

Path Planning Based on Traversability Evaluation from Occupancy-Elevation Grid Maps

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Abstract—Path planning in Robotics is a fundamental point towards autonomy, and it refers to the capability to estimate safe and collision free paths from a point of interest to another. This paper introduces a path planning approach for autonomous navigation based on Occupancy-Elevation Grid (OEG) maps. OEG maps provide probabilistic information about the existence and the height of obstacles in an environment. Thus, in this proposed approach, probabilistic information about the terrain are taken from a OEG map in order to an autonomous vehicle compute a feasible path. In addition, the vehicle must consider its physical characteristics and navigation capability to proceed with the path planning process, verifying if it is possible to traverse specific places in its environment. Experiments carried out with real data demonstrate that the proposed approach yields useful results for vehicles autonomous navigation.

Index Terms—Path planning, Occupancy-Elevation grid maps, Traversability.

I. INTRODUCTION

Autonomous agents, like autonomous robots, must be able to collect information about their environment; considering these information, they must make some decision in order to decide how to proceed, facing what exists in the environment; and have to actuate in order to perform the previous decision, towards completing their mission [1].

A fundamental task to an agent achieve autonomy, is to plan collision free motions for complex bodies from a start to a goal position among a collection of obstacles. Sometimes, this question is thought as a relatively simple problem, but this geometric path planning problem is computationally hard. Recent advances in this topic have taken into account additional points that are inherited from mechanical and sensor limitations of real robots such as uncertainties, feedback, and differential constraints, which further complicate the development of automated planners [2].

In order to proceed with a path planning algorithm, it is fundamental that an environment representation, that is, a map, be available to the robot [3]. There is a set of approaches for representing computationally an environment. Occupancy grid map is one of the most used approaches. It is an interesting approach for path planning since it represents, in a rough way, the metrics of the environment considering uncertainties. The mapped environment is represented as a regular 2D or 3D grid composed of cells that contains, each one, an occupancy

probability value [4]. Therefore, all places of the environment can be classified into occupied by an obstacle, free, or not mapped, which can be considered for estimating a safe path.

However, in some cases, it is possible to face with robots or vehicles with a robust mechanism of locomotion that can provide them the capability of traversing some kind of obstacles. Thinking about these situations, the Occupancy-Elevation Grid Map (OEG Map) was proposed in [5][6], which is an alternative approach for environment mapping with three-dimensional information using a compact structure.

The OEG representation consists of a two-dimensional grid where each cell stores a probabilistic estimation of occupancy, the height of the terrain and its variance. Therefore, OEG maps can be applied for robot navigation with classification of terrain traversability, in a path planning process, aiming shorter and safe path estimation. The occupancy probability, the height of the obstacles and their variance, can be used to allow the robot decide if it is feasible to traverse a determined spot, taking into account its locomotion skills and hardware characteristics like, its own height and wheels diameter.

Thus, in this paper we propose a path planning algorithm that is based on a defined traversability measurement, which is a function of the information available in an Occupancy-Elevation Grid map, that provides a coherent probabilistic way to represent an environment under sensory uncertainties. In this way, we explicitly take into account sensory uncertainties in the path planning process, in order to estimate a safe and efficient path, considering not only occupancy information, as proposed by traditional algorithms, but also elevation measurements to identify traversable areas. We use the *wavefront expansion algorithm*, that is an efficient and simple-to-implement technique to find routes in fixed-size cell array [7], which is the case of the OEG maps. In order to validate the proposed approach, preliminary experiments with real maps are presented.

This paper is organized as follow: section II introduces the Occupancy-Elevation Grid map with its formulation. Section III provides the methodology used for computing traversability values from OEG maps. Section V presents the path planner adopted in this work. Section VI presents some preliminary experiments. Finally, some considerations and future works are presented in section VII.

