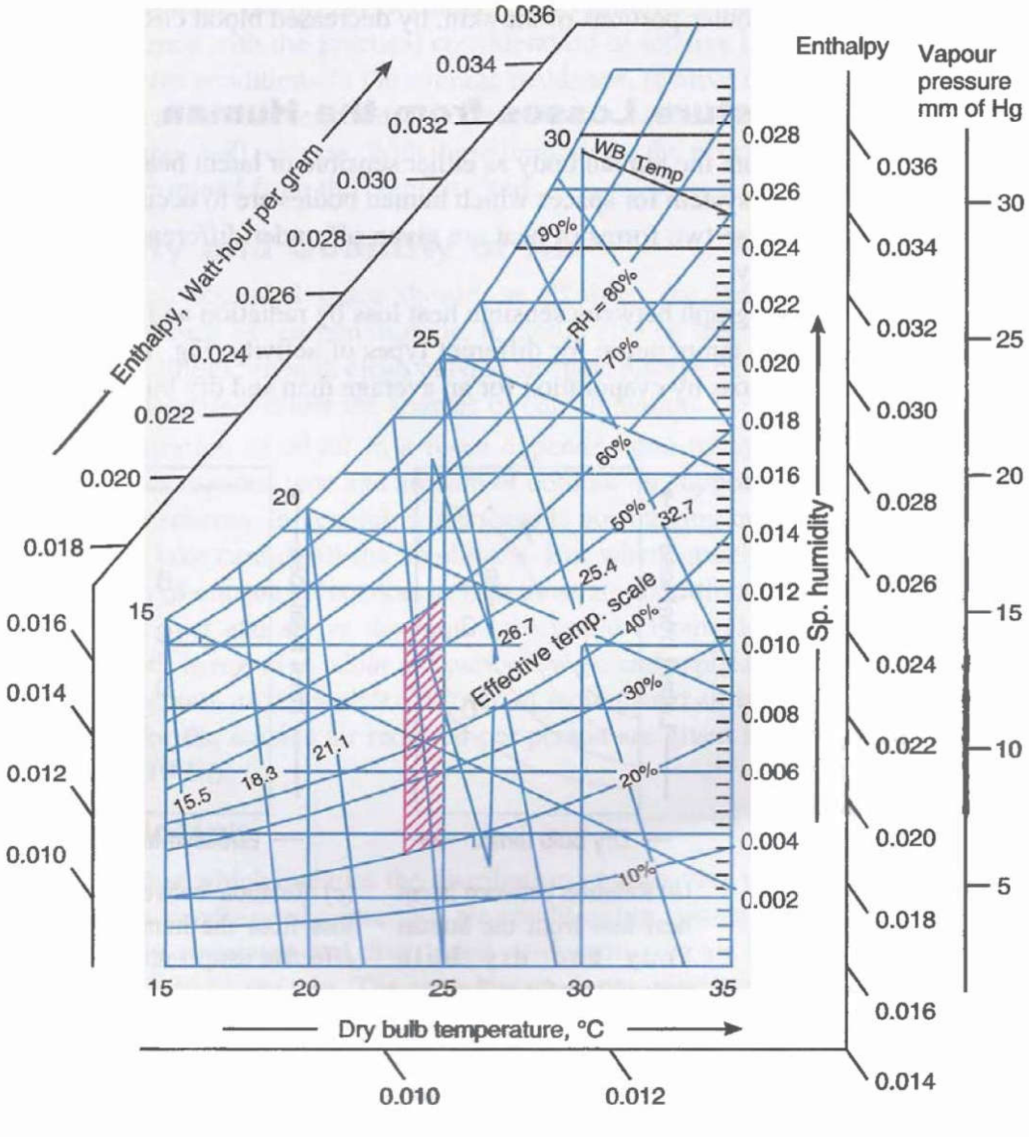


Fig. 17.3. Modified comfort chart.

Since the body has a thermal efficiency of 20 per cent, therefore the remaining 80 per cent of the heat must be rejected to the surrounding environment, otherwise accumulation of heat results which causes discomfort. The rate and the manner of rejection of heat is controlled by the automatic regulation system of a human body.

