PROJECT:MID EVALS

Numerical Algorithms

Analysing Cart Pole Swing up (Z6)

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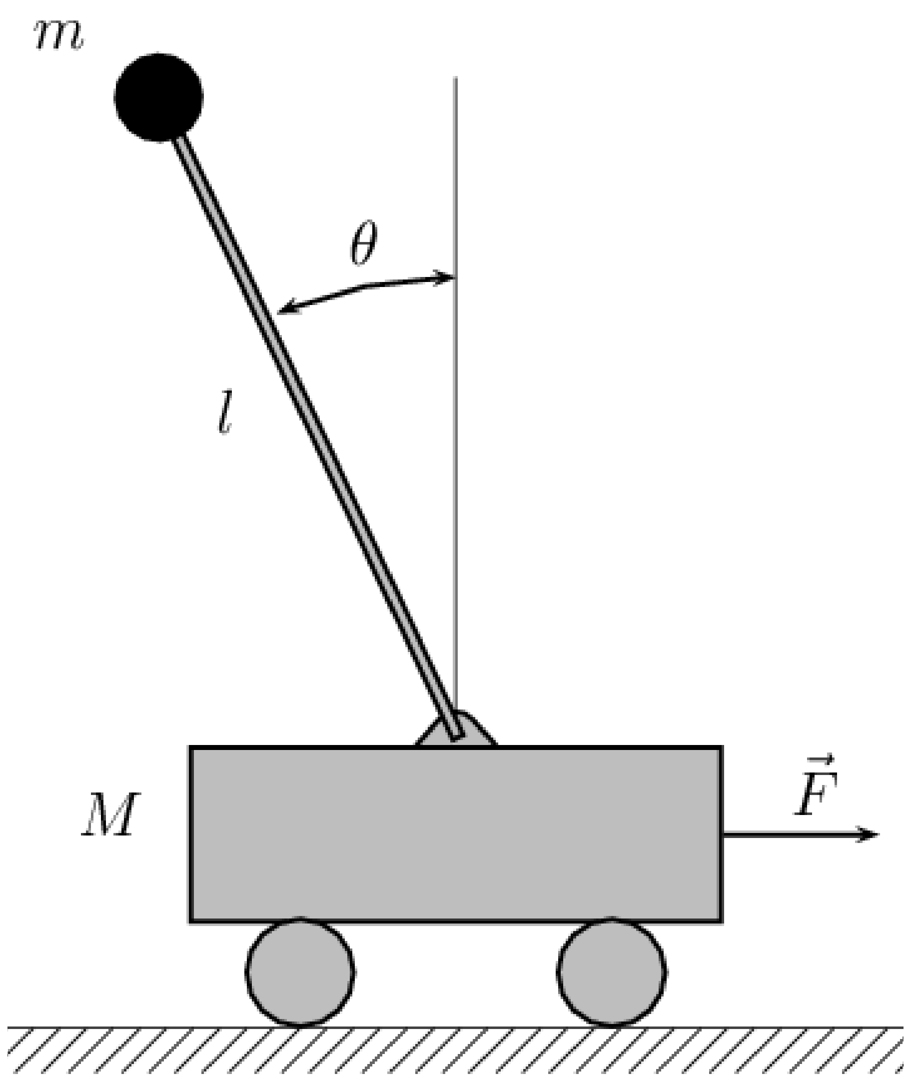
THEORY

- The premise of the problem is to perform the swing up maneuver while minimizing the Objective Function (J).
- We have to find the optimal control force function u(t).

$$J = \int_0^T u^2(\tau)\, d\tau.$$

- What can we solve ? Non Linear Program (NLP)
- 'fmincon' in Matlab can solve NLP

$$\begin{aligned} &\text{minimize } f(x) \\ &\text{subject to } g_i(x) \leq 0 \text{ for each } i \in \{1, \dots, m\} \\ &\qquad\qquad h_j(x) = 0 \text{ for each } j \in \{1, \dots, p\} \\ &\qquad\qquad x \in X. \end{aligned}$$



Approach

- Formalizing the system :

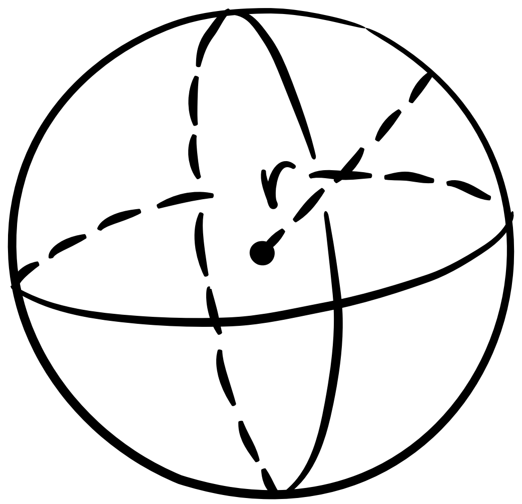
$$\ddot{q}_1 = \frac{\ell\, m_2 \sin(q_2)\, \dot{q}_2^2 + u + m_2\, g\, \cos(q_2)\, \sin(q_2)}{m_1 + m_2\left(1 - \cos^2(q_2)\right)}$$

