UNIT I
Quantum Physics and Introduction to solids

1Q. Write any two properties of matter waves.

1M - BL - L1 - C112.1

ANS (a) If heavier is the particle, smaller is the wavelength i.e., $\lambda \propto \frac{1}{m}$

(b) If smaller is the velocity of the particle, greater is the wavelength i.e., $\lambda \propto \frac{1}{v}$

(c) If $v = 0$, $\lambda \rightarrow \infty$. This means if particle is at rest, wavelength can not be determined.

(d) If $v = \infty$, $\lambda = 0$. This implies that when velocity of the particle is infinity wavelength can not be determined.

(e) Velocity of matter waves is greater than the speed of light.

(f) Matter waves are not physical (mechanical or electromagnetic waves) waves.

(g) Matter waves are neither radiated nor absorbed.

(h) Matter waves require medium.

2 Q. What is dual nature of matter and radiation?

1M - BL - L2 - C112.2

ANS: Either matter or radiation behaves like particle or wave depending on the situa[tion]. This is known as dual nature of matter and radiation. This hypothesis is proposed De-Broglie.

But it should be noted that particle natu[re] or wave nature can not be exhibited by either matter or radiation simultaneously.

3 Q. Mention the types of symmetry in solids.

1M - BL - L2 - C112.2

ANS. The following are the types of symmetry in solids

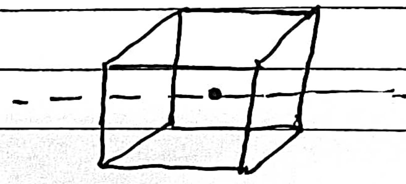
(a) Center of symmetry or line of symmetry

(b) Translational symmetry or plane of symmetry

(c) Rotational Symmetry or axis of symmetry.

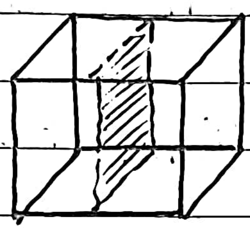
(a) Center of symmetry or point symmetry or line of symmetry

" If an imaginary line is drawn through a point inside the crystal such that it intersects the faces of the crystal at equal distances on either side of the point, then the crystal is said to have center of symmetry



(b) Translational symmetry or plane of symmetry.

"An imaginary plane passing through the crystal which cuts the crystal into two equal parts such that one part is the mirror image of the other part."



(c) Axis of symmetry or rotational symmetry.

"An imaginary line through which when a crystal is rotated, and it it appears more than once as it was in begining in a complete rotation (360°) then the crystal is said to have rotational or axis of symmetry"

4Q. Define photoelectric effect and write photoelectric equation (Einstein's photoelectric equation)

IM-BL-L1-C112.1

Photoelectric effect: The phenomenon of ejection of electrons from a metal surface when a suitable frequency inicidents up on it is called photoelectric effect.

The following mathematical expression is given by Einstein in order explain photoelectric emission phenomenon which is known as "Einsten's photo electric equation"

$$h\nu = \frac{1}{2} m v_{max}^2 + \phi$$

where $h\nu$ is the energy of incident

v_{max} is the maximum velocity of elec

ϕ – work function of the metal surfa

The above equation can also be writte

$$h\nu = KE_{max} + \phi.$$

5Q. **Write the physical significance of wave function.** 1M–BL–L1–C112.1

The wave function ψ has no direct physical meaning. As ψ is complex wave function $\psi\psi^* = |\psi|^2$ represents the probability of finding the particle in a given volume.

$$\Rightarrow \int_0^\infty |\psi|^2 dx\,dy\,dz = 1.$$

1Q. **Write the differences between matter waves and electromagnetic waves.**

3M–BL–L1–C112.1

matter waves	em waves
1. Matter waves are generated by both charged and un charged particles	1. EM waves are generated by only accelerating charged particles

2. Velocity of matter waves is not constant. It changes with the speed of particle	2. The velocity of EM is constant in a homogeneous medium.
3. Matter waves do not propagate in space. They require medium.	3. Electromagnetic waves propagate in space as well as in medium.
4. Matter waves are neither radiated nor absorbed in a medium	4. Electromagnetic waves are either radiated or absorbed by the matter.
5. $\lambda = h/p$	5. $\lambda = c/\nu$
6. Velocity of matter waves is greater than the speed of light.	6. Velocity of em wave is equal to speed of light in space or vacuum.

2 Q. <u>Write the laws of photoelectric effect.</u>

3 M – BL – L1 – C112.1

1. The velocity of emitted electrons is independent of the intensity of incident light and depends only on frequency of incident light no matter how high or low is the intensity

2. Photo electric current (or the number photo electrons emitted per second) is proportional to the intensity of incident light.

3. For a given metal, there exists a cert minimum frequency of incident light which the electron emission takes place, b which the electron emission completely s This frequency of incident light is called 'threshold frequency'.

4. Photoelectron emission is an instantaneous process. So there is no time lagging b/w the incidente of light and emission of elec

3Q. Compute the work function of sodium metal, if its threshold wavelength is 5040 A°.

3 M-BL-L2-C112.2

Sol: given data

$$\lambda = 5040 A^\circ = 5040 \times 10^{-10} m$$

$$h = 6.625 \times 10^{-34} Js$$

$$c = 3 \times 10^{8} m/s$$

work function

$$\phi = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{5040 \times 10^{-10}} \text{ Joules}$$

$$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{5040 \times 10^{-10} \times 1.6 \times 10^{-19}} eV$$

$$\approx 2.46 eV.$$

Write the main postulates of Sommerfeld free electron theory of metals.

3 M — BL - L1 — C112.1

Sommerfeld free e^- theory also known as quantum free $\bar{e}$ theory. To solve the difficulties faced by the classical free $\bar{e}$ theory which was proposed by Drude-Lorentz, in 1928 Sommerfeld developed quantum free e^- theory. In developing the postulates he has used the concepts of quantum mechanics and Fermi-Dirac distribution statistics.

The following are the assumptions (postulates) of Sommefeld quantum free $\bar{e}$ theory:

1) The free $\bar{e}^s$ in a metal can have only descrete (quantized) energy values. Electrons are considered to be moving in a 3-D potential well.
2) The electrons (free) obey Pauli's exclusion principle. According to this, there can not be more than 'two electrons' in any energy level. In otherwords, no two electrons have the same energy state as they have different quantum numbers.
3) The potential due to positive immobile ion cores is uniform throughout the crystal lattice which will be considered as zero for solving eigen values.
4) The force of attraction between $\bar{e}^s$ and lattice ions, the force of repulsion between the electrons is negligible.

