Sail Manual

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Contents

1 Sail syntax

l	::=			Source location
	|			
annot	::=			
id	::=			Identifier
	|	*x*		
	|	(**deinfix** *x*)		remove infix status
	|	**bool**	M	Built in type identifiers
	|	**bit**	M	
	|	**unit**	M	
	|	**nat**	M	
	|	**string**	M	
	|	**range**	M	
	|	**atom**	M	
	|	**vector**	M	
	|	**list**	M	
	|	**set**	M	
	|	**reg**	M	
	|	*to_num*	M	Built in function identifiers
	|	*to_vec*	M	
kid	::=			variables with kind, ticked to differntiate from program variables
	|	'*x*		
base_kind	::=			base kind
	|	**Type**		kind of types
	|	**Nat**		kind of natural number size expressions

	|	**Order**		kind of vector order specifications
	|	**Effect**		kind of effect sets
kind	::=			kinds
	|	$\mathit{base\_kind}_1 \rightarrow \ldots \rightarrow \mathit{base\_kind}_n$		
nexp	::=			expression of kind Nat, for vector sizes and origins
	|	*kid*		variable
	|	*num*		constant
	|	$\mathit{nexp}_1 * \mathit{nexp}_2$		product
	|	$\mathit{nexp}_1 + \mathit{nexp}_2$		sum
	|	$\mathit{nexp}_1 - \mathit{nexp}_2$		subtraction, error for nexp1 to be smaller than nexp2
	|	$2 * * \mathit{nexp}$		exponential
	|	**neg** *nexp*		For internal use
	|	(*nexp*)	S	
order	::=			vector order specifications, of kind Order
	|	*kid*		variable
	|	**inc**		increasing (little-endian)
	|	**dec**		decreasing (big-endian)
	|	(*order*)	S	
base_effect	::=			effect
	|	**rreg**		read register
	|	**wreg**		write register
	|	**rmem**		read memory
	|	**wmem**		write memory
	|	**wmea**		signal effective address for writing memory
	|	**wmv**		write memory, sending only value
	|	**barr**		memory barrier

	|	**depend**		dynamic footprint
	|	**undef**		undefined-instruction exception
	|	**unspec**		unspecified values
	|	**nondet**		nondeterminism from intra-instruction parallelism
	|	**escape**		Tracking of expressions and functions that might call exit
	|	**lset**		Local mutation happend; not user-writable
effect	::=			effect set, of kind Effects
	|	*kid*		
	|	$\{base\_effect_1, .. , base\_effect_n\}$		effect set
	|	**pure**	M	sugar for empty effect set
	|	$effect_1 \uplus .. \uplus effect_n$	M	meta operation for combining sets of effects
typ	::=			Type expressions, of kind **Type**
	|	_		Unspecified type
	|	*id*		Defined type
	|	*kid*		Type variable
	|	$typ_1 \rightarrow typ_2$ **effect** *effect*		Function type (first-order only in user code)
	|	$(typ_1, ... , typ_n)$		Tuple type
	|	$id\langle typ\_arg_1, .. , typ\_arg_n\rangle$		type constructor application
	|	(typ)	S	
	|	$[\vert nexp \vert]$	S	sugar for `range<0, nexp>`
	|	$[\vert nexp : nexp' \vert]$	S	sugar for `range< nexp, nexp’>`
	|	$[: nexp :]$	S	sugar for `atom<nexp>` which is special case of `range<nexp,nexp>`
	|	$typ[nexp]$	S	sugar for vector indexed by [$nexp$]
	|	$typ[nexp : nexp']$	S	sugar for vector indexed by [$nexp..nexp'$]
	|	$typ[nexp <: nexp']$	S	sugar for increasing vector indexed as above
	|	$typ[nexp :> nexp']$	S	sugar for decreasing vector indexed as above
typ_arg	::=			Type constructor arguments of all kinds

	|	*nexp*	
	|	*typ*	
	|	*order*	
	|	*effect*	
n_constraint	::=		constraint over kind **Nat**
	|	$nexp = nexp'$	
	|	$nexp \geq nexp'$	
	|	$nexp \leq nexp'$	
	|	*kid* **IN** $\{num_1, \ldots, num_n\}$	
kinded_id	::=		optionally kind-annotated identifier
	|	*kid*	identifier
	|	*kind kid*	kind-annotated variable
quant_item	::=		Either a kinded identifier or a nexp constraint for a typquant
	|	*kinded_id*	An optionally kinded identifier
	|	*n_constraint*	A constraint for this type
typquant	::=		type quantifiers and constraints
	|	**forall** $quant\_item_1, \ldots, quant\_item_n$.	
	|		sugar, omitting quantifier and constraints
typschm	::=		type scheme
	|	*typquant typ*	
name_scm_opt	::=		Optional variable-naming-scheme specification for variables of defined type
	|		
	|	[**name** = *regexp*]	

type_def	::=		Type definition body
	|	$\textbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \mathit{typschm}$	type abbreviation
	|	$\textbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \textbf{const}\,\textbf{struct}\ \mathit{typquant}\{\mathit{typ}_1\ \mathit{id}_1; \ldots; \mathit{typ}_n\ \mathit{id}_n\ ;^?\}$	struct type definition
	|	$\textbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \textbf{const}\,\textbf{union}\ \mathit{typquant}\{\mathit{type\_union}_1; \ldots; \mathit{type\_union}_n\ ;^?\}$	union type definition
	|	$\textbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \textbf{enumerate}\,\{\mathit{id}_1; \ldots; \mathit{id}_n\ ;^?\}$	enumeration type definition
	|	$\textbf{typedef}\ \mathit{id} = \textbf{register}\,\textbf{bits}\,[\mathit{nexp} : \mathit{nexp}']\{\mathit{index\_range}_1 : \mathit{id}_1; \ldots; \mathit{index\_range}_n : \mathit{id}_n\}$	register mutable bitfield type definition
type_union	::=		Type union constructors
	|	*id*	
	|	*typ id*	
index_range	::=		index specification, for bitfields in register types
	|	*num*	single index
	|	$\mathit{num}_1..\mathit{num}_2$	index range
	|	$\mathit{index\_range}_1, \mathit{index\_range}_2$	concatenation of index ranges
lit	::=		Literal constant
	|	()	() : **unit**
	|	**bitzero**	**bitzero** : **bit**
	|	**bitone**	**bitone** : **bit**
	|	**true**	**true** : **bool**
	|	**false**	**false** : **bool**
	|	*num*	natural number constant
	|	*hex*	bit vector constant, C-style
	|	*bin*	bit vector constant, C-style

	|	**undefined**	constant representing undefined values
	|	*string*	string constant
$;^?$	::=		Optional semi-colon
	|		
	|	;	
pat	::=		Pattern
	|	*lit*	literal constant pattern
	|	_	wildcard
	|	$(pat$ **as** $id)$	named pattern
	|	$(typ)pat$	typed pattern
	|	*id*	identifier
	|	$id(pat_1, .., pat_n)$	union constructor pattern
	|	$\{fpat_1; ... ; fpat_n\ ;^?\}$	struct pattern
	|	$[pat_1, .., pat_n]$	vector pattern
	|	$[num_1 = pat_1, .., num_n = pat_n]$	vector pattern (with explicit indices)
	|	$pat_1 : .... : pat_n$	concatenated vector pattern
	|	$(pat_1, ..., pat_n)$	tuple pattern
	|	$[\|\|pat_1, .., pat_n\|\|]$	list pattern
	|	(pat)	S
fpat	::=		Field pattern
	|	$id = pat$	
exp	::=		Expression
	|	$\{exp_1; ... ; exp_n\}$	block
	|	**nondet** $\{exp_1; ... ; exp_n\}$	nondeterminisitic block, expressions evaluate in an unspecified order, or concurrently
	|	*id*	identifier
	|	*lit*	literal constant

$\vert$	$(\mathit{typ})\,\mathit{exp}$		cast
$\vert$	$\mathit{id}(\mathit{exp}_1,\, ..\,,\, \mathit{exp}_n)$		function application
$\vert$	$\mathit{id}\ \mathit{exp}$	S	No extra parens needed when exp is a tuple
$\vert$	$\mathit{exp}_1\ \mathit{id}\ \mathit{exp}_2$		infix function application
$\vert$	$(\mathit{exp}_1,\, ...\,,\, \mathit{exp}_n)$		tuple
$\vert$	**if** exp_1 **then** exp_2 **else** exp_3		conditional
$\vert$	**if** exp_1 **then** exp_2	S	
$\vert$	**foreach** $(\mathit{id}$ **from** exp_1 **to** exp_2 **by** exp_3 **in** $\mathit{order})\,\mathit{exp}_4$		loop
$\vert$	**foreach** $(\mathit{id}$ **from** exp_1 **to** exp_2 **by** $\mathit{exp}_3)\,\mathit{exp}_4$	S	
$\vert$	**foreach** $(\mathit{id}$ **from** exp_1 **to** $\mathit{exp}_2)\,\mathit{exp}_3$	S	
$\vert$	**foreach** $(\mathit{id}$ **from** exp_1 **downto** exp_2 **by** $\mathit{exp}_3)\,\mathit{exp}_4$	S	
$\vert$	**foreach** $(\mathit{id}$ **from** exp_1 **downto** $\mathit{exp}_2)\,\mathit{exp}_3$	S	
$\vert$	$[\mathit{exp}_1,\, ...\,,\, \mathit{exp}_n]$		vector (indexed from 0)
$\vert$	$[\mathit{num}_1 = \mathit{exp}_1,\, ...\,,\, \mathit{num}_n = \mathit{exp}_n\ \mathit{opt\_default}]$		vector (indexed consecutively)
$\vert$	$\mathit{exp}[\mathit{exp}']$		vector access
$\vert$	$\mathit{exp}[\mathit{exp}_1..\mathit{exp}_2]$		subvector extraction
$\vert$	$[\mathit{exp}$ **with** $\mathit{exp}_1 = \mathit{exp}_2]$		vector functional update
$\vert$	$[\mathit{exp}$ **with** $\mathit{exp}_1 : \mathit{exp}_2 = \mathit{exp}_3]$		vector subrange update (with vector)
$\vert$	$\mathit{exp} : \mathit{exp}_2$		vector concatenation
$\vert$	$[\vert\vert\mathit{exp}_1,\, ..\,,\, \mathit{exp}_n\vert\vert]$		list
$\vert$	$\mathit{exp}_1 :: \mathit{exp}_2$		cons
$\vert$	$\{\mathit{fexps}\}$		struct
$\vert$	$\{\mathit{exp}$ **with** $\mathit{fexps}\}$		functional update of struct
$\vert$	$\mathit{exp}.\mathit{id}$		field projection from struct
$\vert$	**switch** $\mathit{exp}\{$ **case** pexp_1 ... **case** $\mathit{pexp}_n\}$		pattern matching
$\vert$	$\mathit{letbind}$ **in** exp		let expression
$\vert$	$\mathit{lexp} := \mathit{exp}$		imperative assignment
$\vert$	**exit** exp		expression to halt all current execution, potentially calling a system, trap, or interrupt handler with e
$\vert$	**assert** $(\mathit{exp}, \mathit{exp}')$		expression to halt with error, when the first expression is true, reporting the optional string as an err

	$\mid$	(exp)	S	
	$\mid$	$(\mathit{annot})\mathit{exp}$		This is an internal cast, generated during type checking that will resolve into a syntactic cast after
	$\mid$	annot		This is an internal use for passing nexp information to library functions, postponed for constraint solving
	$\mid$	$\mathit{annot}, \mathit{annot}'$		This is like the above but the user has specified an implicit parameter for the current function
	$\mid$	**comment** *string*		For generated unstructured comments
	$\mid$	**comment** *exp*		For generated structured comments
	$\mid$	**let** $\mathit{lexp} = \mathit{exp}$ **in** exp'		This is an internal node for compilation that demonstrates the scope of a local mutable variable
	$\mid$	**let** $\mathit{pat} = \mathit{exp}$ **in** exp'		This is an internal node, used to distinguised some introduced lets during processing from original ones
	$\mid$	**return** (exp)		For internal use to embed into monad definition
lexp	::=			lvalue expression
	$\mid$	id		identifier
	$\mid$	$\mathit{id}(\mathit{exp}_1, \,..\,, \mathit{exp}_n)$		memory write via function call
	$\mid$	$\mathit{id}\ \mathit{exp}$	S	
	$\mid$	$(\mathit{typ})\mathit{id}$		
	$\mid$	$\mathit{lexp}[\mathit{exp}]$		vector element
	$\mid$	$\mathit{lexp}[\mathit{exp}_1..\mathit{exp}_2]$		subvector
	$\mid$	$\mathit{lexp}.\mathit{id}$		struct field
fexp	::=			Field-expression
	$\mid$	$\mathit{id} = \mathit{exp}$		
fexps	::=			Field-expression list
	$\mid$	$\mathit{fexp}_1; \,...\,; \mathit{fexp}_n\ ;^?$		
opt_default	::=			Optional default value for indexed vectors, to define a defualt value for any unspecified positions in a sparse map
	$\mid$			
	$\mid$	; **default** $= \mathit{exp}$		
pexp	::=			Pattern match

	$\mid$	$pat \rightarrow exp$	
$tannot\_opt$	$::=$		Optional type annotation for functions
	$\mid$	$typquant\ typ$	
rec_opt	$::=$		Optional recursive annotation for functions
	$\mid$		non-recursive
	$\mid$	**rec**	recursive
$effect\_opt$	$::=$		Optional effect annotation for functions
	$\mid$		sugar for empty effect set
	$\mid$	**effect** $effect$	
$funcl$	$::=$		Function clause
	$\mid$	$id\ pat = exp$	
$fundef$	$::=$		Function definition
	$\mid$	**function** $rec\_opt\ tannot\_opt\ effect\_opt\ funcl_1$ **and** ... **and** $funcl_n$	
$letbind$	$::=$		Let binding
	$\mid$	**let** $typschm\ pat = exp$	value binding, explicit type (pat must be total)
	$\mid$	**let** $pat = exp$	value binding, implicit type (pat must be total)
val_spec	$::=$		Value type specification
	$\mid$	**val** $typschm\ id$	
	$\mid$	**val extern** $typschm\ id$	
	$\mid$	**val extern** $typschm\ id = string$	Specify the type and id of a function from Lem, where the string must provide an explicit path to the requi
$default\_spec$	$::=$		Default kinding or typing assumption

	|	**default** *base_kind kid*	
	|	**default Order** *order*	
	|	**default** *typschm id*	
scattered_def	::=		Function and type union definitions that can be spread across a file. Each one must end in **id**
	|	**scattered function** *rec_opt tannot_opt effect_opt id*	scattered function definition header
	|	**function clause** *funcl*	scattered function definition clause
	|	**scattered typedef** *id name_scm_opt* = **const union** *typquant*	scattered union definition header
	|	**union** *id* **member** *type_union*	scattered union definition member
	|	**end** *id*	scattered definition end
reg_id	::=		
	|	*id*	
alias_spec	::=		Register alias expression forms. Other than where noted, each id must refer to an unaliased register of type
	|	*reg_id.id*	
	|	*reg_id*[*exp*]	
	|	*reg_id*[*exp..exp*′]	
	|	*reg_id* : *reg_id*′	
dec_spec	::=		Register declarations
	|	**register** *typ id*	
	|	**register alias** *id* = *alias_spec*	
	|	**register alias** *typ id* = *alias_spec*	
def	::=		Top-level definition
	|	*type_def*	type definition
	|	*fundef*	function definition

	|	*letbind*	value definition
	|	*val_spec*	top-level type constraint
	|	*default_spec*	default kind and type assumptions
	|	*scattered_def*	scattered function and type definition
	|	*dec_spec*	register declaration
	|	*dec_comm*	generated comments
defs	::=		Definition sequence
	|	$\mathit{def}_1 \,..\, \mathit{def}_n$	

