Sail Manual

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1 Introduction

This is a manual describing the Sail specification language, its common library, compiler, interpreter and type system. However it is currently in early stages of being written, so questions to the developers are highly encouraged.

2 Tips for Writing Sail specifications

This section attempts to offer advice for writing Sail specifications that will work well with the Sail executable interpreter and compilers.

These tips use ideomatic Sail code; the Sail syntax is formally defined in following section.

Some tips might also be advice for good ways to specify instructions; this will come from a combination of users and Sail developers.

- Declare memory access functions as one read, one write announce, and one write value for each kind of access.

  For basic user-mode instructions, there should be the need for only one memory read and two memory write function. These should each be declared using `val extern` and should have effect `wmem` and `rmem` accordingly.

  Commonly, a memory read function will take as parameters a size (an number below 32) and an address and return a bit vector with length (8 * size). The sequential and concurrent interpreters both only read and write memory as a factor of bytes.

- Declare a default vector order

  Vectors can be either decreasing or increasing, i.e. if we have a vector a with elements [1,2,3] then in an increasing specification the 1 is accessed with `a[0]` but with `a[2]` in a decreasing system. Early in your specification, it is beneficial to clarity to say default Ord inc or default Ord dec.

- Vectors don't necessarily begin indexing at 0 or n-1

  Without any additional specification, a vector will begin indexing at 0 in an increasing spec and n-1 in a decreasing specification. A type declaration can reset this first position to any number.

  Importantly, taking a slice of a vector does not reset the indexes. So if `a = [1,2,3,4]` in an increasing system the slice `a[2 ..3]` generates the vector `[3,4]` and the 3 is indexed at 2 in either vector.

- Be precise in numeric types.

  While Sail includes very wide types like int and nat, consider that for bounds checking, numeric operations, and and clear understanding, these really are unbounded quantities. If you know that a number in the specification will range only between 0 and 32, 0 and 4, -32 to 32, it is better to use a specific range type such as [|32|].

  Similarly, if you don't know the range precisely, it may also be best to remain polymorphic and let Sail's type resolution work out bounds in a particular use rather than removing all bounds; to do this, use [:'n:] to say that it will polymorphically take some number.

- Use bit vectors for registers.

  Sail the language will readily allow a register to store a value of any type. However, the Sail executable interpreter expects that it is simulating a uni-processor machine where all registers are bit vectors.

  A vector of length one, such as a can read the element of a either with `a` or `a[0]`.

- Have functions named decode and execute to evaluate instructions.

  The sail interpreter is hard-wired to look for functions with these names.

- Type annotations are necessary to read the contents of a register into a local variable.

  The code `x := GPR[4]`, where GPR is a vector of general purpose registers, will store a local reference to the fourth general purpose register, not the contents of that registe, i.e. this will not read the register. To read the register contents into a local variable, the type is required explicitly so `(bit[64]) x := GPR[4]` reads the register contents into x. The type annotation may be on either side of the assignment.

3 Sail syntax

l	::=			Source location
	|			
annot	::=			
id	::=			Identifier
	|	x		
	|	(**deinfix** x)		remove infix status
	|	**bool**	M	Built in type identifiers
	|	**bit**	M	
	|	**unit**	M	
	|	**nat**	M	
	|	**string**	M	
	|	**range**	M	
	|	**atom**	M	

	$\vert$	**vector**	M	
	$\vert$	**list**	M	
	$\vert$	**set**	M	
	$\vert$	**reg**	M	
	$\vert$	*to_num*	M	Built in function identifiers
	$\vert$	*to_vec*	M	
	$\vert$	**msb**	M	
kid	::=			variables with kind, ticked to differntiate from program variables
	$\vert$	$'x$		
base_kind	::=			base kind
	$\vert$	**Type**		kind of types
	$\vert$	**Nat**		kind of natural number size expressions
	$\vert$	**Order**		kind of vector order specifications
	$\vert$	**Effect**		kind of effect sets
kind	::=			kinds
	$\vert$	$base\_kind_1 \to \ldots \to base\_kind_n$		
nexp	::=			expression of kind Nat, for vector sizes and origins
	$\vert$	*id*		identifier, bound by def Nat x = nexp
	$\vert$	*kid*		variable
	$\vert$	*num*		constant
	$\vert$	$nexp_1 * nexp_2$		product
	$\vert$	$nexp_1 + nexp_2$		sum
	$\vert$	$nexp_1 - nexp_2$		subtraction
	$\vert$	$2 * * nexp$		exponential
	$\vert$	**neg** *nexp*		For internal use
	$\vert$	$(nexp)$	S	

order	::=			vector order specifications, of kind Order
	|	*kid*		variable
	|	**inc**		increasing (little-endian)
	|	**dec**		decreasing (big-endian)
	|	(*order*)	S	
base_effect	::=			effect
	|	**rreg**		read register
	|	**wreg**		write register
	|	**rmem**		read memory
	|	**wmem**		write memory
	|	**wmea**		signal effective address for writing memory
	|	**wmv**		write memory, sending only value
	|	**barr**		memory barrier
	|	**depend**		dynamic footprint
	|	**undef**		undefined-instruction exception
	|	**unspec**		unspecified values
	|	**nondet**		nondeterminism from intra-instruction parallelism
	|	**escape**		Tracking of expressions and functions that might call exit
	|	**lset**		Local mutation happend; not user-writable
	|	**lret**		Local return happened; not user-writable
effect	::=			effect set, of kind Effects
	|	*kid*		
	|	$\{base\_effect_1, .., base\_effect_n\}$		effect set
	|	**pure**	M	sugar for empty effect set
	|	$effect_1 \uplus .. \uplus effect_n$	M	meta operation for combining sets of effects
typ	::=			Type expressions, of kind **Type**
	|	_		Unspecified type

	$\mid$	id		Defined type
	$\mid$	kid		Type variable
	$\mid$	$typ_1 \rightarrow typ_2$ **effect** $effect$		Function type (first-order only in user code)
	$\mid$	$(typ_1, \ldots., typ_n)$		Tuple type
	$\mid$	$id\langle typ\_arg_1, .., typ\_arg_n \rangle$		type constructor application
	$\mid$	(typ)	S	
	$\mid$	$[\vert nexp \vert]$	S	sugar for range<0, nexp>
	$\mid$	$[\vert nexp : nexp' \vert]$	S	sugar for range< nexp, nexp'>
	$\mid$	$[: nexp :]$	S	sugar for atom<nexp> which is special case of range<nexp,nexp>
	$\mid$	$typ[nexp]$	S	sugar for vector indexed by [$nexp$]
	$\mid$	$typ[nexp : nexp']$	S	sugar for vector indexed by [$nexp..nexp'$]
	$\mid$	$typ[nexp <: nexp']$	S	sugar for increasing vector indexed as above
	$\mid$	$typ[nexp :> nexp']$	S	sugar for decreasing vector indexed as above
typ_arg	::=			Type constructor arguments of all kinds
	$\mid$	$nexp$		
	$\mid$	typ		
	$\mid$	$order$		
	$\mid$	$effect$		
$n\_constraint$	::=			constraint over kind **Nat**
	$\mid$	$nexp = nexp'$		
	$\mid$	$nexp \geq nexp'$		
	$\mid$	$nexp \leq nexp'$		
	$\mid$	kid **IN** $\{num_1, \ldots, num_n\}$		
$kinded\_id$	::=			optionally kind-annotated identifier
	$\mid$	kid		identifier
	$\mid$	$kind\ kid$		kind-annotated variable

quant_item	::=		Either a kinded identifier or a nexp constraint for a typquant
	|	*kinded_id*	An optionally kinded identifier
	|	*n_constraint*	A constraint for this type
typquant	::=		type quantifiers and constraints
	|	**forall** $quant\_item_1, \ldots, quant\_item_n$.	
	|		sugar, omitting quantifier and constraints
typschm	::=		type scheme
	|	*typquant typ*	
name_scm_opt	::=		Optional variable-naming-scheme specification for variables of defined type
	|		
	|	[**name** = *regexp*]	
type_def	::=		Type definition body
	|	**typedef** *id name_scm_opt* = *typschm*	type abbreviation
	|	**typedef** *id name_scm_opt* = **const struct** $typquant\{typ_1\ id_1; \ldots; typ_n\ id_n\ ;^?\}$	struct type definition
	|	**typedef** *id name_scm_opt* = **const union** $typquant\{type\_union_1; \ldots; type\_union_n\ ;^?\}$	union type definition
	|	**typedef** *id name_scm_opt* = **enumerate** $\{id_1; \ldots; id_n\ ;^?\}$	enumeration type definition
	|	**typedef** *id* = **register bits** $[nexp : nexp']\{index\_range_1 : id_1; \ldots; index\_range_n : id_n\}$	register mutable bitfield type definition
type_union	::=		Type union constructors
	|	*id*	