5) The distribution of velocities, K.E among the free electrons obey Fermi-Dirac statistics. The probability distribution function in FD statistics is given by

$$f(E) = 1 \Big/ e^{h\nu/K_BT} + 1$$

5Q. Explain Heisenberg's Uncertainity Principle

3M - BL - L2 - C112.2

ANS: Matter waves are not physical waves. Matter waves are represented by a group of waves called "wave packet". If the width of wave packet is smaller, number of wave pulses in the wave packet is less. Therefore wave length can not be determined precisely. Similarly if the width of wave packet is larger wavelength can be determined, but the location of particle can't be determined accurately. On the otherhand radiation (light) showed both particle property and wave nature.

Based on the above interpretations, Heisenberg formulated a principle known as Heisenberg's Uncertainity principle.

According to this principle,

"It is impossible to determine the position and momentum of a particle simultaneously"

If Δx represents the uncertainity in the position of a particle and ΔP represents the uncertainity in the momentum, then mathematically uncertainity principle is given as

$$\Delta x . \Delta p \geq \frac{h}{4\pi}$$

where 'h' is plank's constant

$$h = 6.624 \times 10^{-34} Js$$

Δx and Δp are called conjugate physical parameters.

Similary, if ΔE is the uncertainity in the measurement of energy and Δt is the uncertainity in the measurement of time, then

$$\Delta E . \Delta t \geq \frac{h}{4\pi}$$

And if ΔJ represents the uncertainity in angularmomentum and $\Delta \theta$ represents uncertainity in the angular displacement, then

$$\Delta J . \Delta \theta \geq \frac{h}{4\pi}$$

The significance of HUP is:

(a) It helps us to understand why electrons can not exist inside the nucleus.

(b) It helps us to calculate greatest velocity and largest frequency of radiation.

(c) It helps us to understand dual nature.

(d) It helps us to calculate the possible energies of α, β, γ radiations.

1Q Derive Schrödinger time independent wav equation

5M - BL - L3 - C112.3

ANS Schrödinger wave equation is a fundemen equation in quantum mechanics which describes the dynamical behaviour of micr scopic particles such as e^-s, protons ---etc using boundary conditions like Newton's laws of motion used for macroscopic particle

Consider a second order partial differential equation for the motion of plane wave along x-axis as is given by

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \quad \text{——} \; (1)$$

where 'y' is the displacement and 'v' is veloci

If 'ψ' is the displacement (amplitude) of the matter waves, then eq n (1) becomes

$$\frac{d^2\psi}{dt^2} = v^2 \frac{d^2\psi}{dx^2} \quad \text{——} \; (2)$$

The solution of eq (2) is given by

$$\psi = \psi_0 \, Sin(\omega t - kx) \quad \text{——} \; (3)$$

ψ_0 - maximum amplitude of matter waves

and $\omega = 2\pi\nu$, $K = \frac{2\pi}{\lambda}$

ν - frequency of matter waves

differentiating eq ③ twice w.r.t time we get

$$\frac{d^2\psi}{dt^2} = -\psi 4\pi^2\nu^2$$

but $\nu = \frac{v}{\lambda}$

$$\Rightarrow \frac{d^2\psi}{dt^2} = -\psi 4\pi^2 \frac{v^2}{\lambda^2} \quad \text{④}$$

from eq ② and ④

$$\cancel{v^2}\frac{d^2\psi}{dx^2} = -\psi \frac{4\pi^2}{\lambda^2}\cancel{v^2}$$

$$\Rightarrow \frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Now let us use $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\Rightarrow \lambda^2 = \frac{h^2}{m^2v^2}$$

using the above,

$$\frac{d^2\psi}{dx^2} + 4\pi^2\frac{m^2v^2}{h^2}\psi = 0 \quad \text{⑤}$$

Now for an e^- moving through a potential difference of V volts, the total energy E is given by

$$E = P.E + K.E$$

$$\Rightarrow E = V + \frac{1}{2}mv^2$$

$$E - V = \frac{1}{2}mv^2$$

$$2(E-V) = mv^2$$

(or $m^2v^2 = 2m(E-V)$ — (6)

Substituting (6) in (5) we get

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2}{h^2}2m(E-V)\psi = 0$$

(or $\frac{d^2\psi}{dx^2} + \frac{8m\pi^2}{h^2}(E-V)\psi = 0$

by letting $\hbar = h/2\pi$, we get

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{— (7)}$$

The above equation is known as time independent SWE in 1-D.

If $\bar{e}$ assumed to be moving through a constant potential field, then V can be taken as zero, i.e., $V=0$. In this case $\bar{e}$ is said to have only KE and hence it is a free particle. The eq (7) becomes

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}E\psi = 0 \quad \text{— (8)}$$

In 3-D,

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{2m}{\hbar^2}E\psi=0$$

$$\Rightarrow \nabla^2\psi+\frac{2m}{\hbar}E\psi=0$$

where $\nabla^2\psi=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}$

and $\nabla=\partial/\partial x+\partial/\partial y+\frac{\partial}{\partial z}$ called

Laplacian operator.

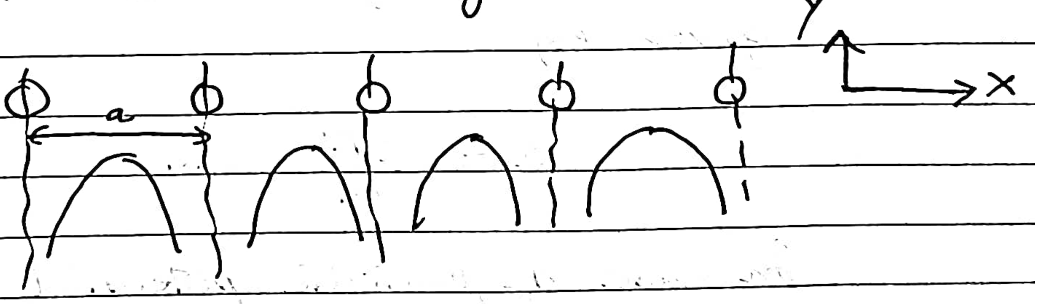
2Q. Define periodic potential and write schrödinger wave equation for a periodic potential function using bloch theorem. Derive an expression for effective mass of an electron.

