MA6.101
Probability And Statistics **ASSIGNMENT 2** Due by : 11:59 PM, Oct 21

Instructions

- There are a total of 10 questions and need to attempt all the questions.
- Each question is worth of 5 points. (Total -> 4 x 10 = 40)

Problem 1

Tharun being poor at shooting takes 10 shots at a target and has probability 0.2 of hitting the target with each shot, independently of all the other shots. Let X be the number of hits.

1. Calculate the PMF of X.
2. Find the expectation and variance of X.
3. Suppose Tharun has to pay $3 to enter the shooting range and he gets $2 for each hit. Let Y be his profit. Find the expectation and variance of Y.
4. Now let's assume that Tharun enters the shooting range for free and gets the number of dollars that is equal to the square of the number of hits. Let Z be his profit. Find the expectation of Z.

Problem 2

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} 0.1, & \text{for } x = 0.2 \\ 0.2, & \text{for } x = 0.4 \\ 0.2, & \text{for } x = 0.5 \\ 0.3, & \text{for } x = 0.8 \\ 0.2, & \text{for } x = 1 \\ 0, & \text{otherwise} \end{cases}$$

1. Find range of the random variable X.
2. Find P(X $\leq$ 0.5).
3. Find P(0.25 $<$ X $<$ 0.75).
4. Find expectation and variance of X.

Problem 3

Let X be a random variable with the following PMF

$$P_X(x) = \begin{cases} 0.2, & \text{for } x = 0 \\ 0.2, & \text{for } x = 1 \\ 0.3, & \text{for } x = 2 \\ 0.3, & \text{for } x = 3 \\ 0, & \text{otherwise} \end{cases}$$

If $Y = X(X-1)(X-2)$, find the PMF of Y.

Problem 4

A player is randomly dealt 13 cards from a standard 52 card deck.

1. What is the probability the 13th card dealt is a king.
2. What is the probability the 13th card dealt is the first king dealt.

Problem 5

The number of customers arriving at a grocery store is a Poisson random variable. On average 20 customers arrive per hour. Let X be the number of customers arriving from 1 pm to 3 pm. What is the value of $P(15 < X < 25)$.

Problem 6

You write a software program over and over, and each time there is probability p that it works correctly, independently of previous attempts. What is the expectation and variance of X, the number of tries until the program works correctly.

Problem 7

Let $X \sim Poisson(\alpha)$ and $Y \sim Poisson(\beta)$ be two independent random variables. If new random variable $Z = X + Y$, then find the PMF of Z.

Problem 8

A particular professor is known for his arbitrary grading policies. Each paper receives a grade from the set { A, A-, B, B-, C, C- }, with equal probability, independently of other papers. How many papers do you expect to hand in before you receive each possible grade at least once ?

Problem 9

You are visiting the rain forest, but unfortunately your insect repellent has run out. As a result at each second, a mosquito lands on you with probability 0.5. If one lands, with probability 0.2 bites you and with probability 0.8 it never bothers you, independently of other mosquitoes. What is expected time and variance of the time between successive mosquito bites ?

Problem 10

Let X and Y be independent random variables. Random variable X has mean μ_X and variance σ_X^2 , and random variable Y has mean μ_Y and variance σ_Y^2. Let $Z = 3X^2 + 4Y$. Find the mean and variance of Z in terms of the means and variances of X and Y.