Engineering Mathematics-II (BAS-203)

Unit 2 Laplace Transform

PRACTISE SHEET -I

Que1. Find the Laplace Transform of $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ t-1, & 2 < t < 3 \\ 7, & t > 3 \end{cases}$

Que2. Find $(i)\ L(sin^3\ 2t)$ $(ii)\ L(\sin 2t \cos 3t)$ $(iii)\ L(\cosh at \sin bt)$ $(iv)\ L\left\{\left((1+te^{-t})\right)^3\right\}$

Que3. (i) If $L(cos^2 t) = \frac{s^2+2}{s(s^2+4)}$ Find $L(cos^2 at)$

(ii) If $L\left(\frac{\sin t}{t}\right) = \tan^{-1}\left(\frac{1}{s}\right)$ Find $L\left(\frac{\sin at}{t}\right)$

Que4. . Find the Laplace Transform of $f(t) = \sin\sqrt{t}$. Hence find $L\left(\frac{cos\sqrt{t}}{\sqrt{t}}\right)$

Que5. Find the Laplace Transform of following functions

$(i)\ f(t) = t\sin at$ $(ii)\ f(t) = t\cosh at$ $(iii)\ f(t) = t^2 e^t \sin 4t$ $(iv)\ f(t) = t^2 e^{-2t}\cos t$

Que6. Find $(i)\ L\left(\frac{\cos at - \cos bt}{t}\right)$ $(ii)\ L\left(\frac{e^{at} - \cos bt}{t}\right)$ $(iii)\ L\left(\frac{1-\cos t}{t^2}\right)$ $(iv)\ L\left(\frac{e^{-4t}\sin 3t}{t}\right)$

Que7. Find the Laplace Transform of following functions

$(i)\ f(t) = \int_0^t \frac{\sin t}{t}\ dt$ $(ii)\ f(t) = \int_0^t \frac{1-\cos 2t}{t}\ dt$ $(iii)\ f(t) = \int_0^t e^{-t}\cos t\ dt$

Que8. Using Laplace Transform , evaluate the following integrals

$(i)\ f(t) = \int_0^\infty \frac{e^{-t}\sin\sqrt{3}t}{t}\ dt$ $(ii)\ f(t) = \int_0^\infty \frac{e^{-2t}-e^{-4t}}{t}\ dt$

Que9. (i) Prove that $\int_{t=0}^\infty \int_{u=0}^t e^{-t}\frac{\sin u}{u}\ du\ dt = \frac{\pi}{4}$

(ii)Find the Laplace Transform of function $f(x) = x^3 \sin x$. Hence Prove that

$\int_0^\infty e^{-t}\ t^3 \sin t\ dt = 0$ **[2022-23]**

Que10. Find the Laplace Transform of following functions

$(i)\ f(t) = t^2\ u(t-2)$ $(ii)\ f(t) = \sin t\ u(t-4)$

Que11. Express the following functions in terms of unit step function and find its Laplace Transform

$(i) f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$ $(ii) f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 4t, & t > 2 \end{cases}$

Que12 Find the Laplace Transform of following Periodic functions

$(i)\ f(t) = \begin{cases} \sin at, & 0 < t \le \frac{\pi}{a} \\ 0, & \frac{\pi}{a} < t < \frac{2\pi}{a} \end{cases}$ $(ii) f(t) = \begin{cases} t, & 0 < t \le a \\ 2a - t, & a < t < 2a \end{cases}$

Answers

Ans1. $\frac{2}{s^3} - \frac{e^{-2s}}{s^3}(2 + 3s + 3s^2) + \frac{e^{-3s}}{s^2}(5s - 1)$

Ans2. $(i)\ \frac{48}{(s^2+4)(s^2+36)}$ $(ii)\ \frac{2(s^2-5)}{(s^2+25)(s^2+1)}$ $(iii)\ \frac{1}{2}\left[\frac{b}{(s-a)^2+b^2} + \frac{b}{(s+a)^2+b^2}\right]$ $(iv)\ \frac{1}{s} + \frac{6}{(s+3)^4} + \frac{3}{(s+1)^2} + \frac{6}{(s+2)^3}$

