Probability and Statistics (MA6.101)

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Probability

Random Walks

1D

A walk is called a simple random walk in 1D if there is an equal probability of going left or right.

Consider the probability that a person ends up at $x = 0$ at the i^{th} step of a simple random walk.
The choice tree for this process resembles a Galton board; we see that the probability is ${}^iC_{\frac{i}{2}}\frac{1}{2^i}$ (for even i) for a simple walk.

If the probability of going to the right is, say, q, then the probability becomes

$$ {}^iC_{\frac{i}{2}} \cdot q^{\frac{i}{2}}(1-q)^{\frac{i}{2}} $$

2D

Let Alice start at the bottom left corner of a 5×5 lattice and Bob at the top right. Alice walks one edge right or up every second; Bob walks left or down. Consider the probability that they meet.

It is clear that they cannot meet anywhere except along the non-principal diagonal, and only after each has taken 5 steps. Thus the total number of ways Alice and Bob can reach the same vertex on the diagonal is given by $1^2 + 5^2 + 10^2 + 5^2 + 1^2 = 252$, out of 1024 total paths. Thus the probability is $\frac{252}{1024} = \frac{63}{256}$.

Conditional Probability

Consider the probability that for two unbiased dice $D1$ and D_2, the probability that D_1 is even given that D_2 is odd. We restrict our sample space to the events where D_2 is odd, and calculate the probability as before. It comes out to be $\frac{1}{2}$.

Similarly, consider the probability that $D_1 = 2$, given that $D_1 + D_2 = 4$. This has a probability of $\frac{1}{3}$, since the sample space is only $(D_1, D_2) \in \{(1,3), (2,2), (3,1)\}$.

The confitional probability of an event E given that an event F occurs is denoted $P(E|F)$. It can be calculated as

$$P(E|F) = \frac{P(E \cap F)}{P(F)}.$$

The law of total probability states that $P(E) = P(E|F)P(F) + P(E|F^c)P(F^c)$. This can be proved by noting that $U = F \cup F^c$, and therefore $P(E) = P((E \cap F) \cup (E \cap F^c)) = P(E \cap F) + P(E \cap F^c)$. From this the law follows directly.