II. OCCUPANCY-ELEVATION GRID - OEG MAP

In this section the formulation of the OEG maps is revisited, as exposed in [6]. OEG is defined as a regular two-dimensional grid, \mathbf{M} , of N cells, with three-dimensional information, in which each cell n stores the occupancy probability $p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t})$, the height or elevation $\mu_{0:t,n}$ and its variance $\sigma_{0:t,n}^2$ for a mapped spot. Eq. (1) describes mathematically a OEG map.

$$\mathbf{M} = \{ \langle p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t}), \mu_{0:t,n}, \sigma_{0:t,n}^2 \rangle, \quad n = 1, \dots, N \} \quad (1)$$

where $\mathbf{s}_{0:t}$ is the historic of the robot poses and $\mathbf{z}_{0:t}$ is the historic of sensor readings estimated considering the temporal evolution up to the instant t . Elevation values are described by a normal distribution $\mathcal{N}(\mu_{0:t}, \sigma_{0:t}^2)$ and the occupancy probability $p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t})$ is given by a probability density. Fig. 1 illustrates this representation.

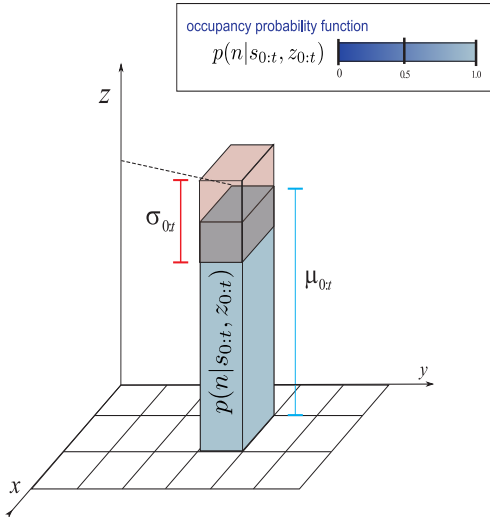


Figure 1: Occupancy-Elevation Grid composition. Each cell stores occupancy $p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t})$, elevation $\mu_{0:t}$ and variance $\sigma_{0:t}^2$ values.

This representation brings some characteristics:

Three-dimensional modeling: OEG maps allow the representation of three-dimensional information of the mapped area.

Compactness: OEG map stores 3D information in a 2D grid. This means a significant decrease of memory requirement to represent three-dimensional data.

Updatable: It is possible to update the map (occupancy, elevation and variance values) whenever new sensory readings are provided. This is an important issue in dynamic environments.

Probabilistic approach: Probabilistic tools provide a coherent way for modeling data under uncertainties effect.

A. Occupancy Value Updating

In order to determine the occupancy probability $p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t})$ of each cell n of the map \mathbf{M} , we used a

Gaussian based approach, in which the occupancy probability value is calculated by the probability density function (2).

$$p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t}) = 1 - \frac{1}{1 + e^{L_{0:t,n}}} \quad (2)$$

where

$$L_{0:t,n} = L_{0:t-1,n} + \log \frac{p(n|\mathbf{s}_t, \mathbf{z}_t)}{1 - p(n|\mathbf{s}_t, \mathbf{z}_t)} - \log \frac{1 - p(n)}{p(n)} \quad (3)$$

A \log notation is used to avoid numerical instabilities. The occupancy of a cell $p(n|\mathbf{s}_{0:t}, \mathbf{z}_{0:t})$, depends on all poses of the robot up to time t , $\mathbf{s}_{0:t}$, and all sensors readings, $\mathbf{z}_{0:t}$. The probability $p(n|\mathbf{s}_t, \mathbf{z}_t)$ is the inverse sensor model that is specified according to the used sensor and $p(n)$ is the prior occupancy value for the cell n assigned before any sensory measurement.

In practice, when occupancy grid map is used for navigation, a threshold on the occupancy probability value calculated by (2) is applied. A cell n is considered occupied when the threshold is reached and is considered free otherwise. In this work, the elevation is also considered as an important variable in order to estimate a navigation path.

B. Elevation Value Updating

Elevation and its error for each cell are estimated with a Kalman filter based formulation, which considers a new measurement, h_t , with its uncertainty represented by σ_t^2 at time t . Those values are calculated by a sensor model for calculating the elevation (z-coordinate) of a spot, taking into account 3D sensor information. Some model are proposed in [6][8].

Equations (4) and (5), derived from Kalman formulation, are applied to calculate the elevation, $\mu_{0:t}$ and its variance $\sigma_{0:t}^2$ respectively, for a cell n , considering all sensor readings up to time t .

$$\mu_{0:t,n} = \frac{\sigma_{t,n}^2 \mu_{0:t-1,n} + \sigma_{0:t-1,n}^2 h_t}{\sigma_{0:t-1,n}^2 + \sigma_{t,n}^2} \quad (4)$$

$$\sigma_{0:t,n}^2 = \frac{\sigma_{0:t-1,n}^2 \sigma_{t,n}^2}{\sigma_{0:t-1,n}^2 + \sigma_{t,n}^2} \quad (5)$$

Fig. 3 illustrates the OEG map estimated from the Canadian Planetary Emulation Terrain 3D Mapping Dataset ¹, which covers a circular workspace area 40m in diameter, shown in Fig. 2.