5M-BL-L1-C112.1

ANS: A potential field in a crystal lattice defined by the following function is called period potential

$$V(x)=V(x+na)$$

where 'a' is the periodicity of the lattice and n is an integer



For a periodic potential the SWE modifies as given below

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}\left(E - V(x)\right)\psi = 0$$

where $V(x) = V(x + na)$

The solution above equation is of the form

$$\psi(x) = \psi_0 e^{\pm ikx} u_k(x)$$

where $u_k(x)$ is periodic function such that $u_k(x) = u_k(x+a)$ and where $u_k(x)$ is called bloch function.

finally the solution becomes

$$\psi(x+a) = \psi_0 e^{\pm ik(x+a)} u_k(x+$$

Effective mass of an e^- :

"The mass of an e^- in a periodic potential due to an applied external field is called effective mass"

If E is an electric field applied, the acceleration of an e^- in the field is given by

$$a = \frac{eE}{m} \quad \text{—①}$$

In case of periodic potential field we can write eq ① as

$$a = \frac{eE}{m^*} \quad \text{—— (2)}$$

When $\bar{e}$ moves through a periodic potential it moves with matter waves. Hence free e^- associates with wave packet. Then

$$V_g = \frac{d\omega}{dk} \quad \text{—— (3)}$$ where V_g is the group velocity.

but $\omega = 2\pi\nu$

$d\omega = 2\pi\, d\nu$

$$\Rightarrow V_g = \frac{2\pi\, d\nu}{dk}$$

Again $\quad \varepsilon = h\nu$

$$\Rightarrow \quad d\varepsilon/dk = h \frac{d\nu}{dk}$$

$$\Rightarrow \quad V_g = \frac{2\pi}{h} \frac{d\varepsilon}{dk}$$

$$dV_g/dt = \frac{2\pi}{h} \frac{d^2\varepsilon}{dk\, dt}$$

$$\Rightarrow dV_g/dt = \frac{2\pi}{h} \left(\frac{d^2\varepsilon}{dk^2}\right) \frac{dk}{dt} \quad \text{—— (4)}$$

Now $\quad p = \hbar k$

$$dp/dt = \hbar \frac{dk}{dt}$$

but $dp/dt = F$ Newton's second law

$$\Rightarrow \quad dk/dt = \frac{1}{\hbar} F \quad \text{—— (5)}$$

from (4) and (5) $\quad dV_g/dt = \frac{2\pi}{h} \times \frac{1}{\hbar} F \left(\frac{d^2\varepsilon}{dk^2}\right)$

$$\Rightarrow \frac{dv_g}{dt} = \frac{1}{\hbar^2} \frac{d^2\varepsilon}{dk^2} (F)$$

but $\frac{dv_g}{dt} = a$ and $F = eE$

$$\Rightarrow a = \frac{1}{\hbar^2}\left(\frac{d^2\varepsilon}{dk^2}\right) eE \quad — ⑥$$

from eq ② and ⑥

$$\boxed{m^* = \frac{\hbar^2}{(d^2\varepsilon/dk^2)}} \quad — ⑦$$

3Q. Derive an expression for De-Broglie wavelength

5M − BL − L3 − C112.3

ANS: According to De-Broglie, material particles are associated with matter waves when they are in motion. The wavelength of matter waves is given by

$$\lambda = \frac{h}{p}$$

h − planks constant

p − linear momentum.

Derivation: (a) According to Eienstein's mass energy relation

$$E = mc^2 \quad — ①$$ c − speed of light

(b) According to plank's quantum hypothesis

$$E = h\nu$$

but $c = \nu\lambda$

$$\Rightarrow \nu = \frac{c}{\lambda}$$

$$\Rightarrow E = h\frac{c}{\lambda} \quad \text{--- (2)}$$

from (1) and (2)

$$mc^2 = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{h}{mc}$$

If 'v' is the velocity of the particle then

$$\lambda = \frac{h}{mv}$$

$$\therefore \lambda = \frac{h}{p}$$

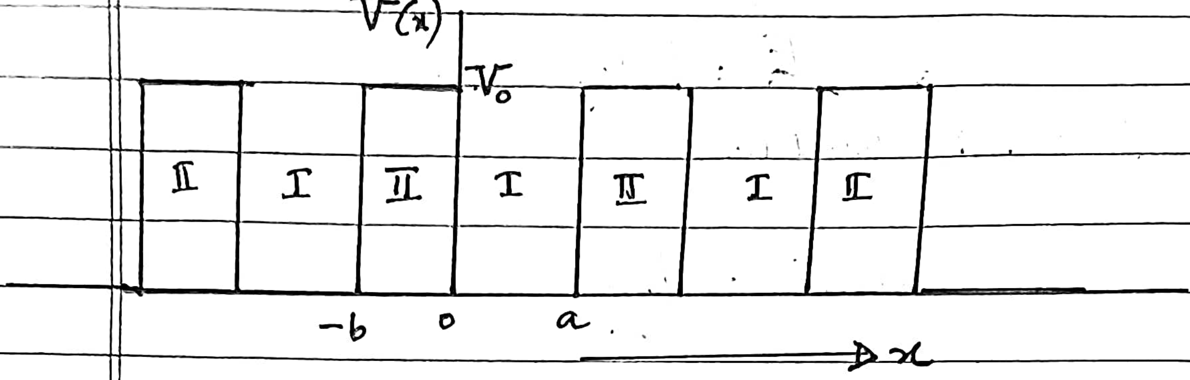
Q. Explain Kronig-Penney model for the motion of electron in a crystal lattice. Also discuss the feautures of this model.

5M - BL - L2 - CU2.2

To determine allowed energy of electron in a periodic crystal lattice, Kronig and Penney suggested a simplified model in which periodic potential consists of alternate potential wells and potential barriers in rectangular form as shown in Figure.

The periodicity of the lattice is considered as "a+b"

Here 'a' is the width of the potential well and 'b' is the width of the barrier.



