UNIT-3

①

a) Define Alternative Hypothesis

Sol:- Any hypothesis which contradicts the Null Hypothesis is called an Alternative Hypothesis. It is denoted by H_1.

b) Define Critical Region.

Sol:- A region corresponding to a statistic t in the sample space 's' which leads to the rejection of H_0 is called Critical region (or) Rejection region.

c) Define type I and type II errors.

Sol:- Type I error : Reject H_0 when it is true. If the Null Hypothesis H_0 is true but it is rejected by test procedure, then the error made is called Type I error (or) α-error.

Type II error : Accept H_0 when it is wrong. i.e accept H_0 when H_1 is true. If the Null Hypothesis is false but it is accepted by test procedure. then the error made is called Type II error (or) β-error.

d) Write the four important tests to test the significance under large sample tests.

Sol:- 1. Testing of significance for single proportion

2. Testing of significance for difference of proportions.

3. Testing of significance for single mean

4. Testing of significance for difference of means.

e) Derive Critical Values of Z for both two tailed and Single tailed tests at 1%, 5% and 10% level of Significance.

Sol:-

Level of Significance	1%	5%	10%
Critical Values for two tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.96$	$\lvert Z_\alpha \rvert = 1.645$
Critical Values for right tailed test	$Z_\alpha = 2.33$	$Z_\alpha = 1.64$	$Z_\alpha = 1.28$
Critical Values for left tailed test	$Z_\alpha = -2.33$	$Z_\alpha = -1.645$	$Z_\alpha = -1.28$

2

a) A Sample of 64 Students has a mean weight of 70kg. Can this be regarded as a Sample from a population which mean weight 56Kgs and Standard deviation 25Kgs.

Sol:- Given that,

The mean of the population $\mu = 56$ Kgs

Standard deviation $\sigma = 25$ Kg

mean of Sample $\bar{x} = 70$ Kgs

Sample Size $n = 64$

Null Hypothesis (H_0): A Sample of 64 Students with mean weight of 70kgs can be regarded as a Sample from a population with mean weight 56 Kgs. $\mu = 56$

Alternative Hypothesis (H_1): Sample cannot be regarded as one coming from the population

$\mu \neq 56$

Level of Significance : 0.05 (assumed)

The test Statistics is $z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{70 - 56}{\left[\frac{25}{\sqrt{64}}\right]} = 4.48$

Conclusion:

As $|z| > 1.645$,

The Null hypothesis H_0 is rejected.

b) A die is tossed 960 times and it falls with 5 upwards 184 times. Is the die unbiased at a level of Significance of 0.01.

Sol:-

n = no. of trails = 960

P = probability of throwing '5' with one die

$P = 1/6$

$q = 1 - p = 1 - \frac{1}{6}$

$q = 5/6$

Mean $= (\mu) = \frac{960}{6} = 160$ (np)

Standard deviation $(\sigma) = \sqrt{npq}$

$= \sqrt{960 \times 1/6 \times 5/6}$

$= \sqrt{\frac{800}{6}}$

$= \sqrt{133.3} = 11.54$

X = no. of successes = 184

Null Hypothesis (H_0): The ~~coin~~ die is unbiased ($\mu = \mu_0$)

Alternative hypothesis (H_1): The die is biased ($\mu \neq \mu_0$)

Level of Significane (α): 0.01 (for 1%)

The test Statistic is $z = \frac{x - \mu}{\sigma} = \frac{184 - 160}{11.54} \Rightarrow \frac{24}{11.54}$

$$Z = 2.07$$

The 'z' value at 1% level of significance is 2.58.

Conclusion: As $|z| < 2.58$, the null hypothesis H_0 is accepted at 1% level of significance.

$\therefore$ The die is unbiased at 1% level of significance.

c) Among 900 people in a state, 90 are found to be chapatti eaters. Construct 99% confidence interval for the true proportion.

Sol:- n = Sample size = 900

P = proportion of Chappathi eater = $\frac{90}{900} = 0.1$

$q = 1 - p = 1 - 0.1 = 0.9$

The 99% confidence interval is

$$\left(P - 3\sqrt{\frac{pq}{n}}, \; P + 3\sqrt{\frac{pq}{n}}\right)$$

$$= \left(0.1 - 3\sqrt{\frac{0.1 \times 0.9}{900}}, \; 0.1 + 3\sqrt{\frac{0.1 \times 0.9}{900}}\right)$$

$$= (0.1 - 0.03, \; 0.1 + 0.03)$$

$$= (0.07, 0.13)$$

d) In big city 325 mean out of 600 men were found to be smokers. Does this information support the conclusion that the majority of men in this city are smokers?