In this workspace, gravel was distributed to emulate scaled planetary hills and ridges, providing characteristic natural, unstructured terrain. Figs. 3a, 3b and 3c show the occupancy probabilities, elevation values and variances of the elevation values, respectively, of the scenario seen in Fig. 2.

¹Courtesy of the Autonomous Space Robotics Lab (ASRL) at the University of Toronto Institute for Aerospace Studies, available at <http://asrl.utias.utoronto.ca/datasets/3dmap>

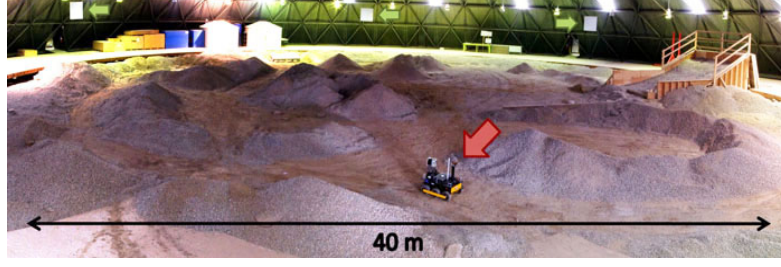


Figure 2: Mapped area.

III. TRAVERSABILITY ESTIMATION

As aforementioned, each cell of an OEG map, that represents a mapped spot, provides three information namely, the occupancy probability $O_{0:t,n}$, the elevation of the mapped site $\mu_{0:t,n}$ and its variance $\sigma_{0:t,n}$. Based on these information we proposed the following formulation in order to estimate a traversability measurement, that is, a value that will indicate if a determined spot is traversable or not.

Let $P_T(n)$ be defined as the traversability probability of a cell n in the OEG map. This value will be used for the path planner estimating a feasible, safe and efficient path. $P_T(n)$ is mathematically specified as described in Eq. (6).

$$P_T(n) = 1 - p(n|s_{0:t}, z_{0:t}) \cdot p_h(\mu_{0:t,n} \geq h) \quad (6)$$

where $p(n|s_{0:t}, z_{0:t})$ is the occupancy probability of the cell n , calculated by Eq. (2) and $p_h(\mu_{0:t,n} \geq h) \in [0, 1]$ can be defined as the probability of the elevation of the cell n be higher than a possible robot traversable height h . In this equation we consider that $p(n|s_{0:t}, z_{0:t})$ and $p_h(\mu_{0:t,n} \geq h)$ are probabilistically independent.

The probability $p_h(\mu_{0:t,n} \geq h)$ is described as a cumulative distribution function (CDF), as shown in Eq. (7).

$$p_h(\mu_{0:t,n} \geq h) = \begin{cases} 1, & \text{if } \mu_{0:t,n} \geq h - \sigma_{0:t,n} \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

This equation returns 1 if the elevation mean of the cell n is greater than the maximum traversable height h minus the sensory uncertainty $\sigma_{0:t,n}$ over the cell elevation. This ensures that the robot will overcome only possible obstacles. And 0 is returned, in the case that the obstacle can be traversable, considering its elevation mean $\mu_{0:t,n}$. The traversable height threshold h must be valued according to the robot mechanical capabilities for overcoming obstacles.

In this way, we are able to compute the traversability probability for each cell $n \in \mathbf{M}$, considering occupancy probability and elevation probability, both based on the information stored in the OEG map.

IV. GRID ADAPTATION

The traditional approach for path planning on occupancy grid maps, is to set up a threshold value for the occupancy probability [9]. Cells on the map with occupancy probability

bigger than the threshold are considered obstacles, and otherwise, they are acceptable as free spaces, where a vehicle may navigate.

