Questions from Latest Examination Papers

DIFFERENTIAL EQUATIONS

1. Determine the differential equation whose set of independent solution is $\{e^x, xe^x, x^2e^x\}$. (AKTU 2021, 2017)

2. Solve: $(D+1)^3 y = 2e^{-x}$ (AKTU 2017)

3. Solve $(D^2 - 2D + 4)y = e^x \cos x + \sin x \cos 3x$. (AKTU 2017)

4. Solve the simultaneous differential equations:

$$\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = y \text{ and } \frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 25x + 16e^t.$$ (AKTU 2017)

5. Use variation of parameter method to solve the differential equation $x^2y'' + xy' - y = x^2e^x$. (AKTU 2017)

6. State the criterion for linearly independent solutions of the homogeneous linear nth order differential equation. (AKTU 2021)

7. Solve: $\frac{d^2x}{dt^2} + \frac{dy}{dt} + 3x = e^{-t}, \frac{d^2y}{dt^2} - 4\frac{dx}{dt} + 3y = \sin 2t.$ (AKTU 2021)

8. Use the variation of parameter method to solve the differential equation

$$(D^2 - 1)y = 2(1 - e^{-2x})^{-1/2}.$$ (AKTU 2020)

9. Solve: $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x).$ (AKTU 2021)

10. Solve: $x\frac{d^2y}{dx^2} + (4x^2 - 1)\frac{dy}{dx} + 4x^3y = 2x^3.$ (AKTU 2017)

11. Solve by change of independent variable method

$$(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x).$$ (AKTU 2021)

12. Solve the equations

$$t\frac{dy}{dt} + x = 0 \text{ and } t\frac{dx}{dt} + y = 0 \text{ given } x(0) = 1 \text{ and } y(-1) = 0.$$ (AKTU 2019)

13. Calculate order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}$.

14. Find particular integral of $(D-2)^2y = 8e^{2x}$. (AKTU 2020)

15. Solve by changing independent variable the differential equation

$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log(1+x).$ *(AKTU 2021)*

16. Solve the following simultaneous differential equations

$\frac{dx}{dt}=3x+2y, \frac{dy}{dt}=5x+3y$ *(AKTU 2020)*

17. Solve the differential equations $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=\frac{e^{-x}}{x+2}$. *(AKTU 2021)*

18. Find the particular integral of $(4D^2+4D-3)y=e^{2x}$, where $D\equiv\frac{d}{dx}$. *(AKTU 2017)*

19. Solve the differential equation:

$\frac{d^2y}{dx^2}+y=0$; given that $y(0)=2$ and $y\left(\frac{\pi}{2}\right)=-2$. *(AKTU 2017)*

20. Solve the following simultaneous differential equations

$\frac{dx}{dt}=-wy, \frac{dy}{dt}=wx$

Also show that the point (x, y) lies on a circle. *(AKTU 2017)*

21. Apply the method of variation of parameters to solve the following differential equations:
$\frac{d^2y}{dx^2}+y=\tan x$ *(AKTU 2017)*

22. Solve: $(D-2)^2y=8(e^{2x}+\sin 2x+x^2)$ *(AKTU 2017)*

23. Find the P.I of $\frac{d^2y}{dx^2}+4y=\sin 2x$. *(AKTU 2018)*

24. Solve simultaneous equations $\frac{dx}{dt}=3y, \frac{dy}{dt}=3x$. *(AKTU 2018)*

25. Solve $\frac{d^2y}{dx^2}+y=\tan x$ by method of variation of parameter. *(AKTU 2018)*

LAPLACE TRANSFORM

26. Find inverse Laplace transform of $\frac{s+8}{s^2+4s+5}$. *(AKTU 2017)*

27. If $L\{F(\sqrt{t})\}=\frac{e^{-1/s}}{s}$, *find* $L\{e^{-t}F(3\sqrt{t})\}$. *(AKTU 2017)*

28. Draw the graph and find the Laplace transform of the triangular wave function of period 2π given $F(t)=\begin{cases} t, & 0<t\le\pi \\ 2\pi-t, & \pi<t<2\pi \end{cases}$. *(AKTU 2017)*

29. State convolution theorem and hence find inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$.

30. Solve the following differential equation using Laplace transform $\frac{d^3y}{dt^3}-3\frac{d^2y}{dt^2}+3\frac{dy}{dt}-y=t^2e^t$

where $y(0)=1, y'(0)=0$ and $y''(0)=-2$. *(AKTU 2017)*

31. Find the Laplace transform of $F(t)=e^t t^{-1/2}$. *(AKTU 2021)*

32. Find the function whose Laplace transform is $\frac{e^{-\pi s}}{S^2+2}$. (AKTU 2021)

33. State Convolution Theorem and hence evaluate $L^{-1}\left[\frac{S}{(S^2+1)(S^2+4)}\right]$.

34. Find the laplace transform of the rectified semi-wave function defined by

$$f(t)=\begin{cases}\sin wt, & 0<t\le\frac{\pi}{w}\\ 0, & \frac{\pi}{w}<t\frac{2\pi}{w}\end{cases}.$$