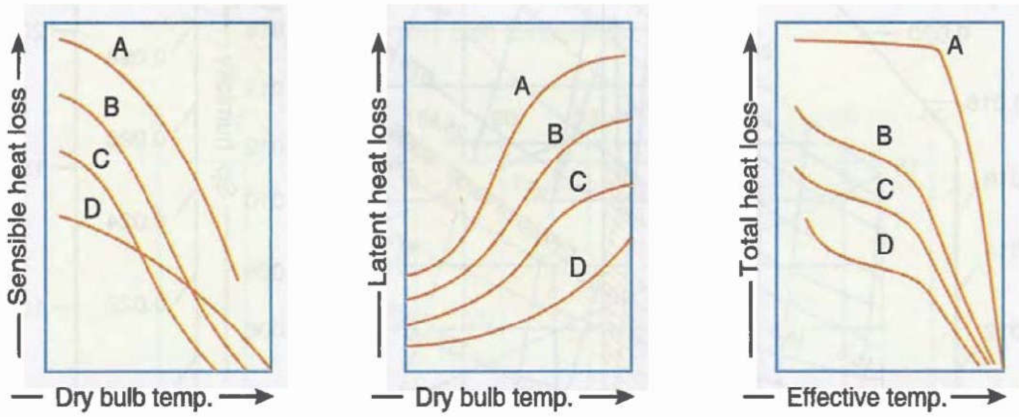
In order to effect the loss of heat from the body to produce cold, the body may react to bring more blood to the capillaries in the skin. The heat losses from the skin, now, may take place by radiation, convection and by evaporation. When the process of radiation or convection or both fails to produce necessary loss of heat, the sweat glands become more active and more moisture is deposited on the skin, carrying heat away as it evaporates . It may be noted that when the temperature of surrounding air and objects is below the blood temperature, the heat is removed by radiation and convection. On the other hand, when the temperature of surrounding air is above the blood temperature, the heat is removed by evaporation only. In case the body fails to throw off the requisite amount of heat, the blood temperature rises. This results in the accumulation of heat which will cause discomfort.

The human body attempts to maintain its temperature when exposed to cold by the withdrawal of blood from the outer portions of the skin, by decreased blood circulation and by an increased rate of metabolism.

17.8 Heat and Moisture Losses from the Human Body

The heat is given off from the human body as either sensible or latent heat or both. In order to design any air-conditioning system for spaces which human bodies are to occupy, it is necessary to know the rates at which these two forms of heat are given off under different conditions of air temperature and bodily activity.

Fig. 17.4 (*a*) shows the graph between sensible heat loss by radiation and convection for an average man and the dry bulb temperature for different types of activity. Fig. 17.4 (*b*) shows the graph between the latent heat loss by evaporation for an average man and dry bulb temperature for different type of activity.



(*a*) Relation between sensible heat loss from the human body and dry bulb temperature for still air.

(*b*) Relation between latent heat loss from the human body and dry bulb temperature for still air.

(*c*) Relation between total heat loss from the human body and effective temperature for still air.

Fig. 17.4

The total heat loss from the human body under varying effective temperatures is shown in Fig. 17.4 (*c*). From curve *D*, which applies to men at rest, we see that from about 19°C to 30°C effective temperature, the heat loss is constant. At the lower effective temperature, the heat dissipation increases which results in a feeling of coolness. At higher effective temperature, the ability to lose heat rapidly decreases resulting in severe discomfort. The curves *A, B, C* and *D* shown in Fig. 17.4 represents as follows :

Curve *A* — Men working at the rate of 90 kN-m / h

Curve *B* — Men working at the rate of 45 kN-m / h

Curve *C* — Men working at the rate of 22.5 kN-m / h

Curve *D* — Men at rest.

17.9 Moisture Content of Air

We have seen in Art. 17.5 that the dry bulb temperature, relative humidity and air motion are inter-related. The moisture content of outside air during winter is generally low and it is above the average during summer, because the capacity of the air to carry moisture is dependent upon its dry bulb temperature. This means that in winter, if the cold outside air having a low moisture content leaks into the conditioned space, it will cause a low relative humidity unless moisture is added to the air by the process of humidification. In summer, the reverse will take place unless moisture is removed from the inside air by the dehumidification process. Thus, while designing an air- conditioning system, the proper dry bulb temperature for either summer or winter must be selected in accordance with the practical consideration of relative humidities which are feasible. In general, for winter conditions in the average residence, relative humidities above 35 to 40 per cent are not practical. In summer comfort cooling, the air of the occupied space should not have a relative humidity above 60 per cent. With these limitations, the necessary dry bulb temperature for the air may be determined from the comfort chart.

17.10 Quality and Quantity of Air

The air in an occupied space should, at all times, be free from toxic, unhealthful or disagreeable fumes such as *carbon dioxide. It should also be free from dust and odour. In order to obtain these conditions, enough clean outside air must always be supplied to an occupied space to counteract or adequately dilute the sources of contamination.

The concentration of odour in a room depends upon many factors such as dietary and hygienic habits of occupants, type and amount of outdoor air supplied, room volume per occupant and types of odour sources. In general, when there is no smoking in a room, 1 m^3/min per person of outside air will take care of all the conditions. But when smoking takes place in a room, 1.5 m^3/min per person of outside air is necessary. In most air-conditioning systems, a large amount of air is recirculated over and above the required amount of outside air to satisfy the minimum ventilation conditions in regard to odour and purity. For general application, a minimum of 0.3 m^3/min of outside air per person, mixed with 0.6 m^3/min of recirculated air is good. The recommended and minimum values for the outside air required per person are given in Chapter 19 on cooling load estimation (Table 19.10).