Sol:- n = Size of Sample = 600

P = Sample Proportion of Smokers = $\frac{325}{600} = 0.54$

P = population proportion of smokers $= \frac{1}{2} = 0.5$

$Q = 1 - P = 0.5$

Null hypothesis (H_0): $P = 0.5$ i.e, population of Smokers and non-smokers are equally popular.

Alternative Hypothesis (H_1): $P > 0.5$

Level of Significance: $\alpha = 0.05$ (5%) assumed.

Test Statistic:

$$z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.54 - 0.5}{\sqrt{(0.5)(0.5)/600}}$$

$$= \frac{0.04 \times \sqrt{600}}{0.5} = 1.95$$

The critical value z_α at 5% level of Significance for right-tailed test is 1.645

Conclusion:

As $|z| > 1.645$, the Null Hypothesis H_0 is rejected.

∴ The Smokers are majority in that city.

e) A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test his claim at 5% level of significance.

Sol:- n = Sample size = 200

No. of pieces confirmed to specification $= 200 - 18$
$= 182$

p = Sample proportion of confirmed to specification

$$= \frac{182}{200} = 0.91$$

P = population proportion of confirmed to Specification

$= 95\% = 0.95$

$Q = 1 - P = 1 - 0.95$

$Q = 0.05$

Null Hypothesis : $P = 0.95$ i.e, the proportion of pieces confirmed to Specification is 95%.

Alternative Hypothesis: $P > 0.95$

Level of Significance : $\alpha = 0.05 (5\%)$ assumed.

Test Statistic:

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.91 - 0.95}{\sqrt{\frac{(0.95)(0.05)}{200}}} = \frac{-0.04\sqrt{200}}{0.217}$$

$$= -2.60$$

$|Z| = 2.60$

The Critical Value Z_α at 5% Level of Significance for right tailed test is 1.645

Conclusion:

As $|Z| > 1.645$, the Null Hypothesis H_0 is rejected.

$\therefore$ Atleast 95% of the equipment confirmed to Specification.

③

a) Write the procedure of Testing Hypothesis

Procedure for testing a Hypothesis

Step 1: Statement (or) assumption of Hypothesis

There are two types of Hypothesis

(i) NULL Hypothesis

(ii) Alternative hypothesis

NULL HYPOTHESIS: A Null hypothesis is denoted by H_0. In step 1, setup a null hypothesis H_0 taking into consideration the nature of the problem and data involved.

Alternative hypothesis: It is denoted by H_1. Setup alternative hypothesis H_1, so that we could decide whether we should use one tailed (or) two tailed test.

Step 2:- Level of Significance:

Select the appropriate level of significance depending on reliability of estimates and permissible risk. That is suitable α is selected in advance R is not given in the problem (usually we choose 5% level of significance)

Step 3:- Test Statistic:

Compute the test statistic $Z = \frac{t - E(t)}{S.E\ of\ t}$ under the Null Hypothesis

Step 4:- Conclusion:

We compare the computed value of the test statistic Z with the critical value Z_α at given level of significance.

If $|z| < z_\alpha$, then we conclude that it is not significant and hence the null hypothesis is rejected.

b) The mean life of a sample of 10 electric light bulbs was found to be 1456 hours with standard deviation of 423 hours. A second sample of 17 bulbs choosen from a different batch showed a mean life of 1280 hours with standard deviation of 398 hours. Is there significant difference between the means of two batches?

Sol:- Let μ_1 and μ_2 be the means of two batches.

n_1 = first sample size = 10

n_2 = second sample size = 17

$\bar{x}_1$ = first sample mean life = 1456 hours

$\bar{x}_2$ = second sample mean life = 1280 hours

σ_1 = first batch S.D = 423 hours

σ_2 = second batch S.D = 398 hours.

Null Hypothesis (H_0): $\mu_1 = \mu_2$ i.e there is no significant difference between the means of two batches.

Alternative Hypothesis (H_1): $\mu_1 \neq \mu_2$

Level of Significance: $\alpha = 0.05$ (5%) assumed

Test Statistic: $$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{1456 - 1280}{\sqrt{\frac{(923)^2}{10} + \frac{(398)^2}{17}}}$$

$$= 1.609$$

The Critical Value Z_α at 5% level of Significance is 1.96

Conclusion:

As $|z| < 1.96$ the Null Hypothesis is accepted at 5% level of Significance

∴ There is no Significant difference between mean of two batches.

C) A Sample of 400 items is taken from a Population whose Standard deviation is 10. The mean of the Sample is 40. Test whether the Sample has come from a population with mean 38. Also Calculate 95% Confidence Interval for the population.

Sol:- n = Sample Size = 400

$\bar{x}$ = Sample mean = 40

μ = population mean = 38

σ = S.D of population = 10.

(i) Null Hypothesis (H_0): $\mu = 38$, that is the Sample mean drawn from a population with mean 38.

