SR 22

ACVC

PART – A (10x1M = 10M)

Note: Answer all Questions. Each Question carries equal marks.

1) Define vector differential operator.
2) Define curl of a vector.
3) For what value of p, the vector $\vec{f} = (x+3y)\vec{i} + (y-2z)\vec{j} + (x+pz)\vec{k}$ is solenoidal.
4) Define divergence of a vector
5) Define line integral.
6) State Greens Theorem in a plane.
7) Define surface integral.
8) Define Indefinite integral.
9) Write the Jacobian for the transformation from Cartesian to polar coordinates.
10) Find $\int_0^{\frac{\pi}{2}} \int_0^a r^2 \, dr d\theta$.
11) Define gradient of a scalar point function.
12) Define scalar potential.
13) Define solenoidal vector.
14) Define vector point function.
15) Define surface integral.
16) State Stoke's theorem.
17) State Gauss Divergence Theorem
18) Define volume integral.
19) Write the Jacobian of the transformation from Cartesian to polar coordinates.
20) Find the value of $\int_0^2 \int_0^x y \, dxdy$.
21) Define curl of a vector.
22) What is solenoidal vector.
23) Define directional derivative.
24) Define vector differential operator.
25) Define volume integral.
26) State Gauss Divergence Theorem

27) Define Work done by force.
28) State Stoke's theorem.
29) Write the relations between cartesian coordinates and polar coordinates.
30) Find the value of $\int_0^2 \int_0^y x\, dxdy$.

PART – B (20M)

Question (s)
1) Find div $\vec{f}$, where $\vec{f} = r^n \vec{r}$. Find n if it is solenoidal.
2) Find the directional derivative of $f = x^2 - y^2 + 2z^2$ at the point P(1,2,3) in the direction of the line PQ where Q=(5,0,4).
3) Find the constants a, b, c such that the vector $\vec{A} = (x+2y+az)\vec{i} + (bx-3y-z)\vec{j} + (4x+cy+2z)\vec{k}$ is irrotational. Also find ϕ such that $\vec{A} = \nabla\phi$.
4) Find $\int \vec{F}.\vec{n}ds$, where $\vec{F} = z\vec{i} + x\vec{j} - 3y^2z\vec{k}$ and S is the surface $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.
5) If $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$, find $\int \vec{F}.\vec{n}ds$, where S is the surface of the cube bounded by $x = 0, x = a, y = 0, y = a, z = 0$.
6) Verify Green's theorem in plane for $\oint (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the region bounded by $y = \sqrt{x}$ and $y = x^{2.}$
7) Find the volume of the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.
8) Evaluate the following integral by changing the order of integration $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dxdy$.

Question (s)
9) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in a direction of the normal to the surface $3xy^2 + y = z$ at $(0,1,1)$.
10) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 47 = 0$ at the point $(4, -3, 2)$.
11) Show that the vector $(x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ is irrotational and find its scalar potential.
12) Find $\int_s \vec{F}.\vec{n}ds$ where $\vec{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and s is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant.

13) Find the work done in moving a particle in the force field $\vec{F}=3x^2\vec{i}+\vec{j}+z\vec{k}$ along the straight line joining $(0,0,0)$ and $(2,1,3)$
14) Verify Gauss's divergence theorem for $\vec{F}=(x^3-yz)\vec{i}-2x^2y\vec{j}+z\vec{k}$ taken over the surface of the cube bounded by the planes $x=y=z=a$ and coordinate planes.
15) Evaluate $\iiint z^2dxdydz$ taken over the volume bounded by $x^2+y^2=a^2, x^2+y^2=z$ and $z=0$.
16) Find $\int_0^a\int_x^a x^2\, dxdy$ by changing the order of integration.

Question (s)
17) **Discuss angle between the normal to the surface $xy=z^2$at the points (4,1,2) and (3,3,-3)**
18) Define irrotational vector. Also if $\vec{f}=\vec{r}$, then prove that $\vec{f}$ is irrotational.
19) If $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ is the position vector of any point in space and $r=\lvert\vec{r}\rvert$ Then prove that the vector $\frac{\vec{r}}{r^3}$ is solenoidal and the vector $r^n\vec{r}$ is irrotational.
20) A vector field is given by $\vec{F}=(\sin y)\vec{i}+x(1+\cos y)\vec{j}$, evaluate the line integral over the circular path $x^2+y^2=a^2, z=0$.
21) If $\vec{F}=2xz\vec{i}-x\vec{j}+y^2\vec{k}$. Then find $\int_V \vec{F}dv$, where V is the region bounded by the surfaces $x=0, x=2, y=0, y=6, z=x^2, z=4$.
22) Verify Stoke's theorem for $\vec{F}=-y^3\vec{i}+x^3\vec{j}$, where S is the circular disc $x^2+y^2\leq 1, z=0$.
23) Find the area between the circle $r=2\ sin\ \theta$ and $r=4\ sin\ \theta$.
24) Change the order of integration in $\int_0^3\int_1^{\sqrt{4-y}}(x+y)\ dxdy$.

ALL THE BEST