(i) for $0 < x < a$ $\quad V(x) = 0$ Region
(ii) for $-b < x < 0$ $\quad V(x) = V_0$ Region
Applying SWES in region I and II,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} E\psi = 0 \qquad (0 < x < a$$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2} [E - V_0]\psi = 0 \qquad (-b < x <$$

Let $\frac{2m}{\hbar^2} E = \alpha^2$ and $\frac{2m}{\hbar^2}(V_0 - E) = \beta^2$

The above equation can be written as

$$\frac{d^2\psi}{dx^2} + \alpha^2\psi = 0 \quad \text{--- ①} \quad (0 < x < a)$$

$$\frac{d^2\psi}{dx^2} - \beta^2\psi = 0 \quad \text{--- ②} \quad (-b < x < a)$$

From Bloch theorem, the solutions of above equations become

$$\psi(x) = \psi_0 e^{\pm ikx} u_k(x)$$

where $u_k(x) = u_k(x+a)$

Solving equations ① and ② by applying boundary conditions, we get a final expression of the following form

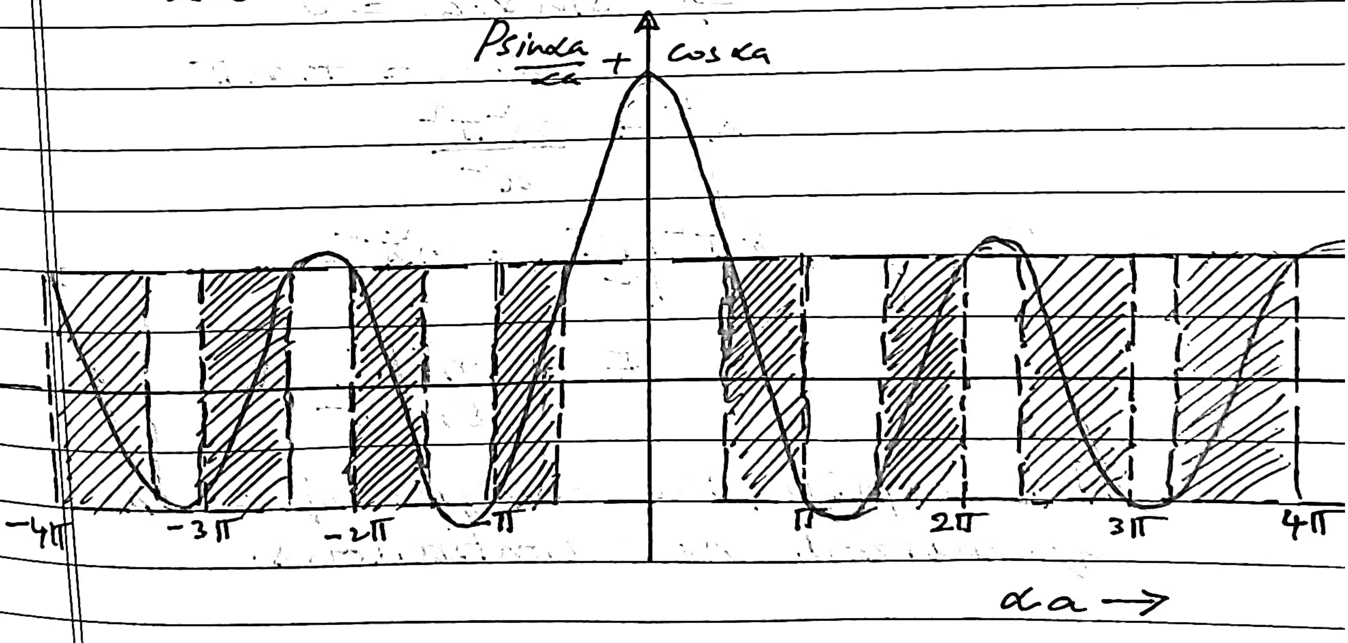
$$\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \quad \text{--- ③}$$

where 'P' is called barrier strength and is given by

$$P = \frac{m V_0 a b}{\hbar^2} \quad \text{and} \quad \alpha = \sqrt{\frac{8\pi^2 m E}{h^2}}$$

The RHS of equation ③ varies between +1 and −1. LHS of equation ③ depends on barrier strength or scattering power.

The following graph is plotted between $\frac{P \sin \alpha a}{\alpha a} + \cos \alpha a$ and αa for $P = \frac{3}{2}\pi$



Inferences (Features)

1) The energy spectrum of e^- in a per potential consists of alternated allowe energy regions called 'allowed bands' (shaded regions) and forbidden energy regions called 'forbidden bands' (unsha regions)

2) As αa increases, the width of allowe bands increases and width of forbidde bands decreases.

3) If P is large, width of allowed band decreases and width of forbidden increases.

4) If $P \to \infty$, $\frac{\sin \alpha a}{\alpha a} = 0$

$$\Rightarrow \sin \alpha a = 0$$

$$\alpha a = \pm n\pi$$

$$\alpha^2 = \frac{n^2\pi^2}{a^2}$$

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$$

$$\Rightarrow E = \frac{n^2 h^2}{8ma^2}$$

This implies, the energies of e^- are quantized not continuous.

for $n = 1$, $E_n = E_1$
for $n = 2$, $E_n = E_2$ ...
so $E_2 = 4E_1$, $E_3 = 9E_1$, $E_4 = 16E_1$ ...

5) If $P = 0$ $\quad \cos \alpha a = \cos ka$

$$\alpha a = ka$$

$$\Rightarrow \alpha = k$$

(or) $\alpha^2 = k^2$

$$\frac{2m}{\hbar^2} E = k^2$$

$$E = \frac{\hbar^2 k^2}{2m}$$

but $p = \hbar k \Rightarrow E = \frac{p^2}{2m}$

(or) $E = \frac{1}{2} m v^2$

Hence in this situation, electron behaves as free particle. This mean binding energy of e^- tends to zero.

Q Draw E-k diagram for an electron in a periodic potential lattice. From the graph explain Brillouin zones.

5M - BL - L2 - C112.2

ANS Brillouin zones are the boundaries that are marked by the values of propagation vector 'k' in which the electron can have allowed values without diffraction. Since 'k' is a vector it has different values in different directions.

