CO4	Remember the concept of Beta and Gamma function; analyze area and volume and Dirichlet's Theorem in multiple integral (K1, K4).			
Q. No.	**Module-04 (Multiple Integration) Question Description**	**CO**	**Marks**	**BLT**
	DOUBLE INTEGRATION			
1.	Evaluate the following integrals (a) $\int_0^1\int_1^2 xy(1+x+y)dydx$ (b) $\int_1^a\int_1^b \frac{dydx}{xy}$ (c) $\int_0^1\int_0^{x^2} e^{y/x}dydx$ (d) $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\cos\theta dr d\theta$	**4**	**7**	**K4**
2.	Evaluate the integrals $\int_0^1\int_0^{\sqrt{1+x^2}} \frac{dydx}{(1+x^2+y^2)}$.	**4**	**7**	**K4**
3.	Evaluate the integrals $\iint (x+y)^2 dxdy$ over the area bounded by ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	**4**	**7**	**K4**
4.	When the region R of integration is the triangle given by $y=0, y=x$ and $x=1,$ then prove that $\iint_R \sqrt{4x^2-y^2}\,dxdy=\frac{1}{3}\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right).$	**4**	**7**	**K4**
5.	Evaluate the following integration $\iint_R \sqrt{xy-y^2}\,dxdy$ where R is a triangle with vertices (0, 0), (10,2) and (1,2).	**4**	**7**	**K4**
6.	Evaluate the integral $\int\int(x^2+y^2)dxdy$ over the region in the positive quadrant For which $x+y\le 1$.	**4**	**7**	**K4**
7.	Evaluate the integral $\iint(x^2+y^2)dxdy$ throughout the area enclosed by the Curves $y=4x,\ x+y=3, y=0$ and $y=2$.	**4**	**7**	**K4**
8.	Let D be the region in the first quadrant bounded by the curves $x\,y=16$, $x=y$, $y=0$ and $x=8$. Sketch the region of integration of the given integral $\iint_D x^2\,dx\,dy$ and evaluate it by expressing it as an appropriate repeated integral.	**4**	**7**	**K4**
9.	Evaluate $\int_0^{\pi/2}\int_0^{a\cos\theta} r\left(\sqrt{a^2-r^2}\right)dr\,d\theta$	**4**	**7**	**K4**
10.	Evaluate $\iint r\sin\theta\,dr\,d\theta$ over the area of the cardioid $r=a(1+\cos\theta)$ above the initial line	**4**	**7**	**K4**
11.	Evaluate $\iint \frac{r\,dr\,d\theta}{\sqrt{(a^2+r^2)}}$ over one loop of the lemniscate $r^2=a^2\cos 2\theta$	**4**	**7**	**K4**
12.	Evaluate $\iint r^3\,dr\,d\theta$ over the area included between the circles $r=2\sin\theta$ and $r=4\sin\theta$	**4**	**7**	**K4**
	TRIPLE INTEGRAL			
13.	Evaluate the integral $\int_1^e\int_1^{\log y}\int_1^{e^x} \log z\,dzdxdy$	**4**	**7**	**K4**
14.	Evaluate the integral $\int_0^a\int_0^{a-x}\int_0^{a-x-y^x}(x+y+z)dzdydx$	**4**	**7**	**K4**

15.	Evaluate $\iiint_R (x+y+z)\,dx\,dy\,dz$ where R: $0\le x\le 1, 1\le y\le 2, 2\le z\le 3.$ area	**4**	**7**	**K4**
15.	Evaluate the integral $\iiint_R (x-2y+z)dzdydx$, where R is the region determined by $0\le x\le 1,\ 0\le y\le x^2,\ 0\le z\le x+y.$	**4**	**7**	**K4**
16.	Evaluate the integral $\int_0^{\pi/2}\int_0^{a\cos\theta}\int_0^{\sqrt{a^2-r^2}} r\,dzdr\,d\theta$	**4**	**7**	**K4**
	BETA AND GAMMA FUNCTIONS			
17.	Define Gamma and Beta functions. Prove that $\gamma(n+1)=n\gamma(n)\ and\ \gamma(n+1)=n!$	**4**	**2**	**K2**
18.	Prove that $\gamma(n)=k^n\int_0^\infty e^{-kx}x^{n-1}dx\ \ and\ \ hence\,calculate\int_0^\infty e^{-3x}x^5dx$	**4**	**2**	**K3**
19.	Prove that $\gamma(n)=\frac{1}{n}\int_0^\infty e^{-x^{1/n}}\ dx\ \ and\ \ hence\ prove that\ \gamma(1/2)=2\int_0^\infty e^{-x^2}dx=\sqrt{\pi}$	**4**	**7**	**K3**
