Assignment 1
MA3.101: Linear Algebra
Spring 2021

March 5, 2021

1 Questions

1. Which of the following pairs of sets V and F form a valid vector space? Prove all your claims. (Addition and multiplication operations are defined as usual arithmetic on real numbers. Note that you have to check if V is a vector space over F. All symbols have the usual meaning.)

   (a) $V = \mathbb{R}$ and $F = \mathbb{N}$

   (b) $V = \mathbb{Q}$ and $F = \mathbb{R}$

   (c) $V = \mathbb{R}$ and $F = \mathbb{Q}$

   (d) $V = \mathbb{R}$ and $F = \mathbb{C}$

2. Prove that the set $F = \{0, 1\}$ is a field if we define addition as the boolean **XOR** and multiplication as the boolean **AND** gates.

3. Let V be a vector space of $n \times n (n \geq 2)$ matrices over an arbitrary field F. Which of the following sets of matrices A in V are subspaces of V?

   (a) Set of all invertable matrices A.

   (b) Set of all non-invertable matrices A.

   (c) All A such that $AB = BA$ for a fixed matrix B in V.

   (d) All idempotent matrices A

4. **S**uppose V is a vector space of all functions $f : \mathbb{R} \to \mathbb{R}$. Which of the following sets of functions are subspaces of V?

   (a) all continuous functions.

   (b) f such that $f(x^2) = f(x)^2$.

   (c) $f(3) = 1 + f(-5)$

5. Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. Show that if S_2 is linearly independent then S_1 is linearly independent.

6. Let V and W be vector spaces over a field F. Let $Z = [(v, w), v \in V, w \in W]$. Prove that Z is a vector space over the field F with the operations: $(v_1, w_1) + (v_2, w_2) = (v_1 + v_2, w_1 + w_2)$ and $c(v_1, w_1) = (cv_1, cw_1)$.

7. Let u, v and w be distinct vectors of a vector space V. Show that if $[u, v, w]$ is a basis for V then $[u + v + w, v + w, w]$ is also a basis for V.

8. Let f and g be the functions from R to R (R is the set of all real numbers) defined by $f(t) = e^{rt}$ and $g(t) = e^{st}$, where $r \neq s$. Prove that f and g are linearly independent.

9. Let $S = [(1, 1, 0), (1, 0, 1), (0, 1, 1)]$. Find out whether the set generates R^3 (R is the set of all real numbers)and linearly independent or not.

10. Give an example to show that union of two subspaces of a vector space may not be a subspace of the vector space.