ABES ENGINEERING COLLEGE, GHAZIABAD (032)

DEPARTMENT OF AS & H

B. TECH SEM-I (2022-23)

Engineering Mathematics - I (BAS-103)

Tutorial Sheet: UNIT-4 (Multiple Integration)

Q.1- Evaluate $\int_0^1 \int_0^{x^2} e^{y/x}\, dxdy$. Ans - 1/2

Q.2- Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$ and hence show that $\int_0^\infty e^{-x^2} dx = \frac{1}{2}\sqrt{\pi}$. Ans - $\frac{1}{4}\pi$

Q.3- Evaluate $\iint ydxdy$ over the area bounded by $x = 0, y = x^2, x + y = 2$ in first quadrant. Ans - $\frac{16}{15}$

Q.4- Change of order of the integration and hence evaluate $\int_0^1 \int_{x^2}^{2-x} xydydx$. Ans - 3/8

Q.5- Changing the order of integration evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dxdy$. Ans - 1

Q.6- Evaluate $\int_{x=-c}^{c} \int_{y=-b}^{b} \int_{z=-a}^{a} (x^2 + y^2 + z^2) dzdydx$ Ans - $\frac{8}{3}abc(a^2 + b^2 + c^2)$

Q.7- Evaluate $\iiint_v zdxdydz$ where the region of integration V is a cylinder which is bounded by the following curve $z = 0, z = 1, x^2 + y^2 = 4$. Ans - 2π

Q.8- Evaluate the following by changing into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2\sqrt{x^2 + y^2}\, dxdy$. Ans - $\frac{\pi a^5}{20}$

Q.9- Using the transformation $x + y = u, y = uv$, prove that $\iint xy(1 - x - y)^{1/2} dxdy = \frac{2\pi}{105}$ taken over the area of triangle bounded by the lines $x = 0,\ y = 0,\ x + y = 1$.

Q.10- Using the transformation $u = x - y, v = x + y$, prove that $\iint \cos\frac{x-y}{x+y} dxdy = \frac{1}{2}sin1$, where R is bounded by $x = 0,\ y = 0,\ x + y = 1$..

Q11. Evaluate

(i) $\Gamma -15/2$ (ii) $\int_0^\infty e^{-h^2x^2} dx$ (iii) $\int_{-\infty}^{\infty} \cos\frac{\pi x^2}{2} dx$ (iv) $\int_0^{\pi/2} \sin^6\theta\, d\theta$

Ans. (i) $\frac{2^8\sqrt{\pi}}{15\times13\times11\times9\times7\times5\times3}$ (ii) $\frac{\sqrt{\pi}}{2h}$ (iii) 1 (iv) $\frac{5\pi}{32}$

Q12. Find the value of $\iiint \log(x+y+z) dxdydz$ the integral extending overall positive and zero values of x, y, z subject to the condition $x + y + z < 1$.

Ans. – 1/18