In our case, a threshold value will be set up for the traversability probability estimation, and it will obey the mechanical capabilities of the vehicle used in the navigation, as described by Eq. (7). In this way, the OEG map will be transformed in a bitmap [9], where each cell n acquires a binary value, 1 or 0, indicating if it is traversable or not, respectively. Fig. 4 exposes the traversability probability values computed from the OEG map shown in Fig. 3, considering that our robot can overcome obstacles until 0.15m height, and Fig. 5b exhibits the respective bitmap.

V. PATH PLANNING

We use the bitmap resulting from the aforementioned step in order to proceed with the path planning. Following, we introduce some concepts to clarify the process.

A. Obstacles and the Configuration Space

We will follow a similar idea as the proposed in [10] to formalize the path-planning problem. Let us denote the configuration space, or C-space, Q , of the robot system as the space of all possible configurations $n = [x, y]$ of the system, where x and y are coordinates of the grid map. The C-space is a lightly modified version of the bitmap, in which the robot is considered as a rigid body that occupies exactly one cell with configuration $R(n) = [x_r, y_r]$ and obstacles are increased by the number of cells that are equivalent to the radius of a circle that encompasses the robot.

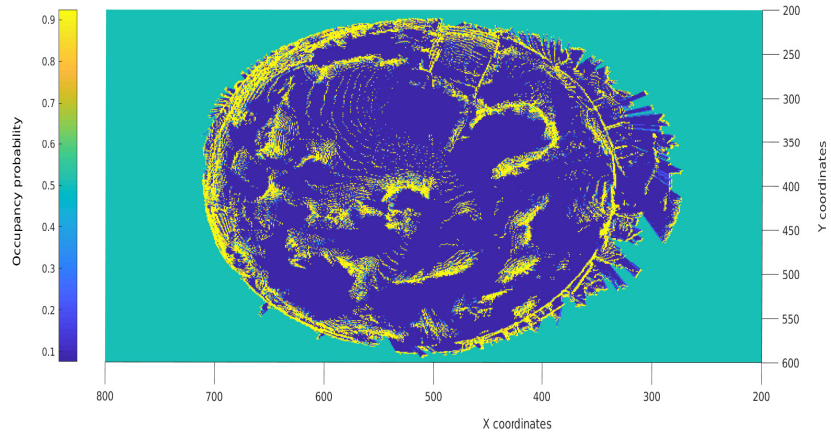
Therefore, the path-planning problem can be defined as the problem of determining a set of configurations $n \in Q$, such that no configuration of the path causes a collision between robot, $R(n)$, and obstacles. We define a configuration space obstacle QO_i to be a set of cells n at which the robot intersects an obstacle WO_i in the workspace (Eq. (8)).

$$QO_i = \{n \in Q | R(n) \cap WO_i \neq \emptyset\} \quad (8)$$

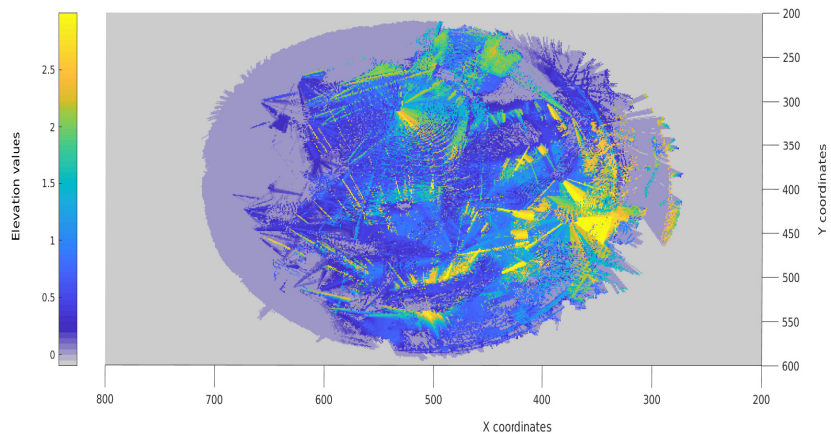
The free space Q_{free} is the set of configurations (or cells) at which the robot does not intersect any obstacle (Eq. (9)).

