Sail Manual

Kathryn E Gray, Gabriel Kerneis, Peter Sewell

February 25, 2016

Contents

1 Introduction

This is a manual describing the Sail specification language, its common library, compiler, interpreter and type system. However it is currently in early stages of being written, so questions to the developers are highly encouraged.

2 Sail syntax

l	::=			Source location
	|			
annot	::=			
id	::=			Identifier
	|	x		
	|	(**deinfix** x)		remove infix status
	|	**bool**	M	Built in type identifiers
	|	**bit**	M	
	|	**unit**	M	
	|	**nat**	M	
	|	**string**	M	
	|	**range**	M	
	|	**atom**	M	
	|	**vector**	M	
	|	**list**	M	
	|	**set**	M	
	|	**reg**	M	
	|	*to_num*	M	Built in function identifiers
	|	*to_vec*	M	
kid	::=			variables with kind, ticked to differntiate from program variables

	$\vert$	$'x$	
$base\_kind$	::=		base kind
	$\vert$	**Type**	kind of types
	$\vert$	**Nat**	kind of natural number size expressions
	$\vert$	**Order**	kind of vector order specifications
	$\vert$	**Effect**	kind of effect sets
$kind$	::=		kinds
	$\vert$	$base\_kind_1 \rightarrow \ldots \rightarrow base\_kind_n$	
$nexp$	::=		expression of kind Nat, for vector sizes and origins
	$\vert$	kid	variable
	$\vert$	num	constant
	$\vert$	$nexp_1 * nexp_2$	product
	$\vert$	$nexp_1 + nexp_2$	sum
	$\vert$	$nexp_1 - nexp_2$	subtraction, error for nexp1 to be smaller than nexp2
	$\vert$	$2 * * nexp$	exponential
	$\vert$	**neg** $nexp$	For internal use
	$\vert$	$(nexp)$ S	
$order$	::=		vector order specifications, of kind Order
	$\vert$	kid	variable
	$\vert$	**inc**	increasing (little-endian)
	$\vert$	**dec**	decreasing (big-endian)
	$\vert$	$(order)$ S	
$base\_effect$	::=		effect
	$\vert$	**rreg**	read register
	$\vert$	**wreg**	write register

	$\mid$	**rmem**		read memory
	$\mid$	**wmem**		write memory
	$\mid$	**wmea**		signal effective address for writing memory
	$\mid$	**wmv**		write memory, sending only value
	$\mid$	**barr**		memory barrier
	$\mid$	**depend**		dynamic footprint
	$\mid$	**undef**		undefined-instruction exception
	$\mid$	**unspec**		unspecified values
	$\mid$	**nondet**		nondeterminism from intra-instruction parallelism
	$\mid$	**escape**		Tracking of expressions and functions that might call exit
	$\mid$	**lset**		Local mutation happend; not user-writable
effect	::=			effect set, of kind Effects
	$\mid$	*kid*		
	$\mid$	$\{base\_effect_1, .., base\_effect_n\}$		effect set
	$\mid$	**pure**	M	sugar for empty effect set
	$\mid$	$effect_1 \uplus .. \uplus effect_n$	M	meta operation for combining sets of effects
typ	::=			Type expressions, of kind **Type**
	$\mid$	$\_$		Unspecified type
	$\mid$	*id*		Defined type
	$\mid$	*kid*		Type variable
	$\mid$	$typ_1 \rightarrow typ_2$ **effect** *effect*		Function type (first-order only in user code)
	$\mid$	$(typ_1, ..., typ_n)$		Tuple type
	$\mid$	$id\langle typ\_arg_1, .., typ\_arg_n\rangle$		type constructor application
	$\mid$	(typ)	S	
	$\mid$	$[\vert nexp \vert]$	S	sugar for range<0, nexp>
	$\mid$	$[\vert nexp : nexp' \vert]$	S	sugar for range< nexp, nexp'>
	$\mid$	$[: nexp :]$	S	sugar for atom<nexp> which is special case of range<nexp,nexp>
	$\mid$	$typ[nexp]$	S	sugar for vector indexed by [*nexp*]

	|	$typ[nexp : nexp']$	S	sugar for vector indexed by [$nexp..nexp'$]
	|	$typ[nexp <: nexp']$	S	sugar for increasing vector indexed as above
	|	$typ[nexp :> nexp']$	S	sugar for decreasing vector indexed as above
typ_arg	::=			Type constructor arguments of all kinds
	|	*nexp*		
	|	*typ*		
	|	*order*		
	|	*effect*		
n_constraint	::=			constraint over kind **Nat**
	|	$nexp = nexp'$		
	|	$nexp \geq nexp'$		
	|	$nexp \leq nexp'$		
	|	*kid* **IN** $\{num_1, \ldots, num_n\}$		
kinded_id	::=			optionally kind-annotated identifier
	|	*kid*		identifier
	|	*kind kid*		kind-annotated variable
quant_item	::=			Either a kinded identifier or a nexp constraint for a typquant
	|	*kinded_id*		An optionally kinded identifier
	|	*n_constraint*		A constraint for this type
typquant	::=			type quantifiers and constraints
	|	**forall** $quant\_item_1, \ldots, quant\_item_n$.		
	|			sugar, omitting quantifier and constraints
typschm	::=			type scheme

	$\vert$	*typquant typ*	
name_scm_opt	::=		Optional variable-naming-scheme specification for variables of defined type
	$\vert$		
	$\vert$	[**name** = *regexp*]	
type_def	::=		Type definition body
	$\vert$	**typedef** *id name_scm_opt* = *typschm*	type abbreviation
	$\vert$	**typedef** *id name_scm_opt* = **const struct** $typquant\{typ_1\ id_1; \ldots; typ_n\ id_n\ ;^?\}$	struct type definition
	$\vert$	**typedef** *id name_scm_opt* = **const union** $typquant\{type\_union_1; \ldots; type\_union_n\ ;^?\}$	union type definition
	$\vert$	**typedef** *id name_scm_opt* = **enumerate** $\{id_1; \ldots; id_n\ ;^?\}$	enumeration type definition
	$\vert$	**typedef** *id* = **register bits** $[nexp : nexp']\{index\_range_1 : id_1; \ldots; index\_range_n : id_n\}$	register mutable bitfield type definition
type_union	::=		Type union constructors
	$\vert$	*id*	
	$\vert$	*typ id*	
index_range	::=		index specification, for bitfields in register types
	$\vert$	*num*	single index
	$\vert$	$num_1..num_2$	index range
	$\vert$	$index\_range_1, index\_range_2$	concatenation of index ranges
lit	::=		Literal constant
	$\vert$	()	() : **unit**
	$\vert$	**bitzero**	**bitzero** : **bit**

	|	**bitone**	**bitone** : **bit**
	|	**true**	**true** : **bool**
	|	**false**	**false** : **bool**
	|	*num*	natural number constant
	|	*hex*	bit vector constant, C-style
	|	*bin*	bit vector constant, C-style
	|	**undefined**	constant representing undefined values
	|	*string*	string constant
$;^?$	::=		Optional semi-colon
	|		
	|	;	
pat	::=		Pattern
	|	*lit*	literal constant pattern
	|	_	wildcard
	|	$(pat\ \mathbf{as}\ id)$	named pattern
	|	$(typ)pat$	typed pattern
	|	*id*	identifier
	|	$id(pat_1, .., pat_n)$	union constructor pattern
	|	$\{fpat_1; ...; fpat_n\ ;^?\}$	struct pattern
	|	$[pat_1, .., pat_n]$	vector pattern
	|	$[num_1 = pat_1, .., num_n = pat_n]$	vector pattern (with explicit indices)
	|	$pat_1 : .... : pat_n$	concatenated vector pattern
	|	$(pat_1, ..., pat_n)$	tuple pattern
	|	$[\|\|pat_1, .., pat_n\|\|]$	list pattern
	|	(pat) S	
fpat	::=		Field pattern
	|	$id = pat$	

exp	::=			Expression
	|	$\{exp_1; \ldots; exp_n\}$		block
	|	**nondet** $\{exp_1; \ldots; exp_n\}$		nondeterminisitic block, expressions evaluate in an unspecified order, or concurrently
	|	id		identifier
	|	lit		literal constant
	|	$(typ)exp$		cast
	|	$id(exp_1, .., exp_n)$		function application
	|	$id\ exp$	S	No extra parens needed when exp is a tuple
	|	$exp_1\ id\ exp_2$		infix function application
	|	$(exp_1, \ldots., exp_n)$		tuple
	|	**if** exp_1 **then** exp_2 **else** exp_3		conditional
	|	**if** exp_1 **then** exp_2	S	
	|	**foreach** $(id$ **from** exp_1 **to** exp_2 **by** exp_3 **in** $order)exp_4$		loop
	|	**foreach** $(id$ **from** exp_1 **to** exp_2 **by** $exp_3)exp_4$	S	
	|	**foreach** $(id$ **from** exp_1 **to** $exp_2)exp_3$	S	
	|	**foreach** $(id$ **from** exp_1 **downto** exp_2 **by** $exp_3)exp_4$	S	
	|	**foreach** $(id$ **from** exp_1 **downto** $exp_2)exp_3$	S	
	|	$[exp_1, \ldots, exp_n]$		vector (indexed from 0)
	|	$[num_1 = exp_1, \ldots, num_n = exp_n\ opt\_default]$		vector (indexed consecutively)
	|	$exp[exp']$		vector access
	|	$exp[exp_1..exp_2]$		subvector extraction
	|	$[exp$ **with** $exp_1 = exp_2]$		vector functional update
	|	$[exp$ **with** $exp_1 : exp_2 = exp_3]$		vector subrange update (with vector)
	|	$exp : exp_2$		vector concatenation
	|	$[\|\|exp_1, .., exp_n\|\|]$		list
	|	$exp_1 :: exp_2$		cons
	|	$\{fexps\}$		struct
	|	$\{exp$ **with** $fexps\}$		functional update of struct
	|	$exp.id$		field projection from struct

	$\vert$	$\mathbf{switch}\ exp\{\ \mathbf{case}\ pexp_1 \ldots \mathbf{case}\ pexp_n\}$		pattern matching
	$\vert$	$letbind\ \mathbf{in}\ exp$		let expression
	$\vert$	$lexp := exp$		imperative assignment
	$\vert$	$\mathbf{exit}\ exp$		expression to halt all current execution, potentially calling a system, trap, or interrupt handler with exp
	$\vert$	$\mathbf{assert}\ (exp, exp')$		expression to halt with error, when the first expression is true, reporting the optional string as an error
	$\vert$	(exp)	S	
	$\vert$	$(annot)exp$		This is an internal cast, generated during type checking that will resolve into a syntactic cast after
	$\vert$	$annot$		This is an internal use for passing nexp information to library functions, postponed for constraint solving
	$\vert$	$annot, annot'$		This is like the above but the user has specified an implicit parameter for the current function
	$\vert$	$\mathbf{comment}\ string$		For generated unstructured comments
	$\vert$	$\mathbf{comment}\ exp$		For generated structured comments
	$\vert$	$\mathbf{let}\ lexp = exp\ \mathbf{in}\ exp'$		This is an internal node for compilation that demonstrates the scope of a local mutable variable
	$\vert$	$\mathbf{let}\ pat = exp\ \mathbf{in}\ exp'$		This is an internal node, used to distinguised some introduced lets during processing from original ones
	$\vert$	$\mathbf{return}\ (exp)$		For internal use to embed into monad definition
$lexp$	$::=$			lvalue expression
	$\vert$	id		identifier
	$\vert$	$id(exp_1, .., exp_n)$		memory write via function call
	$\vert$	$id\ exp$	S	
	$\vert$	$(typ)id$		
	$\vert$	$lexp[exp]$		vector element
	$\vert$	$lexp[exp_1..exp_2]$		subvector
	$\vert$	$lexp.id$		struct field
$fexp$	$::=$			Field-expression
	$\vert$	$id = exp$		
$fexps$	$::=$			Field-expression list
	$\vert$	$fexp_1; \ldots ; fexp_n\ ;^?$		

opt_default	::=		Optional default value for indexed vectors, to define a defualt value for any unspecified positions in a sparse map
	|		
	|	; **default** $=$ *exp*	
pexp	::=		Pattern match
	|	*pat* $\rightarrow$ *exp*	
tannot_opt	::=		Optional type annotation for functions
	|	*typquant typ*	
rec_opt	::=		Optional recursive annotation for functions
	|		non-recursive
	|	**rec**	recursive
effect_opt	::=		Optional effect annotation for functions
	|		sugar for empty effect set
	|	**effect** *effect*	
funcl	::=		Function clause
	|	*id pat* $=$ *exp*	
fundef	::=		Function definition
	|	**function** *rec_opt tannot_opt effect_opt* $funcl_1$ **and** ... **and** $funcl_n$	
letbind	::=		Let binding
	|	**let** *typschm pat* $=$ *exp*	value binding, explicit type (*pat* must be total)
	|	**let** *pat* $=$ *exp*	value binding, implicit type (*pat* must be total)
val_spec	::=		Value type specification

	|	**val** *typschm id*	
	|	**val extern** *typschm id*	
	|	**val extern** *typschm id* = *string*	Specify the type and id of a function from Lem, where the string must provide an explicit path to the requ
default_spec	::=		Default kinding or typing assumption
	|	**default** *base_kind kid*	
	|	**default Order** *order*	
	|	**default** *typschm id*	
scattered_def	::=		Function and type union definitions that can be spread across a file. Each one must end in **id**
	|	**scattered function** *rec_opt tannot_opt effect_opt id*	scattered function definition header
	|	**function clause** *funcl*	scattered function definition clause
	|	**scattered typedef** *id name_scm_opt* = **const union** *typquant*	scattered union definition header
	|	**union** *id* **member** *type_union*	scattered union definition member
	|	**end** *id*	scattered definition end
reg_id	::=		
	|	*id*	
alias_spec	::=		Register alias expression forms. Other than where noted, each id must refer to an unaliased register of type v
	|	*reg_id.id*	
	|	*reg_id*[*exp*]	
	|	*reg_id*[*exp*..*exp*′]	
	|	*reg_id* : *reg_id*′	
dec_spec	::=		Register declarations
	|	**register** *typ id*	
	|	**register alias** *id* = *alias_spec*	

