UNIT-2

1. a) Binomial Distribution :

A random Variable 'x' has a binomial distri-bution if it assumes only non-negetive values and its probability density function is given by

$$p(X=r) = p(r) = \begin{cases} {}^nC_r p^r q^{n-r} & , r=0,1,2,\dots n \\ 0 & , \text{otherwise} \end{cases}$$

$\therefore q = 1-p$

b) Write the Condition of Binomial distribution

1. There are n independent trials
2. Each trial has only 2 possible outcomes
3. The probability of two outcomes remains Constant

c) Define poisson Distribution.

For random Variable 'x' if it assumes non-negetive Values and its probability density function is given by

$$p(x=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!} & , x=0,1,2\dots \\ 0 & , \text{otherwise} \end{cases}$$

d) Write the Applications of Normal Distribution

1. The normal distribution Can be used to approx Binomial and poisson distribution
2. It has Extrensive use in Sampling theory. It help us to Estimate parameter has Statistics and to find Confident limits of the parameters
3. It has a wide use in Statistical hypothesis and tests Significance in which it is always assumed that the population from which the Sample have been drawn Should have normal distribution

c) The mean and variance of a binomial distribution are 4 and 4/3 respectively. Find $P(x \geq 1)$

Given

Mean of Binomial distribution = 4

$\Rightarrow np = 4 \rightarrow (1)$

Variance of BD $= \frac{4}{3}$

$\Rightarrow npq = \frac{4}{3} \rightarrow (2)$

$$\frac{(2)}{(1)} = \frac{\cancel{np}q}{\cancel{np}} = \frac{4/3}{4}$$

$$q = \frac{4}{12} = \frac{1}{3}$$

$$P = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$\Rightarrow np = 4$

$$n \times \frac{2}{3} = 4$$

$\therefore n = 6$

Binomial distribution is

$$p(r) = {}^{n}C_{r}\, p^{r} q^{n-r}, \quad r = 0, 1, 2 \ldots n$$

$$p(r) = {}^{6}C_{r} \left(\frac{2}{3}\right)^{r} \left(\frac{1}{3}\right)^{6-r}, \quad r = 0, 1, 2 \ldots n$$

$$P(x \geq 1) = 1 - p(x < 1)$$

$$= 1 - p(0)$$

$$= 1 - {}^{6}C_{0} \left(\frac{2}{3}\right)^{0} \left(\frac{1}{3}\right)^{6-0}$$

$$= 1 - (1)(1)\frac{1}{3^6}$$

$$= 1 - \frac{1}{3^6} = 0.998$$

3) A fair Coin is tossed 10 times. Find probability of atleast 6 heads

Given n = no. of trials = 10

p = probability of getting head = $\frac{1}{2}$

$q = 1-p = \frac{1}{2}$

Binomial distribution is

$p(r) = {}^{n}c_r \, p^r q^{n-r}, \quad r = 0, 1, 2, \ldots n$

$p(r) = {}^{10}c_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{10-r}$

$= {}^{10}c_r \left(\frac{1}{2}\right)^{r+10-r}$

$= {}^{10}c_r \times \frac{1}{1024}, \quad r = 0, 1, 2 \ldots n$

Probability of getting atleast 6 heads

$p(x \geq 6) = p(6) + p(7) + p(8) + p(9) + p(10)$

$= \frac{1}{1024} {}^{10}c_6 + \frac{1}{1024} {}^{10}c_7 + \frac{1}{1024} {}^{10}c_8 + \frac{1}{1024} {}^{10}c_9 + \frac{1}{1024} {}^{10}c_{10}$

$= \frac{1}{1024} \left({}^{10}c_6 + {}^{10}c_7 + {}^{10}c_8 + {}^{10}c_9 + {}^{10}c_{10} \right)$

$= \frac{386}{1024} = 0.376$

b) If probability of a defective bolt is $\frac{1}{8}$, find

i) mean

ii) The Varience of the distribution of defective bolts of 640

Sol) n = no. of bolts = 640

P = probability of defective Bolt = $1/8$

$\Rightarrow q = 1 - P$

$= 1 - \frac{1}{8} = \frac{7}{8}$

Mean of Binomial distribution is

$\mu = np$

$\mu = 640 \times \frac{1}{8} = 80$

Varience of Binomial distribution is

$a^2 = npq$

$= \cancel{640}^{\cancel{80}^{10}} \times \frac{1}{\cancel{8}_1} \times \frac{7}{\cancel{8}_1}$

