ABES ENGINEERING COLLEGE, GHAZIABAD (032)

B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-5: VECTOR CALCULUS

QUESTION BANK

1. Find a unit vector normal to the surface $x^2y + 2xz = 4$ at the point $(2, -2, 3)$.
2. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.
3. Find the directional derivative of $\emptyset = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x\log z - y^2 + 4 = 0$ at $(2, -1, 1)$.
4. In what direction from $(3, 1, -2)$ is the directional derivative of $\emptyset = x^2y^2z^4$ maximum and what is its magnitude ?
5. Find the directional derivative of $\frac{1}{r^2}$ in the direction of $\vec{r}$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
6. Find $\vec{\nabla}\log r^n$.
7. Find the divergence and curl of the vector $\vec{R} = (x^2 + yz)\hat{i} + (y^2 + zx)\hat{j} + (z^2 + xy)\hat{k}$.
8. Show that vector $\vec{V} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$ is solenoidal.
9. Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find the velocity potential $\emptyset$ such that $\vec{A} = \vec{\nabla}\emptyset$.
10. Find the directional derivative of $\vec{\nabla}.(\vec{\nabla}\emptyset)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $\emptyset = 2x^3y^2z^4$.
11. Find the total work done by a force $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ in moving a point from (0,0) to (a,b) along the rectangle bounded by the lines $x = 0, x = a, y = 0$ & $y = b$. Answer: -2ab^2
12. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\hat{i} + (2xz - y)\hat{j} + z\hat{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.
13. Use divergence theorem to Evaluate $\iint_S (xdydz + ydzdx + zdxdy)$, where S is the portion of the plane $x + 2y + 3z = 6$ which lies in the first octant.
14. Verify the divergence theorem for $\vec{F} = (x^3 - yz)\hat{i} - 2x^2y\hat{j} + 2\hat{k}$ taken over the cube bounded by the planes $x = 0, x = a, y = 0, y = a, z = 0, z = a$.
15. Verify the Stoke's theorem for the function $\vec{F} = xy^2\hat{i} + y\hat{j} + z^2x\hat{k}$ for the surface of a rectangular lamina bounded by arc $x = 0, y = 0, x = a, y = b$.
16. Verify Green's theorem by evaluating $\int_C [(x^3 - xy^3)dx + (y^2 - 2xy)dy]$, where C is the square having the vertices at the point (0,0), (2,0), (2,2)&(0,2). pg. 22
17. Verify Green's theorem in the plane for $\int_C [(xy + y^2)dx + x^2dy]$, where C is closed curve of the region bounded by $y = x$ and $y = x^2$.
18. **Using Green's** theorem to evaluate $\int_C [2y^2dx + 3xdy]$, where C is the boundary of the closed region bounded by $y = x$ and $= x^2$.

ANSWERS

1. $\frac{-\hat{i}+2\hat{j}+2\hat{k}}{3}$ 2. $\theta = cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$ 3. $-3\sqrt{2}$ 4.$96\left(\hat{i} + 3\hat{j} - 3\hat{k}\right), 96\sqrt{19}$

5. $-\frac{2}{r^3}$ 6.$\frac{n\vec{r}}{r^2}$ 7.$2(x+y+z); \vec{0}$ 9.$\emptyset = 3x^2y + xz^3 - zy + c$

10.$\frac{1724}{\sqrt{21}}$ 11.$\frac{a^3}{3} - ab^2$ 12. 16 13. 18

18 .$\frac{7}{30}$