17.11 Air Motion

The air motion which includes the distribution of air is very important to maintain uniform temperature in the conditioned space. No air conditioning system is satisfactory unless the air handled is properly circulated and distributed. Ordinarily, the air velocity in the occupied zone should not exceed 8 to 12 m / min. The air velocities in the space above the occupied zone should be very high in order to produce good distribution of air in the occupied zone, provided that the air in motion does not produce any objectionable noise. The flow of air should be preferably towards the faces of the individuals rather than from the rear in the occupied zone. Also for the proper and perfect distribution of air in the air-conditioned space, down flow should be preferred instead of up flow.

The air motion without proper air distribution produces local cooling sensation known as *draft*.

17.12 Cold and Hot Surfaces

The cold or hot objects in a conditioned space may cause discomfort to the occupants. A single glass of large area when exposed to the outdoor air during winter will produce discomfort

* The atmospheric air contains 0.03% to 0.04% by volume of carbon dioxide and it should not increase 0.6% which is necessary for proper functioning of respiratory system. The carbon dioxide, in excess of 2% dilutes oxygen contents and makes breathing difficult. When the carbon dioxide exceeds 6%, breathing is very difficult and 10% carbon dioxide causes loss of consciousness. A normal man at rest in breathing, exhales about 0.015 to 0.018 m^3/h of carbon dioxide.

to the occupants of a room by absorbing heat from them by radiation. On the other hand, a ceiling that is warmer than the room air during summer causes discomfort. Thus, in the designing of an air conditioning system, the temperature of the surfaces to which the body may be exposed must be given considerable importance.

A jump duct allows for balanced pressure between rooms and improved comfort conditions.

17.13 Air Stratification

When air is heated, its density decreases and thus it rises to the upper part of the confined space. This results in a considerable variation in the temperatures between the floor and ceilng levels. The movement of the air to produce the temperature gradient from floor to ceiling is termed as *air stratification.* In order to achieve comfortable conditions in the occupied space, the air conditioning system must be designed to reduce the air stratification to a minimum.

17.14 Factors Affecting Optimum Effective Temperature

The important factors which affect the optimum effective temperature are as follows :

1. *Climatic and seasonal differences.* It is a known fact that the people living in colder climates feel comfortable at a lower effective temperatures than those living in warmer regions. There is a relationship between the optimum indoor effective temperature and the optimum outdoor temperature, which changes with seasons. We see from the comfort chart (Fig. 17.1) that in winter, the optimum effective temperature is 19°C whereas in summer this temperature is 22°C.

2. *Clothing.* It is another important factor which affects the optimum effective temperature. It may be noted that the person with light clothings need less optimum temperature than a person with heavy clothings.

3. *Age and Sex.* We have already discussed that the women of all ages require higher effective temperature (about 0.5°C) than men. Similar is the case with young and old people. The children also need higher effective temperature than adults. Thus, the maternity halls are always kept at an effective temperature of 2 to 3°C higher than the effective temperature used for adults.

4. *Duration of stay.* It has been established that if the stay in a room is shorter (as in the case of persons going to banks), then higher effective temperature is required than that needed for long stay (as in the case of persons working in an office).

5. *Kind of activity.* When the activity of the person is heavy such as people working in a factory, dancing hall, then low effective temperature is needed than for the people sitting in cinema hall or auditorium.

6. *Density of occupants.* The effect of body radiant heat from person to person particularly in a densely occupied space like auditorium is large enough which require a slight lower effective temperature.

17.15 Inside Summer Design Conditions

The following table shows the recommended inside design conditions for summer comfort cooling :

Table 17.1. Recommended inside design conditions.

Outside DBT(°C)	*Occupancy over 40 minutes*				*Occupancy below 40 minutes*			
	DBT (°C)	*WBT (°C)*	*RH (%)*	*ET (°C)*	*DBT (°C)*	*WBT (°C)*	*RH (%)*	*ET (°C)*
26.5	23.9	18.3	60	21.7	24.4	18.9	61	22.2
	25	17.2	47	21.7	25.6	17.8	47	22.2
	26.1	16.1	35	21.7	26.7	16.7	36	22.2
29.5	24.4	18.9	61	22.2	25	19.4	61	22.8
	25.6	17.8	47	22.2	26.1	18.3	48	22.8
	26.7	16.7	36	22.2	27.2	17.2	36	22.8
32.5	25	19.4	61	22.8	25.6	20.6	64	23.3
	26.1	18.3	48	22.8	26.7	19.4	52	23.3
	27.2	17.2	36	22.8	27.8	18.3	40	23.3
35.5	25.6	20.6	64	23.3	26.1	21.1	65	23.9
	26.7	19.4	52	23.3	27.2	20	52	23.9
	27.8	18.3	40	23.3	28.3	18.9	41	23.9
37.5	26.1	21.1	65	23.9	27.2	21.7	63	24.4
	27.2	20	52	23.9	28.3	20.6	50	24.4
	28.3	18.9	41	23.9	29.5	19.4	38	24.4
40.5	26.6	21.7	65	24.2	27.2	22.2	65	24.7
	27.8	20.6	52	24.2	28.3	21.1	54	24.7
	28.9	19.4	42	24.2	29.5	20	41	24.7

17.16 Outside Summer Design Conditions

The following table shows the outside summer design conditions for important cities in India.

