$\lambda = -1$: $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = -\begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$\Rightarrow 4x_0 + 2y_0 = -x_0 \Rightarrow 5x_0 + 2y_0 = 0$

& $2x_0 + 4y_0 = -y_0 \Rightarrow 2x_0 + 5y_0 = 0$

of which the only soln is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

~~$\lambda = 5$~~ $\lambda = 1$: done in part (i).

$\exists s \in \mathbb{R} : v_0 = \begin{bmatrix} s \\ -s \end{bmatrix}$

$\lambda = -5$: $\begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = -5 \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$

$8x_0 + 2y_0 = -5x_0 \Rightarrow 13x_0 + 2y_0 = 0$

& $2x_0 + 8y_0 = -5y_0 \Rightarrow 2x_0 + 13y_0 = 0$

which has the solution $v_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ only.

$\therefore$ The set v_0 is $\left\{ \begin{bmatrix} s \\ -s \end{bmatrix} \mid s \in \mathbb{R} \right\}$ //.