Engineering Mathematics-II (BAS 203)
Unit 1 (Ordinary Differential Equation of Higher Order)
Tutorial 1

Que1. Find the order and degree of following differential equations

$(\boldsymbol{i})\left(\frac{d^3y}{dx^3}\right)^4 - 6x^2\left(\frac{dy}{dx}\right)^8 = 0 \quad (\boldsymbol{ii})\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2} = \left(1+\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)^2\right)^{\frac{3}{2}}$

Que2. Determine the differential equation whose set of independent solution is $\{\mathrm{e}^{\mathrm{x}}, \mathrm{xe}^{\mathrm{x}}, \mathrm{x}^2\mathrm{e}^{\mathrm{x}}\}$ **[2017-18]**

Que3.Solve the following Differential Equations

$(\boldsymbol{i})\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0 \quad (\boldsymbol{ii})\frac{d^4y}{dx^4} - 7\frac{d^3y}{dx^3} + 15\frac{d^2y}{dx^2} - 13\frac{dy}{dx} + 4y = 0$

$(\boldsymbol{iii})\frac{d^4y}{dx^4} + 81y = 0 \qquad (\boldsymbol{iv})(D^2+1)^3(D^2+D+1)^2 = 0$

$(\boldsymbol{v})(D^2-6D+25)^2 = 0 \qquad (\boldsymbol{vi})(D^2-4D+1)^2 = 0$

Que4.Solve the following Differential Equations

$(\boldsymbol{i})\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2} + 6\frac{\mathrm{dy}}{\mathrm{dt}} + 9\mathrm{y} = 5\mathrm{e}^{3\mathrm{t}}$ **[2015-16]** $(\boldsymbol{ii})(\mathrm{D}^3 - 2\mathrm{D}^2 + 4\mathrm{D} - 8)\mathrm{y} = 8$ where $\mathrm{D} = \frac{\mathrm{d}}{\mathrm{dt}}$

$(\boldsymbol{iii})\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sinh x \quad (\boldsymbol{iv})(D-2)(D+1)^2\, y = e^{2x} + e^x$

$(\boldsymbol{v})\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = \mathrm{e}^{\mathrm{x}}\cosh 2x$

Que5.Solve the following Differential Equations

$(\boldsymbol{i})\mathrm{Solve}(\mathrm{D}^2+4)\mathrm{y} = sin^2 2x$ with conditions $y(0) = 0\,, y'(0) = 0$

$(\boldsymbol{ii})\frac{d^2y}{dx^2} + 5\frac{dy}{dx} - 6y = cos2x + \sin 3x \quad (\boldsymbol{iii})\frac{d^2y}{dx^2} + y = \sin 2x \sin x$

$(\boldsymbol{iv})(\mathrm{D}^2 - 2\mathrm{D} + 2)\mathrm{y} = \sinh x + \sin\sqrt{2}\,x \quad (\boldsymbol{v})(D^3+1)\, y = cos^2\left(\frac{x}{2}\right) + e^{-x}$

Que6.Solve the following Differential Equations

$(\boldsymbol{i})(\mathrm{D}^2 - 4\mathrm{D} + 3)\mathrm{y} = x^3 \quad (\boldsymbol{ii})(\mathrm{D}^3 - 1)\mathrm{y} = 3\mathrm{x}^4 - 2\mathrm{x}^3$ **[2015-16]**

$(\boldsymbol{iii})\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = \cos x + x^2 \quad (\boldsymbol{iv})\frac{d^4y}{dx^4} + 4y = x^4$

Que7.Solve the following Differential Equations

$(\boldsymbol{i})\ (\mathrm{D}^4 - 1)\mathrm{y} = \cos x \cosh x \quad (\boldsymbol{ii})(\mathrm{D}^2 - 5\mathrm{D} + 6)\mathrm{y} = \mathrm{e}^{\mathrm{x}}\sin x$

$(\boldsymbol{iii})(\mathrm{D}^2 - 2\mathrm{D} + 4)\mathrm{y} = e^x \cos x + \sin x \cos 3x$ **[2017-18]**

$(\boldsymbol{iv})(\mathrm{D}^2 + 2\mathrm{D} + 1)\mathrm{y} = x\, cos\, x \quad (\boldsymbol{v})$ Solve $(\mathrm{D}^2 - 4\mathrm{D} + 4)\mathrm{y} = 8\mathrm{x}^2\mathrm{e}^{2\mathrm{x}}\sin 2\mathrm{x}$

Que8.Solve the following Differential Equations

$(\boldsymbol{i})\ \frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}^2} + 4\mathrm{y} = \sec 2\mathrm{x}$ $(\boldsymbol{ii})(\mathrm{D}^2 + 2\mathrm{D} + 2)\mathrm{y} = \mathrm{e}^{-\mathrm{x}}\sec^3\mathrm{x}$ **[2016-17]**

$(\boldsymbol{iii})\frac{d^2y}{dx^2} + y = x - \cot x$

Answers

Que1 $(\boldsymbol{i})\, Order\ 3\,, Degree\ 4 \qquad (\boldsymbol{ii})\ Order\ 2\,, Degree\ 2$

Que2. $(D-1)^3\, y = 0$

Que3 $(\boldsymbol{i})c_1e^x + c_2e^{2x} + c_3e^{3x}$ $(\boldsymbol{ii})\ (c_1 + c_2x + c_3x^2)\ e^x + c_4e^{4x}$