	$\vert$	**register alias** $typ\ id = alias\_spec$	
def	::=		Top-level definition
	$\vert$	$type\_def$	type definition
	$\vert$	$fundef$	function definition
	$\vert$	$letbind$	value definition
	$\vert$	val_spec	top-level type constraint
	$\vert$	$default\_spec$	default kind and type assumptions
	$\vert$	$scattered\_def$	scattered function and type definition
	$\vert$	dec_spec	register declaration
	$\vert$	dec_comm	generated comments
$defs$	::=		Definition sequence
	$\vert$	$def_1\ ..\ def_n$	

3 Sail primitive types and functions

built_in_types ::= Type Kind

| **bit** : **Typ**
| **unit** : **Typ**
| $\textbf{forall}\,\textbf{Nat}\,'n.\,\textbf{Nat}\,'m.\,\textbf{range}\ <'n,'m>:\ \textbf{Nat} \rightarrow \textbf{Nat} \rightarrow \textbf{Typ}$
| $\textbf{forall}\,\textbf{Nat}\,'n.\,\textbf{atom}\ <'n>:\ \textbf{Nat} \rightarrow \textbf{Typ}$ — singleton number, instead of range¡'n,'n¿
| $\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,\ \textbf{Order}\,'o,\ \textbf{Typ}\,'t.\,\textbf{vector}\,\langle 'n,'m,'o,'t\rangle :\ \textbf{Nat} \rightarrow \textbf{Nat} \rightarrow \textbf{Order} \rightarrow \textbf{Typ}$
| $\textbf{forall}\,\textbf{Typ}\,'t.\,\textbf{register}\,\langle 't\rangle :\ \textbf{Typ} \rightarrow \textbf{Typ}$
| $\textbf{forall}\,\textbf{Typ}\,'t.\,\textbf{reg}\,\langle 't\rangle :\ \textbf{Typ} \rightarrow \textbf{Typ}$ — internal reference cell
| $\textbf{forall}\,\textbf{Nat}\,'n.\,\textbf{implicit}\ <'n>:\ \textbf{Nat} \rightarrow \textbf{Typ}$ — see Kathy for explanation

built_in_type_abbreviations ::=

| $\textbf{bool} \Rightarrow \textbf{bit}$
| $\textbf{nat} \Rightarrow [|0..pos\_infinity|]$
| $\textbf{int} \Rightarrow [|neg\_infinity..pos\_infinity|]$
| $uint8 \Rightarrow [|0..2**8|]$
| $uint16 \Rightarrow [|0..2**16|]$
| $uint32 \Rightarrow [|0..2**32|]$
| $uint64 \Rightarrow [|0..2**32|]$

functions ::= Built-in functions: all have effect pure, all order polymorphic

| $\textbf{val}\,\textbf{forall}\,\textbf{Typ}\,'a.'a \rightarrow \textbf{unit}\ :\ \textbf{ignore}$
| $\textbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow [|'n+'o..'m+'p|] : +$ — arithmetic addition
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\ \textbf{bit}\,['n]) \rightarrow \textbf{bit}\,['n] : +$ — unsigned vector addition
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\ \textbf{bit}\,['n]) \rightarrow (\,\textbf{bit}\,['n],\ \textbf{bit}\,,\ \textbf{bit}\,) : +$ — unsigned vector addition with overflow, carry out
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\ \textbf{bit}\,['n]) \rightarrow \textbf{bit}\,['n] : +\_s$ — signed vector addition
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\ \textbf{bit}\,['n]) \rightarrow (\,\textbf{bit}\,['n],\ \textbf{bit}\,,\ \textbf{bit}\,) : +\_s$ — signed vector addition with overflow, carry out
| $\textbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow [|'n-'o..'m-'p|] : -$ — arithmetic subtraction
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\ \textbf{bit}\,['n]) \rightarrow \textbf{bit}\,['n] : -$ — unsigned vector subtraction
| $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\ \textbf{bit}\,['n]) \rightarrow (\,\textbf{bit}\,['n],\ \textbf{bit}\,,\ \textbf{bit}\,) : -$ — unsigned vector subtraction with overflow, carry out

$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : -\_s$	signed vector subtraction
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(bit['n],\,\mathbf{bit}\,['n]) \to (\,\mathbf{bit}\,['n],\,\mathbf{bit}\,,\,\mathbf{bit}\,) : -\_s$	signed vector subtraction with overflow, carry out
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 'o..'p\rvert]) \to [\lvert 'n *' o..'m *' p\rvert] : *$	arithmetic multiplication
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *$	unsigned vector multiplication
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *\_s$	signed vector multiplication
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 1..'p\rvert]) \to [\lvert 0..'p-1\rvert] : \mathbf{mod}$	arithmetic modulo
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : \mathbf{mod}$	unsigned vector modulo
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 1..'p\rvert]) \to [\lvert 'q..'r\rvert] : \mathbf{quot}$	arithmetic integer division
$\mid$	$\mathbf{val\,forall\,Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] : \mathbf{quot}$	unsigned vector division
$\mid$	$\mathbf{val\,forall\,Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] : quot\_s$	signed vector division
$\mid$	$\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Nat}\,'n.('a['n] \to [:' n :]) : \mathbf{length}$	
$\mid$	$\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,' n \le' m.(\,\mathbf{implicit}\,\langle 'm\rangle,' a['n]) \to' a['m] : \mathbf{mask}$	reduce size of vector, dropping MSBits. Type system supplies implicit p
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\equiv$	vector equality
$\mid$	$\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Typ}\,'b.('a,' b) \to \mathbf{bit}\ :\equiv$	
$\mid$	$\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Typ}\,'b.('a,' b) \to \mathbf{bit}\ :! =$	
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 'o..'p\rvert]) \to \mathbf{bit}\ : \langle$	
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ : \langle$	unsigned less than
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :<\ \_s$	
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 'o..'p\rvert]) \to \mathbf{bit}\ :\rangle$	
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\rangle$	unsigned greater than
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :>\ \_s$	
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 'o..'p\rvert]) \to \mathbf{bit}\ :\le$	
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\le$	unsigned less than or eq
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :<=\ \_s$	
$\mid$	$\mathbf{val}\,([\lvert 'n..'m\rvert],[\lvert 'o..'p\rvert]) \to \mathbf{bit}\ :\ge$	
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :\ge$	unsigned greater than or eq
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \to \mathbf{bit}\ :>=\ \_s$	
$\mid$	$\mathbf{val\,bit}\ \to\ \mathbf{bit}\ :$	bit negation
$\mid$	$\mathbf{val\,forall\,Nat}\,'n.\,\mathbf{bit}\,['n] \to \mathbf{bit}\,['n] :$	bitwise negation

$|$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to \textbf{bit}\ :|$ bitwise or

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : |$

$|$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to \textbf{bit}\ : \&$ bitwise and

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : \&$

$|$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to \textbf{bit}\ : \uparrow$ bitwise xor

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : \uparrow$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,,[|'n|]) \to \textbf{bit}\,['n] : \uparrow\uparrow$ duplicate bit into a vector

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[|'m|]) \to \textbf{bit}\,['n] :<<$ left shift

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[|'m|]) \to \textbf{bit}\,['n] :>>$ right shift

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[|'m|]) \to \textbf{bit}\,['n] :<<<$ rotate

functions_with_coercions ::=

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n]) \to [|2 **'\,n|] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to \textbf{bit}\,['n] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.([|'o..'p|],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *'n|] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,) \to \textbf{bit}\,['n] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : +$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n]) \to [|2 **'\,n|] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to \textbf{bit}\,['n] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.([|'o..'p|],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *'n|] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,) \to \textbf{bit}\,['n] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : +\_s$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to \textbf{bit}\,['n] : -$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.([|'o..'p|],\,\textbf{bit}\,['n]) \to \textbf{bit}\,['n] : -$

$|$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\,\textbf{Nat}\,'o,\,\textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[|'o..'p|]) \to [|'o..'p + 2 * *'n|] : -$

4 Tips for Writing Sail specifications

This section attempts to offer advice for writing Sail specifications that will work well with the Sail executable interpreter and compilers.

Some tips might also be advice for good ways to specify instructions; this will come from a combination of users and Sail developers.

- Be precise in numeric types.

  While Sail includes very wide types like int and nat, consider that for bounds checking, numeric operations, and and clear understanding, these really are unbounded quantities. If you know that a number in the specification will range only between 0 and 32, 0 and 4, -32 to 32, it is better to use a specific range type such as [—32—].

  Similarly, if you don't know the range precisely, it may also be best to remain polymorphic and let Sail's type resolution work out bounds in a particular use rather than removing all bounds; to do this, use [:'n:] to say that it will polymorphically take some number.

- Use bit vectors for registers.

  Sail the language will readily allow a register to store a value of any type. However, the Sail executable interpreter expects that it is simulating a uni-processor machine where all registers are bit vectors.

  A vector of length one, such as a can read the element a either with `a` or `a[0]`.

- Have functions named decode and execute to evaluate instructions.

  The sail interpreter is hard-wired to look for functions with these names.

5 Sail type system

5.1 Internal type syntax

k	::=		Internal kinds
	|	K_Typ	
	|	K_Nat	
	|	K_Ord	
	|	K_Efct	
	|	$K\_Lam(k_0 \,..\, k_n \rightarrow k')$	
	|	K_infer	Representing an unknown kind, inferred by context

$t,\ u$	::=			Internal types
	|	x		
	|	$'x$		
	|	$t_1 \rightarrow t_2$ *effect*		
	|	$(t_1,\ \ldots.,\ t_n)$		
	|	$x\langle t\_args \rangle$		
	|	$t \mapsto t_1$		
	|	**register** $\langle t\_arg \rangle$	S	
	|	**range** $\langle ne\ ne' \rangle$	S	
	|	**atom** $\langle ne \rangle$	S	
	|	**vector** $\langle ne\ ne'\ order\ t \rangle$	S	
	|	**list** $\langle t \rangle$	S	
	|	**reg** $\langle t \rangle$	S	
	|	**implicit** $\langle ne \rangle$	S	
	|	**bit**	S	
	|	*string*	S	
	|	**unit**	S	
	|	$t[t\_arg_1 / tid_1 \mathinner{..} t\_arg_n / tid_n]$	M	
optx	::=			
	|	x		
	|			
tag	::=			Data indicating where the identifier arises and thus information necessary in compilation
	|	**None**		
	|	**Intro**		Denotes an assignment and lexp that introduces a binding
	|	**Set**		Denotes an expression that mutates a local variable
	|	**Global**		Globally let-bound or enumeration based value/variable
	|	**Ctor**		Data constructor from a type union
	|	**Extern** *optx*		External function, specied only with a val statement

	$\mid$	**Default**		Type has come from default declaration, identifier may not be bound locally
	$\mid$	**Spec**		
	$\mid$	**Enum** num		
	$\mid$	**Alias**		
	$\mid$	$Unknown\_path\ optx$		Tag to distinguish an unknown path from a non-analysis non deterministic path
ne	$::=$			internal numeric expressions
	$\mid$	$'x$		
	$\mid$	num		
	$\mid$	**infinity**		
	$\mid$	$ne_1 * ne_2$		
	$\mid$	$ne_1 + \ldots + ne_n$		
	$\mid$	$ne_1 - ne_2$		
	$\mid$	$2 ** ne$		
	$\mid$	$(-ne)$		
	$\mid$	**zero**	S	
	$\mid$	**one**	S	
	$\mid$	**bitlength** (bin)	M	
	$\mid$	**bitlength** (hex)	M	
	$\mid$	**count** $(num_0 \ldots num_i)$	M	
	$\mid$	**length** $(pat_1 \ldots pat_n)$	M	
	$\mid$	**length** $(exp_1 \ldots exp_n)$	M	
t_arg	$::=$			Argument to type constructors
	$\mid$	t		
	$\mid$	ne		
	$\mid$	$effect$		
	$\mid$	$order$		
	$\mid$	**fresh**	M	

t_args	::=			Arguments to type constructors
	|	$t\_arg_1 \ldots t\_arg_n$		
nec	::=			Numeric expression constraints
	|	$ne \leq ne'$		
	|	$ne = ne'$		
	|	$ne \geq ne'$		
	|	$'x\ \mathbf{IN}\ \{num_1, \ldots, num_n\}$		
	|	$nec_0 \mathinner{..} nec_n \rightarrow nec'_0 \ldots nec'_m$		
	|	$nec_0 \ldots nec_n$		
Σ^{N}	::=			nexp constraint lists
	|	$\{nec_1, \mathinner{..}, nec_n\}$		
	|	${\Sigma^{\mathrm{N}}}_1 \uplus \mathinner{..} \uplus {\Sigma^{\mathrm{N}}}_n$	M	
	|	**consistent_increase** $ne_1\ ne'_1 \ldots ne_n\ ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always larger than the su
	|	**consistent_decrease** $ne_1\ ne'_1 \ldots ne_n\ ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always smaller than the
	|	**resolve** (Σ^{N})		
E^{D}	::=			Environments storing top level information, such as defined abbreviations, records, enumerations, and kin
	|	$\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle$		
	|	ϵ		
	|	$E^{\mathrm{D}} \uplus E^{\mathrm{D}\prime}$		
$kinf$	::=			Whether a kind is default or from a local binding
	|	k		
	|	k **default**		
tid	::=			A type identifier or type variable
	|	id		
	|	kid		

