UNIT-I

Descrite and continuous Random variable (1)

4. Discrete random variable : A random variable x which can take only a finite no. of discrete values in an interval of domain is called a discrete random variable In other words, if the random variable takes the values only on the set $\{0, 1, 2, ---, n\}$ is called a Discrete Random variable.

continuous Random variable : A random variable x which can to be values continuously i.e which takes all possible values in a given interval is called a continuous random variable.

1b) Define expectation and variance of discrete random variable

Sol :- Expectation :- suppose a random variable x assumus the values $x_1, x_2, ---x_n$ with respective probabilities $p_1, p_2, --p_n$. Then the Mathematical Expectation or mean or expected values of x, denoted by E(X), is defined as the sum of products of different values of x corresponding probabilities $[\therefore E(X) = x_1 p_1 + x_2 p_2 + -- + x_n p_n]$

variance :- variance of the probability distribution of a random variable x is the mathimatical expectation of $[x - E(x)]^2$

Then, $var(x) = E[x - E(x)]^2$

1c) A random sample with replacement of size 2 is taken from $S = \{1, 2, 3\}$ let the random variable x denote the sum of the numbers taken. write the probability distribution.

(Sol: $S = \{(1,1)(1,2)(1,3)(2,1)(2,2),(2,3)(3,1),(3,2)(3,3)\}$

UNIT-I / ②

$X(S) = (2,3,4,5,6)$

$P(2) = P(X=2) = P[(1,1)] = \frac{1}{9}$

$P(3) = P(X=3) = P[(1,2);(2,1)] = \frac{2}{9}$

$P(4) = P(X=4) = P[(1,3);(2,2);(3,1)] = \frac{3}{9}$

$P(5) = P(X=5) = P[(2,3);(3,2)] = \frac{2}{9}$

$P(6) = P(X=6) = P[(3,3)] = \frac{1}{9}$

The probability distibution of x is.

x	2	3	4	5	6
$P(x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

1d) Define mean and Variance of continuous random variable

Sol: let $f(x)$ be the probability density function of a continuous random variable x Then.

Mean: Mean of a distribution is given by $\mu = E(x) = \int_{-\infty}^{\infty} x f(x) dx$.

If x is defined from a to b, then $\mu = E(x) = \int_a^b x f(x) dx$.

Variable:- Variance of a distribution is given by.

$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx$ (or) $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$

Suppose that the variable x is defined from a to b. Then

$\sigma^2 = \int_a^b (x-\mu)^2 f(x) dx$ or $\sigma^2 = \int_a^b x^2 f(x) dx - \mu^2$

1e) If a random variable has the probability density $f(x)$ as $f(x) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & x \leq 1 \end{cases}$ find the probability between 1 and 3

$$P(1 \le X \le 3) = \int_1^3 f(x)\,dx = \int_1^3 2e^{-2x}\,dx$$

$$= 2\left(\frac{e^{-2x}}{-2}\right)_1^3 = e^{-2} = e^{-1}$$

2(a) prove that if x is a discrete random variable and k is constant then $E(X+K) = E(X)+K$

Suppose x is random variable & k is constant

From the definition of expectation,

$$E(X) = \sum_{i=1}^{n} x_i p_i$$

$$E(X+k) = \sum_{i=1}^{n} (x_i + k) p_i$$

$$= \sum_{i=1}^{n} (x_i p_i + k p_i)$$

$$= \sum_{i=1}^{n} x_i p_i + \sum_{i=1}^{n} k p_i$$

$$= \sum_{i=1}^{n} x_i p_i + k \sum_{i=1}^{n} p_i$$

$$= E(X) + k(1) \qquad \left(\because \sum_{i=1}^{n} P_i = 1\right)$$

$$= E(X) + k$$

$$\therefore E(X+K) = E(X) + k$$

2(b) A fair coin is a tossed while a head of five tails occurs, find the expected member E of tosses of the coin? (4)

Sol^n If head occurs first time there will be only one loss.

On the otherhand, if first one is tail, Second occurs If head occurs there will be only two losses.