Table 17.2. Outside summer design conditions.

City	*DBT (°C)*	*WBT (°C)*	*RH (%)*	*ET (°C)*
Agra	41.5	22	17	29.7
Ahmedabad	43.2	26.4	37	32.5
Ahmednagar	42.2	31.1	45	29.5
Alibagh	31.6	25.8	63	28.4
Aligarh	42.2	28.3	35	29.7
Allahabad	41.7	25	26.5	30.8
Ambala	43.3	26.7	29	29.5
Amritsar	40.1	28.1	—	29.9
Banaras	40.8	26.1	31	31.9
Bangalore	32.9	24.7	52	28.1
Baroda	40.3	29.1	45	31.7
Bhopal	40.2	23.2	24	29.7
Chandigarh	40.1	23.9	27	29.9
Chennai	38.5	28.6	47	25.6
Cochin	35	27.8	59	28.1
Coimbatore	34.7	27.5	58	30
Cuttack	40.6	30.6	48	32
Darjeeling	17.2	14.5	77	16.8
Dehradun	40.6	26.7	35	27.8
Delhi	40.4	23.9	26	29.9
Guwahati	30.9	25.4	62	27.6
Hyderabad	39.5	26.6	37	30.8
Indore	39.4	25.1	32	29.7
Jaipur	40.8	21.7	18	28.6
Jamnagar	37.8	27.2	44	30.6
Jamshedpur	39.4	28.4	45	31
Jodhpur	43.3	26.7	29	30.6
Kanpur	41.2	21.5	17	28.6
Kathmandu	29.4	23.9	65	24.9
Kolkata	35.3	27.9	60	30.3
Lucknow	42.8	28.3	34	30.3
Ludhiana	40.1	27.9	41	29.9
Madurai	38.3	25.6	38	30.6
Mahabaleshwar	28.8	19.2	40	24.2
Mangalore	35.6	27.8	60	29
Mumbai	32.8	26.7	64	29

City	DBT (°C)	WBT (°C)	RH (%)	ET (°C)
Mysore	33.3	25.6	52	29.2
Nagpur	42.6	24.6	27	30.6
Patna	37.9	26.7	42	30.5
Pune	37.1	25.6	40	30.5
Puri	32	27.8	75	29.5
Rajpur	43.3	28.3	34	30.5
Rajkot	40.5	28.9	43	31.2
Roorkee	39	26.7	40	29.5
Shillong	29.4	21.1	48	26.6
Shrinagar	25	18.4	55	22.2
Simla	22.9	12.5	28	19.2
Surat	36.3	21.6	26	30.5
Trivandrum	30.7	29.4	90	29.7
Tiruchirappalli	38.6	25.6	32	30.2
Visakhapatnam	33.3	26.9	39	29.3

QUESTIONS

1. Explain in brief as to how the human body reacts to changes in temperature of environment. Also explain the effect of activities on the heat load calculation for comfort application.
2. Distinguish clearly between heat stroke, heat exhaustion and heat cramp.
3. State the factors that determine human comfort.
4. Define the term 'effective temperature' and explain its significance in the design of air conditioning systems.
5. What is 'effective temperature' ? What factors affect effective temperature ?
6. Sketch 'comfort chart' and show on it the 'comfort zone' ?
7. Explain clearly the different stages of human body defence against variations of weather conditions during summer and winter.
8. Discuss, briefly, the factors which govern the optimum effective temperature for comfort.

OBJECTIVE TYPE QUESTIONS

1. A fasting, weak or sick man will have .... metabolic heat production.
   (*a*) less (*b*) more
2. When the temperature of the surroundings is higher than the temperature of the body, then the heat loss by convection from the body to the surroundings will be
   (*a*) positive (*b*) negative (*c*) zero (*d*) none of these
3. The human body feels comfortable when the heat stored in the body is
   (*a*) positive (*b*) negative (*c*) zero (*d*) none of these
4. The degree of warmth or cold felt by a human body depends mainly on
   (*a*) dry bulb temperature (*b*) relative humidity
   (*c*) air velocity (*d*) all of these
5. The index which corelates the combined effects of air temperature, relative humidity and air velocity on the human body is known as
   (*a*) mean radiant temperature (*b*) effective temperature
   (*c*) dew point temperature (*d*) none of these