$$\ddot{q}_2 = -\frac{\ell\, m_2 \cos(q_2)\, \sin(q_2)\, \dot{q}_2^2 + u\, \cos(q_2) + (m_1 + m_2)\, g\, \sin(q_2)}{\ell\, m_1 + \ell m_2\left(1 - \cos^2(q_2)\right)}$$

$$\boldsymbol{x} = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \qquad \dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, u) = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}$$

$$\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$y = mx + b$$



$$V = \frac{4}{3}\pi r^3$$

Approach

- Discretizing the equations :

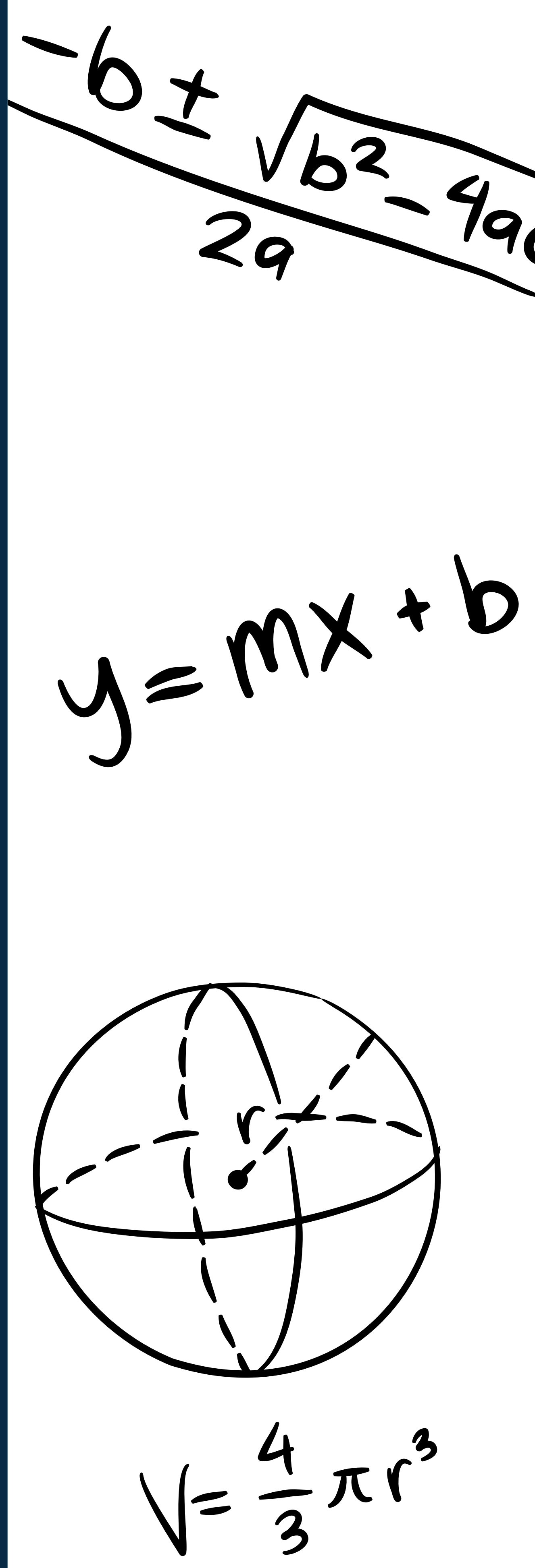
$$J = \sum_{k=0}^{N-1} \frac{h_k}{2} \left(u_k^2 + u_{k+1}^2 \right)$$

$$\frac{1}{2} h_k \left(\boldsymbol{f}_{k+1} + \boldsymbol{f}_k \right) = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k, \qquad k \in 0, \ldots, (N-1)$$

- What are we tying to find? The decision variables:

$$\boldsymbol{x}_0, \ldots, \boldsymbol{x}_N \quad u_0, \ldots, u_N$$

- Add boundary conditions
- Setup is ready to be passed into a NLP solver



Hermite Simpson

- Using quadratic instead of linear splines.
- We also have collocation points in the midpoint of each segment.

$$J = \sum_{k=0}^{N-1} \frac{h_k}{6}\left(u_k^2 + 4u_{k+\frac{1}{2}}^2 + u_{k+1}^2\right)$$