(ii) Alternative Hypothesis (H_1) = $\mu \neq 38$,

(iii) level of Significance (α) = 0.05 (5%) assumed

(iv) Test Statistic : $z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$

$$= \frac{40-38}{\left(10/\sqrt{400}\right)} = \frac{2}{10/20} = 4.$$

The Critical Value z_α at 5% Level of Significance is 1.96

Conclusion:

As $|z| > 1.96$, the Null hypothesis is rejected.

$\therefore$ The Sample has not been drawn from a Population with mean 38

The 95% confidence interval is

$$\left(\bar{x} - 1.96\left(\frac{\sigma}{\sqrt{n}}\right), \bar{x} + 1.96\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$= (39.02, 40.98)$$

d) An ambulance Service claims that it takes on the average less than 10 minutes to reach its destination in emergency Calls. A Sample of 36 calls has a mean of 11 minutes and the Variance of 16 minutes. Test the Claim at 0.05 level of Significance.

Sol:- n = Sample Size = 36

$\bar{x}$ = Sample mean = 11

$\bar{\mu}$ = population mean = 10

σ = S.D of population = $\sqrt{16} = 4$

[Since Variance = $\sigma^2 = 16$]

(i) Null Hypothesis: $\mu = 10$, i.e it takes on average less than 10 mins.

Alternative Hypothesis (H_1): $\mu < 10$

Level of Significance $\alpha = 0.05$ (5%)

Test Statistic : $z = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$

$$= \frac{11-10}{(4/6)} = \frac{6}{4} = \frac{3}{2} = 1.5$$

The critical value Z_α at 5% level of significance for one tailed AH is 1.645

Conclusion:

As $|z| < 1.645$, the Null Hypothesis (NH) is accepted.

∴ The Ambulance was reached the destination on the average of 10 times.

e) In a random sample of 1000 persons from town A, 400 are found to be consumers of wheat. In a sample of 800 from town B, 400 are found to be consumers of wheat. Do these data reveals at significant difference between town A and town B, so far as the proportion of wheat consumers is concerned?

Sol :- Let

P_1 and P_2 be population proportion of town A and town B.

n_1 = first sample size in town A = 1000

n_2 = Second sample size in town B = 800

P_1 = Sample proportion in town A = $\frac{400}{1000} = 0.4$

P_2 = Sample proportion in town B = $\frac{400}{800} = 0.5$

④

a) I) A Sample of 900 members has a mean of 3.4 cms and S.D 2.61 cms. Is this samples have been taken from a large population of mean 3.25 cm and S.D 2.61 cms. If the population is normal and its mean is unknown. Find the 95% fiducial limits of true mean.

II) In a Sample of 1000 people in Karnataka 540 are rice eaters and the rest are wheat are equally popular in this State at 1% level of Significance?

Sol:-

I) n = Sample Size = 900

$\bar{x}$ = Sample mean = 3.4

μ = population mean = 3.25

σ = S.D of population = 2.61

(i) Null Hypothesis : $\mu = 3.25$, is the sample been drawn from a population with mean 3.28

(ii) Alternative Hypothesis : $\mu \neq 3.25$

(iii) Level of Significance : $\alpha = 0.05$ [5%] assumed

(iv) Test Statistic $Z = \dfrac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$

$$= \frac{3.4-3.25}{(2.61/\sqrt{900})} = \frac{0.65}{2.61/30}$$

$$= \frac{19.5}{2.61} = 1.7241$$

The critical value Z_α at 5% level of Significance is 1.96

Conclusion :

As $|Z| < Z_\alpha$ the Null hypothesis is accepted at 5% level of Significance

$\therefore$ The Sample has been drawn from a population with mean 3.25

The 95% fiducial limits are

$$\left(\bar{x} - 1.96\left(\frac{\sigma}{\sqrt{n}}\right), \bar{x} + 1.96\left(\frac{\sigma}{\sqrt{n}}\right)\right)$$

$$= (3.57052, 3.22948)$$

$\therefore$ The 95% fiducial limits are 3.57 ± 3.229

II) n = Sample Size = 1000

p = Sample prop of rice eaters $= \frac{540}{1000} = 0.54$

P = population prop of rice eaters $= \frac{1}{2} = 0.5$

then $Q = 1 - P$
$= 0.5$

Null Hypothesis: $P = 0.5$

i.e both rice and wheat eaters are equally popular in the State.