$\boxed{a^2 = 70}$

c) Derive mean of normal distribution

Proof

We know that

the normal distribution is

$$f(x:b:a) = \frac{1}{a\sqrt{2\pi}} e^{-1/2\left(\frac{x-b}{a}\right)^2}$$

where, b is mean, a is standard deviation of Normal distribution

The mean of Normal distribution is

$$\mu = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\mu = \int_{-\infty}^{\infty} x \left(\frac{1}{a\sqrt{2\pi}} e^{-1/2\left(\frac{x-b}{a}\right)^2}\right) dx$$

$$= \frac{1}{a\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-1/2\left(\frac{x-b}{a}\right)^2} dx$$

Put, $z = \frac{x-b}{a}$ then $dz = \frac{1}{a} dx$

$x = az + b$ & $a\,dz = dx$

$$\mu = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + b) e^{-\frac{z^2}{2}} (\sigma\, dz)$$

$$= \frac{\sigma}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} [\sigma z \cdot e^{-z^2/2} + b e^{-z^2/2}]\, dz$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2} dz + \frac{b}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz$$

$$= \frac{a}{\sqrt{2\pi}} (0) + \frac{b}{\sqrt{2\pi}} \left[2 \int_0^{\infty} e^{-z^2/2} dz \right]$$

$[\because z e^{-z/2}$ is odd, $e^{-z^2/2}$ is Even$]$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + b \cdot 2 \int_0^{\infty} e^{-z^2/2} dz \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[2b \sqrt{\frac{\pi}{2}} \right]$$

$$= \frac{1}{\sqrt{2}\cancel{\sqrt{\pi}}} \left[2b \frac{\cancel{\sqrt{\pi}}}{\sqrt{2}} \right]$$

$$= \frac{2b}{2} = b$$

$\therefore$ The mean of the normal distribution is 'b'

d) 20 % of items produced from a factory are defective Find the probability that in a Sample of 5 Choosen at random

(i) None is defective

(ii) one is defective

(iii) lies between 1 and 4

Sol Given $P =$ probability of defective items

$= 20\%$

$= 0.2$

$x = 1 \qquad x!$

Then $q = 1 - p$
$= 1 - 0.2$
$= 0.8$

n = no. of items = 5

Binomial distribution is

$$p(r) = {}^{n}C_{r}\, p^{r} q^{n-r}, \quad r = 0, 1, 2 \ldots n$$

$$p(r) = {}^{5}C_{r}\,(0.2)^{r}(0.8)^{5-r}, \quad r = 0, 1 \ldots 5$$

(i) probability of no defective items is:

$$p(0) = {}^{5}C_{0}\,(0.2)^{0}(0.8)^{5-0}$$
$$= (1)(1)(0.8)^{5}$$
$$= 0.327$$

(ii) probability of one defective item is

$$p(1) = {}^{5}C_{1}\,(0.2)^{1}(0.8)^{5-1}$$
$$= 5(0.2)(0.8)^{4}$$
$$= 0.409$$

(iii) Probability that it lies between 1 and 4

$$p(1 < x < 4) = p(2) + p(3)$$
$$= {}^{5}C_{2}\,(0.2)^{2}(0.8)^{5-2} + {}^{5}C_{3}\,(0.2)^{3}(0.8)^{5-3}$$
$$= 10(0.04)(0.8)^{3} + 10(0.008)(0.8)^{2}$$
$$= 0.236$$

e) If x is a normal variate with mean 30 and S.D is 5 Find probabilities (i) $26 \le x \le 40$ (ii) $P(x \ge 45)$

