1) a) Define Binomial Distribution.

sol: A Random variable x which takes two values 0 and 1 with probability q and p respectively i.e $P(x=0)=q$ & $P(x=1) P$ $q=1-p$ is called a Bernoulli's discrete random variable and is said to have a bermulli's distribution.

(1b) write the conditions of Binomial distribution

sol: The conditions are :

1) The number of observation n is fixed
2) Each observation is independent
3) Each observation represents one of two outcomes ("success" or failure").
4) The probability of 'sucess' p is the same for each outcomes.

(1c) Define poisson Distribution.

sol: Poisson Distribution can be derived as a limiting case of binomial distribution under the following condition.

i) p the probability of occurence of an event is every small
ii) 'n' is very large when 'n' is no.of trails i.e $n \to \infty$
iii) np is a finite quantity say $np=\lambda$, is called the Parameter of the poisson distribution

(4) Write the applications of Normal distributions. (2)

A: normal distribution plays a very important role in statistical theory because of the following reasons:

1. Data obtained from psychological, physical and Biological ~~mean~~. measurement approximately follow normal distribution. IQ Scores, height and weight of individuals etc.., are examples of measurement which are normally distributed or nearly so.
Since the normal distribution is a limitiy case of the Bionomial distribution for exceptionally large numbers, it is applicable to many applied problems in kinetic theory of gases and fluctuations in the magnitube of an electric current.
for large samples, any statistics (ie sample mean, sample S.D approximately follows normal distribution and as such it can be studied with the help of normal curve

(e) The mean and variable of a binomial distribution are 4 and $\frac{4}{3}$ respectively. find $P(X \geq 1)$

Sol: Given that

$$\text{mean} = 4$$

$$np = 4 \rightarrow (1)$$

The variance of binomial distribution is

$$npq = \frac{4}{3} \rightarrow (2)$$

$$\frac{2}{1} \cdot \frac{\cancel{n}\cancel{p}q}{\cancel{n}\cancel{p}} = \frac{\cancel{4}/3}{\cancel{4}} \Rightarrow q = \frac{1}{3}$$

N.K.T

$$p = 1 - q \Rightarrow p = 1 - \frac{1}{3}$$

$$\boxed{p = \frac{2}{3}}$$

Substitute np values in eq ①

(3)

$np = 4$

$n\left(\frac{2}{3} = 4\right)$

$\Rightarrow n = 6$

The binomial distribution is

$P(x=r) = {}^{n}C_r \, p^r, q^{n-r}, r = 0,1,2 \ldots n$

${}^{6}C_r \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{6-r}, r = 0,1,2,\ldots 6$

$P(x \geq 1) = 1 - P(x<1)$

$= 1 - P(x=0)$

$= 1 - \left[{}^{6}C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^{6-0}\right)$

$= 1 - (1)(1)\left(\frac{1}{3}\right)$

$= 1 - \frac{1}{3^6}$

$= 0.998$

3 marks.

8(a) A fair coin is tossed ten times. find the probability of getting at least 6 heads.

Sol: Given that $n = 10$

The probability of getting head is $p = 1/2$

Then the probability of getting tail is $q = 1 - p = 1/2$

let x be the no. of heads

Then the binomial distribution is

$P(x=r) = P(r)^n = C_r \, p^r, q^{n-r}, r = 0,1,2,\ldots n$

$P(x=r) = {}^{10}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r}, r = 0,1,2 \ldots 10$

$= {}^{10}C_r \left(\frac{1}{2}\right)^{10}$

$P(X=r) = \frac{1}{2^{10}} \cdot {}^{10}C_r \Rightarrow r = 0, 1, 2 \ldots 10$

(4).

(i) 6 heads

$P(X \geq 6) = P(X=6) + P(X=7) + P(X=8) + P(X=9) + P(X=10)$

$= {}^{10}C_6 \frac{1}{2^{10}} + (0.171)$

$= \frac{2^{10}}{2^{10}} + 0.171$

$= 0.376$

2 b) If the probability of a defective bolt is $\frac{1}{8}$,

(i) find mean.

(ii) The variance for the distribution of defective bolts of 640.