$$Q_{free} = Q \setminus (\cup_i QO_i) \quad (9)$$

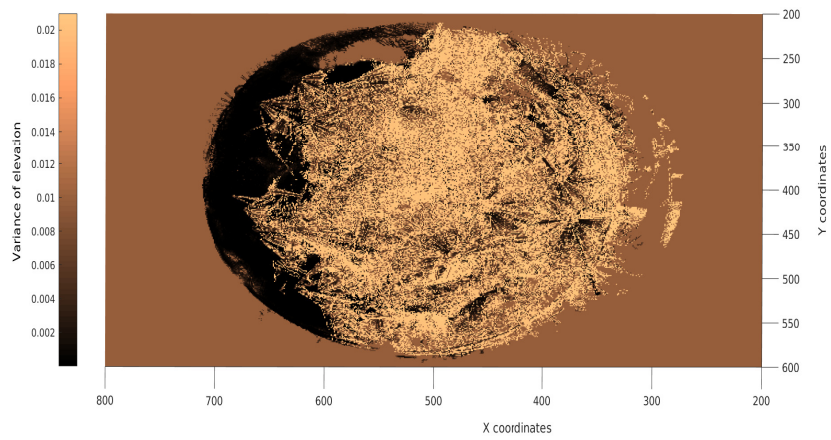
Thus, a *free path* is a set of configurations (or cells) n_i , such that, $\forall n_i \in Q_{free}$.



(a) Occupancy probability values.



(b) Elevation values.



(c) Variance of the elevation values.

Figure 3: Estimated OEG map for outdoor environment.

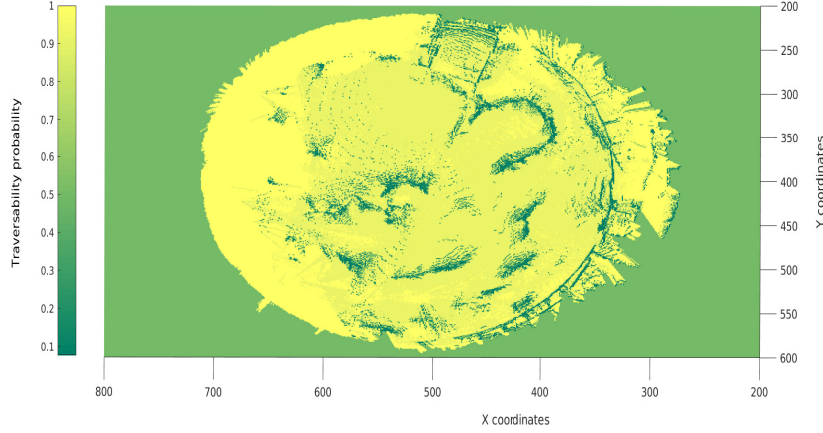


Figure 4: Estimated traversability probability map for path planning.

B. Path planning algorithm

For path planning, we used the *wavefront expansion algorithm*, which is an instance of breadth-first search algorithm. This algorithm is an efficient and simple-to-implement technique to find routes in fixed-size cell array [7]. It employs wavefront expansion from the goal configuration (or cell) $n_g = [x_g, y_g]$ outward, making for each cell n its Manhattan distance to the goal [11]. This operation continues until the initial configuration $n_i = [x_i, y_i]$ is reached.

At this point, the path planner can recover a specific solution trajectory by linking together cells that are adjacent and always closer to the goal [7].

VI. EXPERIMENTS

In order to validate our proposal, we have performed some experiments, in which we can evaluate, by comparing, the planned path with the OEG map information (Figs. 3a, 3b and 3c) and with only occupancy probability information (Fig. 3a).

In these experiments, the grid maps were constructed from the Canadian Planetary Emulation Terrain 3D Mapping Dataset, with 0.1m of cell resolution. We considered that our robot could overcome obstacles until 0.15m height.

Fig. 5 exhibits the resultant planned paths. Fig. 5a shows the bitmap acquired from the occupancy probability information, shown in Fig. 3a, and Fig. 5b exposes the bitmap estimated from the OEG map information (Figs. 3a, 3b and 3c). Circles in red represent initial positions and the green ones are final positions. Ellipses in red highlight obstacles in the path of the robot during the experiments that were overcome.

In the first experiment, we established that the initial position would be $p_{ini} = (50m, 52m)$ and the final position should be $p_{fin} = (59m, 25m)$. Fig. 5a exhibits the resultant path for the traditional approach. The dashed line in red indicates the planned path. Analyzing the mean path distance metric [3], according to this result, the robot would travel a distance of 39.12m. Fig. 5b shows the planned path using

OEG map information. The dashed line in magenta represents the resultant path, and in this case, the robot should travel a distance of 35.09m. The proposed approach resulted in a path, approximately 4.03m less than the approach using only occupancy information.