The relation between 'k' and 'E' of an el in a periodic potential is given by

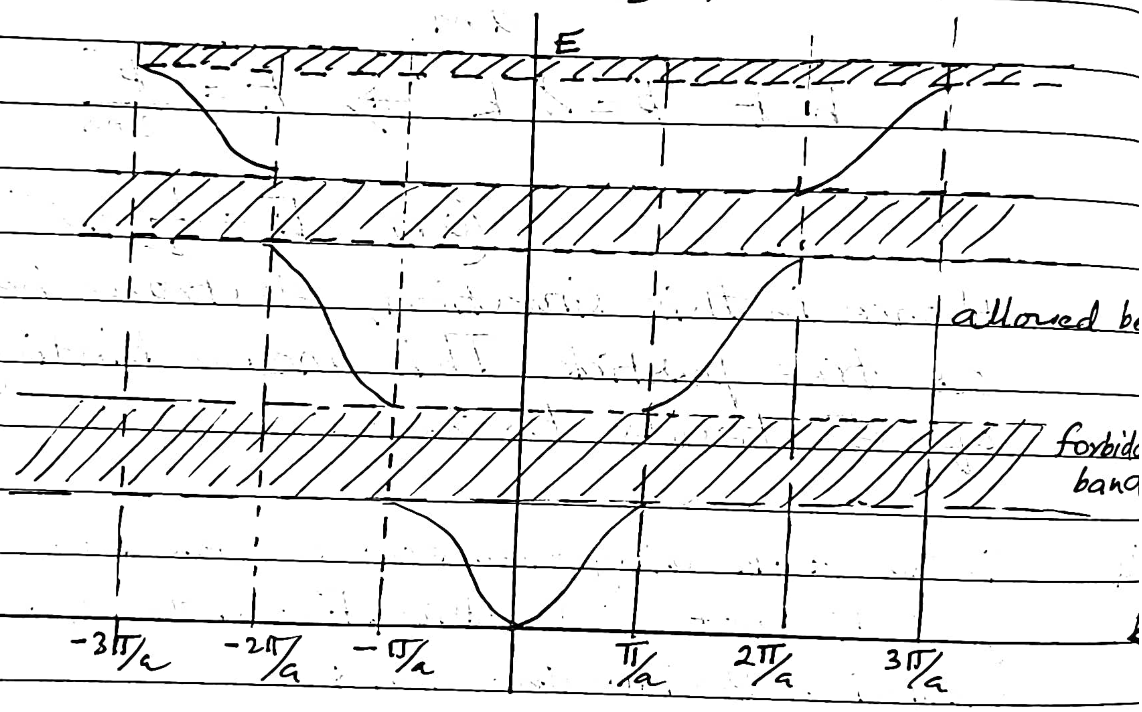
$$E = \frac{n^2 h^2}{8ma^2} \quad \text{—— (1)}$$

but $k = \pm \frac{n\pi}{a}$ (since $\alpha = k$)

—— (2)

from (1) & (2) $E = \frac{k^2 h^2}{8m\pi^2}$ —— (3)

A plot is made between the total energy E and the wave vector 'k' for various valu of k with $n = \pm 1, \pm 2, \ldots$



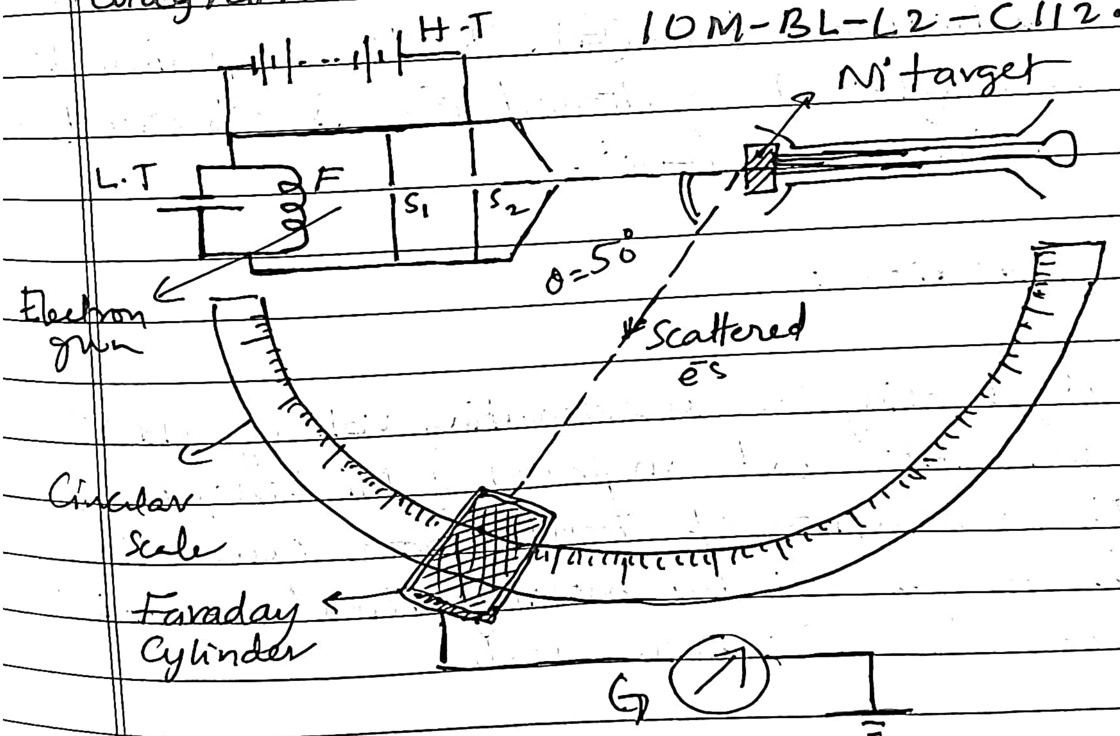
For $k = \pm n\pi/a$, the curve is a parabola with discontinuties.

It is clear that the graph shows a discontinuity in the electron energy from $k=0$ to $\pm \pi/a$ and scatters.

The range of allowed energy values between π/a to $-\pi/a$ is called first Brillouin zones.
The range of allowed energy values from π/a to $2\pi/a$ and then from $-\pi/a$ to $-2\pi/a$ is called second Brillouin zone and so on..
This implies electron can go from B.Z to another B-Z when it is supplied with energy equal to forbidden gap.

Q Describe the construction, working of Davisson-Germer Experiment with the help of neat diagram.

IOM-BL-L2-C112.2



The figure shows the construction of Davisson-Germer Experiment for the proof of existence of matter waves.

In the figure, a filament is heated up by a Low tension battery to emit e̅s by thermoionic emission. A high tension battery is connected across the evaculated glass tube to accelerate the ejected e̅s. Electrons travel through slits S_1 and S_2 in the form of fine pencil beam and incidence on the 'Ni' target. The angle(θ) of scattered e̅s is measured with the help of circular scale. The intensity of scattered e̅s is measured by means ionization current caused in the Faraday cylinder (FC) which can move on the circular scale and connected to a Galvanometer (G). In the experiment the angle of scattered e̅s is observed to be between 29°–80°.