E^{K}	::=			Kind environments
	|	$\{ \mathit{tid}_1 \mapsto \mathit{kinf}_1 , \,..\, , \mathit{tid}_n \mapsto \mathit{kinf}_n \}$		
	|	$E^{\mathrm{K}}{}_1 \uplus \,..\, \uplus E^{\mathrm{K}}{}_n$	M	In a unioning kinf, k default u k results in k (i.e. the default is locally forgotten)
	|	$E^{\mathrm{K}} \setminus E^{\mathrm{K}}{}_1 \,..\, E^{\mathrm{K}}{}_n$	M	
tinf	::=			Type variables, type, and constraints, bound to an identifier
	|	t		
	|	$E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathit{tag}, t$		
E^{A}	::=			
	|	$\{ \mathit{tid}_1 \mapsto \mathit{tinf}_1 , \,..\, , \mathit{tid}_n \mapsto \mathit{tinf}_n \}$		
	|	$E^{\mathrm{A}}_1 \uplus \,..\, \uplus E^{\mathrm{A}}_n$		
$\mathit{field\_typs}$	::=			Record fields
	|	$\mathit{id}_1 : t_1 , \,..\, , \mathit{id}_n : t_n$		
E^{R}	::=			Record environments
	|	$\{ \{ \mathit{field\_typs}_1 \} \mapsto \mathit{tinf}_1 , \,..\, , \{ \mathit{field\_typs}_n \} \mapsto \mathit{tinf}_n \}$		
	|	$E^{\mathrm{R}}_1 \uplus \,..\, \uplus E^{\mathrm{R}}_n$	M	
$\mathit{enumerate\_map}$	::=			
	|	$\{ \mathit{num}_1 \mapsto \mathit{id}_1 \,...\, \mathit{num}_n \mapsto \mathit{id}_n \}$		
E^{E}	::=			Enumeration environments
	|	$\{ t_1 \mapsto \mathit{enumerate\_map}_1 , \,..\, , t_n \mapsto \mathit{enumerate\_map}_n \}$		
	|	$E^{\mathrm{E}}_1 \uplus \,..\, \uplus E^{\mathrm{E}}_n$		
E^{T}	::=			Type environments
	|	$\{ \mathit{id}_1 \mapsto \mathit{tinf}_1 , \,..\, , \mathit{id}_n \mapsto \mathit{tinf}_n \}$		
	|	$\{ \mathit{id} \mapsto \textbf{overload}\ \mathit{tinf}\ \mathit{conformsto} : \mathit{tinf}_1 , \,...\, , \mathit{tinf}_n \}$		

	$\mid$	$(E^{\mathrm{T}}{}_1 \uplus \ldots. \uplus E^{\mathrm{T}}{}_n)$	M
	$\mid$	$\uplus E^{\mathrm{T}}{}_1 \mathinner{..} E^{\mathrm{T}}{}_n$	M
	$\mid$	$E^{\mathrm{T}} \setminus id_1 \mathinner{..} id_n$	M
	$\mid$	$(E^{\mathrm{T}}{}_1 \cap \ldots. \cap E^{\mathrm{T}}{}_n)$	M
	$\mid$	$\cap E^{\mathrm{T}}{}_1 \mathinner{..} E^{\mathrm{T}}{}_n$	M
ts	$::=$		
	$\mid$	$t_1, \mathinner{..}, t_n$	
E	$::=$		Definition environment and lexical environment
	$\mid$	$\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle$	
	$\mid$	ϵ	M
	$\mid$	$E \uplus E'$	
I	$::=$		Information given by type checking an expression
	$\mid$	$\langle \Sigma^{\mathrm{N}}, \mathit{effect} \rangle$	
	$\mid$	I_ϵ	Empty constraints, effect
	$\mid$	$I_1 \uplus I_2$	
	$\mid$	$I_1 \uplus \mathinner{..} \uplus I_n$	Unions the constraints and effect
$\mathit{formula}$	$::=$		
	$\mid$	$\mathit{judgement}$	
	$\mid$	$\mathit{formula}_1 \quad \mathinner{..} \quad \mathit{formula}_n$	
	$\mid$	$E^{\mathrm{K}}(\mathit{tid}) \triangleright \mathit{kinf}$	Kind lookup
	$\mid$	$E^{\mathrm{A}}(\mathit{tid}) \triangleright \mathit{tinf}$	
	$\mid$	$E^{\mathrm{T}}(\mathit{id}) \triangleright \mathit{tinf}$	Type lookup
	$\mid$	$E^{\mathrm{T}}(\mathit{id}) \triangleright \textbf{overload}\ \mathit{tinf} : \mathit{tinf}_1 \ldots \mathit{tinf}_n$	Overloaded type lookup
	$\mid$	$E^{\mathrm{K}}(\mathit{tid}) < -\mid k$	Update the kind associated with id to k
	$\mid$	$E^{\mathrm{R}}(\mathit{id}_0 \mathinner{..} \mathit{id}_n) \triangleright t, ts$	Record lookup
	$\mid$	$E^{\mathrm{R}}(t) \triangleright \mathit{id}_0 : t_0 \mathinner{..} \mathit{id}_n : t_n$	Record looup by type

| $E^{E}(t) \triangleright enumerate\_map$ Enumeration lookup by type
| $\mathbf{dom}\,(E^{T}{}_1) \cap \mathbf{dom}\,(E^{T}{}_2) = \emptyset$
| $\mathbf{dom}\,(E^{K}{}_1) \cap \mathbf{dom}\,(E^{K}{}_2) = \emptyset$
| $\mathbf{disjoint\,doms}\,(E^{T}{}_1, \, ..., E^{T}{}_n)$ Pairwise disjoint domains
| $id \notin \mathbf{dom}\,(E^{K})$
| $id \notin \mathbf{dom}\,(E^{T})$
| $id_0 : t_0 .. id_n : t_n \subset id'_0 : t'_0 .. id'_i : t'_i$
| $num_1 < ... < num_n$
| $num_1 > ... > num_n$
| $exp_1 \equiv exp_2$
| $E^{K}{}_1 \equiv E^{K}{}_2$
| $E^{K}{}_1 \approx E^{K}{}_2$
| $E^{T}{}_1 \equiv E^{T}{}_2$
| $E^{R}_1 \equiv E^{R}_2$
| $E^{E}_1 \equiv E^{E}_2$
| $E^{D}{}_1 \equiv E^{D}{}_2$
| $E_1 \equiv E_2$
| $\Sigma^{N}{}_1 \equiv \Sigma^{N}{}_2$
| $id \equiv' id$
| $x_1 \neq x_2$
| $lit_1 \neq lit_2$
| $I_1 \equiv I_2$
| $effect_1 \equiv effect_2$
| $t_1 \equiv t_2$
| $ne \equiv ne'$
| $kid \equiv fresh\_kid(E^{D})$

5.2 Type relations

$\boxed{E^{K} \vdash_t t\,\mathbf{ok}}$ Well-formed types

$$\frac{E^{K}(\text{'}x) \triangleright K\_Typ}{E^{K} \vdash_t \text{'}x\,\mathbf{ok}} \quad \text{CHECK\_T\_VAR}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \quad E^{\mathrm{K}}(\text{'}x) <-| K\_Typ}{E^{\mathrm{K}} \vdash_t \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_T\_VARINFER}$$

$$\frac{E^{\mathrm{K}} \vdash_t t_1\ \mathbf{ok} \quad E^{\mathrm{K}} \vdash_t t_2\ \mathbf{ok} \quad E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_t t_1 \to t_2\ \mathit{effect}\ \mathbf{ok}} \quad \text{CHECK\_T\_FN}$$

$$\frac{E^{\mathrm{K}} \vdash_t t_1\ \mathbf{ok} \quad .... \quad E^{\mathrm{K}} \vdash_t t_n\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_t (t_1, \ .... \ , t_n)\ \mathbf{ok}} \quad \text{CHECK\_T\_TUP}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \quad E^{\mathrm{K}}, k_1 \vdash t\_arg_1\ \mathbf{ok} \quad .. \quad E^{\mathrm{K}}, k_n \vdash t\_arg_n\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_t x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle\ \mathbf{ok}} \quad \text{CHECK\_T\_APP}$$

$\boxed{E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok}}$ Well-formed effects

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Efct}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_EF\_VAR}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \quad E^{\mathrm{K}}(\text{'}x) <-| K\_Efct}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_EF\_VARINFER}$$

$$\frac{}{E^{\mathrm{K}} \vdash_e \{base\_effect_1, \ .. \ , base\_effect_n\}\ \mathbf{ok}} \quad \text{CHECK\_EF\_SET}$$

$\boxed{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_N\_VAR}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \quad E^{\mathrm{K}}(\text{'}x) <-| K\_Nat}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_N\_VARINFER}$$

$$\frac{}{E^{\mathrm{K}} \vdash_n num\ \mathbf{ok}} \quad \text{CHECK\_N\_NUM}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne_1\,\mathbf{ok} \quad E^{\mathrm{K}} \vdash_n ne_2\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_n ne_1 + ne_2\,\mathbf{ok}} \quad \text{CHECK\_N\_SUM}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne_1\,\mathbf{ok} \quad E^{\mathrm{K}} \vdash_n ne_2\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_n ne_1 * ne_2\,\mathbf{ok}} \quad \text{CHECK\_N\_MULT}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\,\mathbf{ok}}{E^{\mathrm{K}} \vdash_n 2 ** ne\,\mathbf{ok}} \quad \text{CHECK\_N\_EXP}$$

$\boxed{E^{\mathrm{K}} \vdash_o order\,\mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Ord}{E^{\mathrm{K}} \vdash_o \text{'}x\,\mathbf{ok}} \quad \text{CHECK\_ORD\_VAR}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \quad E^{\mathrm{K}}(\text{'}x) < -|K\_Ord}{E^{\mathrm{K}} \vdash_o \text{'}x\,\mathbf{ok}} \quad \text{CHECK\_ORD\_VARINFER}$$

$\boxed{E^{\mathrm{K}}, k \vdash t\_arg\,\mathbf{ok}}$ Well-formed type arguments kind check matching the application type variable

$$\frac{E^{\mathrm{K}} \vdash_t t\,\mathbf{ok}}{E^{\mathrm{K}}, K\_Typ \vdash t\,\mathbf{ok}} \quad \text{CHECK\_TARGS\_TYP}$$

$$\frac{E^{\mathrm{K}} \vdash_e effect\,\mathbf{ok}}{E^{\mathrm{K}}, K\_Efct \vdash effect\,\mathbf{ok}} \quad \text{CHECK\_TARGS\_EFF}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\,\mathbf{ok}}{E^{\mathrm{K}}, K\_Nat \vdash ne\,\mathbf{ok}} \quad \text{CHECK\_TARGS\_NAT}$$

$$\frac{E^{\mathrm{K}} \vdash_o order\,\mathbf{ok}}{E^{\mathrm{K}}, K\_Ord \vdash order\,\mathbf{ok}} \quad \text{CHECK\_TARGS\_ORD}$$

$\boxed{E^{\mathrm{K}} \vdash kind \leadsto k}$

$$\frac{}{E^{\mathrm{K}} \vdash \mathbf{Type} \leadsto K\_Typ} \quad \text{CONVERT\_KIND\_TYP}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{quant\_item} \leadsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}}$ Convert source quantifiers to kind environments and constraints

$$\frac{E^{\mathrm{K}} \vdash \mathit{kind} \leadsto k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{kind}\ \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \text{CONVERT\_QUANTS\_KIND}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \text{CONVERT\_QUANTS\_NOKIND}$$

$$\frac{\begin{array}{c}\vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2\end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 = \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 = \mathit{ne}_2\}} \quad \text{CONVERT\_QUANTS\_EQ}$$

$$\frac{\begin{array}{c}\vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2\end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 \geq \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 \geq \mathit{ne}_2\}} \quad \text{CONVERT\_QUANTS\_GTEQ}$$

$$\frac{\begin{array}{c}\vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2\end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 \leq \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 \leq \mathit{ne}_2\}} \quad \text{CONVERT\_QUANTS\_LTEQ}$$

$$\frac{}{E^{\mathrm{D}} \vdash \text{'}x\ \mathbf{IN}\ \{\mathit{num}_1, \ldots, \mathit{num}_n\} \leadsto \{\,\}, \{\text{'}x\ \mathbf{IN}\ \{\mathit{num}_1, \ldots, \mathit{num}_n\}\}} \quad \text{CONVERT\_QUANTS\_IN}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}$ Convert source types with typeschemes to internal types and kind environments

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{E^{\mathrm{D}} \vdash\ \mathit{typ} \leadsto t, \{\,\}, \{\,\}} \quad \text{CONVERT\_TYPSCHM\_NOQUANT}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash \mathit{quant\_item}_1 \leadsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash \mathit{quant\_item}_n \leadsto E^{\mathrm{K}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{K}} \equiv E^{\mathrm{K}}{}_1 \uplus \ldots \uplus E^{\mathrm{K}}{}_n \\ E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}, \{\,\}, \{\,\}, \{\,\}\rangle \vdash \mathit{typ} \leadsto t\end{array}}{E^{\mathrm{D}} \vdash \mathbf{forall}\ \mathit{quant\_item}_1, \ldots, \mathit{quant\_item}_n.\ \mathit{typ} \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n} \quad \text{CONVERT\_TYPSCHM\_QUANT}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}$ Convert source types to internal types

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright \mathit{K\_Typ}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \text{'}x \leadsto \text{'}x} \quad \text{CONVERT\_TYP\_VAR}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \leadsto x} \quad \text{CONVERT\_TYP\_ID}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash typ_1 \leadsto t_1 \\ E^{\mathrm{D}} \vdash typ_2 \leadsto t_2 \end{array}}{E^{\mathrm{D}} \vdash typ_1 \to typ_2 \,\mathbf{effect}\, effect \leadsto t_1 \to t_2\ effect} \quad \text{CONVERT\_TYP\_FN}$$

$$\frac{E^{\mathrm{D}} \vdash typ_1 \leadsto t_1 \quad .... \quad E^{\mathrm{D}} \vdash typ_n \leadsto t_n}{E^{\mathrm{D}} \vdash (typ_1, \, ... \, , typ_n) \leadsto (t_1, \, ... \, , t_n)} \quad \text{CONVERT\_TYP\_TUP}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \,..\, k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash typ\_arg_1 \leadsto t\_arg_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash typ\_arg_n \leadsto t\_arg_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle typ\_arg_1, \, .. \, , typ\_arg_n\rangle \leadsto x\langle t\_arg_1 \,..\, t\_arg_n\rangle} \quad \text{CONVERT\_TYP\_APP}$$

$\boxed{E^{\mathrm{D}}, k \vdash typ\_arg \leadsto t\_arg}$ Convert source type arguments to internals

$$\frac{E^{\mathrm{D}} \vdash typ \leadsto t}{E^{\mathrm{D}}, K\_Typ \vdash typ \leadsto t} \quad \text{CONVERT\_TARG\_TYP}$$

