Probability and Statistics (MA6.101)

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Taught by Prof. Pawan Kumar

Probability (contd.)

Saint Petersberg Paradox

If a coin is tossed repeatedly until it shows heads, and we earn 2^n rupees if heads shows up on the n^{th} toss. This gives us an infinite expected value. Paradoxically, therefore, we should be willing to pay an arbitrarily large amount to play. However, in practice, one is not willing to do so.

Distributions

Uniform Distribution

A uniform distribution on $\{1, \dots, n\}$ is defined as

$$P(x) = \begin{cases} \frac{1}{n} & x \in \{1, \dots, n\} \\ 0 & \text{otherwise.} \end{cases}$$

Bernoulli Distribution

X is called a Bernoulli random variable with parameter p (denoted X Bernoulli(p)), if

$$P(x) = \begin{cases} p & x = 0 \\ 1 - p & x = 1. \end{cases}$$

Geometric Distribution

X is called a geometric random variable with parameter p (denoted X Geometric(p)), if

$$P(k) = \begin{cases} p(1-p)^{k-1} & k \geq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Binomial Distribution

X is called a binomial random variable with parameters n, p (denoted X Binomial(n, p)), if

$$P(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & k \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Binomial(n, p) is a sum of n independent Bernoulli(p) random variables.

Poisson Distribution

The Poisson distribution is the limit of the binomial as $n \to \infty$.

$$\begin{aligned} P(X = k) &= \lim_{n\to\infty} \binom{n}{k} \left(\frac{\lambda}{n}\right)^k \left(1 - \frac{\lambda}{n}\right)^{n-k} \\ &= \lim_{n\to\infty} \frac{n \cdot (n-1) \cdots (n-k+1)}{n^k} \frac{\lambda^k}{k!} \frac{(1-\frac{\lambda}{n})^n}{(1-\frac{\lambda}{n})^k} \\ &= \frac{e^{-\lambda}\lambda^k}{k!}. \end{aligned}$$