Suppose Second one is also tail third occurs If head occurs there will be three losses & soon

If head occurs there will be three losses & soon

$P(1) = P(H) = \frac{1}{2}$, $P(2) = P(TH) = \frac{1}{4}$, $P(3) = P(TTH) = \frac{1}{8}$

$P(4) = P(TTTH) = \frac{1}{16}$, $P(5) = P(TTTH) + P(TTTTH)$

$= \frac{1}{32} + \frac{1}{32} = \frac{1}{16}$

The Probability distribution of X is

X	1	2	3	4	5
P(x)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{16}$

$\therefore E(x) = \sum_{i=1}^{n} P_i x_i = 1\left(\frac{1}{2}\right) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{16}\right) + 5\left(\frac{1}{16}\right)$

$= \frac{31}{36} = 1.937$

(2C) If the probability density of random variable is given by

$f(x) = \{ k(1-x^2), 0<x<1, 0$ Otherwise find value of k. (5).

Ans) Given $f(x) \begin{cases} k(1-x^2), & 0<x<1 \\ 0, & \text{Otherwise} \end{cases}$ (5)

we know that

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

i.e, $\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$

i.e, $0 + \int_{0}^{1} k(1-x^2)dx + 0 = 1$

i.e $k\left(x - \frac{x^3}{3}\right)_0^1 = 1$ (or) $k\left(1 - \frac{1}{3}\right) = 1$

$k = 3/2$

(2(a)) If x is Continuous random variable and $y = ax+b$ Prove that $E(Y) = aE(x)+b$ and $V(Y) = a^2 \cdot V(X)$. where v stands for variance and a, b are constants

Ans) $E(ax+b) = \int_{-\infty}^{\infty} (ax+b) f(x)dx$

$\left[\because E[\phi(x)] = \int_{-\infty}^{\infty} \phi f(x)(x) dx\right]$

$$= a\int_{-\infty}^{\infty} x f(x)dx + b\int_{-\infty}^{\infty} f(x)dx$$ 6 10

$af(x) + b(1)$

$= aE(x) + b$

$E(y) = aE(x) + b$ —— ①

where $y = ax + b$ —— ②

② - ① gives $y - E(y) = a[x - E(x)]$

Squaring. $[y - E(y)]^2 = a^2[x - E(x)]^2$

Taking expectation of both sides, we get

$E\{[y - E(y)]^2\} = a^2 E\{[x - F(x)]^2\}$

$\therefore V(y) = a^2 V(x)$

2(e) A random variable x is defined as the sum of the numbers on the faces when two dice are thrown. find the mean of x.

Sample Space of two dies is

$\{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (2,1)\ (2,2)\ (2,3)\ (2,4)$
$(2,5)\ (2,6)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\ (4,1)\ (4,2)$
$(4,3)\ (4,4)\ (4,5)\ (4,6)\ (5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

let x denote $x(a,b) = a+b \ \forall (a,b) \in S$

$x(1,1)=2$ $\quad x(2,1)=3$ $\quad x(3,1)=4$ $\quad x(4,1)=5$

$x(1,2)=3$ $\quad x(2,2)=4$ $\quad x(3,2)=5$ $\quad x(4,2)=6$

$x(1,3)=4$ $\quad x(2,3)=5$ $\quad x(3,3)=6$ $\quad x(4,3)=7$

$x(1,4)=5$ $\quad x(2,4)=6$ $\quad x(3,4)=7$ $\quad x(4,4)=8$

$x(1,5)=6$ $\quad x(2,5)=7$ $\quad x(3,5)=8$ $\quad x(4,5)=9$

$x(1,6)=7$ $\quad x(2,6)=8$ $\quad x(3,6)=9$ $\quad x(4,6)=10$

$x(5,1)=6$ $\quad x(6,1)=7$

$x(5,2)=7$ $\quad x(6,2)=8$

$x(5,3)=8$ $\quad x(6,3)=9$

$x(5,4)=9$ $\quad x(6,4)=10$

$x(5,5)=10$ $\quad x(6,5)=11$

$x(5,6)=11$ $\quad x(6,6)=12$

∴ The Possible Values are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

The Probability Distribution is

x	2	3	4	5	6	7	8	9	10	11	12
$f(x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