$\boxed{\vdash nexp \leadsto ne}$ Convert and normalize numeric expressions

$$\frac{}{\vdash \text{'}x \leadsto \text{'}x} \quad \text{CONVERT\_NEXP\_VAR}$$

$$\frac{}{\vdash num \leadsto num} \quad \text{CONVERT\_NEXP\_NUM}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2 \end{array}}{\vdash nexp_1 * nexp_2 \leadsto ne_1 * ne_2} \quad \text{CONVERT\_NEXP\_MULT}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2 \end{array}}{\vdash nexp_1 + nexp_2 \leadsto ne_1 + ne_2} \quad \text{CONVERT\_NEXP\_ADD}$$

$$\frac{\vdash nexp \leadsto ne}{\vdash 2 * *nexp \leadsto 2 ** ne} \quad \text{CONVERT\_NEXP\_EXP}$$

$\boxed{E^{\mathrm{D}} \vdash t \approx t'}$

$$\frac{E^{\mathrm{K}} \vdash_t t\, \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx t} \quad \textsc{conforms\_to\_refl}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_1 \approx t_2 \\ E^{\mathrm{D}} \vdash t_2 \approx t_3 \end{array}}{E^{\mathrm{D}} \vdash t_1 \approx t_3} \quad \textsc{conforms\_to\_trans}$$

$$\frac{}{E^{\mathrm{D}} \vdash \text{'}x \approx t} \quad \textsc{conforms\_to\_var}$$

$$\frac{}{E^{\mathrm{D}} \vdash t \approx \text{'}x} \quad \textsc{conforms\_to\_var2}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u \approx t \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \approx t} \quad \textsc{conforms\_to\_abbrev}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx u \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx x} \quad \textsc{conforms\_to\_abbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash t_n \approx u_n}{E^{\mathrm{D}} \vdash (t_1,\, ...\,, t_n) \approx (u_1,\, ...\,, u_n)} \quad \textsc{conforms\_to\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash t\_arg_1 \approx t\_arg'_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash t\_arg_n \approx t\_arg'_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_n\rangle} \quad \textsc{conforms\_to\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx u[t\_arg'_1/tid_1 \mathinner{..} t\_arg'_m/tid_m] \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x'\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_app}\mathrm{Abbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u[t\_arg_1/tid_1 \mathinner{..} t\_arg_n/tid_n] \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x'\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_app}\mathrm{Abbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{register}\,\langle t \rangle \approx u} \quad \textsc{conforms\_to\_register}$$

$$\boxed{E^{\mathrm{D}}, k \vdash t\_arg \approx t\_arg'}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash t \approx t'} \quad \textsc{targconforms\_typ}$$

$$\frac{}{E^{\mathrm{D}}, K\_Nat \vdash ne \approx ne'} \quad \textsc{targconforms\_nexp}$$

$$\boxed{\sigma_{conformsto(t,t')}(tinflist) \triangleright tinflist'}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_i \approx t_i' \\ E^{\mathrm{D}} \vdash t_j' \approx t_j \\ \sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ tinf_0' \mathinner{..} tinf_n') \triangleright \epsilon \end{array}}{\sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t_i' \to t_j'\ effect\ tinf_0' \mathinner{..} tinf_n') \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t_i' \to t_j'} \quad \textsc{so\_full}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_i \approx t_i' \\ \sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t_i' \to t\ effect\ tinf_0' \mathinner{..} tinf_n') \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t_i' \to t} \quad \textsc{so\_parm}$$

$$\boxed{E^{\mathrm{D}} \vdash t \precapprox t', \Sigma^{\mathrm{N}}}$$

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash t \precapprox t, \{\,\}} \quad \textsc{consistent\_typ\_refl}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_1 \precapprox t_2, {\Sigma^{\mathrm{N}}}_1 \\ E^{\mathrm{D}} \vdash t_2 \precapprox t_3, {\Sigma^{\mathrm{N}}}_2 \end{array}}{E^{\mathrm{D}} \vdash t_1 \precapprox t_3, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2} \quad \textsc{consistent\_typ\_trans}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash u \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash x \precapprox t, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \textsc{consistent\_typ\_abbrev}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash t \precapprox u, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash t \precapprox x, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \textsc{consistent\_typ\_abbrev2}$$

$$\frac{}{E^{\text{D}} \vdash \text{'}x \precapprox t, \{\,\}} \quad \text{CONSISTENT\_TYP\_VAR}$$

$$\frac{}{E^{\text{D}} \vdash t \precapprox \text{'}x, \{\,\}} \quad \text{CONSISTENT\_TYP\_VAR2}$$

$$\frac{E^{\text{D}} \vdash t_1 \precapprox u_1, \Sigma^{\text{N}}{}_1 \quad .... \quad E^{\text{D}} \vdash t_n \precapprox u_n, \Sigma^{\text{N}}{}_n}{E^{\text{D}} \vdash (t_1, \, ... \, , t_n) \precapprox (u_1, \, ... \, , u_n), \Sigma^{\text{N}}{}_1 \uplus \, ... \, \uplus \Sigma^{\text{N}}{}_n} \quad \text{CONSISTENT\_TYP\_TUP}$$

$$\frac{}{E^{\text{D}} \vdash \mathbf{range}\,\langle ne_1\ ne_2 \rangle \precapprox \mathbf{range}\,\langle ne_3\ ne_4 \rangle, \{ne_3 \leq ne_1, ne_2 \leq ne_4\}} \quad \text{CONSISTENT\_TYP\_RANGE}$$

$$\frac{}{E^{\text{D}} \vdash \mathbf{atom}\,\langle ne \rangle \precapprox \mathbf{range}\,\langle ne_1\ ne_2 \rangle, \{ne_1 \leq ne, ne \leq ne_2\}} \quad \text{CONSISTENT\_TYP\_ATOMRANGE}$$

$$\frac{}{E^{\text{D}} \vdash \mathbf{atom}\,\langle ne_1 \rangle \precapprox \mathbf{atom}\,\langle ne_2 \rangle, \{ne_1 = ne_2\}} \quad \text{CONSISTENT\_TYP\_ATOM}$$

$$\frac{}{E^{\text{D}} \vdash \mathbf{range}\,\langle ne_1\ ne_2 \rangle \precapprox \mathbf{atom}\,\langle \text{'}x \rangle, \{ne_1 \leq \text{'}x, \text{'}x \leq ne_2\}} \quad \text{CONSISTENT\_TYP\_RANGEATOM}$$

$$\frac{E^{\text{D}} \vdash t \precapprox t', \Sigma^{\text{N}}}{E^{\text{D}} \vdash \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t \rangle \precapprox \mathbf{vector}\,\langle ne_3\ ne_4\ order\ t' \rangle, \{ne_1 = ne_3, ne_2 = ne_3\} \uplus \Sigma^{\text{N}}} \quad \text{CONSISTENT\_TYP\_VECTOR}$$

$$\frac{\begin{array}{l} E^{\text{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \\ \langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle, k_1 \vdash t\_arg_1 \precapprox t\_arg'_1, \Sigma^{\text{N}}{}_1 \quad .. \quad \langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle, k_n \vdash t\_arg_n \precapprox t\_arg'_n, \Sigma^{\text{N}}{}_n \end{array}}{\langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n \rangle \precapprox x\langle t\_arg'_1 \mathinner{..} t\_arg'_n \rangle, \Sigma^{\text{N}}{}_1 \uplus \mathinner{..} \uplus \Sigma^{\text{N}}{}_n} \quad \text{CONSISTENT\_TYP\_APP}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\text{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_m \mapsto kinf_m\}, \Sigma^{\text{N}}, tag, u \\ \langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n \rangle \precapprox u[t\_arg'_1/tid_1 \mathinner{..} t\_arg'_m/tid_m], \Sigma^{\text{N}}{}_2 \end{array}}{\langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n \rangle \precapprox x'\langle t\_arg'_1 \mathinner{..} t\_arg'_m \rangle, \Sigma^{\text{N}} \uplus \Sigma^{\text{N}}{}_2} \quad \text{CONSISTENT\_TYP\_APPABBREV}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\text{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_m \mapsto kinf_m\}, \Sigma^{\text{N}}, tag, u \\ \langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash u[t\_arg'_1/tid_1 \mathinner{..} t\_arg'_m/tid_m] \precapprox x\langle t\_arg_1 \mathinner{..} t\_arg_n \rangle, \Sigma^{\text{N}}{}_2 \end{array}}{\langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash x'\langle t\_arg'_1 \mathinner{..} t\_arg'_m \rangle \precapprox x\langle t\_arg_1 \mathinner{..} t\_arg_n \rangle, \Sigma^{\text{N}} \uplus \Sigma^{\text{N}}{}_2} \quad \text{CONSISTENT\_TYP\_APPABBREV2}$$

$$\boxed{E^{\text{D}}, k \vdash t\_arg \precapprox t\_arg', \Sigma^{\text{N}}}$$

$$\frac{E^{\text{D}} \vdash t \precapprox t', \Sigma^{\text{N}}}{E^{\text{D}}, K\_Typ \vdash t \precapprox t', \Sigma^{\text{N}}} \quad \text{TARG\_CONSISTENT\_TYP}$$

$$\frac{}{E^{\mathrm{D}}, K\_Nat \vdash ne \precapprox ne', \{ne = ne'\}} \quad \textsc{targ\_consistent\_nexp}$$

$$\boxed{E^{\mathrm{D}}, t' \vdash exp : t \triangleright t'', exp', \Sigma^{\mathrm{N}}, effect}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, u_1 \vdash id_1 : t_1 \triangleright u_1, exp_1, \Sigma^{\mathrm{N}}{}_1, effect_1 \quad .. \quad E^{\mathrm{D}}, u_n \vdash id_n : t_n \triangleright u_n, exp_n, \Sigma^{\mathrm{N}}{}_n, effect_n \\ exp' \equiv \mathbf{switch}\, exp\{\, \mathbf{case}\, (id_1, \,..\,, id_n) \to (exp_1, \,..\,, exp_n)\} \end{array}}{E^{\mathrm{D}}, (u_1, \,..\,, u_n) \vdash exp : (t_1, \,..\,, t_n) \triangleright (u_1, \,..\,, u_n), exp', \Sigma^{\mathrm{N}}{}_1 \uplus \,..\, \uplus \Sigma^{\mathrm{N}}{}_n, \mathbf{pure}} \quad \textsc{coerce\_typ\_tuple}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}} \\ exp' \equiv (annot) exp \end{array}}{E^{\mathrm{D}}, \mathbf{vector}\, \langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \mathbf{vector}\, \langle ne_3\; ne_4\; order\; u\rangle \triangleright \mathbf{vector}\, \langle ne_3\; ne_4\; order\; t\rangle, exp', \Sigma^{\mathrm{N}} \uplus \{ne_2 = ne_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_vectorUpdateStart}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash u \precapprox \mathbf{bit}, \Sigma^{\mathrm{N}} \\ exp' \equiv to\_num\; exp \end{array}}{E^{\mathrm{D}}, \mathbf{range}\, \langle ne_1\; ne_2\rangle \vdash exp : \mathbf{vector}\, \langle ne_3\; ne_4\; order\; u\rangle \triangleright \mathbf{range}\, \langle ne_1\; ne_2\rangle, exp', \Sigma^{\mathrm{N}} \uplus \{ne_1 = \mathbf{zero}, ne_2 \geq 2 ** ne_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toNum}$$

$$\frac{exp' \equiv to\_vec\; exp}{E^{\mathrm{D}}, \mathbf{vector}\, \langle ne_1\; ne_2\; order\; \mathbf{bit}\rangle \vdash exp : \mathbf{range}\, \langle ne_3\; ne_4\rangle \triangleright \mathbf{vector}\, \langle ne_1\; ne_2\; order\; \mathbf{bit}\rangle, exp', \{ne_3 = \mathbf{zero}, ne_4 \leq 2 ** ne_2\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash typ \leadsto t \\ exp' \equiv (typ) exp \\ E^{\mathrm{D}}, u \vdash exp' : t \triangleright t', exp'', \Sigma^{\mathrm{N}}, \mathbf{pure} \end{array}}{E^{\mathrm{D}}, u \vdash exp : \mathbf{register}\, \langle t\rangle \triangleright t', exp'', \Sigma^{\mathrm{N}}, \{\mathbf{rreg}\}} \quad \textsc{coerce\_typ\_readReg}$$

$$\frac{exp' \equiv exp[numZero]}{E^{\mathrm{D}}, \mathbf{bit} \vdash exp : \mathbf{vector}\, \langle ne_1\; ne_2\; order\; \mathbf{bit}\rangle \triangleright \mathbf{bit}, exp', \{ne_1 = \mathbf{one}\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_accessVecBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathbf{range}\, \langle \mathbf{zero}\, \mathbf{one}\rangle \precapprox \mathbf{range}\, \langle ne_1\; ne_2\rangle, \Sigma^{\mathrm{N}} \\ exp' \equiv \mathbf{switch}\, exp\{\, \mathbf{case}\, \mathbf{bitzero} \to numZero\; \mathbf{case}\, \mathbf{bitone} \to numOne\} \end{array}}{E^{\mathrm{D}}, \mathbf{range}\, \langle ne_1\; ne_2\rangle \vdash exp : \mathbf{bit} \triangleright \mathbf{range}\, \langle ne_1\; ne_2\rangle, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_bitToNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathbf{range}\, \langle ne_1\; ne_2\rangle \precapprox \mathbf{range}\, \langle \mathbf{zero}\, \mathbf{one}\rangle, \Sigma^{\mathrm{N}} \\ exp' \equiv \mathbf{switch}\, exp\{\, \mathbf{case}\, numZero \to \mathbf{bitzero}\; \mathbf{case}\, numOne \to \mathbf{bitone}\} \end{array}}{E^{\mathrm{D}}, \mathbf{bit} \vdash \mathbf{range} : \mathbf{range}\, \langle ne_1\; ne_2\rangle \triangleright \mathbf{bit}, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_numToBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\, \overline{num_i \mapsto id_i}^{\,i} \,\} \\ exp' \equiv \mathbf{switch}\, exp\{\, \overline{\mathbf{case}\, num_i \to id_i}^{\,i} \,\} \\ ne_3 \equiv \mathbf{count}\ (\, \overline{num_i}^{\,i} \,) \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, x \vdash exp : \mathbf{range}\, \langle ne_1\ ne_2 \rangle \triangleright x, exp', \{ne_1 \leq \mathbf{zero}, ne_2 \leq ne_3\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toEnumerate}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\, \overline{num_i \mapsto id_i}^{\,i} \,\} \\ exp' \equiv \mathbf{switch}\, exp\{\, \overline{\mathbf{case}\, id_i \to num_i}^{\,i} \,\} \\ ne_3 \equiv \mathbf{count}\ (\, \overline{num_i}^{\,i} \,) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash \mathbf{range}\, \langle \mathbf{zero}\, ne_3 \rangle \lessapprox \mathbf{range}\, \langle ne_1\ ne_2 \rangle, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, \mathbf{range}\, \langle ne_1\ ne_2 \rangle \vdash exp : x \triangleright \mathbf{range}\, \langle \mathbf{zero}\, ne_3 \rangle, exp', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromEnumerate}$$

$$\frac{E^{\mathrm{D}} \vdash t \lessapprox u, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, u \vdash exp : t \triangleright t, exp, \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_eq}$$

