Diffraction

Diffraction at a single slit.

Derivations →.

Diagram:—

① Path difference $= a\sin\theta$

② Phase difference $= \frac{2\pi}{\lambda} a\sin\theta$.

③ Phase difference between the waves from any two successive parts of the slit AB (n parts)

$$\frac{1}{n}\left(\frac{2\pi}{\lambda}a\sin\theta\right) = \delta$$

④ Resultant amplitude at P

$$R = \frac{A\sin\alpha}{\alpha}.$$

$A = na$, $\boxed{\alpha = \frac{\pi a\sin\theta}{\lambda}}$

⑤ Resultant intensity $I = R^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

⑥ Position of Principal maxima.

$$R = \frac{A}{\alpha}\sin\alpha = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \cdots\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \cdots\right]$$

$\boxed{R \text{ max}}$ when $\boxed{\alpha = 0}$ or $\boxed{\theta = 0}$

Single slit

⑦ Position of minima.

$$\sin\alpha = 0 \quad , \quad \alpha \neq 0$$

$$\boxed{\alpha = \pm n\pi} \quad n = 1, 2, 3, \ldots$$

$$a \sin\theta = \pm n\lambda$$

⑧ Secondary maxima

$$\frac{dI}{d\alpha} = 0$$

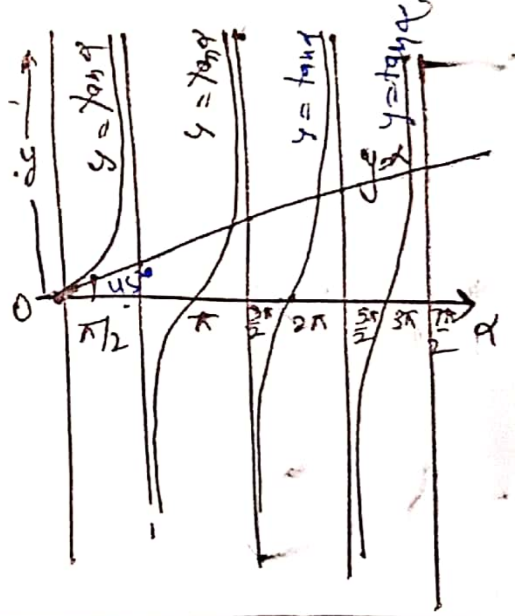
$$\frac{d}{d\alpha} = \frac{d}{d\alpha}\left(A^2 \frac{\sin^2\alpha}{\alpha^2}\right) = A^2\, 2\, \frac{\sin\alpha}{\alpha}\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

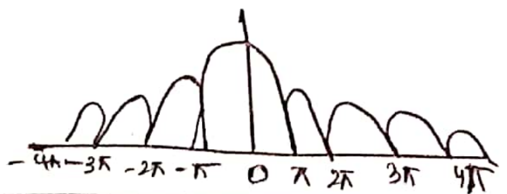
$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha = \tan\alpha = y$$

$$\boxed{\therefore \alpha = \pm\frac{(2n+1)\pi}{2}} \quad n = 1, 2, 3, \ldots$$

$$a\sin\theta = \pm\frac{(2n+1)\lambda}{2}$$





$\alpha = 0 \quad I = I_0$, $I_1 = A^2 \frac{\left(\sin\frac{3\pi}{2}\right)^2}{\left(\frac{3\pi}{2}\right)^2} = \frac{4}{9\pi^2}A^2 = \frac{A^2}{22} = \frac{I_0}{22}$

I_1 is 4.5% of I_0 or the intensity of first secondary maximum is about (1/22)th of intensity of central maximum.

$I_2 = A^2 \frac{(\sin 5\pi/2)^2}{(5\pi/2)^2} = \frac{4}{25\pi^2}A^2 = \frac{A^2}{62} = \frac{I_0}{62}$; I_2 is 1.61% of I_0. or the intensity of second secondary maxima is about (1/62)th of I_0.

(because α / n is very small)

$na = A$, we obtain $R = A \dfrac{\sin\alpha}{\alpha}$ ...(8)

$$\left(\because \quad \text{For large } n, \frac{n\delta}{2} \approx \frac{(n-1)\delta}{2}\right) \qquad ...(9)$$

hase of n simple harmonic vibrations of equal progression.

