Engineering Mathematics-II (BAS-203)
Unit 2 Laplace Transform
Tutorial 4

Que1. Find the Laplace Transform of $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t-1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$

Que2. Find $(i)\ L(sin^3\ 2t)\ \ (ii)\ L(\sin 2t \cos 3t)\ \ (iii)\ L(\cosh at \sin bt)\ \ (iv)\ L\left\{\left((1+te^{-t})\right)^3\right\}$

Que3. (i) If $L(cos^2 t) = \frac{s^2+2}{s(s^2+4)}$ Find $L(cos^2 at)$

(ii) If $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ Find $L\left(\frac{\sin at}{t}\right)$

Que4. . Find the Laplace Transform of $f(t) = \sin\sqrt{t}$. Hence find $L\left(\frac{cos\sqrt{t}}{\sqrt{t}}\right)$

Que5. Find the Laplace Transform of following functions

$(i)\ f(t) = t\sin at\ \ (ii)\ \ f(t) = t\cosh at\ \ \ (iii)\ \ f(t) = t^2 e^t \sin 4t\ \ (iv)\ \ f(t) = t^2 e^{-2t}\cos t$

Que6. Find $(i)\ L\left(\frac{\cos at - \cos bt}{t}\right)\ (ii)\ L\left(\frac{e^{at} - \cos bt}{t}\right)\ (iii)\ L\left(\frac{1-\cos t}{t^2}\right)\ (iv)\ L\left(\frac{e^{-4t}\sin 3t}{t}\right)$

Que7. Find the Laplace Transform of following functions

$(i)\ f(t) = \int_0^t \frac{\sin t}{t}\ dt\ \ (ii)\ \ f(t) = \int_0^t \frac{1-\cos 2t}{t}\ dt\ \ \ (iii)\ \ f(t) = \int_0^t e^{-t}\cos t\ \ dt$

Que8. Using Laplace Transform , evaluate the following integrals

$(i)\ f(t) = \int_0^\infty \frac{e^{-t}\sin\sqrt{3}t}{t}\ dt\ \ (ii)\ \ f(t) = \int_0^\infty \frac{e^{-2t} - e^{-4t}}{t}\ dt$

Que9. (i) Prove that $\int_{t=0}^\infty \int_{u=0}^t e^{-t}\frac{\sin u}{u}\ du\ dt = \frac{\pi}{4}$

(ii)Find the Laplace Transform of function $f(x) = x^3 \sin x$. Hence Prove that

$\int_0^\infty e^{-t}\ \ t^3 \sin t\ \ dt = 0$ **[2022-23]**

Que10. Find the Laplace Transform of following functions

$(i)\ f(t) = t^2\ u(t-2)\ (ii)\ \ f(t) = \sin t\ \ u(t-4)$

Que11. Express the following functions in terms of unit step function and find its Laplace Transform

$(i) f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}\ \ (ii) f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & t > 2 \end{cases}$

Que12 Find the Laplace Transform of following Periodic functions

$(i)\ f(t) = \begin{cases} \sin at, & 0 < t \le \frac{\pi}{a} \\ 0, & \frac{\pi}{a} < t < \frac{2\pi}{a} \end{cases}\qquad (ii) f(t) = \begin{cases} t, & 0 < t \le a \\ 2a - t, & a < t < 2a \end{cases}$

Answers

Ans1. $\frac{2}{s^3} - \frac{e^{-2s}}{s^3}(2 + 3s + 3s^2) + \frac{e^{-3s}}{s^2}(5s - 1)$

Ans2. $(i)\ \frac{48}{(s^2+4)(s^2+36)}\ (ii)\ \frac{2(s^2-5)}{(s^2+25)(s^2+1)}\ \ (iii)\ \frac{1}{2}\left[\frac{b}{(s-a)^2+b^2} + \frac{b}{(s+a)^2+b^2}\right]\ (iv)\ \frac{1}{s} + \frac{6}{(s+3)^4} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3}$

Ans3. $(i)\ \frac{s^2+2a^2}{s(s^2+4a^2)}\ (ii)\ \tan^{-1}\left(\frac{a}{s}\right)$

Ans4. $\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}e^{-\frac{1}{4s}}\ ,\ \sqrt{\frac{\pi}{s}}e^{-\frac{1}{4s}}$

Ans5. $(i)\ \frac{2as}{(s^2+a^2)^2}\ \ (ii)\ \frac{s^2+a^2}{(s^2-a^2)^2}\ \ \ (iii)\ \frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}\ \ (iv)\ \frac{2(s^3+6s^2+9s+2)}{(s^2+4s+5)^3}$

Ans6. $(i) -\frac{1}{2}log\left(\frac{s^2+a^2}{s^2+b^2}\right)\ (ii)\ \frac{1}{2}log\left(\frac{s^2+b^2}{(s-a)^2}\right)\ (iii)\ \cot^{-1}s + \frac{s}{2}log\left(\frac{s^2}{s^2+1}\right)\ \ (iv)\ \tan^{-1}\left(\frac{3}{s+4}\right)$

Ans7. $(i)\ \frac{1}{s}\cot^{-1}s\ \ \ (ii)\ \ \frac{1}{2s}\log\left(1 + \frac{4}{s^2}\right)\ (iii)\ \ \frac{s+1}{s(s^2+2s+2)}$

Ans 8. $(i)\ \frac{\pi}{3}\ \ (ii)\ \log 2$

Ans 9. $(ii)\ \frac{24\,s\,(s^2-1)}{(s^2+1)^4}$

Ans10. $(i)\ \ \frac{e^{-2s}}{s^3}(4s^2 + 4s + 2)\ \ \ (ii)\ \frac{e^{-4s}}{s^2+1}(\cos 4 + s\sin 4)$

Ans11. $(i)\ \ \frac{e^{-s}}{s^2}(1 - e^{-s})^2\ \ \ (ii)\ \frac{2(1-e^{-2s})}{s^3} + \frac{4e^{-2s}}{s}$

Ans12. $(i)\ \ \frac{a}{\left(1-e^{-\frac{\pi s}{a}}\right)(s^2+a^2)}\ \ \ (ii)\ \frac{1}{s^2}\tanh\frac{as}{2}$