Sol:- Given that $n = 640$

The probability of a defective bolt $p = \frac{1}{8}$

W.K.T $q = 1 - p$

$= 1 - \frac{1}{8}$

$= \frac{9}{8}$

i) Mean $\mu = np$

$\mu = \cancel{640}^{80} \left(\frac{1}{\cancel{8}}\right)$

$\mu = 80$

ii) Variance $\sigma^2 = npq$

$= 640 \left(\frac{1}{8}\right)\left(\frac{9}{8}\right)$

$= 90$

Mean of normal discontinous (2c)

(2c) The normal distribution is $f(x;b,\sigma) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma^2}}$

(5)

The mean of normal distribution is

$$\mu = \int_{-\infty}^{\infty} x f(x)dx - \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-b)^2}{2\sigma^2}} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2}\left(\frac{x-b}{\sigma}\right)^2} dx$$

Put $z = \frac{x-b}{\sigma}$

Hence, $x = \sigma z + b$ and $dz = \frac{1}{\sigma} dx$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + b) e^{-1/2(z^2)} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sigma \int_{-\infty}^{\infty} z e^{-z^2/2} dz + b \int_{-\infty}^{\infty} e^{-z^2/2} dz\right]$$

$\therefore z e^{-z^2/2}$ is odd and $e^{-z/2}$ is even

$$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_{0}^{a} f(x)dx & f \text{ is even} \\ 0 & f \text{ is odd} \end{cases}$$

$$= \frac{1}{\sqrt{2\pi}} \left[\sigma(0) + b\left(2\int_{0}^{\infty} e^{-z^2/2} dz\right)\right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + 2b\sqrt{\frac{\pi}{2}}\right] \quad \left[\because \int_{0}^{\infty} e^{-z^2/2} dz = \sqrt{\pi/2}\right]$$

$$= \frac{1}{\sqrt{2\pi}} \left(2b \cdot \frac{\sqrt{\pi}}{\sqrt{2}}\right) = \frac{2b}{2} = b$$

$\therefore$ mean $= b$

20) 20% of items produced from a factory are defective. Find the probability that in a sample of 5 choosen at random.

i) none is defective ii) one is defective. (6)

iii) lies between 1 and 4.

Sol: Given that $n=5$.

The probability of getting defective item $p = 20\%$
$= 0.2$.

The probability of getting non defective item $q = 1-p$
$= 1-0.2$
$= 0.8$

The binomial distribution is $p(x=r) = n_{C_r} p^r q^{n-r}$

$P(x=r) = 5_{C_r} (0.2)^r (0.8)^{5-r}, \; r = 0, 1, 2 \ldots 5$

et x denotes the no. of defective items

i) $p(x=0) = 5_{C_0} (0.2)^0 (0.8)^{5-0}$

$= 0.32768$

i) $p(x=1) = 5_{C_1} (0.2)(0.8)^{5-1}$

$= 5_{C_1} (0.2)^1 (0.8)^4$

$= 0.4096$.

iii) $p(1<x<4) = p(x=2)+p(x=3)$

$= 5c_2(0.2)^2(0.8)^3 + 5c_3(0.2)^3(0.p)^2$ (7).

$= 0.204 + 0.05$

$= 0.255$

(2e). If is a normal variate with mean 30 and S.D. 5. Find the probabilities that

(i) $26 \le x \le 40$ (ii) $p(x \ge 45)$

Sol:- Given that mean $\mu = 30$ standard deviation $\sigma = 5$.

W.K.T. z-score is $z = \frac{x-\mu}{\sigma} = \frac{\lambda - 30}{5}$

i) when $x = 26$ then $z = \frac{26-30}{5} = -0.8 = z_1$ (say)

when $x = 40$ then $z = \frac{40-30}{5} = 2 = z_2$ (say)

$\therefore z_1 < 0$ and $z_2 > 0$

$P(26 \le x \le 40) = p(z_1 < z \le z_2)$

$= A(z_2) + A(z_1)$

$= A(2) + A(0.8)$

$= 0.4772 + 0.2881$ ($A(-0.8) = A(0.8)$)

$= 0.7653.$

ii) $x \ge 45$

when $x = 45$ then $z = \frac{45-30}{5} = 3 = z_1$ (say)

$\therefore z_1 > 0$

$\therefore p(x \ge 45) = p(z \ge 3)$

$= 0.5 - A(z_1)$

$= 0.5 - A(3)$

$= 0.0 - 0.4987$

$= 0.0013$

8

5 marks

3a) Six dice are thrown 729 times. How many times do you except at least three dice to show a 5 or 6.