Sol Given that

Mean is $\mu = 30$

Standard deviation is $\sigma = 5$

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We know

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{x-30}{5}$$

(i) for $x=26$, $z = \frac{26-30}{5} = -0.8 = z_1 < 0$

for $x=40$, $z = \frac{40-30}{5} = 2 = z_2 > 0$

$z_1 < 0$ & $z_2 > 0$

$\therefore P(26 \le x \le 40) = P(-0.8 \le x \le 2)$

$= A(2) + A(-0.8)$ $[\because A(-0.8) = A(0.8)]$

$= A(2) + A(0.8)$

$= 0.4772 + 0.2881$

$= 0.7653$

(ii) for $x=45$, $z = \frac{45-30}{5} = 3 = z_1 > 0$

$\therefore P(x \ge 45) = P(x \ge 3)$

$= 0.5 - A(3)$

$= 0.5 - 0.4986$

$= 0.0014$

3a) Six dice are thrown 729 times. How many times do you Except atleast three dice to show a 5 or 6?

Given

no. of trials $= 6$

$P=$ probability of getting 5 or 6 in one throw

$= \frac{2}{6} = 1/3$

$q = 1-p = 1-\frac{1}{3} = \frac{2}{3}$

Binomial distribution is

$P(r) = {}^nC_r\, p^r q^{n-r}$, $r = 0, 1, 2 \ldots n$

$$p(r) = {}^6C_r \left(\frac{1}{3}\right)^r \left(\frac{2}{3}\right)^{6-r}, \quad r = 0, 1, 2 \dots, 6$$

Probability of getting 5 or 6 atleast 3 times on dice is

$$P(x \geq 3) = 1 - P(x < 3)$$

$$= 1 - (P(0) + P(1) + P(2))$$

$$= 1 - \left[{}^6C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^6 + {}^6C_1 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^5 + {}^6C_2 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^4\right]$$

$$= 1 - \left[\left(\frac{2}{3}\right)^6 + 6\left(\frac{1}{3}\right)\left(\frac{2}{5}\right)^5 + 15\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4\right]$$

$$= 0.319$$

The Expected number of Such Cases in 729 times is

$$= 729 \times P(x \geq 3)$$

$$= 729 \times 0.319$$

$$= 232.9$$

$$\simeq 233$$

b) Derive mean and Variance of poisson distribution

Proof: the poisson distribution is

$$p(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2 \dots$$

The mean of the distribution is

$$\mu = \sum_{x=0}^{\infty} x\, p(x)$$

$$= \sum_{x=1}^{\infty} x\, p(x)$$

$$= \sum_{x=1}^{\infty} x \cdot \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\cancel{x}\, \lambda^x}{\cancel{x}(x-1)!}$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

Put, $y = x-1$, then $x = y+1$

$$\mu = e^{-\lambda} \sum_{y+1=1}^{\infty} \frac{\lambda^{y+1}}{y!}$$

$$= e^{-\lambda} \sum_{y=0}^{\infty} \frac{\lambda^y \cdot \lambda}{y!}$$

$$= e^{-\lambda} \cdot \lambda \sum_{y=0}^{\infty} \frac{\lambda^y}{y!} \quad \left[\because e^{\lambda} = \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}\right]$$

$$= e^{-\lambda} \cdot \lambda (e^{\lambda})$$

$$= e^{(-\lambda+\lambda)} \cdot \lambda = \lambda e^{0}$$

$$\boxed{\therefore \mu = \lambda}$$

$\therefore$ The mean of poisson distribution is 'λ'

Varience

proof:

The poisson distribution is

$$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2 \ldots$$

The Variance of the distribution is

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \sum_{x=0}^{\infty} x^2 p(x) - \mu^2$$

$$= \sum_{x=1}^{\infty} x^2 p(x) - \mu^2$$

$$= \sum_{x=1}^{\infty} x^2 \frac{e^{-\lambda}\lambda^x}{x!} - \mu^2$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \frac{x^2 \cdot \lambda^x}{x(x-1)!} - \mu^2$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} x \cdot \frac{\lambda^x}{(x-1)!} - \mu^2$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} (x-1+1) \frac{\lambda^\infty}{(x-1)!} - \mu^2$$

$$= e^{-\lambda} \sum_{x=1}^{\infty} \left[(x-1) \frac{\lambda^x}{(x-1)!} + \frac{\lambda^x}{(x-1)!} \right] - \mu^2$$

$$= e^{-\lambda} \left[\sum_{x=2}^{\infty} \cancel{(x-1)} \frac{\lambda^x}{\cancel{(x-1)}(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right] - \mu^2$$

$$= e^{-\lambda} \left[\sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} + \sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} \right] - \mu^2 \longrightarrow (1)$$