The mean $\mu = \sum_{i=1}^{n} x_i p_i$

$$= \sum_{i=1}^{2} 2\left(\frac{1}{36}\right)+3\left(\frac{2}{36}\right)+4\left(\frac{3}{36}\right)+5\left(\frac{4}{36}\right)+6\left(\frac{5}{36}\right)+7\left(\frac{6}{36}\right)$$

$$+8\left(\frac{5}{36}\right)+9\left(\frac{4}{36}\right)+10\left(\frac{3}{36}\right)+11\left(\frac{2}{36}\right)+12\left(\frac{1}{36}\right)$$

$$\Rightarrow \frac{2}{36}+\frac{6}{36}+\frac{12}{36}+\frac{20}{36}+\frac{30}{36}+\frac{42}{36}+\frac{40}{36}+\frac{36}{36}+\frac{30}{36}+\frac{22}{36}+\frac{12}{36}$$

$$\Rightarrow \frac{2+6+12+20+30+42+40+36+30+22+12}{36}$$

$\Rightarrow 252/36 = 7$ ∴ The mean (μ) of $x = 7$

3) a) A player tosses two fair Coin. (8)
He wins Rs 100/- If a head appears
Rs 200/- If two heads appears. On
the Other hand. He loses Rs 500/-
If no head appears. Determine the expected value
E of the game and is the game favourable
to the player.

Ans) let
$S = \{HH, TH, HT, TT\}$ be sample Space let
x denotes the value of money that he
get (or) lose

Hence $(P(X=-500)) = 1/4$

$P(X=100) = 2/4$

$P(X=200) = 1/4$

The probability distribution is

x	-500	100	200
P(x)	1/4	2/4	1/4

Total Out comes = 1

The expected value is

$P(x) = (-500)(1/4) + 100(2/4) + 200(\frac{1}{4})$

$= -25 < 0$ ∴ The game isn't favourable

(3b) A continuous random variables of probability density function (9)

(5)

$f(x) = \{ k x e^{-x\lambda}$, for $x \geq 0$, $\lambda > 0$, 0, other wise

$$f(x) = \begin{cases} k x e^{-x\lambda} & \text{for } x \geq 0, \lambda > 0 \\ 0, & \text{Otherwise} \end{cases}$$

Determine :-

i, k ii, Mean iii, Variance

Ans) Given that the function is

$$f(x) = \begin{cases} k x e^{-\lambda x} & \text{for } x \geq 0, \lambda > 0 \\ 0 & , \text{Otherwise} \end{cases}$$

we know that $\int_{-\infty}^{\infty} f(x)\, dx = 1$

i, $\int_{-\infty}^{0} f(x)\, dx + \int_{0}^{\infty} f(x)\, dx = 1$

$$\int_{-\infty}^{0} 0\, dx + \int_{0}^{\infty} k x e^{-\lambda x} dx = 1$$

$$= 0 + k \int_{0}^{\infty} x e^{-\lambda x} dx = 1 \qquad \left[\because \int f g\, dx = f\int g - f'\iint g + f''\iiint g \ldots\right]$$

$$= 0 + k \int_{0}^{\infty} \underset{f}{x}\, \underset{g}{e^{-\lambda x}}\, dx = 1$$

(10)

$$\Rightarrow k\left[\frac{xe^{-\lambda x}}{-\lambda} - (1)\frac{e^{-\lambda x}}{(-\lambda)^2}\right]_0 = 1$$

$$k\left[\left(\frac{x}{\lambda} - \frac{1}{\lambda^2}\right) - e^{-\lambda x}\right]_0^\infty = 1$$

$$k\left[0 - \left(\infty - \frac{1}{\lambda^2}\right)(1)\right]_0^1 = 1$$

$$k\left(\frac{1}{\lambda^2}\right) = 1 \qquad \Rightarrow k = \lambda^2$$

$$\therefore f(x) = \begin{cases} \lambda x e^{-\lambda x} & \text{for } x \geq 0, \lambda > 0 \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{Mean, } \mu = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$\int_{\infty}^{0} x(0)\, dx + \int_0^\infty x\left(\lambda^2 \cdot x e^{-\lambda x}\right) dx$$

$$= 0 + \lambda^2 \int_0^\infty \underset{f}{x^2}\, \underset{g}{e^{-\lambda x}}\, dx$$

$$\lambda^2 \left[x^2 \frac{e^{-\lambda x}}{-\lambda} - (2x)\frac{e^{-\lambda x}}{(-1)^2} + \frac{2e^{-\lambda x}}{(-\lambda)^3}\right]_0^\infty$$

$$\lambda^2\left[\left(-\frac{x^2}{\lambda} - \frac{2x}{\lambda^2} - \frac{2}{\lambda^3}\right)e^{-\lambda x}\right]_0^\infty$$

$$\Rightarrow \lambda^2\left[0 - \left(0 - 0 - \frac{2}{\lambda^3}\right)(1)\right]$$

$$= \lambda^2\left(\frac{2}{\lambda^3}\right) = \frac{2}{\lambda} \qquad \therefore \text{Mean} = \frac{2}{\lambda}$$

