2\. RTP: $A_{n\times n}$ is invertible $\Leftrightarrow$ the reduced row echelon form of A is I_n.

(i) $\Rightarrow$.

Let A be an invertible $n\times n$ matrix.

Then, A has nonzero determinant, i.e. $\det(A) \neq 0$.

We know that $\det(EB) = \det(E)\det(B)$ for all ~~inverti~~ elementary matrices E. —①

Further, $\det(E) \in \{1, -1, k\}$, $k \neq 0$, i.e. $\det(E)$ is nonzero. —②

Consider the reduction of A to its RREF R:

$$A = E_1 E_2 \cdots E_k R.$$

Then, $\det(A) = \det(E_1 E_2 \cdots E_k R)$

$= \det(E_1)\det(E_2)\cdots\det(E_k)\det(R)$

[repeated application of ①]

$= c\det(R)$ for some $c \neq 0$ [by ②]

Hence, $\det(A) \neq 0 \Leftrightarrow \det(R) \neq 0$.

But $\det(R) \neq 0 \rightarrow R = I_n$, since R is in RREF.

Therefore, A is invertible $\Rightarrow$ its RREF is I_n.

(ii) $\Leftarrow$

Let A's RREF be I_n.

This means that there exist elementary matrices E_i such that

$$(E_1 E_2 \cdots E_k) A = I_n. \quad \text{③}$$

However, all E_i ~~has are in~~ have nonzero determinant and are invertible. Therefore we can say that

$$A = (E_k^{-1} \cdots E_2^{-1} E_1^{-1}) I_n,$$

from which we obtain

$$A(E_1 E_2 \cdots E_k) = I_n \quad \text{④}.$$

Letting $B = E_1 E_2 \cdots E_k$, we see that $BA = I_n$ [③]

& $AB = I_n$ [④],

which means that $B = A^{-1}$, i.e. A is invertible

Hence A's RREF is $I_n \Rightarrow$ A is invertible, QED.