Quiz 1

Automata Theory Monsoon 2021, IIIT Hyderabad

October 27, 2021

Total Points: 20 Time: 45 mins

General Instructions: FSM stands for finite state machine. DFA stands for deterministic finite automata. NFA stands for non-deterministic finite automata. PDA stands for Push Down Automata. a^* is the Kleene Star operation. $a^+ = a^* \setminus \{\epsilon\}$, where ϵ is the empty string.

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1. [2 points] Show that any language L containing only finitely many strings is regular.

2. [4 points] Let B and C be languages over $\Sigma = \{0, 1\}$. Define

$$B \overset{1}{\leftarrow} C = \{w \in B \mid \exists\, y \in C, \text{ such that strings } w \text{ and } y \text{ contain equal number of } 1s\}$$

   Prove that the class of regular languages is closed under $\overset{1}{\leftarrow}$.

3. [2 points] Convert the following language into its Chomsky normal form and write the number of production rules.

$$S \to AB$$
$$A \to B$$
$$A \to a$$
$$B \to Bb$$

   [N.B: Do not have rules that are redundant. For example don't add $S_0 \to S$ since S itself can be the start symbol.]

4. [2 points] Show that the grammar G whose rules are as follows, is ambiguous.

$$S \to aS|aSbS|c$$

5. [3 points] For a symbol a, define $a^+ = \{a, aa, \cdots\}$. Is the language $L = \{wcw^R | w, c \in \{a, b\}^+\}$ regular? If your answer is yes, write the equivalent regular expression and if no, prove that L is not regular using pumping lemma.

6. [3 points] Is $L = \{a^n b^{4n} | n \geq 1\}$ context-free? If so, (i) write the corresponding context free grammar for this language and (ii) draw the PDA that recognizes L.

7. [4 points] The language L consisting of all strings having an equal number of $0's$ and $1's$ is context-free. State whether the following languages are regular or context-free. Draw the corresponding automaton to support your answer (DFA/PDA).

(i) $L_1 = \{w|\ |\#0's - \#1's| \leq 1\}$

(ii) $L_2 = \{w|\ |\#0's - \#1's| \leq 1,\ \forall \text{ prefixes of } w\}$

Note: In 7 the string 00001111 will belong to L_1 but will not belong to L_2 since 00001, a prefix of 00001111, does not satisfy $|\#0's - \#1's| \leq 1$.