2 Sail primitive types and functions

built_in_types ::= Type Kind

| $\mathbf{bit}\ :\ \mathbf{Typ}$

| $\mathbf{unit}\ :\ \mathbf{Typ}$

| $\mathbf{forall}\,\mathbf{Nat}\,'n.\,\mathbf{Nat}\,'m.\,\mathbf{range}\ <'n,'m>:\ \mathbf{Nat}\ \to\ \mathbf{Nat}\ \to\ \mathbf{Typ}$

| $\mathbf{forall}\,\mathbf{Nat}\,'n.\,\mathbf{atom}\ <'n>:\ \mathbf{Nat}\ \to\ \mathbf{Typ}$ singleton number, instead of range¡'n,'n¿

| $\mathbf{forall}\,\mathbf{Nat}\,'n,\ \mathbf{Nat}\,'m,\ \mathbf{Order}\,'o,\ \mathbf{Typ}\,'t.\,\mathbf{vector}\ \langle 'n,'m,'o,'t\rangle\ :\ \mathbf{Nat}\ \to\ \mathbf{Nat}\ \to\ \mathbf{Order}\ \to\ \mathbf{Typ}$

| $\mathbf{forall}\,\mathbf{Typ}\,'t.\,\mathbf{register}\ \langle 't\rangle\ :\ \mathbf{Typ}\ \to\ \mathbf{Typ}$

| $\mathbf{forall}\,\mathbf{Typ}\,'t.\,\mathbf{reg}\ \langle 't\rangle\ :\ \mathbf{Typ}\ \to\ \mathbf{Typ}$ internal reference cell

| $\mathbf{forall}\,\mathbf{Nat}\,'n.\,\mathbf{implicit}\ <'n>:\ \mathbf{Nat}\ \to\ \mathbf{Typ}$ see Kathy for explanation

built_in_type_abbreviations ::=

| $\mathbf{bool}\ \Rightarrow\ \mathbf{bit}$

| $\mathbf{nat}\ \Rightarrow\ [|0..pos\_infinity|]$

| $\mathbf{int}\ \Rightarrow\ [|neg\_infinity..pos\_infinity|]$

| $uint8 \Rightarrow [|0..2**8|]$

| $uint16 \Rightarrow [|0..2**16|]$

| $uint32 \Rightarrow [|0..2**32|]$

| $uint64 \Rightarrow [|0..2**32|]$

functions ::= Built-in functions: all have effect pure, all order polymorphic

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a.'a \to\ \mathbf{unit}\ :\ \mathbf{ignore}$

| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to [|'n+'o..'m+'p|] : +$ arithmetic addition

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\ \mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to\ \mathbf{bit}\,['n] : +$ unsigned vector addition

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\ \mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to (\ \mathbf{bit}\,['n],\ \mathbf{bit}\ ,\ \mathbf{bit}\ ) : +$ unsigned vector addition with overflow, carry out

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\ \mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to\ \mathbf{bit}\,['n] : +\_s$ signed vector addition

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(bit['n],\ \mathbf{bit}\,['n]) \to (\ \mathbf{bit}\,['n],\ \mathbf{bit}\ ,\ \mathbf{bit}\ ) : +\_s$ signed vector addition with overflow, carry out

| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \to [|'n-'o..'m-'p|] : -$ arithmetic subtraction

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\ \mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to\ \mathbf{bit}\,['n] : -$ unsigned vector subtraction

| $\mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\ \mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to (\ \mathbf{bit}\,['n],\ \mathbf{bit}\ ,\ \mathbf{bit}\ ) : -$ unsigned vector subtraction with overflow, carry out

$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] : -\_s$	signed vector subtraction
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(bit['n],\ \mathbf{bit}\,['n]) \to (\,\mathbf{bit}\,['n],\ \mathbf{bit}\,,\ \mathbf{bit}\,) : -\_s$	signed vector subtraction with overflow, carry out
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 'o..'p\vert]) \to [\vert 'n *' o..'m *' p\vert] : *$	arithmetic multiplication
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *$	unsigned vector multiplication
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\,[2 *' n] : *\_s$	signed vector multiplication
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 1..'p\vert]) \to [\vert 0..'p-1\vert] :\ \mathbf{mod}$	arithmetic modulo
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\,['n] :\ \mathbf{mod}$	unsigned vector modulo
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 1..'p\vert]) \to [\vert 'q..'r\vert] :\ \mathbf{quot}$	arithmetic integer division
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\ \mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] :\ \mathbf{quot}$	unsigned vector division
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n,\ \mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['m]) \to \mathbf{bit}\,['n] : quot\_s$	signed vector division
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\ \mathbf{Nat}\,'n.('a['n] \to [:' n :]) :\ \mathbf{length}$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\ \mathbf{Nat}\,'n,\ \mathbf{Nat}\,'m,' n \leq' m.(\,\mathbf{implicit}\,\langle 'm\rangle,' a['n]) \to' a['m] :\ \mathbf{mask}$	reduce size of vector, dropping MSBits. Type system supplies implicit p
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :\equiv$	vector equality
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\ \mathbf{Typ}\,'b.('a,' b) \to \mathbf{bit}\ :\equiv$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Typ}\,'a,\ \mathbf{Typ}\,'b.('a,' b) \to \mathbf{bit}\ :! =$	
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 'o..'p\vert]) \to \mathbf{bit}\ : \langle$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ : \langle$	unsigned less than
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :< \_s$	
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 'o..'p\vert]) \to \mathbf{bit}\ :\rangle$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :\rangle$	unsigned greater than
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :> \_s$	
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 'o..'p\vert]) \to \mathbf{bit}\ :\leq$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :\leq$	unsigned less than or eq
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :<= \_s$	
$\mid\ \mathbf{val}\,([\vert 'n..'m\vert],[\vert 'o..'p\vert]) \to \mathbf{bit}\ :\geq$	
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :\geq$	unsigned greater than or eq
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.(\,\mathbf{bit}\,['n],\ \mathbf{bit}\,['n]) \to \mathbf{bit}\ :>= \_s$	
$\mid\ \mathbf{val}\,\mathbf{bit}\ \to \mathbf{bit}\ :$	bit negation
$\mid\ \mathbf{val}\,\mathbf{forall}\,\mathbf{Nat}\,'n.\,\mathbf{bit}\,['n] \to \mathbf{bit}\,['n] :$	bitwise negation

	$\vert$	$\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,)\rightarrow\ \textbf{bit}\ :\vert$	bitwise or
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:\vert$	
	$\vert$	$\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,)\rightarrow\ \textbf{bit}\ :\&$	bitwise and
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:\&$	
	$\vert$	$\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,)\rightarrow\ \textbf{bit}\ :\uparrow$	bitwise xor
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:\uparrow$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,,[\vert'n\vert])\rightarrow\ \textbf{bit}\,['n]:\uparrow\uparrow$	duplicate bit into a vector
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m\leq'\,n.(\,\textbf{bit}\,['n],[\vert'm\vert])\rightarrow\ \textbf{bit}\,['n]:<<$	left shift
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m\leq'\,n.(\,\textbf{bit}\,['n],[\vert'm\vert])\rightarrow\ \textbf{bit}\,['n]:>>$	right shift
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m\leq'\,n.(\,\textbf{bit}\,['n],[\vert'm\vert])\rightarrow\ \textbf{bit}\,['n]:<<<$	rotate
functions_with_coercions	$::=$		
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n])\rightarrow[\vert 2**'\,n\vert]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow\ \textbf{bit}\,['n]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\vert'o..'p\vert],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow[\vert'o..'p+2**'n\vert]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,)\rightarrow\ \textbf{bit}\,['n]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:+$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n])\rightarrow[\vert 2**'\,n\vert]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow\ \textbf{bit}\,['n]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\vert'o..'p\vert],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow[\vert'o..'p+2**'n\vert]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,)\rightarrow\ \textbf{bit}\,['n]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:+\_s$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow\ \textbf{bit}\,['n]:-$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\vert'o..'p\vert],\,\textbf{bit}\,['n])\rightarrow\ \textbf{bit}\,['n]:-$	
	$\vert$	$\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\vert'o..'p\vert])\rightarrow[\vert'o..'p+2**'n\vert]:-$	

3 Sail type system

3.1 Internal type syntax

$$
\begin{array}{llll}
k & ::= & & \text{Internal kinds} \\
 & | & K\_Typ & \\
 & | & K\_Nat & \\
 & | & K\_Ord & \\
 & | & K\_Efct & \\
 & | & K\_Lam(k_0 \mathrel{..} k_n \to k') & \\
 & | & K\_infer & \quad \text{Representing an unknown kind, inferred by context}
\end{array}
$$

$$
\begin{array}{lllll}
t,\ u & ::= & & & \text{Internal types} \\
 & | & x & & \\
 & | & \text{'}x & & \\
 & | & t_1 \to t_2\ \mathit{effect} & & \\
 & | & (t_1,\ ....\ ,t_n) & & \\
 & | & x\langle t\_args \rangle & & \\
 & | & t \mapsto t_1 & & \\
 & | & \textbf{register}\ \langle t\_arg \rangle & \mathsf{S} & \\
 & | & \textbf{range}\ \langle ne\ ne' \rangle & \mathsf{S} & \\
 & | & \textbf{atom}\ \langle ne \rangle & \mathsf{S} & \\
 & | & \textbf{vector}\ \langle ne\ ne'\ order\ t \rangle & \mathsf{S} & \\
 & | & \textbf{list}\ \langle t \rangle & \mathsf{S} & \\
 & | & \textbf{reg}\ \langle t \rangle & \mathsf{S} & \\
 & | & \textbf{implicit}\ \langle ne \rangle & \mathsf{S} & \\
 & | & \textbf{bit} & \mathsf{S} & \\
 & | & \mathit{string} & \mathsf{S} & \\
 & | & \textbf{unit} & \mathsf{S} & \\
 & | & t[t\_arg_1/tid_1 \mathrel{..} t\_arg_n/tid_n] & \mathsf{M} &
\end{array}
$$

optx	::=			
	$\mid$	x		
	$\mid$			
tag	::=			Data indicating where the identifier arises and thus information necessary in compilation
	$\mid$	**None**		
	$\mid$	**Intro**		Denotes an assignment and lexp that introduces a binding
	$\mid$	**Set**		Denotes an expression that mutates a local variable
	$\mid$	**Global**		Globally let-bound or enumeration based value/variable
	$\mid$	**Ctor**		Data constructor from a type union
	$\mid$	**Extern** *optx*		External function, specied only with a val statement
	$\mid$	**Default**		Type has come from default declaration, identifier may not be bound locally
	$\mid$	**Spec**		
	$\mid$	**Enum** *num*		
	$\mid$	**Alias**		
	$\mid$	$Unknown\_path optx$		Tag to distinguish an unknown path from a non-analysis non deterministic path
ne	::=			internal numeric expressions
	$\mid$	$'x$		
	$\mid$	*num*		
	$\mid$	**infinity**		
	$\mid$	$ne_1 * ne_2$		
	$\mid$	$ne_1 + \ldots + ne_n$		
	$\mid$	$ne_1 - ne_2$		
	$\mid$	$2 ** ne$		
	$\mid$	$(-ne)$		
	$\mid$	**zero**	S	
	$\mid$	**one**	S	
	$\mid$	**bitlength** (bin)	M	
	$\mid$	**bitlength** (hex)	M	

	|	**count** $(num_0 \ldots num_i)$	M	
	|	**length** $(pat_1 \ldots pat_n)$	M	
	|	**length** $(exp_1 \ldots exp_n)$	M	
t_arg	::=			Argument to type constructors
	|	t		
	|	ne		
	|	$effect$		
	|	$order$		
	|	**fresh**	M	
t_args	::=			Arguments to type constructors
	|	$t\_arg_1 \ldots t\_arg_n$		
nec	::=			Numeric expression constraints
	|	$ne \leq ne'$		
	|	$ne = ne'$		
	|	$ne \geq ne'$		
	|	$'x\,\mathbf{IN}\,\{num_1, \ldots, num_n\}$		
	|	$nec_0 \,..\, nec_n \rightarrow nec'_0 \ldots nec'_m$		
	|	$nec_0 \ldots nec_n$		
Σ^{N}	::=			nexp constraint lists
	|	$\{nec_1, \,..\,, nec_n\}$		
	|	${\Sigma^{\mathrm{N}}}_1 \uplus \,..\, \uplus {\Sigma^{\mathrm{N}}}_n$	M	
	|	**consistent_increase** $ne_1\, ne'_1 \ldots ne_n\, ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always larger than the su
	|	**consistent_decrease** $ne_1\, ne'_1 \ldots ne_n\, ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always smaller than the c
	|	**resolve** (Σ^{N})		
E^{D}	::=			Environments storing top level information, such as defined abbreviations, records, enumerations, and kin

| $\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle$
| ϵ
| $E^{\mathrm{D}} \uplus E^{\mathrm{D}\prime}$

$kinf$ ::= Whether a kind is default or from a local binding
| k
| k **default**

tid ::= A type identifier or type variable
| id
| kid

E^{K} ::= Kind environments
| $\{tid_1 \mapsto kinf_1, \, .. \, , tid_n \mapsto kinf_n\}$
| $E^{\mathrm{K}}{}_1 \uplus \, .. \, \uplus E^{\mathrm{K}}{}_n$ M In a unioning kinf, k default u k results in k (i.e. the default is locally forgotten)
| $E^{\mathrm{K}} \setminus E^{\mathrm{K}}{}_1 \, .. \, E^{\mathrm{K}}{}_n$ M

$tinf$ ::= Type variables, type, and constraints, bound to an identifier
| t
| $E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t$

E^{A} ::=
| $\{tid_1 \mapsto tinf_1, \, .. \, , tid_n \mapsto tinf_n\}$
| $E^{\mathrm{A}}_1 \uplus \, .. \, \uplus E^{\mathrm{A}}_n$