	$\vert$	*typ id*	
index_range	::=		index specification, for bitfields in register types
	$\vert$	*num*	single index
	$\vert$	$num_1..num_2$	index range
	$\vert$	$index\_range_1, index\_range_2$	concatenation of index ranges
lit	::=		Literal constant
	$\vert$	()	() : **unit**
	$\vert$	**bitzero**	**bitzero** : **bit**
	$\vert$	**bitone**	**bitone** : **bit**
	$\vert$	**true**	**true** : **bool**
	$\vert$	**false**	**false** : **bool**
	$\vert$	*num*	natural number constant
	$\vert$	*hex*	bit vector constant, C-style
	$\vert$	*bin*	bit vector constant, C-style
	$\vert$	**undefined**	constant representing undefined values
	$\vert$	*string*	string constant
$;^?$	::=		Optional semi-colon
	$\vert$		
	$\vert$	;	
pat	::=		Pattern
	$\vert$	*lit*	literal constant pattern
	$\vert$	_	wildcard
	$\vert$	$(pat$ **as** $id)$	named pattern
	$\vert$	$(typ)pat$	typed pattern
	$\vert$	*id*	identifier
	$\vert$	$id(pat_1, .., pat_n)$	union constructor pattern

	|	$\{fpat_1 ; \ldots ; fpat_n \; ;^?\}$		struct pattern
	|	$[pat_1 , \ldots , pat_n]$		vector pattern
	|	$[num_1 = pat_1 , \ldots , num_n = pat_n]$		vector pattern (with explicit indices)
	|	$pat_1 : \ldots : pat_n$		concatenated vector pattern
	|	$(pat_1 , \ldots , pat_n)$		tuple pattern
	|	$[\|\|pat_1 , \ldots , pat_n\|\|]$		list pattern
	|	(pat)	S	
fpat	::=			Field pattern
	|	$id = pat$		
exp	::=			Expression
	|	$\{exp_1 ; \ldots ; exp_n\}$		block
	|	**nondet** $\{exp_1 ; \ldots ; exp_n\}$		nondeterminisitic block, expressions evaluate in an unspecified order, or concurrently
	|	id		identifier
	|	lit		literal constant
	|	$(typ)exp$		cast
	|	$id(exp_1 , \ldots , exp_n)$		function application
	|	$id\ exp$	S	No extra parens needed when exp is a tuple
	|	$exp_1\ id\ exp_2$		infix function application
	|	$(exp_1 , \ldots , exp_n)$		tuple
	|	**if** exp_1 **then** exp_2 **else** exp_3		conditional
	|	**if** exp_1 **then** exp_2	S	
	|	**foreach** $(id$ **from** exp_1 **to** exp_2 **by** exp_3 **in** $order)exp_4$		loop
	|	**foreach** $(id$ **from** exp_1 **to** exp_2 **by** $exp_3)exp_4$	S	
	|	**foreach** $(id$ **from** exp_1 **to** $exp_2)exp_3$	S	
	|	**foreach** $(id$ **from** exp_1 **downto** exp_2 **by** $exp_3)exp_4$	S	
	|	**foreach** $(id$ **from** exp_1 **downto** $exp_2)exp_3$	S	
	|	$[exp_1 , \ldots , exp_n]$		vector (indexed from 0)
	|	$[num_1 = exp_1 , \ldots , num_n = exp_n\ opt\_default]$		vector (indexed consecutively)

	$\mid$	$exp[exp']$	vector access
	$\mid$	$exp[exp_1..exp_2]$	subvector extraction
	$\mid$	$[exp\ \mathbf{with}\ exp_1 = exp_2]$	vector functional update
	$\mid$	$[exp\ \mathbf{with}\ exp_1 : exp_2 = exp_3]$	vector subrange update (with vector)
	$\mid$	$exp : exp_2$	vector concatenation
	$\mid$	$[\|\|exp_1, .., exp_n\|\|]$	list
	$\mid$	$exp_1 :: exp_2$	cons
	$\mid$	$\{fexps\}$	struct
	$\mid$	$\{exp\ \mathbf{with}\ fexps\}$	functional update of struct
	$\mid$	$exp.id$	field projection from struct
	$\mid$	$\mathbf{switch}\ exp\{\ \mathbf{case}\ pexp_1 \ldots \mathbf{case}\ pexp_n\}$	pattern matching
	$\mid$	$letbind\ \mathbf{in}\ exp$	let expression
	$\mid$	$lexp := exp$	imperative assignment
	$\mid$	$\mathbf{sizeof}\ nexp$	Expression to return the value of the nexp variable or expression at run time
	$\mid$	$\mathbf{exit}\ exp$	expression to halt all current execution, potentially calling a system, trap, or interrupt handler with exp
	$\mid$	$\mathbf{return}\ exp$	expression to end current function execution and return the value of exp from the function; this can be used
	$\mid$	$\mathbf{assert}\,(exp, exp')$	expression to halt with error, when the first expression is false, reporting the optional string as an error
	$\mid$	(exp)	S
	$\mid$	$(annot)\,exp$	This is an internal cast, generated during type checking that will resolve into a syntactic cast after
	$\mid$	$annot$	This is an internal use for passing nexp information to library functions, postponed for constraint solving
	$\mid$	$\mathbf{sizeof}\ annot$	For sizeof during type checking, to replace nexp with internal n
	$\mid$	$annot, annot'$	This is like the above but the user has specified an implicit parameter for the current function
	$\mid$	$\mathbf{comment}\ string$	For generated unstructured comments
	$\mid$	$\mathbf{comment}\ exp$	For generated structured comments
	$\mid$	$\mathbf{let}\ lexp = exp\ \mathbf{in}\ exp'$	This is an internal node for compilation that demonstrates the scope of a local mutable variable
	$\mid$	$\mathbf{let}\ pat = exp\ \mathbf{in}\ exp'$	This is an internal node, used to distinguised some introduced lets during processing from original ones
	$\mid$	$return\_int(exp)$	For internal use to embed into monad definition
$lexp$	$::=$		lvalue expression
	$\mid$	id	identifier

	|	$id(exp_1, .., exp_n)$		memory write via function call
	|	$id\ exp$	S	
	|	$(typ)id$		
	|	$(lexp_0, .., lexp_n)$		set multiple at a time, a check will ensure it's not memory
	|	$lexp[exp]$		vector element
	|	$lexp[exp_1..exp_2]$		subvector
	|	$lexp.id$		struct field
fexp	::=			Field-expression
	|	$id = exp$		
fexps	::=			Field-expression list
	|	$fexp_1; \ldots; fexp_n\ ;^?$		
opt_default	::=			Optional default value for indexed vectors, to define a defualt value for any unspecified positions in a sparse map
	|			
	|	; **default** $= exp$		
pexp	::=			Pattern match
	|	$pat \rightarrow exp$		
tannot_opt	::=			Optional type annotation for functions
	|	$typquant\ typ$		
rec_opt	::=			Optional recursive annotation for functions
	|			non-recursive
	|	**rec**		recursive
effect_opt	::=			Optional effect annotation for functions
	|			sugar for empty effect set

	|	**effect** *effect*	
funcl	::=		Function clause
	|	*id pat* = *exp*	
fundef	::=		Function definition
	|	**function** *rec_opt tannot_opt effect_opt* $funcl_1$ **and** ... **and** $funcl_n$	
letbind	::=		Let binding
	|	**let** *typschm pat* = *exp*	value binding, explicit type (*pat* must be total)
	|	**let** *pat* = *exp*	value binding, implicit type (*pat* must be total)
val_spec	::=		Value type specification
	|	**val** *typschm id*	
	|	**val extern** *typschm id*	
	|	**val extern** *typschm id* = *string*	Specify the type and id of a function from Lem, where the string must provide an explicit path to the requ
default_spec	::=		Default kinding or typing assumption
	|	**default** *base_kind kid*	
	|	**default Order** *order*	
	|	**default** *typschm id*	
scattered_def	::=		Function and type union definitions that can be spread across a file. Each one must end in **id**
	|	**scattered function** *rec_opt tannot_opt effect_opt id*	scattered function definition header
	|	**function clause** *funcl*	scattered function definition clause
	|	**scattered typedef** *id name_scm_opt* = **const union** *typquant*	scattered union definition header
	|	**union** *id* **member** *type_union*	scattered union definition member

	$\mid$	**end** *id*	scattered definition end
reg_id	::=		
	$\mid$	*id*	
alias_spec	::=		Register alias expression forms. Other than where noted, each id must refer to an unaliased register of type ve
	$\mid$	$reg\_id.id$	
	$\mid$	$reg\_id[exp]$	
	$\mid$	$reg\_id[exp..exp']$	
	$\mid$	$reg\_id : reg\_id'$	
dec_spec	::=		Register declarations
	$\mid$	**register** *typ id*	
	$\mid$	**register alias** $id = alias\_spec$	
	$\mid$	**register alias** $typ\ id = alias\_spec$	
def	::=		Top-level definition
	$\mid$	*kind_def*	definition of named kind identifiers
	$\mid$	*type_def*	type definition
	$\mid$	*fundef*	function definition
	$\mid$	*letbind*	value definition
	$\mid$	*val_spec*	top-level type constraint
	$\mid$	*default_spec*	default kind and type assumptions
	$\mid$	*scattered_def*	scattered function and type definition
	$\mid$	*dec_spec*	register declaration
	$\mid$	*dec_comm*	generated comments
defs	::=		Definition sequence
	$\mid$	$def_1\ ..\ def_n$	

4 Sail primitive types and functions

built_in_types	::=		Type Kind
	|	**bit** : **Typ**	
	|	**unit** : **Typ**	
	|	**forall Nat** $'n$. **Nat** $'m$. **range** $<'\,n,'\,m>$: **Nat** → **Nat** → **Typ**	
	|	**forall Nat** $'n$. **atom** $<'\,n>$: **Nat** → **Typ**	singleton number, instead of range$<'\,n,'\,$r
	|	**forall Nat** $'n$, **Nat** $'m$, **Order** $'o$, **Typ** $'t$. **vector** $\langle 'n,'\,m,'\,o,'\,t\rangle$: **Nat** → **Nat** → **Order** → **Typ**	
	|	**forall Typ** $'a$. **option** $\langle 'a\rangle$: **Typ** → **Typ**	
	|	**forall Typ** $'t$. **register** $\langle 't\rangle$: **Typ** → **Typ**	
	|	**forall Typ** $'t$. **reg** $\langle 't\rangle$: **Typ** → **Typ**	internal reference cell
	|	**forall Nat** $'n$. **implicit** $<'\,n>$: **Nat** → **Typ**	To add to a function val specification ind

built_in_type_abbreviations	::=		
	|	**bool** ⇒ **bit**	
	|	**nat** ⇒ $[\|0..pos\_infinity\|]$	
	|	**int** ⇒ $[\|neg\_infinity..pos\_infinity\|]$	
	|	$uint8 \Rightarrow [\|0..2**8\|]$	
	|	$uint16 \Rightarrow [\|0..2**16\|]$	
	|	$uint32 \Rightarrow [\|0..2**32\|]$	
	|	$uint64 \Rightarrow [\|0..2**64\|]$	

functions	::=		Built-in functions: all have effect pure, all order polymorphic
	|	**val forall Typ** $'a.'a$ → **unit** : **ignore**	
	|	**val forall Typ** $'a.'a$ → **option** $\langle 'a\rangle$: **Some**	
	|	**val forall Typ** $'a$. **unit** → **option** $\langle 'a\rangle$: **None**	
	|	**val** $([:'\,n:],[:'\,m:]) \to [\|'n+'\,m\|] : +$	arithmetic addition
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) → **bit** $['n]$: +	unsigned vector addition
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) → (**bit** $['n]$, **bit** , **bit**) : +	unsigned vector addition with overflow, carry out
	|	**val forall Nat** $'n$.(**bit** $['n]$, **bit** $['n]$) → **bit** $['n]$: $+\_s$	signed vector addition
	|	**val forall Nat** $'n$.($bit['n]$, **bit** $['n]$) → (**bit** $['n]$, **bit** , **bit**) : $+\_s$	signed vector addition with overflow, carry out

| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow [|'n -' o..'m -' p|] : -$ — arithmetic subtraction
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\,['n] : -$ — unsigned vector subtraction
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow (\,\mathbf{bit}\,['n],\,\mathbf{bit}\,,\,\mathbf{bit}\,) : -$ — unsigned vector subtraction with overflow, carry out
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\,['n] : -\_s$ — signed vector subtraction
| $\mathbf{val\,forall\,Nat}\,'n.(bit['n],\,\mathbf{bit}\,['n]) \rightarrow (\,\mathbf{bit}\,['n],\,\mathbf{bit}\,,\,\mathbf{bit}\,) : -\_s$ — signed vector subtraction with overflow, carry out
| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow [|'n *' o..'m *' p|] : *$ — arithmetic multiplication
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\,[2 *' n] : *$ — unsigned vector multiplication
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\,[2 *' n] : *\_s$ — signed vector multiplication
| $\mathbf{val}\,([|'n..'m|],[|1..'p|]) \rightarrow [|0..'p-1|] : \,\mathbf{mod}$ — arithmetic modulo
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\,['n] : \,\mathbf{mod}$ — unsigned vector modulo
| $\mathbf{val}\,([|'n..'m|],[|1..'p|]) \rightarrow [|'q..'r|] : \,\mathbf{quot}$ — arithmetic integer division
| $\mathbf{val\,forall\,Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \rightarrow \,\mathbf{bit}\,['n] : \,\mathbf{quot}$ — unsigned vector division
| $\mathbf{val\,forall\,Nat}\,'n,\,\mathbf{Nat}\,'m.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['m]) \rightarrow \,\mathbf{bit}\,['n] : quot\_s$ — signed vector division
| $\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Nat}\,'n.('a['n] \rightarrow [:' n :]) : \,\mathbf{length}$
| $\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Nat}\,'n,\,\mathbf{Nat}\,'m,' n \leq' m.(\,\mathbf{implicit}\,\langle 'm\rangle,' a['n]) \rightarrow' a['m] : \,\mathbf{mask}$ — reduce size of vector, dropping MSBits. Type system supplies implicit p
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :\equiv$ — vector equality
| $\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Typ}\,'b.('a,' b) \rightarrow \,\mathbf{bit}\, :\equiv$
| $\mathbf{val\,forall\,Typ}\,'a,\,\mathbf{Typ}\,'b.('a,' b) \rightarrow \,\mathbf{bit}\, :! =$
| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow \,\mathbf{bit}\, : \langle$
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, : \langle$ — unsigned less than
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :< \_s$
| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow \,\mathbf{bit}\, :\rangle$
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :\rangle$ — unsigned greater than
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :> \_s$
| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow \,\mathbf{bit}\, :\leq$
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :\leq$ — unsigned less than or eq
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :<= \_s$
| $\mathbf{val}\,([|'n..'m|],[|'o..'p|]) \rightarrow \,\mathbf{bit}\, :\geq$
| $\mathbf{val\,forall\,Nat}\,'n.(\,\mathbf{bit}\,['n],\,\mathbf{bit}\,['n]) \rightarrow \,\mathbf{bit}\, :\geq$ — unsigned greater than or eq

	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to \textbf{bit}\ :>=\_s$	
	$\mid$ $\textbf{val}\,\textbf{bit}\ \to\ \textbf{bit}\ :$	bit negation
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.\,\textbf{bit}\,['n] \to\ \textbf{bit}\,['n] :$	bitwise negation
	$\mid$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to\ \textbf{bit}\ : \mid$	bitwise or
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : \mid$	
	$\mid$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to\ \textbf{bit}\ : \&$	bitwise and
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : \&$	
	$\mid$ $\textbf{val}\,(\,\textbf{bit}\,,\,\textbf{bit}\,) \to\ \textbf{bit}\ : \uparrow$	bitwise xor
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,['n],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : \uparrow$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(\,\textbf{bit}\,,[\lvert 'n\rvert]) \to\ \textbf{bit}\,['n] : \uparrow\uparrow$	duplicate bit into a vector
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[\lvert 'm\rvert]) \to\ \textbf{bit}\,['n] :<<$	left shift
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[\lvert 'm\rvert]) \to\ \textbf{bit}\,['n] :>>$	right shift
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'m,'\,m \leq'\,n.(\,\textbf{bit}\,['n],[\lvert 'm\rvert]) \to\ \textbf{bit}\,['n] :<<<$	rotate
functions_with_coercions	$::=$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n]) \to [\lvert 2 **'\,n\rvert] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to\ \textbf{bit}\,['n] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\lvert 'o..'p\rvert],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to [\lvert 'o..'p + 2**'n\rvert] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,) \to\ \textbf{bit}\,['n] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : +$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n.(bit['n],\,\textbf{bit}\,['n]) \to [\lvert 2 **'\,n\rvert] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to\ \textbf{bit}\,['n] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\lvert 'o..'p\rvert],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to [\lvert 'o..'p + 2**'n\rvert] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,['n],\,\textbf{bit}\,) \to\ \textbf{bit}\,['n] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n(\,\textbf{bit}\,,\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : +\_s$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to\ \textbf{bit}\,['n] : -$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.([\lvert 'o..'p\rvert],\,\textbf{bit}\,['n]) \to\ \textbf{bit}\,['n] : -$	
	$\mid$ $\textbf{val}\,\textbf{forall}\,\textbf{Nat}\,'n,\ \textbf{Nat}\,'o,\ \textbf{Nat}\,'p.(\,\textbf{bit}\,['n],[\lvert 'o..'p\rvert]) \to [\lvert 'o..'p + 2**'n\rvert] : -$	

5 Sail type system

5.1 Internal type syntax

k	::=			Internal kinds
	|	K_Typ		
	|	K_Nat		
	|	K_Ord		
	|	K_Efct		
	|	$K\_Lam(k_0 \mathinner{..} k_n \rightarrow k')$		
	|	K_infer		Representing an unknown kind, inferred by context
$t,\ u$	::=			Internal types
	|	x		
	|	$'x$		
	|	$t_1 \rightarrow t_2$ *effect*		
	|	$(t_1,\ \ldots.,\ t_n)$		
	|	$x\langle t\_args\rangle$		
	|	$t \mapsto t_1$		
	|	**register** $\langle t\_arg\rangle$	S	
	|	**range** $\langle ne\ ne'\rangle$	S	
	|	**atom** $\langle ne\rangle$	S	
	|	**vector** $\langle ne\ ne'\ order\ t\rangle$	S	
	|	**list** $\langle t\rangle$	S	
	|	**reg** $\langle t\rangle$	S	
	|	**implicit** $\langle ne\rangle$	S	
	|	**bit**	S	
	|	*string*	S	
	|	**unit**	S	
	|	$t[t\_arg_1/tid_1 \mathinner{..} t\_arg_n/tid_n]$	M	

optx	::=		
	|	x	
	|		
tag	::=		Data indicating where the identifier arises and thus information necessary in compilation
	|	**None**	
	|	**Intro**	Denotes an assignment and lexp that introduces a binding
	|	**Set**	Denotes an expression that mutates a local variable
	|	**Tuple**	Denotes an assignment with a tuple lexp
	|	**Global**	Globally let-bound or enumeration based value/variable
	|	**Ctor**	Data constructor from a type union
	|	**Extern** *optx*	External function, specied only with a val statement
	|	**Default**	Type has come from default declaration, identifier may not be bound locally
	|	**Spec**	
	|	**Enum** *num*	
	|	**Alias**	
	|	$Unknown\_path\, optx$	Tag to distinguish an unknown path from a non-analysis non deterministic path
ne	::=		internal numeric expressions
	|	x	
	|	'x	
	|	*num*	
	|	**infinity**	
	|	$ne_1 * ne_2$	
	|	$ne_1 + \ldots + ne_n$	
	|	$ne_1 - ne_2$	
	|	$2 ** ne$	
	|	$(-ne)$	
	|	**zero** S	
	|	**one** S	

	|	**bitlength** (bin)	M	
	|	**bitlength** (hex)	M	
	|	**count** $(num_0 \ldots num_i)$	M	
	|	**length** $(pat_1 \ldots pat_n)$	M	
	|	**length** $(exp_1 \ldots exp_n)$	M	
t_arg	::=			Argument to type constructors
	|	t		
	|	ne		
	|	$effect$		
	|	$order$		
	|	**fresh**	M	
t_args	::=			Arguments to type constructors
	|	$t\_arg_1 \ldots t\_arg_n$		
nec	::=			Numeric expression constraints
	|	$ne \leq ne'$		
	|	$ne = ne'$		
	|	$ne \geq ne'$		
	|	$'x$ **IN** $\{num_1, \ldots, num_n\}$		
	|	$nec_0 \,..\, nec_n \rightarrow nec'_0 \ldots nec'_m$		
	|	$nec_0 \ldots nec_n$		
Σ^{N}	::=			nexp constraint lists
	|	$\{nec_1, \,..\, , nec_n\}$		
	|	$\Sigma^{\mathrm{N}}{}_1 \uplus \,..\, \uplus \Sigma^{\mathrm{N}}{}_n$	M	
	|	**consistent_increase** $ne_1\, ne'_1 \ldots ne_n\, ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always larger than the su
	|	**consistent_decrease** $ne_1\, ne'_1 \ldots ne_n\, ne'_n$	M	Generates constraints from pairs of constraints, where the first of each pair is always smaller than the
	|	**resolve** (Σ^{N})		