Sol: p = probability of occurence of 5 or 6 in one thrown $= 2/6 = \frac{1}{3}$

$p(x=r) = n_{c_r} p^r q^{n-r} = 6c_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}$

$r = 0, 1, 2 \ldots 6$

$\therefore q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$ and $n = 6$.

The probability of getting at least three dice to show a 5 or 6.

$= p(x \geq 3) = p(x=3) + p(x=4) + p(x=5) + p(x=6)$

$= {}^6c_3 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^2 + {}^6c_4 \left(\frac{1}{4}\right)^4 \left(\frac{2}{3}\right)^2 + {}^6c_5 \left(\frac{1}{3}\right)^5 \left(\frac{2}{3}\right) + {}^6c_6 \left(\frac{1}{3}\right)^6$

$= \frac{1}{(3)^6} [160 + 60 + 12 + 1] = \frac{233}{729}$

$\therefore$ The excepted number of such cases in 729 times

$= 729 \left(\frac{233}{729}\right) = 233.$

3b) Derive mean and variance of the poission distibution.

3b) Mean of the poissons distribution:-

→ The poission distribution is $P(X=r) = \frac{e^{-\lambda} \cdot \lambda^r}{r!}$

The mean of poisson distribution is

$$E(x) = \sum_{r=0}^{\infty} r\,p(r)$$

$$= \sum_{r=1}^{\infty} r\,p(r)$$

$$= \sum_{r=1}^{\infty} r \cdot \frac{e^{-\lambda} \cdot \lambda^r}{r!}$$

$$= \sum_{(r-1)=0}^{\infty} \cancel{r} \cdot \frac{e^{-\lambda} \cdot \lambda^r}{\cancel{r}(r-1)!}$$

$$= \sum_{(r-1)=0}^{\infty} \frac{e^{-\lambda} \cdot \lambda^r}{(r-1)!}$$

Put $x = r - 1$

then $r = x + 1$

$$\Rightarrow \sum_{x=0}^{\infty} \frac{e^{-\lambda} \cdot \lambda^{x+1}}{x!} \Rightarrow \sum_{x=0}^{\infty} \frac{e^{-\lambda} \cdot \lambda^{x} \cdot \lambda}{x!}$$

$$\Rightarrow \lambda e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \Rightarrow \lambda e^{-\lambda} \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \Rightarrow \lambda e^{-\lambda} (e^{\lambda})$$

$$\left[\because e^{\lambda} = \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} \right]$$

$$\Rightarrow \lambda e^{-\lambda + \lambda} \Rightarrow \lambda e^{0} \Rightarrow \lambda$$

$$\boxed{\therefore E(X) = \lambda}$$

$\therefore$ The mean of P.D is $\lambda = np$

(3b)(i)

<u>To Derive the variance of poission distributing.</u> (11)

The poisson distribution is

$$P(r) = \frac{e^{-\lambda}\lambda^r}{r!} \quad r = 0, 1, 2, \ldots$$

Variance of poission distribution is

$$\text{Variance}(x) = \sum_{r=0}^{\infty} r^2 p(r) - \mu^2$$

$$= \sum_{r=1}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} r^2 \frac{e^{-\lambda}\lambda^r}{r(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} r \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} (r-1+1) \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=1}^{\infty} \left[(r-1) \frac{e^{-\lambda}\lambda^r}{(r-1)!} + \frac{e^{-\lambda}\lambda^0}{(r-1)!} \right] - \lambda^2$$

$$= \sum_{r=2}^{\infty} (r-1) \frac{e^{-\lambda}\lambda^r}{(r-1)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

$$= \sum_{r=2}^{\infty} (r-1) \frac{e^{-\lambda}\lambda^r}{(r-1)(r-2)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!}) - \lambda^2$$

$$= \left(\sum_{r=2}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-2)!} + \sum_{r=1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} \right) - \cancel{\lambda}^2$$

(12)

$$= \sum_{r-2=0}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-2)!} + \sum_{r-1}^{\infty} \frac{e^{-\lambda}\lambda^r}{(r-1)!} - \lambda^2$$

put $x = r-2$ in 1st expression and $y = r-1$ in 2nd expression.