$\boxed{t \vdash lit : t' \Rightarrow exp, \Sigma^{\mathrm{N}}}$ Typing literal constants, coercing to expected type t

$$\frac{}{\mathbf{range}\, \langle ne\ ne' \rangle \vdash num : \mathbf{atom}\, \langle num \rangle \Rightarrow num, \{ne \leq num, num \leq ne'\}} \quad \textsc{check\_lit\_num}$$

$$\frac{}{\mathbf{vector}\, \langle ne\ ne'\ order\ \mathbf{bit} \rangle \vdash num : \mathbf{atom}\, \langle num \rangle \Rightarrow to\_vec\, num, \{num + \mathbf{one} \leq 2 ** ne'\}} \quad \textsc{check\_lit\_numToVec}$$

$$\frac{}{\mathbf{bit} \vdash numZero : \mathbf{atom}\, \langle \mathbf{zero} \rangle \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \textsc{check\_lit\_numbitzero}$$

$$\frac{}{\mathbf{bit} \vdash numOne : \mathbf{atom}\, \langle \mathbf{one} \rangle \Rightarrow \mathbf{bitone}, \{\,\}} \quad \textsc{check\_lit\_numbitone}$$

$$\frac{}{string \vdash string : string \Rightarrow string, \{\,\}} \quad \textsc{check\_lit\_string}$$

$$\frac{ne \equiv \mathbf{bitlength}\, (hex)}{\mathbf{vector}\, \langle ne_1\ ne_2\ order\ \mathbf{bit} \rangle \vdash hex : \mathbf{vector}\, \langle ne_1\ ne\ order\ \mathbf{bit} \rangle \Rightarrow hex, \{ne = ne_2\}} \quad \textsc{check\_lit\_hex}$$

$$\frac{ne \equiv \mathbf{bitlength}\, (bin)}{\mathbf{vector}\, \langle ne_1\ ne_2\ order\ \mathbf{bit} \rangle \vdash bin : \mathbf{vector}\, \langle ne_1\ ne\ order\ \mathbf{bit} \rangle \Rightarrow bin, \{ne = ne_2\}} \quad \textsc{check\_lit\_bin}$$

$$\frac{}{\mathbf{unit} \vdash () : \mathbf{unit} \Rightarrow \mathbf{unit}, \{\,\}} \quad \textsc{check\_lit\_unit}$$

$$\frac{}{\textbf{bit} \vdash \textbf{bitzero} : \textbf{bit} \Rightarrow \textbf{bitzero}, \{\,\}} \quad \textsc{check\_lit\_bitzero}$$

$$\frac{}{\textbf{bit} \vdash \textbf{bitone} : \textbf{bit} \Rightarrow \textbf{bitzero}, \{\,\}} \quad \textsc{check\_lit\_bitone}$$

$$\frac{}{t \vdash \textbf{undefined} : t \Rightarrow \textbf{undefined}, \{\,\}} \quad \textsc{check\_lit\_undef}$$

$\boxed{E, t \vdash pat : t' \vartriangleright pat', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Typing patterns, building their binding environment

$$\frac{\begin{array}{l} lit \neq \textbf{undefined} \\ t \vdash lit : u \Rightarrow lit', \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash lit : u \vartriangleright lit', \{\,\}, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \textsc{check\_pat\_lit}$$

$$\frac{}{E, t \vdash \_ : t \vartriangleright \_, \{\,\}, \{\,\}} \quad \textsc{check\_pat\_wild}$$

$$\frac{\begin{array}{l} E, t \vdash pat : u \vartriangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ id \notin \textbf{dom}\,(E^{\mathrm{T}}{}_1) \end{array}}{E, t \vdash (pat\,\textbf{as}\,id) : u \vartriangleright (pat'\,\textbf{as}\,id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_as}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t' \vdash pat : t \vartriangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ E^{\mathrm{T}}(id) \vartriangleright \{\,\}, \{\,\}, \textbf{Default}, t' \\ E^{\mathrm{D}} \vdash t' \precapprox u, \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash (pat\,\textbf{as}\,id) : t \vartriangleright (pat'\,\textbf{as}\,id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t'\}), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \textsc{check\_pat\_asDefault}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash typ \rightsquigarrow t \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat : t \vartriangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash (typ)pat : t \vartriangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_typ}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \vartriangleright \{tid_1 \mapsto kinf_1, \ldots, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \textbf{Ctor}, (u'_1, \ldots, u'_n) \to x\langle t\_arg_1 \ldots t\_arg_m\rangle\,\textbf{pure} \\ (u_1, \ldots, u_n) \to x\langle t\_args'\rangle\,\textbf{pure} \equiv (u'_1, \ldots, u'_n) \to x\langle t\_args\rangle\,\textbf{pure}[t\_arg_1/tid_1 \ldots t\_arg_m/tid_m] \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_1 \vdash pat_1 : t_1 \vartriangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_n \vdash pat_n : t_n \vartriangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}} \vdash x\langle t\_args'\rangle \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(pat_1, \ldots, pat_n) : x\langle t\_args'\rangle \vartriangleright id(pat'_1, \ldots, pat'_n), \uplus E^{\mathrm{T}}{}_1 \ldots E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n} \quad \textsc{check\_pat\_constr}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \rhd \{tid_1 \mapsto kinf_1, \,..\,, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, \mathbf{unit} \rightarrow x\langle t\_arg_1 \,..\, t\_arg_m\rangle\, \mathbf{pure} \\ \mathbf{unit} \rightarrow x\langle t\_args'\rangle\, \mathbf{pure} \equiv \mathbf{unit} \rightarrow x\langle t\_args\rangle\, \mathbf{pure}[t\_arg_1/tid_1 \,..\, t\_arg_m/tid_m] \\ E^{\mathrm{D}} \vdash x\langle t\_args'\rangle \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id : t \rhd id, \{\,\}, \Sigma^{\mathrm{N}}} \quad \text{CHECK\_PAT\_IDENTCONSTR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \rhd \{\,\}, \{\,\}, \mathbf{Default}, t \\ E^{\mathrm{D}} \vdash t \lessapprox u, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash id : t \rhd id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \text{CHECK\_PAT\_VARDEFAULT}$$

$$\frac{}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id : t \rhd id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \{\,\}} \quad \text{CHECK\_PAT\_VAR}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \rhd x\langle t\_args\rangle, (\,\overline{t_i}^{\,i}\,) \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash pat_i : u_i \rhd pat'_i, E^{\mathrm{T}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \mathbf{disjoint\ doms}\,(\,\overline{E^{\mathrm{T}}{}_i}^{\,i}\,) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_args\rangle \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\,\overline{id_i = pat_i}^{\,i}\,;^?\} : x\langle t\_args\rangle \rhd \{\,\overline{id_i = pat'_i}^{\,i}\,;^?\}, \uplus\,\overline{E^{\mathrm{T}}{}_i}^{\,i}, \Sigma^{\mathrm{N}} \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}} \quad \text{CHECK\_PAT\_RECORD}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \rhd pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \rhd pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}\prime}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}\prime}{}_n \\ ne_4 \equiv \mathbf{length}\,(pat_1 \ldots pat_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_1\, ne_2\, order\, t\rangle \vdash [pat_1, \ldots, pat_n] : \mathbf{vector}\,\langle ne_3\, ne_4\, order\, u\rangle \rhd [pat'_1, \ldots, pat'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime} \uplus \{ne_2 = ne_4\}} \quad \text{CHECK\_PAT\_VECTOR}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \rhd pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \rhd pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}\prime}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}\prime}{}_n \\ ne_4 \equiv \mathbf{length}\,(pat_1 \ldots pat_n) \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ num_1 < \ldots < num_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_1\, ne_2\, \mathbf{inc}\, t\rangle \vdash [num_1 = pat_1, \ldots, num_n = pat_n] : \mathbf{vector}\,\langle ne_3\, ne_4\, \mathbf{inc}\, t\rangle \rhd [num_1 = pat'_1, \ldots, num_n = pat'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{ne_1 \le num_1, ne_2 \ge ne_4\} \uplus \Sigma}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, \Sigma^{\mathrm{N}\prime}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \precapprox t, \Sigma^{\mathrm{N}\prime}{}_n \\ \mathit{ne}_4 \equiv \mathbf{length}\,(\mathit{pat}_1 \ldots \mathit{pat}_n) \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ \mathit{num}_1 > \ldots > \mathit{num}_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathbf{dec}\, t\rangle \vdash [\mathit{num}_1 = \mathit{pat}_1, \ldots, \mathit{num}_n = \mathit{pat}_n] : \mathbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \mathbf{dec}\, t\rangle \triangleright [\mathit{num}_1 = \mathit{pat}'_1, \ldots, \mathit{num}_n = \mathit{pat}'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{\mathit{ne}_1 \geq \mathit{num}_1, \mathit{ne}_2 \geq \mathit{ne}_4\} \uplus}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle \mathit{ne}''_1\, \mathit{ne}'''_1\, \mathit{order}\, t\rangle \vdash \mathit{pat}_1 : \mathbf{vector}\,\langle \mathit{ne}''_1\, \mathit{ne}'_1\, \mathit{order}\, u_1\rangle \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle \mathit{ne}''_n\, \mathit{ne}'''_n\, \mathit{order}\, t\rangle \vdash \mathit{pat}_1 : \mathbf{vector}\,\langle \mathit{ne}''_n\, \mathit{ne}'_n\, \mathit{order}\, u_1\rangle \triangleright \mathit{pat}'_n, E \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, \Sigma^{\mathrm{N}\prime}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \precapprox t, \Sigma^{\mathrm{N}\prime}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\, t\rangle \vdash \mathit{pat}_1 : \ldots : \mathit{pat}_n : \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_4\, \mathit{order}\, t\rangle \triangleright \mathit{pat}'_1 : \ldots : \mathit{pat}'_n, (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{\mathit{ne}'_1 + \ldots + \mathit{ne}'_n \leq \mathit{ne}_2\} \uplus \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}}$$

$$\frac{\begin{array}{l} E, t_1 \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots. \quad E, t_n \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots., E^{\mathrm{T}}{}_n) \end{array}}{E, (t_1, \ldots., t_n) \vdash (\mathit{pat}_1, \ldots., \mathit{pat}_n) : (u_1, \ldots., u_n) \triangleright (\mathit{pat}'_1, \ldots., \mathit{pat}'_n), (E^{\mathrm{T}}{}_1 \uplus \ldots. \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}}{}_1 \uplus \ldots. \uplus \Sigma^{\mathrm{N}}{}_n} \quad \text{CHECK\_PAT\_TUP}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, \Sigma^{\mathrm{N}\prime}{}_1 \quad .. \quad E^{\mathrm{D}} \vdash u_n \precapprox t, \Sigma^{\mathrm{N}\prime}{}_n \\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}\prime}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle \vdash [||\mathit{pat}_1, .., \mathit{pat}_n||] : \mathbf{list}\,\langle t\rangle \triangleright [||\mathit{pat}'_1, .., \mathit{pat}'_n||], (E^{\mathrm{T}}{}_1 \uplus .. \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \text{CHECK\_PAT\_LIST}$$

$\boxed{E, t \vdash \mathit{exp} : t' \triangleright \mathit{exp}', I, E^{\mathrm{T}}}$ Typing expressions, collecting nexp constraints, effects, and new bindings