E^{D}	::=			Environments storing top level information, such as defined abbreviations, records, enumerations, and kinds
	|	$\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle$		
	|	ϵ		
	|	$E^{\mathrm{D}} \uplus E^{\mathrm{D}\prime}$		
kinf	::=			Whether a kind is default or from a local binding
	|	k		
	|	k **default**		
tid	::=			A type identifier or type variable
	|	id		
	|	kid		
E^{K}	::=			Kind environments
	|	$\{ tid_1 \mapsto kinf_1, .., tid_n \mapsto kinf_n \}$		
	|	$E^{\mathrm{K}}{}_1 \uplus .. \uplus E^{\mathrm{K}}{}_n$	M	In a unioning kinf, k default u k results in k (i.e. the default is locally forgotten)
	|	$E^{\mathrm{K}} \setminus E^{\mathrm{K}}{}_1 .. E^{\mathrm{K}}{}_n$	M	
tinf	::=			Type variables, type, and constraints, bound to an identifier
	|	t		
	|	$E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t$		
E^{A}	::=			
	|	$\{ tid_1 \mapsto tinf_1, .., tid_n \mapsto tinf_n \}$		
	|	$E^{\mathrm{A}}_1 \uplus .. \uplus E^{\mathrm{A}}_n$		
field_typs	::=			Record fields
	|	$id_1 : t_1, .., id_n : t_n$		
E^{R}	::=			Record environments

	$\mid$	$\{\{\mathit{field\_typs}_1\} \mapsto \mathit{tinf}_1, \,..\,, \{\mathit{field\_typs}_n\} \mapsto \mathit{tinf}_n\}$		
	$\mid$	$E^{\mathrm{R}}_1 \uplus \,..\, \uplus E^{\mathrm{R}}_n$	M	
$\mathit{enumerate\_map}$	$::=$			
	$\mid$	$\{\mathit{num}_1 \mapsto \mathit{id}_1 \ldots \mathit{num}_n \mapsto \mathit{id}_n\}$		
E^{E}	$::=$			Enumeration environments
	$\mid$	$\{t_1 \mapsto \mathit{enumerate\_map}_1, \,..\,, t_n \mapsto \mathit{enumerate\_map}_n\}$		
	$\mid$	$E^{\mathrm{E}}_1 \uplus \,..\, \uplus E^{\mathrm{E}}_n$		
E^{T}	$::=$			Type environments
	$\mid$	$\{\mathit{id}_1 \mapsto \mathit{tinf}_1, \,..\,, \mathit{id}_n \mapsto \mathit{tinf}_n\}$		
	$\mid$	$\{\mathit{id} \mapsto \textbf{overload}\ \mathit{tinf}\ \mathit{conformsto} : \mathit{tinf}_1, \ldots, \mathit{tinf}_n\}$		
	$\mid$	$({E^{\mathrm{T}}}_1 \uplus \ldots. \uplus {E^{\mathrm{T}}}_n)$	M	
	$\mid$	$\uplus {E^{\mathrm{T}}}_1 \,..\, {E^{\mathrm{T}}}_n$	M	
	$\mid$	$E^{\mathrm{T}} \setminus \mathit{id}_1 \,..\, \mathit{id}_n$	M	
	$\mid$	$({E^{\mathrm{T}}}_1 \cap \ldots. \cap {E^{\mathrm{T}}}_n)$	M	
	$\mid$	$\cap {E^{\mathrm{T}}}_1 \,..\, {E^{\mathrm{T}}}_n$	M	
ts	$::=$			
	$\mid$	$t_1, \,..\,, t_n$		
E	$::=$			Definition environment and lexical environment
	$\mid$	$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle$		
	$\mid$	ϵ	M	
	$\mid$	$E \uplus E'$		
I	$::=$			Information given by type checking an expression
	$\mid$	$\langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle$		
	$\mid$	I_ϵ		Empty constraints, effect

	|	$I_1 \uplus I_2$	
	|	$I_1 \uplus .. \uplus I_n$	Unions the constraints and effect
formula	::=		
	|	*judgement*	
	|	$formula_1 \quad .. \quad formula_n$	
	|	$E^{\mathrm{K}}(tid) \triangleright kinf$	Kind lookup
	|	$E^{\mathrm{A}}(tid) \triangleright tinf$	
	|	$E^{\mathrm{A}}(tid) \triangleright ne$	
	|	$E^{\mathrm{T}}(id) \triangleright tinf$	Type lookup
	|	$E^{\mathrm{T}}(id) \triangleright \mathbf{overload}\, tinf : tinf_1 \ldots tinf_n$	Overloaded type lookup
	|	$E^{\mathrm{K}}(tid) < -\vert k$	Update the kind associated with id to k
	|	$E^{\mathrm{R}}(id_0 \,..\, id_n) \triangleright t, ts$	Record lookup
	|	$E^{\mathrm{R}}(t) \triangleright id_0 : t_0 \,..\, id_n : t_n$	Record looup by type
	|	$E^{\mathrm{E}}(t) \triangleright enumerate\_map$	Enumeration lookup by type
	|	$\mathbf{dom}\,(E^{\mathrm{T}}{}_1) \cap \mathbf{dom}\,(E^{\mathrm{T}}{}_2) = \emptyset$	
	|	$\mathbf{dom}\,(E^{\mathrm{K}}{}_1) \cap \mathbf{dom}\,(E^{\mathrm{K}}{}_2) = \emptyset$	
	|	$\mathbf{disjoint\,doms}\,(E^{\mathrm{T}}{}_1, \ldots., E^{\mathrm{T}}{}_n)$	Pairwise disjoint domains
	|	$id \notin \mathbf{dom}\,(E^{\mathrm{K}})$	
	|	$id \notin \mathbf{dom}\,(E^{\mathrm{T}})$	
	|	$id_0 : t_0 \,..\, id_n : t_n \subset id'_0 : t'_0 \,..\, id'_i : t'_i$	
	|	$num_1 < \ldots < num_n$	
	|	$num_1 > \ldots > num_n$	
	|	$exp_1 \equiv exp_2$	
	|	$E^{\mathrm{K}}{}_1 \equiv E^{\mathrm{K}}{}_2$	
	|	$E^{\mathrm{K}}{}_1 \approx E^{\mathrm{K}}{}_2$	
	|	$E^{\mathrm{T}}{}_1 \equiv E^{\mathrm{T}}{}_2$	
	|	$E^{\mathrm{R}}_1 \equiv E^{\mathrm{R}}_2$	
	|	$E^{\mathrm{E}}_1 \equiv E^{\mathrm{E}}_2$	
	|	$E^{\mathrm{D}}{}_1 \equiv E^{\mathrm{D}}{}_2$	

$$
\begin{array}{ll}
| & E_1 \equiv E_2 \\
| & \Sigma^{\mathrm{N}}{}_1 \equiv \Sigma^{\mathrm{N}}{}_2 \\
| & id \equiv' id \\
| & x_1 \neq x_2 \\
| & lit_1 \neq lit_2 \\
| & I_1 \equiv I_2 \\
| & effect_1 \equiv effect_2 \\
| & t_1 \equiv t_2 \\
| & ne \equiv ne' \\
| & kid \equiv fresh\_kid(E^{\mathrm{D}})
\end{array}
$$

5.2 Type relations

$\boxed{E^{\mathrm{K}} \vdash_t t\, \mathbf{ok}}$ Well-formed types

$$
\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Typ}{E^{\mathrm{K}} \vdash_t \text{'}x\, \mathbf{ok}} \quad \text{CHECK\_T\_VAR}
$$

$$
\frac{\begin{array}{l} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Typ \end{array}}{E^{\mathrm{K}} \vdash_t \text{'}x\, \mathbf{ok}} \quad \text{CHECK\_T\_VARINFER}
$$

$$
\frac{\begin{array}{l} E^{\mathrm{K}} \vdash_t t_1\, \mathbf{ok} \\ E^{\mathrm{K}} \vdash_t t_2\, \mathbf{ok} \\ E^{\mathrm{K}} \vdash_e effect\, \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_t t_1 \rightarrow t_2\ effect\, \mathbf{ok}} \quad \text{CHECK\_T\_FN}
$$

$$
\frac{E^{\mathrm{K}} \vdash_t t_1\, \mathbf{ok} \quad .... \quad E^{\mathrm{K}} \vdash_t t_n\, \mathbf{ok}}{E^{\mathrm{K}} \vdash_t (t_1, \ ...., t_n)\, \mathbf{ok}} \quad \text{CHECK\_T\_TUP}
$$

$$
\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \,..\, k_n \rightarrow K\_Typ) \\ E^{\mathrm{K}}, k_1 \vdash t\_arg_1\, \mathbf{ok} \quad .. \quad E^{\mathrm{K}}, k_n \vdash t\_arg_n\, \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_t x\langle t\_arg_1 \,..\, t\_arg_n\rangle\, \mathbf{ok}} \quad \text{CHECK\_T\_APP}
$$

$\boxed{E^{\mathrm{K}} \vdash_e effect\, \mathbf{ok}}$ Well-formed effects

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Efct}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_EF\_VAR}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Efct \end{array}}{E^{\mathrm{K}} \vdash_e \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_EF\_VARINFER}$$

$$\frac{}{E^{\mathrm{K}} \vdash_e \{\mathit{base\_effect}_1, .., \mathit{base\_effect}_n\}\ \mathbf{ok}} \quad \text{CHECK\_EF\_SET}$$

$\boxed{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n x\ \mathbf{ok}} \quad \text{CHECK\_N\_ID}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Nat}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_N\_VAR}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Nat \end{array}}{E^{\mathrm{K}} \vdash_n \text{'}x\ \mathbf{ok}} \quad \text{CHECK\_N\_VARINFER}$$

$$\frac{}{E^{\mathrm{K}} \vdash_n \mathit{num}\ \mathbf{ok}} \quad \text{CHECK\_N\_NUM}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_n ne_1 + ne_2\ \mathbf{ok}} \quad \text{CHECK\_N\_SUM}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_n ne_1 - ne_2\ \mathbf{ok}} \quad \text{CHECK\_N\_MINUS}$$

$$\frac{\begin{array}{c} E^{\mathrm{K}} \vdash_n ne_1\ \mathbf{ok} \\ E^{\mathrm{K}} \vdash_n ne_2\ \mathbf{ok} \end{array}}{E^{\mathrm{K}} \vdash_n ne_1 * ne_2\ \mathbf{ok}} \quad \text{CHECK\_N\_MULT}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{E^{\mathrm{K}} \vdash_n 2 ** ne\ \mathbf{ok}} \quad \text{CHECK\_N\_EXP}$$

