DSA Assignment - 2

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1. The signals are all discrete. Thus, if a signal is periodic,

$$x(n) = x(n+T) \quad (1)$$

is solvable for nonzero integer T, and the fundamental period is the smallest value of T that solves (1).

(a) $x[n] = \cos(0.03\pi n)$

$x(n) = x(n+T)$

$\Rightarrow \cos(0.03\pi(n+T)) = \cos(0.03\pi n)$

$\Rightarrow \cancel{0.03\pi n} + 0.03\pi T = \cancel{0.03\pi n} + 2k\pi$

$\Rightarrow T = \dfrac{2k}{0.03}$

which has as the integer solutions $\{200, 400, 600 \ldots\}$.

$\Rightarrow x(n)$ is periodic

w/ fundamental pd. 200.

(b) $x[n] = \cos\left(\frac{\pi n^2}{4}\right)$

$x(n) = x(n+T) \Rightarrow \cos\left(\frac{\pi n^2}{4}\right) = \cos\left(\frac{\pi(n+T)^2}{4}\right)$

$\Rightarrow 2k\pi + \frac{\pi n^2}{4} = \frac{\pi(n^2 + T^2 + 2nT)}{4}$

$\Rightarrow 8k = \frac{T^2 + 2nT}{\cancel{4}} \Rightarrow (T+n)^2 = 8k + n^2$

$\Rightarrow T = \sqrt{8k+n^2} - n$

which depends on n. Thus this eq^n cannot be solved.

$\Rightarrow x(n)$ is aperiodic

(c) $x[n] = 5$

$x(n) = x(n+T) \Rightarrow 5 = 5 \longrightarrow$ true $\forall T$

$\therefore$ The function is periodic

w/ fundamental period $\underline{\underline{1}}$.

(d) $x(n) = \cos(5\pi n) + \cos(\frac{4}{5}\pi n)$

$x(n) = x(n+T) \Rightarrow \cos(5\pi n) + \cos(\frac{4}{5}\pi n)$

$= \cos(5\pi(n+T)) + \cos(\frac{4}{5}\pi(n+T))$

$\Rightarrow \cos(5\pi n) - \cos(5\pi(n+T)) = -\cos(\frac{4}{5}\pi n) + \cos(\frac{4}{5}\pi(n+T))$

$\Rightarrow \cancel{2}\sin(10\pi(2n+T))\sin(5\pi T) = \cancel{2}\sin(\frac{4}{3}\pi(2n+T)) \cdot \sin(\frac{4}{5}\pi T)$

(d) $x[n] = \cos(5\pi n) + \cos(\frac{4}{5}\pi n) = x_1(n) + x_2(n)$

$x_1(n) = x_1(n+T_1)$

$\Rightarrow \cos(5\pi n) = \cos(5\pi(n+T_1))$

$\Rightarrow \cancel{5\pi n} + 2k\pi = \cancel{5\pi n} + 5\pi T_1$

$\Rightarrow T_1 = \frac{2k}{5}$

$\Rightarrow T_1 \in \{2, 4, 6 \ldots\}$

$x_2(n) = x_2(n+T_2)$

$\Rightarrow \cos(\frac{4}{5}\pi n) = \cos(\frac{4}{5}\pi(n+T_2))$

$\Rightarrow \cancel{\frac{4}{5}\pi n} + 2k\pi = \cancel{\frac{4}{5}\pi n} + \frac{4}{5}\pi T_2$

$\Rightarrow T_2 = \frac{5k}{2}$

$\Rightarrow T_2 = \{5, 10, 15 \ldots\}$

$\therefore$ each of x_1 & x_2 are periodic. ~~The period~~

$\Rightarrow x$ is periodic

w/ period $T = \min(T_1 \cap T_2) = 10$

(e) $x[n] = \sin(5\pi n + 2)$

$x[n] = x[n+T] \Rightarrow \sin(5\pi n + 2) = \sin(5\pi(n+T) + 2)$

$\Rightarrow \cancel{5\pi n} + \cancel{2} + 2k\pi = \cancel{5\pi n} + 5\pi T + \cancel{2}$

$\Rightarrow T = \frac{2k}{5} \Rightarrow T \in \{2, 4, 6 \ldots\}$

$\therefore x(n)$ is periodic w/ fundamental period 2.

