~~Let~~ ~~(Given:~~ ~~[illegible]~~

Given: $\dim U = n$

$U = V \oplus W$

$\dim V' > V$

RTP: $V' \cap W \neq \{0\}$

Proof:

Let $v_1, \dots, v_k$ form a basis of V

$w_1, \dots, w_{n-k}$ " " " W

$u_1, \dots, u_m$ " " " V', where $m > k$.

Consider the set $\{u_1, \dots, u_m\} \cap \{w_1, \dots, w_{n-k}\}$.
If it is nonempty, we are done. Let, if poss.,
$\{u_1, \dots, u_m\} \cap \{w_1, \dots, w_{n-k}\} = \emptyset$.

We know, then, that $\{u_1, \dots, u_m\} \cup \{w_1, \dots, w_{n-k}\}$
contains $> n$ vectors. $[n-k+m > n-k+k]$.
Therefore it is linearly dependent. Hence there are a_i, b_i s.t. [not all zero] s.t.

$$\underbrace{a_1 u_1 + \dots + a_m u_m}_{\alpha} + \underbrace{b_1 w_1 + \dots + b_{n-k} w_k}_{\beta} = 0.$$

If $\alpha = 0$, then all $a_i = 0$. This means that $\beta = 0$, so all $b_i = 0$ as well, which is a contradiction.
Therefore $\alpha \neq 0$. — ⓐ

However, $\alpha = -\beta$

$$= -b_1 w_1 + (-b_2) w_2 + \cdots + (-b_{n-k}) w_{k},$$

i.e. α is a linear combination of W's basis vectors.

$\Rightarrow \alpha \in W$. — ①

~~the~~ But by definition $\alpha = a_1 u_1 + \cdots + a_m u_m$ is a linear combination of V''s basis vectors.

$\Rightarrow \alpha \in V'$. — ②

And it is proved above [③] that $\alpha \neq 0$.

From ①, ② & ③, $V' \cap W \neq \{0\}$. QED.