Linear Regression and Logistic Regression are two fundamental algorithms in machine learning, each serving a distinct purpose:

1. **Linear Regression:**
   - **Definition:** Linear Regression is a supervised learning algorithm used for predicting a continuous outcome variable (dependent variable) based on one or more predictor variables (independent variables) with a linear relationship. It assumes that there is a linear relationship between the independent variables and the dependent variable.
   - **Use case:** It is commonly applied in scenarios where the goal is to establish a predictive relationship, such as predicting house prices based on features like square footage, number of bedrooms, etc.

   - **Mathematical Representation:** For a simple linear regression with one predictor variable, the relationship can be represented as:
     \(Y = mx + b \)
     where \(Y \) is the dependent variable, \(x \) is the independent variable, \(m \) is the slope, and \(b \) is the intercept.

   - **Objective:** The objective of linear regression is to find the best-fitting line that minimizes the sum of squared differences between the observed and predicted values.

2. **Logistic Regression:**
   - **Definition:** Despite its name, Logistic Regression is used for binary classification problems, where the outcome variable is categorical and has two classes (0 or 1, True or False, Yes or No). It models the probability that a given input belongs to a particular category.
   - **Use case:** It is commonly used in scenarios such as spam detection (classifying emails as spam or not spam) or medical diagnosis (classifying patients as having a particular condition or not).

   - **Mathematical Representation:** Logistic Regression uses the logistic function (sigmoid function) to model the probability. For a single predictor variable, the logistic regression equation can be represented as:
     \(P(Y=1) = \frac{1}{1 + e^{-(mx + b)}} \)
     where \(P(Y=1) \) is the probability of the positive class, \(x \) is the independent variable, \(m \) is the slope, and \(b \) is the intercept.

   - **Objective:** The objective of logistic regression is to find the optimal parameters (weights) that maximize the likelihood of observing the given set of outcomes.

In summary, while Linear Regression is used for predicting continuous outcomes, Logistic Regression is used for binary classification problems where the outcome is categorical. Both are widely used and form the basis for more complex machine learning models.

Linear Regression and Logistic Regression are both fundamental machine learning algorithms, but they serve different purposes and are applied in different types of problems.

Linear Regression:

Purpose:
- Linear Regression is used for predicting a continuous outcome variable (also called dependent variable) based on one or more predictor variables (independent variables).
- It establishes a linear relationship between the input features and the target variable.

Model:
- The linear regression model is represented as \(y = b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_n \cdot x_n + \varepsilon \), where:
  - \(y \) is the dependent variable.
  - \(x_1, x_2, \ldots, x_n \) are the independent variables.
  - \(b_0, b_1, \ldots, b_n \) are the coefficients to be learned.
  - \(\varepsilon \) is the error term.

Training:
- The model is trained by minimizing a cost function, often the mean squared error, using techniques like gradient descent.

Application:
- Predicting house prices, stock prices, sales, etc.

Logistic Regression:

Purpose:
- Logistic Regression is used for binary classification problems where the outcome variable is categorical with two classes (0 or 1).
- It models the probability that an instance belongs to a particular category.

Model:
- The logistic regression model is represented as \(h(x) = \frac{1}{1 + e^{-(b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_n \cdot x_n)}} \), where:
  - \(h(x) \) is the predicted probability that \(y = 1 \).
  - \(x_1, x_2, \ldots, x_n \) are the independent variables.
  - \(b_0, b_1, \ldots, b_n \) are the coefficients to be learned.

Training:
- The model is trained by maximizing the likelihood function (or minimizing the negative log-likelihood) using techniques like gradient descent.

Application:

- Spam detection, medical diagnosis, credit scoring, etc.

Key Differences:

1. **Output:**
   - Linear Regression predicts a continuous output.
   - Logistic Regression predicts the probability of belonging to a certain class (binary output).

2. **Model Representation:**
   - Linear Regression uses a linear equation to represent the relationship between variables.
   - Logistic Regression uses the logistic function to model the probability of a binary outcome.

3. **Objective Function:**
   - Linear Regression minimizes the mean squared error.
   - Logistic Regression maximizes the likelihood function (or minimizes the negative log-likelihood).

4. **Application:**
   - Linear Regression is suitable for regression problems.
   - Logistic Regression is suitable for binary classification problems.

Both algorithms play crucial roles in machine learning, with Linear Regression addressing regression tasks and Logistic Regression addressing binary classification tasks.

Linear Regression:

Model Representation:
In linear regression, we model the relationship between the dependent variable y and the independent variable x by assuming a linear relationship:

$$ y = b_0 + b_1 \cdot x + \varepsilon $$

- y is the dependent variable we're trying to predict.
- x is the independent variable.
- b_0 is the y-intercept (constant term).
- b_1 is the slope (coefficient) associated with the independent variable.
- ε is the error term, representing the difference between the actual and predicted values.

Objective Function (Cost Function):
The goal is to find the values of b_0 and b_1 that minimize the sum of squared differences between the predicted and actual values. The cost function $J(b_0, b_1)$ is defined as:

\[J(b_0, b_1) = \frac{1}{2m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)})^2 \]

Here, \(m\) is the number of training examples, \(h(x^{(i)})\) is the predicted value given by the linear equation, and \((x^{(i)}, y^{(i)})\) are the training examples.

Optimization (Gradient Descent):
Minimizing the cost function is achieved using an optimization algorithm like gradient descent. The update rules for \(b_0\) and \(b_1\) are given by:

\[b_0 = b_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)}) \]

\[b_1 = b_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \]

Here, \(\alpha\) is the learning rate.

Logistic Regression:

Model Representation:
Logistic regression is used for binary classification, where the dependent variable \(y\) is binary (0 or 1). It models the probability that \(y = 1\) using the logistic function:

\[g(z) = \frac{1}{1 + e^{-z}} \]

The logistic regression model is expressed as:

\[h(x) = g(b_0 + b_1 \cdot x_1 + b_2 \cdot x_2 + \ldots + b_n \cdot x_n) \]

- \(h(x)\) is the predicted probability that \(y = 1\).
- \(g(z)\) is the logistic function.
- \(b_0, b_1, \ldots, b_n\) are the parameters.
- \(x_1, x_2, \ldots, x_n\) are the independent variables.

Objective Function (Cost Function):
The cost function for logistic regression is the negative log-likelihood. For a single training example, the cost is given by:

\[J(b) = -[y \log(h(x)) + (1 - y) \log(1 - h(x))] \]

For the entire dataset:

\[J(b) = -\frac{1}{m} \sum_{i=1}^{m}[y^{(i)} \log(h(x^{(i)})) + (1 - y^{(i)}) \log(1 - h(x^{(i)}))] \]

Optimization (Gradient Descent):

Gradient descent is used to minimize the cost function. The update rule for logistic regression parameters is:

$$ b_j = b_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h(x^{(i)}) - y^{(i)}) \cdot x_j^{(i)} $$

Here, α is the learning rate, m is the number of training examples, $h(x^{(i)})$ is the predicted probability, $y^{(i)}$ is the actual label, and $x_j^{(i)}$ is the j-th feature of the i-th example.

These detailed mathematical representations showcase how linear and logistic regression models are built and optimized to make predictions and classify data points.