$\boxed{E^{\mathrm{K}} \vdash_o \mathit{order}\ \mathbf{ok}}$ Well-formed numeric expressions

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Ord}{E^{\mathrm{K}} \vdash_o \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ord\_var}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(\text{'}x) \triangleright K\_infer \\ E^{\mathrm{K}}(\text{'}x) < -|K\_Ord \end{array}}{E^{\mathrm{K}} \vdash_o \text{'}x\ \mathbf{ok}} \quad \textsc{check\_ord\_varInfer}$$

$$\frac{}{E^{\mathrm{K}} \vdash_o \mathbf{inc}\ \mathbf{ok}} \quad \textsc{check\_ord\_inc}$$

$$\frac{}{E^{\mathrm{K}} \vdash_o \mathbf{dec}\ \mathbf{ok}} \quad \textsc{check\_ord\_dec}$$

$\boxed{E^{\mathrm{K}}, k \vdash t\_arg\ \mathbf{ok}}$ Well-formed type arguments kind check matching the application type variable

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Typ \vdash t\ \mathbf{ok}} \quad \textsc{check\_targs\_typ}$$

$$\frac{E^{\mathrm{K}} \vdash_e \mathit{effect}\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Efct \vdash \mathit{effect}\ \mathbf{ok}} \quad \textsc{check\_targs\_eff}$$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Nat \vdash ne\ \mathbf{ok}} \quad \textsc{check\_targs\_nat}$$

$$\frac{E^{\mathrm{K}} \vdash_o \mathit{order}\ \mathbf{ok}}{E^{\mathrm{K}}, K\_Ord \vdash \mathit{order}\ \mathbf{ok}} \quad \textsc{check\_targs\_ord}$$

$\boxed{E^{\mathrm{K}} \vdash \mathit{kind} \leadsto k}$

$$\frac{}{E^{\mathrm{K}} \vdash \mathbf{Type} \leadsto K\_Typ} \quad \textsc{convert\_kind\_Typ}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{quant\_item} \leadsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}$ Convert source quantifiers to kind environments and constraints

$$\frac{E^{\mathrm{K}} \vdash \mathit{kind} \leadsto k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash \mathit{kind}\ \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \textsc{convert\_quants\_kind}$$

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \text{'}x \leadsto \{\text{'}x \mapsto k\}, \{\,\}} \quad \text{CONVERT\_QUANTS\_NOKIND}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \leadsto ne_1\\ \vdash nexp_2 \leadsto ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 = nexp_2 \leadsto \{\,\}, \{ne_1 = ne_2\}} \quad \text{CONVERT\_QUANTS\_EQ}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \leadsto ne_1\\ \vdash nexp_2 \leadsto ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 \geq nexp_2 \leadsto \{\,\}, \{ne_1 \geq ne_2\}} \quad \text{CONVERT\_QUANTS\_GTEQ}$$

$$\frac{\begin{array}{c}\vdash nexp_1 \leadsto ne_1\\ \vdash nexp_2 \leadsto ne_2\end{array}}{E^{\mathrm{D}} \vdash nexp_1 \leq nexp_2 \leadsto \{\,\}, \{ne_1 \leq ne_2\}} \quad \text{CONVERT\_QUANTS\_LTEQ}$$

$$\frac{}{E^{\mathrm{D}} \vdash \text{'}x\,\mathbf{IN}\,\{num_1, \ldots, num_n\} \leadsto \{\,\}, \{\text{'}x\,\mathbf{IN}\,\{num_1, \ldots, num_n\}\}} \quad \text{CONVERT\_QUANTS\_IN}$$

$\boxed{E^{\mathrm{D}} \vdash typschm \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}$ Convert source types with typeschemes to internal types and kind environments

$$\frac{E^{\mathrm{D}} \vdash typ \leadsto t}{E^{\mathrm{D}} \vdash \; typ \leadsto t, \{\,\}, \{\,\}} \quad \text{CONVERT\_TYPSCHM\_NOQUANT}$$

$$\frac{\begin{array}{l}E^{\mathrm{D}} \vdash quant\_item_1 \leadsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad E^{\mathrm{D}} \vdash quant\_item_n \leadsto E^{\mathrm{K}}{}_n, \Sigma^{\mathrm{N}}{}_n\\ E^{\mathrm{K}} \equiv E^{\mathrm{K}}{}_1 \uplus \ldots \uplus E^{\mathrm{K}}{}_n\\ E^{\mathrm{D}} \uplus \langle E^{\mathrm{K}}, \{\,\}, \{\,\}, \{\,\}\rangle \vdash typ \leadsto t\end{array}}{E^{\mathrm{D}} \vdash \mathbf{forall}\, quant\_item_1, \ldots, quant\_item_n.\, typ \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n} \quad \text{CONVERT\_TYPSCHM\_QUANT}$$

$\boxed{E^{\mathrm{D}} \vdash typ \leadsto t}$ Convert source types to internal types

$$\frac{E^{\mathrm{K}}(\text{'}x) \triangleright K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \text{'}x \leadsto \text{'}x} \quad \text{CONVERT\_TYP\_VAR}$$

$$\frac{E^{\mathrm{K}}(x) \triangleright K\_Typ}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \leadsto x} \quad \text{CONVERT\_TYP\_ID}$$

$$\frac{\begin{array}{c}E^{\mathrm{D}} \vdash typ_1 \leadsto t_1\\ E^{\mathrm{D}} \vdash typ_2 \leadsto t_2\end{array}}{E^{\mathrm{D}} \vdash typ_1 \rightarrow typ_2\, \mathbf{effect}\, effect \leadsto t_1 \rightarrow t_2\, effect} \quad \text{CONVERT\_TYP\_FN}$$

$$\frac{E^{\mathrm{D}} \vdash typ_1 \leadsto t_1 \quad .... \quad E^{\mathrm{D}} \vdash typ_n \leadsto t_n}{E^{\mathrm{D}} \vdash (typ_1, \; .... \;, typ_n) \leadsto (t_1, \; .... \;, t_n)} \quad \textsc{convert\_typ\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \,..\, k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, k_1 \vdash typ\_arg_1 \leadsto t\_arg_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, k_n \vdash typ\_arg_n \leadsto t\_arg_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle \vdash x \langle typ\_arg_1, \,..\,, typ\_arg_n \rangle \leadsto x \langle t\_arg_1 \,..\, t\_arg_n \rangle} \quad \textsc{convert\_typ\_app}$$

$\boxed{E^{\mathrm{D}}, k \vdash typ\_arg \leadsto t\_arg}$ Convert source type arguments to internals

$$\frac{E^{\mathrm{D}} \vdash typ \leadsto t}{E^{\mathrm{D}}, K\_Typ \vdash typ \leadsto t} \quad \textsc{convert\_targ\_typ}$$

$\boxed{\vdash nexp \leadsto ne}$ Convert and normalize numeric expressions

$$\frac{}{\vdash x \leadsto x} \quad \textsc{convert\_nexp\_id}$$

$$\frac{}{\vdash \text{'}x \leadsto \text{'}x} \quad \textsc{convert\_nexp\_var}$$

$$\frac{}{\vdash num \leadsto num} \quad \textsc{convert\_nexp\_num}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2 \end{array}}{\vdash nexp_1 * nexp_2 \leadsto ne_1 * ne_2} \quad \textsc{convert\_nexp\_mult}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2 \end{array}}{\vdash nexp_1 + nexp_2 \leadsto ne_1 + ne_2} \quad \textsc{convert\_nexp\_add}$$

$$\frac{\begin{array}{c} \vdash nexp_1 \leadsto ne_1 \\ \vdash nexp_2 \leadsto ne_2 \end{array}}{\vdash nexp_1 - nexp_2 \leadsto ne_1 - ne_2} \quad \textsc{convert\_nexp\_sub}$$

$$\frac{\vdash nexp \leadsto ne}{\vdash 2 * *nexp \leadsto 2 ** ne} \quad \textsc{convert\_nexp\_exp}$$

$\boxed{E^{\mathrm{D}} \vdash_n ne \approx ne'}$

$$\frac{E^{\mathrm{K}} \vdash_n ne\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_n ne \approx ne} \quad \text{CONFORMS\_TO\_NE\_REFL}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash_n ne_1 \approx ne_2 \\ E^{\mathrm{D}} \vdash_n ne_2 \approx ne_3 \end{array}}{E^{\mathrm{D}} \vdash_n ne_1 \approx ne_3} \quad \text{CONFORMS\_TO\_NE\_TRANS}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx ne_1}{E^{\mathrm{D}} \vdash_n ne_1 \approx ne_2} \quad \text{CONFORMS\_TO\_NE\_ASSOC}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright ne \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_n ne \approx ne' \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_n x \approx ne'} \quad \text{CONFORMS\_TO\_NE\_ABBREV}$$

$$\frac{num \equiv num'}{E^{\mathrm{D}} \vdash_n num \approx num'} \quad \text{CONFORMS\_TO\_NE\_CONSTANTS}$$

$$\frac{}{E^{\mathrm{D}} \vdash_n ne \approx ne'} \quad \text{CONFORMS\_TO\_NE\_REST}$$

$\boxed{E^{\mathrm{D}} \vdash t \approx t'}$ Relate t and t' when t can be used where t' is expected without consideration for non-constant nats