Consider,

$$\sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!}$$

Put, $z = x-2$ then $x = z+2$

$$\sum_{x=2}^{\infty} \frac{\lambda^x}{(x-2)!} = \sum_{z+2}^{\infty} \frac{\lambda^{z+2}}{z!}$$

$$= \sum_{z=0}^{\infty} \frac{\lambda^2 \cdot \lambda^z}{z!}$$

$$= \lambda^2 \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \quad \left[\because e^{\lambda} = \sum_{z=0}^{\infty} \frac{\lambda^z}{z!} \right]$$

$$= \lambda^2 e^{\lambda}$$

Consider,

$$\sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!}$$

Put, $y = x-1$ then $x = y+1$

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$$\sum_{x=1}^{\infty} \frac{\lambda^x}{(x-1)!} = \sum_{y+1=1}^{\infty} \frac{\lambda^{y+1}}{y!}$$

$$= \sum_{y=0}^{\infty} \frac{\lambda^y \cdot \lambda}{y!}$$

$$= \lambda \sum_{y=0}^{\infty} \frac{\lambda^y}{y!}$$

$$= \lambda \cdot e^{\lambda}$$

from eq (1)

$$\sigma^2 = e^{-\lambda}\left[\lambda^2 e^{\lambda} + \lambda e^{\lambda}\right] - \mu^2$$

$$\sigma^2 = e^{-\lambda} \cdot e^{\lambda}\left[\lambda^2 + \lambda\right] - \lambda^2$$

$$\sigma^2 = e^{-\lambda+\lambda}\left[\lambda^2 + \lambda\right] - \lambda^2$$

$$\sigma^2 = e^{0}\left(\lambda^2 + \lambda\right) - \lambda^2$$

$$\sigma^2 = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$\boxed{\therefore \sigma^2 = \lambda}$$

So the variance of poisson distribution is λ

Q. If poisson distribution is such that $p(x=1) \cdot \frac{3}{2} = p(x=3)$ Find (i) $p(x \geq 1)$

(ii) $p(x \leq 3)$

(iii) $p(2 \leq x \leq 5)$

The poisson distribution is

$$p(x) = \frac{e^{-\lambda} \lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$$

Given

$$p(x=1) \frac{3}{2} = p(x=3)$$

$$\frac{e^{-\lambda} \lambda^1}{1!} \times \frac{3}{2} = \frac{e^{-\lambda} \lambda^3}{3!}$$

$$\lambda \cdot \frac{3}{\cancel{\lambda}} = \frac{\lambda^3}{\cancel{6}3}$$

$$\lambda^3 = 9\lambda$$

$$\lambda^3 - 9\lambda = 0$$

$$\lambda(\lambda^2 - 9) = 0$$

$\lambda = 0$ | $\lambda^2 - 9 = 0$

$$\Rightarrow \lambda^2 = 9$$

$$\lambda = \pm 3$$

Since, $\lambda > 0$; $\lambda = 3$

$$P(X = x) = \frac{e^{-3} 3^x}{x!}, \quad x = 0, 1, 2, \ldots$$

(i) $P(X \geq 1) = 1 - P(X < 1)$

$$= 1 - P(0)$$

$$= 1 - \frac{e^{-3} 3^0}{0}$$

$$= 1 - e^{-3} = 0.950$$

(ii) $P(X \leq 3) = P(0) + P(1) + P(2) + P(3)$

$$= \frac{e^{-3} 3^0}{0!} + \frac{e^{-3} 3^1}{1!} + \frac{e^{-3} 3^2}{2!} + \frac{e^{-3} 3^3}{3!}$$

$$= e^{-3}\left(1 + 3 + \frac{9}{2} + \frac{27}{6}\right)$$

$$= 0.647$$

(iii) $P(2 \leq X \leq 5) = P(2) + P(3) + P(4) + P(5)$

$$= \frac{e^{-3} \cdot 3^2}{2!} + \frac{e^{-3} \cdot 3^3}{3!} + \frac{e^{-3} \cdot 3^4}{4!} + \frac{e^{-3} \cdot 3^5}{5!}$$