$(\boldsymbol{iii})\ y = e^{\frac{3\sqrt{2}}{2}x}\left[c_1\ cos\left(\frac{3\sqrt{2}}{2}x\right) + c_2\sin\left(\frac{3\sqrt{2}}{2}x\right)\right] + e^{\frac{-3\sqrt{2}}{2}x}\left[c_3\ cos\left(\frac{3\sqrt{2}}{2}x\right) + c_4\sin\left(\frac{3\sqrt{2}}{2}x\right)\right]$

$(\boldsymbol{iv})\ \ y = e^{0x}[(c_1 + c_2x + c_3x^2)cos\ x + (c_4 + c_5x + c_6x^2)sin\ x]$

$+e^{\frac{-x}{2}}\left[(c_7 + c_8x)cos\left(\frac{\sqrt{3}}{2}x\right) + (c_9 + c_{10}x)cos\left(\frac{\sqrt{3}}{2}x\right)\right]$

$(\boldsymbol{v})y = e^{3x}[(c_1 + c_2x)cos\ 4x + (c_3 + c_4x)sin\ 4x]$

$(\boldsymbol{vi})y = e^{2x}\left[(c_1 + c_2x)cosh\sqrt{3}x + (c_3 + c_4x)sinh\sqrt{3}x\right]$

Que4.$(\boldsymbol{i})y = (c_1 + c_2t)e^{-3t} + \frac{5}{36}e^{3t}$ $(\boldsymbol{ii})y = c_1e^{2t} + c_2\cos 2t + c_3\sin 2t - 1$

$(\boldsymbol{iii})\ y = e^{-x}[c_1\ cos\ x + c_2sin\ x] + \frac{1}{10}e^x - \frac{1}{2}e^{-x}$ $(\boldsymbol{iv})y = c_1e^{2x} + (c_2 + c_3x)e^{-x} + \frac{x}{9}e^{2x} - \frac{1}{4}e^x$

$(\boldsymbol{v})y = c_1e^{2x} + c_2e^{-2x} + \frac{1}{10}xe^{3x} - \frac{1}{8}e^{-x}$

Que5 $(\boldsymbol{i})\ y = -\frac{1}{6}\cos 2x + \frac{1}{8} + \frac{1}{24}\cos 4x$

$(\boldsymbol{ii})y = c_1e^x + c_2e^{-6x} - \frac{1}{30}(\cos 3x + \sin 3x) - \frac{1}{20}(\cos 2x - \sin 2x)$

$(\boldsymbol{iii})y = c_1\ cos\ x + c_2sin\ x + \frac{1}{16}(4x\ \sin x + \cos 3x)$

$(\boldsymbol{iv})y = e^x[c_1\ cos\ x + c_2sin\ x] + \frac{1}{2}e^x - \frac{1}{10}e^{-x} + \frac{1}{2\sqrt{2}}\cos\sqrt{2}x$

$(\boldsymbol{v})y = c_1e^{-x} + e^{\frac{1}{2}x}\left[c_2\ cos\left(\frac{\sqrt{3}}{2}x\right) + c_3\sin\left(\frac{\sqrt{3}}{2}x\right)\right] + \frac{1}{2} + \frac{1}{4}(\cos x - \sin x) + \frac{x}{3}e^{-x}$

Que6 $(\boldsymbol{i})y = c_1e^x + c_2e^{3x} + \frac{1}{27}(9x^3 + 36x^2 + 78x + 80)$

$(\boldsymbol{ii})y = c_1e^x + e^{-\frac{1}{2}x}\left[c_2\ cos\left(\frac{\sqrt{3}}{2}x\right) + c_3\sin\left(\frac{\sqrt{3}}{2}x\right)\right] - (3x^4 - 2x^3 + 72x - 12)$

$(\boldsymbol{iii})\ y = e^x\left[c_1\ cos\left(\sqrt{2}x\right) + c_2\sin\left(\sqrt{2}x\right)\right] + \frac{1}{4}(\cos x - \sin x) + \frac{1}{3}\left(x^2 + \frac{4}{3}x + \frac{2}{9}\right)$

$(\boldsymbol{iv})\ \ y = e^x[c_1\ cos\ x + c_2sin\ x] + e^{-x}[c_3\ cos\ x + c_4sin\ x] + \frac{1}{4}(x^4 - 6)$

Que7 $(\boldsymbol{i})\ y = c_1e^x + c_2e^{-x} + [c_3\ cos\ x + c_4sin\ x] - \frac{1}{5}cos\ x\ cosh\ x$

$(\boldsymbol{ii})y = c_1e^{2x} + c_2e^{3x} + \frac{1}{10}e^x(3\cos x + 3\sin x)$

$(\boldsymbol{iii})\ y = e^x\left[c_1\ cos\left(\sqrt{3}x\right) + c_2\sin\left(\sqrt{3}x\right)\right] + \frac{1}{2}e^x\cos x - \frac{1}{8}\cos 2x + \frac{1}{104}(2\cos 2x - 3\sin 4x)$

$(\boldsymbol{iv})y = e^{-x}[c_1 + c_2\ x] + \frac{1}{2}\cos x + \frac{1}{2}(x-1)\ sinx$

$(\boldsymbol{v})\ \ y = e^{2x}[c_1 + c_2\ x] - e^{2x}[4x\cos 2x + \ (2x^2 - 3)\sin 2x]$

Que8. $(\boldsymbol{i})y = c_1\cos 2x + c_2\sin 2x + \frac{1}{2}\left(x\ \sin 2x + \frac{1}{2}\cos 2x\log\cos 2x\right)$

$(\boldsymbol{ii})y = e^{-x}\left(c_1\cos x + c_2\sin x + \frac{\sin x\tan x}{2}\right)$

$(\boldsymbol{iii})y = c_1\cos x + c_2\sin x + x - sin\ x\ log(cosecx - cotx)$