MA6.101
Probability And Statistics **ASSIGNMENT 1** Due by : 11:59 PM, Sep 28

Instructions

- There are a total of 10 questions and need to attempt all the questions.
- Each question is worth of 3 points. (Total -> 3 x 10 = 30)

Problem 1

Two dice are rolled. 3 events A,B,C are defined as A = 'sum of two dice equals 3', B = 'sum of two dice equals 8', C = 'at least one of the dice shows a 2'.

1. What is P(A|C)?
2. What is P(B|C)?
3. Are A and C independent? What about B and C?

Problem 2

Alex is one of the 2 siblings of his/her parents. Find out the probability of him/her having a sister, given that:

1. Alex is a girl.
2. Alex is a boy.

Problem 3

We are given 100 red and 100 blue candies. We also have 2 jars, in which we have to fill all the candies. Now we want to pick out a candy at random from any one of the 2 jars (both jars have equal probability of being selected). Find out the distribution of the candies in the 2 jars such that the probability of drawing a blue candy is maximized.

Problem 4

Out of 3 biased coins (with probability of heads as 0.2, 0.6, and 0.4 respectively), one coin is selected and tossed 4 times.

1. Find P(HTHT).

2. Given that the coin toss results are Head, Tail, Head, Tail respectively, find out the probability that the coin with probability of head equal to 0.2 was selected.

Problem 5

Let A and B be two events. Suppose P(A) = 0.45, $P(A \cap B) = 0.15$ and $P(AC \cap BC) = 0.45$. What is P(B)?

Problem 6

For three events A, B, and C, we know that,

1. A and C are independent,
2. B and C are independent,
3. A and B are disjoint,
4. $P(A \cup C) = 2/3, P(B \cup C) = 3/4, P(A \cup B \cup C) = 11/12$.

Find P(A),P(B), and P(C).

Problem 7

A biased coin (with probability of obtaining a Head equal to p > 0) is tossed repeatedly and independently until the first head is observed. Compute the probability that the first head appears at an even numbered toss using total probability theorem.

Problem 8

Let C_1, C_2, ...C_M be a disjoint partition of the sample space S, and A and B be two events. Suppose we know that,

1. A and B are conditionally independent given C_i ∀ i ∈ 1, 2, ..., M;
2. B is independent of all Ci's.

Prove that A and B are independent.

Problem 9

Consider a test to detect a disease that 0.1 percent of the population have. The test is 99 percent effective in detecting an infected person. However, the test gives a false positive result in 0.5 percent of cases. If a person tests positive for the disease what is the probability that they actually have it?

Problem 10

A man is known to speak the truth 3 out of 5 times. He throws a die and reports that the number obtained is a two. Find the probability that the number obtained is actually a two.