MA6.101

Probability And Statistics **ASSIGNMENT 4** Due by : 11:59 PM, Nov 17

Problem 1

Suppose that buses arrive are scheduled to arrive at a bus stop at noon but are always X minutes late, where X is an exponential random variable with probability density function $f_X(x) = \lambda e^{-\lambda x}$. Suppose that you arrive at the bus stop precisely at noon.

(a) Compute the probability that you have to wait for more than five minutes for the bus to arrive.

(b) Suppose that you have already waiting for 10 minutes. Compute the probability that you have to wait an additional five minutes or more.

Problem 2

Let R be the rate at which customers are served in a queue. Suppose that R is exponential with pdf $f(r) = 2e^{-2r}$ on $[0, \infty)$.

Find the pdf of the waiting time per customer T = 1/R.

Problem 3

Suppose that the cdf of X is given by:

$$F(a) = \begin{cases} 0 \text{ for } a < 0 \\ 1 \text{ for } 0 \leq a < 2\ 5 \\ 2 \text{ for } 2 \leq a < 4\ 5 \\ 1 \text{ for } a \geq 4. \end{cases}$$

Determine the pmf of X.

Problem 4

(a) Suppose that X has probability density function $f_X(x) = \lambda e^{-\lambda x}$ for $x \geq 0$. Compute the cdf, $F_X(x)$.

(b)) If $Y = X^2$, compute the pdf and cdf of Y.

Problem 5

Let X have range [0,3] and density $f_X(x) = kx^2$. Let $Y = X^3$.

(a) Find k and the cumulative distribution function of X.

(b) Compute E(Y).

(c) Compute Var(Y).

(e) Find the probability density function $f_Y(y)$ for Y.

Problem 6

Let X be the result of rolling a fair 4-sided die. Let Y be the result of rolling a fair 6-sided die. Let Z be the average of X and Y.

(a) Find the standard deviation of X, of Y, and of Z.

(b) Now consider a game; you win 2X dollars if X>Y and lose 1 dollar otherwise. After playing this game 60 times, what is your expected total gain (positive) or loss (negative)?

Problem 7

Let X be a continuous random variable with PDF:

$$f_X(x) = \begin{cases} x+\frac{1}{2} & ; \quad 0 \le x \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Find $E(X^n)$, where $n \in N$.

Problem 8

Let X be a continuous random variable with PDF:

$$f_X(x) = \begin{cases} x^2\left(2x + \frac{3}{2}\right) & ; \quad 0 < x \le 1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

If $Y = \frac{2}{X} + 3$, find Var(Y).