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \{\mathit{tid}_0 \mapsto \mathit{kinf}_0, .., \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \{\,\}, \mathbf{Ctor}, \mathbf{unit} \rightarrow x\langle \mathit{t\_args}\rangle\, \mathbf{pure} \\ u \equiv x\langle \mathit{t\_args}\rangle[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{id} : x \triangleright \mathit{id}, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, \{\,\}} \quad \text{CHECK\_EXP\_UNARYCTOR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, tag, u \\ E^{\mathrm{D}}, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}}, effect\rangle, \{\,\}} \quad \textsc{check\_exp\_localVar}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \,..\, , tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u' \\ u \equiv u'[t\_arg_1/tid_1 \,..\, t\_arg_n/tid_n] \\ E^{\mathrm{D}}, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}\prime}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, effect\rangle, \{\,\}} \quad \textsc{check\_exp\_otherVar}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \,..\, , tid_n \mapsto kinf_n\}, \{\,\}, \mathbf{Ctor}, t'' \to x\langle t\_args\rangle\ \mathbf{pure} \\ t' \to u\ \mathbf{pure} \equiv t'' \to x\langle t\_args\rangle\ \mathbf{pure}[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t' \vdash exp : u' \triangleright exp, \langle \Sigma^{\mathrm{N}\prime}, effect\rangle, E^{\mathrm{T}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp) : t \triangleright id(exp'), \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}, effect\rangle, \{\,\}} \quad \textsc{check\_exp\_Ctor}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \,..\, , tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ u_i \equiv (\mathbf{implicit}\ \langle ne\rangle, t_0, \,..\, , t_m) \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_0, \,..\, , t_m) \vdash (exp_0, \,..\, , exp_m) : u_i' \triangleright (exp_0', \,..\, , exp_m'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash id(annot, exp_0', \,..\, , exp_m') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp_0, \,..\, , exp_m) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appImplicit}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \,..\, , tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash id(exp') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\,\{tid_0 \mapsto kinf_0, \,..\, , tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \ldots tinf_n \\ u[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ \texttt{<<no parses (char 3): sel***ect (conformsto(ui', t)) of tinf1 ... tinfn gives tinf >>} \\ \langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t \vdash id(exp) : t' \triangleright exp'', I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp) : u_j \triangleright exp'', I \uplus I' \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \ldots, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 \ldots t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash exp_1'\ id\ exp_2' : u_j \triangleright u_j', exp, \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_infix\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\, \{tid_0 \mapsto kinf_0, \ldots, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \ldots tinf_n \\ u[t\_arg_0/tid_0 \ldots t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ \texttt{<<no parses (char 3): sel***ect (conformsto(ui', t)) of tinf1 ... tinfn gives tinf >>} \\ \langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t' \triangleright exp, I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_infix\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \triangleright x\langle t\_args\rangle, \overline{t_i}^{\,i} \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash exp_i : u_i \triangleright exp_i', \langle \Sigma^{\mathrm{N}}{}_i, effect_i\rangle, E^{\mathrm{T}}}^{\,i} \\ \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u_i \precapprox t_i, \Sigma^{\mathrm{N}\prime}_i}^{\,i} \\ \Sigma^{\mathrm{N}} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}\prime}_i}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\overline{id_i = exp_i}^{\,i}\, ;^?\} : x\langle t\_args\rangle \triangleright \{\overline{id_i = exp_i'}^{\,i}\, ;^?\}, \uplus\langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, \uplus\, \overline{effect_i}^{\,i}\rangle, \{\,\}} \quad \textsc{check\_exp\_record}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash exp : x\langle t\_args\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E^{\mathrm{R}}(x\langle t\_args\rangle) \triangleright \overline{id_n' : t_n'}^{\,n} \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash exp_i : u_i \triangleright exp_i', I_i, E^{\mathrm{T}}}^{\,i} \\ \overline{id_i : t_i}^{\,i} \subset \overline{id_n' : t_n'}^{\,n} \\ \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u_i \precapprox t_i, \Sigma^{\mathrm{N}\prime}_i}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{exp\ \mathbf{with}\ \overline{id_i = exp_i}^{\,i}\, ;^?\} : x\langle t\_args\rangle \triangleright \{exp'\ \mathbf{with}\ \overline{id_i = exp_i'}^{\,i}\ \}, I \uplus \overline{I_i}^{\,i}, E^{\mathrm{T}}} \quad \textsc{check\_exp\_recup}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1 : u_1 \triangleright exp_1', I_1, E^{\mathrm{T}\prime} \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_n : u_n \triangleright exp_n', I_n, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \precapprox t, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{length}\,(exp_1 \ldots exp_n) \equiv ne \\ \Sigma^{\mathrm{N}} \equiv \{ne = ne_2\} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle \vdash [exp_1, \ldots, exp_n] : \mathbf{vector}\,\langle ne_1\ num\ order\ t\rangle \triangleright [exp_1', \ldots, exp_n'], \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle \uplus I_1 \uplus \ldots \uplus I_n, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vector}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; order\; t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_1\; ne'_1\; \mathbf{inc}\; u\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_2\; ne'_2\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_3\; ne'_3\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \end{array}}{E, t \vdash exp_1[exp_2] : u \triangleright exp'_1[exp'_2], I_1 \uplus I_2 \uplus \langle\{ne_1 \le ne_3, ne_3 + ne'_3 \le ne_1 + ne'_1\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vectorgetInc}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; order\; t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_1\; ne'_1\; \mathbf{dec}\; u\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_2\; ne'_2\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_3\; ne'_3\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \end{array}}{E, t \vdash exp_1[exp_2] : u \triangleright exp'_1[exp'_2], I_1 \uplus I_2 \uplus \langle\{ne_1 \ge ne_3, ne_3 + (-ne'_3) \le ne_1 + (-ne'_1)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vectorgetDec}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne'_1\; \mathbf{inc}\; t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_2\; ne'_2\; \mathbf{inc}\; u\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\; ne'_3\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_4\; ne'_4\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\; ne'_5\rangle \vdash exp_3 : \mathbf{range}\,\langle ne_6\; ne'_6\rangle \triangleright exp'_3, I_3, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\; ne'_7\; \mathbf{inc}\; u\rangle \triangleright exp'_1[exp'_2 : exp'_3], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \ge ne_4, ne \le ne'_4, ne' \le ne_4 + ne'_6, ne_4 \le ne_2, ne_4 + ne'_6 \le ne'_2\}, \mathbf{pure}}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne'_1\; \mathbf{dec}\; t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_2\; ne'_2\; \mathbf{dec}\; u\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\; ne'_3\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_4\; ne'_4\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\; ne'_5\rangle \vdash exp_3 : \mathbf{range}\,\langle ne_6\; ne'_6\rangle \triangleright exp'_3, I_3, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\; ne'_7\; \mathbf{dec}\; u\rangle \triangleright exp'_1[exp'_2 : exp'_3], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \le ne_4, ne \ge ne'_4, ne' \le ne'_6 + (-ne_4), ne'_4 \ge ne_2, ne'_6 + (-ne_4) \le ne}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle \vdash exp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne'_1\; ne'_2\rangle \vdash exp_1 : \mathbf{range}\,\langle ne_3\; ne_4\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, t \vdash exp_2 : u \triangleright exp'_2, I_2, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle \vdash [exp\; \mathbf{with}\; exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; u\rangle \triangleright [exp'\; \mathbf{with}\; exp'_1 = exp'_2], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \le ne_3, ne_2 \ge ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vectorU}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle \vdash exp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne'_1\; ne'_2\rangle \vdash exp_1 : \mathbf{range}\,\langle ne_3\; ne_4\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, t \vdash exp_2 : u \triangleright exp'_2, I_2, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle \vdash [exp\; \mathbf{with}\; exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; u\rangle \triangleright [exp'\; \mathbf{with}\; exp'_1 = exp'_2], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \ge ne_3, ne_2 \ge ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vector}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, \mathbf{vector}\,\langle ne_9\; ne_{10}\; \mathbf{inc}\; t\rangle \vdash exp_3 : \mathbf{vector}\,\langle ne_{11}\; ne_{12}\; \mathbf{inc}\; u\rangle \triangleright exp'_3, I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_3 \le ne_5, ne_3 + ne_4 \le ne_7, ne_{12} = ne_8 + (-ne_6), ne_6 + \mathbf{one} \le ne_8\}, \mathbf{pure}\rangle \end{array}}{E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash [exp\; \mathbf{with}\; exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \triangleright [exp'\; \mathbf{with}\; exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vecrangeupInc}$$

$$\frac{\begin{array}{l} E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{inc}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_5\rangle \vdash exp_1 : \textbf{atom}\,\langle ne_6\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_7\rangle \vdash exp_2 : \textbf{atom}\,\langle ne_8\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, u \vdash exp_3 : u' \triangleright exp'_3, I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle \{ne_3 \leq ne_5, ne_3 + ne_4 \leq ne_7\}, \textbf{pure}\rangle \end{array}}{E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash [exp\; \textbf{with}\; exp_1 : exp_2 = exp_3] : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{inc}\; u\rangle \triangleright [exp'\; \textbf{with}\; exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupvalu}$$

$$\frac{\begin{array}{l} E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{dec}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_5\rangle \vdash exp_1 : \textbf{atom}\,\langle ne_6\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_7\rangle \vdash exp_2 : \textbf{atom}\,\langle ne_8\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, \textbf{vector}\,\langle ne_9\; ne_{10}\; \textbf{dec}\; t\rangle \vdash exp_3 : \textbf{vector}\,\langle ne_{11}\; ne_{12}\; \textbf{dec}\; u\rangle \triangleright exp'_3, I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle \{ne_5 \leq ne_3, ne_3 + (-ne_4) \leq ne_6 + (-ne_8), ne_8 + \textbf{one} \leq ne_6\}, \textbf{pure}\rangle \end{array}}{E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash [exp\; \textbf{with}\; exp_1 : exp_2 = exp_3] : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{dec}\; u\rangle \triangleright [exp'\; \textbf{with}\; exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupDec}$$

$$\frac{\begin{array}{l} E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{dec}\; u\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_5\rangle \vdash exp_1 : \textbf{atom}\,\langle ne_6\rangle \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \textbf{atom}\,\langle ne_7\rangle \vdash exp_2 : \textbf{atom}\,\langle ne_8\rangle \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, u \vdash exp_3 : u' \triangleright exp'_3, I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle \{ne_5 \leq ne_3, ne_3 + (-ne_4) \leq ne_6 + (-ne_8), ne_8 + \textbf{one} \leq ne_6\}, \textbf{pure}\rangle \end{array}}{E, \textbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash [exp\; \textbf{with}\; exp_1 : exp_2 = exp_3] : \textbf{vector}\,\langle ne_3\; ne_4\; \textbf{dec}\; u\rangle \triangleright [exp'\; \textbf{with}\; exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupval}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(x\langle t\_args\rangle) \triangleright \overline{id_i : t_i}^{\,i}\; id : u\; \overline{id'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t'' \vdash exp : x\langle t\_args\rangle \triangleright exp', I, E^{\mathrm{T}} \\ E^{\mathrm{D}}, t \vdash exp'.id : u \triangleright t', exp'_1, \Sigma^{\mathrm{N}'}, effect \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash exp.id : u \triangleright exp'_1, I \uplus \langle \Sigma^{\mathrm{N}'}, effect\rangle, E^{\mathrm{T}}}\;\textsc{check\_exp\_field}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'' \vdash exp : u \triangleright exp', I, E^{\mathrm{T}} \\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash pat_i : u'_i \triangleright pat'_i, E^{\mathrm{T}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \overline{\langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_i), E^{\mathrm{D}}\rangle, t \vdash exp_i : u''_i \triangleright exp'_i, I_i, E^{\mathrm{T}}{}'_i}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \textbf{switch}\; exp\{\,\overline{\textbf{case}\; pat_i \to exp_i}^{\,i}\,\} : u \triangleright \textbf{switch}\; exp'\{\,\overline{\textbf{case}\; pat'_i \to exp'_i}^{\,i}\,\}, I \uplus \overline{I_i \uplus \langle \Sigma^{\mathrm{N}}{}_i, \textbf{pure}\rangle}^{\,i}, E^{\mathrm{T}}}\;\textsc{check\_exp\_case}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'' \vdash \mathit{exp} : u \triangleright \mathit{exp}', I, E^{\mathrm{T}} \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t' \\ E^{\mathrm{D}}, t' \vdash \mathit{exp}' : u \triangleright u', \mathit{exp}'', \Sigma^{\mathrm{N}}, \mathit{effect} \\ E^{\mathrm{D}}, t \vdash \mathit{exp}'' : t' \triangleright u'', \mathit{exp}''', \Sigma^{\mathrm{N}\prime}, \mathit{effect}'\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash (\mathit{typ})\mathit{exp} : t \triangleright \mathit{exp}''', I \uplus \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, \mathit{effect} \uplus \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_TYPED}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{letbind} \triangleright \mathit{letbind}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}}, \mathit{effect}, \{\,\} \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{letbind}\ \mathbf{in}\ \mathit{exp} : t \triangleright \mathit{letbind}'\ \mathbf{in}\ \mathit{exp}', \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LET}$$

$$\frac{E, t_1 \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad .... \quad E, t_n \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n}{E, (t_1, \ ...., t_n) \vdash (\mathit{exp}_1, \ ...., \mathit{exp}_n) : (u_1, \ ...., u_n) \triangleright (\mathit{exp}'_1, \ ...., \mathit{exp}'_n), I_1 \uplus \ .... \ \uplus I_n, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_TUP}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n \\ E^{\mathrm{D}} \vdash u_1 \gtrapprox t, {\Sigma^{\mathrm{N}}}_1 \quad .. \quad E^{\mathrm{D}} \vdash u_n \gtrapprox t, {\Sigma^{\mathrm{N}}}_n\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle \vdash [||\mathit{exp}_1, \ .., \mathit{exp}_n||] : \mathbf{list}\,\langle u\rangle \triangleright [||\mathit{exp}'_1, \ .., \mathit{exp}'_n||], \langle {\Sigma^{\mathrm{N}}}_1 \uplus \ .. \ \uplus {\Sigma^{\mathrm{N}}}_n, \mathbf{pure}\rangle \uplus I_1 \uplus \ .. \ \uplus I_n, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LIST}$$

$$\frac{\begin{array}{l}E, \mathbf{bit} \vdash \mathit{exp}_1 : \mathbf{bit} \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}\prime} \\ E, t \vdash \mathit{exp}_2 : u_1 \triangleright \mathit{exp}'_2, I_2, {E^{\mathrm{T}}}_2 \\ E, t \vdash \mathit{exp}_3 : u_2 \triangleright \mathit{exp}'_3, I_3, {E^{\mathrm{T}}}_3 \\ E^{\mathrm{D}} \vdash u_1 \gtrapprox t, {\Sigma^{\mathrm{N}}}_1 \\ E^{\mathrm{D}} \vdash u_2 \gtrapprox t, {\Sigma^{\mathrm{N}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathbf{if}\ \mathit{exp}_1\ \mathbf{then}\ \mathit{exp}_2\ \mathbf{else}\ \mathit{exp}_3 : u \triangleright \mathbf{if}\ \mathit{exp}'_1\ \mathbf{then}\ \mathit{exp}'_2\ \mathbf{else}\ \mathit{exp}'_3, \langle {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2, \mathbf{pure}\rangle \uplus I_1 \uplus I_2 \uplus I_3, ({E^{\mathrm{T}}}_2 \cap {E^{\mathrm{T}}}_3)} \quad \text{CHECK\_EXP\_IF}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2\rangle \vdash \mathit{exp}_1 : \mathbf{range}\,\langle \mathit{ne}_7\ \mathit{ne}_8\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_3\ \mathit{ne}_4\rangle \vdash \mathit{exp}_2 : \mathbf{range}\,\langle \mathit{ne}_9\ \mathit{ne}_{10}\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{range}\,\langle \mathit{ne}_5\ \mathit{ne}_6\rangle \vdash \mathit{exp}_3 : \mathbf{range}\,\langle \mathit{ne}_{11}\ \mathit{ne}_{12}\rangle \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}} \\ \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_4\rangle\}), E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathit{exp}_4 : t \triangleright \mathit{exp}'_4, I_4, E^{\mathrm{T}\prime}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}_1\ \mathbf{to}\ \mathit{exp}_2\ \mathbf{by}\ \mathit{exp}_3)\mathit{exp}_4 : t \triangleright \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}'_1\ \mathbf{to}\ \mathit{exp}'_2\ \mathbf{by}\ \mathit{exp}'_3)\mathit{exp}'_4, I_1 \uplus I_2 \uplus I_3 \uplus I_4 \uplus \langle \{\mathit{ne}_1 \leq \mathit{ne}_3 + \mathit{ne}_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXI}$$