6. The effective temperature .... with decrease in relative humidity at the same dry bulb temperature.
   (*a*) decreases (*b*) increases
7. For comfort, all men and women above 40 years of age prefer .... effective temperature than the persons below 40 years of age.
   (*a*) higher (*b*) lower
8. The heat production from a normal healthy man when asleep is about
   (*a*) 20 watts (*b*) 40 watts (*c*) 60 watts (*d*) 80 watts
9. In summer comfort cooling, the air of the occupied space should not have a relative humidity above
   (*a*) 30% (*b*) 40% (*c*) 50% (*d*) 60%
10. The optimum effective temperature for human comfort is
    (*a*) higher in winter than in summer (*b*) lower in winter than in summer
    (*c*) same in winter and summer (*d*) not dependent on season

ANSWERS

1. (*a*)	2. (*b*)	3. (*c*)	4. (*d*)	5. (*b*)
6. (*a*)	7. (*a*)	8. (*c*)	9. (*d*)	10. (*b*)

CHAPTER

22

Applications of Refrigeration and Air Conditioning

1. Introduction.
2. Domestic Refrigerator and Freezer.
3. Defrosting in Refrigerators.
4. Controls in Refrigerator.
5. Room Air Conditioner.
6. Water Coolers.
7. Capacity of Water Coolers.
8. Applications of Air Conditioning in Industry.
9. Refrigerated Trucks.
10. Marine Air Conditioning.
11. Ice Manufacture.
12. Cooling of Milk (Milk Processing).
13. Cold Storages.
14. Quick Freezing.
15. Cooling and Heating of Foods.
16. Freeze Drying.
17. Heat and Mass Transfer through the Dried Material.

22.1 Introduction

Over the span of last few decades, refrigeration industry has grown into full-fledged industry in developed or northern countries. The refrigeration has become as essential feature rather than a luxury. The refrigeration has brought much more laurels and comforts to human beings than any other devices of human comfort. We can see the use of refrigeration and air conditioning practically in all spheres and walks of life. The application of refrigeration can be classified in the following six categories :

1. Domestic,
2. Commercial,
3. Industrial,
4. Marine,
5. Air conditioning, and
6. *Food preservation.

22.2 Domestic Refrigerator and Freezer

Now-a-days, the refrigerator has become an essential part of a household rather than a luxury. It is used for preserving food and thereby reducing waste. The primary function of a refrigerator or freezer is to provide food storage space maintained at low temperature for the preservation of food. Its essential secondary function is the formation of ice cubes for domestic consumption. They are usually specified by the internal gross volume and the deep freezer's volume. A storage temperature of 0°C to 4°C (273 K to 277 K) is satisfactory for the preservation of most of the fresh foods. For the short term storage of frozen foods (such as in a domestic refrigerator), temperatures much below the freezing point are required. The freezers are generally provided at the top portion of the refrigerator space. In some refrigerators, freezers are provided at bottom. This arrangement seems to be based on the heat transfer considerations but it may be noted that the time taken to cool products kept at upper portion would be more.


Domestic Refrigerator.

The refrigerators may be single-door, double-door top freezer, double-door bottom freezer, and side-by-side door freezer. The double-door refrigerators are very commonly used now-a-days because of the need for larger storage space and better preservation of frozen foods. These refrigerators are divided into two separate compartments, one for the fresh food or general items and the other for the storage of frozen foods. Since the requirement of food or general items in a day is quite high, therefore the frequency of opening the door is also high, but the daily use of frozen items is very much limited so with double-door system, the freezer space is not subjected to wide temperature variations. This helps in maintaining a stable temperature for the preservation of the frozen foods.

The mechanical vapour compression cycle as well as absorption cycle may be adopted for domestic refrigerators and freezers, but the mechanical vapour compression system is actually used over absorption system, because of its compactness and more efficient use of electrical energy, as shown in Fig. 22.1. The refrigerants used are generally R-12 or R-22. The compressor is mounted at the bottom of the refrigerator frame. The power of compressor can vary according to size of the refrigerator (*i.e.* 75 W, 92 W, 125 W, 180 W and 370 W etc.). The condenser is put at the back about 40 to 60 mm away from the cabinet. The condenser may be either chassis type or tube and wire type. In the former, the condenser tube is mounted on a metal sheet which acts as fins. The tube and wire type condensers are quite simple in which few tubes are held tightly under wire frame from both sides. These wires act as cylindrical fins increasing the rate of heat transfer. The capillary tube is kept in contact with the evaporator inlet pipe. A drier is connected between the receiver and the evaporator to eliminate traces of moisture if any. In some cases, the temperature of two cabinets of the refrigerator have to be controlled independently. Under such circumstances, independent compressors and cooling coils are used.

The evaporator coil is wrapped around the freezer in a suitable manner to give efficient heat transfer. Sometimes, the freezer chamber is made from a pair of sheet joined together in such a way that the passage between the sheets act as an evaporator coil. The cooling of lower space is

* Food preservation has been discussed, in detail, in Chapter 13.

