Automata Theory
Quiz 1

Abhinav S Menon

2020114001

1. Since the language contains only finitely many strings, we can construct a regex for it simply by connecting all the strings by the '+' operator.
More concretely, if $L = \{a_1, \ldots, a_k\}$, then we can let $R = a_1 + a_2 + \cdots + a_k$, which makes $L(R) = L$.
This shows that L is regular.

2. Consider the NFAs for B and C. Let them be called N_B and N_C.
First, construct a new NFA N_C', which accepts all strings that have the same number of 1s as some string $y \in C$. This can be done by: ~~replacing every~~
- leaving 1-transitions unchanged
- leaving ε-transitions unchanged
- adding ε to 0-transitions.

Now, construct a new NFA N, with states labelled (q_i, q_j), where $q_i \in Q_B$ and $q_j \in Q_C$, and
$\cancel{\delta((q_i, q_j), a) = \delta_B(q_i, a) \cap \delta_C(q_j, a)}$

$$\delta((q_i, q_j), a) = \{(q_i', q_j') \mid q_i' \in \delta_A(q_i, a), \; q_j' \in \delta_B(q_j, a)\}.$$

Effectively, a string passed to N will be accepted iff it is accepted by both N_B and N_C'. Clearly, the set of such strings is $B \cap C' = B \xleftarrow{1} C$. $\therefore$ Thus, we have constructed an NFA for $B \xleftarrow{1} C$, and it is therefore regular, QED.

3. First, we eliminate ~~$\$$~~ $A \to B$.

$S \to AB \mid BB$

$A \to a \mid Bb$

$B \to Bb$.

Next, we eliminate $A \to Bb$ and $B \to Bb$.

$S \to AB \mid BB$

$A \to a \mid BX$

$B \to BX$

$X \to b$.

This is the grammar's CNF. It has no redundant rules. The no. of rules is $\underline{\underline{6}}$.

4. Consider the string $aacbc \in L(G)$. It ~~can be~~ has ~~der~~ the 2 following leftmost-rule derivations:

$S \to aSbS$	$S \to aS$
$\to aaSbS$	$\to aaSbS$
$\to aacbS$	$\to aacbS$
$\to aacbc$	$\to aacbc$.

Thus, the grammar G is ambiguous.

5. $L = \{ wcw^R \mid w, c \in \{a, b\}^+ \}$ is a regular language.

Its equivalent regex is

$$R = \left(a\left((a+b)(a+b)^*\right)a\right)$$

$$+ \left(b\left((a+b)(a+b)^*\right)b\right).$$

This can be more concisely written as

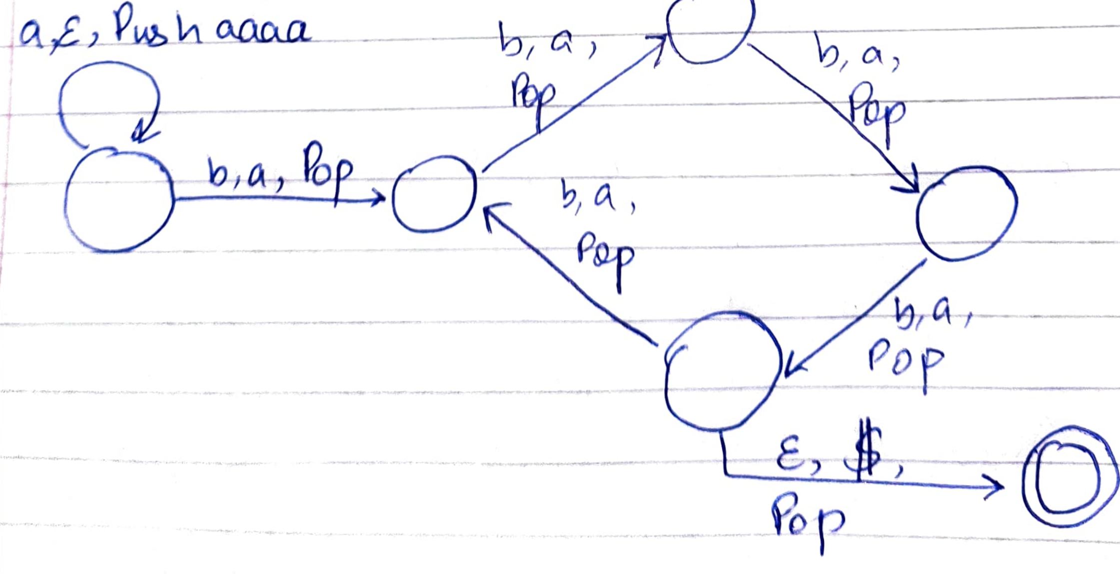
$$R = (a(a+b)^+a) + (b(a+b)^+b)$$

Clearly, all $x \in L(R)$ also are in L, letting $w = w^R = a$ or b, and $c =$ the middle part.

Conversely, all $x = wcw^R \in L$ are in $L(R)$, as they must have the same first & last characters (a or b) and the middle part could be anything.

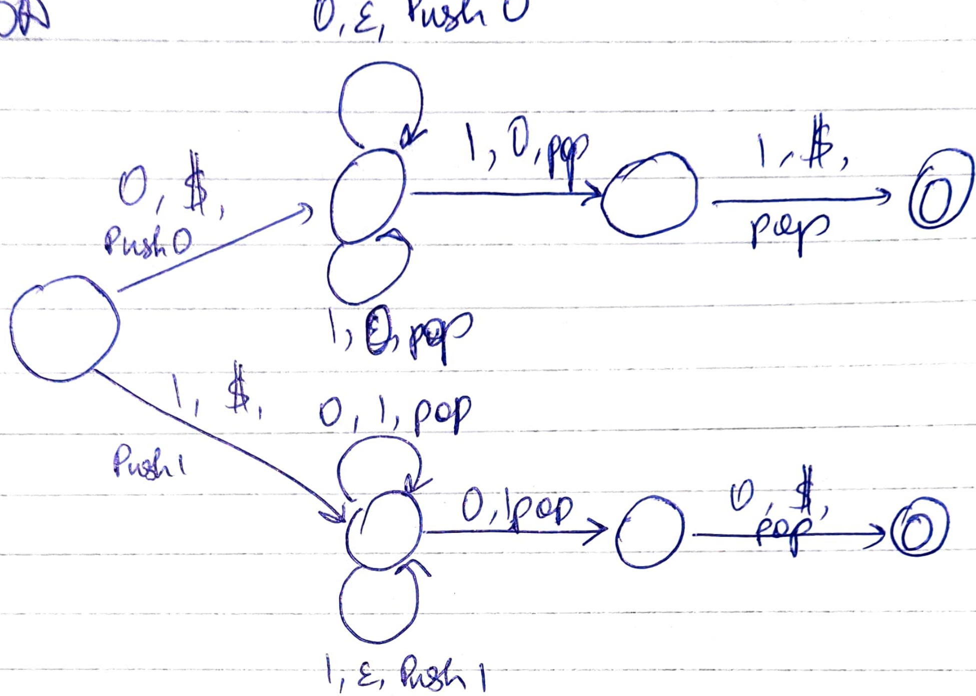
6. (i) $S \rightarrow aSbbbb \mid \varepsilon$.

(ii) The equivalent PDA is



7. (i) L1 is context free.

PDA



(ii) L_2 is context free.