Data and Its Applications (CS4.301)

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Assignment 3

George Paul (2021121006)
Abhijith Anil (2020101030)
Abhinav S Menon (2020114001)

Question 1

Question 1.1

The given set of functional dependencies is possible for all $n > 1$. For $n = 1$, the RHS of the first dependency is the null set, which is not allowed.

Question 1.2

We assume that there are no FDs which cannot be inferred from the given dependencies.

Each FD's LHS is a superkey (since the RHS includes all attributes except the LHS), and by assumption, has no subset which is also a superkey; therefore it is also a key. The number of FDs is given by solving the equation

$$\frac{(i-1)i}{2} + i \leq n$$

for i. By completing the square, we see that

$$\left(i + \frac{1}{2}\right)^2 - \frac{1}{4} = 2n.$$

Rearranging, we get

$$i = \left\lfloor \sqrt{2n + \frac{1}{4}} - \frac{1}{2} \right\rfloor.$$

The keys are the LHSs of each of the dependencies.

Question 1.3

The given relation is in BCNF, since all the relations are of the form $X \to Y$, where X is a key (see question 1.2).

Question 1.4

Since the relation is already in BCNF (see question 1.3), the given set of dependencies is itself the cover, *i.e.*, none of them can be inferred from the rest.

Question 2

We can rewrite the given set of dependencies as

$$\begin{aligned} A_1 &\to A_2 \cdots A_n \\ A_2 &\to A_1 A_3 \cdots A_n \\ &\vdots \\ A_n &\to A_1 \cdots A_{n-1}. \end{aligned}$$

Or equivalently,

$$A_i \to \{A_j \mid j \neq i\}, 1 \leq i \leq n$$

Question 2.1

From above, each of the LHSs of the FDs is a singleton superkey and therefore a key. Thus R has n keys.

Question 2.2

The given relation is in BCNF, since all the relations are of the form $X \to Y$, where X is a key (see question 2.1).

Question 2.3

Since the relation is already in BCNF (see question 2.2), the given set of dependencies is itself the cover, *i.e.*, none of them can be inferred from the rest.