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx t} \quad \text{CONFORMS\_TO\_REFL}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}} \vdash t_1 \approx t_2 \\ E^{\mathrm{D}} \vdash t_2 \approx t_3 \end{array}}{E^{\mathrm{D}} \vdash t_1 \approx t_3} \quad \text{CONFORMS\_TO\_TRANS}$$

$$\frac{}{E^{\mathrm{D}} \vdash {}'x \approx t} \quad \text{CONFORMS\_TO\_VAR}$$

$$\frac{}{E^{\mathrm{D}} \vdash t \approx {}'x} \quad \text{CONFORMS\_TO\_VAR2}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u \approx t \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x \approx t} \quad \text{CONFORMS\_TO\_ABBREV}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx u \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash t \approx x} \quad \textsc{conforms\_to\_abbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash t_n \approx u_n}{E^{\mathrm{D}} \vdash (t_1,\ ....\ , t_n) \approx (u_1,\ ....\ , u_n)} \quad \textsc{conforms\_to\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \mathinner{..} k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash t\_arg_1 \approx t\_arg'_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash t\_arg_n \approx t\_arg'_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_n\rangle} \quad \textsc{conforms\_to\_app}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{atom}\,\langle ne\rangle \approx \mathbf{range}\,\langle ne_1\ ne_2\rangle} \quad \textsc{conforms\_to\_atom}$$

$$\frac{}{E^{\mathrm{D}} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \approx \mathbf{atom}\,\langle ne\rangle} \quad \textsc{conforms\_to\_atom2}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx u[t\_arg'_1/tid_1 \mathinner{..} t\_arg'_m/tid_m] \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x'\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_appAbbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \mathinner{..}, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash u[t\_arg_1/tid_1 \mathinner{..} t\_arg_n/tid_n] \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash x'\langle t\_arg_1 \mathinner{..} t\_arg_n\rangle \approx x\langle t\_arg'_1 \mathinner{..} t\_arg'_m\rangle} \quad \textsc{conforms\_to\_appAbbrev2}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{register}\,\langle t\rangle \approx u} \quad \textsc{conforms\_to\_register}$$

$$\frac{E^{\mathrm{D}} \vdash t \approx u}{E^{\mathrm{D}} \vdash \mathbf{reg}\,\langle t\rangle \approx u} \quad \textsc{conforms\_to\_reg}$$

$\boxed{E^{\mathrm{D}}, k \vdash t\_arg \approx t\_arg'}$

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash t \approx t'} \quad \textsc{targconforms\_typ}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne \approx ne'}{E^{\mathrm{D}}, K\_Nat \vdash ne \approx ne'} \quad \textsc{targconforms\_nexp}$$

$\boxed{E^{\mathrm{D}} \vdash_c t \approx t'}$ Relate t and t' when t can be used where t' is expected upto applying coercions to t

$$\frac{E^{\mathrm{D}} \vdash t \approx t'}{E^{\mathrm{D}} \vdash_c t \approx t'} \quad \textsc{conforms\_to\_upto\_coerce\_base}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx \mathbf{one}}{E^{\mathrm{D}} \vdash_c \mathbf{bit} \approx \mathbf{vector}\,\langle ne\; ne_2\; order\; \mathbf{bit}\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_bitToVec}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne_2 \approx \mathbf{one}}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\; ne_2\; order\; \mathbf{bit}\rangle \approx \mathbf{bit}} \quad \textsc{conforms\_to\_upto\_coerce\_vecToBit}$$

$$\frac{}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\; ne_2\; order\; \mathbf{bit}\rangle \approx \mathbf{atom}\,\langle ne_3\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_vecToAtom}$$

$$\frac{}{E^{\mathrm{D}} \vdash_c \mathbf{vector}\,\langle ne\; ne_2\; order\; \mathbf{bit}\rangle \approx \mathbf{range}\,\langle ne_3\; ne_4\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_vecToRange}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x \approx \mathbf{range}\,\langle ne_1\; ne_2\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_enumToRange}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c \mathbf{range}\,\langle ne_1\; ne_2\rangle \approx x} \quad \textsc{conforms\_to\_upto\_coerce\_rangeToEnum}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x \approx \mathbf{atom}\,\langle ne\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_enumToAtom}$$

$$\frac{E^{\mathrm{E}}(x) \triangleright enumerate\_map}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c \mathbf{atom}\,\langle ne\rangle \approx x} \quad \textsc{conforms\_to\_upto\_coerce\_atomToEnum}$$

$$\frac{E^{\mathrm{D}} \vdash_c t_1 \approx u_1 \quad .... \quad E^{\mathrm{D}} \vdash_c t_n \approx u_n}{E^{\mathrm{D}} \vdash_c (t_1, \,....\,, t_n) \approx (u_1, \,....\,, u_n)} \quad \textsc{conforms\_to\_upto\_coerce\_tup}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \triangleright K\_Lam(k_1 \,..\, k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_1 \vdash_c t\_arg_1 \approx t\_arg'_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, k_n \vdash_c t\_arg_n \approx t\_arg'_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1 \,..\, t\_arg_n\rangle \approx x\langle t\_arg'_1 \,..\, t\_arg'_n\rangle} \quad \textsc{conforms\_to\_upto\_coerce\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \,..\,, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1 \,..\, t\_arg_n\rangle \approx u[t\_arg'_1/tid_1 \,..\, t\_arg'_m/tid_m] \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x\langle t\_arg_1 \,..\, t\_arg_n\rangle \approx x'\langle t\_arg'_1 \,..\, t\_arg'_m\rangle} \quad \text{CONFORMS\_TO\_UPTO\_COERCE\_APPABBREV}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \triangleright \{tid_1 \mapsto kinf_1, \,..\,, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c u[t\_arg_1/tid_1 \,..\, t\_arg_n/tid_n] \approx x\langle t\_arg'_1 \,..\, t\_arg'_m\rangle \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash_c x'\langle t\_arg_1 \,..\, t\_arg_n\rangle \approx x\langle t\_arg'_1 \,..\, t\_arg'_m\rangle} \quad \text{CONFORMS\_TO\_UPTO\_COERCE\_APPABBREV2}$$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx u}{E^{\mathrm{D}} \vdash_c \mathbf{register}\,\langle t\rangle \approx u} \quad \text{CONFORMS\_TO\_UPTO\_COERCE\_REGISTER}$$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx u}{E^{\mathrm{D}} \vdash_c \mathbf{reg}\,\langle t\rangle \approx u} \quad \text{CONFORMS\_TO\_UPTO\_COERCE\_REG}$$

$\boxed{E^{\mathrm{D}}, k \vdash_c t\_arg \approx t\_arg'}$

$$\frac{E^{\mathrm{D}} \vdash_c t \approx t'}{E^{\mathrm{D}}, K\_Typ \vdash_c t \approx t'} \quad \text{TARGCONFORMS\_COERCE\_TYP}$$

$$\frac{E^{\mathrm{D}} \vdash_n ne \approx ne'}{E^{\mathrm{D}}, K\_Nat \vdash_c ne \approx ne'} \quad \text{TARGCONFORMS\_COERCE\_NEXP}$$

$\boxed{\sigma_{conformsto(t,t')}(tinflist) \triangleright tinflist'}$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ E^{\mathrm{D}} \vdash t'_j \approx t_j \\ \sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \,..\, tinf_m\ tinf'_0 \,..\, tinf'_n) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{full}(t_i,t_j)}(tinf_0 \,..\, tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j\ effect\ tinf'_0 \,..\, tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t'_j} \quad \text{SO\_FULL}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash t_i \approx t'_i \\ \sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \,..\, tinf_m) \triangleright \epsilon \end{array}}{\sigma_{\mathbf{parm}(t_i,t_j)}(tinf_0 \,..\, tinf_m\ E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t\ effect\ tinf'_0 \,..\, tinf'_n) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, tag, t'_i \to t} \quad \text{SO\_PARM}$$

$\boxed{E^{\mathrm{D}}, widening \vdash t \precapprox t' : t'', \Sigma^{\mathrm{N}}}$ t is consistent with t' if they match if t can be used where t' is needed after the constraints are solved, with no coercions needed. t" is t

$$\frac{E^{\mathrm{K}} \vdash_t t\ \mathbf{ok}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \textit{widening} \vdash t \precapprox t : t, \{\,\}} \quad \text{CONSISTENT\_TYP\_REFL}$$

$$\frac{\begin{array}{c} E^{\mathrm{D}}, \textit{widening} \vdash t_1 \precapprox t_3 : t_4, {\Sigma^{\mathrm{N}}}_1 \\ E^{\mathrm{D}}, \textit{widening} \vdash t_4 \precapprox t_2 : t_5, {\Sigma^{\mathrm{N}}}_2 \end{array}}{E^{\mathrm{D}}, \textit{widening} \vdash t_1 \precapprox t_2 : t_5, {\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2} \quad \text{CONSISTENT\_TYP\_TRANS}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \textit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \textit{widening} \vdash u \precapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \textit{widening} \vdash x \precapprox t : t', \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \text{CONSISTENT\_TYP\_ABBREV}$$

$$\frac{\begin{array}{l} E^{\mathrm{A}}(x) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \textit{tag}, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \textit{widening} \vdash t \precapprox u : t', {\Sigma^{\mathrm{N}}}' \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, \textit{widening} \vdash t \precapprox x : t', \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_1} \quad \text{CONSISTENT\_TYP\_ABBREV2}$$

$$\frac{}{E^{\mathrm{D}}, \textit{widening} \vdash {}'x \precapprox t : t, \{\,\}} \quad \text{CONSISTENT\_TYP\_VAR}$$

$$\frac{}{E^{\mathrm{D}}, \textit{widening} \vdash t \precapprox {}'x : t, \{\,\}} \quad \text{CONSISTENT\_TYP\_VAR2}$$

$$\frac{E^{\mathrm{D}}, \textit{widening} \vdash t_1 \precapprox u_1 : u'_1, {\Sigma^{\mathrm{N}}}_1 \quad .... \quad E^{\mathrm{D}}, \textit{widening} \vdash t_n \precapprox u_n : u'_n, {\Sigma^{\mathrm{N}}}_n}{E^{\mathrm{D}}, \textit{widening} \vdash (t_1, \, ..., t_n) \precapprox (u_1, \, ..., u_n) : (u'_1, \, ..., u'_n), {\Sigma^{\mathrm{N}}}_1 \uplus \; .... \; \uplus {\Sigma^{\mathrm{N}}}_n} \quad \text{CONSISTENT\_TYP\_TUP}$$

$$\frac{}{E^{\mathrm{D}}, \textit{widening} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \precapprox \mathbf{range}\,\langle ne_3\ ne_4\rangle : \mathbf{range}\,\langle ne_3\ ne_4\rangle, \{ne_3 \leq ne_1, ne_2 \leq ne_4\}} \quad \text{CONSISTENT\_TYP\_RANGE}$$

$$\frac{}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle ne\rangle \precapprox \mathbf{range}\,\langle ne_1\ ne_2\rangle : \mathbf{atom}\,\langle ne\rangle, \{ne_1 \leq ne, ne \leq ne_2\}} \quad \text{CONSISTENT\_TYP\_ATOMRANGE}$$