(f) $x(n) = \cos(n+\pi)$

$$x(n) = x(n+T) \Rightarrow \cos(n+\pi) = \cos(n+T+\pi)$$

$$\Rightarrow \cancel{n} + \cancel{\pi} + 2k\pi = \cancel{n} + T + \cancel{\pi}$$

$$\Rightarrow T = 2k\pi$$

which can never be an integer $\therefore x[n]$ is aperiodic.

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2. (a) $x(n) = e^{\left(\frac{a\pi n}{b}\right)j} = \cos\left(\frac{a\pi n}{b}\right) + j\sin\left(\frac{a\pi n}{b}\right)$

we know that $\cos\left(\frac{a\pi(-n)}{b}\right) = \cos\left(-\frac{a\pi n}{b}\right) = \cos\left(\frac{a\pi n}{b}\right)$

and $j\sin\left(\frac{a\pi(-n)}{b}\right) = j\sin\left(-\frac{a\pi n}{b}\right) = -j\sin\left(\frac{a\pi n}{b}\right)$

$$\Rightarrow x_e(n) = \cos\left(\frac{a\pi n}{b}\right), \quad x_o(n) = j\sin\left(\frac{a\pi n}{b}\right)$$

(b). $x[n] = a\cos(b\pi n + 1)$

$$= a\cos(b\pi n)\cos(1) - a\sin(b\pi n)\sin(1)$$

we know that $a\cos(b\pi(-n))\cos(1) = a\cos(b\pi n)\cos(1)$

and $-a\sin(b\pi(-n))\sin(1) = -(-a\sin(b\pi n)\sin(1))$

$$\Rightarrow x_e(n) = a\cos(b\pi n)\cos(1), \quad x_o(n) = -a\sin(b\pi n)\sin(1)$$

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3. (a) $x(n) = \left(\frac{1}{4}\right)^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^{2n} (u(n))^2 = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^{2n}$$

[assuming $u(n) = 1, n \geq 0$]

$$= \sum_{n=0}^{\infty} \left(\frac{1}{16}\right)^n = \frac{1}{1-\frac{1}{16}} = \frac{16}{15} < \infty$$

$\Rightarrow x$ is an energy signal.

(b). $x(n) = a^n u(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} a^{2n} (u(n))^2 = \sum_{n=0}^{\infty} (a^2)^n$$

$= \frac{1}{1-a^2}$, which is finite for all nonzero $a \in \mathbb{R}$.

$\Rightarrow x(n)$ is an energy signal if $a \neq 0$.

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} a^{2n} = 0.$$

$$= \cancel{\lim_{N\to\infty} \frac{a^{2N}-1}{a^2-1} \cdot \frac{1}{(2N+1)}}$$

$\therefore x$ is a ~~power~~ power signal if $a = 0$

(c) $x(n) = a^n \delta(n)$

[assuming $\delta(0) = 1$ & $\delta(n) = 0$ for $n \neq 0$]

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} a^{2n} (\delta(n))^2 = a^{2(0)} = 1 < \infty$$

$\therefore x$ is an energy signal.

(d) $x(n) = \sin\left(\frac{n\pi}{4}\right)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=-\infty}^{\infty} \left|\sin^2\left(\frac{n\pi}{4}\right)\right| = 2\sum_{n=0}^{\infty} \sin^2\left(\frac{n\pi}{4}\right)$$

$$\cancel{= 2\left[\sum_{n=4k} \sin^2\left(\frac{4k\pi}{4}\right) + \sum_{n=4k+}\right]}$$