Ans3. $(i)\ \frac{s^2+2a^2}{s(s^2+4a^2)}$ $(ii)\ \tan^{-1}\left(\frac{a}{s}\right)$

Ans4. $\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}e^{-\frac{1}{4s}}$, $\sqrt{\frac{\pi}{s}}e^{-\frac{1}{4s}}$

Ans5. $(i)\ \frac{2as}{(s^2+a^2)^2}$ $(ii)\ \frac{s^2+a^2}{(s^2-a^2)^2}$ $(iii)\ \frac{8(3s^2-6s-13)}{(s^2-2s+17)^3}$ $(iv)\ \frac{2(s^3+6s^2+9s+2)}{(s^2+4s+5)^3}$

Ans6. $(i) -\frac{1}{2}log\left(\frac{s^2+a^2}{s^2+b^2}\right)$ $(ii)\ \frac{1}{2}log\left(\frac{s^2+b^2}{(s-a)^2}\right)$ $(iii)\ \cot^{-1}s + \frac{s}{2}log\left(\frac{s^2}{s^2+1}\right)$ $(iv)\ \tan^{-1}\left(\frac{3}{s+4}\right)$

Ans7. $(i)\ \frac{1}{s}\cot^{-1}s$ $(ii)\ \frac{1}{2s}\log\left(1 + \frac{4}{s^2}\right)$ $(iii)\ \frac{s+1}{s(s^2+2s+2)}$

Ans 8. $(i)\ \frac{\pi}{3}$ $(ii)\ \log 2$

Ans 9. $(ii)\ \frac{24\,s\,(s^2-1)}{(s^2+1)^4}$

Ans10. $(i)\ \frac{e^{-2s}}{s^3}(4s^2 + 4s + 2)$ $(ii)\ \frac{e^{-4s}}{s^2+1}(\cos 4 + s\sin 4)$

Ans11. $(i)\ \frac{e^{-s}}{s^2}(1 - e^{-s})^2$ $(ii)\ \frac{2(1-e^{-2s})}{s^3} + \frac{4e^{-2s}}{s}$

Ans12. $(i)\ \frac{a}{\left(1-e^{-\frac{\pi s}{a}}\right)(s^2+a^2)}$ $(ii)\ \frac{1}{s^2}\tanh\frac{as}{2}$

Engineering Mathematics-II (BAS-203)
Unit 2 Laplace Transform

PRACTISE SHEET - II

Que1. Find the Inverse Laplace Transform of following functions

$(i)\ \frac{1}{(p-2)^2+1}$ $(ii)\ \frac{6}{2p-3}-\frac{3+4p}{9p^2-16}+\frac{8-6p}{16p^2+9}$ $(iii)\ \frac{14p+10}{49p^2+28p+13}$

Que2. Find $(i)L^{-1}\left[\frac{1}{p^2(p+1)}\right]$ $(ii)\ L^{-1}\left[\frac{p^2+3}{p(p^2+9)}\right]$ $(iii)\ L^{-1}\left[\frac{(p^2+2)}{p(p^2+4)}\right]$

Que3. Find the Inverse Laplace Transform of following functions

$(i)\ \frac{p-1}{p^2-6p+25}$ $(ii)\ \frac{p}{p^2+4p+13}$ $(iii)\ \frac{1}{9p^2+6p+1}$

Que4. Find $(i)L^{-1}\left[\frac{e^{-p}}{(p+1)^3}\right]$ $(ii)\ L^{-1}\left[\frac{\pi e^{-p}+pe^{-p/2}}{p^2+\pi^2}\right]$ $(iii)\ L^{-1}\left[\frac{e^{-\pi p/2}+e^{-3\pi p/2}}{p^2+1}\right]$