(AKTU 2020)

35. Using Laplace transform, evaluate the integral $\int_0^\infty \frac{e^{-2t}-e^{-4t}}{t}dt$ (AKTU 2021)

36. Find the Laplace transform of t^3e^{-3t}. (AKTU 2017)

37. State change of scale property of Laplace transform.

38. Write the Laplace equation in two dimensions. (AKTU 2017)

39. Find the Laplace transform of the following periodic function with period $\frac{2\pi}{w}$.

$$F(t)=\begin{cases}\sin wt, & 0<t\le\frac{\pi}{w}\\ 0, & \frac{\pi}{w}<t<\frac{2\pi}{w}\end{cases}$$

(AKTU 2017)

40. Express the following function in term of unit step function and find its

Laplace transform: $F(t)=\begin{cases}2 & if\, 0<t<\pi\\ 0 & if\, \pi<t<2\pi\\ \sin t & if\, t>2\pi\end{cases}$ (AKTU 2017)

41. Evaluate by using convolution theorem: $L^{-1}\left\{\frac{1}{p(p^2-a^2)}\right\}$ (AKTU 2017)

42. Solve Laplace equation in a rectangle in the $0<x<m$ and $0<y<n$ satisfying the following boundary conditions $u(x, 0) = 0$, $u(x, n) = 0$, $u(0, y) = 0$ and $u(m, y) = ky(n-y)$ (AKTU 2018)

SEQUENCE AND SERIES

43. Test the series $\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{1}{n}$. (AKTU 2021)

44. Test the series $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\ldots\ldots\ldots$ (AKTU 2021)

45. Discuss the convergence of sequence $(1, 2^1, 2^2, 2^3, 2^4 \ldots\ldots\ldots)$. (AKTU 2020)

46. Examine the series for convergence or divergence $1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\ldots\ldots\ldots$ (AKTU 2021)

47. Discuss the convergence of sequence $a_n = \frac{2n}{n^2+1}$. *(AKTU 2018)*

48. State D' Alembert's test. Test the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}\ldots\ldots+\frac{x^n}{n^2+1}+\ldots\ldots\ldots$ *(AKTU 2019)*

FOURIER SERIES

49. Obtain half range cosine series for e^x the function $f(t)=\begin{cases} 2t, & 0<t<1 \\ 2(2-t), & 1<t<2 \end{cases}$ *(AKTU 2017)*

50. Obtain Fourier series for the function $f(x)=\begin{cases} x, & -\pi<x\le 0 \\ -x, & 0<x<\pi \end{cases}$ and hence show that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi^2}{8}$. *(AKTU 2017)*

51. Find the constant term when $f(x)=1+|x|$ is expanded in Fourier series in the interval $(-3, 3)$. *(AKTU 2017)*

52. If $f(x) = 1$, $0 < x < \pi$ is expanded in half range since series then find the value of b_n. *(AKTU 2020)*

53. Obtain Fourier series for $f(x)=\begin{cases} \pi x, & 0\le x\le 1 \\ \pi(2-x), & 1\le x\le 2 \end{cases}$ *(AKTU 2021)*

54. Find half range Fourier sine series for $f(x)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \frac{\pi}{2}<x<\pi \end{cases}$ *(AKTU 2019)*

55. Find the Fourier constant a_n for $f(x)= x \cos x$ in the invterval $(-\pi, \pi)$. *(AKTU 2019)*

56. Obtain Fourier series for $f(x)=\begin{cases} \pi x, & 0\le x\le 1 \\ \pi(2-x), & 1\le x\le 2 \end{cases}$. *(AKTU 2021)*

57. Obtain the Fourier series for the function $f(x)=x^2, -\pi\le x\le\pi$. Also show that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots.=\frac{\pi^2}{6}$ *(AKTU 2017)*

58. Find the Fourier half range cosine series for the function:

$$f(x)=\begin{cases} 1, & 0<x<\frac{\pi}{2} \\ 0, & \frac{\pi}{2}<x<\pi \end{cases}$$ *(AKTU 2017)*

59. Find the half range Fourier sine series for $f(x)=x \; in -\pi<x<\pi$. *(AKTU 2018)*

60. Find the Fourier series of $f(x)=x\cos\left(\frac{\pi x}{l}\right) in -l\le x\le l.$. *(AKTU 2018)*

61. Find the Fourier series expansion of $f(x)=x\sin x$ in $-\pi<x<\pi$. *(AKTU 2018)*

62. Find the Fourier constant a_1 of $f(x)=x^2, -\pi\le x\le\pi$. *(AKTU 2018)*

63. Find half range since series of $f(x)=\begin{cases} x, & 0<x<2 \\ 4-x, & 2<x<4 \end{cases}$. *(AKTU 2017)*

64. Find the Fourier series of $f(x)=x\sin x, -\pi\le x\le\pi$. *(AKTU 2018)*

COMPLEX VARIABLES

65. Show that $f(z)=z+2\bar{z}$ is not analytic anywhere in the complex plane.

66. Find the image of $|z-2i|=2$ under the mapping $w=\frac{1}{z}$. *(AKTU 2020)*

67. Expand $f(z)=e^{z/(z-2)}$ in a Laurent series about the point $z=2$. *(AKTU 2019)*

68. Discuss the nature of singularity of $\frac{\cot \pi z}{(z-a)^2}$ at $z=a$ and $z=\infty$. *(AKTU 2020)*

69. If $f(z)=u+iv$ is an analytic function, $f(z)$ in term of z if $u-v=\frac{e^y-\cos x+\sin x}{\cosh y-\cos x}$ when $f\left(\frac{\pi}{2}\right)=\frac{3-i}{2}$. *(AKTU 2020)*