but, $$\sum_{n=0}^{7} \sin^2\left(\frac{n\pi}{4}\right) = \sin^2(0) + \sin^2\left(\frac{\pi}{4}\right) + \sin^2\left(\frac{\pi}{2}\right) + \sin^2\left(\frac{3\pi}{4}\right) + \sin^2(\pi) + \sin^2\left(\frac{5\pi}{4}\right) + \sin^2\left(\frac{3\pi}{2}\right) + \sin^2\left(\frac{7\pi}{4}\right)$$

$$= 0 + \frac{1}{2} + 1 + \frac{1}{2} + 0 + \frac{1}{2} + 1 + \frac{1}{2} = 4$$

and the values repeat thereafter $\Rightarrow E \to \infty$

$$P = \lim_{n\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2 = \lim_{n\to\infty} \frac{2}{2N+1} \sum_{n=0}^{N} \sin^2\left(\frac{\pi n}{4}\right)$$

from above, $\sum_{n=0}^{N} \sin^2\left(\frac{\pi n}{4}\right) \to \frac{N}{8} \cdot 4$ [as the sum is 4 for N=8
8 for N=

$$= \frac{N}{2}$$

$$\therefore P = \lim_{N\to\infty} \frac{2 \cdot \frac{N}{2}}{2N+1} = \lim_{N\to\infty} \frac{N}{2N+1} = \frac{1}{2} < \infty$$

$\therefore x$ is a power signal.

4. $x_a(t) = 5\sin(200\pi t)$.

clearly, this signal has time period $T = \frac{1}{100}$ s

$$\Rightarrow f_m = \frac{1}{T} = 100 \text{ Hz}$$

$\therefore$ the min. sampling rate $f_s = 2f_m = 200$ Hz.

$f_s = 250$ Hz

we know that $x_d(n) = x_a\left(\frac{n}{f_s}\right)$.

$$= 5\sin\left(200\pi \cdot \frac{n}{f_s}\right)$$

$$= 5\sin\left(200\pi \cdot \frac{n}{250}\right)$$

$$= 5\sin\left(\frac{4}{5}\pi n\right)$$

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5. $x_a(t) = 3\cos(80\pi t) + 5\sin(40\pi t) - 10\cos(160\pi t)$.

$$T_1 = \frac{1}{40} \qquad T_2 = \frac{1}{20} \qquad T_3 = \frac{1}{80}$$

$\Rightarrow T = \text{lcm} = \frac{1}{20}$ s. $\Rightarrow f_m = 20$ Hz

$\therefore$ the nyquist rate $f_s = 2f_m = 40$ Hz.