$$\frac{\begin{array}{l}E, t \vdash \mathit{exp}_1 : u \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{list}\,\langle t\rangle \vdash \mathit{exp}_2 : \mathbf{list}\,\langle u\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{list}\,\langle t\rangle \vdash \mathit{exp}_1 :: \mathit{exp}_2 : \mathbf{list}\,\langle u\rangle \triangleright \mathit{exp}'_1 :: \mathit{exp}'_2, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_CONS}$$

$$\frac{t \vdash \mathit{lit} : u \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}{E, t \vdash \mathit{lit} : u \triangleright \mathit{exp}, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LIT}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I, {E^{\mathrm{T}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \{\mathit{exp}\} : \mathbf{unit} \rhd \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \quad \textsc{check\_exp\_blockbase}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \{\, \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \rhd \{\, \overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \{\mathit{exp};\ \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \rhd \{\mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \textsc{check\_exp\_blockrec}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I, {E^{\mathrm{T}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\mathit{exp}\} : \mathbf{unit} \rhd \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \quad \textsc{check\_exp\_nondetbase}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\, \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \rhd \{\, \overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\mathit{exp};\ \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \rhd \{\mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \textsc{check\_exp\_nondetrec}$$

$$\frac{\begin{array}{c}E, t \vdash \mathit{exp} : u \rhd \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ E \vdash \mathit{lexp} : t \rhd \mathit{lexp}', I_2, {E^{\mathrm{T}}}_2\end{array}}{E, \mathbf{unit} \vdash \mathit{lexp} := \mathit{exp} : \mathbf{unit} \rhd \mathit{lexp}' := \mathit{exp}', I \uplus I_2, {E^{\mathrm{T}}}_2} \quad \textsc{check\_exp\_assign}$$

$\boxed{E \vdash \mathit{lexp} : t \rhd \mathit{lexp}', I, E^{\mathrm{T}}}$ Check the left hand side of an assignment

$$\frac{E^{\mathrm{T}}(\mathit{id}) \rhd \mathbf{register}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \rhd \mathit{id}, \langle\{\,\}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \rhd \mathbf{reg}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \rhd \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wlocl}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \rhd t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \rhd \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_var}$$

$$\frac{\mathit{id} \notin \mathbf{dom}\,(E^{\mathrm{T}})}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \rhd \mathit{id}, I_\epsilon, \{\mathit{id} \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnew}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(\mathit{id}) \rhd \mathbf{register}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (\mathit{typ})\mathit{id} : t \rhd \mathit{id}, \langle\Sigma^{\mathrm{N}}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}\mathrm{Cast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \mathbf{reg}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash typ \leadsto u \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (typ)id : t \triangleright id, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wloclCast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright t \\ E^{\mathrm{D}} \vdash typ \leadsto u \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (typ)id : t \triangleright id, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_varCast}$$

$$\frac{\begin{array}{l} id \not\in \mathbf{dom}\,(E^{\mathrm{T}}) \\ E^{\mathrm{D}} \vdash typ \leadsto t \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (typ)id : t \triangleright id, I_{\epsilon}, \{id \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnewCast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, t_1 \to t\,\{\,\overline{base\_effect_i}^{\,i}\,, \mathbf{wmem},\ \overline{base\_effect'_j}^{\,j}\ \} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash exp : u_1 \triangleright exp', I, {E^{\mathrm{T}}}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash id(exp) : t \triangleright id(exp'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wmem}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wmem}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle ne\rangle \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E \vdash lexp : \mathbf{vector}\,\langle ne_1\, ne_2\, \mathbf{inc}\, t\rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle\{ne_1 \leq ne, ne_1 + ne_2 \geq ne\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitInc}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle ne\rangle \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E \vdash lexp : \mathbf{vector}\,\langle ne_1\, ne_2\, \mathbf{dec}\, t\rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle\{ne \leq ne_1, ne_1 + (-ne_2) \leq ne\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitDec}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle ne_1\rangle \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2\rangle \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E \vdash lexp : \mathbf{vector}\,\langle ne_3\, ne_4\, \mathbf{inc}\, t\rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\, ne_2 + (-ne_1)\, \mathbf{inc}\, t\rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne_3 \leq ne_1, ne_3 + ne_4 \leq ne_2 + (-ne_1)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wsliceInc}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle ne_1\rangle \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2\rangle \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E \vdash lexp : \mathbf{vector}\,\langle ne_3\, ne_4\, \mathbf{inc}\, t\rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\, ne_2 + (-ne_1)\, \mathbf{inc}\, t\rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne_1 \leq ne_3, ne_3 + (-ne_4) \leq ne_1 + (-ne_2)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wsliceDec}$$

$$\frac{\begin{array}{c} E^{\mathrm{R}}(x\langle t\_args\rangle) \rhd \overline{id_i : t_i}^{\,i}\ id : t\ \overline{id'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash lexp : x\langle t\_args\rangle \rhd lexp', I, E^{\mathrm{T}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash lexp.id : t \rhd lexp'.id, I, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_WRECORD}$$

$\boxed{E \vdash letbind \rhd letbind', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}, effect, E^{\mathrm{K}}}$ Build the environment for a let binding, collecting index constraints

$$\frac{\begin{array}{l} \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typschm \rightsquigarrow t, E^{\mathrm{K}}{}_2, \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash pat : u \rhd pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash exp : u' \rhd exp', \langle \Sigma^{\mathrm{N}}{}_2, effect\rangle, E^{\mathrm{T}}{}_2 \\ \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u' \gtrapprox u, \Sigma^{\mathrm{N}}{}_3 \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{let}\ typschm\ pat = exp \rhd \mathbf{let}\ typschm\ pat' = exp', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2 \uplus \Sigma^{\mathrm{N}}{}_3, effect, E^{\mathrm{K}}{}_2} \quad \text{CHECK\_LETBIND\_VAL\_ANNOT}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat : u \rhd pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \\ \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}}\rangle, u \vdash exp : u' \rhd exp', \langle \Sigma^{\mathrm{N}}{}_2, effect\rangle, E^{\mathrm{T}}{}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{let}\ pat = exp \rhd \mathbf{let}\ pat' = exp', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2, effect, \{\,\}} \quad \text{CHECK\_LETBIND\_VAL\_NOANNOT}$$

$\boxed{E^{\mathrm{D}} \vdash type\_def \rhd E}$ Check a type definition

$$\frac{E^{\mathrm{D}} \vdash typschm \rightsquigarrow t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ id\ name\_scm\_opt = typschm \rhd \langle\{\,\}, \langle\{\,\}, \{id \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_ABBREV}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash typ_1 \rightsquigarrow t_1 \quad .. \quad E^{\mathrm{D}} \vdash typ_n \rightsquigarrow t_n \\ E^{\mathrm{R}} \equiv \{\{id_1 : t_1, .., id_n : t_n\} \mapsto x\} \end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ name\_scm\_opt = \mathbf{const\ struct}\ \{typ_1\ id_1; ..; typ_n\ id_n\ ;^?\} \rhd \langle\{\,\}, \langle\{x \mapsto K\_Typ\}, \{\,\}, E^{\mathrm{R}}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_RECORD}$$

$$\frac{\begin{array}{l} \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash quant\_item_i \rightsquigarrow E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i} \\ \langle E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_1 \rightsquigarrow t_1 \quad .. \quad \langle E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_n \rightsquigarrow t_n \\ \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\} \equiv \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i} \\ E^{\mathrm{R}}_1 \equiv \{\{id_1 : t_1, .., id_n : t_n\} \mapsto \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\}, \uplus \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}, \mathbf{None}, x\langle x'_1 .. x'_m\rangle\} \\ E^{\mathrm{K}}{}'_1 \equiv \{x \mapsto K\_Lam(k_1 .. k_m \to K\_Typ)\} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ name\_scm\_opt = \mathbf{const\ struct\ forall}\ \overline{quant\_item_i}^{\,i}.\{typ_1\ id_1; ..; typ_n\ id_n\ ;^?\} \rhd \langle\{\,\}, \langle E^{\mathrm{K}}{}', \{\,\}, E^{\mathrm{R}}_1, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_QUANT\_RECORD}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}} \equiv \{id_1 \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_1 \to x\ \mathbf{pure}, \ldots, id_n \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_n \to x\ \mathbf{pure}\} \\ E^{\mathrm{K}}{}_1 \equiv \{x \mapsto K\_Typ\} \\ \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_1 \rightsquigarrow t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_n \rightsquigarrow t_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ name\_scm\_opt = \mathbf{const\ union}\ \{typ_1\ id_1; \ldots; typ_n\ id_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}{}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_UNION}$$

$$\frac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash quant\_item_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\ \{x'_1 \mapsto k_1, \ldots, x'_m \mapsto k_m\} \equiv \uplus \overline{{E^{\mathrm{K}}}_i}^{i}\\ {E^{\mathrm{K}}}' \equiv \{x \mapsto K\_Lam(k_1 \ldots k_m \to K\_Typ)\} \uplus \overline{{E^{\mathrm{K}}}_i}^{i}\\ \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}', E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_1 \leadsto t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}', E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_n \leadsto t_n\\ t \equiv x\langle x'_1 \ldots x'_m\rangle\\ E^{\mathrm{T}} \equiv \{id_1 \mapsto {E^{\mathrm{K}}}', \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}, \mathbf{Ctor}, t_1 \to t\, \mathbf{pure}, \ldots, id_n \mapsto {E^{\mathrm{K}}}', \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}, \mathbf{Ctor}, t_n \to t\, \mathbf{pure}\}\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ id\ name\_scm\_opt = \mathbf{const\ union\ forall}\ \overline{quant\_item_i}^{i}.\{typ_1\ id_1; \ldots; typ_n\ id_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle {E^{\mathrm{K}}}', \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_QUANT\_UNION}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}} \equiv \{id_1 \mapsto x, \ldots, id_n \mapsto x\}\\ E^{\mathrm{E}} \equiv \{x \mapsto \{num_1 \mapsto id_1 \ldots num_n \mapsto id_n\}\}\end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ name\_scm\_opt = \mathbf{enumerate}\,\{id_1; \ldots; id_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle\{id \mapsto K\_Typ\}, \{\,\}, \{\,\}, E^{\mathrm{E}}\rangle\rangle} \quad \text{CHECK\_TD\_ENUMERATE}$$

$\boxed{E \vdash fundef \rhd fundef', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Check a function definition

$$\frac{\begin{array}{l}E^{\mathrm{T}}(id) \rhd {E^{\mathrm{K}}}', {\Sigma^{\mathrm{N}}}', \mathbf{Global}, t_1 \to t\ effect\\ \overline{E^{\mathrm{D}} \vdash quant\_item_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\ {\Sigma^{\mathrm{N}}}'' \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}\\ {E^{\mathrm{K}}}' \equiv \overline{{E^{\mathrm{K}}}_i}^{i}\\ {E^{\mathrm{D}}}_1 \equiv \langle {E^{\mathrm{K}}}', \{\,\}, \{\,\}, \{\,\}\rangle \uplus E^{\mathrm{D}}\\ {E^{\mathrm{D}}}_1 \vdash typ \leadsto u\\ {E^{\mathrm{D}}}_1 \vdash u \precapprox t, {\Sigma^{\mathrm{N}}}_2\\ \overline{\langle E^{\mathrm{T}}, {E^{\mathrm{D}}}_1\rangle, t_1 \vdash pat_j : u_j \rhd pat'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}}}'''_j}^{j}\\ \overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_j), {E^{\mathrm{D}}}_1\rangle, u \vdash exp_j : u' \rhd exp'_j, \langle {\Sigma^{\mathrm{N}}}''''_j, effect'_j\rangle, {E^{\mathrm{T}}}'_j}^{j}\\ {\Sigma^{\mathrm{N}}}''''' \equiv {\Sigma^{\mathrm{N}}}_2 \uplus \overline{{\Sigma^{\mathrm{N}}}'''_j \uplus {\Sigma^{\mathrm{N}}}''''_j}^{j}\\ effect \equiv \uplus \overline{effect'_j}^{j}\\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,({\Sigma^{\mathrm{N}}}' \uplus {\Sigma^{\mathrm{N}}}'' \uplus {\Sigma^{\mathrm{N}}}''''')\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\ rec\ forall}\ \overline{quant\_item_i}^{i} .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{j} \rhd \mathbf{function\ rec\ forall}\ \overline{quant\_item_i}^{i} .\ typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}} \quad \text{CHECK\_FD\_REC}$$

$$
\dfrac{\begin{array}{l}
E^{\mathrm{T}}(\mathit{id}) \rhd E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect} \\
E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\
E^{\mathrm{D}} \vdash u \lessapprox t, {\Sigma^{\mathrm{N}}}_2 \\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \rhd \mathit{pat}', {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime\prime}}_j}^{j} \\
\overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_j), E^{\mathrm{D}}\rangle, u \vdash \mathit{exp}_j : u'_j \rhd \mathit{exp}'_j, \langle {\Sigma^{\mathrm{N}\prime\prime\prime}}_j, \mathit{effect}'_j\rangle, {E^{\mathrm{T}\prime}}_j}^{j} \\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j} \\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,({\Sigma^{\mathrm{N}}}_2 \uplus \Sigma^{\mathrm{N}\prime} \uplus \overline{{\Sigma^{\mathrm{N}\prime\prime}}_j \uplus {\Sigma^{\mathrm{N}\prime\prime\prime}}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \rhd \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}\quad \textsc{check\_fd\_rec\_function2}
$$

