

26/09/22

PROBABILITY & STATISTICS

1. In probability distribution, the sum of all probabilities of  $n=1$  time,

$$k + 8k + 3k + k^2 = 1$$

$$\Rightarrow k^2 + 12k + 1$$

$$k^2 + 12k - 1 = 0 \quad (1)$$

Solving the quadratic equation (1) we have,

$$k = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-12 \pm \sqrt{(12)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{-12 \pm 2\sqrt{37}}{2} = -6 \pm \sqrt{37}$$

Since, the value of probability can't be negative:

$$k = -6 + \sqrt{37}$$

$$k = 0.0828$$

$x$	0	1	2	3
$P(x)$	0.0828	0.6624	0.2484	0.0069

To find the mean:

$$\bar{x} = \sum_{i=0}^3 x_i f_i(x)$$

$$= (0 \times 0.0828) + (1 \times 0.6624) + (2 \times 0.2484) + (3 \times 0.0069)$$

$$\bar{x} = 1.1799$$

2. If  $x$  is binomial with ~~mean~~  $(4, 1/3)$  then find  $P(x < 3)$  &  $P(2 < x < 5)$

$B(4, 1/3) \Rightarrow n=4, p=1/3 \text{ \& } (1-p)=2/3$

Hence, the probability distribution is.

$x$	0	1	2	3	4
$P(x)$	${}^4C_0 (1/3)^0 (2/3)^4$	${}^4C_1 (1/3)^1 (2/3)^3$	${}^4C_2 (1/3)^2 (2/3)^2$	${}^4C_3 (1/3)^3 (2/3)^1$	${}^4C_4 (1/3)^4 (2/3)^0$

$x$	0	1	2	3	4
$P(x)$	0.1975	0.3951	0.2963	0.0988	0.0123

$$P(x < 3) = P(x=0) + P(x=1) + P(x=2) = 0.8889$$

$$\begin{aligned} P(2 < x < 5) &= P(x=3) + P(x=4) \\ &= 0.0988 + 0.0123 \\ &= 0.1111 \end{aligned}$$



27/09/22

1. If  $u$  is normal with mean 5 & variance 9, find  $P(u > 3)$  &  $P(6 > u > 3)$

⇒ In this case,

$$\mu = 5, \sigma = \sqrt{9} = 3$$

For any particular value of  $u$   
 $z = \frac{u - \mu}{\sigma}$

for  $u < 3$

$$\Rightarrow u = 3, \mu = 5, \sigma = 3$$

$$\text{Thus, } z = \frac{3 - 5}{3} = -0.6667$$

Using a table of Standard normal probability,

$$P(0.6667) = 0.2514$$

$$P(u < 3) = 1 - P(u > 3) = 0.7486$$

for  $P(3 < u < 6)$

$$P(3 < u) = 0.2514$$

$$\begin{aligned} P(u < 6) &= 1 - P(u > 6) \\ &= 1 - \phi(0.333) \\ &= 0.3707 \end{aligned}$$

$$\begin{aligned} P(3 < u < 6) &= P(u < 6) - P(3 < u) \\ &= 0.3707 - 0.2514 \\ &= 0.1193 \end{aligned}$$

2. If  $\bar{x}$  is normal with mean 102  
variance 100, then find  $P(u < 15)$  &  
 $P(13 < u < 14)$ .

⇒

In this case,

$$\mu = 10, \sigma = \sqrt{100} = 10$$

for any  $u$ ,

$$z = \frac{u - \mu}{\sigma}$$

for  $P(u < 15)$

$$\Rightarrow u = 15, \mu = 10, \sigma = 10$$

$$z = \frac{15 - 10}{10} = \frac{1}{2} = 0.5$$

Using the value of standard normal  
probability;  $\Phi = 0.6915$

$$P(u < 15) = 0.6915$$

for  $P(13 < u < 14)$ ,

$$P(13 < u) = 1 - \Phi\left(\frac{13 - 10}{10}\right)$$

$$= 1 - \Phi(0.3)$$

$$= 1 - 0.6179 = 0.3821$$

$$P(u < 14) = \Phi\left(\frac{14 - 10}{10}\right) = 0.6554$$

$$P(13 < u < 14) = P(u < 14) - P(13 < u)$$

$$= 0.6554 - 0.3821$$

$$= 0.2733$$