$$x_d(n) = x_a\left(\frac{n}{f_s}\right) = x_a\left(\frac{n}{40}\right)$$

$$= 3\cos(2\pi n) + 5\sin(\pi n) - 10\cos(4\pi n)$$

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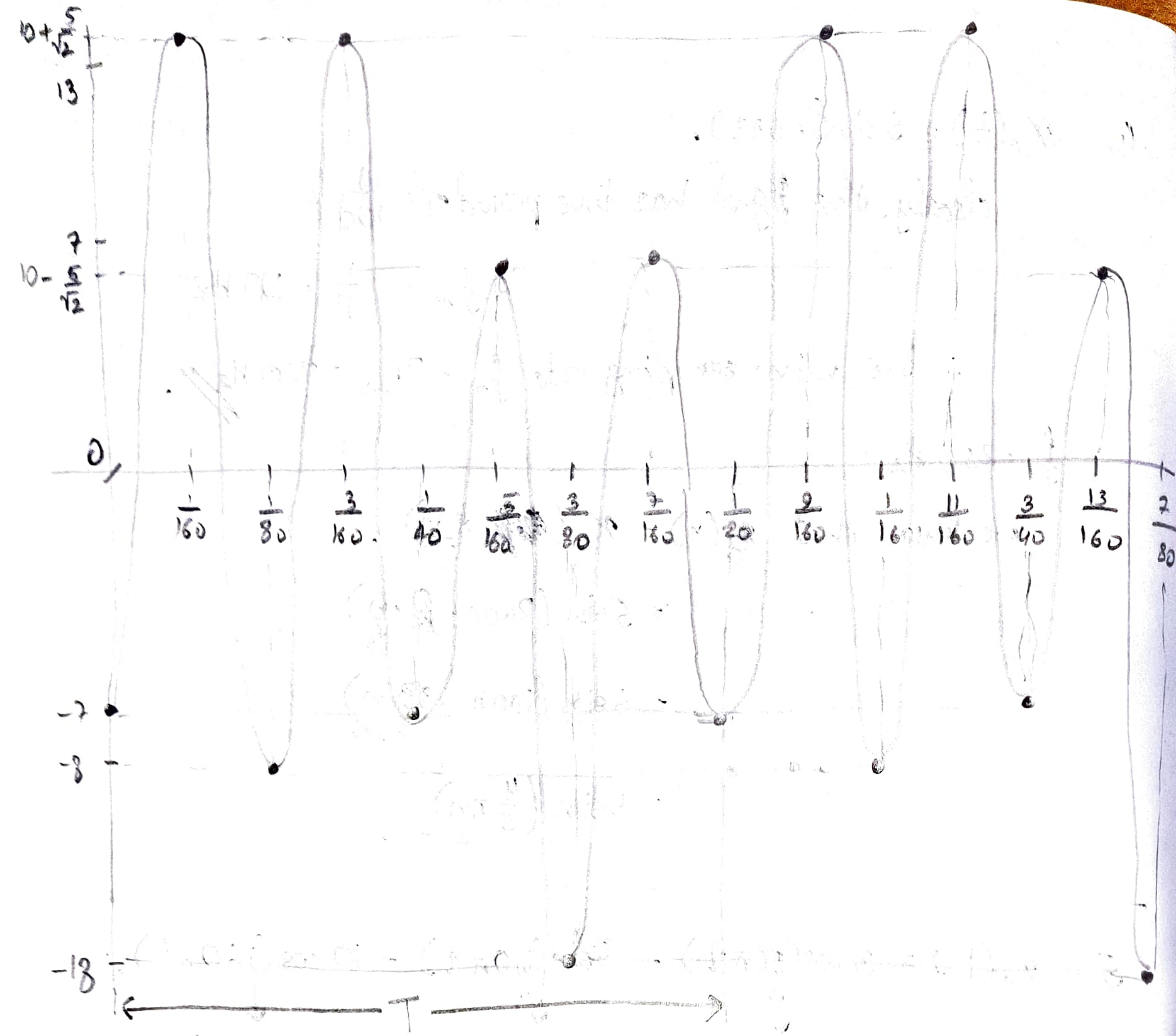
5. $x_a(t) = 3\cos(80\pi t) + 5\sin(40\pi t) - 10\cos(160\pi t)$

$$T_1 = 1/40 \qquad T_2 = 1/20 \qquad T_3 = 1/80$$

$$\Rightarrow f_1 = 40 \text{ Hz} \qquad f_2 = 20 \text{ Hz} \qquad f_3 = 80 \text{ Hz}$$

$\Rightarrow f_{max} = 80$ Hz $\Rightarrow f_s = 160$ Hz.

$$\therefore x_d(n) = x_a\left(\frac{n}{160}\right) = 3\cos\left(\frac{1}{2}\pi n\right) + 5\sin\left(\frac{1}{4}\pi n\right) - 10\cos(\pi n)$$



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6. $x_a(t) = 5\cos(2000\pi t) + 5\sin(6000\pi t) - 7\cos(12000\pi t)$

$$f_1 = 1000\,Hz \qquad f_2 = 3000\,Hz \qquad f_3 = 6000\,Hz$$

$$\Rightarrow f_{max} = 6000\,Hz \Rightarrow f_s = 12000\,Hz$$

if $f_s = 5000\,Hz$,

$$x_d(n) = x_a\left(\frac{n}{f_s}\right) = x_a\left(n/5000\right)$$

$$= 5\cos\left(\tfrac{2}{5}\pi n\right) + 5\sin\left(\tfrac{6}{5}\pi n\right) - 7\cos\left(\tfrac{12}{5}\pi n\right)$$

