Lecture-29

Unit-I
Differential Calculus-I

Contents:- Curve Tracing in Polar Co-ordinates.

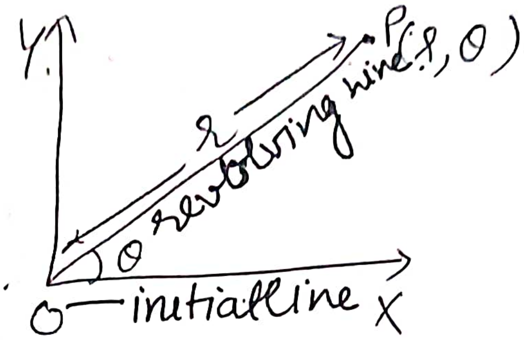
Polar-Co-ordinates:-

OX - horizontal line is called initial line.

OP is called revolving line making an angle θ.

polar co-ordinates of pt. $P(r, \theta)$

O is called pole, θ → vertical angle, r - radius vector



Working Process:- To plot polar curves.

1. **Symmetry:** 1) If $f(-\theta, r) = f(\theta, r)$ then symmetrical about initial line.
   2) If $f(\theta, -r) = f(\theta, r)$, then curve is symmetrical about pole.
   3) If $f(-\theta, -r) = f(\theta, r) \Rightarrow$ then curve is symmetrical about line passing through pole and ⊥ to initial line $\theta = \frac{\pi}{2}$
   4) for $\theta = \alpha$, if $f(2\alpha - \theta, r) = f(\theta, r)$ then symmetrical about line $\theta = \alpha$.
2. **origin:-** If at $r = 0$, we get real values of θ, then curve passes through pole or origin.
3. **Tangent at pole:** At $r = 0$ the values of θ gives tangent at the pole.
4. **Asymptote:** If at $\theta \to \theta_1$ (any real value) r tends to ∞ then there exist asymptote. $r\sin(\theta - \alpha) = \frac{1}{f'(\alpha)}$

   condition ① $f(\theta) = \frac{1}{r}$ (in this form)

   put $f(\theta) = 0$, $\theta = \alpha_1, \alpha_2, \ldots \alpha_n$.

   at these values of θ

   asymptotes are $r\sin(\theta - \alpha) = \frac{1}{f'(\alpha)}$
5. **Nature of curve or point of Intersection** - Find max and minimum points on the curve at the values of θ, region.
6. Construct the table
7. plot the curve.

Q.1 Trace the curve $r = a(1-\cos\theta)$ [Cardi

Sol. Symmetry 1) $f(-r,\theta) \neq f(r,\theta)$, curve is not symm about pole.

2) $f(r,-\theta) = f(r,\theta) \Rightarrow$ curve is symmetrical about initial line.

Origin at $r=0$. pole.   r becomes 0 $a(1-\cos\theta)=0$ $\therefore$ curve passes through $\Rightarrow \cos\theta = 1$. $\theta = 0, 2\pi$. (real values exist)

3. Tangent at origin:- put $r=0$

$\Rightarrow a(1-\cos\theta)=0$ $\Rightarrow 1-\cos\theta = 0$.

$\cos\theta = 1$.

$\boxed{\theta = 0, 2\pi}$

$\theta = 0, 2\pi$ are tangents to the curve at origin.

4. Asymptote:- $r = a(1-\cos\theta)$

$|\cos\theta| \leq 1$ $\forall\theta$ (for all)   r can never tends to ∞. Hence curve has no asymptotes.

5. Nature of the curve, and region. $\because |\cos\theta| \leq 1$.

$\Rightarrow r \leq 2a$.

$-1 \leq \cos\theta \leq 1$

At $\theta = 0$ $\Rightarrow r = 0$

At $\theta = -\pi$ $\Rightarrow r = 2a$.

$\boxed{\theta = 0, \pi}$

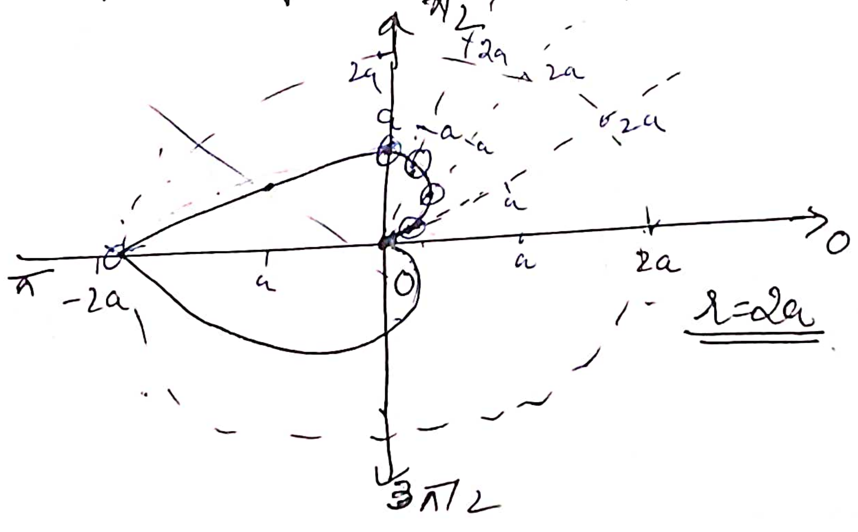
$\Rightarrow$ curve lies b/w. $0 \leq r \leq 2a$

