



ABES Engineering College, Ghaziabad
Department of AS&H

Session: 2023-24

Semester: II

Section: All

Course Code: BAS-203

Course Name: Engg.Maths-II

Tutorial-4 (Complex Variable - Differentiation)

S.No.	KL, CO	Question
1	K3, CO4	Find the constants a,b,c such that the function $f(z)$ where $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic. Express $f(z)$ in terms of z .
2	K3, CO4	Show that $f(z) = z z $ is not analytic anywhere.
3	K3, CO4	Show that function $f(z) = \frac{x^3y^5(x+iy)}{x^6+y^{10}}$, $z \neq 0$, $f(0) = 0$, is not analytic at the origin even though it satisfies Cauchy-Riemann equations at the origin.
4	K3, CO4	Show that $v(x,y) = e^{-x}(x\cos y + y\sin y)$ is harmonic. Find its harmonic conjugate without Milne's method.
5	K3, CO4	Find the regular function $f(z)$ in terms of z whose imaginary part is $\frac{x-y}{x^2-y^2}$ using Milne's Thomson method.
6	K3, CO4	If $f(z) = u + iv$ is an analytic function, find $f(z)$ in terms of z if $u - v = e^x(\cos y - \sin y)$
7	K3, CO4	Find the image of $ z - 2i = 2$ under the mapping $w = \frac{1}{z}$.
8	K3, CO4	Find the bilinear transformation which maps the point $z = i, -i, 1$ into the points $w = 0, 1, \infty$.
9	K3, CO4	Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Hence find the image of $ z < 1$.

10	K3, CO4	Prove that $w = \frac{z}{1-z}$ maps the upper half of the Z plane onto the upper half of the w plane.
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Answers:

1. $a = \frac{-1}{2}; b = -2; c = \frac{1}{2}; f(z) = \frac{-1}{2}(2+i)z^2.$

4. $u(x, y) = e^{-x}(x \sin y - y \cos y) + c.$

5. $\frac{1+i}{z} + c$

6. $e^z + c.$

7. $4v + 1 = 0$

8. $w = \frac{(i-1)z+(i+1)}{-2z+2}$

9. $w = \frac{i-z}{i+z}; u > 0$