Que5. Find the Inverse Laplace Transform of following functions

$(i)\log\left[\frac{p+a}{p+b}\right]$ $(ii)\log\left[\frac{p^2-1}{p^2}\right]$ $(iii)\ \tan^{-1}\frac{2}{p^2}$ $(iv)\ \frac{2ap}{(p^2+a^2)^2}$ $(v)\ \frac{1}{(p^2+a^2)^2}$

Que6. Find the Inverse Laplace Transform of following functions by using partial fractions

$(i)\ \frac{5p+3}{(p-1)(p^2+2p+5)}$ $(ii)\ \frac{p}{p^4+4a^4}$ $(iii)\ \frac{2p^2-6p+5}{p^3-6p^2+11p-6}$

Que7. Hence find the Inverse Laplace Transform by using the Convolution theorem.

$(i)\ \frac{p^2}{(p^2+a^2)(p^2+b^2)}, a\neq b$ $(ii)\ \frac{p}{(p^2+1)(p^2+4)}$ $(iii)\ \frac{16}{(p+2)^2(p-2)}$ $(iv)\ \frac{1}{(p^2+1)p^3}$ $(v)\ \frac{p^2}{p^4-a^4}$

$(vi)\ \frac{1}{p^2(p+1)^2}$ **[2022-23]**

Answers

Ans1. $(i)e^{2t}\sin t$ $(ii)3e^{\frac{3}{2}t}-\frac{1}{4}\sinh\frac{4}{3}t-\frac{4}{9}\cosh\frac{4}{3}t+\frac{2}{3}\sin\frac{3}{4}t-\frac{3}{8}\cos\frac{3}{4}t$ $(iii)\ \frac{2}{7}e^{-\frac{2}{7}t}\left(\cos\frac{3}{7}t+\sin\frac{3}{7}t\right)$

Ans2. $(i)t-1+e^{-t}$ $(ii)\ \frac{1}{a}(1-e^{-at})$ $(iii)\ cos^2t$

Ans3. $(i)\ e^{3t}\cos 4t+\frac{1}{2}e^{3t}\sin 4t$ $(ii)\ e^{-2t}\cos 3t-\frac{2}{3}e^{-2t}\sin 3t$ $(iii)\ \frac{t}{9}\ e^{-\frac{t}{3}}$

Ans4. $(i)e^{-(t-1)}\frac{(t-1)^2}{2!}\ U(t-1)$ $(ii)\sin\pi t\left[U\left(t-\frac{1}{2}\right)-U(t-1)\right]$ $(iii)\ \cot t\left[U\left(t-\frac{3\pi}{2}\right)-U\left(t-\frac{\pi}{2}\right)\right]$

Ans5. $(i)\ \frac{e^{-bt}-e^{-at}}{t}$ $(ii)\ \frac{2}{t}(1-\cosh t)$ $(iii)\ \frac{2}{t}\sin t\sinh t$ (iv) t sin at $(v)\frac{\sin at-at\cos at}{2a^3}$

Ans6. $(i)\ \ e^t-e^{-t}\left(\cos 2t-\frac{3}{2}\sin 2t\right)$ $(ii)\frac{1}{2a^2}\sin at\sinh at$ $(iii)\frac{1}{2}e^t-e^{2t}+\frac{5}{2}e^{3t}$

Ans7. $(i)\ \frac{a\sin at-b\sin bt}{a^2-b^2}$ $(ii)\ \frac{1}{3}(\cos t-\cos 2t)$ $(iii)\ e^{2t}-e^{-2t}(1+4t)$

$(iv)\frac{t^2}{2}+\cos t-1$ $(v)\ \frac{1}{2a}(\sinh at+\sin at)$ $(vi)\ (t+2)e^{-t}+t-2$

Engineering Mathematics-II (BAS-203)

