$$A = E_1 E_2 \cdots E_k R.$$

Then, $\det(A) = \det(E_1 E_2 \cdots E_k R)$

$= \det(E_1)\det(E_2)\cdots\det(E_k)\det(R)$ [repeated application of 1]

$= c\det(R)$ for some $c \neq 0$ [by 2]

Hence, $\det(A) \neq 0 \Leftrightarrow \det(R) \neq 0$.

But $\det(R) \neq 0 \rightarrow R = I_n$, since R is in RREF.

Therefore, A is invertible $\Rightarrow$ its RREF is I_n.

(ii) $\Leftarrow$

Let A's RREF be I_n.

This means that there exist elementary matrices E_i such that

$$(E_1 E_2 \cdots E_k) A = I_n. \quad \text{— } 3$$

However, all E_i ~~has are is~~ have nonzero determinant and are invertible. Therefore we can say that

$$A = (E_k^{-1} \cdots E_2^{-1} E_1^{-1}) I_n,$$

from which we obtan

$$A(E_1 E_2 \cdots E_k) = I_n \text{ — } 4.$$

Letting $B = E_1 E_2 \cdots E_k$, we see that $BA = I_n$ [3]

& $AB = I_n$ [4],

which means that $B = A^{-1}$, i.e. A is invertible

Hence A's RREF is $I_n \Rightarrow$ A is invertible, QED.