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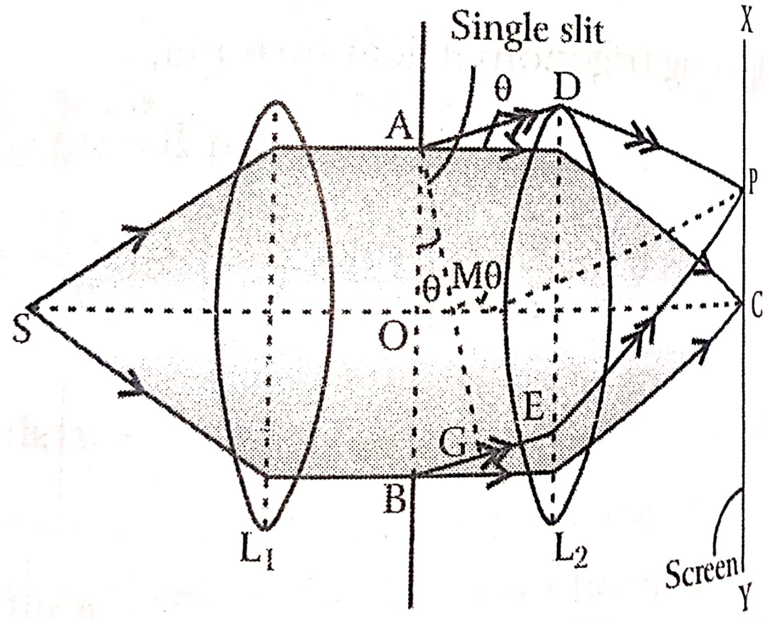

Fig. 3

of decreasing intensity is obtained on both the attern can be explained as follows:

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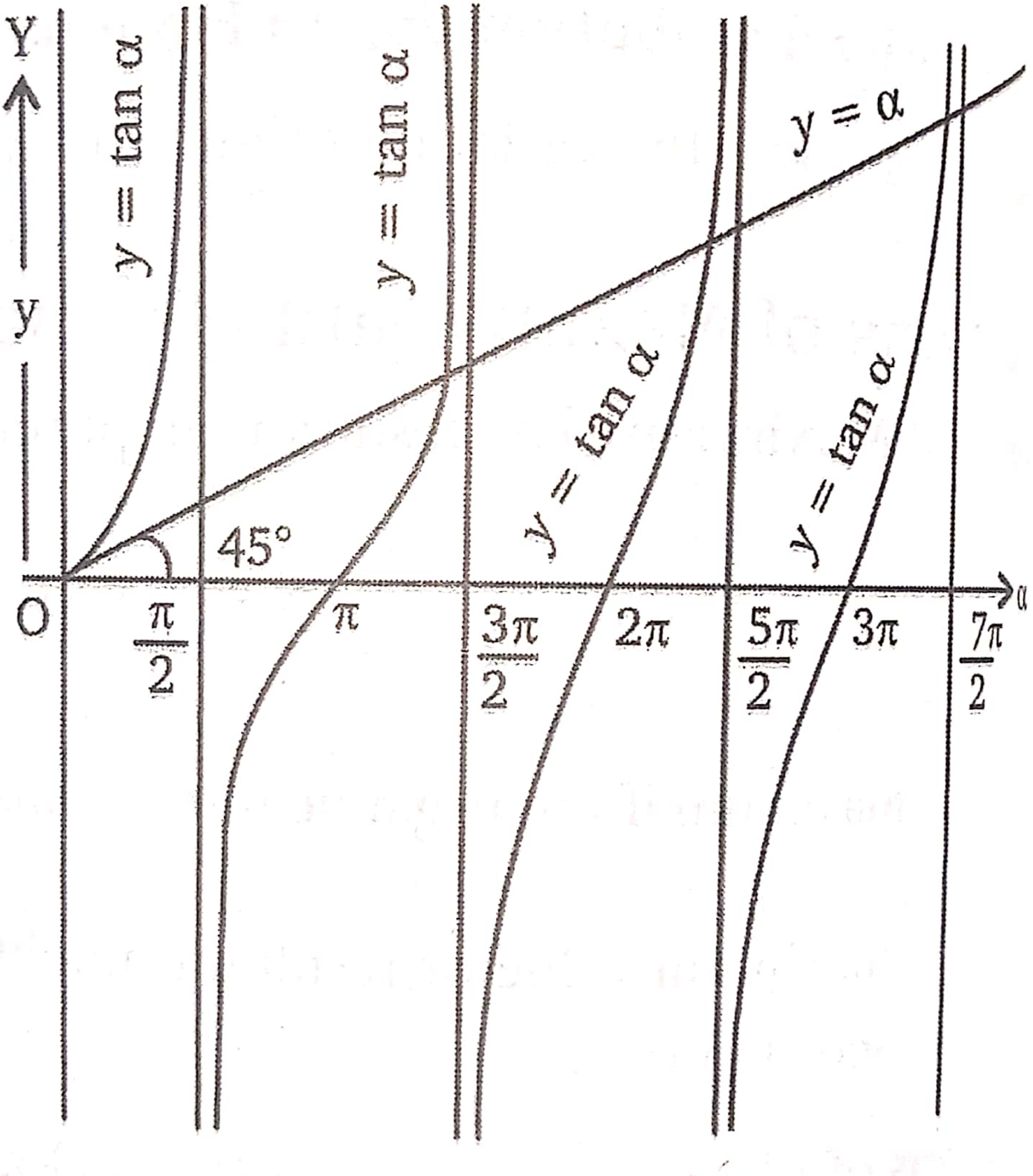
Y
y
$y = \tan \alpha$
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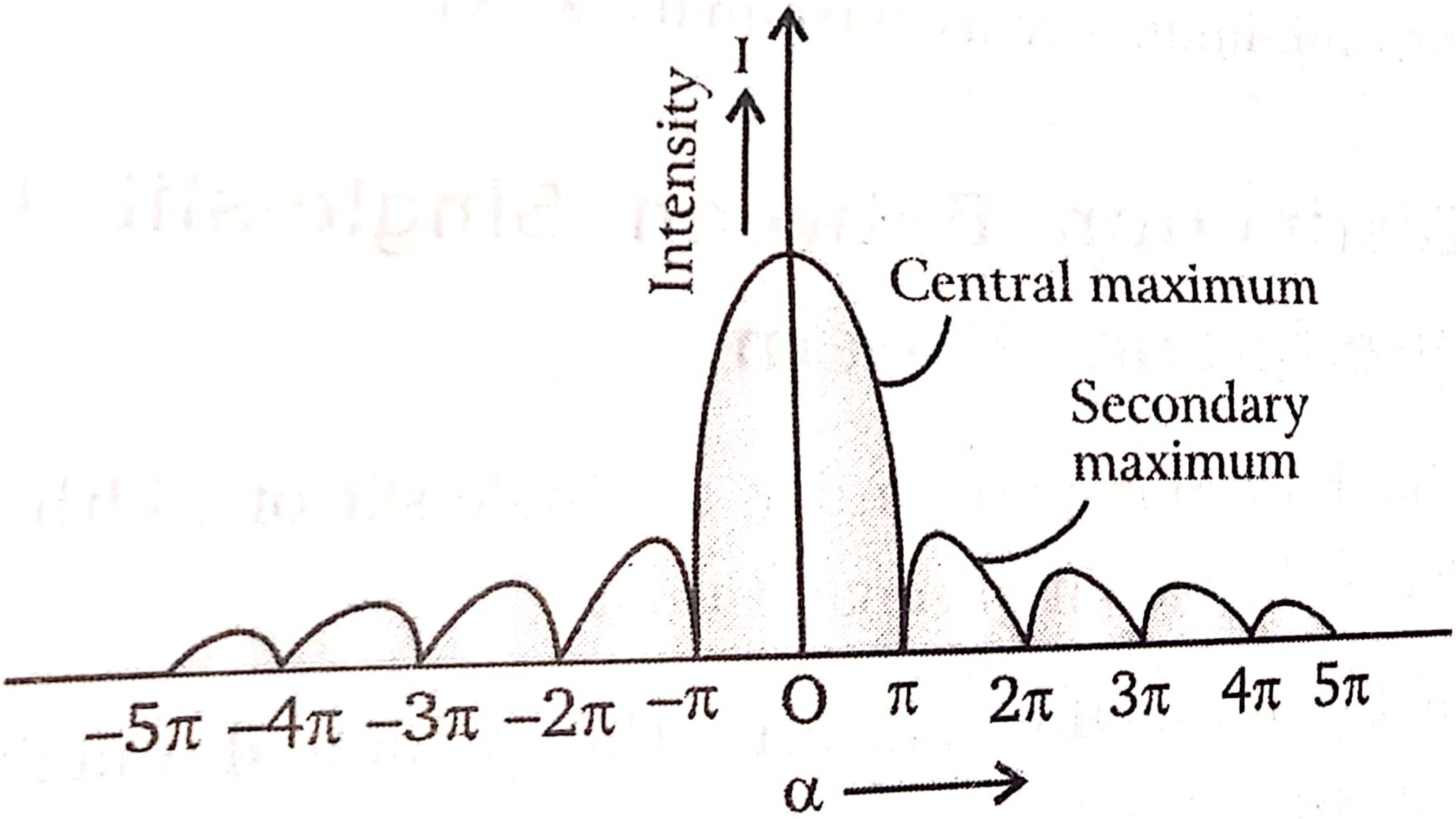
$y = \alpha$
$y = \tan \alpha$
$y = \tan \alpha$
45°
O
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
$\frac{5\pi}{2}$
3π
$\frac{7\pi}{2}$
α

Fig. 4


Intensity
I
Central maximum
Secondary maximum
−5π −4π −3π −2π −π O π 2π 3π 4π 5π
α