DSA Assignment-3

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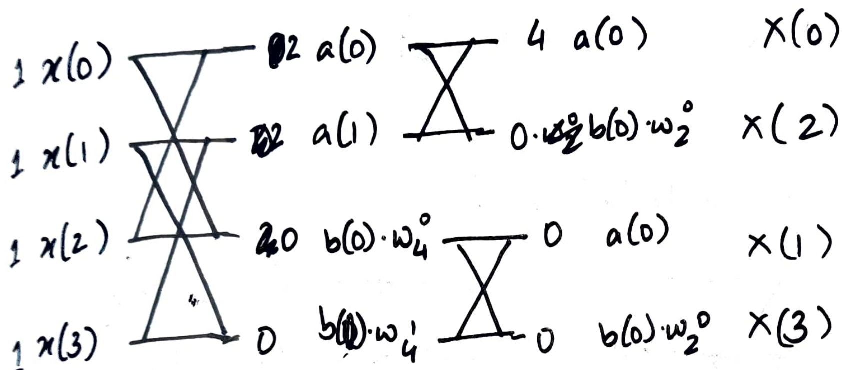
1. $x(n) = \{1, 1, 1, 1\}$
   $h(n) = \{1, 0, 1, 0\}$

We wish to find $y(n) = x(n) * h(n)$

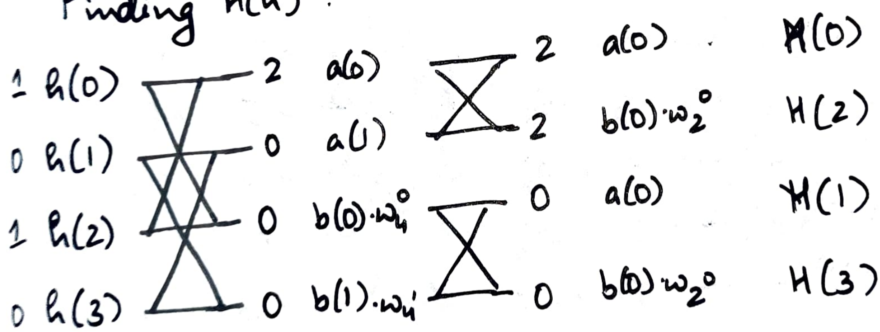
We take DFT on both sides : $Y(n) = X(n) \times H(n)$.

Finding $X(n)$:



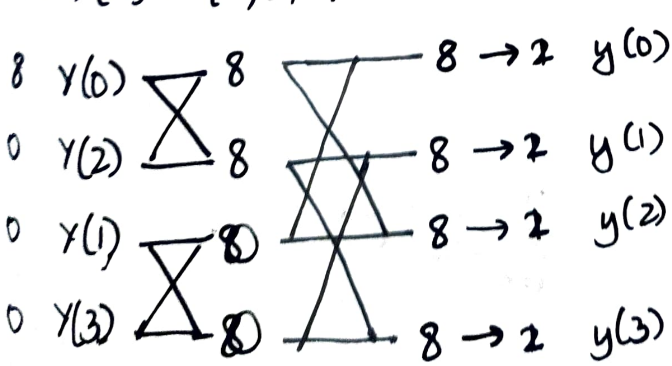
$\therefore X(n) = \{4, 0, 0, 0\}$

Finding $H(n)$:



$\therefore H(n) = \{2, 0, 2, 0\}$

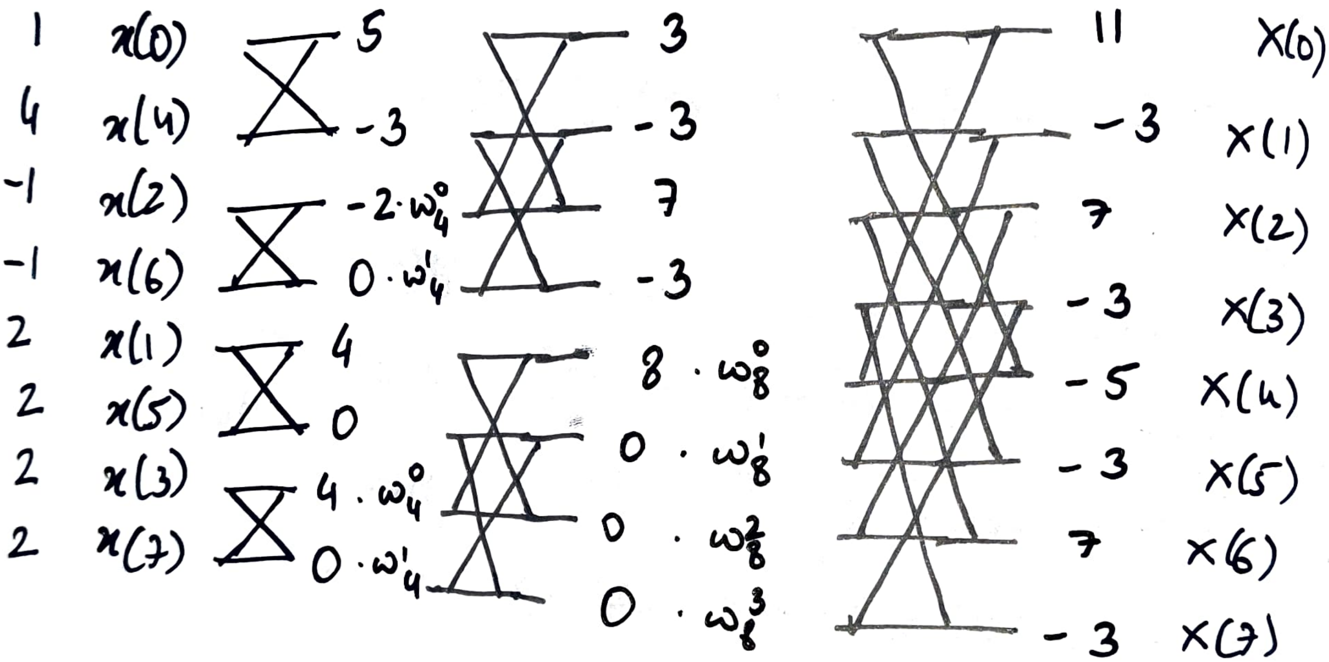
$\therefore Y(n) = \{8, 0, 0, 0\}$



$\therefore y(n) = \{2, 2, 2, 2\}$ //

2.

2. $x(n) = \{1, 2, -1, 2, 4, 2, -1, 2\}$



$\therefore X(n) = \{11, -3, 7, -3, -5, -3, 7, -3\}$

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3. (i) $x(n) = \left(\frac{1}{4}\right)^n u(n)$

$$X(z) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{4}\right)^n u(n) \cdot z^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n}$$

$$= \frac{z}{z - \frac{1}{4}}$$

The RoC is $|z| > |p| \Rightarrow |z| > 1/4$.

(ii) $x(n) = [5(2^n) - 4(3^n)]u(n)$

$= 5\cdot 2^n u(n) - 4\cdot 3^n u(n) = x_1(n) - x_2(n)$

$X_1(z) = \sum_{n=-\infty}^{\infty} 5\cdot 2^n u(n) z^{-n} = 5\cdot \frac{z}{z-2}$; R_1 : $|z| > 2$

$X_2(z) = \sum_{n=-\infty}^{\infty} 4\cdot 3^n u(n) z^{-n} = 4\cdot \frac{z}{z-3}$; R_2 : $|z| > 3$

$\therefore X(n) = \frac{5z}{z-2} - \frac{4z}{z-3} = \frac{z(z-7)}{(z-2)(z-3)}$; $R : |z| > 3$.

(iii) $x(n) = na^n u(n)$

we know that $a^n u(n) \xrightarrow{z} \frac{z}{z-a} = \frac{1}{1-\frac{a}{z}}$

$\Rightarrow na^n u(n) \xrightarrow{z} -z\cdot \frac{d}{dz}\left(\frac{1}{1-\frac{a}{z}}\right)$

$= -z\cdot -\left(\frac{1}{1-\frac{a}{z}}\right)^2 \cdot \frac{a}{z^2}$

$X(z) = \frac{1}{\frac{z}{a}-1}$; $R : |z| > |a|$

(iv) $x(n) = \{3, 4, \underset{\uparrow}{8}, 7, 0, 4\}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n)\cdot z^{-n} = 3z^2 + 4z + 8 + \frac{7}{z} + \frac{4}{z^3}$; $R : \mathbb{C} - \{0, \infty\}$

(v): $x(n) = a^n u(n) + b^n u(-n-2)$

$= x_1(n) + x_2(n)$

$X_1(z) = \sum_{n=-\infty}^{\infty} a^n u(n)\cdot z^{-n} = \frac{z}{z-a}$; $|z| > |a|$

$X_2(z) : -b^n u(-n-1) \rightarrow \frac{z}{z-b}$; $|z| < |b|$

$\Rightarrow b^n u(-n-1) \rightarrow -\frac{z}{z-b} \Rightarrow b^n u(-n-2) \rightarrow \frac{z\cdot z^{-1}}{b-z}$

$$\therefore X(z) = \frac{z}{z-a} + \frac{1}{b-z} \quad ; \quad R: |a| < z < |b|$$

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4. $x_1(n) = 2\delta(n) - \delta(n-1)$
   $x_2(n) = 4\delta(n) + 3\delta(n-1)$