$$\frac{}{E^{\mathrm{D}}, (\mathbf{none}, \_) \vdash \mathbf{atom}\,\langle ne_1\rangle \precapprox \mathbf{atom}\,\langle ne_2\rangle : \mathbf{atom}\,\langle ne_2\rangle, \{ne_1 = ne_2\}} \quad \text{CONSISTENT\_TYP\_ATOM}$$

$$\frac{\textit{num}_1 \ < \ \textit{num}_2}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle \textit{num}_1\rangle \precapprox \mathbf{atom}\,\langle \textit{num}_2\rangle : \mathbf{range}\,\langle \textit{num}_1\ \textit{num}_2\rangle, \{\,\}} \quad \text{CONSISTENT\_TYP\_ATOMWIDENCONST}$$

$$\frac{\textit{num}_2 \ < \ \textit{num}_1}{E^{\mathrm{D}}, (\mathbf{nums}, \_) \vdash \mathbf{atom}\,\langle \textit{num}_1\rangle \precapprox \mathbf{atom}\,\langle \textit{num}_2\rangle : \mathbf{range}\,\langle \textit{num}_2\ \textit{num}_1\rangle, \{\,\}} \quad \text{CONSISTENT\_TYP\_ATOMWIDENCONST2}$$

$$\frac{}{E^{\mathrm{D}}, \textit{widening} \vdash \mathbf{range}\,\langle ne_1\ ne_2\rangle \precapprox \mathbf{atom}\,\langle {}'x\rangle : \mathbf{atom}\,\langle {}'x\rangle, \{ne_1 \leq {}'x, {}'x \leq ne_2\}} \quad \text{CONSISTENT\_TYP\_RANGEATOM}$$

$$\frac{E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash t \precapprox t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, (\_, \mathbf{none}) \vdash \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \precapprox \mathbf{vector}\,\langle ne_3\; ne_4\; order\; t'\rangle : \mathbf{vector}\,\langle ne_3\; ne_4\; order\; t''\rangle, \{ne_2 = ne_4, ne_1 = ne_3\} \uplus \Sigma^{\mathrm{N}}} \quad \textsc{consistent\_typ\_vector}$$

$$\frac{E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash t \precapprox t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, (\_, \mathbf{vectors}) \vdash \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \precapprox \mathbf{vector}\,\langle ne_3\; ne_4\; order\; t'\rangle : \mathbf{vector}\,\langle ne_3\; ne_4\; order\; t''\rangle, \{ne_2 = ne_4\} \uplus \Sigma^{\mathrm{N}}} \quad \textsc{consistent\_typ\_vectorWiden}$$

$$\frac{\begin{array}{l} E^{\mathrm{K}}(x) \rhd K\_Lam(k_1 \mathrel{..} k_n \to K\_Typ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, k_1 \vdash t\_arg_1 \precapprox t\_arg'_1, {\Sigma^{\mathrm{N}}}_1 \quad .. \quad \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening, k_n \vdash t\_arg_n \precapprox t\_arg'_n, {\Sigma^{\mathrm{N}}}_n \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle \precapprox x\langle t\_arg'_1 \mathrel{..} t\_arg'_n\rangle : x\langle t\_arg'_1 \mathrel{..} t\_arg'_n\rangle, {\Sigma^{\mathrm{N}}}_1 \uplus \mathrel{..} \uplus {\Sigma^{\mathrm{N}}}_n} \quad \textsc{consistent\_typ\_app}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \rhd \{tid_1 \mapsto kinf_1, \mathrel{..}, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle \precapprox u[t\_arg'_1/tid_1 \mathrel{..} t\_arg'_m/tid_m] : t, {\Sigma^{\mathrm{N}}}_2 \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle \precapprox x'\langle t\_arg'_1 \mathrel{..} t\_arg'_m\rangle : x'\langle t\_arg'_1 \mathrel{..} t\_arg'_m\rangle, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_2} \quad \textsc{consistent\_typ\_appAbbrev}$$

$$\frac{\begin{array}{l} x' \neq x \\ E^{\mathrm{A}}(x') \rhd \{tid_1 \mapsto kinf_1, \mathrel{..}, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, tag, u \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u[t\_arg'_1/tid_1 \mathrel{..} t\_arg'_m/tid_m] \precapprox x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle : t, {\Sigma^{\mathrm{N}}}_2 \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash x'\langle t\_arg'_1 \mathrel{..} t\_arg'_m\rangle \precapprox x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle : x\langle t\_arg_1 \mathrel{..} t\_arg_n\rangle, \Sigma^{\mathrm{N}} \uplus {\Sigma^{\mathrm{N}}}_2} \quad \textsc{consistent\_typ\_appAbbrev2}$$

$\boxed{E^{\mathrm{D}}, widening, k \vdash t\_arg \precapprox t\_arg', \Sigma^{\mathrm{N}}}$

$$\frac{E^{\mathrm{D}}, widening \vdash t \precapprox t' : t'', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, widening, K\_Typ \vdash t \precapprox t', \Sigma^{\mathrm{N}}} \quad \textsc{targ\_consistent\_typ}$$

$$\frac{}{E^{\mathrm{D}}, widening, K\_Nat \vdash ne \precapprox ne', \{ne = ne'\}} \quad \textsc{targ\_consistent\_nexp}$$

$\boxed{E^{\mathrm{D}}, widening, t' \vdash exp : t \rhd t'', exp', \Sigma^{\mathrm{N}}, effect}$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, widening, u_1 \vdash id_1 : t_1 \rhd u_1, exp_1, {\Sigma^{\mathrm{N}}}_1, effect_1 \quad .. \quad E^{\mathrm{D}}, widening, u_n \vdash id_n : t_n \rhd u_n, exp_n, {\Sigma^{\mathrm{N}}}_n, effect_n \\ exp' \equiv \mathbf{switch}\; exp\{\,\mathbf{case}\,(id_1, \mathrel{..}, id_n) \to (exp_1, \mathrel{..}, exp_n)\} \end{array}}{E^{\mathrm{D}}, widening, (u_1, \mathrel{..}, u_n) \vdash exp : (t_1, \mathrel{..}, t_n) \rhd (u_1, \mathrel{..}, u_n), exp', {\Sigma^{\mathrm{N}}}_1 \uplus \mathrel{..} \uplus {\Sigma^{\mathrm{N}}}_n, \mathbf{pure}} \quad \textsc{coerce\_typ\_tuple}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u \precapprox t : t', \Sigma^{\mathrm{N}} \\ exp' \equiv (annot)exp \end{array}}{E^{\mathrm{D}}, widening, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash exp : \mathbf{vector}\,\langle ne_3\; ne_4\; order\; u\rangle \rhd \mathbf{vector}\,\langle ne_3\; ne_4\; order\; t'\rangle, exp', \Sigma^{\mathrm{N}} \uplus \{ne_2 = ne_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_vectorUpdateStart}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, (\mathbf{none}, \mathbf{none}) \vdash u \lessapprox \mathbf{bit} : \mathbf{bit}, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathit{to\_num}\ \mathit{exp} \end{array}}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp} : \mathbf{vector}\ \langle \mathit{ne}_3\ \mathit{ne}_4\ \mathit{order}\ u \rangle \triangleright \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \mathit{exp}', \Sigma^{\mathrm{N}} \uplus \{\mathit{ne}_1 = \mathbf{zero}, \mathit{ne}_2 \geq 2 ** \mathit{ne}_4\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toNum}$$

$$\frac{\mathit{exp}' \equiv \mathit{to\_vec}\ \mathit{exp}}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{vector}\ \langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{exp} : \mathbf{range}\ \langle \mathit{ne}_3\ \mathit{ne}_4 \rangle \triangleright \mathbf{vector}\ \langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle, \mathit{exp}', \{\mathit{ne}_3 = \mathbf{zero}, \mathit{ne}_4 \leq 2 ** \mathit{ne}_2\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t \\ \mathit{exp}' \equiv (\mathit{typ})\mathit{exp} \\ E^{\mathrm{D}}, \mathit{widening}, u \vdash \mathit{exp}' : t \triangleright t', \mathit{exp}'', \Sigma^{\mathrm{N}}, \mathbf{pure} \end{array}}{E^{\mathrm{D}}, \mathit{widening}, u \vdash \mathit{exp} : \mathbf{register}\ \langle t \rangle \triangleright t', \mathit{exp}'', \Sigma^{\mathrm{N}}, \{\mathbf{rreg}\}} \quad \textsc{coerce\_typ\_readReg}$$

$$\frac{\mathit{exp}' \equiv \mathbf{msb}\ (\mathit{exp})}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{bit} \vdash \mathit{exp} : \mathbf{vector}\ \langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \triangleright \mathbf{bit}, \mathit{exp}', \{\mathit{ne}_1 = \mathbf{one}\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_accessVecBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, \mathit{widening} \vdash \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one} \rangle \lessapprox \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle : t, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \mathbf{case}\ \mathbf{bitzero} \to \mathit{numZero}\ \mathbf{case}\ \mathbf{bitone} \to \mathit{numOne}\} \end{array}}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp} : \mathbf{bit} \triangleright \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_bitToNum}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, \mathit{widening} \vdash \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \lessapprox \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one} \rangle : t, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \mathbf{case}\ \mathit{numZero} \to \mathbf{bitzero}\ \mathbf{case}\ \mathit{numOne} \to \mathbf{bitone}\} \end{array}}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{bit} \vdash \mathit{exp} : \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \triangleright \mathbf{bit}, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_numToBit}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash \mathbf{atom}\ \langle \mathit{ne} \rangle \lessapprox \mathbf{range}\ \langle \mathbf{zero}\ \mathbf{one} \rangle : t, \Sigma^{\mathrm{N}} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \mathbf{case}\ \mathit{numZero} \to \mathbf{bitzero}\ \mathbf{case}\ \mathit{numOne} \to \mathbf{bitone}\} \end{array}}{E^{\mathrm{D}}, \mathit{widening}, \mathbf{bit} \vdash \mathit{exp} : \mathbf{atom}\ \langle \mathit{ne} \rangle \triangleright \mathbf{bit}, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_numToBitAtom}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\ \overline{\mathit{num}_i \mapsto \mathit{id}_i}^{\,i}\ \} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \overline{\mathbf{case}\ \mathit{num}_i \to \mathit{id}_i}^{\,i}\ \} \\ \mathit{ne}_3 \equiv \mathbf{count}\ (\ \overline{\mathit{num}_i}^{\,i}\ ) \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, \mathit{widening}, x \vdash \mathit{exp} : \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \triangleright x, \mathit{exp}', \{\mathit{ne}_1 \leq \mathbf{zero}, \mathit{ne}_2 \leq \mathit{ne}_3\}, \mathbf{pure}} \quad \textsc{coerce\_typ\_toEnumerate}$$