$$= 5\cos\left(\tfrac{2}{5}\pi n\right) + 5\sin\left(\left(2 - \tfrac{4}{5}\right)\pi n\right) - 7\cos\left(\left(2 + \tfrac{2}{5}\right)\pi n\right)$$

$$= 5\cos\left(\tfrac{2}{5}\pi n\right) - 5\sin\left(\tfrac{4}{5}\pi n\right) - 7\cos\left(\tfrac{2}{5}\pi n\right)$$

$$= -5\sin\left(\tfrac{4}{5}\pi n\right) - 2\cos\left(\tfrac{2}{5}\pi n\right)$$

To reconstruct,

$$x_r(t) = \sum_{k=-\infty}^{\infty} x_d[k] \cdot \operatorname{sinc}\left(\frac{t-kT}{T}\right)$$

for $x_d(t)$, $T = 5s$.

$$\therefore x_r(t) = \sum_{k=-\infty}^{\infty} -\left(5\sin\left(\frac{4}{5}\pi k\right) + 2\cos\left(\frac{2}{5}\pi k\right)\right) \cdot \operatorname{sinc}\left(\frac{t}{5} - k\right)$$

7.
$$x(t) = \begin{cases} \sin(\pi t/4), & 0 \le t \le 4 \\ 4 - t, & 4 \le t \le 6 \\ t - 8, & 6 \le t \le 9 \\ 1, & 9 \le t \le 10. \end{cases}$$

$f_s = 1000$ Hz.

$l = -2V$; $u = 2V$; $n = 2$.

(a) The sampling points are $\{0, \frac{1}{1000}s, \frac{2}{1000}s, \frac{3}{1000}s, \ldots\} \ni t$.

0, $\sin\left(\frac{\pi}{4000}\right)$, $\sin\left(\frac{\pi}{2000}\right)$, $\sin\left(\frac{3\pi}{4000}\right)$ ...

(b)

2V – 1V	11
1V – 0V	10
0V – −1V	01
−1V – −2V	00

~~(c) $n = -3$: $x(nT) = \sin\left(-\frac{3\pi}{4000}\right) \in [-1, 0)$: 01~~

~~$n = -2$: $x(nT) = \sin(-2\pi/4000) \in [-1, 0)$: 01~~

~~$n = -1$: $x(nT) = \sin(-\pi/4000) \in [-1, 0)$: 01~~

~~$n = 0$: $x(nT) =$~~

(c) $n = 0$: $x(nT) = \sin(0) \in [0, 1)$: 10.

$n = 1$: $x(nT) = \sin(\pi/4000) \in [0, 1)$: 10

$n = 2$: $x(nT) = \sin(2\pi/4000) \in [0, 1)$: 10

$\vdots$

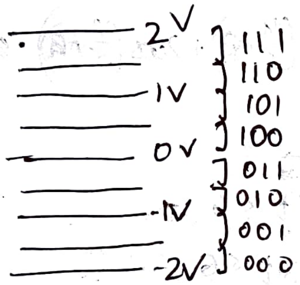
(d) u starts as 101010...

(e) Bitrate $= n \cdot f_s = 2000$ bits/s.

(f). max quantisation error = $\frac{\text{step size}}{2}$ = 0.5V.

(g) $n = 3 \Rightarrow$ step size = $\frac{4}{8} = 0.5$ V.

Quant^n intervals:



Digital words:

$n = 0, 1, 2$: 100

Bit stream : 100100100...

Bitrate : $n f_s = 3000$ bits/s.

Quant^n error : $\frac{0.5}{2} = 0.25$ V

(h) 3 bit Quant^n is better as the error is reduced by a factor of 2 while bitrate only increases 1.5x.

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8. (a) $y(n) = \frac{a}{x(n)}$

let $x'(n) = x(n-k)$.

$T(x') = y'$

$y'(n) = \frac{a}{x'(n)} = \frac{a}{x(n-k)} = y(n-k)$.

