Probability and Statistics (MA6.101)

Monsoon 2021, IIIT Hyderabad
19 November, Friday (Lecture 23)

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Probability

Some Properties of Expectation

If X, Y are two RVs and g, h are functions of X and Y, then

$$E[g(X)h(Y) \mid X] = g(X)E[h(Y) \mid X].$$

The Law of Iterated Expectations states that if X, Y are two RVs, then

$$E[X] = E[E[X \mid Y]].$$

If X, Y are two independent RVs, then

- $E[X \mid Y] = E[X]$
- $E[g(X) \mid Y] = E[g(X)]$
- $E[XY] = E[X]E[Y]$
- $E[g(X)h(Y)] = E[g(X)]E[h(Y)]$

Conditional Variance

If X, Y are two RVs, then the conditional variance of X given $Y = y$, is

$$\text{Var}(X \mid Y = y) = E[X^2 \mid Y] = \mu_{X|Y}(x^2).$$

The Law of Total Variance states that if X, Y are two random variables, then

$$\text{Var}(X) = E[\text{Var}(X \mid Y)] + \text{Var}(X \mid Y).$$

Joint Probability Density Function

Two RVs X, Y are jointly continuous if there exists a nonnegative function $f_{XY} : \mathbb{R}^2 \to \mathbb{R}^2$ such that for any $A \subseteq \mathbb{R}^2$, we have

$$P((X, Y) \in A) = \int\int_A f_{XY}(x, y)dxdy.$$

The function f_{XY} is the joint probability density function.

The marginal continuous PDF is analogously defined as

$$f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x, y)dy.$$

Conditional PDF

The conditional PDF of X given $Y = y$ is $f_{X|Y}(X \mid y) = \frac{f_{XY}(x,y)}{f_Y(y)}$.\$

This can be integrated to find the conditional CDF and the conditional probability of any range A.