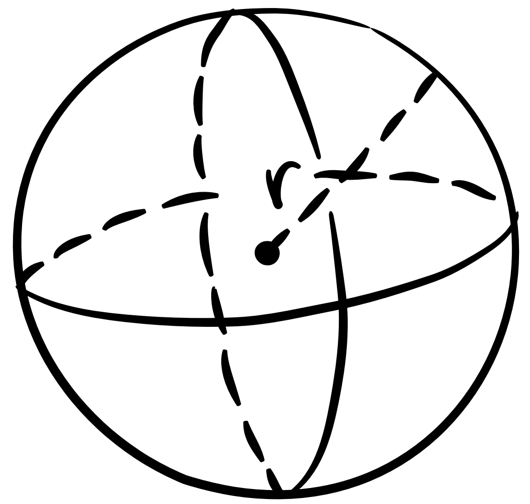
$$\frac{h_k}{6}\left(\boldsymbol{f}_k + 4\boldsymbol{f}_{k+\frac{1}{2}} + \boldsymbol{f}_{k+1}\right) = \boldsymbol{x}_{k+1} - \boldsymbol{x}_k$$

- Interpolation at midpoints:

$$\boldsymbol{x}_{k+\frac{1}{2}} = \frac{1}{2}\left(\boldsymbol{x}_k + \boldsymbol{x}_{k+1}\right) + \frac{h_k}{8}\left(\boldsymbol{f}_k - \boldsymbol{f}_{k+1}\right), \qquad k \in 0, \ldots, (N-1)$$

$$\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$y = mx + b$$



$$V = \frac{4}{3}\pi r^3$$

Interpreting the Results

- The NLP solver returns the optimal values of the decision variables:

$$\boldsymbol{x}_0, \ldots, \boldsymbol{x}_N \quad u_0, \ldots, u_N$$

- We still have to convert them into the continuous functions x(t), u(t) :

$$\boldsymbol{u}(t) \approx \boldsymbol{u}_k + \frac{\tau}{h_k}\left(\boldsymbol{u}_{k+1} - \boldsymbol{u}_k\right)$$

$$\boldsymbol{x}(t) \approx \boldsymbol{x}_k + \boldsymbol{f}_k \tau + \frac{\tau^2}{2h_k}\left(\boldsymbol{f}_{k+1} - \boldsymbol{f}_k\right)$$

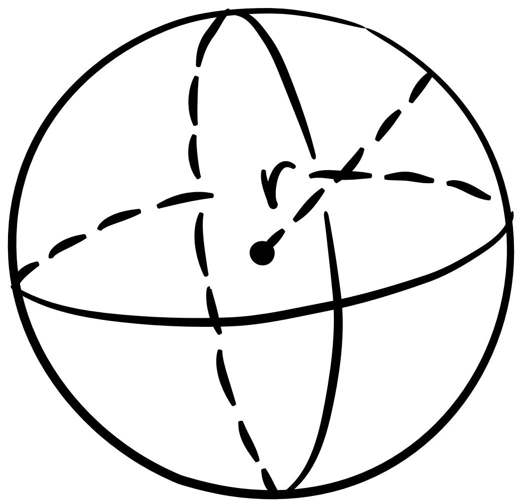
- For Hermite Simpson, use quadratic and cubic splines :

$$\boldsymbol{u}(t) = \frac{2}{h_k^2}\left(\tau - \tfrac{h_k}{2}\right)\left(\tau - h_k\right)\boldsymbol{u}_k - \frac{4}{h_k^2}\left(\tau\right)\left(\tau - h_k\right)\boldsymbol{u}_{k+\frac{1}{2}} + \frac{2}{h_k^2}\left(\tau\right)\left(\tau - \tfrac{h_k}{2}\right)\boldsymbol{u}_{k+1}$$

$$\boldsymbol{x}(t) = \boldsymbol{x}_k + \boldsymbol{f}_k\left(\frac{\tau}{h_k}\right) + \frac{1}{2}\left(-3\boldsymbol{f}_k + 4\boldsymbol{f}_{k+\frac{1}{2}} - \boldsymbol{f}_{k+1}\right)\left(\frac{\tau}{h_k}\right)^2 + \frac{1}{3}\left(2\boldsymbol{f}_k - 4\boldsymbol{f}_{k+\frac{1}{2}} + 2\boldsymbol{f}_{k+1}\right)\left(\frac{\tau}{h_k}\right)^3.$$

