Engineering Mathematics-II (BAS-203)
Unit 2 Laplace Transform
Tutorial 5

Que1. Find the Inverse Laplace Transform of following functions

$(i)\ \frac{1}{(p-2)^2+1}$ $(ii)\ \frac{6}{2p-3}-\frac{3+4p}{9p^2-16}+\frac{8-6p}{16p^2+9}$ $(iii)\ \frac{14p+10}{49p^2+28p+13}$

Que2. Find $(i)L^{-1}\left[\frac{1}{p^2(p+1)}\right]$ $(ii)\ L^{-1}\left[\frac{p^2+3}{p(p^2+9)}\right]$ $(iii)\ L^{-1}\left[\frac{(p^2+2)}{p(p^2+4)}\right]$

Que3. Find the Inverse Laplace Transform of following functions

$(i)\ \frac{p-1}{p^2-6p+25}$ $(ii)\ \frac{p}{p^2+4p+13}$ $(iii)\ \frac{1}{9p^2+6p+1}$

Que4. Find $(i)L^{-1}\left[\frac{e^{-p}}{(p+1)^3}\right]$ $(ii)\ L^{-1}\left[\frac{\pi e^{-p}+pe^{-p/2}}{p^2+\pi^2}\right]$ $(iii)\ L^{-1}\left[\frac{e^{-\pi p/2}+e^{-3\pi p/2}}{p^2+1}\right]$

Que5. Find the Inverse Laplace Transform of following functions

$(i)\log\left[\frac{p+a}{p+b}\right]$ $(ii)\log\left[\frac{p^2-1}{p^2}\right]$ $(iii)\ \tan^{-1}\frac{2}{p^2}$ $(iv)\ \frac{2ap}{(p^2+a^2)^2}$ $(v)\ \frac{1}{(p^2+a^2)^2}$

Que6. Find the Inverse Laplace Transform of following functions by using partial fractions

$(i)\ \frac{5p+3}{(p-1)(p^2+2p+5)}$ $(ii)\ \frac{p}{p^4+4a^4}$ $(iii)\ \frac{2p^2-6p+5}{p^3-6p^2+11p-6}$

Que7. Hence find the Inverse Laplace Transform by using the Convolution theorem.

$(i)\ \frac{p^2}{(p^2+a^2)(p^2+b^2)}, a\neq b$ $(ii)\ \frac{p}{(p^2+1)(p^2+4)}$ $(iii)\ \frac{16}{(p+2)^2(p-2)}$ $(iv)\ \frac{1}{(p^2+1)p^3}$ $(v)\ \frac{p^2}{p^4-a^4}$

$(vi)\ \frac{1}{p^2(p+1)^2}$ **[2022-23]**

Answers

Ans1. $(i)e^{2t}\sin t$ $(ii)3e^{\frac{3}{2}t}-\frac{1}{4}\sinh\frac{4}{3}t-\frac{4}{9}\cosh\frac{4}{3}t+\frac{2}{3}\sin\frac{3}{4}t-\frac{3}{8}\cos\frac{3}{4}t$ $(iii)\ \frac{2}{7}e^{-\frac{2}{7}t}\left(\cos\frac{3}{7}t+\sin\frac{3}{7}t\right)$

Ans2. $(i)t-1+e^{-t}$ $(ii)\ \frac{1}{a}(1-e^{-at})$ $(iii)\ cos^2 t$

Ans3. $(i)\ e^{3t}\cos 4t+\frac{1}{2}e^{3t}\sin 4t$ $(ii)\ e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$ $(iii)\ \frac{t}{9}\ e^{-\frac{t}{3}}$

Ans4. $(i)e^{-(t-1)}\frac{(t-1)^2}{2!}U(t-1)$ $(ii)\sin\pi t\left[U\left(t-\frac{1}{2}\right)-U(t-1)\right]$ $(iii)\ \cot t\left[U\left(t-\frac{3\pi}{2}\right)-U\left(t-\frac{\pi}{2}\right)\right]$

Ans5. $(i)\ \frac{e^{-bt}-e^{-at}}{t}$ $(ii)\ \frac{2}{t}(1-\cosh t)$ $(iii)\ \frac{2}{t}\sin t\sinh t$ (iv) t sin at $(v)\frac{\sin at-at\cos at}{2a^3}$

Ans6. $(i)\ \ e^t-e^{-t}\left(\cos 2t-\frac{3}{2}\sin 2t\right)$ $(ii)\frac{1}{2a^2}\sin at\sinh at$ $(iii)\frac{1}{2}e^t-e^{2t}+\frac{5}{2}e^{3t}$

Ans7. $(i)\ \frac{a\sin at-b\sin bt}{a^2-b^2}$ $(ii)\ \frac{1}{3}(\cos t-\cos 2t)$ $(iii)\ e^{2t}-e^{-2t}(1+4t)$

$(iv)\frac{t^2}{2}+\cos t-1$ $(v)\ \frac{1}{2a}(\sinh at+\sin at)$ $(vi)\ (t+2)e^{-t}+t-2$