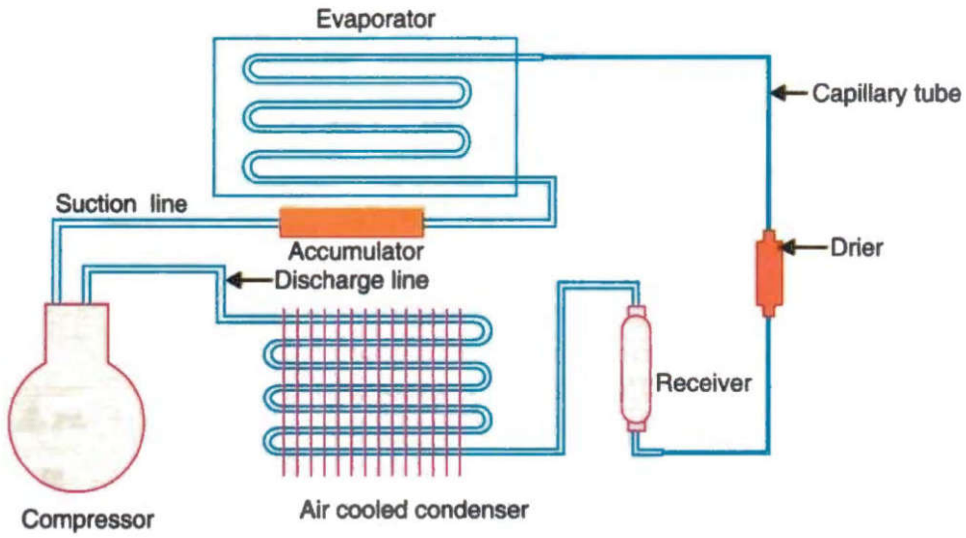


Fig. 22.1. Refrigeration system for a domestic refrigerator.

accomplished by free convection (due to density gradient). The thermostatic sensing element is provided to the evaporator coil which can control temperature in the freezer upto −15°C in steps or continuously depending upon the type of controlling switch employed.

The refrigerator body is provided with good quality insulation in order to prevent heat transfer into the system. Usually 60 to 100 mm thick glass loose-fill fibre or glass rolls or thermocole is used since the conductivity of these insulating materials are quite low. In freezers where temperature has to be maintained quite low (− 40°C), the insulation thickness may be about 200 mm.

22.3 Defrosting in Refrigerators

Since the evaporator in a refrigerator operate at temperatures below 0°C (*i.e.* below the freezing point of water), therefore it is subjected to the accumulation of frost or ice. The lower the evaporator temperature, the thicker will be the frost formation. The frost acts as an insulation that impedes the heat transfer to the evaporator. This leads to further thickening of frost as the temperature tends to go down because of decreased heat transfer rate. This ultimately leads to substantial reduction in the evaporator capacity and the system efficiency. Thus, the removal of frost or defrosting the evaporator at regular intervals is an absolute necessity.

One of the simplest method of defrosting is by manually putting 'Off' the refrigerator and restarting only after complete defrosting of the evaporator. Now-a-days, the refrigerators are provided with push button defrost thermostats. A push button is provided in the centre of the thermostat knob. The defrosting can be initiated by simply pressing the push button which causes the leverage in the thermostat to break and keeps the electrical contact of the thermostat open until the evaporator temperature rises above freezing point and defrosting takes place. The refrigerator returns to normal functioning automatically, once the defrosting is complete.

In a double-door refrigerator, the evaporator in the fresh food or general storage compartment is generally designed for natural cycle defrost. The defrosting takes place every time the compressor switches off the thermostat, as the storage temperature is above the freezing point. However, the defrosting may not be always complete. Over a period of continuous use, some amount of residual frost after the defrost cycle can get collected, particularly on the lower portion of the evaporator. Such accumulation of frost due to the residual frost of the natural defrost cycle has to be removed periodically by manually stopping the machine. The freezer compartment is not provided with automatic defrost system, as the temperature of the frozen food should not be allowed to go above the freezing point. Therefore, manual defrosting has to be carried out periodically. Obviously, for defrosting, the frozen foods should be removed or defrosting should be carried out when the freezer

Freezer plus three fridge compartments including drinks fridge.

is empty. The defrost water from the evaporator flows to a condensate pan provided below the evaporator of the fresh food compartment. From there it drains into a tray in the compressor compartment. The water collected in the tray will evaporate due to the hot environment in the compressor compartment. It may be necessary to empty the tray manually once in a while, as the water accumulates in the tray.

