Probability and Statistics (MA6.101)

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Probability

Bayes’ Theorem

For any events E and F with nonzero probabilities, we have

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}.$$

This theorem is provable from the relation between conditional probability and probability of intersection.

P(D|P) = P(P|D)P(D)/P(P) = 0.98*0.005/P(P) P(P) = P(P|D)P(D) + P(P|D’)P(D’) = 0.98*0.005 + 0.01*0.995

The Monty Hall Problem

There are three doors, one of which has a car and the other two have goats. A participant picks a door without opening it; a host then opens another door, revealing a car. Should the participant switch?

Yes, because
* if they picked a car first (which had a probability of $\frac{1}{3}$), they will lose (probability of getting car 0) * if they picked a goat first (which had a probability of $\frac{2}{3}$), they will win (probability of getting car 1) Therefore the probability of winning if the participant switches is $\frac{1}{3} \cdot 0 + \frac{2}{3} \cdot 1 = \frac{2}{3}$.

Alternatively, suppose the participant walks away after the host reveals a goat and a new participant comes in. For the new participant, the chance is 50-50; the question is, did the old participant have any information that made one door more likely? Yes, because they knew that the door they picked had only a $\frac{1}{3}$ chance of having a car. Therefore the other door is more likely to have a car.