$field\_typs$ ::= Record fields
| $id_1 : t_1, \, .. \, , id_n : t_n$

E^{R} ::= Record environments
| $\{\{field\_typs_1\} \mapsto tinf_1, \, .. \, , \{field\_typs_n\} \mapsto tinf_n\}$

	$\mid$	$E^{\mathrm{R}}_1 \uplus \, .. \, \uplus E^{\mathrm{R}}_n$	M	
$\mathit{enumerate\_map}$	$::=$			
	$\mid$	$\{\mathit{num}_1 \mapsto \mathit{id}_1 \ldots \mathit{num}_n \mapsto \mathit{id}_n\}$		
E^{E}	$::=$			Enumeration environments
	$\mid$	$\{t_1 \mapsto \mathit{enumerate\_map}_1, \, .. \, , t_n \mapsto \mathit{enumerate\_map}_n\}$		
	$\mid$	$E^{\mathrm{E}}_1 \uplus \, .. \, \uplus E^{\mathrm{E}}_n$		
E^{T}	$::=$			Type environments
	$\mid$	$\{\mathit{id}_1 \mapsto \mathit{tinf}_1, \, .. \, , \mathit{id}_n \mapsto \mathit{tinf}_n\}$		
	$\mid$	$\{\mathit{id} \mapsto \textbf{overload}\, \mathit{tinf}\ \mathit{conformsto} : \mathit{tinf}_1, \ldots, \mathit{tinf}_n\}$		
	$\mid$	$(E^{\mathrm{T}}{}_1 \uplus \ldots. \uplus E^{\mathrm{T}}{}_n)$	M	
	$\mid$	$\uplus E^{\mathrm{T}}{}_1 \, .. \, E^{\mathrm{T}}{}_n$	M	
	$\mid$	$E^{\mathrm{T}} \setminus \mathit{id}_1 \, .. \, \mathit{id}_n$	M	
	$\mid$	$(E^{\mathrm{T}}{}_1 \cap \ldots. \cap E^{\mathrm{T}}{}_n)$	M	
	$\mid$	$\cap E^{\mathrm{T}}{}_1 \, .. \, E^{\mathrm{T}}{}_n$	M	
ts	$::=$			
	$\mid$	$t_1, \, .. \, , t_n$		
E	$::=$			Definition environment and lexical environment
	$\mid$	$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle$		
	$\mid$	ϵ	M	
	$\mid$	$E \uplus E'$		
I	$::=$			Information given by type checking an expression
	$\mid$	$\langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle$		
	$\mid$	I_ϵ		Empty constraints, effect
	$\mid$	$I_1 \uplus I_2$		

	|	$I_1 \uplus .. \uplus I_n$	Unions the constraints and effect
formula	::=		
	|	*judgement*	
	|	$formula_1 \quad .. \quad formula_n$	
	|	$E^{\mathrm{K}}(tid) \triangleright kinf$	Kind lookup
	|	$E^{\mathrm{A}}(tid) \triangleright tinf$	
	|	$E^{\mathrm{T}}(id) \triangleright tinf$	Type lookup
	|	$E^{\mathrm{T}}(id) \triangleright \textbf{overload}\ tinf : tinf_1 ... tinf_n$	Overloaded type lookup
	|	$E^{\mathrm{K}}(tid) < -\vert k$	Update the kind associated with id to k
	|	$E^{\mathrm{R}}(id_0 .. id_n) \triangleright t, ts$	Record lookup
	|	$E^{\mathrm{R}}(t) \triangleright id_0 : t_0 .. id_n : t_n$	Record looup by type
	|	$E^{\mathrm{E}}(t) \triangleright enumerate\_map$	Enumeration lookup by type
	|	$\mathbf{dom}\,({E^{\mathrm{T}}}_1) \cap \mathbf{dom}\,({E^{\mathrm{T}}}_2) = \emptyset$	
	|	$\mathbf{dom}\,({E^{\mathrm{K}}}_1) \cap \mathbf{dom}\,({E^{\mathrm{K}}}_2) = \emptyset$	
	|	$\mathbf{disjoint\,doms}\,({E^{\mathrm{T}}}_1, ..., {E^{\mathrm{T}}}_n)$	Pairwise disjoint domains
	|	$id \notin \mathbf{dom}\,(E^{\mathrm{K}})$	
	|	$id \notin \mathbf{dom}\,(E^{\mathrm{T}})$	
	|	$id_0 : t_0 .. id_n : t_n \subset id'_0 : t'_0 .. id'_i : t'_i$	
	|	$num_1 < ... < num_n$	
	|	$num_1 > ... > num_n$	
	|	$exp_1 \equiv exp_2$	
	|	${E^{\mathrm{K}}}_1 \equiv {E^{\mathrm{K}}}_2$	
	|	${E^{\mathrm{K}}}_1 \approx {E^{\mathrm{K}}}_2$	
	|	${E^{\mathrm{T}}}_1 \equiv {E^{\mathrm{T}}}_2$	
	|	$E^{\mathrm{R}}_1 \equiv E^{\mathrm{R}}_2$	
	|	$E^{\mathrm{E}}_1 \equiv E^{\mathrm{E}}_2$	
	|	${E^{\mathrm{D}}}_1 \equiv {E^{\mathrm{D}}}_2$	
	|	$E_1 \equiv E_2$	
	|	${\Sigma^{\mathrm{N}}}_1 \equiv {\Sigma^{\mathrm{N}}}_2$	

$|\quad id \equiv' id$
$|\quad x_1 \neq x_2$
$|\quad lit_1 \neq lit_2$
$|\quad I_1 \equiv I_2$
$|\quad \mathit{effect}_1 \equiv \mathit{effect}_2$
$|\quad t_1 \equiv t_2$
$|\quad ne \equiv ne'$
$|\quad kid \equiv fresh\_kid(E^{\mathrm{D}})$

3.2 Type relations

$\boxed{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}$ Well-formed types

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Typ}{E^{\mathrm{K}} \vdash_t \text{'}x\ \mathbf{ok}} \quad \textsc{check\_t\_var}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Typ \end{array}}{E^{\mathrm{K}} \vdash_t \text{'}x\ \mathbf{ok}} \quad \textsc{check\_t\_varInfer}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_t t_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_t t_2\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_t t_1 \to t_2\ \mathit{effect}\ \mathbf{ok}} \quad \textsc{check\_t\_fn}$$

$$\frac{E^{\mathrm{K}} \vdash_t t_1\ \mathbf{ok} \quad .... \quad E^{\mathrm{K}} \vdash_t t_n\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_t (t_1,\ ....,\ t_n)\ \mathbf{ok}} \quad \textsc{check\_t\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \\ E^{\mathrm{K}}, k_1 \vdash t\_arg_1\ \mathbf{ok} \quad .. \quad E^{\mathrm{K}}, k_n \vdash t\_arg_n\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_t x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle\ \mathbf{ok}} \quad \textsc{check\_t\_app}$$

$\boxed{E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok}}$ Well-formed effects

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Efct}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ef\_var}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Efct \end{array}}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ef\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_e \{base\_effect_1, .., base\_effect_n\}\ \mathbf{ok}} \quad \textsc{check\_ef\_set}$$

$\boxed{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \textsc{check\_n\_var}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Nat \end{array}}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \textsc{check\_n\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_n num\ \mathbf{ok}} \quad \textsc{check\_n\_num}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_n ne_1 + ne_2\ \mathbf{ok}} \quad \textsc{check\_n\_sum}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_n ne_1 * ne_2\ \mathbf{ok}} \quad \textsc{check\_n\_mult}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n 2 ** ne\ \mathbf{ok}} \quad \textsc{check\_n\_exp}$$

$\boxed{E^{\mathrm{K}} \vdash_o order\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Ord}{E^{\mathrm{K}} \vdash_o \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ord\_var}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Ord \end{array}}{E^{\mathrm{K}} \vdash_o \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ord\_varInfer}$$

$\boxed{E^{\mathrm{K}}, k \vdash t\_arg\ \mathbf{ok}}$ Well-formed type arguments kind check matching the application type variable

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Typ \vdash t\ \mathbf{ok}} \quad \textsc{check\_targs\_typ}$$

$$\frac{E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Efct \vdash \mathit{effect}\ \mathbf{ok}} \quad \textsc{check\_targs\_eff}$$

$$\frac{E^{\mathrm{K}} \vdash_n \mathit{ne}\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Nat \vdash \mathit{ne}\ \mathbf{ok}} \quad \textsc{check\_targs\_nat}$$

$$\frac{E^{\mathrm{K}} \vdash_o \mathit{order}\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Ord \vdash \mathit{order}\ \mathbf{ok}} \quad \textsc{check\_targs\_ord}$$

$\boxed{E^{\mathrm{K}} \vdash \mathit{kind} \leadsto k}$

$$\frac{}{E^{\mathrm{K}} \vdash \mathbf{Type} \leadsto K\_Typ} \quad \textsc{convert\_kind\_Typ}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{quant\_item} \leadsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}$ Convert source quantifiers to kind environments and constraints

$$\frac{E^{\mathrm{K}} \vdash \mathit{kind} \leadsto k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash \mathit{kind}\ \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \textsc{convert\_quants\_kind}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \textsc{convert\_quants\_nokind}$$

$$\frac{\begin{array}{c} \vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2 \end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 = \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 = \mathit{ne}_2\}} \quad \textsc{convert\_quants\_eq}$$

$$\frac{\begin{array}{c} \vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2 \end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 \geq \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 \geq \mathit{ne}_2\}} \quad \textsc{convert\_quants\_gteq}$$

$$\frac{\begin{array}{c} \vdash \mathit{nexp}_1 \leadsto \mathit{ne}_1 \\ \vdash \mathit{nexp}_2 \leadsto \mathit{ne}_2 \end{array}}{E^{\mathrm{D}} \vdash \mathit{nexp}_1 \leq \mathit{nexp}_2 \leadsto \{\,\}, \{\mathit{ne}_1 \leq \mathit{ne}_2\}} \quad \textsc{convert\_quants\_lteq}$$

$$\frac{}{E^{\mathrm{D}} \vdash \text{'}x\,\mathbf{IN}\,\{\mathit{num}_1,\,\dots,\,\mathit{num}_n\} \leadsto \{\,\},\{\text{'}x\,\mathbf{IN}\,\{\mathit{num}_1,\,\dots,\,\mathit{num}_n\}\}}\quad \textsc{convert\_quants\_in}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}$ Convert source types with typeschemes to internal types and kind environments

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{E^{\mathrm{D}} \vdash\ \mathit{typ} \leadsto t, \{\,\}, \{\,\}}\quad \textsc{convert\_typschm\_noquant}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathit{quant\_item}_1 \leadsto {E^{\mathrm{K}}}_1, {\Sigma^{\mathrm{N}}}_1 \quad \dots \quad E^{\mathrm{D}} \vdash \mathit{quant\_item}_n \leadsto {E^{\mathrm{K}}}_n, {\Sigma^{\mathrm{N}}}_n \\ E^{\mathrm{K}} \equiv {E^{\mathrm{K}}}_1 \uplus \dots \uplus {E^{\mathrm{K}}}_n \\ E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}, \{\,\}, \{\,\}, \{\,\}\rangle \vdash \mathit{typ} \leadsto t \end{array}}{E^{\mathrm{D}} \vdash \mathbf{forall}\,\mathit{quant\_item}_1,\,\dots,\,\mathit{quant\_item}_n.\,\mathit{typ} \leadsto t, E^{\mathrm{K}}, {\Sigma^{\mathrm{N}}}_1 \uplus \dots \uplus {\Sigma^{\mathrm{N}}}_n}\quad \textsc{convert\_typschm\_quant}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}$ Convert source types to internal types

$$\frac{E^{\mathrm{K}}(\text{'}x) \rhd K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \text{'}x \leadsto \text{'}x}\quad \textsc{convert\_typ\_var}$$

$$\frac{E^{\mathrm{K}}(x) \rhd K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \leadsto x}\quad \textsc{convert\_typ\_id}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash \mathit{typ}_1 \leadsto t_1 \\ E^{\mathrm{D}} \vdash \mathit{typ}_2 \leadsto t_2 \end{array}}{E^{\mathrm{D}} \vdash \mathit{typ}_1 \to \mathit{typ}_2\,\mathbf{effect}\,\mathit{effect} \leadsto t_1 \to t_2\ \mathit{effect}}\quad \textsc{convert\_typ\_fn}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ}_1 \leadsto t_1 \quad \dots. \quad E^{\mathrm{D}} \vdash \mathit{typ}_n \leadsto t_n}{E^{\mathrm{D}} \vdash (\mathit{typ}_1,\,\dots.\,,\mathit{typ}_n) \leadsto (t_1,\,\dots.\,,t_n)}\quad \textsc{convert\_typ\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \rhd K\_Lam(k_1\,..\,k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash \mathit{typ\_arg}_1 \leadsto \mathit{t\_arg}_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash \mathit{typ\_arg}_n \leadsto \mathit{t\_arg}_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle \mathit{typ\_arg}_1,\,..\,,\mathit{typ\_arg}_n\rangle \leadsto x\langle \mathit{t\_arg}_1\,..\,\mathit{t\_arg}_n\rangle}\quad \textsc{convert\_typ\_app}$$

$\boxed{E^{\mathrm{D}}, k \vdash \mathit{typ\_arg} \leadsto \mathit{t\_arg}}$ Convert source type arguments to internals

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{E^{\mathrm{D}}, K\_Typ \vdash \mathit{typ} \leadsto t}\quad \textsc{convert\_targ\_typ}$$

$\boxed{\vdash \mathit{nexp} \leadsto \mathit{ne}}$ Convert and normalize numeric expressions

$$\frac{}{\vdash \text{'}x \leadsto \text{'}x} \quad \text{CONVERT\_NEXP\_VAR}$$

$$\frac{}{\vdash num \leadsto num} \quad \text{CONVERT\_NEXP\_NUM}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2\end{array}}{\vdash nexp_1 * nexp_2 \leadsto ne_1 * ne_2} \quad \text{CONVERT\_NEXP\_MULT}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2\end{array}}{\vdash nexp_1 + nexp_2 \leadsto ne_1 + ne_2} \quad \text{CONVERT\_NEXP\_ADD}$$

$$\frac{\vdash nexp \leadsto ne}{\vdash 2 * *nexp \leadsto 2 ** ne} \quad \text{CONVERT\_NEXP\_EXP}$$