70. Evaluate by contour integration: $\int_0^{2\pi} e^{-\cos\theta}\cos(n\theta+\sin\theta)\,d\theta; n\in I$. *(AKTU 2019)*

71. Prove that $w=\frac{z}{1-z}$ maps the upper half of the z-plane onto upper half of the w-plane. What is the image of the circle $|z|=1$ under this tranformation? *(AKTU 2021)*

72. Find a bilinear transformation which maps the point $i, -i, 1$ of the z-plane into $0, 1, \infty$ of the w-plane respectively. *(AKTU 2019)*

73. Evaluate $\oint_C \frac{e^z}{z(1-z)^3}dz$, where c is (*i*) $|z|=\frac{1}{2}$ (*ii*) $|z-1|=\frac{1}{2}$ (*iii*) $|z|=2$ *(AKTU 2021)*

74. Find the Taylor's and Laurent's series which reprsent the function $\frac{z^2-1}{(z+2)(z+3)}$ when

    (*i*) $|z|<2$ (*ii*) $2<|z|<3$ (*iii*) $|z|>3$. *(AKTU 2021)*

75. Define harmonic function. *(AKTU 2021)*

76. Find the points of invariant of the transformation $w=\frac{2z+3}{z+2}$. *(AKTU 2020)*

77. Sate Cauchy integral theorem. *(AKTU 2021)*

78. Discuss the singularity of $\sin\left(\frac{1}{z-a}\right)$.

79. Examine the nature of the function

$$f(z)=\begin{cases}\frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z\neq 0\\ 0, & z=0\end{cases}$$ in the region including the origin. *(AKTU 2021)*

80. Evaluate, $\frac{1}{2\pi i}\oint_C \frac{z^2-z+1}{z-a}dz$, where $C\equiv|z-1|=\frac{1}{2}$. *(AKTU 2020)*

81. Define an analytic function. If $f(z)=u+iv$ is an analytic function find $f(z)$ in term of z if $u-v=e^x(\cos y-\sin y)$. *(AKTU 2017)*

82. Find the image of circle $|z-1|=1$ in the complex plane under the mapping $w=1/z$. *(AKTU 2020)*

83. Find Laurent series expansion of $\frac{1-\cos z}{z^3}$ about the point z = 0 is. *(AKTU 2021)*

84. Find residue at each pole of the function and hence using Cauchy residue theorem evaluate integral $\frac{4+3z}{(z-2)(z-3)}dz$, where C: $|z| = 1$. *(AKTU 2020)*

85. Show that the function defined by $f(z)=\sqrt{|xy|}$ is not regular at the origin, although the Cauchy-Riemann equations are satisfied there. *(AKTU 2018)*

86. Show that complex function $f(z) = z^3$ is analytic. *(AKTU 2018)*

87. Define Conformal mapping. *(AKTU 2018)*

88. Evaluate $\int_0^{1+i}(x^2-iy)dz$ along the path $y = x$. *(AKTU 2018)*

89. Find residue of $f(z)=\frac{\cos z}{z(z+5)}$ at $z = 0$. *(AKTU 2018)*

90. Show that $u=x^4-6x^2y^2+y^4$ is harmonic function. Find complex function $f(z)$ whose u is a real part. *(AKTU 2018)*

91. Expand $f(z)=\frac{1}{(z-1)(z-2)}$ in regions

    (*i*) $1<|z|<2$ (*ii*) $2<|z|$ *(AKTU 2018)*

92. Let $f(z)=\frac{x^2y^5(x+iy)}{x^4+y^{10}}$ when $z \neq 0$, $f(z) = 0$ when $z = 0$. Prove that Cauchy Riemann satisfies at $z = 0$ but function is not differentiable at $z = 0$. *(AKTU 2018)*

93. Find Mobius transformation that maps point $z = 0, -i, 2i$ into the points $w=5i,\infty,-\frac{i}{3}$ respectively. *(AKTU 2018)*

94. Using Cauchy Integral formula evaluate $\int_c \frac{\sin z}{(z^2+25)^2}dz$ where c is circle $|z| = 8$. *(AKTU 2018)*

95. Apply residue theorem to evaluate $\int_{-\infty}^{\infty}\frac{x^2dx}{(x^2+1)(x^2+4)}$. *(AKTU 2018)*