$$= e^{-3}\left[\frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \frac{243}{120}\right]$$

$$= 0.716$$

d) Fit a poisson distribution to the following data

x	0	1	2	3	4	5
f	142	156	69	27	5	1

Sol

n = no. of trials = 5

N = total frequency $= \sum f_i$

$= 400$

The mean is $\mu = \frac{\sum x_i f_i}{\sum f_i}$

$$= \frac{0(142)+1(156)+2(69)+3(27)+4(5)+5(1)}{400}$$

$$= \frac{400}{400} = 1$$

The mean of poisson distribution is $\lambda = \mu = 1$

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0,1,2 \ldots$$

$$P(X=x) = \frac{e^{-1} 1^{x}}{x!}, \quad x = 0,1,2 \ldots$$

then,

$$P(0) = \frac{e^{-1}(1)^0}{0!} = e^{-1} = 0.367$$

$$P(1) = \frac{e^{-1}.1^1}{1!} = e^{-1} = 0.367$$

$$P(2) = \frac{e^{-1}.1^2}{2!} = \frac{e^{-1}}{2} = 0.183$$

$$P(3) = \frac{e^{-1}.1^3}{3!} = \frac{e^{-1}}{6} = 0.061$$

$P(4) = \frac{e^{-1} \cdot 1^4}{4!} = \frac{e^{-1}}{24} = 0.015$

$P(5) = \frac{e^{-1} 1^5}{5!} = \frac{e^{-1}}{120} = 0.003$

x	f	$P(x)$	Expected frequency $NP(x)$
0	142	0.367	$400 \times P(0)$ $= 146.8 \simeq 147$
1	156	0.367	$400 \times P(1)$ $= 146.8 \simeq 147$
2	69	0.183	$400 \times P(2)$ $= 72.58 \simeq 73$
3.	27	0.061	$400 \times P(3)$ $= 24.53 \simeq 25$
4.	5	0.010	$400 \times P(4)$ $= 6.13 \simeq 6$
5.	1	0.003	$400 \times P(5)$ $= 1.2 \simeq 1$

e) Find the mean and Varience of the distribution In a normal distribution, 7% of the items are under 35 and 89% are over 63.

Let

μ be the mean and σ be the Standard distribution of normal Curve

Given,

$P(x < 35) = 7\% = 0.07$

$P(x < 63) = 89\% = 0.89$

Then, $P(x \geq 63) = 1 - P(x < 63)$

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$= 1 - 0.89$

$= 0.11$

from fig,

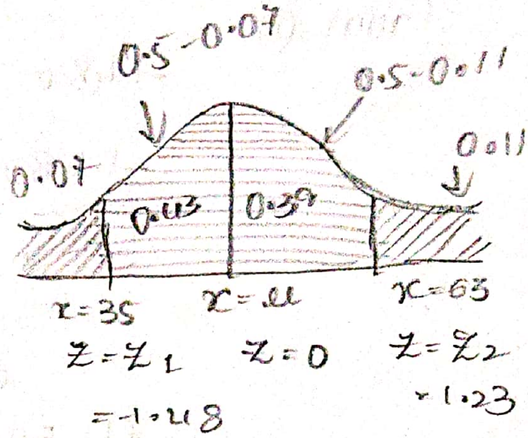
we have

$P(z < z_1) = 0.43$

$\Rightarrow z_1 = -1.48$

$P(z < z_2) = 0.39$

$z_2 = 1.23$



We know,

$$z = \frac{x-\mu}{\sigma}$$

for,

$x = 35 \Rightarrow z = \frac{35-\mu}{\sigma} \rightarrow (1)$

for, $x = 63 \Rightarrow z = \frac{63-\mu}{\sigma} \rightarrow (2)$

from (1)

$$-1.48 = \frac{35-\mu}{\sigma}$$

$$\Rightarrow -1.48\sigma = 35-\mu \rightarrow (3)$$

from (2)

