Probability and Statistics (MA6.101)

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Probability

Joint Probability

Joint Probability Mass Function

The joint PMF of two discrete random variables X and Y is denoted $P_{XY}(x, y)$ and defined $P(X = x, Y = y)$.

The joint range for X and Y is $R_{XY} = \{(x, y) \mid P_{XY} > 0\}$, *i.e.*, $R_{XY} \subset R_X \times R_Y$.

The sum of joint probabilities must sum to 1.

To compute the probability of some event set $A \subset \mathbb{R}^2$, we take the sum:

$$P((X, Y) \in A) = \sum_{(x_i, y_j) \in A \cap R_{XY}} P_{XY}(x_i, y_j).$$

We can also obtain the PMF of X from its joint PMF with Y as

$$P_X(x) = \sum_{y_j \in R_Y} P_{XY}(x, y_j).$$

We call the PMF obtained in this way the marginal PMF of X.

Joint Cumulative Distribution Function

Analogously, we have

$$F_{XY}(x, y) = P(X \leq x, Y \leq y).$$

This applies to both discrete and continuous variables.

Note that we have the property $F_{XY}(\infty, \infty) = 1$, and $F_{XY}(-\infty, y) = 0$.

For the marginal CDF, we can find

$$F_X(x) = F_{XY}(x, \infty).$$

For the probability of ranges, we can use the following result:

$$P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{XY}(x_1, y_1) + F_{XY}(x_2, y_2) - F_{XY}(x_1, y_2) - F_{XY}(x_2, y_1).$$

Independent Random Variables

If X, Y are two RVs, they are independent if

$$\forall x, y : P_{XY}(x, y) = P_X(x) P_Y(y).$$

Note that if X and Y are independent, then

$$P(x_i | y_j) = P_X(x_i).$$