$\boxed{+\pi \leq \theta \leq 0^\circ}$

6. Special points / Table:-

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/4$	π
r	0	.134a	.293a	0.5a	a		2a

$\because$ curve is symmetrical about initial line.



$r = 2a$

Trace the curve $r^2 = a^2 \cos 2\theta$. UPTU-2012, 14

Symmetry :- 1) $f(-r,\theta) = f(r,\theta)$, curve is symmetrical about pole.

2) $f(r,-\theta) = f(r,\theta)$; curve is symmetrical about initial line.

3) $f(-r,-\theta) = f(r,\theta)$ ∴ curve is symmetrical about both.

put $\theta = \pi/2$ $f(\pi-\theta, r) = f(\theta, r) \Rightarrow$ curve is symmetrical about line $\theta = \pi/2$.

2. Origin. put $r = 0$. $r^2 = a^2 \cos 2\theta = 0$ ∴ curve ~~does not~~

$\cos 2\theta = 0 \Rightarrow 2\theta = \pm\pi/2$ $\theta = \pm\pi/4$

∵ real values of θ exists ∴ curve passes through pole.

3. Tangent at origin. put $r = 0$ $\Rightarrow \theta = \pm\pi/4$, $= \pm 3\pi/4$ are tangents to the curve.

4. Asymptote. $r^2 = a^2 \cos 2\theta$. ∵ $|\cos 2\theta| \leq 1$

$r^2 \leq a^2 \Rightarrow r \leq \pm a$ $-a < r < a$

r does not tends to infinity. ∴ curve does not have any asymptote.

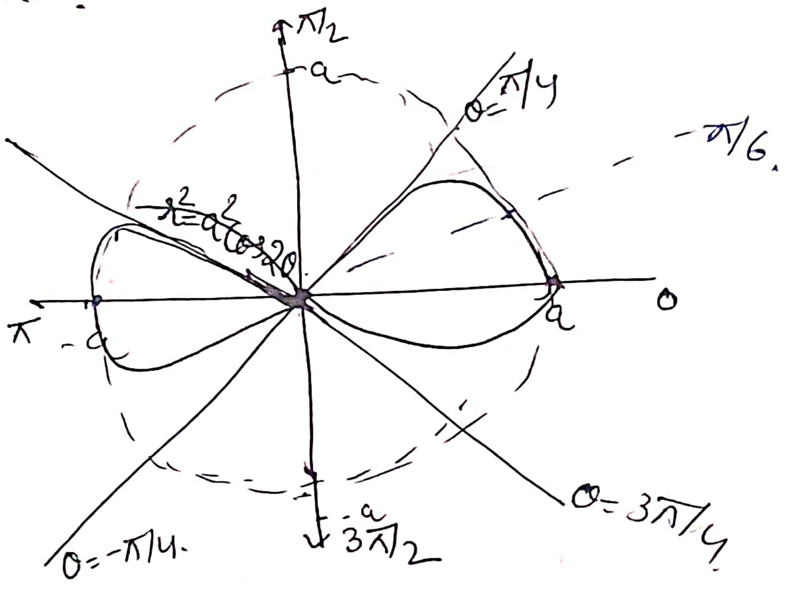
5. Nature of the curve :- ∵ $|\cos\theta| \leq 1 \Rightarrow |\cos 2\theta| \leq 1$

$\Rightarrow -a < r < a$.

$a r = |a|$ is ~~the~~ region of the curve.

6. Special points or table.

θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	$3\pi/4$	π
r	$\pm a$	$\pm\frac{a}{2}$	0	imag	0		



Problems for Practice!

UPTU-2013

1. Trace the curve $r = a(1+\cos\theta)$
2. Trace the curve $r = a(1-\sin\theta)$
3. Trace the curve $r = a(1+\sin\theta)$
4. Trace the curve $r = a\sin 2\theta$.
5. Trace the curve $r = a\cos\theta$. $r = a\sin\theta$
6. Trace the curve $r = 2\cos\theta$

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