$$1.23 = \frac{63-\mu}{\sigma}$$

$$\Rightarrow 1.23\sigma = 63-\mu \rightarrow (4)$$

$$\frac{(3)}{(4)} = \frac{-1.48\sigma}{1.23\sigma} = \frac{35-\mu}{63-\mu}$$

$$(-1.203)(63-\mu) = 35-\mu$$

$$-75.789 + 1.203\mu = 35-\mu$$

$$(1 + 1.203)\mu = 35 + 75.789$$

$$2.203\mu = 110.789$$

$$\mu = \frac{110.789}{2.203}$$

$$= 50.29$$

$$\simeq 50.3$$

From ①

$$-1.48\sigma = 35 - \mu$$

$$-1.48\sigma = 35 - 50.3$$

$$\sigma = \frac{-15.3}{-1.48}$$

$$\sigma = 10.3$$

$$\mu = 50.3 \ \& \ \sigma = 10.3$$

a) I) Out of 800 families with 5 children each, how many would you Expect to have a) 3 boys b) 5 girls c) Either 2 or 3 boys? Assume Equal probabilities for boys & girls

II) Derive mean and Varience of Binomial distribution

Sol I)

$n =$ no. of Children $= 5$

$P =$ probability of boy $= \frac{1}{2}$

$$q = 1 - P$$

$$= 1 - \frac{1}{2} = \frac{1}{2}$$

Binomial distribution is

$$P(r) = {}^{n}C_r \, p^r q^{n-r}, \ r = 0, 1, 2 - - - n$$

$$P(r) = {}^{5}C_r \left(\tfrac{1}{2}\right)^r \left(\tfrac{1}{2}\right)^{5-r}, \ r = 0, 1, - - - 5$$

$$= {}^{5}C_r \left(\tfrac{1}{2}\right)^{r+5-r}$$

$$= \frac{1}{32} \, {}^{5}C_r, \ r = 0, 1, 2 - - -, 5$$

(i) Probability of having 3 Boys is

$$P(3) = \frac{1}{32}\,{}^5C_3$$
$$= 10\left(\frac{1}{32}\right)$$
$$= 0.3125$$

(ii) probability of having 3 boys is

$$P(3) = \frac{1}{32}\,{}^5C_3$$
$$= 10 \times \frac{1}{32}$$
$$= 0.3125$$

(ii) Probability of having 5 girls is

$$P(5) = \frac{1}{32}\,{}^5C_1$$
$$= \frac{1}{32} = 0.03125$$

(iii) Probability of having either 2 or 3 boys is

$$P(2 \text{ or } 3) = P(2) + P(3)$$
$$= \frac{1}{32}\,{}^5C_2 + \frac{1}{32}\,{}^5C_3$$
$$= \frac{10}{32} + \frac{10}{32}$$
$$= \frac{20}{32} = 0.625$$

The no. of families having 3 boys is

$$= 800 \times P(3) = 800 \times \frac{10}{32}$$
$$= 250$$

The no. of families having 5 girls is

$$= 800 \times P(5) = 800 \times \frac{1}{32}$$

$$= 25$$

The no. of families having atleast either 2 (or) 3 boys is

$$= 800 \times P(2 \text{ or } 3) = 800 \times \frac{20}{32}$$

$$= 500$$

II) Derive mean and variance of Binomial distribution

Proof:

The binomial distribution is

$$P(r) = {}^{n}C_{r}\, p^{r} q^{n-r}, \quad r = 0, 1, 2, \ldots n$$

$$\& \quad q = 1 - p$$

The mean is $\mu = \sum_{r=0}^{n} r\,p(r)$

Then $\mu = 0 \cdot P(0) + 1 \cdot P(1) + 2 \cdot P(2) + 3 \cdot P(3) + 4 P(4) + \ldots + n P(n)$

$$= 1P(1) + 2P(2) + 3P(3) + \ldots + nP(n)$$

$$= {}^{n}C_{1} p^{1} q^{n-1} + 2({}^{n}C_{2} p^{2} q^{n-2}) + 3({}^{n}C_{3} p^{3} q^{n-3}) + \ldots + n({}^{n}C_{n} p^{n} q^{n-n})$$

$$= np\left(q^{n-1} + (n-1) p q^{n-2} + \frac{(n-1)(n-2)}{2} p^{3} q^{n-3} + \ldots p^{n-1}\right)$$