Unit 2 Laplace Transform

PRACTISE SHEET - III

Que1. Solve the following Differential equation by using Laplace Transform

$\frac{d^3y}{dt^3} + 2\frac{d^2y}{dt^2} - \frac{dy}{dt} - 2y = 0$, $y = \frac{dy}{dt} = 0$ and $\frac{d^2y}{dt^2} = 6$ *when* $t = 0$

Que2. A particle moves in a line so that its displacement x from a fixed point O at any time t, is given by $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 5x = 80 \sin 5t$

Using Laplace Transform,find its displacement at any time t, if intially particle is at rest at $x = 0$

Que3. Solve the intial value problem by using Laplace Transform

$y'' + y' - 2y = 1 - 2x$ given that $y = 0$, $y' = 4$ *when* $x = 0$

Que4. Solve the following Differential equation by using Laplace Transform

$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = e^{-x} \sin x$, where $y(0) = 0$, $y'(0) = 1$

Que5. Apply Laplace Transform to solve the equation

$\frac{d^2y}{dt^2} + y = t \cos 2t$, given that $y = \frac{dy}{dt} = 0$ for $t = 0$

Que6. By using Laplace Transform, find the solution of intial value problem

$y'' + 9y = 9\,u(t-3)$ given that $y(0) = y'(0) = 0$ where $u(t-3)$ is unit step function

Que7. Use Laplace Transform , solve the following differential equation

$\frac{d^2y}{dx^2} + y = 6 \cos 2x$, given that $y(0) = 3$ & $y'(0) = 1$ **[2022-23]**

Que8. Solve the following simultaneous differential equations by Laplace Transform

$3\frac{dx}{dt} - y = 2t$, $\frac{dx}{dt} + \frac{dy}{dt} - y = 0$, given that $x = y = 0$ when $t = 0$

Que9. Use Laplace Transform to solve

$\frac{dx}{dt} + y = \sin t$, $\frac{dy}{dt} + x = \cos t$, given that $x = 2, y = 0$ when $t = 0$

Que10. Solve the following simultaneous differential equations by Laplace Transform

$\frac{dx}{dt} + 4\frac{dy}{dt} - y = 0$, $\frac{dx}{dt} + 2y = e^t$, with condition $x = y = 0$ when $t = 0$

Que11. Use Laplace Transform to solve

$\frac{dx}{dt} - y = e^t$, $\frac{dy}{dt} + x = \sin t$, given that $x = 1, y = 0$ when $t = 0$

Answers

Ans1. $y = e^{t} - 3e^{-t} + 2e^{-2t}$

Ans2. $x = 2e^{-2t}(\cos t + \sin t) - 2(\cos 5t + \sin 5t)$

Ans3. $y = e^{x} - e^{-2x} + x$

Ans4. $y = \frac{1}{3}e^{-x}(\sin x + \sin 2x)$

Ans5. $y = -\frac{5}{9}\sin t + \frac{4}{9}\sin 2t - \frac{t}{3}\cos 2t$

Ans6. $y = [1 - \cos 3(t-3)]\, u(t-3)$

Ans7. $y = 5\cos x + \sin x - 2\cos 2x$

Ans8. $y = t + \frac{3}{2} - \frac{3}{2}e^{\frac{2t}{3}}$, $x = \frac{t^2}{2} + \frac{t}{2} - \frac{3}{4}e^{\frac{2t}{3}} + \frac{3}{4}$

Ans9. $x = e^{-t} + e^{t}$, $y = \sin t + e^{-t} - e^{t}$

Ans10. $x = \frac{1}{3} - \frac{5}{7}e^{-t} + \frac{8}{21}e^{\frac{3}{4}t}$, $y = \frac{1}{7}\left(e^{-t} - e^{\frac{3}{4}t}\right)$

Ans11. $x = \frac{1}{2}(e^{t} + \cos t + 2\sin t - t\cos t)$, $y = \frac{1}{2}(t\sin t - e^{t} + \cos t - \sin t)$