$\boxed{E^{\mathrm{D}} \vdash t \approx t'}$

$$\frac{E^{\mathrm{K}} \vdash_t t\, \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx t} \quad \text{CONFORMS\_TO\_REFL}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}} \vdash t_1 \approx t_2 \\ E^{\mathrm{D}} \vdash t_2 \approx t_3\end{array}}{E^{\mathrm{D}} \vdash t_1 \approx t_3} \quad \text{CONFORMS\_TO\_TRANS}$$

$$\frac{}{E^{\mathrm{D}} \vdash \text{'}x \approx t} \quad \text{CONFORMS\_TO\_VAR}$$

$$\frac{}{E^{\mathrm{D}} \vdash t \approx \text{'}x} \quad \text{CONFORMS\_TO\_VAR2}$$

$$\frac{\begin{array}{l}E^{\mathrm{A}}(x) \rhd u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u \approx t\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \approx t} \quad \text{CONFORMS\_TO\_ABBREV}$$

$$\frac{\begin{array}{l}E^{\mathrm{A}}(x) \rhd u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx u\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx x} \quad \text{CONFORMS\_TO\_ABBREV2}$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash t_n \approx u_n}{E^{\mathrm{D}} \vdash (t_1,\, ...\,, t_n) \approx (u_1,\, ...\,, u_n)} \quad \textsc{conforms\_to\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash t\_arg_1 \approx t\_arg'_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash t\_arg_n \approx t\_arg'_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_n\rangle} \quad \textsc{conforms\_to\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\, ..\,, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx u[t\_arg'_1/tid_1 \mathinner{..} t\_arg'_m/tid_m] \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x'\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_appAbbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1,\, ..\,, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u[t\_arg_1/tid_1 \mathinner{..} t\_arg_n/tid_n] \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x'\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_appAbbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{register}\,\langle t\rangle \approx u} \quad \textsc{conforms\_to\_register}$$

$$\boxed{E^{\mathrm{D}}, k \vdash t\_arg \approx t\_arg'}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash t \approx t'} \quad \textsc{targconforms\_typ}$$

$$\frac{}{E^{\mathrm{D}}, K\_Nat \vdash ne \approx ne'} \quad \textsc{targconforms\_nexp}$$

$$\boxed{\sigma_{conformsto(t,t')}(tinflist) \triangleright tinflist'}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ E^{\mathrm{D}} \vdash t'_j \approx t_j \\ \sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ tinf'_0 \mathinner{..} tinf'_n) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j\ effect\ tinf'_0 \mathinner{..} tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j} \quad \textsc{so\_full}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ \sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \mathinner{..} tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t\ effect\ tinf'_0 \mathinner{..} tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t} \quad \textsc{so\_parm}$$

$\boxed{E^{\mathrm{D}} \vdash t \lessapprox t', \Sigma^{\mathrm{N}}}$

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \lessapprox t, \{\,\}}\quad \textsc{consistent\_typ\_refl}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_1 \lessapprox t_2, {\Sigma^{\mathrm{N}}}_1 \\ E^{\mathrm{D}} \vdash t_2 \lessapprox t_3, {\Sigma^{\mathrm{N}}}_2 \end{array}}{E^{\mathrm{D}} \vdash t_1 \lessapprox t_3, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2}\quad \textsc{consistent\_typ\_trans}$$

$$\frac{\begin{array}{c} E^{\mathrm{A}}(x) \rhd \{\,\}, {\Sigma^{\mathrm{N}}}_1, \mathit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \lessapprox t, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1}\quad \textsc{consistent\_typ\_abbrev}$$

$$\frac{\begin{array}{c} E^{\mathrm{A}}(x) \rhd \{\,\}, {\Sigma^{\mathrm{N}}}_1, \mathit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \lessapprox u, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \lessapprox x, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1}\quad \textsc{consistent\_typ\_abbrev}2$$

$$\frac{}{E^{\mathrm{D}} \vdash {}'x \lessapprox t, \{\,\}}\quad \textsc{consistent\_typ\_var}$$

$$\frac{}{E^{\mathrm{D}} \vdash t \lessapprox {}'x, \{\,\}}\quad \textsc{consistent\_typ\_var}2$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \lessapprox u_1, {\Sigma^{\mathrm{N}}}_1 \quad .... \quad E^{\mathrm{D}} \vdash t_n \lessapprox u_n, {\Sigma^{\mathrm{N}}}_n}{E^{\mathrm{D}} \vdash (t_1,\ ....\,, t_n) \lessapprox (u_1,\ ....\,, u_n), {\Sigma^{\mathrm{N}}}_1 \uplus\ ....\ \uplus {\Sigma^{\mathrm{N}}}_n}\quad \textsc{consistent\_typ\_tup}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2\rangle \lessapprox \mathbf{range}\,\langle \mathit{ne}_3\ \mathit{ne}_4\rangle, \{\mathit{ne}_3 \leq \mathit{ne}_1, \mathit{ne}_2 \leq \mathit{ne}_4\}}\quad \textsc{consistent\_typ\_range}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{atom}\,\langle \mathit{ne}\rangle \lessapprox \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2\rangle, \{\mathit{ne}_1 \leq \mathit{ne}, \mathit{ne} \leq \mathit{ne}_2\}}\quad \textsc{consistent\_typ\_atomRange}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{atom}\,\langle \mathit{ne}_1\rangle \lessapprox \mathbf{atom}\,\langle \mathit{ne}_2\rangle, \{\mathit{ne}_1 = \mathit{ne}_2\}}\quad \textsc{consistent\_typ\_atom}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2\rangle \lessapprox \mathbf{atom}\,\langle {}'x\rangle, \{\mathit{ne}_1 \leq {}'x, {}'x \leq \mathit{ne}_2\}}\quad \textsc{consistent\_typ\_rangeAtom}$$

$$\frac{E^{\mathrm{D}} \vdash t \lessapprox t', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ t\rangle \lessapprox \mathbf{vector}\,\langle \mathit{ne}_3\ \mathit{ne}_4\ \mathit{order}\ t'\rangle, \{\mathit{ne}_1 = \mathit{ne}_3, \mathit{ne}_2 = \mathit{ne}_3\} \uplus \Sigma^{\mathrm{N}}}\quad \textsc{consistent\_typ\_vector}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathrel{..} k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash \mathit{t\_arg}_1 \lessapprox \mathit{t\_arg}'_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash \mathit{t\_arg}_n \lessapprox \mathit{t\_arg}'_n, \Sigma^{\mathrm{N}}{}_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle \mathit{t\_arg}_1 \mathrel{..} \mathit{t\_arg}_n\rangle \lessapprox x\langle \mathit{t\_arg}'_1 \mathrel{..} \mathit{t\_arg}'_n\rangle, \Sigma^{\mathrm{N}}{}_1 \uplus \mathrel{..} \uplus \Sigma^{\mathrm{N}}{}_n} \quad \textsc{consistent\_typ\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{\mathit{tid}_1 \mapsto \mathit{kinf}_1, \mathrel{..}, \mathit{tid}_m \mapsto \mathit{kinf}_m\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle \mathit{t\_arg}_1 \mathrel{..} \mathit{t\_arg}_n\rangle \lessapprox u[\mathit{t\_arg}'_1/\mathit{tid}_1 \mathrel{..} \mathit{t\_arg}'_m/\mathit{tid}_m], \Sigma^{\mathrm{N}}{}_2 \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle \mathit{t\_arg}_1 \mathrel{..} \mathit{t\_arg}_n\rangle \lessapprox x'\langle \mathit{t\_arg}'_1 \mathrel{..} \mathit{t\_arg}'_m\rangle, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_2} \quad \textsc{consistent\_typ\_appAbbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{\mathit{tid}_1 \mapsto \mathit{kinf}_1, \mathrel{..}, \mathit{tid}_m \mapsto \mathit{kinf}_m\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u[\mathit{t\_arg}'_1/\mathit{tid}_1 \mathrel{..} \mathit{t\_arg}'_m/\mathit{tid}_m] \lessapprox x\langle \mathit{t\_arg}_1 \mathrel{..} \mathit{t\_arg}_n\rangle, \Sigma^{\mathrm{N}}{}_2 \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x'\langle \mathit{t\_arg}'_1 \mathrel{..} \mathit{t\_arg}'_m\rangle \lessapprox x\langle \mathit{t\_arg}_1 \mathrel{..} \mathit{t\_arg}_n\rangle, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_2} \quad \textsc{consistent\_typ\_appAbbrev2}$$

$$\boxed{E^{\mathrm{D}}, k \vdash \mathit{t\_arg} \lessapprox \mathit{t\_arg}', \Sigma^{\mathrm{N}}}$$

$$\frac{E^{\mathrm{D}} \vdash t \lessapprox t', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, K\_Typ \vdash t \lessapprox t', \Sigma^{\mathrm{N}}} \quad \textsc{targ\_consistent\_typ}$$

$$\frac{}{E^{\mathrm{D}}, K\_Nat \vdash \mathit{ne} \lessapprox \mathit{ne}', \{\mathit{ne} = \mathit{ne}'\}} \quad \textsc{targ\_consistent\_nexp}$$

$$\boxed{E^{\mathrm{D}}, t' \vdash \mathit{exp} : t \triangleright t'', \mathit{exp}', \Sigma^{\mathrm{N}}, \mathit{effect}}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, u_1 \vdash \mathit{id}_1 : t_1 \triangleright u_1, \mathit{exp}_1, \Sigma^{\mathrm{N}}{}_1, \mathit{effect}_1 \quad .. \quad E^{\mathrm{D}}, u_n \vdash \mathit{id}_n : t_n \triangleright u_n, \mathit{exp}_n, \Sigma^{\mathrm{N}}{}_n, \mathit{effect}_n \\ \mathit{exp}' \equiv \mathbf{switch}\,\mathit{exp}\{\,\mathbf{case}\,(\mathit{id}_1, \mathrel{..}, \mathit{id}_n) \to (\mathit{exp}_1, \mathrel{..}, \mathit{exp}_n)\} \end{array}}{E^{\mathrm{D}}, (u_1, \mathrel{..}, u_n) \vdash \mathit{exp} : (t_1, \mathrel{..}, t_n) \triangleright (u_1, \mathrel{..}, u_n), \mathit{exp}', \Sigma^{\mathrm{N}}{}_1 \uplus \mathrel{..} \uplus \Sigma^{\mathrm{N}}{}_n, \mathbf{pure}} \quad \textsc{coerce\_typ\_tuple}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv (\mathit{annot})\mathit{exp} \end{array}}{E^{\mathrm{D}}, \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\, t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \mathit{order}\, u\rangle \triangleright \mathbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \mathit{order}\, t\rangle, \mathit{exp}', \Sigma^{\mathrm{N}} \uplus \{\mathit{ne}_2 = \mathit{ne}_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_vectorUpdateStart}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash u \lessapprox \mathbf{bit}, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathit{to\_num}\,\mathit{exp} \end{array}}{E^{\mathrm{D}}, \mathbf{range}\,\langle \mathit{ne}_1\, \mathit{ne}_2\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \mathit{order}\, u\rangle \triangleright \mathbf{range}\,\langle \mathit{ne}_1\, \mathit{ne}_2\rangle, \mathit{exp}', \Sigma^{\mathrm{N}} \uplus \{\mathit{ne}_1 = \mathbf{zero}, \mathit{ne}_2 \geq 2 ** \mathit{ne}_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toNum}$$

$$\frac{\mathit{exp}' \equiv \mathit{to\_vec}\,\mathit{exp}}{E^{\mathrm{D}}, \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\, \mathbf{bit}\rangle \vdash \mathit{exp} : \mathbf{range}\,\langle \mathit{ne}_3\, \mathit{ne}_4\rangle \triangleright \mathbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\, \mathbf{bit}\rangle, \mathit{exp}', \{\mathit{ne}_3 = \mathbf{zero}, \mathit{ne}_4 \leq 2 ** \mathit{ne}_2\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t \\ \mathit{exp}' \equiv (\mathit{typ})\mathit{exp} \\ E^{\mathrm{D}}, u \vdash \mathit{exp}' : t \triangleright t', \mathit{exp}'', \Sigma^{\mathrm{N}}, \mathbf{pure} \end{array}}{E^{\mathrm{D}}, u \vdash \mathit{exp} : \mathbf{register}\,\langle t \rangle \triangleright t', \mathit{exp}'', \Sigma^{\mathrm{N}}, \{\mathbf{rreg}\}} \quad \textsc{coerce\_typ\_readReg}$$

$$\frac{\mathit{exp}' \equiv \mathit{exp}[\mathit{numZero}]}{E^{\mathrm{D}}, \mathbf{bit} \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \triangleright \mathbf{bit}, \mathit{exp}', \{\mathit{ne}_1 = \mathbf{one}\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_accessVecBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathbf{range}\,\langle \mathbf{zero}\,\mathbf{one} \rangle \lessapprox \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \mathbf{case}\,\mathbf{bitzero} \to \mathit{numZero}\ \mathbf{case}\,\mathbf{bitone} \to \mathit{numOne}\} \end{array}}{E^{\mathrm{D}}, \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp} : \mathbf{bit} \triangleright \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_bitToNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \lessapprox \mathbf{range}\,\langle \mathbf{zero}\,\mathbf{one} \rangle, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \mathbf{case}\,\mathit{numZero} \to \mathbf{bitzero}\ \mathbf{case}\,\mathit{numOne} \to \mathbf{bitone}\} \end{array}}{E^{\mathrm{D}}, \mathbf{bit} \vdash \mathbf{range} : \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \triangleright \mathbf{bit}, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_numToBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\ \overline{\mathit{num}_i \mapsto \mathit{id}_i}^{\,i}\ \} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \overline{\mathbf{case}\,\mathit{num}_i \to \mathit{id}_i}^{\,i}\ \} \\ \mathit{ne}_3 \equiv \mathbf{count}\ (\ \overline{\mathit{num}_i}^{\,i}\ ) \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, x \vdash \mathit{exp} : \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \triangleright x, \mathit{exp}', \{\mathit{ne}_1 \le \mathbf{zero}, \mathit{ne}_2 \le \mathit{ne}_3\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toEnumerate}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\ \overline{\mathit{num}_i \mapsto \mathit{id}_i}^{\,i}\ \} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \overline{\mathbf{case}\,\mathit{id}_i \to \mathit{num}_i}^{\,i}\ \} \\ \mathit{ne}_3 \equiv \mathbf{count}\ (\ \overline{\mathit{num}_i}^{\,i}\ ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash \mathbf{range}\,\langle \mathbf{zero}\,\mathit{ne}_3 \rangle \lessapprox \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp} : x \triangleright \mathbf{range}\,\langle \mathbf{zero}\,\mathit{ne}_3 \rangle, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromEnumerate}$$

$$\frac{E^{\mathrm{D}} \vdash t \lessapprox u, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, u \vdash \mathit{exp} : t \triangleright t, \mathit{exp}, \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_eq}$$