$$= np(q+p)^{n-1}$$

$$= np(1)^{n-1} \quad [\because p + q = 1]$$

$$\mu = np$$

Varience

Proof : The binomial distribution is

$$P(r) = {}^nC_r\, p^r q^{n-r}, \quad r = 0,1,2,---n$$

The Variance of B.D is

$$\sigma^2 = E[x^2] - (E[x])^2$$

$$= \sum_{r=0}^{n} r^2 P(r) - \mu^2$$

$$= \sum_{r=0}^{n} (r^2 + r - r) P(r) - \mu^2$$

$$= \sum_{r=0}^{n} (r(r-1) + r) P(r) - \mu^2$$

$$= \sum_{r=0}^{n} (r(r-1)P(r) + rP(r)) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1) P(r) + \sum_{r=0}^{n} rP(r) - \mu^2$$

$$= \sum_{r=0}^{n} r(r-1) P(r) + \mu - \mu^2$$

$$= (0)(-1)P(0) + (1)(0)P(1) + (2)(1)P(2) + (3)(2)P(3) + ----- (n)(n-1)P(n) + \mu - \mu^2$$

$$= 2P(2) + 6P(3) + ------ + n(n-1)P(n) + \mu - \mu^2$$

$$= 2\left({}^nC_2\, p^2 q^{n-2}\right) + 6\left({}^nC_3\, p^3 q^{n-3}\right) + ---- + n(n-1)\left({}^nC_n\, p^n q^{n-n}\right) + \mu - \mu^2$$

$$= n(n-1)p^2\left(q^{n-2} + (n-2)pq^{n-3} + --- + p^{n-2}\right) + \mu - \mu^2$$

$$= n(n-1)p^2 (q+p)^{n-2} + \mu - \mu^2$$

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$= (n^2-n)p^2 + \mu - \mu^2$

$= \cancel{n^2p^2} - np^2 + np - \cancel{(np)^2}$

$= np - np^2$

$= np(1-p)$

$= npq$

$\therefore$ The Variance of Binomial distribution is $= npq$

b) 1) If x poissons Varient Such that $3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$, So find (i) the mean of x
(ii) $P(x \leq 2)$

Given

$3P(x=4) = \frac{1}{2}P(x=2) + P(x=0)$

The poisson distribution is

$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}, \quad x = 0, 1, 2, \ldots$

$3 \cdot \frac{\cancel{e^{-\lambda}}\lambda^4}{4!} = \frac{1}{2}\left(\frac{\cancel{e^{-\lambda}}\lambda^2}{2!}\right) + \frac{\cancel{e^{-\lambda}}\lambda^0}{0!}$

$\frac{\cancel{3} \cdot \lambda^4}{\cancel{24}\ \cancel{8}\ \cancel{4}\ 2} = \frac{\lambda^2}{\cancel{2}} \cdot \frac{1}{\cancel{2}} + 1$

$\frac{\lambda^4}{2} = \lambda^2 + 1$

$(\lambda^2)^2 - 2\lambda^2 - 8 = 0$

$(\lambda^2)^2 - 4\lambda^2 + 2\lambda^2 - 8 = 0$

$\lambda^2(\lambda^2 - 4) + 2(\lambda^2 - 4) = 0$

$$(\lambda^2-4)(\lambda^2+2)=0$$

$\lambda^2=4$ | $\lambda^2+2=0$

$\lambda=\pm 2$ | $\lambda^2=-2 \notin R$

Since $\lambda>0$; $\lambda=2$

$$P(x=x)=\frac{e^{-2}\cdot 2^x}{x!}, \quad x=0,1,2 \ldots$$

(i) Mean $=\lambda=2$

(ii) $P(x\le 2)=P(0)+P(1)+P(2)$

$$=\frac{e^{-2}\cdot 2^0}{0!}+\frac{e^{-2}\cdot 2^1}{1!}+\frac{e^{-2}\cdot 2^2}{2!}$$

$$=e^{-2}(1+2+2)$$

$$=5e^{-2}=0.6766$$

II) If the masses of 300 Students are normally distributed with mean 68kg and Standard deviation 3kg, How many Student have masses (i) greater than 72 kg. (ii) less than (or) Equal to 64kg (iii) between 65 and 71kg inclusive.

