Consider the vector $v_i + u_{\sigma(i)}$ for some permutation σ.

Then $(A^2 + AB + B^2)(v_i + u_{\sigma(i)})$

$$= \lambda_i^2 v_i + A^2 u_{\sigma(i)} + AB v_i + \gamma_{\sigma(i)} A u_{\sigma(i)} + B^2 v_i + \gamma_{\sigma(i)}^2 u_{\sigma(i)}$$

$$= (\lambda_i^2 + \lambda_i \gamma_{\sigma(i)} + \gamma_{\sigma(i)}^2)(v_i + u_{\sigma(i)}).$$

This is because $AB = BA \Rightarrow AB u_i = \gamma_i A u_i = BA u_i$

& $\Rightarrow ABBA v_i = \lambda_i B v_i = AB v_i$

So $Au_i = u_{\sigma(i)}$

& $Bv_i = v_{\sigma(i)}$.