$\boxed{t \vdash \mathit{lit} : t' \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}$ Typing literal constants, coercing to expected type t

$$\frac{}{\mathbf{range}\,\langle \mathit{ne}\ \mathit{ne}' \rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num} \rangle \Rightarrow \mathit{num}, \{\mathit{ne} \le \mathit{num}, \mathit{num} \le \mathit{ne}'\}} \quad \textsc{check\_lit\_num}$$

$$\frac{}{\mathbf{vector}\,\langle \mathit{ne}\ \mathit{ne}'\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num} \rangle \Rightarrow \mathit{to\_vec}\,\mathit{num}, \{\mathit{num} + \mathbf{one} \le 2 ** \mathit{ne}'\}} \quad \textsc{check\_lit\_numToVec}$$

$$\frac{}{\mathbf{bit} \vdash \mathit{numZero} : \mathbf{atom}\,\langle \mathbf{zero} \rangle \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITZERO}$$

$$\frac{}{\mathbf{bit} \vdash \mathit{numOne} : \mathbf{atom}\,\langle \mathbf{one} \rangle \Rightarrow \mathbf{bitone}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITONE}$$

$$\frac{}{\mathit{string} \vdash \mathit{string} : \mathit{string} \Rightarrow \mathit{string}, \{\,\}} \quad \text{CHECK\_LIT\_STRING}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{hex})}{\mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{hex} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\ \mathbf{bit} \rangle \Rightarrow \mathit{hex}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_HEX}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{bin})}{\mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{bin} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\ \mathbf{bit} \rangle \Rightarrow \mathit{bin}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_BIN}$$

$$\frac{}{\mathbf{unit} \vdash () : \mathbf{unit} \Rightarrow \mathbf{unit}, \{\,\}} \quad \text{CHECK\_LIT\_UNIT}$$

$$\frac{}{\mathbf{bit} \vdash \mathbf{bitzero} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITZERO}$$

$$\frac{}{\mathbf{bit} \vdash \mathbf{bitone} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITONE}$$

$$\frac{}{t \vdash \mathbf{undefined} : t \Rightarrow \mathbf{undefined}, \{\,\}} \quad \text{CHECK\_LIT\_UNDEF}$$

$\boxed{E, t \vdash \mathit{pat} : t' \vartriangleright \mathit{pat}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Typing patterns, building their binding environment

$$\frac{\begin{array}{l} \mathit{lit} \neq \mathbf{undefined} \\ t \vdash \mathit{lit} : u \Rightarrow \mathit{lit}', \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash \mathit{lit} : u \vartriangleright \mathit{lit}', \{\,\}, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \text{CHECK\_PAT\_LIT}$$

$$\frac{}{E, t \vdash \_ : t \vartriangleright \_, \{\,\}, \{\,\}} \quad \text{CHECK\_PAT\_WILD}$$

$$\frac{\begin{array}{l} E, t \vdash \mathit{pat} : u \vartriangleright \mathit{pat}', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ \mathit{id} \notin \mathbf{dom}\,(E^{\mathrm{T}}{}_1) \end{array}}{E, t \vdash (\mathit{pat}\ \mathbf{as}\ \mathit{id}) : u \vartriangleright (\mathit{pat}'\ \mathbf{as}\ \mathit{id}), (E^{\mathrm{T}}{}_1 \uplus \{\mathit{id} \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \text{CHECK\_PAT\_AS}$$

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t' \vdash \mathit{pat} : t \rhd \mathit{pat}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}} \\ E^{\mathrm{T}}(\mathit{id}) \rhd \{\,\}, \{\,\}, \mathbf{Default}, t' \\ E^{\mathrm{D}} \vdash t' \precapprox u, {\Sigma^{\mathrm{N}}}' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash (\mathit{pat}\ \mathbf{as}\ \mathit{id}) : t \rhd (\mathit{pat}'\ \mathbf{as}\ \mathit{id}), ({E^{\mathrm{T}}}_1 \uplus \{\mathit{id} \mapsto t'\}), \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}'} \quad \textsc{check\_pat\_asDefault}$$

$$\dfrac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat} : t \rhd \mathit{pat}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash (\mathit{typ})\mathit{pat} : t \rhd \mathit{pat}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_typ}$$

$$\dfrac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \rhd \{\mathit{tid}_1 \mapsto \mathit{kinf}_1, \,..\,, \mathit{tid}_m \mapsto \mathit{kinf}_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, (u'_1, \,..\,, u'_n) \to x\langle \mathit{t\_arg}_1 \,..\, \mathit{t\_arg}_m\rangle\ \mathbf{pure} \\ (u_1, \,..\,, u_n) \to x\langle \mathit{t\_args}'\rangle\ \mathbf{pure} \equiv (u'_1, \,..\,, u'_n) \to x\langle \mathit{t\_args}\rangle\ \mathbf{pure}[\mathit{t\_arg}_1/\mathit{tid}_1 \,..\, \mathit{t\_arg}_m/\mathit{tid}_m] \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_1 \vdash \mathit{pat}_1 : t_1 \rhd \mathit{pat}'_1, {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_n \vdash \mathit{pat}_n : t_n \rhd \mathit{pat}'_n, {E^{\mathrm{T}}}_n, {\Sigma^{\mathrm{N}}}_n \\ \mathbf{disjoint\ doms}\,({E^{\mathrm{T}}}_1, \,..\,, {E^{\mathrm{T}}}_n) \\ E^{\mathrm{D}} \vdash x\langle \mathit{t\_args}'\rangle \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{id}(\mathit{pat}_1, \,..\,, \mathit{pat}_n) : x\langle \mathit{t\_args}'\rangle \rhd \mathit{id}(\mathit{pat}'_1, \,..\,, \mathit{pat}'_n), \uplus {E^{\mathrm{T}}}_1 \,..\, {E^{\mathrm{T}}}_n, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1 \uplus \,..\, \uplus {\Sigma^{\mathrm{N}}}_n} \quad \textsc{check\_pat\_constr}$$

$$\dfrac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \rhd \{\mathit{tid}_1 \mapsto \mathit{kinf}_1, \,..\,, \mathit{tid}_m \mapsto \mathit{kinf}_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, \mathbf{unit} \to x\langle \mathit{t\_arg}_1 \,..\, \mathit{t\_arg}_m\rangle\ \mathbf{pure} \\ \mathbf{unit} \to x\langle \mathit{t\_args}'\rangle\ \mathbf{pure} \equiv \mathbf{unit} \to x\langle \mathit{t\_args}\rangle\ \mathbf{pure}[\mathit{t\_arg}_1/\mathit{tid}_1 \,..\, \mathit{t\_arg}_m/\mathit{tid}_m] \\ E^{\mathrm{D}} \vdash x\langle \mathit{t\_args}'\rangle \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{id} : t \rhd \mathit{id}, \{\,\}, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_identConstr}$$

$$\dfrac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \rhd \{\,\}, \{\,\}, \mathbf{Default}, t \\ E^{\mathrm{D}} \vdash t \precapprox u, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash \mathit{id} : t \rhd \mathit{id}, (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_varDefault}$$

$$\dfrac{}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{id} : t \rhd \mathit{id}, (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto t\}), \{\,\}} \quad \textsc{check\_pat\_var}$$

$$\dfrac{\begin{array}{l} E^{\mathrm{R}}(\,\overline{\mathit{id}_i}^{\,i}\,) \rhd x\langle \mathit{t\_args}\rangle, (\,\overline{t_i}^{\,i}\,) \\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash \mathit{pat}_i : u_i \rhd \mathit{pat}'_i, {E^{\mathrm{T}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \mathbf{disjoint\ doms}\,(\,\overline{{E^{\mathrm{T}}}_i}^{\,i}\,) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle \mathit{t\_args}\rangle \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\,\overline{\mathit{id}_i = \mathit{pat}_i}^{\,i}\,;^?\} : x\langle \mathit{t\_args}\rangle \rhd \{\,\overline{\mathit{id}_i = \mathit{pat}'_i}^{\,i}\,;^?\}, \uplus\,\overline{{E^{\mathrm{T}}}_i}^{\,i}, \Sigma^{\mathrm{N}} \uplus\ \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}} \quad \textsc{check\_pat\_record}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}'}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}'}{}_n \\ \mathit{ne}_4 \equiv \textbf{length}\,(\mathit{pat}_1 \ldots \mathit{pat}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\ t\rangle \vdash [\mathit{pat}_1, \ldots, \mathit{pat}_n] : \textbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \mathit{order}\ u\rangle \triangleright [\mathit{pat}'_1, \ldots, \mathit{pat}'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'} \uplus \{\mathit{ne}_2 = \mathit{ne}_4\}} \quad \text{CHECK\_PAT\_VECTOR}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}'}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}'}{}_n \\ \mathit{ne}_4 \equiv \textbf{length}\,(\mathit{pat}_1 \ldots \mathit{pat}_n) \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ \mathit{num}_1 < \ldots < \mathit{num}_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \textbf{inc}\ t\rangle \vdash [\mathit{num}_1 = \mathit{pat}_1, \ldots, \mathit{num}_n = \mathit{pat}_n] : \textbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \textbf{inc}\ t\rangle \triangleright [\mathit{num}_1 = \mathit{pat}'_1, \ldots, \mathit{num}_n = \mathit{pat}'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{\mathit{ne}_1 \leq \mathit{num}_1, \mathit{ne}_2 \geq \mathit{ne}_4\} \uplus}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_1 : u_1 \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat}_n : u_n \triangleright \mathit{pat}'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}'}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}'}{}_n \\ \mathit{ne}_4 \equiv \textbf{length}\,(\mathit{pat}_1 \ldots \mathit{pat}_n) \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ \mathit{num}_1 > \ldots > \mathit{num}_n \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \textbf{dec}\ t\rangle \vdash [\mathit{num}_1 = \mathit{pat}_1, \ldots, \mathit{num}_n = \mathit{pat}_n] : \textbf{vector}\,\langle \mathit{ne}_3\, \mathit{ne}_4\, \textbf{dec}\ t\rangle \triangleright [\mathit{num}_1 = \mathit{pat}'_1, \ldots, \mathit{num}_n = \mathit{pat}'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{\mathit{ne}_1 \geq \mathit{num}_1, \mathit{ne}_2 \geq \mathit{ne}_4\} \uplus}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}''_1\, \mathit{ne}'''_1\, \mathit{order}\ t\rangle \vdash \mathit{pat}_1 : \textbf{vector}\,\langle \mathit{ne}''_1\, \mathit{ne}'_1\, \mathit{order}\ u_1\rangle \triangleright \mathit{pat}'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}''_n\, \mathit{ne}'''_n\, \mathit{order}\ t\rangle \vdash \mathit{pat}_1 : \textbf{vector}\,\langle \mathit{ne}''_n\, \mathit{ne}'_n\, \mathit{order}\ u_1\rangle \triangleright \mathit{pat}'_n, E \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}'}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}'}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_2\, \mathit{order}\ t\rangle \vdash \mathit{pat}_1 : \ldots : \mathit{pat}_n : \textbf{vector}\,\langle \mathit{ne}_1\, \mathit{ne}_4\, \mathit{order}\ t\rangle \triangleright \mathit{pat}'_1 : \ldots : \mathit{pat}'_n, (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{\mathit{ne}'_1 + \ldots + \mathit{ne}'_n \leq \mathit{ne}_2\} \uplus \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}}$$

$$\frac{\begin{array}{l} E, t_1 \vdash pat_1 : u_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .... \quad E, t_n \vdash pat_n : u_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \,....\,, E^{\mathrm{T}}{}_n) \end{array}}{E, (t_1, \,....\,, t_n) \vdash (pat_1, \,....\,, pat_n) : (u_1, \,....\,, u_n) \triangleright (pat'_1, \,....\,, pat'_n), (E^{\mathrm{T}}{}_1 \uplus \,....\, \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}}{}_1 \uplus \,....\, \uplus \Sigma^{\mathrm{N}}{}_n} \quad \textsc{check\_pat\_tup}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash pat_1 : u_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash pat_n : u_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \,..\,, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, \Sigma^{\mathrm{N}'}{}_1 \quad .. \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, \Sigma^{\mathrm{N}'}{}_n \\ \textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \,..\,, E^{\mathrm{T}}{}_n) \\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \,..\, \uplus \Sigma^{\mathrm{N}}{}_n \\ \Sigma^{\mathrm{N}'} \equiv \Sigma^{\mathrm{N}'}{}_1 \uplus \,..\, \uplus \Sigma^{\mathrm{N}'}{}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \textbf{list}\,\langle t \rangle \vdash [||pat_1, \,..\,, pat_n||] : \textbf{list}\,\langle t \rangle \triangleright [||pat'_1, \,..\,, pat'_n||], (E^{\mathrm{T}}{}_1 \uplus \,..\, \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}} \quad \textsc{check\_pat\_list}$$

$\boxed{E, t \vdash exp : t' \triangleright exp', I, E^{\mathrm{T}}}$ Typing expressions, collecting nexp constraints, effects, and new bindings