Observation:

1) By keeping the accelerating voltage (H.T) constant and angle (θ) of Ni target fixed the variation of ionization current is measured by moving Faraday cylinder over the circular scale.

1) Experiment is repeated by changing the value of accelerating potential and angle of Ni targe(θ), and current is noted.

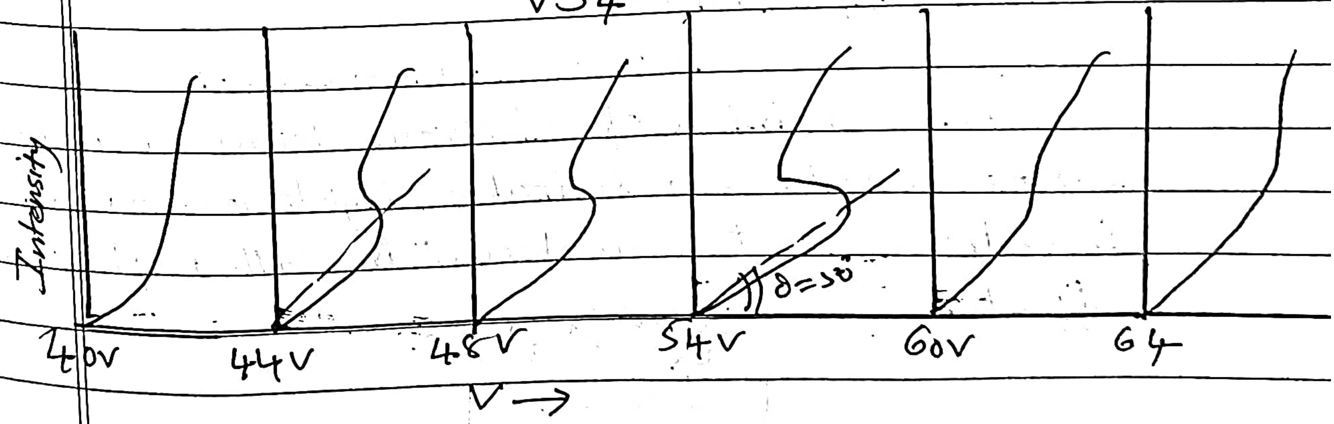
3) It is observed that at θ = 50° and at 44 V a bump begins to start and the bump becomes larger at 54 V indicating the maximum intensity of scattered electrons.

4) After 54 V further increase in applied voltage the bump gradually disappeared.

5) The above observations showed that the variation of intensity of scattered e⁻s is in accordance with the diffraction pattern.

6) The wavelength of e⁻ beam is calculated as below

$$\lambda = \frac{12.26}{\sqrt{54}} \approx 1.67\,\mathring{A}$$



Further the electron wave nature is confirmed by Bragg's condition

$$2d \sin\theta' = n\lambda$$

for $n = 1$

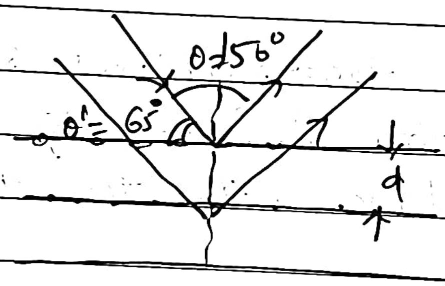
$d = 0.91\ A^{\circ}$ (Ni target)

$\theta' = 65$

$\lambda \approx 1.65\ A^{\circ}$

where 'd' is interplanar spacing

θ' - glancing angle

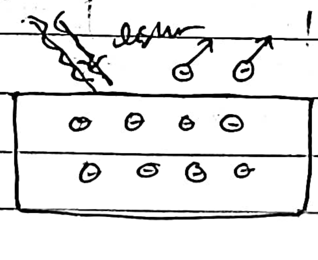


Ni crystal plane

2Q. Define photoelectric effect. Explain the experimental observations in photoelectric effect experiment.

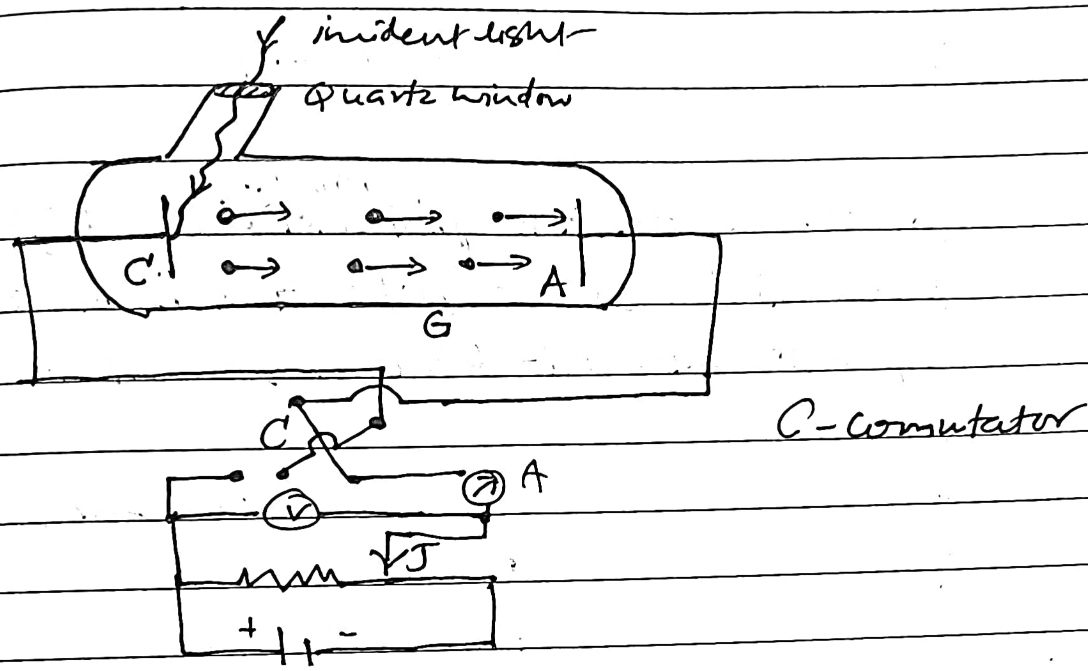
10M - BL - L1 - C112.1

"The emission of electrons from a metal surface when a suitable light or radiation incidence on the surface" is called photoelectric effect.



metal

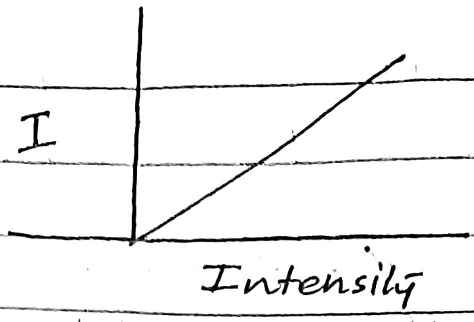
The figure shows the experimental arrangement for the demonstration of photoelectric effect.