22.4 Controls in Refrigerator

The controls are very essential for satisfactory and economical working of any refrigerator. The electrical connection diagram of a domestic refrigerator is shown in Fig. 22.2. The refrigerator is fitted with the following controls :

(a) ***Starting relay.*** The starting relay is used to provide the necessary starting torque required to start the motor. It also disconnects the starting winding of the motor when the motor speed increases. When the compressor motor is to be started, the thermostat is in the closed position. When the electric supply is given, an electric current passes through the running winding of the motor and the starting relay. Due to the flow of electric current through relay coil and due to electromagnetism, its armature is pulled thereby closing the starting winding contacts. The current through starting winding provides the starting torque and the motor starts. As the motor speed increases, the running winding current decreases. The current in the starting relay is no longer able to hold the relay and it gets released thereby opening the starting winding contacts. Thus, the starting winding gets disconnected.

(b) ***Overload protector.*** The basic function of an overload protector is to protect the compressor motor winding from damage due to excessive current, in the event of overloading or due to some fault in the electric circuit. It consists of a bimetallic strip. During the normal working of the compressor, the contacts are closed. Whenever there is any abnormal behaviour (*i.e.* overheating, overcurrent due to fault or overload), the bimetallic strip gets heated and bends, thereby opening the motor contacts, and de-energising it. The overload protector is fitted on the body of the compressor and operates due to the combined action of heat produced when current passes through the bimetallic strip and a heater element, and heat transferred from the compressor body. It may be noted that the abnormal behaviour of the compressor may be due to low voltage, high voltage, high load, low suction pressure, high suction and discharge pressure.

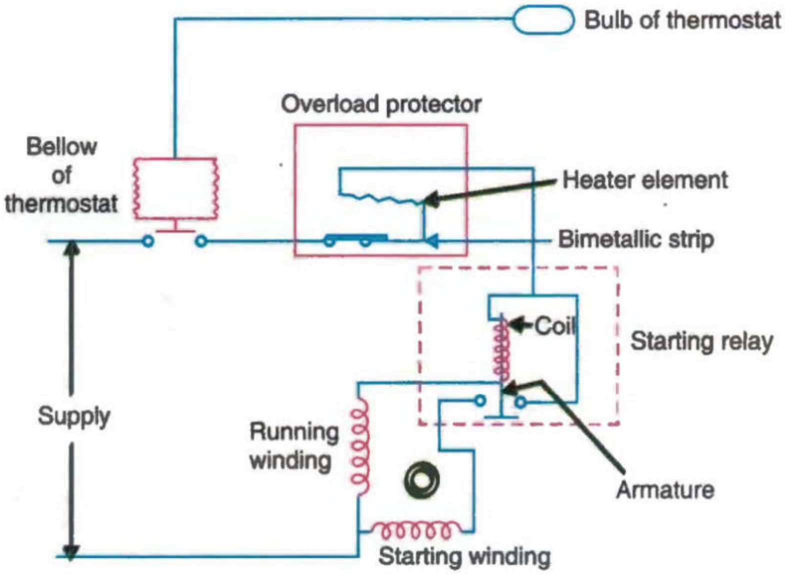


Fig. 22.2. Electrical connection diagram of a domestic refrigerator.

(c) *Thermostat.* A thermostat is used to control the temperature in the refrigerator. The bulb of the thermostat is clamped to the evaporator or freezer. The thermostat bulb is charged with few drops of refrigerant. The thermostat can be set to maintain different temperatures at a time. When the desired temperature is obtained, the bulb of the thermostat senses it, the liquid in it compresses and operate bellows of the thermostat and opens the compressor motor contacts. The temperature at which compressor motor stops is called *cut-out temperature*. When the temperature increases, the liquid in the bulb expands thereby closing the bellows contact of the compressor motor. The temperature at which the compressor motor starts, is called *cut-in temperature*. A thermostat is very crucial in the operation of a refrigerator as the running time of the compressor is reduced considerably thereby cutting the operation cost as well as enhancing the compressor life due to non-continuous working.

22.5 Room Air Conditioner

A room air conditioner is a compact, self contained air-conditioning unit which is normally installed in a window or wall opening of the room and is widely known as window type air conditioner. It works on vapour compression cycle. A complete unit of a room air conditioner consists of the refrigeration system, the control system (thermostat and selector switch), electrical protection system (motor overload switches and winding protection thermostat on the compressor motor), air circulation system (fan motor, centrifugal evaporator blower), ventilation (fresh air damper) and exhaust system.

The refrigeration system consists of a hermetic type compressor, forced air-cooled finned condenser coil, finned cooling coil, capillary tube as the throttling device and a refrigerant drier. The refrigerant used is R-12 or R-22. In hermetic compressors, a winding thermostat is embedded in the compressor motor windings. It puts off the compressor if the winding temperature exceeds the safe limit, thus protecting the winding against high temperature.

The condenser is a continuous coil made of copper tubing with aluminium fins attached to it to increase the heat transfer rate (rejecting heat to atmosphere). A propeller type fan provides the necessary air to cool the refrigerant in the condenser and also exhausts air from the air-conditioned space when the exhaust damper is opened. The evaporator is a cooling coil also made of copper with aluminium fins attached to it to increase the heat transfer rate (taking in heat from the room air).