$$\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$$

$$y = mx + b$$



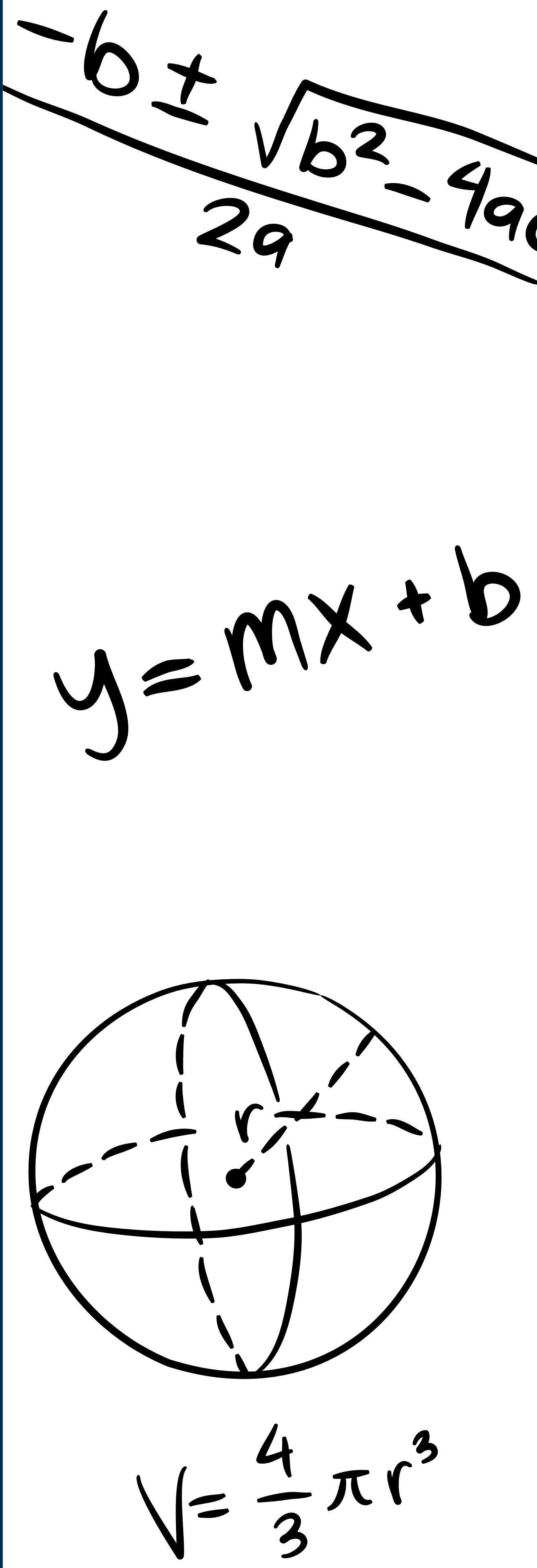
$$V = \frac{4}{3}\pi r^3$$

CODE ...

Running the MATLAB code provided in the following Github repository:
https://github.com/MatthewPeterKelly/OptimTraj

There are 2 optimisation problems solved:
1) Minimum force
2) Minimum time

The minimum time problem is more complex than the minimum force one.



Code ...

The initial conditions and constraints for the problem are:

cart mass = 2 kg

pole mass = 0.5 kg

pole length = 0.5 m

g = 9.81 m/s^2

Min Time

d = 1 m

max force = 50 N

Min Force

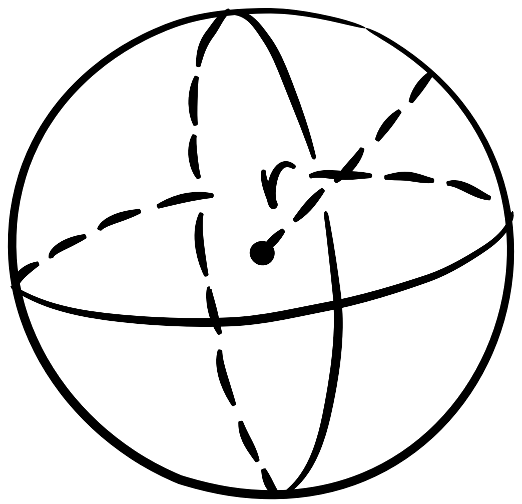
d = 0.8 m

max force = 100 N

duration = 2 s

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

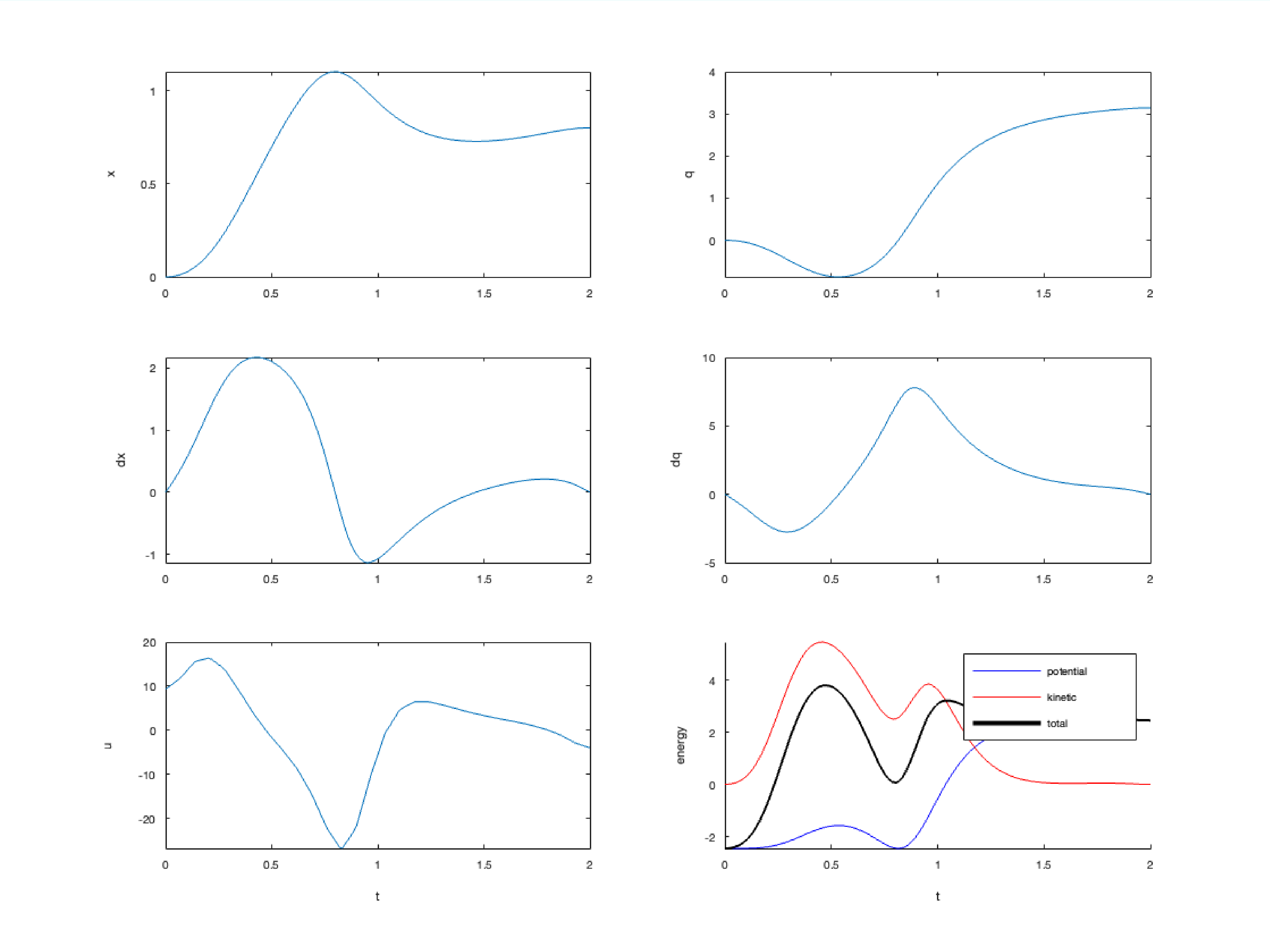
$$y = mx + b$$



$$V = \frac{4}{3}\pi r^3$$

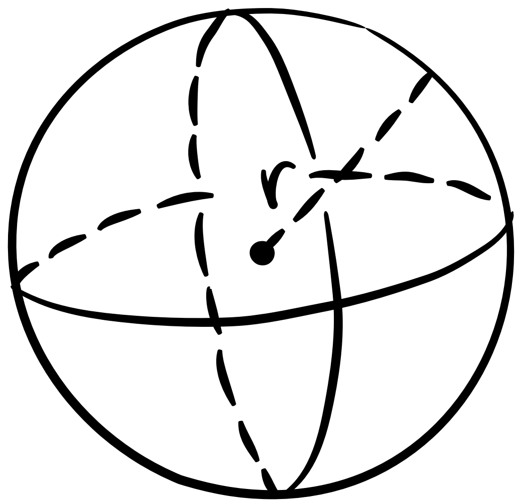
Results...

For minimum force:



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$y = mx + b$$



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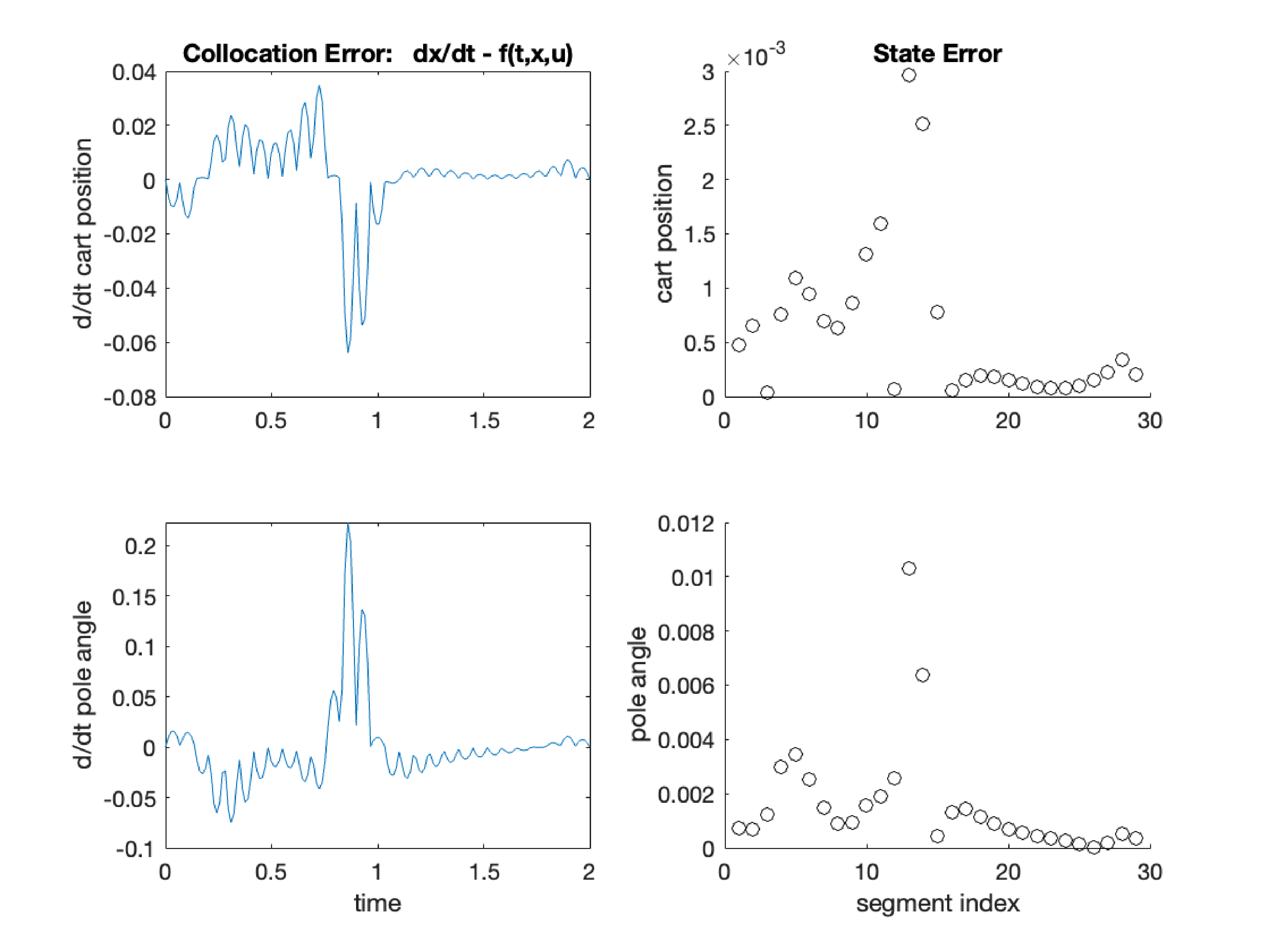
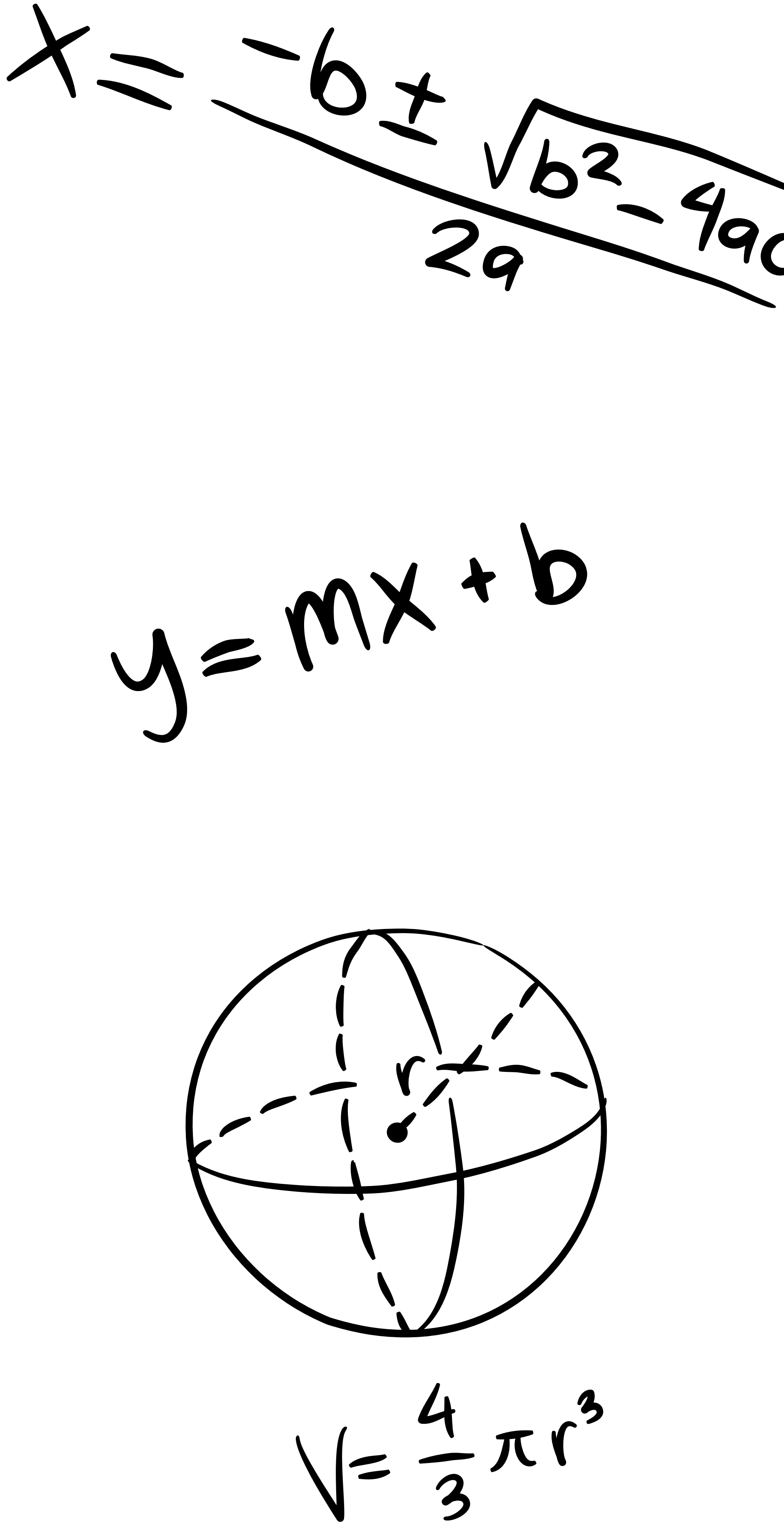
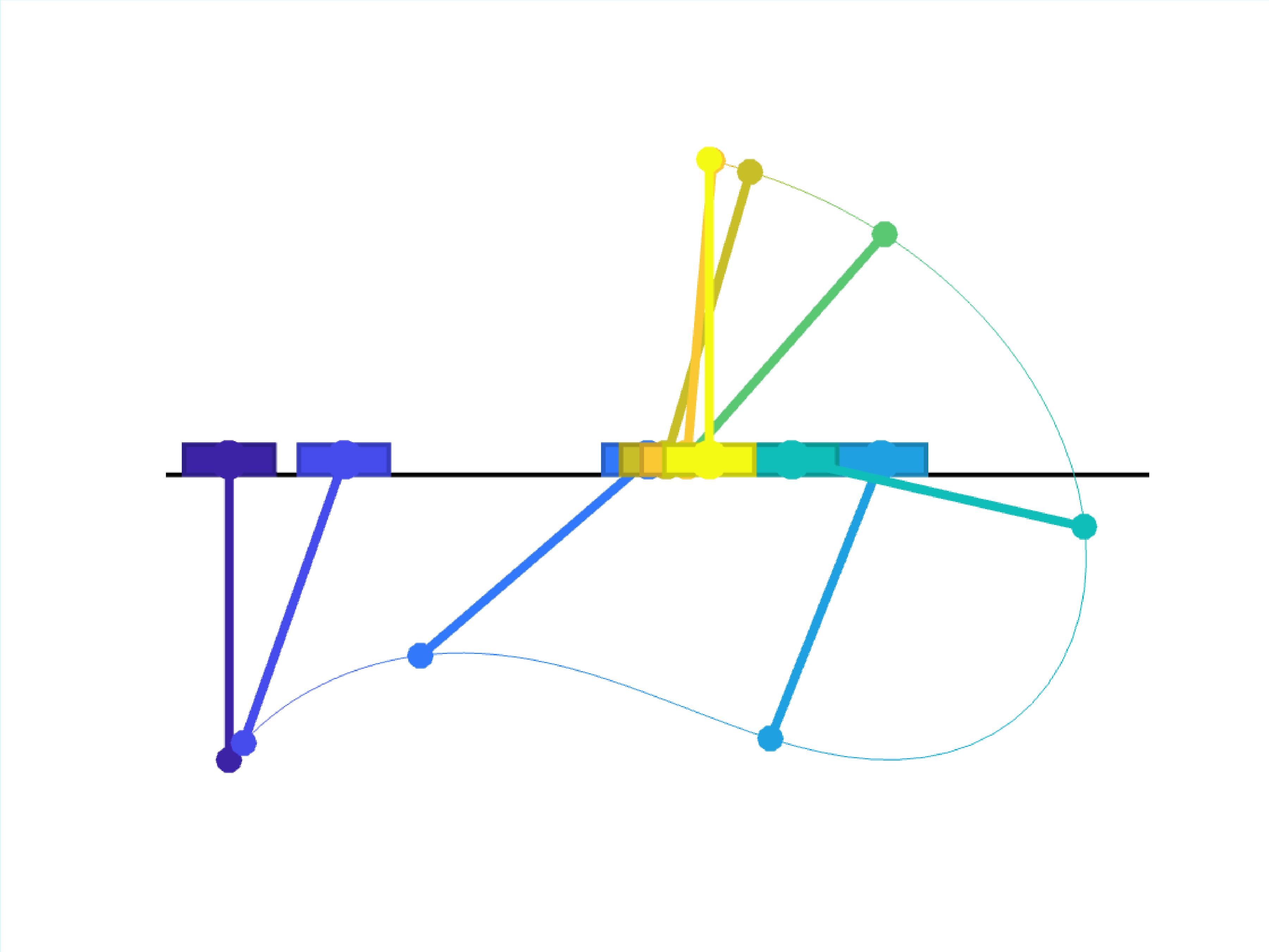