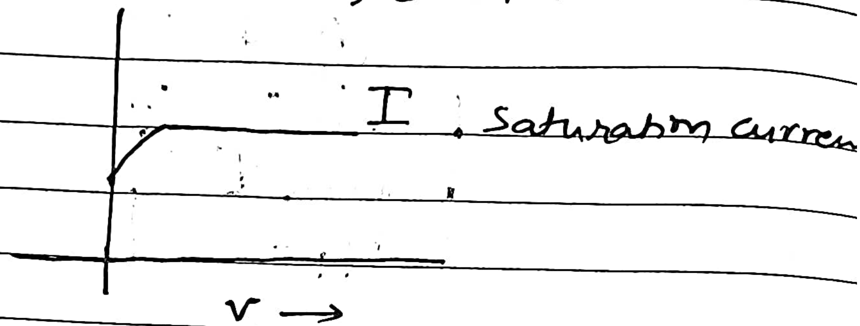
1) The cathode C is photosensitive material and A is metallic anode. G is an evacuated glass tube.
2) When voltage is applied b/w Cathode and anode ejected e^-s reach anode.
3) The function of commutator is to reverse the direction of applied voltage when required.
4) Light or radiation is allowed to fall on cathode through a Quartz window.

Observations:

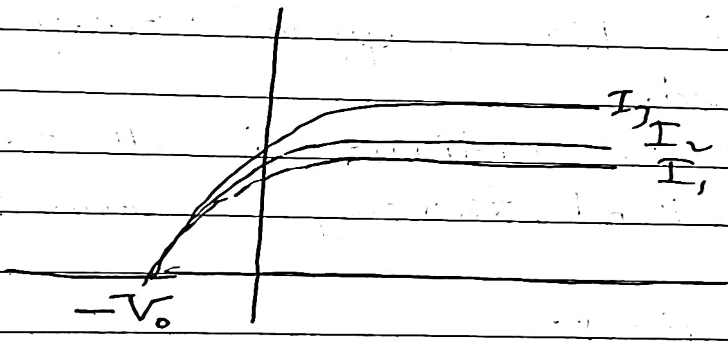
1) When intensity of incident light is increased the photocurrent also increased.



2) At a constant intensity of light, when applied anode potential is increased the photo current becomes constant.



3) When Anode potential is reversed w.r.t cathode, the photocurrent decreases and reaches a zero value at a particular reverse anode potential. This potential at which the photocurrent becomes zero is called stopping potentia

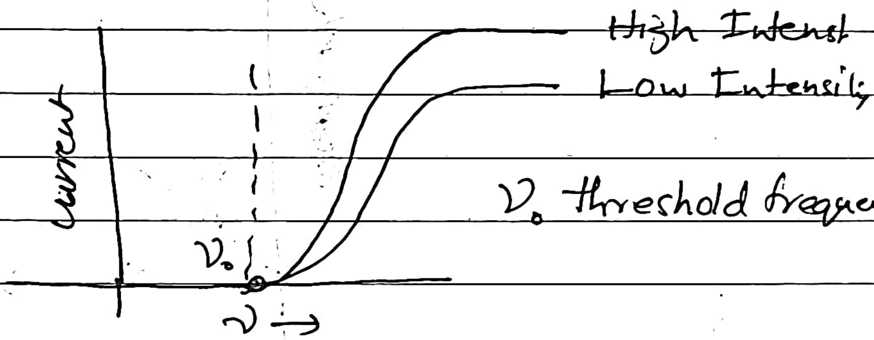


Stopping potential is independent of intensity of incident radiation.

At stopping potential the K.E of electrons is maximum which is equal to work done.

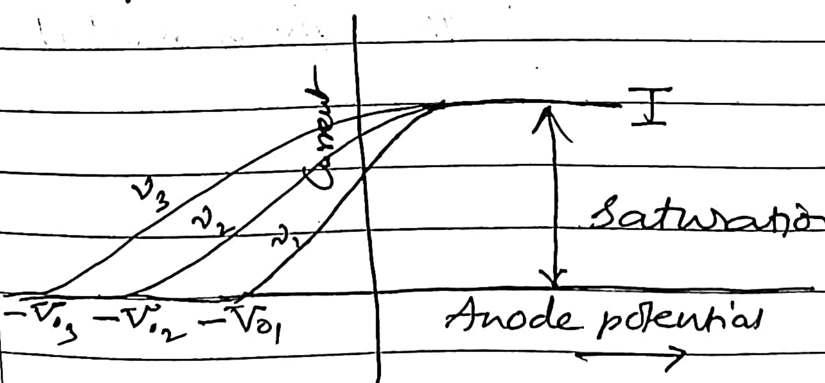
$$\Rightarrow \frac{1}{2} m V_{max}^2 = K.E_{max} = eV_0$$

When intensity of radiation and anode potential kept constant, by changing frequency of incident radiation photo current is measured. The following graph shows that the photo current is zero until a minimum frequency is reached above which the photocurrent increases and saturates.



The minimum frequency of incident radiation at which the photo electron emission takes place is called threshold frequency (ν_0).

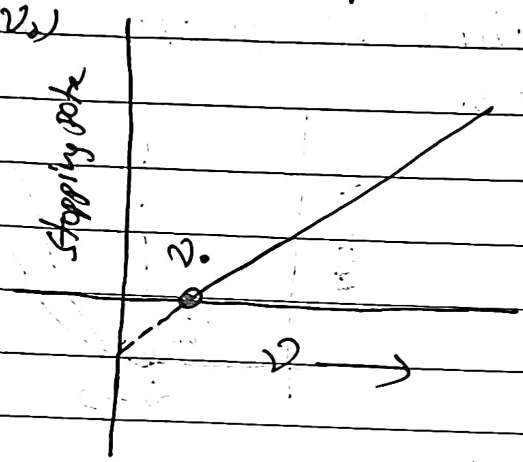
6) By keeping intensity constant, frequency versus stopping potential are measured with photocurrent.



