ABES ENGINEERING COLLEGE, GHAZIABAD (032)

B. TECH FIRST SEMESTER 2023-2024

ENGINEERING MATHEMATICS-I (BAS-103)

UNIT-2: Differential Calculus-I

Question Bank

1. Find the n^{th} derivative of $sinx.sin2x.sin3x$.
2. Find the n^{th} derivative of $\frac{2x+1}{(2x-1)(2x+3)}$.
3. Find the n^{th} derivative of $\dfrac{x}{2x^2+3x+1}$.
4. If $y = xlog\frac{x-1}{x+1}$, show that $y_n = (-1)^{n-2}(n-2)!\left[\frac{x-n}{(x-1)^n} - \frac{x+n}{(x+1)^n}\right]$
5. Find the n^{th} derivative of $tan^{-1}\left(\frac{1+x}{1-x}\right)$.
6. If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2-1)y_{n+2} + (2n+1)xy_{n+1} + (n^2-m^2)y_n = 0$.
7. Apply Leibnitz Theorem to find $y_n \quad if \;\; y = x^{n-1}logx.$
8. If $y = e^{\tan^{-1}x}$, Prove that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$
9. If $y = \left(\frac{1+x}{1-x}\right)^{1/2}$, prove that $(1-x^2)y_n - [2(n-1)x+1]y_{n-1} - (n-1)(n-2)y_{n-2} = 0.$
10. If $x = \tan(\log y)$, Prove that $(1+x^2)y_{n+2} + [2(n+1)x-1]y_{n+1} + n(n+1)y_n = 0$
11. If $y = \sin(a\sin^{-1}x)$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2-a^2)y_n = 0$ and hence find the value of y_n when $x = 0$.
12. If $y = \left[log\left(x+\sqrt{1+x^2}\right)\right]^2$, find $y_n(0)$.
13. If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ and hence find the value of y_n when $x = 0$.
14. If $u = f(r), where\ r^2 = x^2 + y^2\ ,\ prove\ that\ \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f''(r) + \dfrac{1}{r}f'(r)$.
15. If $u = x^2\tan^{-1}\dfrac{y}{x} - y^2\tan^{-1}\dfrac{x}{y}$, then prove that $\dfrac{\partial^2 u}{\partial x\partial y} = \dfrac{x^2-y^2}{x^2+y^2}$.
16. If $z = f(x+ct) + \phi(x-ct)$, show that $\dfrac{\partial^2 z}{\partial t^2} = c^2\dfrac{\partial^2 z}{\partial x^2}$.
17. If $x^2 = au + bv\ ,\ y^2 = au - bv\ ,\ prove\ that\ \left(\dfrac{\partial u}{\partial x}\right)_y . \left(\dfrac{\partial x}{\partial u}\right)_v = \dfrac{1}{2} = \left(\dfrac{\partial v}{\partial y}\right)_x \left(\dfrac{\partial y}{\partial v}\right)_u$.
18. If $u = e^{xyz}$ show that $\dfrac{\partial^3 u}{\partial x\partial y\partial z} = (1+3xyz+x^2y^2z^2)u$.