Assignment 2

Automata Theory Monsoon 2021, IIIT Hyderabad

November 11, 2021

Total marks: 30   Due date: November 21, 2021

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1. [2 points] What is the cardinality of the configuration space of a Turing Machine, in terms of the cardinality of the tape alphabet τ and the set of states Q. For the formal definition of a Turing machine, refer to Wikipedia.

2. [5 points] Construct a Turing machine that takes a string $w \in \{0,1\}^n : n \in \mathbb{N}$ in its input tape and outputs the reverse string w^R in its output tape.

3. [3 points] Let C be a context-free language and R be a regular language. Define $L_1 = C \setminus R$ and $L_2 = R \setminus C$. Prove or disprove the following.

   1. L_1 is context-free.
   2. L_2 is context-free.

4. [4 points] Give an example of a language that is not context-free but satisfies the pumping lemma.

5. [3 points] Show that a non-deterministic push down automata that has access to 2 stacks instead of one, can simulate a Turing Machine.

6. [5 points] Show that a Turing machine M_1 whose work tape has been replaced by an infinite 2D grid on which the tape head can now move left, right, up and down can be simulated by a standard Turing Machine M discussed in class (with one way infinite tape). To help build some intuition, figure 1 shows the tape structure of M_1 and figure 2 for M.

   Note that the operations that the tape head of M_1 can perform are, one the cell below the tape head on the 2D tape, read and move up, read and move down, read and move left, read and move right, write and move up, write and move down, write and move left, write and move right.

7. [4 points] Show that the following language L is undecidable:

   $$L = \{\langle M \rangle : M \text{ is a Turing machine that runs in } 3n^2 + 3n + 3 \text{ time}\}$$

8. [4 points] Determine whether the following languages are context-free or not. If it is context-free, prove it using pumping lemma else write corresponding grammar.

   1. $L = \{a^n b^j c^k : k = jn\}$
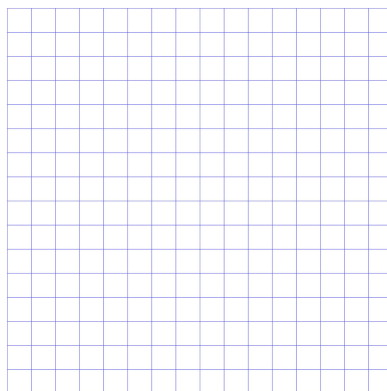   2. $L = \{a^{n!} \mid n \geq 0\}$

Figure 1: Tape of M_1

Figure 2: Tape of M