Machine, Data and Learning (CS7.301)

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Solving Problems by Searching (contd.)

Informed Search Algorithms

Best-first search is an informed search algorithm which uses an *evaluation function* $f(n)$ for each node. This function quantifies the "desirability" of a node. Thus the most desirable unexpanded node is followed.

One special case of this is greedy best-first search, where $f(n)$ is defined to be a heuristic $h(n)$ estimating the cost from n to the goal. This algorithm is neither complete nor optimal. It has time and space complexity $O(b^m)$, but this can be improved by the choice of heuristic.

A^* search is another special case of best-first search. The evaluation function here incorporates the cost of the path up to the current node, *i.e.*, $f(n) = g(n) + h(n)$, where $g(n)$ is the cost of the node and $h(n)$ is the estimated cost from n to the goal.
It is complete (unless there are infinitely many nodes n such that $f(n) \leq f(G)$) and optimal. Its time is exponential.

Heuristics

A heuristic $h(n)$ is *admissible* if it is bounded by the true cost to reach the goal from n, *i.e.*, it never overestimates the cost. In other words, it is optimistic.

It can be proved that if $h(n)$ is admissible, then A^* using tree search is optimal. Let G be the optimal goal and suppose some suboptimal goal G' has been generated in the fringe. Let n be an unexpanded node in the fringe.
Then we have $f(G') > f(G)$, and $h(n) \leq h^*(n)$, the true cost. Adding $g(n)$ to both sides, we have $g(n) + h(n) \leq g(n) + h^*(n)$. This means that $f(n) \leq f(G)$, which in turn means $f(n) < f(G')$. Thus G' will never be expanded.

Furthermore, a heuristic is consistent if for every node n, and every child n' of n generated by a,

$$h(n) \leq c(n, a, n') + h(n')$$

holds, where c is the cost function for a step. If $h(n)$ is consistent, then $f(n') > f(n)$, *i.e.*, $f(n)$ is non-decreasing.

It can be shown that if $h(n)$ is consistent, then A^* using graph search is optimal.

A heuristic $h_1(n)$ is dominated by another $h_2(n)$ is $h_2(n) \geq h_1(n)$ for all n, and both are admissible. This means that $h_2(n)$ is better for a search.