Automata Theory (CS1.302)

Monsoon 2021, IIIT Hyderabad
11 November, Thursday (Lecture 9)

Models of Computation

Turing Machines

Turing machines are a mathematical abstraction of computing devices.
They are FSMs that have access to memory in the form of an infinite tape, which contains the input string followed by blanks.
A TM has a read-write head that can both read from and write to the tape. It can move to the left or right one cell at a time. When the computation reaches an accept/reject state, it halts and accepts or rejects the input string.

A transition in a TM, therefore, has three elements – what is to be read, what is to be written, and whether the head is to move left or right.

Note that a certain TM may never halt on a certain input.

Turing claimed that Turing machines could simulate anything "humanly computable"; he considered a human brain as an FSM.

Formally, a Turing machine is a 7-tuple $(Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$, where

- Q is a finite set called the states.
- Σ is the set of input alphabtes not containing the blank symbol B.
- Γ is the tape alphabet, where $B \in \Gamma$ and $\Sigma subseteq \Gamma$.
- $\delta : Q \times \Gamma \to Q \times \Gamma \times \{L, R\}$ is the transition function.
- $q_0 \in Q$ is the start state.
- $q_{\text{accept}} \in Q$ is the accept state.
- $q_{\text{reject}} \in Q - \{q_{\text{accept}}\}$ is the reject state.

A *configuration* of a TM consists of the current tape contents, the current state and the current head location. At each step, the configuration changes; we say that C_1 yields C_2 if the TM goes from C_1 to C_2 in one step.

A TM M accepts w if there exists a sequence of configurations $C_1, \dots, C_k$, where C_1 is the start configuration, each C_i yields C_{i+1}, and C_k is an accepting configuration. The language of a TM M is $L = \{w \mid M \text{ accepts } w\}$.

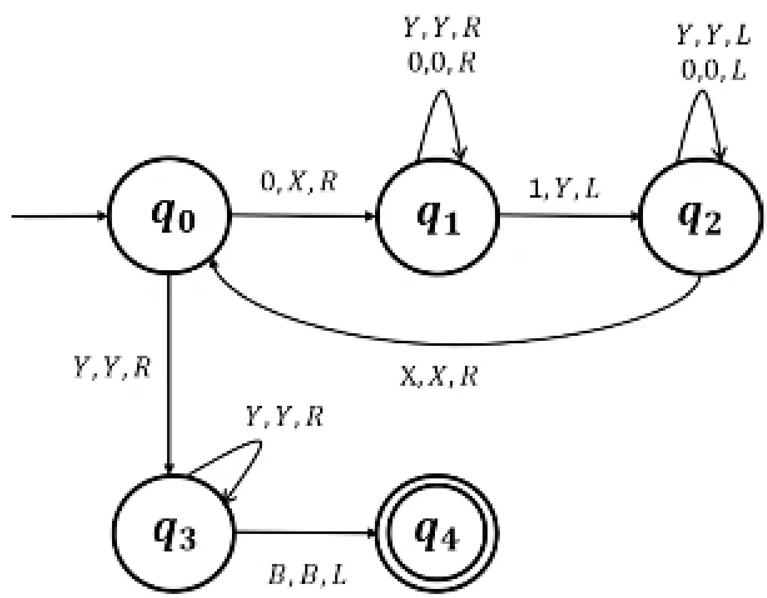

Figure 1: A TM to Accept $L = \{0^n1^n \mid n \geq 1\}$