ABES Engineering College, Ghaziabad

Department of Applied Sciences & Humanities

Session: 2023-24 **Semester: II** **Section: All**

Code: BAS 203 **Course Name: Engineering Mathematics II**

Assignment 4

Date of Assignment: **Date of submission:**

S.No.	KL, CO		Question	Marks
1	K_3,CO4	1.3.1,2.1.3 2.4.4,4.3.4	Given that $(x,y) = x^2 - y^2$ & $v(x,y) = -\left(\frac{y}{x^2+y^2}\right)$. Prove that both u and v are harmonic functions but u+iv is not an analytic function of z.	5
2	K_3,CO4	2.1.3,2.4.1 2.4.4,4.3.4	Find the constants a,b,c such that the function $f(z)$ where $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic. Express f(z) in terms of z.	5
3	K_3,CO4	2.1.3 2.4.1	Using the Cauchy- Reimann equations , show that $f(z) = \|z\|^2$ is not analytic at any point.	5
4	K_3,CO4	4.3.4	Show that $v(x,y) = e^{-x}(xcosy + ysiny)$ is harmonic. Find its harmonic conjugate.	5
5	K_3,CO4	2.4.1,2.4.4	Determine the analytic function $f(z)$ in terms of z whose real part is (i) $\frac{1}{2}log(x^2+y^2)$ (ii) $(x-y)(x^2+4xy+y^2)$	5
6	K_3,CO4	1.3.1,2.1.3 2.4.1,4.3.4	If $f(z) = u + iv$ is an analytic function , find $f(z)$ in terms of z if $u - v = e^x(cosy - siny)$	5
7	K_3,CO4	2.1.3,2.4.1 2.4.4	Find the image of $\|z - 2i\| = 2$ under the mapping $= \frac{1}{z}$	5
8	K_3,CO4	4.3.4	Find the invariant points of transformation $w = \frac{1+z}{1-z}$	5
9	K_3,CO4	2.4.1	Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $\|z\| < 1$.	5
10	K_3,CO4	2.1.3,2.4.1 2.4.4	Show that the transformation $w = i\left(\frac{1-z}{1+z}\right)$ transforms the circle $\|z\| = 1$ onto the real axis of the w plane and the interior of the circle into the upper half of the w-plane	5

Answers

2. $\mathbf{a = \frac{-1}{2}; b = -2; c = \frac{1}{2};}$ $\mathbf{f(z) = \frac{-1}{2}(2+i)z^2}$ **4.** $\mathbf{u(x,y) = e^{-x}(xsiny - ycosy) + c}$

5. **(i)** $logz + c$ **(ii)** $(1-i)z^3 + c$ **6.** $e^z + c$ **7.** $4v + 1 = 0$

8. $\boldsymbol{i, -i}$ **9.** $\mathbf{w = \frac{i-z}{i+z}}$;u>0