20.	Prove that $\beta(m,n)=\int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx\ \ and\ \ hence\,calculate\int_0^\infty \frac{x^8(1-x^6)}{(1+x)^{24}}dx$	**4**	**7**	**K3**
21.	Prove that $\beta(m,n)=\frac{\gamma(m)\gamma(n)}{\gamma(m+n)}, m>0, n>0\ and\ \ hence\,calculate\int_0^\infty \frac{x^4(1+x^6)}{(1+x)^{15}}dx$	**4**	**7**	**K3**
22.	Prove that $\beta(l,m).\beta(l+m,n).\beta(l+m+n,p)=\frac{\Gamma(l)\Gamma(m)\Gamma(n)\Gamma(p)}{\Gamma(l+m+n+p)}$	**4**	**7**	**K3**
23.	$\Pr ovet\ hat\beta(m,n)=\beta(m+1,n)+\beta(m,n+1)$	**4**	**7**	**K3**
24.	Prove that $\int_0^{\frac{\pi}{2}}\sin^{2m-1}\theta\cos^{2n-1}\theta d\theta=\frac{1}{2}\beta(m,n)$ and hence show that $\int_0^{\pi/2}\sin^m\theta\cos^n\theta\,d\theta=\frac{\gamma\left(\frac{m+1)}{2}\right)\gamma\left(\frac{n+1)}{2}\right)}{2\gamma\left(\frac{m+n+2)}{2}\right)}$	**4**	**7**	**K3**
25.	Prove that $\int_0^{\pi/2}\sin^3 x\cos^{5/2}x dx=\frac{8}{77}.$	**4**	**7**	**K3**
26.	Prove following results $(i)\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta=\int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta=\frac{\pi}{\sqrt{2}}\ \ (ii)\int_0^{\pi/2}\frac{d\theta}{\sqrt{\sin\theta}}*\int_0^{\pi/2}\sqrt{\sin\theta}\,d\theta=\pi$	**4**	**7**	**K3**
27.	Using $\int_0^\infty \frac{x^{n-1}}{(1+x)}dx=\frac{\pi}{\sin n\pi}, where\, 0<n<1, prove that\,\gamma(n)\gamma(n-1)=\frac{\pi}{\sin n\pi}$ Also, deduce the followings. $(i)\ \gamma(1/4)\gamma(3/4)\ \ (ii)\ \gamma(1/3)\gamma(2/3)$	**4**	**7**	**K3**
28.	Prove the followings (i) $\int_0^{\pi/2}\tan^n x\,dx=\frac{\pi}{2}\sec\frac{n\pi}{2}$ (ii) $\int_0^1 x^5(1-x^3)^{10}dx=\frac{1}{396}$ $(iii)\int_0^2 x(8-x^3)^{1/3}dx=\frac{16\pi}{9\sqrt{3}}\ \ (iv)\int_0^2\frac{dx}{\sqrt{1+x^4}}dx=\frac{1}{8\sqrt{\pi}}(\gamma 1/4)^2$	**4**	**7**	**K3**
29.	Prove that $\int_0^\infty \sqrt{y}e^{-y^2}dx\times\int_0^\infty\frac{e^{-y^2}}{\sqrt{y}}dx=\frac{\pi}{2\sqrt{2}}.$	**4**	**7**	**K3**

30.	**Duplication Formula** Prove that: $\gamma(m)\gamma(m+1/2)=\frac{\sqrt{\pi}}{(2)^{2m-1}}\gamma(2m), where\ m\ is\ positive.$ Also, show that $\beta(m,m)=2^{1-2m}\ \beta(m,1/2)$	**4**	**7**	**K3**
31.	Evaluate $\int_0^a \frac{x^2}{\sqrt{a-x}}dx$ $\frac{16}{15}a^{5/2}$	**4**	**7**	**K3**
32.	Evaluate the following integrals $(i)\int_0^{\infty} e^{-\sqrt{x}}\ x^{1/4}\ dx \quad (ii)\int_0^1 \left(\frac{x^3}{(1-x^3)}\right)^{1/3} dx$	**4**	**7**	**K3**
33.	Prove that $(i)\frac{\gamma(1/3)\gamma(5/6)}{\gamma(2/3)}=(2)^{1/3}\sqrt{\pi}$	**4**	**7**	**K3**
34.	Assuming $\gamma(n)\gamma(1-n)=\pi\cos ec\, n\pi, 0<n<1, show that \int_0^{\infty}\frac{x^{p-1}}{1+x}dx=\frac{\pi}{\sin\ p\pi}; 0<p<1$	**4**	**7**	**K3**
35.	Use Beta and Gamma functions to solve: $\int_0^1\left(\frac{1}{(1+x4)}\right)dx * \int_0^{\pi/2}\sqrt{\cot\theta}\,d\theta$	**4**	**7**	**K3**
36.	State Dirichlet’s Integral theorem	**4**	**2**	**K1**
37.	Evaluate the integral $\iiint x^{l-1}y^{m-1}z^{n-1}dxdydz$ where x,y,z are all positive but limited by the condition $\left(\frac{x}{a}\right)^p+\left(\frac{y}{b}\right)^q+\left(\frac{z}{c}\right)^r\le 1$.	**4**	**7**	**K1**
38.	The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B and C. Apply Dirichlet’s integral to find the volume of the tetrahedron OABC. Also, find the mass if the density at any point is *k x y z*.	**4**	**7**	**K4**
