Probability and Statistics (MA6.101)

Monsoon 2021, IIIT Hyderabad
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Probability

Higher Order Moments (contd.)

The moment generating function for a variable $X \sim \text{Binomial}(n, p)$ is given by

$$\begin{aligned} M_X(t) &= \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} e^{tx} \\ &= \sum_{x=0}^{n} \binom{n}{x} (e^t p)^x (1-p)^{n-x} \\ &= (e^t + 1 - p)^n. \end{aligned}$$

Similarly for $X \sim \text{Poisson}(p)$, we have

$$\begin{aligned} M_X(t) &= \sum_{x=0}^{\infty} e^{-\lambda} \frac{\lambda^x}{x!} e^{tx} \\ &= \sum_{x=0}^{\infty} e^{-\lambda} \frac{(e^t \lambda)^x}{x!} \\ &= e^{\lambda(e^t - 1)}. \end{aligned}$$

We can also find the variance using the moment generating function: $\text{Var}(X) = M_X''(0) - M_X'(0)^2$.