$\therefore$ the system is time invt.

(b) $y[n] = 3x(n) + 5x(n-2)$

let $x'(n) = x(n-k)$

$T(x') = y'$

$y'(n) = 3x'(n) + 5x'(n-2)$

$= 3x(n-k) + 5x(n-2-k) = y(n-k)$

$\therefore$ the system is time invt.

(c) $y(n) = x(-n)$

$x'(n) = x(n-k)$; $T(x') = y'$

$y'(n) = x'(-n) = x(-n-k) \neq y(n-k)$

$= x(-(n-k))$

$\therefore$ the system is time-variant.

(d) $y(n) = nx(n)$.

$x'(n) = x(n-k)$; $T(x') = y'$

$y'(n) = nx'(n) = nx(n-k) \neq y(n-k) = (n-k)x(n-k)$

$\therefore$ the system is time variant.

(e) $y(n) = x(5n)$

$x'(n) = x(n-k)$; $T(x') = y'$

$y'(n) = x'(5n) = x(5n-k) \neq y(n-k) = x(5(n-k))$

$\therefore$ the system is time variant

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9. (a) $y(n) = 5x(n) + 7x(n-1)$.

let $T(x_1) = y_1$; $T(x_2) = y_2$.

$ay_1(n) + by_2(n) = a(5x_1(n) + 7x_1(n-1)) + b(5x_2(n) + 7x_2(n-1))$

$= 5(ax_1(n) + bx_2(n)) + 7(ax_1(n-1) + bx_2(n-1))$

$= T(ax_1 + bx_2)$.

$\therefore$ the system is linear.

(b) $y(n) = 4x(n) - \dfrac{9}{x(n-1)}$

let $T(x_1) = y_1$; $T(x_2) = y_2$

$ay_1(n) + by_2(n) = a\left(4x_1(n) - \dfrac{9}{x_1(n-1)}\right) + b\left(4x_2(n) - \dfrac{9}{x_2(n-1)}\right)$

$\neq 4(ax_1(n) + bx_2(n)) - \dfrac{9}{ax_1(n-1) + bx_2(n-1)}$

$\therefore$ the system is non linear.

(c) $y(n) = \sum_{m=0}^{N} b_m x(n-m) - \sum_{m=1}^{N} d_m x(n-m)$

let $T(x_1) = y_1$; $T(x_2) = y_2$

$ay_1(n) + by_2(n) = a\left(\sum_{m=0}^{N} b_m x_1(n-m) - \sum_{m=1}^{N} d_m x_1(n-m)\right)$

$+ b\left(\sum_{m=0}^{N} b_m x_2(n-m) - \sum_{m=1}^{N} d_m x_2(n-m)\right)$

$$= \sum_{m=0}^{N} b_m \left(ax_1(n-m) + bx_2(n-m)\right) - \sum_{m=1}^{N} d_m \left(ax_1(n-m) + bx_2(n-m)\right)$$

$$= T\left(ax_1 + bx_2\right)$$

$\therefore$ The system is linear.

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10. (a) $y(n) = ax(n) + bx(n-1)$.

The output $y(n)$ depends only on present input $x(n)$ & past input $x(n-1)$

$\Rightarrow$ The system is causal.

(b) $y(n) = ax(n-1) + bx(n+1)$

The output $y(n)$ depends on past input $x(n-1)$ but also future input $x(n+1)$.

$\Rightarrow$ The system is non-causal.

(c). $y(n) = \sum_{k=0}^{\infty} x(n-k) = x(n) + x(n-1) + x(n-2) + \cdots$

The output $y(n)$ depends only on present input $x(n)$ and past inputs $x(n-k)$, $k \geq 1$.

$\Rightarrow$ The system is causal.

(d) $y(n) = \sum_{k=0}^{\infty} x(n+k) = x(n) + x(n+1) + x(n+2) + \cdots$

The output of the system depends on present input $x(n)$ but also future inputs $x(n+k)$, $k \geq 1$.

$\Rightarrow$ The system is non-causal.