(3 d) for a continous probability density function is given by $f(x) = ce^{-|x|}$ $-\infty < x < \infty$ find the value of c and hence mean & variance (5) (11)

Solⁿ we have $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{\infty} ce^{-|x|}\,dx = 1$$

$$= 2c \int_{0}^{\infty} e^{-|x|}\,dx = 1$$

$$= 2c \int_{0}^{\infty} e^{-x}\,dx = 1$$

$$= 2c\,(-e^{-x})_0^{\infty} = 1 \Rightarrow 2c = 1 \Rightarrow c = 1/2$$

$$f(x) = \frac{1}{2} e^{-|x|}, -\infty, x < \infty$$

i) mean, $\mu = \int_{-\infty}^{\infty} x\, f(x)\,dx = \frac{1}{2}\int_{-\infty}^{\infty} x e^{-|x|}\,dx = 0$

$(\because xe^{-|x|}$ is odd$)$

ii) variance, $\sigma^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$

$$= \int_{-\infty}^{\infty} \frac{-x^2}{2} e^{-|x|}\,dx - 0 \quad (\because x^2 e^{-|x|} \text{ is even})$$

$$= \frac{2}{e}\int_{0}^{\infty} x^2 e^{-x}\,dx = 2$$

(3 e) let x denotes the minimum of two numbers that appear when a pair of fair Dice is thrown once determine the

i) Discrete probability distribution

ii) expectation

Solⁿ let $S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)$
$(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)$

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6) }

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Then

$x(1,1)=1$ $x(2,1)=1$ $x(3,1)=1$

$x(1,2)=1$ $x(2,2)=2$ $x(3,2)=2$

$x(1,3)=1$ $x(2,3)=2$ $x(3,3)=3$

$x(1,4)=1$ $x(2,4)=2$ $x(3,4)=3$

$x(1,5)=1$ $x(2,5)=2$ $x(3,5)=3$

$x(1,6)=1$ $x(2,6)=2$ $x(3,6)=3$

$x(4,1)=1$ $x(5,1)=1$ $x(6,1)=1$

$x(4,2)=2$ $x(5,2)=2$ $x(6,2)=2$

$x(4,3)=3$ $x(5,3)=3$ $x(6,3)=3$

$x(4,4)=4$ $x(5,4)=4$ $x(6,4)=4$

$x(4,5)=4$ $x(5,5)=5$ $x(6,5)=5$

$x(4,6)=4$ $x(5,6)=5$ $x(6,6)=6$

9) The probability distribution is

x	x_i	1	2	3	4	5	6
$p(x=x_i)$	$p(x_i)$	11/36	9/36	7/36	5/36	3/36	1/36

expected value $= 11/36 + 18/36 + 21/36 + 20/36 + 15/36 + 6/36$

$= 1/36(91) = 2.52$

variance $= \sigma^2 \int_{\infty} x^2 f(x)\,dx - \mu^2$

$$= \left[\int_{-\infty}^{\infty} x^2(0)\,dx + \int_0^{\infty} x^2(\lambda^2 x e^{-\lambda x})\,dx\right] - \left(\frac{2}{\lambda}\right)^2$$

$$\left[0 + \lambda^2 \int_0^{\infty} x^3 e^{-\lambda x}\,dx\right] - \frac{4}{\lambda^2}$$

$$= \lambda^2 \left[\frac{x^3 e^{-\lambda x}}{(-\lambda)} - 3x^2 \frac{e^{-\lambda x}}{(-\lambda)^2} + \frac{6x e^{-\lambda x}}{(-\lambda)^3} \frac{6x^{-\lambda x}}{(-\lambda)^4}\right]_0^{\infty} - \frac{4}{\lambda^2}$$

$$= \lambda^2 \left[\left(-\frac{x^3}{\lambda} - \frac{3x^2}{\lambda^2} - \frac{6x}{\lambda^3} - \frac{6}{\lambda^4} \right) e^{-\lambda x} \right]_0^\infty - \frac{4}{\lambda^2}$$