Alternative Hypothesis : $P \neq 0.5$

Level of Significance : $\alpha = 0.01$ [1%] given

Test Statistic : $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{1000}}}$

$$= \frac{0.04}{\frac{0.5}{10\sqrt{10}}} = \frac{0.4\sqrt{10}}{0.5}$$

$$= 2.532$$

The critical value Z_α at level of significance (1%) is 2.58

$Z_\alpha = 2.58$

Conclusion: As $|Z| < 2.58$ the Null hypothesis is accepted

∴ Both rice and wheat eaters are equally popular in Karnataka.

b) I) If two large populations, there are 30% and 25% respectively of fair haired people. Is this difference likely to be hidden in samples of 1200 and 900 respectively from the two populations

II) In a sample of 500 from a village in Rajasthan, 280 are found to be wheat eaters and the rice eaters. Can we assume that both articles are equally popular.

i) Given $n_1 = 1200$, $n_2 = 900$.

P_1 = proportion of fair haired people in the first population $= \frac{3}{10} = 0.3$

P_2 = proportion of fair haired people in the second Population $= \frac{25}{100} = 0.25$

Null Hypothesis (H_0) : Assume that the sample proportions are equal i.e, the difference in populations is likely to be hidden in sampling

i.e, $H: P_1 = Q_2$

Alternative Hypothesis: $H_1: P_1 \neq P_2$

The Test Statistic is $Z = \dfrac{P_1 - P_2}{\sqrt{\frac{P_1 Q_1}{n_1} + \frac{P_2 Q_2}{n_2}}}$

where, $Q_1 = 1 - P_1 = 1 - 0.3 = 0.7$

$Q_2 = 1 - P_2 = 1 - 0.25 = 0.75$

$$\therefore Z = \frac{0.3 - 0.25}{\sqrt{\frac{0.3 \times 0.7}{1200} + \frac{0.25 \times 0.75}{900}}} = \frac{0.05}{0.0195} = 2.55$$

i.e $Z = 2.5$

Since $Z > 1.96$, Therefore we reject the null hypothesis H_0 at 5% level of Significance (Two tailed test) i.e, the Sample proportions are not equal. Thus we conclude that the difference in population proportions is unlikely that the real difference will be hidden.

ii) n = Sample size = 500

p = Sample prop of wheat eaters $= \frac{280}{500} = 0.56$

P = population prop of wheat eaters $= \frac{1}{2} = 0.5$

then $Q = 1 - P$
$= 0.5$

Null Hypothesis: $P = 0.5$, i.e both wheat and rice eaters are found to be equally popular.

Alternative Hypothesis: $P \neq 0.5$

Level of Significance: $\alpha = 0.05$ (5%) assume

Test Statistic: $Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}}$

$$= \frac{0.06}{\frac{0.5}{10\sqrt{5}}} = 2.68$$

The critical value Z_α for 5% level of Significance is 1.96

$$Z_\alpha = 1.96$$

Conclusion: As $|Z| > 1.96$, the Null Hypothesis H_0 is rejected.
Both rice and wheat eaters are not equally popular in rajasthan.

C) I) Experience had shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level.

II) 20 people were affected by a disease and only 18 Survived. Will you reject the Hypothesis that the Survival rate affected by this disease is 85% in favour of the hypothesis that is more at 5% level.

Sol:-

I) n = Sample Size = 400

p = Sample proportion of quality rugs

$$= \frac{50}{400} = \frac{1}{8}$$

P = population proportion of top quality

$$= 20\% = 0.2$$

$$Q = 1 - P = 1 - 0.2 = 0.8$$

Null Hypothesis: $P = 0.2$, i.e 20% of top quality rugs

Alternative Hypothesis: $P \neq 0.2$

Level of Significance: $\alpha = 0.05 (5\%)$ given

Test statistic: $Z = \frac{p - P}{\sqrt{PQ/n}}$

$= \frac{0.125 - 0.2}{\sqrt{\frac{(0.2)(0.8)}{400}}} = \frac{-0.075 \times 20}{0.4} = -3.75$

$\boxed{|Z| = 3.75}$

The critical value Z_α at 5% level of significance is 1.96

Conclusion: As $|Z| > 1.96$, the null hypothesis H_0 is rejected.

∴ 20% of manufacturing product is of top quality is wrong.

II) n = Sample Size = 20

P = population portion of survived people = 85% = 0.85

$Q = 1 - P = 1 - 0.85 = 0.15$

p = ~~Population~~ Sample proportion of survived people $= \frac{18}{20} = 0.9$

Null Hypothesis (H_0): $P = 0.85$ i.e, the survived rate

Alternate Hypothesis (H_1): $P > 0.85$

Level of Significance: $\alpha = 0.05$ (5%) given

Test Statistic: $Z = \frac{p - P}{\sqrt{PQ/n}} = \frac{0.9 - 0.85}{\sqrt{\frac{(0.85)(0.15)}{20}}}$

$= \frac{0.05 \times \sqrt{20}}{0.357} = 0.626$

The critical value Z_α at 5% level of significance is 1.645

Conclusion:

As $|z| < z_\alpha$, the Null hypothesis H_0 is accepted.

$\therefore$ The survival rate of population is 85%.