(i) We wish to find $X(z) = Z(x_1(n) * x_2(n))$
$= X_1(z) \times X_2(z)$

$X_1(z):\ 2\delta(n) - \delta(n-1)$

$$\begin{array}{cc} 2\delta(n) & \delta(n-1) \\ \downarrow & \downarrow \\ 1 & z^{-1}\cdot 1 \\ R: \mathbb{C} & R: \mathbb{C} - \{0\} \end{array}$$

$\therefore X_1(z) = 2 - z^{-1}$; $R: \mathbb{C} - \{0\}$

$X_2(z):\ 4\delta(n) + 3\delta(n-1)$

$$\begin{array}{cc} \delta(n) & \delta(n-1) \\ \downarrow & \downarrow \\ 1 & z^{-1}\cdot 1 \\ R_1: \mathbb{C} & R: \mathbb{C} - \{0\} \end{array}$$

$\therefore X_2(z) = 4 + 3z^{-1}$; $R: \mathbb{C} - \{0\}$

$\therefore X(z) = (2 - z^{-1})(4 + 3z^{-1})$
$= 8 + 2z^{-1} - 3z^{-2}$; $R: \mathbb{C} - \{0\}$

(ii) ~~x(n)~~ $X(z) = (2 - z^{-1})(4 + 3z^{-1}) = 8 + 2z^{-1} - 3z^{-2}$

$\Rightarrow x(n) = \{\underset{\uparrow}{8}, 2, -3\}$
$= 8\delta(n) + 2\delta(n-1) - 3\delta(n-2)$

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5. $H(z) = \dfrac{z+1}{z-0.5}$

(i) $H(z) = 1 + z^{-1}\dfrac{1.5z}{z-0.5}$

$$\begin{array}{cc} 1 & \frac{1.5z}{z-0.5} \\ \downarrow & \downarrow \\ \delta(n) & 1.5\,(0.5)^n u(n) \end{array}$$

$\Rightarrow h(n) = \delta(n) + 1.5(0.5)^{n-1} u(n-1)$.

(ii) $y(n) = $ ~~tra~~ response of $u(n)$. $= u(n) * h(n)$

$$\Rightarrow Y(z) = \left(\frac{z}{z-1}\right) \cdot H(z) = \frac{z \cdot (z+1)}{(z-1)(z-0.5)}$$

$$\begin{array}{r|l} & 1 \\ z^2 - 1.5z + 0.5 & z^2 + z \\ & \underline{z^2 - 1.5z + 0.5} \\ & 2.5z - 0.5 \end{array}$$

$$\therefore Y(z) = 1 + \frac{2.5z - 0.5}{(z-1)(z-0.5)}$$

$A + B = 2.5$

$0.5A + B = 0.5$

$$= 1 + \frac{A = 4}{z-1} + \frac{B = -1.5}{z - 0.5}$$

$$= 1 + z^{-1} \cdot \frac{4z}{z-1} - z^{-1} \cdot \frac{1.5z}{z-0.5}$$

$$\Rightarrow y(n) = \delta(n) + 4u(n-1) - 1.5(0.5)^{n-1} u(n-1).$$

(iii) $x(n) = (0.2)^n u(n)$

$$\Rightarrow X(z) = \frac{z}{z - 0.2}$$

$$Y(z) = \frac{z}{z-0.2} \cdot \frac{z+1}{z-0.5} = \frac{z^2 + z}{z^2 - 0.7z + 0.1}$$

$$= 1 + \frac{1.7z - 0.1}{(z-0.2)(z-0.5)}$$

$$= 1 + \frac{A = \cancel{-0.8}\ 2.5}{z - 0.2} + \frac{B = \cancel{2.5}\ -0.8}{z-0.5}$$

$A + B = 1.7$

$0.5A + 0.2B = 0.1$

$$= 1 + z^{-1} \cdot \frac{2.5z}{z-0.2} - z^{-1} \cdot \frac{0.8z}{z-0.5}$$

$$\Rightarrow y(n) = \delta(n) + 2.5(0.2)^{n-1} u(n-1) - 0.8(0.5)^n u(n-1).$$

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6. $y(n) = 0.2x(n) + x(n-1) + 0.3x(n-3) + 0.5x(n-4)$

Taking z-transform on both sides,

$$Y(z) = 0.2X(z) + z^{-1}X(z) + 0.3z^{-3}X(z) + 0.5z^{-4}X(z)$$

$$\Rightarrow H(z) = \frac{Y(z)}{X(z)} = 0.2 + z^{-1} + 0.3z^{-3} + 0.5z^{-4}$$

$$\Rightarrow h(n) = \{\underset{\uparrow}{0.2}, 1, 0, 0.3, 0.5\}$$