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$$= \lambda^2 \left[0 - \left(0 - 0 - 0 - \frac{6}{\lambda^4} \right)(1) - \frac{4}{\lambda^2} \right.$$

$$= \lambda^2 \left(\frac{6}{\lambda^4} \right) - \frac{4}{\lambda^2}$$

$$= \frac{6}{\lambda^2} - \frac{4}{\lambda^2} = \frac{2}{\lambda^2}$$

$$\therefore \text{variance} = \frac{2}{\lambda^2}$$

3(a)) A random variable x has following probability distribution

x	0	1	2	3	4	5	6	7	8
$p(x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) determine the value of a

ii) find $P(x<3)$, $P(0<x<5)$

sol) (i) W.K.T $\sum_{i=1}^{81} P_i = 1$

$$\Rightarrow a+3a+5a+7a+9a+11a+13a+15a+17a = 1$$

i.e. $81a = 1 \Rightarrow \boxed{a = \frac{1}{81}}$

$\therefore$ The probability distribution is

x	0	1	2	3	4	5	6	7	8
$P(x)$	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{9}{81}$	$\frac{16}{81}$	$\frac{25}{81}$	$\frac{36}{81}$	$\frac{49}{81}$	$\frac{64}{81}$	$\frac{81}{81} = 1$

(ii)

$$P(x<3) = P(X=0) + P(X=1) + P(X=2)$$

$$= a + 3a + 5a + 9a = \frac{9}{81} = \frac{1}{9}$$

$P(x \geq 3) = 1 - P(x < 3) = 1 - \frac{1}{9} = \frac{8}{9}$

$P(0 < x < 5) = P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 3a + 5a + 7a + 9a = 24a = \frac{24}{81} = \frac{8}{27}$

ii) The distribution function $F(x)$ of the ~~distance~~ discrete variate x is defined by $F(x) = P(X \leq x)$

$= \sum_{i=1}^{x} P(x_i)$ where x is any integers.

(4a) A Random variable x has the following probability of

X	0	1	2	3	4	5	6	7
$f(x)$	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(15)

i) Determine K

ii) Evaluate $P(x<6)$, $P(x\geq 6)$, $P(0<x<5)$ & $P(0\leq x\leq 4)$

iii) If $P(x\leq K)>\frac{b}{2}$ find the mean value of K

iv) Determine distribution function of X

v) Mean

vi) Variance.

Sol's we know

$$\sum_{i=0}^{n} p_i = 1$$

$$\Rightarrow 0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$

$$\Rightarrow 10K^2+9K-1=0$$

$$\Rightarrow 10K^2+10K-K-1=0$$

$$\Rightarrow 10K(K+1)-1(K+1)=0$$

$$\Rightarrow (K+1)(10K-1)=0$$

$$\Rightarrow K=-1, \frac{1}{10}$$

$$\boxed{\therefore K=\frac{1}{10}}$$

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i) The probability distribution is

x	0	1	2	3	4	5	6	7
(x)	0	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{1}{100}$	$\frac{2}{100}$	$\frac{17}{100}$

$p(x<6) = 1-p(x\geq 6)$ [$\because p(x<a) = 1-p(x\geq a)$]

$= 1-[p(x=6)+p(x=7)]$

$= 1-\left[\frac{2}{100}+\frac{17}{100}\right] = 1-\frac{19}{100} = 0.8$

$p(x\geq 6) = 1-p(x<6)$. [$\because p(x\geq a) = 1-p(x<a)$]

$= 1-p(x=0)+p(x=1)+p(x=2)+p(x=3)+p(x=4)+$
$p(x=5)$

$= 1-(0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}+\frac{1}{100}) \Rightarrow 1-(\frac{8}{10}+\frac{1}{100})$

$= 1-0.81 = 0.19$

$p(0<x<5) = 1-p(x=x)+p(x=2)+p(x=3)+p_1(x=4)$

$= (\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}) = 1-\frac{1}{10} \Rightarrow \frac{10-8}{10} = \frac{2}{10}$

$p(0<x<5) = p(x=1)+p(x=2)+p(x=3)+p(x=4)$

$= \frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10} \Rightarrow \frac{8}{10} \Rightarrow \frac{\cancel{0.8}\,8}{10} \Rightarrow 0.8$

