Mathematics - I
KAS-103

Module-IV
Multiple Integrals.

Lecture no -44.

Content: Change of Order of Integration

Some of problem can be made easy by change of order of integration.

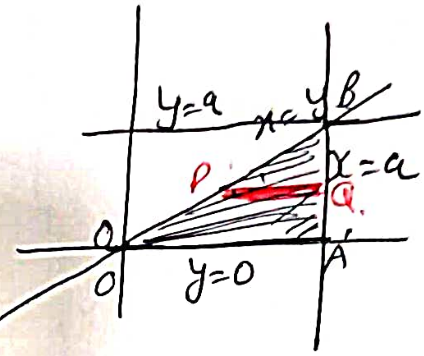
To change order of integration we first draw region under given limits. Then if numerical is of Horizontal strip [limits involve y] we change it into vertical strip [limits involving x].

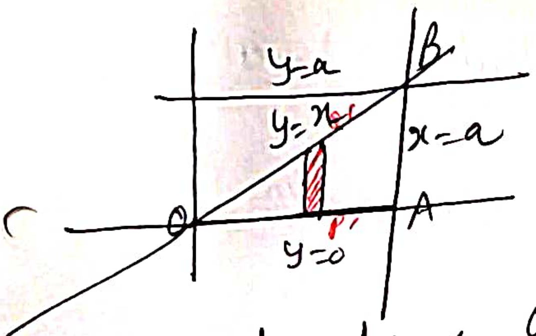
Q.1 Change the order of integration in $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$ and evaluate.

Sol. $R = \{ y \le x \le a ;\ 0 \le y \le a \}$

In order to change.

$R' = \{ 0 \le y \le x ;\ 0 \le x \le a \}$

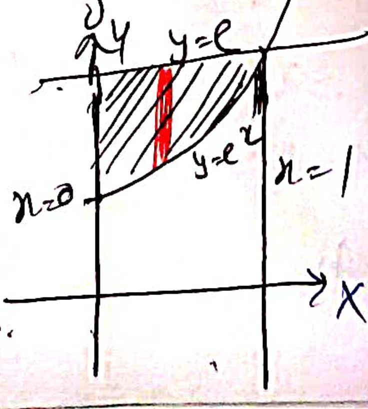


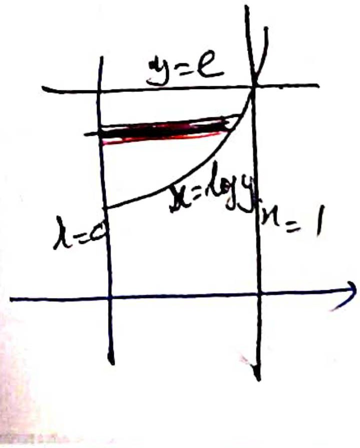


In order to change order of integration OAB taking along vertical strip $0 \le y \le x$, $0 \le x \le a$.

$$\int_0^a \int_0^x \frac{x\,dy\,dx}{x^2+y^2} = \int_0^a x\left[\frac{1}{x}\tan^{-1}\frac{y}{x}\right]_0^x dx = \frac{\pi a}{4}$$

Q.2 $\int_0^1 \int_{e^x}^e \frac{dy\,dx}{\log y}$





$R = \{ 0 \le x \le 1 ;\ e^x \le y \le e \}$

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$$\int_1^e \int_0^{\log y} \frac{dx\,dy}{\log y} = (e-1) \text{ Ans.}$$

Q.3 $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy = \int_0^\infty \int_0^y \frac{e^{-y}}{y}\,dy = 1.$

Q.# $\int_0^\infty \int_0^x x \exp\left(\frac{-x^2}{y}\right) dx\,dy$ evaluate.

$\because$ integral is complex if we integrate w.r.t 'x'

$\therefore$ changing order of integration.