Results...

For minimum force:



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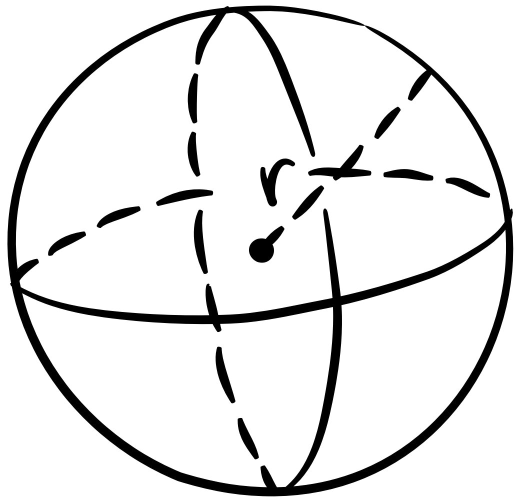

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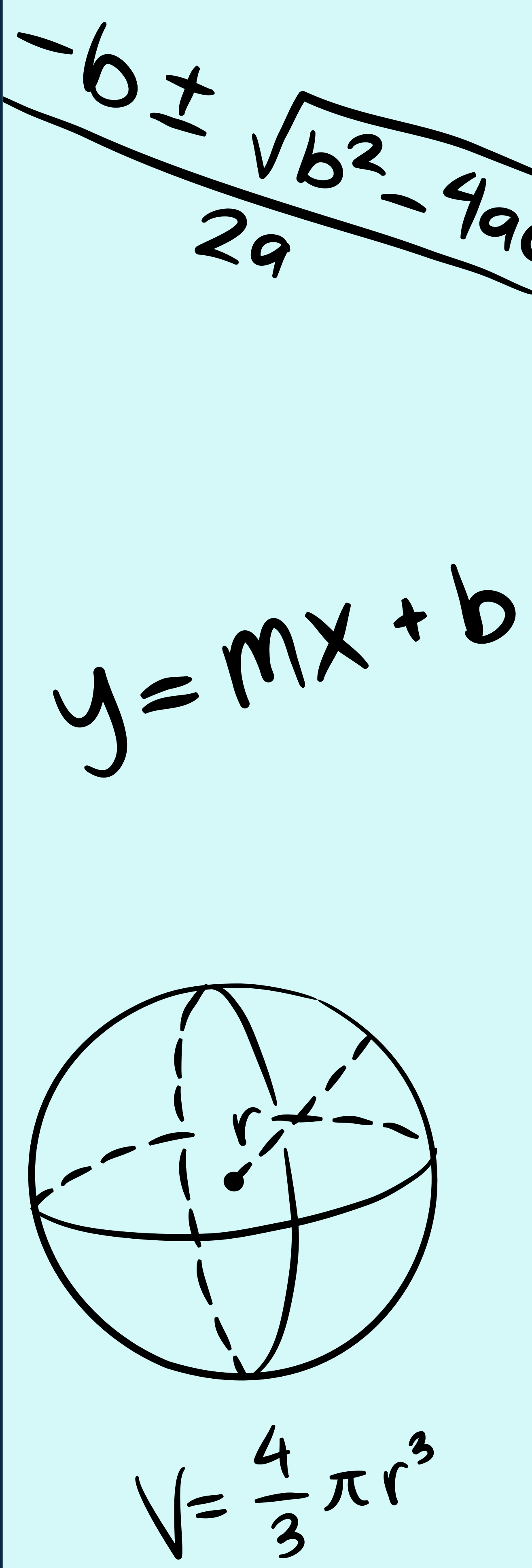
Plan for Final Presentation

Simulate the results to display a video of the trajectory of the system to analyze the model better.

Introduce external perturbations in the system and observe how stable the algorithm is.



Thank You