Engineering Mathematics-II (BAS-203)
Unit 4 Complex Variable- Differentiation
Tutorial 10

Que1. Define analytic function with example. **[2017-18]**

Que2. Give an example of a function in which Cauchy Riemann Equations are satisfied yet the function is not analytic at origin. Justify your answer. **[2017-18]**

Que 3. Write Cauchy-Riemann Equations in Polar form. **[2017-18, [2015-16]**

Que4. Show that the function $f(z) = z + 2\overline{z}$ is not analytic anywhere in the complex plane. **[2021-22]**

Que5. Show that the following functions are not analytic at the origin although satisfies Cauchy Riemann equations at origin.

$(\boldsymbol{i}) f(z) = \begin{cases} \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ **[2016-17]** $(\boldsymbol{ii})\ f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0 \end{cases}$. **[2015-16]**

$(\boldsymbol{iii})\ f(z) = \begin{cases} \frac{x^3y^5(x+iy)}{x^6+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ **[2022-23]** $(\boldsymbol{iv})\ f(z) = \begin{cases} \frac{x^2y^5(x+iy)}{x^4+y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ **[2017-18],[2018-19]**

Que 6. Determine p such that function$f(z) = \frac{1}{2}\log(x^2 + y^2) + i\tan^{-1}\frac{px}{y}$ is an analytic function. Also find $f'(z)$.

Que 7. Prove that the following functions are holomorphic (analytic) and find its derivatives.

$(\boldsymbol{i}) f(z) = \sinh z$ $(\boldsymbol{ii})\ f(z) = z^3$

Que 8. Let $f(z) = u(r,\theta) + iv(r,\theta)$ be an analytic function. If $u = -r^3\, sin3\theta, then\ find\ f(z)$.

Que 9. In two dimensional fluid flow, the stream function is $\psi = -\frac{y}{x^2+y^2}$, Find the velocity potential $\emptyset$.

Que 10. Show that the following functions are harmonic function and find their harmonic conjugate.

$(\boldsymbol{i}) u = x^4 - 6x^2y^2 + y^4$ **[2018-19]** $(\boldsymbol{ii})\ u = \frac{1}{2}log(x^2 + y^2)$

Answers

Ans 3. $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$ And $\frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$

Ans 6. $p = -1$ $f'(z)=\frac{1}{z}$

Ans7.$(\boldsymbol{i})$ $\cosh z$ $(\boldsymbol{ii})\ f(z) = 3z^2$

Ans 8. $f(z) = iz^3 + k$, where k is constant of integration

Ans 9. $\emptyset = \frac{x}{x^2+y^2} + k$, where k is constant of integration

Ans 10 $(\boldsymbol{i}) v = 4x^3y - 4xy^3 + k$, where k is constant of integration

$(\boldsymbol{ii})\ v = \tan^{-1}\frac{y}{x} + k$, where k is constant of integration