Then $r = x+2$ in 1st & $r = y+1$ in 2nd

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x+2}}{x!} + \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^{y+1}}{y!} - \lambda^2$$

$$\sum_{x=0}^{\infty} \frac{e^{-\lambda}\lambda^{x}\lambda^2}{x!} + \sum_{y=0}^{\infty} \frac{e^{-\lambda}\lambda^{y}.\lambda}{y!} - \lambda^2$$

$$= e^{-\lambda}\lambda^2 \sum_{x=0}^{\infty} \frac{\lambda^x}{x!} + e^{-\lambda}\lambda \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} - \lambda^2$$

$$= e^{-\lambda}\lambda^2 e^{\lambda} + e^{-\lambda}.\lambda e^{\lambda} - \lambda^2$$

$$= \lambda^2 e^{-\lambda+\lambda} + \lambda e^{-\lambda+\lambda} - \lambda^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

The variance of poisson distribution is λ.

3(c) If a poisson distribution is such that $P(x=1).\frac{3}{2} = P(x=3)$, find i) $P(x \geq 1)$ ii) $P(x \leq 3)$ and iii) $P(2 \leq x \leq 5)$

Sol: Given $\frac{3}{2}P(x=1) = P(x=3)$

i.e., $\frac{3}{2}\frac{e^{-\lambda}.\lambda^1}{1!} = \frac{e^{-\lambda}.\lambda^3}{3!}$ i.e., $\frac{3\lambda}{2} = \frac{\lambda^3}{6}$

i.e., $\lambda^3 - 9\lambda = 0$ or $\lambda(\lambda^2-9) = 0$ or $\lambda(\lambda-3)(\lambda+3) = 0$

$\therefore \lambda = 0, 3, -3$

$\Rightarrow \lambda = 3 \quad (\because \lambda > 0)$

Hence $P(X=x) = P(x) = \frac{e^{-3}3^x}{x!}$

i) $P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0)$

$= 1 - e^{-3} = 0.950213$

ii) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-3}\left[\frac{3^0}{0!} + \frac{3}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!}\right] = e^{-3}\left[1 + 3 + \frac{9}{2} + \frac{9}{2}\right]$

$= 13e^{-3} = 0.6472318$

iii) $P(2 \leq X \leq 5) = P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$= e^{-3}\left[\frac{3^2}{2!} + \frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!}\right]$

$= 9e^{-3}\left[\frac{1}{2} + \frac{1}{2} + \frac{3}{8} + \frac{9}{40}\right]$

$= 9e^{-3}(1.6) = 0.7169337$

3d) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and standard deviation 11%. Determine

i) How many students got marks above 90%.

ii) what was the highest mark obtained by the lowest 10% of the students.

Sol: Given mean $\mu = 0.78$ and S.D. $\sigma = 0.11$

i) When $x = 0.9$, $z = \frac{x-\mu}{\sigma} = \frac{0.9 - 0.78}{0.11}$

$= 1.07 = Z_1$

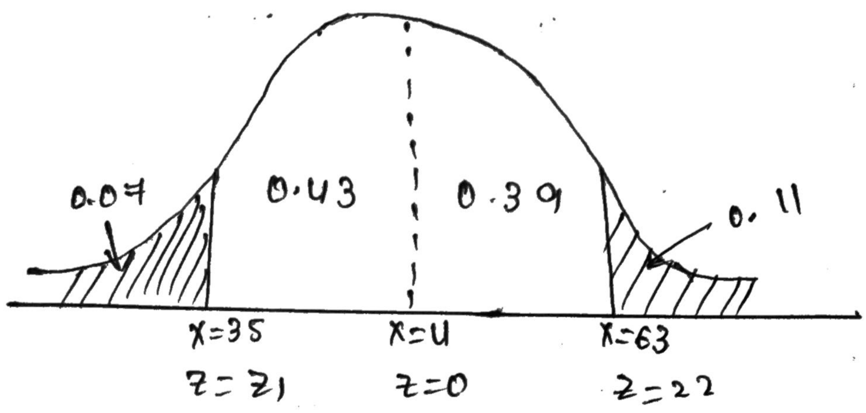
(3e:) Find the mean and variance ~~standard~~ deviation of a normal distribution in which 7% of items are under 35 and 89% are under 63. (14).