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \,..\,, tid_n \mapsto kinf_n\}, \{\,\}, \textbf{Ctor}, \textbf{unit} \to x\langle t\_args \rangle\,\textbf{pure} \\ u \equiv x\langle t\_args \rangle[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : x \triangleright id, \langle \Sigma^{\mathrm{N}}, \textbf{pure} \rangle, \{\,\}} \quad \textsc{check\_exp\_unaryCtor}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, tag, u \\ E^{\mathrm{D}}, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}}, effect \rangle, \{\,\}} \quad \textsc{check\_exp\_localVar}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \,..\,, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u' \\ u \equiv u'[t\_arg_1/tid_1 \,..\, t\_arg_n/tid_n] \\ E^{\mathrm{D}}, t \vdash id : u \triangleright t', exp, \Sigma^{\mathrm{N}'}, effect \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : u \triangleright id, \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}, effect \rangle, \{\,\}} \quad \textsc{check\_exp\_otherVar}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, \,..\,, tid_n \mapsto kinf_n\}, \{\,\}, \textbf{Ctor}, t'' \to x\langle t\_args \rangle\,\textbf{pure} \\ t' \to u\,\textbf{pure} \equiv t'' \to x\langle t\_args \rangle\,\textbf{pure}[t\_arg_0/tid_0 \,..\, t\_arg_n/tid_n] \\ E^{\mathrm{D}} \vdash u \lessapprox t, \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t' \vdash exp : u' \triangleright exp, \langle \Sigma^{\mathrm{N}'}, effect \rangle, E^{\mathrm{T}'} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id(exp) : t \triangleright id(exp'), \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}, effect \rangle, \{\,\}} \quad \textsc{check\_exp\_ctor}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 .. t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ u_i \equiv (\textbf{implicit}\ \langle ne\rangle, t_0, .., t_m) \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_0, .., t_m) \vdash (exp_0, .., exp_m) : u_i' \triangleright (exp_0', .., exp_m'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash id(annot, exp_0', .., exp_m') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp_0, .., exp_m) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appImplicit}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 .. t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash id(exp') : u_j \triangleright u_j', exp'', \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp) : u_j \triangleright exp'', I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \textbf{overload}\ \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \ldots tinf_n \\ u[t\_arg_0/tid_0 .. t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash exp : u_i' \triangleright exp', I, E^{\mathrm{T}\prime} \\ \text{\texttt{<<no parses (char 3): sel***ect (conformsto(ui', t)) of tinf1 ... tinfn gives tinf >>}} \\ \langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t \vdash id(exp) : t' \triangleright exp'', I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash id(exp) : u_j \triangleright exp'', I \uplus I' \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ u[t\_arg_0/tid_0 .. t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ E^{\mathrm{D}}, t \vdash exp_1'\ id\ exp_2' : u_j \triangleright u_j', exp, \Sigma^{\mathrm{N}\prime}, effect' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_infix\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \textbf{overload}\ \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \ldots tinf_n \\ u[t\_arg_0/tid_0 .. t\_arg_n/tid_n] \equiv u_i \to u_j\ effect \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i \vdash (exp_1, exp_2) : u_i' \triangleright (exp_1', exp_2'), I, E^{\mathrm{T}\prime} \\ \text{\texttt{<<no parses (char 3): sel***ect (conformsto(ui', t)) of tinf1 ... tinfn gives tinf >>}} \\ \langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t' \triangleright exp, I', E^{\mathrm{T}\prime\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1\ id\ exp_2 : t \triangleright exp, I \uplus I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_infix\_appOverload}$$

$$\frac{\begin{array}{l}E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \rhd x\langle t\_args\rangle, \overline{t_i}^{\,i}\\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash exp_i : u_i \rhd exp'_i, \langle {\Sigma^{\mathrm{N}}}_i, effect_i\rangle, E^{\mathrm{T}}}^{\,i}\\ \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u_i \lessapprox t_i, {\Sigma^{\mathrm{N}}}'_i}^{\,i}\\ \Sigma^{\mathrm{N}} \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}\\ \Sigma^{\mathrm{N}\prime} \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}'_i}^{\,i}\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t\_args\rangle \rhd \{\overline{id_i = exp'_i}^{\,i}\,;^?\}, \uplus\langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, \uplus\,\overline{effect_i}^{\,i}\rangle, \{\,\}} \quad \textsc{check\_exp\_record}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash exp : x\langle t\_args\rangle \rhd exp', I, E^{\mathrm{T}}\\ E^{\mathrm{R}}(x\langle t\_args\rangle) \rhd \overline{id'_n : t'_n}^{\,n}\\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash exp_i : u_i \rhd exp'_i, I_i, E^{\mathrm{T}}}^{\,i}\\ \overline{id_i : t_i}^{\,i} \subset \overline{id'_n : t'_n}^{\,n}\\ \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u_i \lessapprox t_i, {\Sigma^{\mathrm{N}}}'_i}^{\,i}\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{exp\ \mathbf{with}\ \overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t\_args\rangle \rhd \{exp'\ \mathbf{with}\ \overline{id_i = exp'_i}^{\,i}\,\}, I \uplus \overline{I_i}^{\,i}, E^{\mathrm{T}}} \quad \textsc{check\_exp\_recup}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_1 : u_1 \rhd exp'_1, I_1, E^{\mathrm{T}\prime} \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash exp_n : u_n \rhd exp'_n, I_n, E^{\mathrm{T}\prime}\\ E^{\mathrm{D}} \vdash u_1 \lessapprox t, {\Sigma^{\mathrm{N}}}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash u_n \lessapprox t, {\Sigma^{\mathrm{N}}}_n\\ \mathbf{length}\,(exp_1 \ldots exp_n) \equiv ne\\ \Sigma^{\mathrm{N}} \equiv \{ne = ne_2\} \uplus {\Sigma^{\mathrm{N}}}_1 \uplus \ldots \uplus {\Sigma^{\mathrm{N}}}_n\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle \vdash [exp_1, \ldots, exp_n] : \mathbf{vector}\,\langle ne_1\ num\ order\ t\rangle \rhd [exp'_1, \ldots, exp'_n], \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle \uplus I_1 \uplus \ldots \uplus I_n, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vector}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle ne\ ne'\ order\ t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_1\ ne'_1\ \mathbf{inc}\ u\rangle \rhd exp'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle ne_2\ ne'_2\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_3\ ne'_3\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}\end{array}}{E, t \vdash exp_1[exp_2] : u \rhd exp'_1[exp'_2], I_1 \uplus I_2 \uplus \langle\{ne_1 \leq ne_3, ne_3 + ne'_3 \leq ne_1 + ne'_1\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vectorgetInc}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle ne\ ne'\ order\ t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_1\ ne'_1\ \mathbf{dec}\ u\rangle \rhd exp'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle ne_2\ ne'_2\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_3\ ne'_3\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}\end{array}}{E, t \vdash exp_1[exp_2] : u \rhd exp'_1[exp'_2], I_1 \uplus I_2 \uplus \langle\{ne_1 \geq ne_3, ne_3 + (-ne'_3) \leq ne_1 + (-ne'_1)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_vectorgetDec}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle ne_1\ ne'_1\ \mathbf{inc}\ t\rangle \vdash exp_1 : \mathbf{vector}\,\langle ne_2\ ne'_2\ \mathbf{inc}\ u\rangle \rhd exp'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle ne_3\ ne'_3\rangle \vdash exp_2 : \mathbf{range}\,\langle ne_4\ ne'_4\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle ne_5\ ne'_5\rangle \vdash exp_3 : \mathbf{range}\,\langle ne_6\ ne'_6\rangle \rhd exp'_3, I_3, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle ne\ ne'\ \mathbf{inc}\ t\rangle \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\ ne'_7\ \mathbf{inc}\ u\rangle \rhd exp'_1[exp'_2 : exp'_3], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \geq ne_4, ne \leq ne'_4, ne' \leq ne_4 + ne'_6, ne_4 \leq ne_2, ne_4 + ne'_6 \leq ne'_2\}, \mathbf{pure}}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}'_1\; \mathbf{dec}\; t\rangle \vdash \mathit{exp}_1 : \mathbf{vector}\,\langle \mathit{ne}_2\; \mathit{ne}'_2\; \mathbf{dec}\; u\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle \mathit{ne}_3\; \mathit{ne}'_3\rangle \vdash \mathit{exp}_2 : \mathbf{range}\,\langle \mathit{ne}_4\; \mathit{ne}'_4\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle \mathit{ne}_5\; \mathit{ne}'_5\rangle \vdash \mathit{exp}_3 : \mathbf{range}\,\langle \mathit{ne}_6\; \mathit{ne}'_6\rangle \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}\; \mathit{ne}'\; \mathbf{dec}\; t\rangle \vdash \mathit{exp}_1[\mathit{exp}_2..\mathit{exp}_3] : \mathbf{vector}\,\langle \mathit{ne}_7\; \mathit{ne}'_7\; \mathbf{dec}\; u\rangle \triangleright \mathit{exp}'_1[\mathit{exp}'_2 : \mathit{exp}'_3], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{\mathit{ne} \leq \mathit{ne}_4, \mathit{ne} \geq \mathit{ne}'_4, \mathit{ne}' \leq \mathit{ne}'_6 + (-\mathit{ne}_4), \mathit{ne}'_4 \geq \mathit{ne}_2, \mathit{ne}'_6 + (-\mathit{ne}_4) \leq \mathit{ne}}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}\; \mathit{ne}'\; \mathbf{inc}\; t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathbf{inc}\; u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle \mathit{ne}'_1\; \mathit{ne}'_2\rangle \vdash \mathit{exp}_1 : \mathbf{range}\,\langle \mathit{ne}_3\; \mathit{ne}_4\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, t \vdash \mathit{exp}_2 : u \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}\; \mathit{ne}'\; \mathbf{inc}\; t\rangle \vdash [\mathit{exp}\; \mathbf{with}\; \mathit{exp}_1 = \mathit{exp}_2] : \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathbf{inc}\; u\rangle \triangleright [\mathit{exp}'\; \mathbf{with}\; \mathit{exp}'_1 = \mathit{exp}'_2], I \uplus I_1 \uplus I_2 \uplus \langle\{\mathit{ne}_1 \leq \mathit{ne}_3, \mathit{ne}_2 \geq \mathit{ne}_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\;\textsc{check\_exp\_vectorU}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}\; \mathit{ne}'\; \mathbf{dec}\; t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathbf{dec}\; u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}}\\ E, \mathbf{range}\,\langle \mathit{ne}'_1\; \mathit{ne}'_2\rangle \vdash \mathit{exp}_1 : \mathbf{range}\,\langle \mathit{ne}_3\; \mathit{ne}_4\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, t \vdash \mathit{exp}_2 : u \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}\; \mathit{ne}'\; \mathbf{dec}\; t\rangle \vdash [\mathit{exp}\; \mathbf{with}\; \mathit{exp}_1 = \mathit{exp}_2] : \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathbf{dec}\; u\rangle \triangleright [\mathit{exp}'\; \mathbf{with}\; \mathit{exp}'_1 = \mathit{exp}'_2], I \uplus I_1 \uplus I_2 \uplus \langle\{\mathit{ne}_1 \geq \mathit{ne}_3, \mathit{ne}_2 \geq \mathit{ne}_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}}\;\textsc{check\_exp\_vector}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{inc}\; u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_5\rangle \vdash \mathit{exp}_1 : \mathbf{atom}\,\langle \mathit{ne}_6\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_7\rangle \vdash \mathit{exp}_2 : \mathbf{atom}\,\langle \mathit{ne}_8\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\\ E, \mathbf{vector}\,\langle \mathit{ne}_9\; \mathit{ne}_{10}\; \mathbf{inc}\; t\rangle \vdash \mathit{exp}_3 : \mathbf{vector}\,\langle \mathit{ne}_{11}\; \mathit{ne}_{12}\; \mathbf{inc}\; u\rangle \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}}\\ I_4 \equiv \langle\{\mathit{ne}_3 \leq \mathit{ne}_5, \mathit{ne}_3 + \mathit{ne}_4 \leq \mathit{ne}_7, \mathit{ne}_{12} = \mathit{ne}_8 + (-\mathit{ne}_6), \mathit{ne}_6 + \mathbf{one} \leq \mathit{ne}_8\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash [\mathit{exp}\; \mathbf{with}\; \mathit{exp}_1 : \mathit{exp}_2 = \mathit{exp}_3] : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{inc}\; u\rangle \triangleright [\mathit{exp}'\; \mathbf{with}\; \mathit{exp}'_1 : \mathit{exp}'_2 = \mathit{exp}'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupInc}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{inc}\; u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_5\rangle \vdash \mathit{exp}_1 : \mathbf{atom}\,\langle \mathit{ne}_6\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_7\rangle \vdash \mathit{exp}_2 : \mathbf{atom}\,\langle \mathit{ne}_8\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\\ E, u \vdash \mathit{exp}_3 : u' \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}}\\ I_4 \equiv \langle\{\mathit{ne}_3 \leq \mathit{ne}_5, \mathit{ne}_3 + \mathit{ne}_4 \leq \mathit{ne}_7\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash [\mathit{exp}\; \mathbf{with}\; \mathit{exp}_1 : \mathit{exp}_2 = \mathit{exp}_3] : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{inc}\; u\rangle \triangleright [\mathit{exp}'\; \mathbf{with}\; \mathit{exp}'_1 : \mathit{exp}'_2 = \mathit{exp}'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupvalu}$$

$$\frac{\begin{array}{l}E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{dec}\; u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_5\rangle \vdash \mathit{exp}_1 : \mathbf{atom}\,\langle \mathit{ne}_6\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle \mathit{ne}_7\rangle \vdash \mathit{exp}_2 : \mathbf{atom}\,\langle \mathit{ne}_8\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}\\ E, \mathbf{vector}\,\langle \mathit{ne}_9\; \mathit{ne}_{10}\; \mathbf{dec}\; t\rangle \vdash \mathit{exp}_3 : \mathbf{vector}\,\langle \mathit{ne}_{11}\; \mathit{ne}_{12}\; \mathbf{dec}\; u\rangle \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}}\\ I_4 \equiv \langle\{\mathit{ne}_5 \leq \mathit{ne}_3, \mathit{ne}_3 + (-\mathit{ne}_4) \leq \mathit{ne}_6 + (-\mathit{ne}_8), \mathit{ne}_8 + \mathbf{one} \leq \mathit{ne}_6\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}_1\; \mathit{ne}_2\; \mathit{order}\; t\rangle \vdash [\mathit{exp}\; \mathbf{with}\; \mathit{exp}_1 : \mathit{exp}_2 = \mathit{exp}_3] : \mathbf{vector}\,\langle \mathit{ne}_3\; \mathit{ne}_4\; \mathbf{dec}\; u\rangle \triangleright [\mathit{exp}'\; \mathbf{with}\; \mathit{exp}'_1 : \mathit{exp}'_2 = \mathit{exp}'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\;\textsc{check\_exp\_vecrangeupDe}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ t\rangle \vdash \mathit{exp} : \mathbf{vector}\,\langle \mathit{ne}_3\ \mathit{ne}_4\ \mathbf{dec}\ u\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle \mathit{ne}_5\rangle \vdash \mathit{exp}_1 : \mathbf{atom}\,\langle \mathit{ne}_6\rangle \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle \mathit{ne}_7\rangle \vdash \mathit{exp}_2 : \mathbf{atom}\,\langle \mathit{ne}_8\rangle \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ E, u \vdash \mathit{exp}_3 : u' \triangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle \{\mathit{ne}_5 \leq \mathit{ne}_3, \mathit{ne}_3 + (-\mathit{ne}_4) \leq \mathit{ne}_6 + (-\mathit{ne}_8), \mathit{ne}_8 + \mathbf{one} \leq \mathit{ne}_6\}, \mathbf{pure}\rangle \end{array}}{E, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ t\rangle \vdash [\mathit{exp}\ \mathbf{with}\ \mathit{exp}_1 : \mathit{exp}_2 = \mathit{exp}_3] : \mathbf{vector}\,\langle \mathit{ne}_3\ \mathit{ne}_4\ \mathbf{dec}\ u\rangle \triangleright [\mathit{exp}'\ \mathbf{with}\ \mathit{exp}'_1 : \mathit{exp}'_2 = \mathit{exp}'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_VECRANGEUPVAL}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(x\langle t\_\mathit{args}\rangle) \triangleright \overline{\mathit{id}_i : t_i}^{\,i}\ \mathit{id} : u\ \overline{\mathit{id}'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t'' \vdash \mathit{exp} : x\langle t\_\mathit{args}\rangle \triangleright \mathit{exp}', I, E^{\mathrm{T}} \\ E^{\mathrm{D}}, t \vdash \mathit{exp}'.\mathit{id} : u \triangleright t', \mathit{exp}'_1, {\Sigma^{\mathrm{N}}}', \mathit{effect} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp}.\mathit{id} : u \triangleright \mathit{exp}'_1, I \uplus \langle {\Sigma^{\mathrm{N}}}', \mathit{effect}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_FIELD}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'' \vdash \mathit{exp} : u \triangleright \mathit{exp}', I, E^{\mathrm{T}} \\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash \mathit{pat}_i : u'_i \triangleright \mathit{pat}'_i, {E^{\mathrm{T}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_i), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_i : u''_i \triangleright \mathit{exp}'_i, I_i, {E^{\mathrm{T}}}'_i}^{\,i} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathbf{switch}\ \mathit{exp}\{\,\overline{\mathbf{case}\ \mathit{pat}_i \to \mathit{exp}_i}^{\,i}\,\} : u \triangleright \mathbf{switch}\ \mathit{exp}'\{\,\overline{\mathbf{case}\ \mathit{pat}'_i \to \mathit{exp}'_i}^{\,i}\,\}, I \uplus\ \overline{I_i \uplus \langle {\Sigma^{\mathrm{N}}}_i, \mathbf{pure}\rangle}^{\,i}, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_CASE}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'' \vdash \mathit{exp} : u \triangleright \mathit{exp}', I, E^{\mathrm{T}} \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t' \\ E^{\mathrm{D}}, t' \vdash \mathit{exp}' : u \triangleright u', \mathit{exp}'', \Sigma^{\mathrm{N}}, \mathit{effect} \\ E^{\mathrm{D}}, t \vdash \mathit{exp}'' : t' \triangleright u'', \mathit{exp}''', {\Sigma^{\mathrm{N}}}', \mathit{effect}' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash (\mathit{typ})\mathit{exp} : t \triangleright \mathit{exp}''', I \uplus \langle \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}', \mathit{effect} \uplus \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_TYPED}$$

