
Full Paper Title

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Abstract

1. Introduction

2. Related Work

3. Preliminaries and Problem Setup

Reasoning structure. We model inference as expansion of a directed tree (or a DAG with tie-breaking) $G = (V, E)$. Each node $v \in V$ is a *reasoning step* with textual content x_v ; the root v_0 holds the task statement. Each edge $e = (u \rightarrow v) \in E$ carries a natural-language label L_e and induces a control vector Π_e used to expand the child v . We distinguish two roles: a *labeller* LM Λ that proposes edge labels, and a *tuner* LM Ψ that emits control, with mappings

$$L = \Lambda(P, C), \quad \Pi = \Psi(P, L, C).$$

Here P denotes the parent node text (and any exposed metadata), and C denotes a compact context.

Context C . We keep C compact and measurable. In our setting, C may include:

- **Frontier uncertainty:** summaries such as the median σ across candidate values;
- **Novelty:** nearest-neighbor distances among frontier candidates (embedding or lexical);
- **Depth:** distance from the root;
- **Sibling/frontier summaries:** best (μ, σ) among siblings;
- **Raw label history:** the most recent edge labels as *strings* (from siblings and, optionally, a short frontier window);
- **Budgets:** token usage, retrieval calls, and verification outcomes.

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Control schema Π . The tuner controls a task-agnostic set of fields:

- **Decoding:** temperature, top- p , maximum tokens, repetition penalty;
- **Generation:** `gen_count` $\in \mathbb{N}^+$ (bundle size under this label);
- **Search:** branch quota and an exploration coefficient β ;
- **Retrieval:** mixture weights over indices or corpora;
- **Verification:** number and strictness of checks;
- **(Optional) Selection hint:** `keep_k` $\in \mathbb{N}^+$ (if set, passed to the child-selection module).

Given Π , a downstream selector (agnostic to NLEL) can use scores such as $S = \mu + \beta \sigma$ or a standard ToT culling operator.

Edge labels. Labels are produced by Λ from (P, C) .

Problem instances. An instance consists of a task T , root v_0 text, and an evaluation function producing (μ, σ) for partial answers. Unless noted, we treat G as a tree; extension to DAGs is straightforward by merging isomorphic textual states.

Notation summary.

Symbol	Meaning
P	parent node content (text + exposed metadata)
L	natural-language edge label
C	compact context features (bulleted above)
Λ	labeller LM mapping $(P, C) \rightarrow L$
Ψ	tuner LM mapping $(P, L, C) \rightarrow \Pi$
Π	control vector (decoding, search, retrieval, verification)
μ, σ	value / uncertainty estimates used by the selector
w	retrieval mixture weights over indices/corpora
β	exploration coefficient in selection
c_e, C_t	per-edge and cumulative compute cost
<code>gen_count</code>	generation bundle size (per edge label)

4. Method

4.1. Overview

We propose *Natural Language Edge Labelling* (NLEL), a control layer for structured language-model (LM) reasoning in which each edge carries a natural-language label that specifies *how* the next step should proceed (e.g., “seek a counterexample”, “work backward”, “apply an anthropological lens; probe for defeaters”). A dedicated *tuner* LM reads a tuple (P, L, C) —the parent node P , the edge label L , and the current context C —and maps it directly to a control vector Π that configures decoding, search, retrieval, and verification for the next expansion.

4.2. Inputs, Outputs, and Mapping

Inputs. P is the current parent state (text and optional structure). L is a free-form natural-language directive for the edge. C denotes the remaining state, which can include the partial tree/graph, concise summaries of the frontier and siblings, budget trackers, and verifier configuration.

Output. A control vector Π whose fields actuate the reasoning stack. A task-agnostic schema can include:

- **Decoding:** temperature, top- p , max tokens, repetition penalty;
- **Search:** branch quota, variance/risk coefficient β , and a UCT/exploration constant;
- **Retrieval:** mixture weights over indices or corpora;
- **Verification:** number and strictness of checks.

Mapping. Let $\Psi : (P, L, C) \mapsto \Pi$ denote the tuner mapping. In our prompt-only instantiation (Section ??), Ψ is realized by a JSON parameter emitter that respects a schema with bounds and learns from a compact in-prompt ledger of historical expansions.

4.3. Expansion Procedure

We expand the structure at a parent p in two phases: label emission and bundle generation, followed by a single selection step.

