Final Exam

Automata Theory Monsoon 2021, IIIT Hyderabad

December 4, 2021

Total Points: 25   Time: 90 mins [75 mins for writing + 15 mins for submission]

<u>General Instructions:</u> Upload a scanned PDF of handwritten answers on moodle. No submissions in any form after 10:05 will be evaluated. You should strictly stop writing at 9:45 and start scanning and uploading. DFA is deterministic finite automata. NFA is non-deterministic finite automata. CFL is context free language. If S_1 and S_2 are two strings, $S_1 S_2$ represents their concatenation. All notation is standard from the lectures.

0.0.1 Parsing

1. [3 points] Comment on the Decidability of the Halting problem for Push-down Automata, compare it to that of the Turing Machine. If they don't match, explain why.

2. [4 points] Let R_1, R_2 be a recursive language, and let RE_1, RE_2 be two recursively enumerable languages. Are the following languages recursive or not? Are they recursively enumerable or not? Justify.

   A: $RE_1 \cup RE_2$

   B: $R_1 \cap RE_2$

   C: $R_1 - R_2 = \{x | (x \in R_1 \text{ and } x \notin R_2) \text{ or } (x \in R_2 \text{ and } x \notin R_1)\}$

3. [4 points] For any language L over Σ, the prefix closure of L is defined as:

$$Pre(L) = \{x \in \Sigma^* | \exists y \in \Sigma^* : xy \in L\}$$

   Prove that if L is regular, Pre(L) is regular.

4. [3 points] Define the language EQ_{DFA} as follows:

$$EQ_{DFA} = \{\langle M_1, M_2 \rangle | L(M_1) = L(M_2)\}$$

   where M_1, M_2 are encodings of DFAs as strings. Prove that EQ_{DFA} is recursive.

5. [2 points] Which of the following is the correct order and why?

   A: Regular $\subset$ Context Free $\subset$ Recursive $\subset$ Recursively Enumerable

   B: Regular $\subseteq$ Context Free $\subset$ Recursive $\subseteq$ Recursively Enumerable

   C: Regular $\subset$ Context Free $\subset$ Recursively Enumerable $\subset$ Recursive

D: Regular $\subseteq$ Context Free $\subseteq$ Recursive $\subseteq$ Recursively Enumerable

6. [3 points] Use the pumping lemma for CFLs to show that $L = \{www \mid w \in \{a, b\}^*\}$ is not context free.

7. [4 points] Let L be a CFL and $LEAST(L) = \{y \in L \mid y$ is a proper prefix of $x, \ \forall x \in L\}$. Show that CFLs are not closed under LEAST operation. (A proper prefix X of a string A is a substring such that there exists a non-empty string $S \in \Sigma^+$ such that $XS = A$)

8. [2 points]   A. The regular expression $0^*(10^*)^*$ denotes the same set as
   - (a) $(1^*0)^*1^*$
   - (b) $0^+(0^+10)^*$
   - (c) none of these

   B. Draw an NFA for the same RegEx.