$$\frac{\begin{array}{c} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{letbind} \triangleright \mathit{letbind}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}}, \mathit{effect}, \{\,\} \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_2, {E^{\mathrm{T}}}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{letbind}\ \mathbf{in}\ \mathit{exp} : t \triangleright \mathit{letbind}'\ \mathbf{in}\ \mathit{exp}', \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LET}$$

$$\frac{E, t_1 \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad \ldots. \quad E, t_n \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n}{E, (t_1, \ldots., t_n) \vdash (\mathit{exp}_1, \ldots., \mathit{exp}_n) : (u_1, \ldots., u_n) \triangleright (\mathit{exp}'_1, \ldots., \mathit{exp}'_n), I_1 \uplus \ldots. \uplus I_n, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_TUP}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, {E^{\mathrm{T}}}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_n : u_n \triangleright \mathit{exp}'_n, I_n, {E^{\mathrm{T}}}_n \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, {\Sigma^{\mathrm{N}}}_1 \quad .. \quad E^{\mathrm{D}} \vdash u_n \precapprox t, {\Sigma^{\mathrm{N}}}_n \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle \vdash [||\mathit{exp}_1, .., \mathit{exp}_n||] : \mathbf{list}\,\langle u\rangle \triangleright [||\mathit{exp}'_1, .., \mathit{exp}'_n||], \langle {\Sigma^{\mathrm{N}}}_1 \uplus .. \uplus {\Sigma^{\mathrm{N}}}_n, \mathbf{pure}\rangle \uplus I_1 \uplus .. \uplus I_n, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LIST}$$

$$\frac{\begin{array}{l} E, \mathbf{bit} \vdash \mathit{exp}_1 : \mathbf{bit} \vartriangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}\prime} \\ E, t \vdash \mathit{exp}_2 : u_1 \vartriangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}}{}_2 \\ E, t \vdash \mathit{exp}_3 : u_2 \vartriangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}}{}_3 \\ E^{\mathrm{D}} \vdash u_1 \precapprox t, \Sigma^{\mathrm{N}}{}_1 \\ E^{\mathrm{D}} \vdash u_2 \precapprox t, \Sigma^{\mathrm{N}}{}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash \mathbf{if}\ \mathit{exp}_1\ \mathbf{then}\ \mathit{exp}_2\ \mathbf{else}\ \mathit{exp}_3 : u \vartriangleright \mathbf{if}\ \mathit{exp}'_1\ \mathbf{then}\ \mathit{exp}'_2\ \mathbf{else}\ \mathit{exp}'_3, \langle \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2, \mathbf{pure} \rangle \uplus I_1 \uplus I_2 \uplus I_3, (E^{\mathrm{T}}{}_2 \cap E^{\mathrm{T}}{}_3)} \quad \text{CHECK\_EXP\_IF}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp}_1 : \mathbf{range}\,\langle \mathit{ne}_7\ \mathit{ne}_8 \rangle \vartriangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\,\langle \mathit{ne}_3\ \mathit{ne}_4 \rangle \vdash \mathit{exp}_2 : \mathbf{range}\,\langle \mathit{ne}_9\ \mathit{ne}_{10} \rangle \vartriangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\,\langle \mathit{ne}_5\ \mathit{ne}_6 \rangle \vdash \mathit{exp}_3 : \mathbf{range}\,\langle \mathit{ne}_{11}\ \mathit{ne}_{12} \rangle \vartriangleright \mathit{exp}'_3, I_3, E^{\mathrm{T}} \\ \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{range}\,\langle \mathit{ne}_1\ \mathit{ne}_4 \rangle\}), E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathit{exp}_4 : t \vartriangleright \mathit{exp}'_4, I_4, E^{\mathrm{T}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}_1\ \mathbf{to}\ \mathit{exp}_2\ \mathbf{by}\ \mathit{exp}_3)\mathit{exp}_4 : t \vartriangleright \mathbf{foreach}\,(\mathit{id}\ \mathbf{from}\ \mathit{exp}'_1\ \mathbf{to}\ \mathit{exp}'_2\ \mathbf{by}\ \mathit{exp}'_3)\mathit{exp}'_4, I_1 \uplus I_2 \uplus I_3 \uplus I_4 \uplus \langle \{\mathit{ne}_1 \leq \mathit{ne}_3 + \mathit{ne}_4\}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP}$$

$$\frac{\begin{array}{l} E, t \vdash \mathit{exp}_1 : u \vartriangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{list}\,\langle t \rangle \vdash \mathit{exp}_2 : \mathbf{list}\,\langle u \rangle \vartriangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \end{array}}{E, \mathbf{list}\,\langle t \rangle \vdash \mathit{exp}_1 :: \mathit{exp}_2 : \mathbf{list}\,\langle u \rangle \vartriangleright \mathit{exp}'_1 :: \mathit{exp}'_2, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_CONS}$$

$$\frac{t \vdash \mathit{lit} : u \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}{E, t \vdash \mathit{lit} : u \vartriangleright \mathit{exp}, \langle \Sigma^{\mathrm{N}}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_LIT}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \vartriangleright \mathit{exp}', I, E^{\mathrm{T}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \{\mathit{exp}\} : \mathbf{unit} \vartriangleright \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_BLOCKBASE}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \vartriangleright \mathit{exp}', I_1, E^{\mathrm{T}}{}_1 \\ \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \{\,\overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \vartriangleright \{\,\overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, E^{\mathrm{T}}{}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \{\mathit{exp};\ \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \vartriangleright \{\mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_BLOCKREC}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \vartriangleright \mathit{exp}', I, E^{\mathrm{T}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\mathit{exp}\} : \mathbf{unit} \vartriangleright \{\mathit{exp}'\}, I, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_NONDETBASE}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathit{exp} : \mathbf{unit} \vartriangleright \mathit{exp}', I_1, E^{\mathrm{T}}{}_1 \\ \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\,\overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \vartriangleright \{\,\overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, E^{\mathrm{T}}{}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit} \vdash \mathbf{nondet}\,\{\mathit{exp};\ \overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \vartriangleright \{\mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_NONDETREC}$$

$$\frac{\begin{array}{c} E, t \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ E \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I_2, {E^{\mathrm{T}}}_2 \end{array}}{E, \mathbf{unit} \vdash \mathit{lexp} := \mathit{exp} : \mathbf{unit} \triangleright \mathit{lexp}' := \mathit{exp}', I \uplus I_2, {E^{\mathrm{T}}}_2} \quad \textsc{check\_exp\_assign}$$

$\boxed{E \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I, E^{\mathrm{T}}}$ Check the left hand side of an assignment

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \triangleright \mathit{id}, \langle\{\,\}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wlocl}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_var}$$

$$\frac{\mathit{id} \not\in \mathbf{dom}\,(E^{\mathrm{T}})}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, \{\mathit{id} \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnew}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wreg}\mathrm{C}\textsc{ast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wlocl}\mathrm{C}\textsc{ast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright t \\ E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u \\ E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_var}\mathrm{C}\textsc{ast}$$

$$\frac{\begin{array}{l} \mathit{id} \not\in \mathbf{dom}\,(E^{\mathrm{T}}) \\ E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, I_\epsilon, \{\mathit{id} \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnew}\mathrm{C}\textsc{ast}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, t_1 \to t\,\{\, \overline{\mathit{base\_effect}_i}^{\,i}\,, \mathbf{wmem},\ \overline{\mathit{base\_effect}'_j}^{\,j}\ \} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{exp} : u_1 \triangleright \mathit{exp}', I, {E^{\mathrm{T}}}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{id}(\mathit{exp}) : t \triangleright \mathit{id}(\mathit{exp}'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wmem}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wmem}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle \mathit{ne}\rangle \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_1, E^{\mathrm{T}} \\ E \vdash \mathit{lexp} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\,\mathbf{inc}\,t\rangle \triangleright \mathit{lexp}', I_2, E^{\mathrm{T}} \end{array}}{E \vdash \mathit{lexp}[\mathit{exp}] : t \triangleright \mathit{lexp}'[\mathit{exp}'], I_1 \uplus I_2 \uplus \langle\{\mathit{ne}_1 \leq \mathit{ne}, \mathit{ne}_1 + \mathit{ne}_2 \geq \mathit{ne}\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitInc}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle \mathit{ne}\rangle \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_1, E^{\mathrm{T}} \\ E \vdash \mathit{lexp} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\,\mathbf{dec}\,t\rangle \triangleright \mathit{lexp}', I_2, E^{\mathrm{T}} \end{array}}{E \vdash \mathit{lexp}[\mathit{exp}] : t \triangleright \mathit{lexp}'[\mathit{exp}'], I_1 \uplus I_2 \uplus \langle\{\mathit{ne} \leq \mathit{ne}_1, \mathit{ne}_1 + (-\mathit{ne}_2) \leq \mathit{ne}\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitDec}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle \mathit{ne}_1\rangle \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle \mathit{ne}_2\rangle \vdash \mathit{exp}_2 : u_2 \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ E \vdash \mathit{lexp} : \mathbf{vector}\,\langle \mathit{ne}_3\ \mathit{ne}_4\,\mathbf{inc}\,t\rangle \triangleright \mathit{lexp}', I_3, E^{\mathrm{T}} \end{array}}{E \vdash \mathit{lexp}[\mathit{exp}_1 : \mathit{exp}_2] : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2 + (-\mathit{ne}_1)\,\mathbf{inc}\,t\rangle \triangleright \mathit{lexp}'[\mathit{exp}'_1 : \mathit{exp}'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{\mathit{ne}_3 \leq \mathit{ne}_1, \mathit{ne}_3 + \mathit{ne}_4 \leq \mathit{ne}_2 + (-\mathit{ne}_1)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wsliceInc}$$

$$\frac{\begin{array}{l} E, \mathbf{atom}\,\langle \mathit{ne}_1\rangle \vdash \mathit{exp}_1 : u_1 \triangleright \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle \mathit{ne}_2\rangle \vdash \mathit{exp}_2 : u_2 \triangleright \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ E \vdash \mathit{lexp} : \mathbf{vector}\,\langle \mathit{ne}_3\ \mathit{ne}_4\,\mathbf{inc}\,t\rangle \triangleright \mathit{lexp}', I_3, E^{\mathrm{T}} \end{array}}{E \vdash \mathit{lexp}[\mathit{exp}_1 : \mathit{exp}_2] : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2 + (-\mathit{ne}_1)\,\mathbf{inc}\,t\rangle \triangleright \mathit{lexp}'[\mathit{exp}'_1 : \mathit{exp}'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{\mathit{ne}_1 \leq \mathit{ne}_3, \mathit{ne}_3 + (-\mathit{ne}_4) \leq \mathit{ne}_1 + (-\mathit{ne}_2)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wsliceDec}$$

$$\frac{\begin{array}{l} E^{\mathrm{R}}(x\langle \mathit{t\_args}\rangle) \triangleright \overline{\mathit{id}_i : t_i}^{\,i}\ \mathit{id} : t\ \overline{\mathit{id}'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathit{lexp} : x\langle \mathit{t\_args}\rangle \triangleright \mathit{lexp}', I, E^{\mathrm{T}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathit{lexp}.\mathit{id} : t \triangleright \mathit{lexp}'.\mathit{id}, I, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wrecord}$$