$p(0\leq x\leq 4) = p(x=0)+p(x=x)+p(x=2)+p(x=3)+p(x=4)$

$= 0+\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10} \Rightarrow \frac{\cancel{0}8}{10} \Rightarrow 0.8$

ii) $p(x\leq x)>\frac{1}{2}$ find minimum value of x

$p(x\leq 6) = 1-p(x>6) \Rightarrow 1-\frac{17}{100} = 0.83 > 0.5$

$p(x\leq 5) = 1-p(x>5)$

$= 1-[p(x=5)+p(x=7)]$

$= 1-\left[\frac{2}{100}+\frac{17}{100}\right] = 1-\frac{19}{100} - 0.87 > 0.5$

$p(x \le 4) = 1 - p(x > 4)$

$= 1 - [p(x=5) + p(x=6) + p(x=7))$ (17)

$= 1 - \left[\frac{1}{100} + \frac{2}{100} + \frac{17}{100}\right] = 1 - \frac{26}{100} = 1 - 0.2 \Rightarrow 0.8 > 0.5$

$p(x \le 3) = p(x=0) + p(x=1) + p(x=2) + p(x=3)$

$= 0 + \frac{1}{10} + \frac{2}{10} + \frac{2}{10} \Rightarrow \frac{5}{10} \Rightarrow 0.5$

(v) Mean:

$\text{Mean}(\mu) = \sum_{i=0}^{*} x_i P_i$

$= 0(0) + 1\left(\frac{1}{10}\right) + 2\left(\frac{2}{10}\right) + 3\left(\frac{2}{10}\right) + 4\left(\frac{3}{10}\right) + 5\left(\frac{1}{100}\right) + 6\left(\frac{2}{100}\right)$
$7\left(\frac{17}{100}\right)$

$= \frac{1}{10} + \frac{4}{10} + \frac{6}{10} + \frac{12}{10} + \frac{5}{100} + \frac{12}{100} + \frac{11.9}{100} \Rightarrow \frac{23}{10} + \frac{136}{100}$

$= 2.3 + 1.36 \Rightarrow 3.66 \; [\because \mu = 3.68]$

(vi) Varience

$Var(x) = \sum_{i=0}^{7} x_i^2 P_i = \mu^2$

$= 0^2(0) + 1^2\left(\frac{1}{10}\right) + 2^2\left(\frac{2}{10}\right) + 3^2\left(\frac{2}{10}\right) + 4^2\left(\frac{3}{10}\right) + 5^2\left(\frac{1}{100}\right) + 6^2\left(\frac{2}{100}\right) + 7^2\left(\frac{17}{100}\right)$

$= \frac{1}{10} + \frac{8}{60} + \frac{18}{10} + \frac{48}{10} + \frac{25}{100} + \frac{72}{100} + \frac{833}{100} = 18.39$

$= \frac{75}{00} + 930 - 13.39 = 7.5 + 9.3 - 13.39 = 3.43$

4b(I)

A sample of "4 items" is selected of random from a box containing 12 items of which 5 are defective find the expected number of defective items

Solⁿ: given that

(18)

total no of items = 12

No of defective items = 5

No of non defective items = 7

let x denotes the no. of defective items then the possible value of x are 0, 1, 2, 3, 4

$p(x=0) = p(\text{non defective items}) = \frac{{}^7C_4}{{}^{12}C_4} = \frac{35}{495} = 0.07$

$p(x=1) = p(1 \text{ defective } \& \text{ } 3 \text{ non defective}) = \frac{{}^5C_1 \times {}^7C_3}{{}^{12}C_4} = \frac{175}{495}$

$= 0.35$

$p(x=2) = p(2 \text{ defective } \& \text{ } 2 \text{ non defective}) = \frac{{}^5C_2 \times {}^7C_4}{{}^{12}C_4} \Rightarrow \frac{210}{495}$

$= 0.42$

$p(x=3) = p(3 \text{ defective } \& \text{ } 1 \text{ non defective}) = \frac{{}^5C_3 \times {}^7C_2}{{}^{12}C_4} = \frac{70}{495}$

$= 0.14$

$P(X=4) = P(\text{all defective}) = \frac{5C_4}{12C_4}$ 19

$= \frac{5}{495} = \frac{1}{99}$

The probability Distribution is

$X=x_i$	0	1	2	3	4
$P(X=x_i)$	$7/99$	$35/99$	$42/99$	$14/99$	$1/99$

Expected number of defective items

$= E(x) = \sum x_i P_i$

$= 0 \cdot \frac{7}{99} + 1 \cdot \frac{35}{99} + \cdots + 4 \cdot \frac{1}{99} = \frac{165}{99}$

$= 1.66$

4b(II)