Sol:- let μ be the mean (at $z=0$) and σ the standard deviation of the normal curve. 7% of items are under 35 means the area to the lift of the ordinate $x=35$.

given $P(X<35)=0.07$ and $P(X<63)=0.89$

$P(X>63)=1-P(X<63)=1-0.89=0.11$

The points $x=35$ and $X=63$



when $X=35$, $z=\frac{x-\mu}{\sigma}=\frac{35-\mu}{\sigma}=-z$

when $X=63$, $z=\frac{x-\mu}{\sigma}=\frac{63-\mu}{\sigma}=z_2$

from the figure, we have

$P(0<z<z_2)=0.39 \Rightarrow z_2=1.23$

and $P(0<z<z_1)=0.43 \Rightarrow z_1=1.48$

from (1), we have $\frac{35-\mu}{\sigma}=-1.48$

from (2), we have $\frac{63-\mu}{\sigma}=1.23$

Hence the number of students with Marks more than 90%

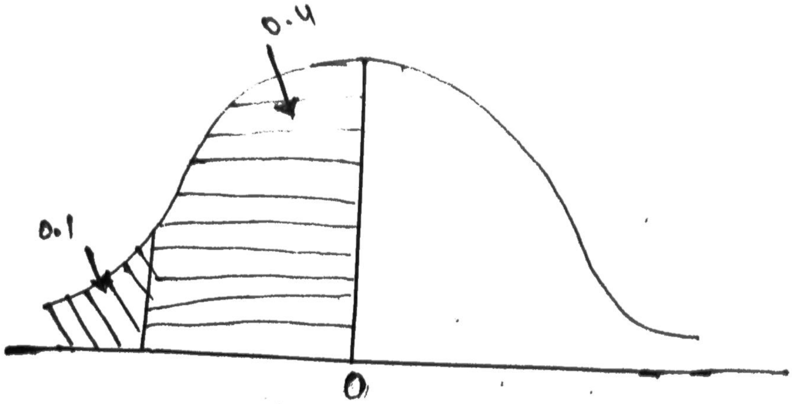
$= 0.1379 \times 1000$

$= 137.9$

$\simeq 138$

(15).

i) The 0.1 area to the left of z corresponds to the lowest 10% of the students



from figure,

$0.4 = 0.5 - 0.1 = 0.5 -$ Area from 0 to z

$\therefore z_1 = -1.28$

Thus $-1.28 = \dfrac{x - \mu}{\sigma} = \dfrac{x - 0.78}{0.11}$

$\Rightarrow x = 0.78 - 1.28\,(0.11) = 0.6392$

Hence the highest mark obtained by the lowest 10% students

$= 0.6392 \times 1000$

$\simeq 64\%$

(4) − (3) gives

$\frac{28}{\sigma} = 2.71 = \sigma = \frac{28}{2.71} = 10.332$

Probability distribution

form (3), $35 - \mu = -1.48(\sigma) = (-1.48)(10.332)$

$= -15.3$

$\mu = 35 + 15.3 = 50.3$

and variance $= \sigma^2 = 106.75$

10marks:

(4a)

i) Out of 800 families with 5 children each, how many would you expect to have a) 3 boys b) 5 girls c) either 2 or 3 boys. d) atleast one boy? Assume equal probabilities for boys and girls.

Sol: let the number of boys in each ~~each~~ family $= x$

P = Probability of each boy $= \frac{1}{2}$

number of children, $n = 5$

The probability distribution is

$P(r) = nC_r p^r q^{n-r} = 5C_2 \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{5-r}$

$= \frac{1}{2^5} \cdot 5C_r$ per family

a) $P(3\text{boys}) = P(r=3) = P(3) = \frac{1}{2^5} \cdot 5C_3$

$= \frac{10}{32} = \frac{5}{16}$ Per family

Thus for 300 families the probability of number of families having 3 boys

(40) (i)

$= \frac{5}{16}(800) = 250$ families.

30

17

(b) $P(5\text{ girls}) = P(\text{no boys}) = P(r=0) = P(0)$

$= \frac{1}{2^5} \cdot {}^5C_0 = \frac{1}{32}$ per family

Thus for 800 families the probability of number of families having 5 girls

$= \frac{1}{32}(800) = 25$ families.

c) $P(\text{either } 2 \text{ or } 3 \text{ boys}) = P(r=2) + P(r=3)$

$= P(2) + P(3)$

$= \frac{1}{2^5}\, {}^5C_2 + \frac{1}{2^5} \cdot {}^5C_3$

$= \frac{1}{2^5}(10+10) = \frac{20}{25} = \frac{4}{5}$ per family.