$\boxed{E \vdash \mathit{letbind} \triangleright \mathit{letbind}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}, \mathit{effect}, E^{\mathrm{K}}}$ Build the environment for a let binding, collecting index constraints

$$\frac{\begin{array}{l} \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_2, \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp} : u' \triangleright \mathit{exp}', \langle {\Sigma^{\mathrm{N}}}_2, \mathit{effect}\rangle, {E^{\mathrm{T}}}_2 \\ \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u' \lessapprox u, {\Sigma^{\mathrm{N}}}_3 \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{let}\,\mathit{typschm}\,\mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\,\mathit{typschm}\,\mathit{pat}' = \mathit{exp}', {E^{\mathrm{T}}}_1, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2 \uplus {\Sigma^{\mathrm{N}}}_3, \mathit{effect}, {E^{\mathrm{K}}}_2} \quad \textsc{check\_letbind\_val\_annot}$$

$$\frac{\begin{array}{l} \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, u \vdash \mathit{exp} : u' \triangleright \mathit{exp}', \langle {\Sigma^{\mathrm{N}}}_2, \mathit{effect}\rangle, {E^{\mathrm{T}}}_2 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{let}\,\mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\,\mathit{pat}' = \mathit{exp}', {E^{\mathrm{T}}}_1, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2, \mathit{effect}, \{\,\}} \quad \textsc{check\_letbind\_val\_noannot}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{type\_def} \rhd E}$ Check a type definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \mathit{typschm} \rhd \langle \{\,\}, \langle \{\,\}, \{\mathit{id} \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_ABBREV}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash \mathit{typ}_1 \leadsto t_1 \quad .. \quad E^{\mathrm{D}} \vdash \mathit{typ}_n \leadsto t_n \\ E^{\mathrm{R}} \equiv \{\{\mathit{id}_1 : t_1, \,..\,, \mathit{id}_n : t_n\} \mapsto x\} \end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \ \mathbf{const\,struct}\,\{\mathit{typ}_1\ \mathit{id}_1;\ ..\,;\mathit{typ}_n\ \mathit{id}_n\ ;^?\} \rhd \langle \{\,\}, \langle \{x \mapsto K\_Typ\}, \{\,\}, E^{\mathrm{R}}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_RECORD}$$

$$\frac{\begin{array}{l} \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \langle E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \leadsto t_1 \quad .. \quad \langle E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \leadsto t_n \\ \{x'_1 \mapsto k_1, \,..\,, x'_m \mapsto k_m\} \equiv \uplus\overline{{E^{\mathrm{K}}}_i}^{\,i} \\ E^{\mathrm{R}}_1 \equiv \{\{\mathit{id}_1 : t_1, \,..\,, \mathit{id}_n : t_n\} \mapsto \{x'_1 \mapsto k_1, \,..\,, x'_m \mapsto k_m\}, \uplus\overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{None}, x\langle x'_1 \,..\, x'_m\rangle\} \\ {E^{\mathrm{K}}}'_1 \equiv \{x \mapsto K\_Lam(k_1 \,..\, k_m \to K\_Typ)\} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \ \mathbf{const\,struct\,forall}\,\overline{\mathit{quant\_item}_i}^{\,i}\,.\{\mathit{typ}_1\ \mathit{id}_1;\ ..\,;\mathit{typ}_n\ \mathit{id}_n\ ;^?\} \rhd \langle \{\,\}, \langle E^{\mathrm{K}\prime}, \{\,\}, E^{\mathrm{R}}_1, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_QUANT\_RECORD}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_1 \to x\ \mathbf{pure}, \,...\,, \mathit{id}_n \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_n \to x\ \mathbf{pure}\} \\ {E^{\mathrm{K}}}_1 \equiv \{x \mapsto K\_Typ\} \\ \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \leadsto t_1 \quad ... \quad \langle E^{\mathrm{K}} \uplus {E^{\mathrm{K}}}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \leadsto t_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \ \mathbf{const\,union}\,\{\mathit{typ}_1\ \mathit{id}_1;\ ...\,;\mathit{typ}_n\ \mathit{id}_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle {E^{\mathrm{K}}}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_UNION}$$

$$\frac{\begin{array}{l} \overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i} \\ \{x'_1 \mapsto k_1, \,...\,, x'_m \mapsto k_m\} \equiv \uplus\overline{{E^{\mathrm{K}}}_i}^{\,i} \\ E^{\mathrm{K}\prime} \equiv \{x \mapsto K\_Lam(k_1 \,...\, k_m \to K\_Typ)\} \uplus \overline{{E^{\mathrm{K}}}_i}^{\,i} \\ \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \leadsto t_1 \quad ... \quad \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \leadsto t_n \\ t \equiv x\langle x'_1 \,...\, x'_m\rangle \\ E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto E^{\mathrm{K}\prime}, \uplus\overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{Ctor}, t_1 \to t\ \mathbf{pure}, \,...\,, \mathit{id}_n \mapsto E^{\mathrm{K}\prime}, \uplus\overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}, \mathbf{Ctor}, t_n \to t\ \mathbf{pure}\} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \ \mathbf{const\,union\,forall}\,\overline{\mathit{quant\_item}_i}^{\,i}\,.\{\mathit{typ}_1\ \mathit{id}_1;\ ...\,;\mathit{typ}_n\ \mathit{id}_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_QUANT\_UNION}$$

$$\frac{\begin{array}{c} E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto x, \,...\,, \mathit{id}_n \mapsto x\} \\ E^{\mathrm{E}} \equiv \{x \mapsto \{\mathit{num}_1 \mapsto \mathit{id}_1 \,...\, \mathit{num}_n \mapsto \mathit{id}_n\}\} \end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \ \mathbf{enumerate}\,\{\mathit{id}_1;\ ...\,;\mathit{id}_n\ ;^?\} \rhd \langle E^{\mathrm{T}}, \langle \{\mathit{id} \mapsto K\_Typ\}, \{\,\}, \{\,\}, E^{\mathrm{E}}\rangle\rangle} \quad \text{CHECK\_TD\_ENUMERATE}$$

$\boxed{E \vdash \mathit{fundef} \rhd \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Check a function definition

$$
\frac{\begin{array}{l}
E^{\mathrm{T}}(\mathit{id}) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect} \\
\overline{E^{\mathrm{D}} \vdash \mathit{quant\_item}_i \leadsto E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i} \\
\Sigma^{\mathrm{N}\prime\prime} \equiv \uplus \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i} \\
E^{\mathrm{K}\prime} \equiv \overline{E^{\mathrm{K}}{}_i}^{\,i} \\
E^{\mathrm{D}}{}_1 \equiv \langle E^{\mathrm{K}\prime}, \{\,\}, \{\,\}, \{\,\} \rangle \uplus E^{\mathrm{D}} \\
E^{\mathrm{D}}{}_1 \vdash \mathit{typ} \leadsto u \\
E^{\mathrm{D}}{}_1 \vdash u \precapprox t, \Sigma^{\mathrm{N}}{}_2 \\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}{}_1 \rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j} \\
\overline{\langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_j), E^{\mathrm{D}}{}_1 \rangle, u \vdash \mathit{exp}_j : u' \triangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime\prime\prime}_j, \mathit{effect}'_j \rangle, E^{\mathrm{T}\prime}_j}^{\,j} \\
\Sigma^{\mathrm{N}\prime\prime\prime\prime\prime} \equiv \Sigma^{\mathrm{N}}{}_2 \uplus \overline{\Sigma^{\mathrm{N}\prime\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime}_j}^{\,j} \\
\mathit{effect} \equiv \uplus \overline{\mathit{effect}'_j}^{\,j} \\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime\prime})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}} \quad \textsc{check\_fd\_rec}
$$

$$
\frac{\begin{array}{l}
E^{\mathrm{T}}(\mathit{id}) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect} \\
E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u \\
E^{\mathrm{D}} \vdash u \precapprox t, \Sigma^{\mathrm{N}}{}_2 \\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}', E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j} \\
\overline{\langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_j), E^{\mathrm{D}} \rangle, u \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime\prime}_j, \mathit{effect}'_j \rangle, E^{\mathrm{T}\prime}_j}^{\,j} \\
\mathit{effect} \equiv \uplus \overline{\mathit{effect}'_j}^{\,j} \\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}{}_2 \uplus \Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}} \quad \textsc{check\_fd\_rec\_function}2
$$

$$\frac{\begin{array}{l}
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\
\Sigma^{\mathrm{N}\prime} \equiv \ \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{i}\\
E^{\mathrm{K}\prime} \equiv E^{\mathrm{K}} \uplus\ \overline{{E^{\mathrm{K}}}_i}^{i}\\
\langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \rightsquigarrow t\\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \vartriangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{j}\\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\
\overline{\langle (E^{\mathrm{T}\prime} \uplus {E^{\mathrm{T}}}_j), \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp}_j : u'_j \vartriangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j}\\
\mathit{effect} \equiv \ \uplus\, \overline{\mathit{effect}'_j}^{j}\\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus\ \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \vartriangleright \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\ \text{C}$$

$$\frac{\begin{array}{l}
E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t\\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \vartriangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime}_j}^{j}\\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \{\,\}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\
\overline{\langle (E^{\mathrm{T}\prime} \uplus {E^{\mathrm{T}}}_j), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_j : u'_j \vartriangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j}\\
\mathit{effect} \equiv \ \uplus\, \overline{\mathit{effect}'_j}^{j}\\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\,rec}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \vartriangleright \mathbf{function\,rec}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}\ \textsc{check\_fd\_rec\_function\_no\_spec}2$$

$$\frac{\begin{array}{l}
E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \textbf{Global}, t_1 \to t\ \mathit{effect} \\
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i} \\
\Sigma^{\mathrm{N}\prime\prime} \equiv \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{i} \\
E^{\mathrm{K}\prime\prime} \equiv \overline{{E^{\mathrm{K}}}_i}^{i} \\
\langle E^{\mathrm{K}\prime\prime} \uplus E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \leadsto u \\
\langle E^{\mathrm{K}\prime\prime} \uplus E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u \precapprox t, {\Sigma^{\mathrm{N}}}_2 \\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{j} \\
\overline{\langle (E^{\mathrm{T}} \setminus id \uplus {E^{\mathrm{T}}}_j), \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j} \\
\Sigma^{\mathrm{N}\prime\prime\prime\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{j} \\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j} \\
\Sigma^{\mathrm{N}} \equiv \textbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime})
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \textbf{function forall}\, \overline{\mathit{quant\_item}_i}^{i}\, .\, \mathit{typ}\, \textbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \textbf{function forall}\, \overline{\mathit{quant\_item}_i}^{i}\, .\, \mathit{typ}\, \textbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}} \quad \text{CHECK\_F}$$

$$\frac{\begin{array}{l}
E^{\mathrm{T}}(id) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \textbf{Global}, t_1 \to t\ \mathit{effect} \\
E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u \\
E^{\mathrm{D}} \vdash u \precapprox t, {\Sigma^{\mathrm{N}}}_2 \\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime}_j}^{j} \\
\overline{\langle (E^{\mathrm{T}} \setminus id \uplus {E^{\mathrm{T}}}_j), E^{\mathrm{D}}\rangle, u \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j} \\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j} \\
\Sigma^{\mathrm{N}} \equiv \textbf{resolve}\,({\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2 \uplus \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{j})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \textbf{function}\ \ \mathit{typ}\, \textbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \textbf{function}\ \ \mathit{typ}\, \textbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}} \quad \text{CHECK\_FD\_FUNCTION2}$$

$$\frac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\ \Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{{\Sigma^{\mathrm{N}}}_i}^{i}\\ E^{\mathrm{K}\prime\prime} \equiv E^{\mathrm{K}} \uplus \overline{{E^{\mathrm{K}}}_i}^{i}\\ \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \rightsquigarrow t\\ \overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{j}\\ E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\ \overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_j), \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}'_j, \langle \Sigma^{\mathrm{N}\prime\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j}\\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j}\\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{j})\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ forall}\, \overline{\mathit{quant\_item}_i}^{i} \,.\, \mathit{typ}\, \mathbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \mathbf{function\ forall}\, \overline{\mathit{quant\_item}_i}^{i} \,.\, \mathit{typ}\, \mathbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}$$

CHECK_

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t\\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime}_j}^{j}\\ E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \Sigma^{\mathrm{N}}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\ \overline{\langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_j), E^{\mathrm{D}}\rangle, t \vdash \mathit{exp}_j : u'_j \triangleright \mathit{exp}', \langle \Sigma^{\mathrm{N}\prime}_j, \mathit{effect}'_j\rangle, E^{\mathrm{T}\prime}_j}^{j}\\ \mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{j}\\ \Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{j})\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function}\ \ \mathit{typ}\, \mathbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{j} \triangleright \mathbf{function}\ \ \mathit{typ}\, \mathbf{effect}\, \mathit{effect}\, \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}}$$

CHECK_FD_FUNCTION_NO_SPEC2

$\boxed{E \vdash \mathit{val\_spec} \triangleright E^{\mathrm{T}}}$ Check a value specification

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val}\, \mathit{typschm}\ \mathit{id} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}, \mathbf{Global}, t\}}$$

CHECK_SPEC_VAL_SPEC

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \rightsquigarrow t, {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val\,extern}\, \mathit{typschm}\ \mathit{id} = \mathit{string} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, t\}}$$

CHECK_SPEC_EXTERN

$\boxed{E^{\mathrm{D}} \vdash \mathit{default\_spec} \triangleright E^{\mathrm{T}}, {E^{\mathrm{K}}}_1}$ Check a default typing specification

$$\frac{E^{\mathrm{K}} \vdash \mathit{base\_kind} \rightsquigarrow k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{default}\, \mathit{base\_kind}\ \text{'}x \triangleright \{\,\}, \{\text{'}x \mapsto k\ \mathbf{default}\}}$$

CHECK_DEFAULT_KIND

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{default}\; \mathit{typschm}\; \mathit{id} \rhd \{\mathit{id} \mapsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}, \mathbf{Default}, t\}, \{\,\}} \quad \textsc{check\_default\_typ}$$

$\boxed{E \vdash \mathit{def} \rhd \mathit{def}', E'}$ Check a definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{type\_def} \rhd E}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathit{type\_def} \rhd \mathit{type\_def}, \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \uplus E} \quad \textsc{check\_def\_tdef}$$

$$\frac{E \vdash \mathit{fundef} \rhd \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}{E \vdash \mathit{fundef} \rhd \mathit{fundef}', E \uplus \langle E^{\mathrm{T}}, \epsilon \rangle} \quad \textsc{check\_def\_fdef}$$

$$\frac{\begin{array}{l} E \vdash \mathit{letbind} \rhd \mathit{letbind}', \{\mathit{id}_1 \mapsto t_1, \,..\,, \mathit{id}_n \mapsto t_n\}, \Sigma^{\mathrm{N}}, \mathbf{pure}, E^{\mathrm{K}} \\ \Sigma^{\mathrm{N}}{}_1 \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}) \end{array}}{E \vdash \mathit{letbind} \rhd \mathit{letbind}', E \uplus \langle \{\mathit{id}_1 \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_1, \,..\,, \mathit{id}_n \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_n\}, \epsilon \rangle} \quad \textsc{check\_def\_vdef}$$

$$\frac{E \vdash \mathit{val\_spec} \rhd E^{\mathrm{T}}}{E \vdash \mathit{val\_spec} \rhd \mathit{val\_spec}, E \uplus \langle E^{\mathrm{T}}, \epsilon \rangle} \quad \textsc{check\_def\_vspec}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{default\_spec} \rhd E^{\mathrm{T}}{}_1, E^{\mathrm{K}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathit{default\_spec} \rhd \mathit{default\_spec}, \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}{}_1, \{\,\}, \{\,\}, \{\,\} \rangle \rangle} \quad \textsc{check\_def\_default}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle \vdash \mathbf{register}\; \mathit{typ}\; \mathit{id} \rhd \mathbf{register}\; \mathit{typ}\; \mathit{id}, \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{register}\, \langle t \rangle\}), E^{\mathrm{D}} \rangle} \quad \textsc{check\_def\_register}$$

$\boxed{E \vdash \mathit{defs} \rhd \mathit{defs}', E'}$ Check definitions, potentially given default environment of built-in library

$$\frac{\begin{array}{l} E \vdash \mathit{def} \rhd \mathit{def}', E_1 \\ E \uplus E_1 \vdash \overline{\mathit{def}_i}^{\,i} \rhd \overline{\mathit{def}'_i}^{\,i}, E_2 \end{array}}{E \vdash \mathit{def}\; \overline{\mathit{def}_i}^{\,i} \rhd \mathit{def}'\; \overline{\mathit{def}'_i}^{\,i}, E_2} \quad \textsc{check\_defs\_defs}$$

4 Sail operational semantics {TODO}