Find the mean & variable of the uniform probability Distribution given by

$f(x) = \frac{1}{n}$ for $x = 1, 2, \ldots, n$ (5M)

Sol: The probability Distribution is

x	1	2	3	n
$f(x)$	$1/n$	$1/n$	$1/n$	$1/n$

i) Mean $= \sum_{i=1}^{n} x_i f(x_i)$

$= 1 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n} + \cdots + n \cdot \frac{1}{n}$

$E(x) = \mu = \frac{1}{n}(1+2+\cdots+n) = \frac{n(n+1)}{n \cdot 2} = \frac{n+1}{2}$

ii) Variance $= \sum_{i=1}^{n} x_i^2 f(x_i) - \mu^2$

$$= 2\cdot\frac{1}{n} + 2^2\frac{1}{n} + 3^2\cdot\frac{1}{n} + \cdots + n^2\cdot\frac{1}{n} - \left(\frac{n+1}{2}\right)^2$$

20

$$= \frac{1}{n}\left[1^2+2^2+\cdots+n^2\right] - \frac{1}{4}(n+1)^2$$

$$= \frac{1}{n}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{1}{4}(n+1)^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4} = \frac{n+1}{2}\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)$$

$$= \frac{(n+1)(n-1)}{12} = \frac{n^2-1}{12}$$

4C(I).

If X is a continuous random variable and k is a constant. Then prove that

$Var(X+k) = Var(X)$

$Var(kX) = k^2 Var(\cancel{X})$ (5M)

We know that

$$Var(X) = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2 = \int_{-\infty}^{\infty} x^2 f(x)dx - \left[\int_{-\infty}^{\infty} x f(x)dx\right]^2$$

$$Var(X+k) = \int_{-\infty}^{\infty}(x+k)^2 f(x)dx - \left[\int_{-\infty}^{\infty}(x+k)f(x)dx\right]^2$$

$$= \int_{-\infty}^{\infty}(x^2+2kx+k^2)f(x)dx = \left[\int_{-\infty}^{\infty} x f(x)dx + k\int_{-\infty}^{\infty} f(x)dx\right]^2$$

$$\int_{-\infty}^{\infty} x^2 f(x)dx + 2k\int_{-\infty}^{\infty} x f(x)dx + k^2\int_{-\infty}^{\infty} f(x)dx$$

$$- \left[\int_{-\infty}^{\infty} x f(x)dx + k\int_{-\infty}^{\infty} f(x)dx\right]^2$$

$= E(X^2) + 2kE(X) + k^2 - [E(X)+k]^2$

$= E(X^2) - [E(X)]^2 = Var(X)$

$Var(kX)$

$= \int_{-\infty}^{\infty} k^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} kx f(x)\,dx\right]^2$

$= k^2 \int_{-\infty}^{\infty} x^2 f(x)\,dx - k^2 \left[\int_{-\infty}^{\infty} x f(x)\,dx\right]^2$

$= k^2 [E(X)^2] = E(X)^2] = k^2 Var(X)$

20) For the Continuous probability $f(x) = kx^2 e^{-x}$ if $x \geq 0$.

HC (II). find i, k ii, Mean iii, Variance

(22)

we take $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\Rightarrow \int_{-\infty}^{\infty} k x^2 e^{-x}\,dx = 1$$

$$\Rightarrow k\left[x^2(-e^{-x}) - 2x(e^{-x}) + 2(-e^{-x})\right]_0^{\infty} = 1$$

$$\Rightarrow k(0+2) = 1 \quad \Rightarrow \quad k = 1/2.$$

Thus $f(x) = 1/2\, x^2 e^{-x}$ if $x \geq 0$

ii, Mean $= \int_{-\infty}^{\infty} x f(x)\,dx = 1/2 \int_0^{\infty} x^3 e^{-x}\,dx.$

$$= \frac{1}{2}\left[-e^{-x}\left[x^3 + 3x^2 + 6x + 6\right]\right]_0^{\infty}$$

$$\Rightarrow 1/2\,(6) = 3.$$

iii, Variance

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

$$= \frac{1}{2}\int_0^{\infty} x^4 e^{-x} - 9$$

$$= \frac{1}{2}\left[-e^{-x}\left[x^4 - 14x^3 + 12x^2 + 24x + 24\right]\right]_0^{\infty} - 9$$

$$\Rightarrow \frac{1}{2}(24) - 9 = 3$$