It is clear from the graph that as the frequency of incident radiation increa[ses] stopping potential increases.

This means the K.E of electrons is a function of incident radiation frequ[ency]

7) When frequency vs stopping potential is plotted, a straight line intersect the x-axis which represents threshold frequency (ν_0)



From the above experimental observations Einstein derived photoelectric equation

$$h\nu = \frac{1}{2} m v_{max}^2 + \phi$$

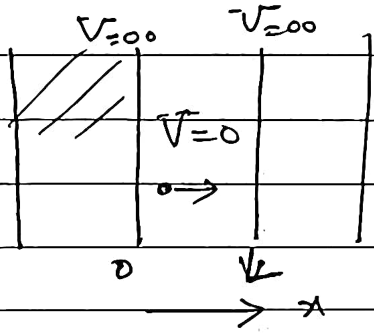
where ϕ is work function of the given metal.

Discuss eigen values and eigen functions for an electron moving in an infinite square well potential box.

10M - BL - L2 - CU2.2

Consider an $\bar{e}$ moving along x-axis in an infinite square well potential as shown in figue. When electron reaches the wall it experience an abrupt force.

L — length of the potential box.



Boundary conditions

$V(x) = 0 \qquad 0 < x < L$

$V(x) = \infty \qquad x \leq 0,\ x \geq L$

Applying SWE

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad 0 < x < L \quad —①$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \qquad \text{where } k = \frac{2mE}{\hbar^2}$$

—②

The solution of the above equation is given as

$$\psi(x) = A\sin kx + B\cos kx \quad —③$$

(a) At $x = 0$ $\psi(x) = 0$

At $x = L$ $\psi(x) = 0$

This gives $B = 0$

$\Rightarrow A\sin(KL) = 0$

(or) $KL = n\pi$

$$k = \frac{n\pi}{L} \quad (n = 1, 2, 3 \ldots)$$

$$\Rightarrow \frac{2mE}{\hbar^2} = \left(\frac{n\pi}{L}\right)^2$$

$$\text{(or)} \quad E = \frac{n^2 h^2}{8mL^2}$$

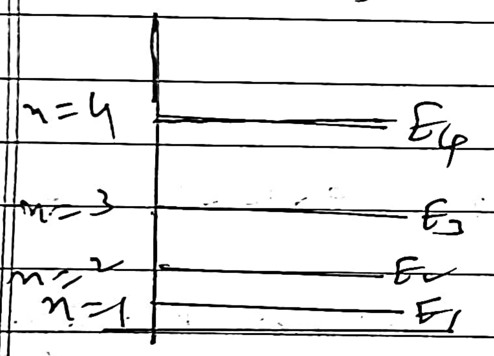
In general $E_n = \frac{n^2 h^2}{8mL^2}$

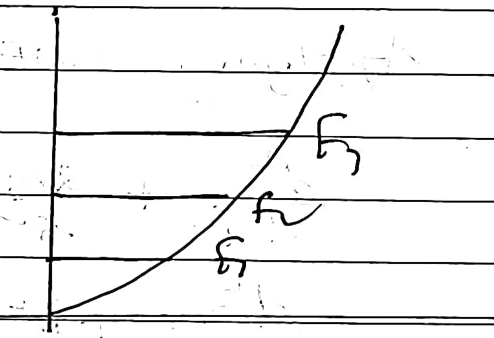
for $n = 1, \quad E_1 = \frac{h^2}{8mL^2}$

$n = 2 \quad E_2 = 4 \frac{h^2}{8mL^2}$

$\Rightarrow E_2 = 4E_1$

Ily $E_3 = 9E_1$ ---etc





Determination of constant A:

When $B = 0$

$$\psi(x) = A \sin\left(\frac{n\pi x}{L}\right)$$

The constant 'A' is determined by using normalised condition.

$$\int_0^L |\psi(x)|^2 \, dx = 1$$

$$\Rightarrow \int_0^L \left\{A \sin\left(\frac{n\pi x}{L}\right)\right\}^2 dx = 1$$

$$\Rightarrow A^2 \int_0^L \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$A^2 \int_0^L \left(\frac{1 - \cos\left(\frac{2n\pi x}{L}\right)}{2}\right) dx = 1$$

$$\Rightarrow \frac{A^2}{2} \left| x - \frac{L}{2\pi n} \sin \frac{2n\pi x}{L} \right|_0^L = 1$$

The second term in the bracket is zero at $x = 0$ and $x = L$

$$\Rightarrow \frac{A^2}{2} L = 1$$

$$(\text{or}) \quad A = \sqrt{2/L}$$

$$\Rightarrow \psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

The above equation represents the eigen (wave) function for an e' in the potential box.

Evaluation of eigen functions:

Case I: $n = 1 \quad \psi_1 = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L}$

at $x=0, x=L$ $\psi_1 = 0$
at $x = \frac{L}{2}$ ψ_1 is m

at $x=0, x=L$ $\psi_1^2 = 0$
at $x=0, x=L$ ψ_1^2 is ma

Case 2: $n=2$, $\psi_2 = \sqrt{\frac{2}{L}} \sin \frac{2\pi x}{L}$

at $x=0, x=L/2, x=L$ $\psi_2 = 0$

at $x = \frac{L}{4}, x = \frac{3L}{4}$ ψ_2 is m

and at $x=0, x=\frac{L}{2}, x=L$ $\psi_2^2 = 0$

at $x = \frac{L}{4}, x = \frac{3L}{4}$ ψ_2^2 is max

Case 3: $n=3$ $\psi_3 = \sqrt{\frac{2}{L}} \sin \frac{3\pi x}{L}$

at $x=0, x=L/3, x=\frac{3L}{4}, x=L$
$\psi_3 = 0$
$\psi_3^2 = 0$

at $x=\frac{L}{6}, x=\frac{L}{2}, x=\frac{5L}{6}$ ψ_3 is max
ψ_3^2 is max

The above results are depicted in the following figure.