Thus for 800 families the probability of number of families having either 2 or 3 boys.

$= \frac{4}{\cancel{5}} \times \cancel{800}^{160} = 640$ families.

4a(II). Mean of the binomial distribution: (8)

→ The binomial PD is given by

$$P(r) = P(X=r) = {}^nC_r \cdot p^r \cdot q^{n-r}, \quad r = 0, 1, 2 \ldots n, \ \& \ q = 1-p$$

$$\text{mean of } X, \ \mu = \sum_{r=0}^{n} r P(r)$$

$$= \sum_{r=0}^{n} r({}^nC_r \cdot p^r \cdot q^{n-r}$$

$$= 0 + 1\ ({}^nC_1 \cdot p^1 q^{n-1}) + 2\ ({}^nC_2\, p^2 q^{n-2}) + 3\ ({}^nC_3\, p^3 q^{n-3}) +$$

$$\ldots + n({}^nC_n \cdot p^n \cdot q^{n-n}).$$

$= npq^{n-1} + 2\left(\frac{n(n+1)}{2} p^2 q^{n-2}\right) + 3\left(\frac{n(n-1)(n-2)}{6} p^3 q^{n-3}\right) + \dots np^n)$

$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2} p^2 q^{n-3} + \dots + p^{n-1}\right)$

$= np\,(p+q)^{n-1}$ ($\because$ by using binomial theorem)

$= np(1)^{n-1} \Rightarrow \boxed{np = \mu}$

(II) Variance of the binomial distribution :-

$\Rightarrow$ The binomial of P.D is given by

$P(r) = P(X=r) = {}^nC_r\, p^r q^{n-r}$, $r = 0, 1, 2, \dots, n$ & $q = 1-p$

Variance of X, $V(X) = \sum_{r=0}^{n} r^n p(r) - \mu^2$

$$= \sum_{r=0}^{n} (r^2 - r + r)\, p(r) - \mu^2$$

$$= \sum_{r=0}^{n} \left(r(r-1)\, p(r) + r\,p(r)\right) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1)\, p(r) + \sum_{r=0}^{n} r\,p(r) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1)\left({}^nC_r\, p^r q^{n-r}\right) + \mu$$

$$= \left[2.({}^nC_2\, p^2 q^{n-2}) + 3.2\,({}^nC_3\, p^3 q^{n-3}) + \dots\right.$$
$$\left. + n(n-1)({}^nC_n\, p^n q^{n-1}) + \mu - \mu^2\right]$$

$$= n(n-1)p^2\left[q^{n-2} + (n-2)pq^{n-3} + \dots + p^{n-2}\right] + \mu - \mu^2$$

$$= (n^2p^2 - np^2)(q+p)^{n-2} + (np) - (np)^2$$

$$= (n^2p^2 - np^2)(1)^{n-2} + np - n^2p^2$$

$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$

$= np(1-p)$

4b $= npq$

Q (I) If x is a poisson variate such that $3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$, find i) The mean of x ii) $P(X \le 2)$

Sol: i) If x is poisson variate with parameter λ

then $P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2 \ldots$ $\lambda > 0$

Given $3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$

$$3 \cdot \frac{e^{-\lambda}\lambda^4}{4!} = \frac{1}{2} \cdot \frac{e^{-\lambda}\lambda^2}{2!} + e^{-\lambda}$$

$$\lambda^4 - 2\lambda^2 - 8 = 0$$

$$(\lambda^2 - 4)(\lambda^2 + 2) = 0$$

$$\lambda = \pm 2 \qquad (\because \lambda > 0)$$

$$\lambda = 2$$

4b (ii) Now we have $P(X=x) = \frac{e^{-2} \cdot 2^x}{x!}$

$$P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= e^{-2} + \frac{1}{4}e^{-1} + \frac{e^{-2} \cdot 2^2}{2!}$$

$$= 3e^{-2} + \frac{1}{4}e^{-1}$$

$$= 3(0.135) + \frac{1}{4}(0.3679)$$

$$= 0.498$$

(21)

4b (II) ⇒ In a sample of 1000 cases. The mean of certain test mean 14 & SD is 2.5. Assuming distribution is normal, and

i) how many students scores between 12 & 15

ii) how many score above 18? iii) how many score below 18?