The room air-conditioner is installed in such a way that the evaporator faces the room. A centrifugal blower is installed behind the cooling coil which sends cool air in the room. A filter is installed on the fresh air entering side of the evaporator to remove any dirt from the air. A damper inside the cabinet regulates the fresh air intake of the room air-conditioner. The quality of fresh air may be varied by adjusting the dampers. If all the air in the room is to be exhausted, the fan control of the unit is set to 'Exhaust' position. The condenser fan or blower exhausts all the air to the atmosphere. Thus, smoke and odour are removed by the condenser fan which draws air through the dampers and exhausts it through the louvers in the rear of the unit.

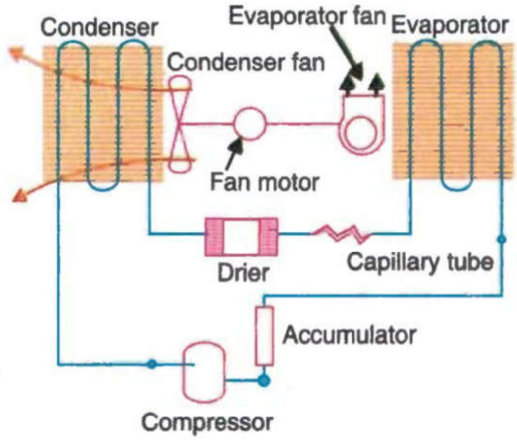
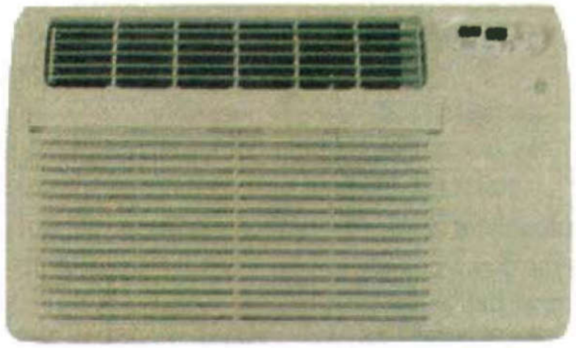


Fig. 22.3. Layout of a room air conditioner.



Room air conditioner.

A thermostat element is located in the return air passage of the unit. It controls the opertion of the compressor based on the return air temperature, which indicates the room temperature. It may be noted that when the required temperature is obtained, the compressor is stopped.

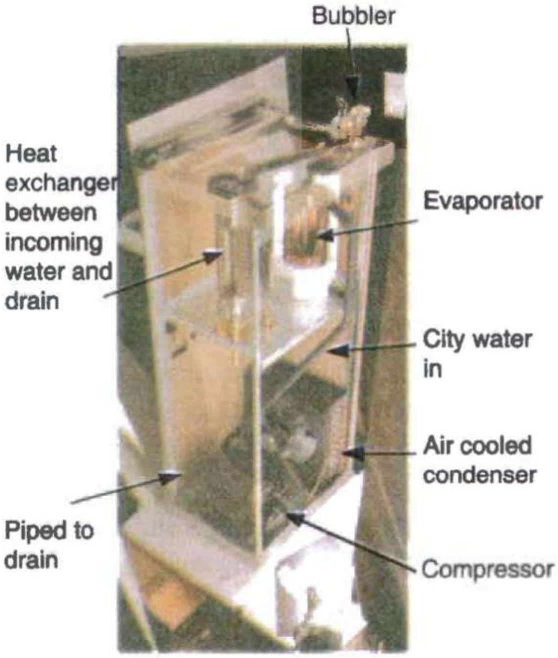
A selector switch often known as master control, controls the compressor motor, condenser fan motor, and evaporator fan motor. When the control switch is in 'Ventilate' position, only evaporator blower motor operates and outside fresh air is supplied in the room which is not cool as the compressor is not working. In the 'Exhaust' position, the condenser fan motor operates and all the room air is exhausted to the atmosphere. In the 'cool' position, all the motors *i.e.* compressor motor, condenser motor, and evaporator motor are in working state and cool air is supplied to the room.

Notes : **1.** By installing a reversing valve, the air-conditioner unit can be used for heating the room during winter. The reversing valve is a two position valve with four ports. The discharge and suction lines of the compressor are connected to two ports. The other two ports are connected to the inlet side of the air-cooled condenser and the suction outlet of the evaporator.

2. The advantage of using a reversing valve for heating is that the energy required for heating the room will be much less than that required for heating with electrical strip heaters.

22.6 Water Coolers

The purpose of a water cooler is to make water available at a constant temperature irrespective of ambient temperature. They are meant to produce cold water at about 7°C to 13°C (280 K to 286 K) for quenching the thirst of the people working in hot environment. The warm or normal water can serve the physical requirement of our system for the proper functioning of the body organs but it does not quench the thirst especially in hot summers. The temperature of cold water is controlled with the help of a thermostatic switch set within 7°C to 13°C range.



Water cooler.