Digital Signal Analysis (CS7.303)

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Systems (contd.)

LTI Systems

LTI systems are those which are both linear and time-invariant. For example, consider the system

$$y(n) = ay(n-1) + x(n).$$

An important characteristic of an LTI system is its *impulse response*, *i.e.*, its output when the input is the impulse $\delta(n)$. It is denoted $h(n)$.

LTI systems have the property that $y(n) = x(n)h(n)$, *i.e.*, the output is the convolution of the input with the impulse response. We will prove this for discrete systems.

It can be shown that

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\delta(n-k),$$

as $\delta(n-k)$ becomes 1 only for $k = n$, which makes the value of the sum $x(n)$.

Now, we apply the system on both sides.
As it is linear, we can consider each term separately.
In each term, we have a constant coefficient $x(k)$ and a time-shifted impulse $\delta(n-k)$.
By time-invariance, however, this is simply $h(n-k)$.

Putting all this together, we get

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k),$$

which is the same as

$$y(n) = x(n)h(n).$$

Note that when taking the convolution, we let n be $l_1 + l_2 - 1$, where l_1 is the number of nonzero values of $x(n)$, and l_2 the number of nonzero values of $h(n)$. This is called a *linear convolution*.

The impulse response of an LTI system also helps us determine if it is causal or not. An LTI system is causal iff

$$h(n) = 0, \forall n < 0.$$

Similarly, an LTI system is stable iff

$$\sum_{k=-\infty}^{\infty} |h(k)|$$

is finite.

Sampling and Quantisation

Sampling and quantisation are two processes carried out on signals. Analog signals are converted to discrete ones by sampling, and discrete ones to digital ones by quantisation.

Sampling

For information to not be lost in the process of sampling, the frequency of sampling f_S has a minimum bound given by the Nyquist criterion:

$$f_S \geq 2f_m,$$

where f_m is the frequency of the analog input signal.

Quantisation

Quantisation involves discretising the output of the signal, given the number of bits N and the range $[l, u]$. The expression for the step size given these parameters is

$$\frac{(u-l)}{2^N}.$$

The maximum quantisation error is given by half the step size.