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Parameter Estimation Assignment

Q1. normal distribution   mean → θ_1 variance → θ_2

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Joint density of $(X_1, X_2, \ldots X_n)$ is

$$L(\theta_1, \theta_2; X_1, X_2, \ldots X_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\frac{(x-\theta_1)^2}{2\theta_2}}$$

Taking log on both the sides

$$\ln[L(\theta_1, \theta_2)] = \ln\left((2\pi\theta_2)^{-n/2} \cdot e^{-\frac{\Sigma(x_i-\theta_1)^2}{2\theta_2}}\right)$$

$$\ln[L(\theta_1, \theta_2)] = -\frac{n}{2}\ln(2\pi\theta_2) - \frac{1}{2\theta_2}\Sigma(x_i-\theta_1)^2$$

for θ_1 ⇒ Differentiate $\ln[L(\theta_1, \theta_2)]$ wrt θ_1

$$\frac{\partial \ln L}{\partial \theta_1} = \frac{1}{\theta_2}\sum_{i=1}^{n}(x_i - \theta_1) = 0$$

$$\boxed{\theta_1 = \frac{\Sigma x_i}{n}}$$ ← Sample Mean

for θ_2 ⇒ Differentiate $\ln[L(\theta_1, \theta_2)]$ wrt θ_2

$$\frac{\partial \ln L}{\partial \theta_2} = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2 = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2}\Sigma(x_i-\theta_1)^2$$

$$\boxed{\theta_2 = \frac{1}{n}\Sigma(x_i-\theta_1)^2}$$ ← Variance

∴ MLE of θ_1 for is $\bar{x}$ (sample mean)

MLE of θ_2 is var(x) (sample variance)

Q2. To find the MLE of θ for a random sample $X_1, X_2, \ldots X_n$ from a Bernoulli distribution with parameter θ and a known m. The likelihood for this scenario is:

$$L(\theta | x_1, x_2, \ldots x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

Since X_i follows a Bernoulli distribution

$$P(X_i = x_i | \theta) = \theta^{x_i} (1-\theta)^{m-x_i} \quad \text{for each } i$$

Taking log on both the sides

$$\ln L(\theta | x_1, x_2, \ldots x_n) = \sum_{i=1}^{n} \ln\left(\theta^{x_i} (1-\theta)^{m-x_i}\right)$$

$$= \sum_{i=1}^{n} \left(x_i \ln\theta + (m - x_i) \ln(1-\theta)\right)$$

Now differentiate wrt θ and set to zero

$$\frac{d}{d\theta}\left(\ln L(\theta | x_1, x_2, \ldots x_n)\right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

$$\frac{\sum_{i=1}^{n} x_i}{\theta} = \frac{n \cdot m - \sum_{i=1}^{n} x_i}{1-\theta}$$

$$\therefore \quad \theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}$$

So Maximum likelihood estimate for θ is

$$\hat{\theta}_{MLE} = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}$$