$\int e^{ay}\,dy = \frac{e^{ay}}{a}$

$$\int_0^\infty \int_y^\infty x e^{\frac{-x^2}{y}}\,dx = \int_0^\infty \frac{x e^{\frac{-x^2}{y}}}{\frac{-2x^2}{y}}\,dx$$

$\frac{x^2}{y} = t$

$x^2 = ty$

$x = \sqrt{ty}$.

limit remains same

$$= \int_0^\infty \left(\frac{-y}{2}\right)[-e^{-y}]\,dy = \int_0^\infty -x e^{\frac{-x^2}{y}}\,dx.$$

$= \frac{1}{2}$. $= \frac{1}{2}$.

Q.4 $\int_0^1 \int_x^1 \sin(y^2)\,dy\,dx = \int_0^1 \int_0^y \sin(y^2)\,dx\,dy = \frac{1}{2} - \frac{\cos 1}{2}$

Q.5 $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ change and evaluate the same.

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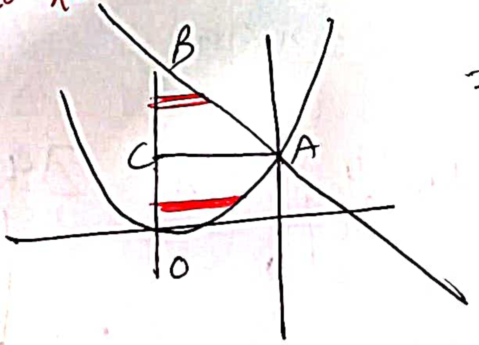


$= I_1 + I_2$

$I_1 = R_1 \{0 \le x \le \sqrt{y};\ 0 \le y \le 1\}$

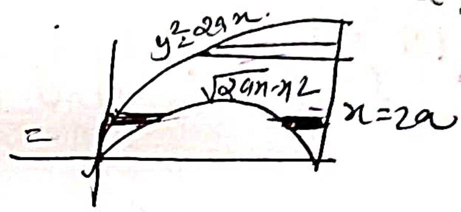
$I_2 = R_2 \{0 \le x \le 2-y,\ 1 \le y \le 2$

$= \frac{3}{8}$ Ans.

$\ldots y\,dy\,dx = \frac{8}{3}.$

Q.7 $$\int_0^1\int_0^{\sqrt{1-x^2}} \frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx = \frac{\pi}{2}\log\left(\frac{e+1}{2}\right)$$

Ts 8 $$\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V(x,y)\,dy\,dx =$$

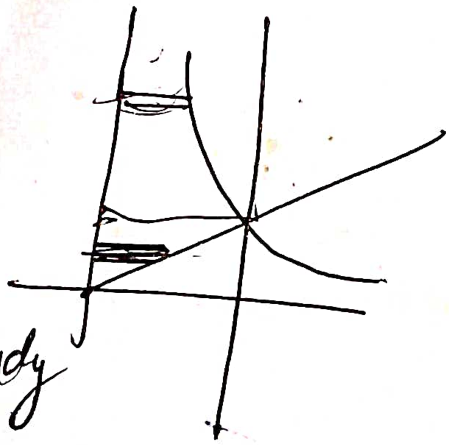


$$= \int_0^a\int_{y^2/2a}^{a-\sqrt{a^2-y^2}} V(x,y)\,dx\,dy + \int_0^a\int_{a+\sqrt{a^2-y^2}}^{2a} V\,dx\,dy + \int_a^{2a}\int_{y^2/2a}^{2a} V(x,y)\,dx\,dy$$

Q.9 $$\int_0^1\int_{x^2}^{2-x} f(x,y)\,dy\,dx \qquad \int_0^1\int_0^{\sqrt{y}} f(x,y)\,dx\,dy + \int_1^2\int_0^{2-y} f(x,y)\,dx\,dy.$$

Q.10 $$\int_0^a\int_{\sqrt{a^2-y^2}}^{y+a} f(x,y)\,dx\,dy = \int_0^a\int_{\sqrt{a^2-x^2}}^{a} f(x,y)\,dy\,dx + \int_a^{2a}\int_{x-a}^{a} f(x,y)\,dy\,dx$$

11 $$\int_0^a\int_x^{a^2/x} f(x,y)\,dy\,dx$$



$$\int_0^a\int_0^y f(x,y)\,dx\,dy + \int_a^{\infty}\int_0^{a^2/y} f(x,y)\,dx\,dy$$