Sol: G.T $\mu = 14$ & $\sigma = 2.5$

let the variable X denote the score in a Test

Then $z = \frac{X - \mu}{\sigma} = \frac{X - 14}{2.5}$

$X = 12$, $z = \frac{12 - 14}{2.5} = -0.8 = +z_1$ (

$X = 15$, $z = \frac{15 - 14}{2.5} = 0.4 = z_2$ (

i) In a sample of]

$P(12 < X < 15) = P[-0.8 < z < 0.4]$

$= A[z_2] + A(z_1)$

$= A[0.4] + A(-0.8)$

$= A(0.4) + A(0.8]$ [(due to symmetry]

$= 0.1554 + 0.2881 = 0.4435$

∴ No. of students score between 12 & 15 is

$= 1000 \times 0.4435 = 443$ (approximates)

ii) When $X = 18$, $z = \frac{18 - 14}{2.5} = 1.6$

$P(X > 18) = P(Z > 1.6) = 0.5 - A(1.6) = 0.5 - 0.4452$

$= 0.0548$

No. of students score above 18

$= 1000 \times 0.0548 = 54.8 = 55$ (Approximately)

(Continue in next page)

23

$P(12<X<15) = P(-0.8<Z<0.4)$

$= A(Z_2) + A(Z_1)$

$= A(0.4) + A(0.8)$

$= A(0.4) + A(0.8)$ (due to symmetry)

$= 0.1554 + 0.2881 = 0.4435$

$\therefore$ No. of students score between 12 & 15 is

$= 1000 \times 0.4435 = 443$ (approximate)

ii) when $x = 18$, $Z = \frac{18-14}{2.5} = 1.6$

$\therefore P(X>18) = P(Z>1.6) = 0.5 - A(1.6) = 0.5 - 0.4452$

$= 0.0548$

No. of students score above 18

$= 1000 \times 0.0548 = 54.8 = 55$ (approximate)

4c).

(i) Seven coins are tossed and the number of heads are noted. The experiment is repeated 128 times and the following distribution is obtained.

No. of heads	0	1	2	3	4	5	6	7	Total
Frequency	7	6	19	35	30	23	7	1	128.

Fit a binomial distribution assuming the coin is unbiased.

The coin is unbiased

$\therefore p = \frac{1}{2}$ $q = \frac{1}{2}$ and $n = 7$

$N = \Sigma f = 7+6+19+35+30+23+7 = 128.$

By binomial distribution $p(x) = {}^nC_x \, p^x q^{n-x}$

we have the recurrence relation

$$p(x+1) = \frac{(n-x)p}{(x+1)q}\, p(x) = \frac{7-x}{x+1}\, p(x) \quad \left[\because n=7 \; \frac{p}{q}=1\right]$$

$$\therefore \; p(0) = {}^7C_0 \, p^0 q^7$$

$$= {}^7C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^7$$

$$= \frac{1}{2^7}$$

No: of heads x	observed frequency	probability $p(x)$	Expected or Theoretical frequency $f(x) = N \cdot p(x)$
0	7	$p(0) = \frac{1}{2^7}$	$f(0) = 128\, p(0) = 128 \times \frac{1}{2^7} = 1$
1	6	$p(1) = 7 \cdot p(0) = \frac{7}{2^7}$	$f(1) = 128 \cdot p(1) = 128 \times \frac{7}{2^7} = 7$
2	10	$p(2) = 3 \cdot p(1) = \frac{21}{2^7}$	$f(2) = 128\, p(2) = 128 \times \frac{21}{2^7} = 21$
3	35	$p(3) = \frac{35}{2^7}$	$f(3) = 128 \cdot p(3) = 35$
4	30	$p(4) = \frac{35}{2^7}$	$f(4) = 128 \cdot p(4) = 35$
5	23	$p(5) = \frac{21}{2^7}$	$f(5) = 128\, p(5) = 21$
6	7	$p(6) = \frac{7}{2^7}$	$f(6) = 128 \cdot p(6) = 7$
7	1	$p(7) = \frac{1}{2^7}$	$f(7) = 128 \cdot p(7) = 1$