1. **Emit labels.** Use the labeller to obtain a set of edge labels for p : $\mathcal{L}_p = \{L_1, \dots, L_m\}$, where each $L_i = \Lambda(P, C)$. The number of labels may be governed by a search quota or policy.
2. **Generate bundles under each label.** For each $L \in \mathcal{L}_p$, obtain control $\Pi = \Psi(P, L, C)$ and generate a bundle of `gen_count` candidate children under L using Π .

3. **Select children (ToT).** Let $\mathcal{B}(L)$ denote the bundle generated under label L . Form the union of all candidates for the parent, $\mathcal{C}_p = \bigcup_{L \in \mathcal{L}_p} \mathcal{B}(L)$, and apply the standard ToT child-selection operator to \mathcal{C}_p . We inherit ToT’s selector as-is.

4. **Update state.** Add survivors to the frontier and update C (budgets, summaries, raw label history strings).

4.4. Prompt-Only JSON Parameter Emitter (JPE)

The tuner LM receives three ingredients in the prompt: (i) a concise *schema* that specifies control fields and bounds; (ii) a *historical ledger* of $(P_i, L_i, C_i) \mapsto \Pi_i$ with outcomes, where rows are tagged as *Pareto* or *dominated* to provide contrastive signals about efficient trade-offs; and (iii) the *current case* (P, L, C) . It emits a single JSON object Π that must validate against the schema. The ledger can be curated with a lightweight objective that balances task success against compute usage and verification reliability (e.g., `success@compute` with penalties for excessive tokens or failed checks).

4.5. Context Features

To keep C compact and measurable, we surface a small set of features that capture the state of search:

- **Frontier uncertainty:** median σ across candidate downstream values (from ensembles, bootstraps, or dropout estimates);
- **Novelty deficit:** median nearest-neighbor distance among frontier candidates (embedding or lexical);
- **Depth:** distance from root (enables exploration annealing and quota schedules);
- **Sibling/frontier summaries:** best (μ, σ) among siblings; raw label history (strings); budget usage.

4.6. Downstream Selection (Agnostic to NLEL)

We inherit the standard ToT child-selection operator and apply it once to the union of all candidates produced for a parent (across labels).

4.7. Stability and Safety

We employ non-intrusive guards: (i) strict schema/bounds validation for emitted JSON; (ii) projection into a trust region around safe defaults to prevent pathological jumps; and (iii) depth-annealed exploration so late-depth expansions remain conservative.

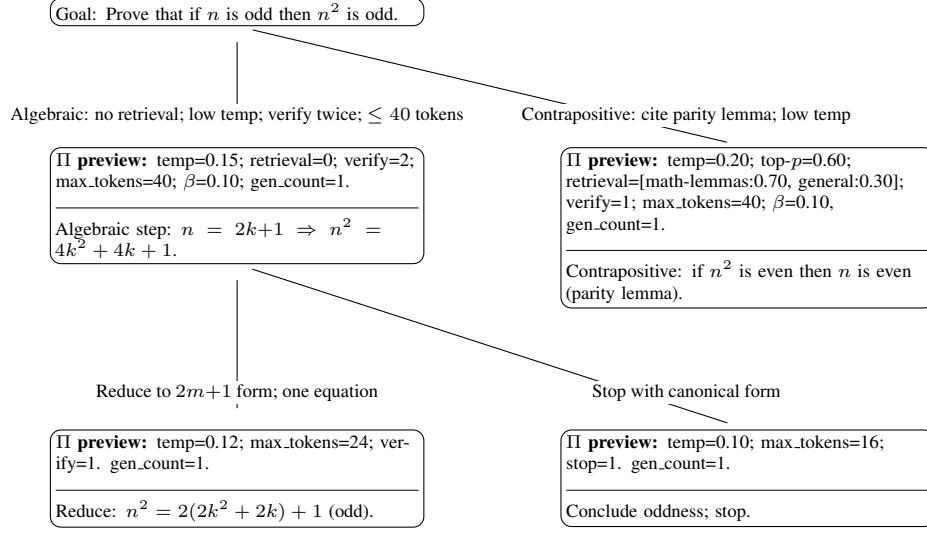


Figure 1. A synthetic example of NLEL being used in a ToT setting. For simplicity, gen_count is set to one for all Π_i .

4.8. Design Notes

NLEL is compatible with a non-reasoning tuner or a reasoning tuner (e.g., CoT/ToT) used *only* as a controller. The child reasoner can be held fixed to cleanly attribute outcomes to the edge label and the control vector Π .

5. Theory (Optional)

6. Experiments

7. Limitations

8. Conclusion

Impact Statement

References

A. Additional Experimental Details

B. Proofs

C. Extra Results