Given

Mean $\mu=68kg$

Standard deviation $=\sigma=3kg$

we know

$$z=\frac{x-\mu}{\sigma}=\frac{x-68}{3}$$

(i) for $x=72$, $z=\frac{72-68}{4}=1.33=z_1$

$\therefore Z_1 > 0$

$P(x > 72) = P(z > 1.33)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082$

$= 0.0918$

The no. of Students who have greater than 72

$= 300 \times 0.0918$

$= 27.5 \simeq 28$

(ii) for $x = 64$, $z = \frac{64 - 68}{4} = -1.33 = z_1$

$\therefore z_1 < 0$

$P(x < 64) = P(z < -1.33)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082 = 0.0918$

The no. of Students who have less than 64 kg

$= 300 \times 0.0918$

$= 27.5$

$\simeq 28$

(iii) for $x = 65$, $z = \frac{65-68}{3} = -1 = z_1 < 0$

for $x = 71$, $z = \frac{71-68}{3} = 1 = z_2 > 0$

$z_1 < 0$ & $z_2 > 0$

$P(65 \leq x \leq 71) = P(-1 \leq x \leq 1)$

$= A(-1) + A(1)$

$= A(1) + A(1) = 2A(1)$

$= 2 \times 0.3413 = 0.6826$

The no. of Students who have mass between 65 & 71

$= 300 \times 0.6826$

$= 204.7 \simeq 205$

c) I) 7 Coins are tossed and the no. of head are no tail the Experiment is repeated 128 times. And the following distribution is obtained

number of heads(x)	0	1	2	3	4	5	6	7	Total
Frequency F	7	6	19	35	30	23	7	1	128

Fit a binomial assuming the coin is unbiased

Given

n = no. of trials = 7

N = total frequency = Σf_i

$\Sigma f_i = 7+6+19+35+30+23+7+1$

$= 128$

Since the Coin is unbiased ; $p = q = \frac{1}{2}$

Binomial distribution is

$P(r) = {}^{n}C_r \, p^r \, q^{n-r}$, $r = 0, 1, 2 \ldots n$

$P(r) = {}^{7}C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{7-r}$, $r = 0, 1, 2, \ldots, 7$

$= {}^{7}C_r \left(\frac{1}{2}\right)^{\cancel{r}+7-\cancel{r}}$

$= \frac{1}{2^7} \cdot {}^{7}C_r = \frac{1}{128} \cdot {}^{7}C_r$

$P(0) = \frac{1}{128} \cdot {}^{7}C_0 = \frac{1}{128}$ $\quad$ $P(4) = \frac{1}{128} \cdot {}^{7}C_4 = \frac{35}{128}$

$P(1) = \frac{1}{128} \cdot {}^{7}C_1 = \frac{7}{128}$ $\quad$ $P(5) = \frac{1}{128} \cdot {}^{7}C_5 = \frac{21}{128}$

$P(2) = \frac{1}{128} \cdot {}^{7}C_2 = \frac{21}{128}$ $\quad$ $P(6) = \frac{1}{128} \cdot {}^{7}C_6 = \frac{7}{128}$

$P(3) = \frac{1}{128} \cdot {}^{7}C_3 = \frac{35}{128}$ $\quad$ $P(7) = \frac{1}{128} \cdot {}^{7}C_7 = \frac{1}{128}$

The Binomial distribution is fitted as

x	observed freq f	Probability $P(x)$	Expected frequency $NP(x)$
0	7	$1/128$	$128 \cdot P(0) = \frac{128}{128} = 1$
1	6	$7/128$	$128 \cdot P(1) = \frac{128 \times 7}{128} = 7$
2	19	$21/128$	$128 \cdot P(2) = \frac{128 \times 21}{128} = 21$
3	35	$35/128$	$128 \cdot P(3) = \frac{128 \times 35}{128} = 35$
4	30	$35/128$	$128 \cdot P(4) = \frac{128 \times 35}{128} = 35$
5	23	$21/128$	$128 \cdot P(5) = \frac{128 \times 21}{128} = 21$
6	7	$7/128$	$128 \cdot P(6) = \frac{128 \times 7}{128} = 7$
7	1	$1/128$	$128 \cdot P(7) = \frac{128}{128} = 1$