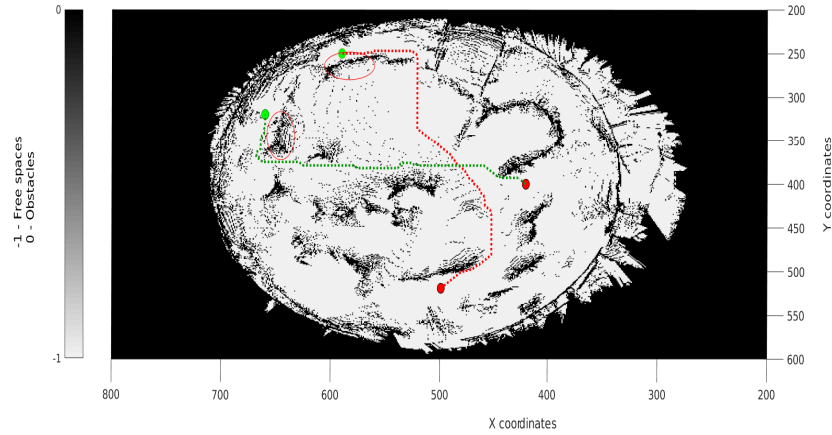
In the second experiment, we set up the initial position in $p_{ini} = (40m, 42m)$ and the final position in $p_{fin} = (36m, 67m)$. The dashed line in green in Fig. 5a displays the planned path using traditional approach (only occupancy information). According to this result, the robot would travel a distance of 31.93m. In Fig. 5b, we have displayed the planned path using OEG map information, represented by the dashed line in orange. In this case, the robot should travel a distance of 27.31m. The proposed approach resulted in a path 4.62m less than the approach using only occupancy information.

In both experiments we obtained a smaller path using OEG map information for path planning. The OEG map made it possible to classify some obstacles as traversable. The path planner analyzed those obstacle as navigable places, calculating shortcuts. Thus, the planner estimated feasible, safe and efficient paths with OEG maps.

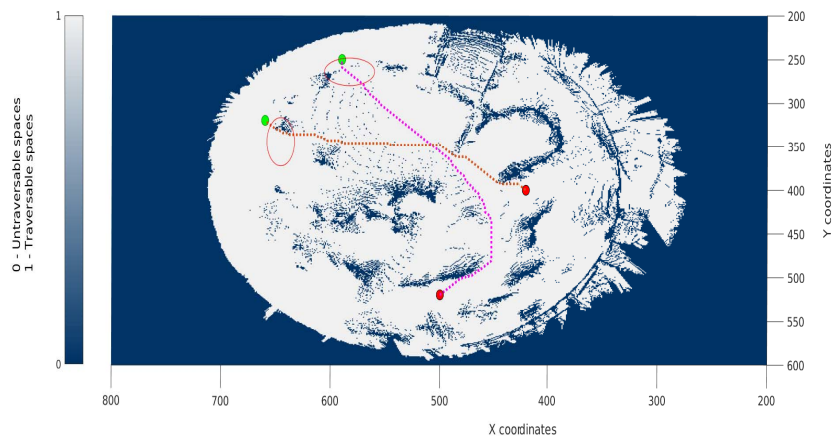
VII. CONCLUSION

This paper presented a path planning approach for autonomous navigation based on Occupancy-Elevation Grid (OEG) maps. OEG maps provide probabilistic information about the existence and the height of obstacles in an environment. The proposed approach used those information in order to estimate traversability values that classify existing obstacles as traversable or not. This classification is used to provide shorter paths in comparison to traditional the approach (that uses only occupancy information), making it possible the reduction of energy cost in a robot mission.

Experiments carried out with real data demonstrated that the proposed approach yields useful results for vehicles autonomous navigation, producing feasible, safe and efficient paths from OEG maps.



(a) Bitmap from occupancy grid.



(b) Bitmap from traversability map.

Figure 5: Bitmaps yielded from occupancy probability and from traversability probability, respectively.

As future works, the proposed method will be improved with more robust path panning algorithms like Probabilistic Road Maps (PRM), Markov Decision Process based planners, Stochastic Functional Gradient based planner [12] to produce more suitable treatment for the robot motion uncertainties.

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