$$
\dfrac{\begin{array}{l}
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i} \\
\Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{i} \\
E^{\mathrm{K}\prime} \equiv E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{i} \\
\langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \rightsquigarrow t \\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \rhd \mathit{pat}'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime\prime}}_j}^{j} \\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\}) \\
\overline{\langle (E^{\mathrm{T}\prime} \uplus {E^{\mathrm{T}}}_j), \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp}_j : u'_j \rhd \mathit{exp}'_j, \langle {\Sigma^{\mathrm{N}\prime\prime\prime}}_j, \mathit{effect}'_j\rangle, {E^{\mathrm{T}\prime}}_j}^{j} \\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j} \\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \overline{{\Sigma^{\mathrm{N}\prime\prime}}_j \uplus {\Sigma^{\mathrm{N}\prime\prime\prime}}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{i}\,.\,\mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \rhd \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{i}\,.\,\mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\quad \textsc{c}
$$

$$
\dfrac{\begin{array}{l}
E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t \\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \rhd \mathit{pat}'_j, {E^{\mathrm{T}}}_j, {\Sigma^{\mathrm{N}\prime}}_j}^{j} \\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \{\,\}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\}) \\
\overline{\langle (E^{\mathrm{T}\prime} \uplus {E^{\mathrm{T}}}_j), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_j : u'_j \rhd \mathit{exp}'_j, \langle {\Sigma^{\mathrm{N}\prime}}_j, \mathit{effect}'_j\rangle, {E^{\mathrm{T}\prime}}_j}^{j} \\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j} \\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{{\Sigma^{\mathrm{N}\prime}}_j \uplus {\Sigma^{\mathrm{N}\prime\prime}}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \rhd \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\quad \textsc{check\_fd\_rec\_function\_no\_spec2}
$$

$$\frac{\begin{array}{l} E^{\text{T}}(id) \triangleright E^{\text{K}\prime}, \Sigma^{\text{N}\prime}, \textbf{Global}, t_1 \to t\ \mathit{effect} \\ \overline{\langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\text{K}}}_i, {\Sigma^{\text{N}}}_i}^{i} \\ \Sigma^{\text{N}\prime\prime} \equiv \uplus \overline{{\Sigma^{\text{N}}}_i}^{i} \\ E^{\text{K}\prime\prime} \equiv \overline{{E^{\text{K}}}_i}^{i} \\ \langle E^{\text{K}\prime\prime} \uplus E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash \mathit{typ} \leadsto u \\ \langle E^{\text{K}\prime\prime} \uplus E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \vdash u \precapprox t, {\Sigma^{\text{N}}}_2 \\ \overline{\langle E^{\text{T}}, \langle E^{\text{K}} \uplus E^{\text{K}\prime\prime}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\text{T}}}_j, \Sigma^{\text{N}\prime\prime}_j}^{j} \\ \overline{\langle (E^{\text{T}} \setminus \mathit{id} \uplus {E^{\text{T}}}_j), \langle E^{\text{K}} \uplus E^{\text{K}\prime\prime}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \rangle, t \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\text{N}\prime\prime\prime}_j, \mathit{effect}'_j \rangle, E^{\text{T}\prime}_j}^{j} \\ \Sigma^{\text{N}\prime\prime\prime\prime} \equiv \uplus \overline{\Sigma^{\text{N}\prime\prime}_j \uplus \Sigma^{\text{N}\prime\prime\prime}_j}^{j} \\ \mathit{effect} \equiv \uplus \overline{\mathit{effect}'_j}^{j} \\ \Sigma^{\text{N}} \equiv \textbf{resolve}\,(\Sigma^{\text{N}\prime} \uplus \Sigma^{\text{N}\prime\prime} \uplus \Sigma^{\text{N}\prime\prime\prime\prime}) \end{array}}{\langle E^{\text{T}}, \langle E^{\text{K}}, E^{\text{A}}, E^{\text{R}}, E^{\text{E}} \rangle \rangle \vdash \textbf{function forall}\,\overline{\mathit{quant\_item}_i}^{i}\,.\,\mathit{typ}\,\textbf{effect}\,\mathit{effect}\,\overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \textbf{function forall}\,\overline{\mathit{quant\_item}_i}^{i}\,.\,\mathit{typ}\,\textbf{effect}\,\mathit{effect}\,\overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\text{T}}, \Sigma^{\text{N}}} \quad \text{CHECK\_F}$$

$$\frac{\begin{array}{l} E^{\text{T}}(id) \triangleright \{\,\}, {\Sigma^{\text{N}}}_1, \textbf{Global}, t_1 \to t\ \mathit{effect} \\ E^{\text{D}} \vdash \mathit{typ} \leadsto u \\ E^{\text{D}} \vdash u \precapprox t, {\Sigma^{\text{N}}}_2 \\ \overline{\langle E^{\text{T}}, E^{\text{D}} \rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, {E^{\text{T}}}_j, \Sigma^{\text{N}\prime}_j}^{j} \\ \overline{\langle (E^{\text{T}} \setminus \mathit{id} \uplus {E^{\text{T}}}_j), E^{\text{D}} \rangle, u \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\text{N}\prime\prime}_j, \mathit{effect}'_j \rangle, E^{\text{T}\prime}_j}^{j} \\ \mathit{effect} \equiv \uplus \overline{\mathit{effect}'_j}^{j} \\ \Sigma^{\text{N}} \equiv \textbf{resolve}\,({\Sigma^{\text{N}}}_1 \uplus {\Sigma^{\text{N}}}_2 \uplus \overline{\Sigma^{\text{N}\prime}_j \uplus \Sigma^{\text{N}\prime\prime}_j}^{j}) \end{array}}{\langle E^{\text{T}}, E^{\text{D}} \rangle \vdash \textbf{function}\ \ \mathit{typ}\,\textbf{effect}\,\mathit{effect}\,\overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \textbf{function}\ \ \mathit{typ}\,\textbf{effect}\,\mathit{effect}\,\overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\text{T}}, \Sigma^{\text{N}}} \quad \text{CHECK\_FD\_FUNCTION2}$$

$$\frac{\begin{array}{l}\overline{\langle E^{\rm K}, E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\rm K}}_i, {\Sigma^{\rm N}}_i}^{i}\\ {\Sigma^{\rm N}}' \equiv \uplus\, \overline{{\Sigma^{\rm N}}_i}^{i}\\ {E^{\rm K}}'' \equiv E^{\rm K} \uplus \overline{{E^{\rm K}}_i}^{i}\\ \langle {E^{\rm K}}'', E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle \vdash \mathit{typ} \leadsto t\\ \overline{\langle E^{\rm T}, \langle {E^{\rm K}}'', E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \rhd \mathit{pat}_j, {E^{\rm T}}_j, {\Sigma^{\rm N}}''_j}^{j}\\ {E^{\rm T}}' \equiv (E^{\rm T} \uplus \{\mathit{id} \mapsto {E^{\rm K}}'', {\Sigma^{\rm N}}', \textbf{Global}, t_1 \to t\ \mathit{effect}\})\\ \overline{\langle (E^{\rm T} \uplus {E^{\rm T}}_j), \langle {E^{\rm K}}'', E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle\rangle, t \vdash \mathit{exp}_j : u'_j \rhd \mathit{exp}'_j, \langle {\Sigma^{\rm N}}''_j, \mathit{effect}'_j\rangle, {E^{\rm T}}'_j}^{j}\\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j}\\ \Sigma^{\rm N} \equiv \textbf{resolve}\,({\Sigma^{\rm N}}' \uplus \overline{{\Sigma^{\rm N}}'_j \uplus {\Sigma^{\rm N}}''_j}^{j})\end{array}}{\langle E^{\rm T}, \langle E^{\rm K}, E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle\rangle \vdash \textbf{function forall}\ \overline{\mathit{quant\_item}_i}^{i}\ .\ \mathit{typ}\ \textbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \rhd \textbf{function forall}\ \overline{\mathit{quant\_item}_i}^{i}\ .\ \mathit{typ}\ \textbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, {E^{\rm T}}', \Sigma^{\rm N}}\ \text{CHECK\_I}$$

$$\frac{\begin{array}{l}E^{\rm D} \vdash \mathit{typ} \leadsto t\\ \overline{\langle E^{\rm T}, E^{\rm D}\rangle, t_1 \vdash \mathit{pat}_j : u_j \rhd \mathit{pat}'_j, {E^{\rm T}}_j, {\Sigma^{\rm N}}'_j}^{j}\\ {E^{\rm T}}' \equiv (E^{\rm T} \uplus \{\mathit{id} \mapsto \{\,\}, \Sigma^{\rm N}, \textbf{Global}, t_1 \to t\ \mathit{effect}\})\\ \overline{\langle (E^{\rm T} \uplus {E^{\rm T}}_j), E^{\rm D}\rangle, t \vdash \mathit{exp}_j : u'_j \rhd \mathit{exp}', \langle {\Sigma^{\rm N}}'_j, \mathit{effect}'_j\rangle, {E^{\rm T}}'_j}^{j}\\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j}\\ \Sigma^{\rm N} \equiv \textbf{resolve}\,(\uplus\, \overline{{\Sigma^{\rm N}}'_j \uplus {\Sigma^{\rm N}}''_j}^{j})\end{array}}{\langle E^{\rm T}, E^{\rm D}\rangle \vdash \textbf{function}\ \ \mathit{typ}\ \textbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \rhd \textbf{function}\ \ \mathit{typ}\ \textbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, {E^{\rm T}}', \Sigma^{\rm N}}\ \text{CHECK\_FD\_FUNCTION\_NO\_SPEC2}$$

$\boxed{E \vdash \mathit{val\_spec} \rhd E^{\rm T}}$ Check a value specification

$$\frac{E^{\rm D} \vdash \mathit{typschm} \leadsto t, {E^{\rm K}}_1, \Sigma^{\rm N}}{\langle E^{\rm T}, E^{\rm D}\rangle \vdash \textbf{val}\ \mathit{typschm}\ \mathit{id} \rhd \{\mathit{id} \mapsto {E^{\rm K}}_1, \Sigma^{\rm N}, \textbf{Global}, t\}}\ \text{CHECK\_SPEC\_VAL\_SPEC}$$

$$\frac{E^{\rm D} \vdash \mathit{typschm} \leadsto t, {E^{\rm K}}_1, \Sigma^{\rm N}}{\langle E^{\rm T}, E^{\rm D}\rangle \vdash \textbf{val extern}\ \mathit{typschm}\ \mathit{id} = \mathit{string} \rhd \{\mathit{id} \mapsto {E^{\rm K}}_1, \Sigma^{\rm N}, \textbf{Extern}\,, t\}}\ \text{CHECK\_SPEC\_EXTERN}$$

$\boxed{E^{\rm D} \vdash \mathit{default\_spec} \rhd E^{\rm T}, {E^{\rm K}}_1}$ Check a default typing specification

$$\frac{E^{\rm K} \vdash \mathit{base\_kind} \leadsto k}{\langle E^{\rm K}, E^{\rm A}, E^{\rm R}, E^{\rm E}\rangle \vdash \textbf{default}\ \mathit{base\_kind}\ \text{'}x \rhd \{\,\}, \{\text{'}x \mapsto k\ \textbf{default}\}}\ \text{CHECK\_DEFAULT\_KIND}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{default}\ \mathit{typschm}\ \mathit{id} \rhd \{\mathit{id} \mapsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}, \mathbf{Default}, t\}, \{\,\}} \quad \text{CHECK\_DEFAULT\_TYP}$$

$\boxed{E \vdash \mathit{def} \rhd \mathit{def}', E'}$ Check a definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{type\_def} \rhd E}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{type\_def} \rhd \mathit{type\_def}, \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \uplus E} \quad \text{CHECK\_DEF\_TDEF}$$

$$\frac{E \vdash \mathit{fundef} \rhd \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}{E \vdash \mathit{fundef} \rhd \mathit{fundef}', E \uplus \langle E^{\mathrm{T}}, \epsilon\rangle} \quad \text{CHECK\_DEF\_FDEF}$$

$$\frac{\begin{array}{l} E \vdash \mathit{letbind} \rhd \mathit{letbind}', \{\mathit{id}_1 \mapsto t_1, \,..\,, \mathit{id}_n \mapsto t_n\}, \Sigma^{\mathrm{N}}, \mathbf{pure}, E^{\mathrm{K}} \\ \Sigma^{\mathrm{N}}{}_1 \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}) \end{array}}{E \vdash \mathit{letbind} \rhd \mathit{letbind}', E \uplus \langle\{\mathit{id}_1 \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_1, \,..\,, \mathit{id}_n \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_n\}, \epsilon\rangle} \quad \text{CHECK\_DEF\_VDEF}$$

$$\frac{E \vdash \mathit{val\_spec} \rhd E^{\mathrm{T}}}{E \vdash \mathit{val\_spec} \rhd \mathit{val\_spec}, E \uplus \langle E^{\mathrm{T}}, \epsilon\rangle} \quad \text{CHECK\_DEF\_VSPEC}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{default\_spec} \rhd E^{\mathrm{T}}{}_1, E^{\mathrm{K}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{default\_spec} \rhd \mathit{default\_spec}, \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}{}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_DEF\_DEFAULT}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{register}\ \mathit{typ}\ \mathit{id} \rhd \mathbf{register}\ \mathit{typ}\ \mathit{id}, \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{register}\,\langle t\rangle\}), E^{\mathrm{D}}\rangle} \quad \text{CHECK\_DEF\_REGISTER}$$

$\boxed{E \vdash \mathit{defs} \rhd \mathit{defs}', E'}$ Check definitions, potentially given default environment of built-in library

$$\frac{\begin{array}{l} E \vdash \mathit{def} \rhd \mathit{def}', E_1 \\ E \uplus E_1 \vdash \overline{\mathit{def}_i}^{\,i} \rhd \overline{\mathit{def}'_i}^{\,i}, E_2 \end{array}}{E \vdash \mathit{def}\ \overline{\mathit{def}_i}^{\,i} \rhd \mathit{def}'\ \overline{\mathit{def}'_i}^{\,i}, E_2} \quad \text{CHECK\_DEFS\_DEFS}$$

6 Sail operational semantics {TODO}