39.	Find the volume of the solid surrounded by the surface $\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}+\left(\frac{z}{c}\right)^{2/3}=1$	**4**	**7**	**K4**
40.	Evaluate $\iiint x^2yz\,dx\,dy\,dz$ throughout the volume bounded by the planes $x=0,\ y=0, z=0\ and\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$	**4**	**7**	**K4**
41.	Show that $\iiint x^2yz\,dx\,dy\,dz=\frac{1}{2}-\frac{5}{16}$ the integral being taken throughout the volume bounded by the planes $x=0, y=0, z=0\ and\ x+y+z=1$.	**4**	**7**	**K4**
42.	Evaluate $\int\int\int_V e^{-(x+y+z)}dxdydz$ where the region of integration is bounded by the planes x=0, y=0, z=0 and x + y + z = a, a > 0 using Liouville’s theorem.	**4**	**7**	**K4**
43.	Prove that $\iiint\frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}=\frac{\pi^2}{8}$, the integral being extended to all positive values of the variables for which the expression is real.	**4**	**7**	**K4**
44.	Evaluate $\iiint\sqrt{\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}}dx\,dy\,dz$ integral being taken over all positive values of x, y, z such that $x^2+y^2+z^2<1$.			

	CHANGE OF ORDER OF INTEGRATION			
45.	Evaluate the following integral by changing the order of integration $\int_0^1\int_{e^x}^{e^2}\frac{dydx}{\log y}$	**4**	**7**	**K4**
46.	Evaluate the integral by changing the order of integration: $\int_0^1\int_{2y}^{2} e^{x^2}\,dxdy$	**4**	**7**	**K4**
47.	Change the order of integration in $\int_0^1\int_{x^2}^{2-x} xydydx$ and hence evaluate the same.	**4**	**7**	**K4**
48.	Evaluate the integral $\int_0^\infty\int_0^x xe^{-\frac{x^2}{y}}\,dydx$ by changing the order of integration.	**4**	**7**	**K4**
49.	Change the order of integration in $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} f(x,y)dydx$	**4**	**7**	**K4**
50.	By changing the order of integration, evaluate the following integration $\int_0^\infty\int_0^\infty e^{-xy}\sin pxdxdy,$ and hence show that $\int_0^\infty \frac{\sin px}{x}dx=\frac{\pi}{2}$.	**4**	**7**	**K4**
	CHANGE OF VARIABLES OF AN INTEGRATION			
51.	Change into polar co-ordinates and evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dydx.$ Hence show that $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$.	**4**	**7**	**K4**
52.	Evaluate the following integral $\int\int\sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}dxdy$ over the positive quadrant of the circle $x^2+y^2=1$.	**4**	**7**	**K4**
53.	Evaluate $\iint\sqrt{(a^2-x^2-y^2)}dx\,dy$, over the semi-circle $x^2+y^2=ax$ in the positive quadrant.	**4**	**7**	**K4**
54.	Evaluate $\iint_R (x+y)^2\,dx\,dy$, where R is the parallelogram in the xy plane with vertices (1, 0), (3, 1), (2, 2), (0, 1), using the transformation $u = x + y$ and $v = x - 2y$.	**4**	**7**	**K4**
55.	Using the transformation $x - y = u$, $x + y = v$, show that $\iint_R \sin\left(\frac{x-y}{x+y}\right)dx\,dy=0$, where R is bounded by the co-ordinate axes and x +y = 1 in the first quadrant.	**4**	**7**	**K4**
56.	Evaluate by changing the variable, $\iint_R (x+y)^2\,dx\,dy$, where R is the region bounded by the parallelogram $x + y = 0$, $x + y = 2$, $3x - 2y = 0$ and $3x - 2y = 2$.	**4**	**7**	**K4**