$$\frac{\begin{array}{l} E^{\mathrm{E}}(x) \triangleright \{\ \overline{\mathit{num}_i \mapsto \mathit{id}_i}^{\,i}\ \} \\ \mathit{exp}' \equiv \mathbf{switch}\ \mathit{exp}\{\ \overline{\mathbf{case}\ \mathit{id}_i \to \mathit{num}_i}^{\,i}\ \} \\ \mathit{ne}_3 \equiv \mathbf{count}\ (\ \overline{\mathit{num}_i}^{\,i}\ ) \\ \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, (\mathbf{nums}, \mathbf{none}) \vdash \mathbf{range}\ \langle \mathbf{zero}\ \mathit{ne}_3 \rangle \lessapprox \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle : t, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}} \rangle, \mathit{widening}, \mathbf{range}\ \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle \vdash \mathit{exp} : x \triangleright \mathbf{range}\ \langle \mathbf{zero}\ \mathit{ne}_3 \rangle, \mathit{exp}', \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \textsc{coerce\_typ\_fromEnumerate}$$

$$\frac{E^{\mathrm{D}}, \mathit{widening} \vdash t \precapprox u : u', \Sigma^{\mathrm{N}}}{E^{\mathrm{D}}, \mathit{widening}, u \vdash \mathit{exp} : t \rhd u', \mathit{exp}, \Sigma^{\mathrm{N}}, \mathbf{pure}} \quad \text{COERCE\_TYP\_EQ}$$

$\boxed{\mathit{widening}, t \vdash \mathit{lit} : t' \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}$ Typing literal constants, coercing to expected type t

$$\frac{}{\mathit{widening}, \mathbf{range}\,\langle \mathit{ne}\ \mathit{ne}' \rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num} \rangle \Rightarrow \mathit{num}, \{\mathit{ne} \leq \mathit{num}, \mathit{num} \leq \mathit{ne}'\}} \quad \text{CHECK\_LIT\_NUM}$$

$$\frac{}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}\ \mathit{ne}'\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{num} : \mathbf{atom}\,\langle \mathit{num} \rangle \Rightarrow \mathit{to\_vec}\,\mathit{num}, \{\mathit{num} + \mathbf{one} \leq 2 ** \mathit{ne}'\}} \quad \text{CHECK\_LIT\_NUMToVEC}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathit{numZero} : \mathbf{atom}\,\langle \mathbf{zero} \rangle \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITZERO}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathit{numOne} : \mathbf{atom}\,\langle \mathbf{one} \rangle \Rightarrow \mathbf{bitone}, \{\,\}} \quad \text{CHECK\_LIT\_NUMBITONE}$$

$$\frac{}{\mathit{widening}, \mathit{string} \vdash \mathit{string} : \mathit{string} \Rightarrow \mathit{string}, \{\,\}} \quad \text{CHECK\_LIT\_STRING}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{hex})}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{hex} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\ \mathbf{bit} \rangle \Rightarrow \mathit{hex}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_HEX}$$

$$\frac{\mathit{ne} \equiv \mathbf{bitlength}\,(\mathit{bin})}{\mathit{widening}, \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}_2\ \mathit{order}\ \mathbf{bit} \rangle \vdash \mathit{bin} : \mathbf{vector}\,\langle \mathit{ne}_1\ \mathit{ne}\ \mathit{order}\ \mathbf{bit} \rangle \Rightarrow \mathit{bin}, \{\mathit{ne} = \mathit{ne}_2\}} \quad \text{CHECK\_LIT\_BIN}$$

$$\frac{}{\mathit{widening}, \mathbf{unit} \vdash () : \mathbf{unit} \Rightarrow \mathbf{unit}, \{\,\}} \quad \text{CHECK\_LIT\_UNIT}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathbf{bitzero} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITZERO}$$

$$\frac{}{\mathit{widening}, \mathbf{bit} \vdash \mathbf{bitone} : \mathbf{bit} \Rightarrow \mathbf{bitzero}, \{\,\}} \quad \text{CHECK\_LIT\_BITONE}$$

$$\frac{}{\mathit{widening}, t \vdash \mathbf{undefined} : t \Rightarrow \mathbf{undefined}, \{\,\}} \quad \text{CHECK\_LIT\_UNDEF}$$

$\boxed{E, t \vdash \mathit{pat} : t' \rhd \mathit{pat}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Typing patterns, building their binding environment

$$\frac{\begin{array}{l} \mathit{lit} \neq \mathbf{undefined} \\ (\mathbf{none}, \mathbf{none}), t \vdash \mathit{lit} : u \Rightarrow \mathit{lit}', \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u \precapprox t : t', \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash \mathit{lit} : t' \rhd \mathit{lit}', \{\,\}, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \text{CHECK\_PAT\_LIT}$$

$$\frac{}{E, t \vdash \_ : t \triangleright \_, \{\,\}, \{\,\}} \quad \textsc{check\_pat\_wild}$$

$$\frac{\begin{array}{l} E, t \vdash pat : u \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ id \notin \mathbf{dom}\,(E^{\mathrm{T}}{}_1) \end{array}}{E, t \vdash (pat\,\mathbf{as}\,id) : u \triangleright (pat'\,\mathbf{as}\,id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_as}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, \mathbf{Default}, t' \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t' \vdash pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \\ E^{\mathrm{D}}, (\mathbf{none}, \mathbf{none}) \vdash t' \precapprox u : u', \Sigma^{\mathrm{N}\prime} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash (pat\,\mathbf{as}\,id) : t \triangleright (pat'\,\mathbf{as}\,id), (E^{\mathrm{T}}{}_1 \uplus \{id \mapsto t'\}), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \textsc{check\_pat\_asDefault}$$

$$\frac{\begin{array}{l} E^{\mathrm{D}} \vdash typ \rightsquigarrow t \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash (typ) pat : t \triangleright pat', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_typ}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \,..\,, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, (u'_1, \,..\,, u'_n) \to x\langle t\_arg_1 \,..\, t\_arg_m \rangle\,\mathbf{pure} \\ (u_1, \,..\,, u_n) \to x\langle t\_args' \rangle\,\mathbf{pure} \equiv (u'_1, \,..\,, u'_n) \to x\langle t\_args \rangle\,\mathbf{pure}[t\_arg_1/tid_1 \,..\, t\_arg_m/tid_m] \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u_1 \vdash pat_1 : t_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u_n \vdash pat_n : t_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n \\ \mathbf{disjoint\,doms}\,(E^{\mathrm{T}}{}_1, \,..\,, E^{\mathrm{T}}{}_n) \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash x\langle t\_args' \rangle \precapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id(pat_1, \,..\,, pat_n) : x\langle t\_args' \rangle \triangleright id(pat'_1, \,..\,, pat'_n), \uplus E^{\mathrm{T}}{}_1 \,..\, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \,..\, \uplus \Sigma^{\mathrm{N}}{}_n} \quad \textsc{check\_pat\_constr}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{tid_1 \mapsto kinf_1, \,..\,, tid_m \mapsto kinf_m\}, \Sigma^{\mathrm{N}}, \mathbf{Ctor}, \mathbf{unit} \to x\langle t\_arg_1 \,..\, t\_arg_m \rangle\,\mathbf{pure} \\ \mathbf{unit} \to x\langle t\_args' \rangle\,\mathbf{pure} \equiv \mathbf{unit} \to x\langle t\_args \rangle\,\mathbf{pure}[t\_arg_1/tid_1 \,..\, t\_arg_m/tid_m] \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash x\langle t\_args' \rangle \precapprox t : t', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : t \triangleright id, \{\,\}, \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_identConstr}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(id) \triangleright \{\,\}, \{\,\}, \mathbf{Default}, t \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash t \precapprox u : u', \Sigma^{\mathrm{N}} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, u \vdash id : t \triangleright id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \Sigma^{\mathrm{N}}} \quad \textsc{check\_pat\_varDefault}$$

$$\frac{}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t \vdash id : t \triangleright id, (E^{\mathrm{T}} \uplus \{id \mapsto t\}), \{\,\}} \quad \textsc{check\_pat\_var}$$

$$
\frac{\begin{array}{l}
E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \triangleright x\langle t\_args\rangle, (\,\overline{t_i}^{\,i}\,)\\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i \vdash pat_i : u_i \triangleright pat'_i, E^{\mathrm{T}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}\\
\textbf{disjoint doms}\,(\,\overline{E^{\mathrm{T}}{}_i}^{\,i}\,)\\
\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, (\textbf{nums}, \textbf{vectors}) \vdash x\langle t\_args\rangle \lessapprox t : t', \Sigma^{\mathrm{N}}
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \{\,\overline{id_i = pat_i}^{\,i}\,;?\} : x\langle t\_args\rangle \triangleright \{\,\overline{id_i = pat'_i}^{\,i}\,;?\}, \uplus\,\overline{E^{\mathrm{T}}{}_i}^{\,i}, \Sigma^{\mathrm{N}} \uplus \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}} \quad \text{CHECK\_PAT\_RECORD}
$$

$$
\frac{\begin{array}{l}
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n\\
\textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n)\\
E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_1 \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_1 \quad \ldots \quad E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_n \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_n\\
ne_4 \equiv \textbf{length}\,(pat_1 \ldots pat_n)\\
\Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n\\
\Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}_n
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle ne_1\, ne_2\, order\ t\rangle \vdash [pat_1, \ldots, pat_n] : \textbf{vector}\,\langle ne_3\, ne_4\, order\ u\rangle \triangleright [pat'_1, \ldots, pat'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime} \uplus \{ne_2 = ne_4\}} \quad \text{CHECK\_PAT\_VECTOR}
$$

$$
\frac{\begin{array}{l}
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n\\
E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_1 \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_1 \quad \ldots \quad E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_n \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_n\\
ne_4 \equiv \textbf{length}\,(pat_1 \ldots pat_n)\\
\textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n)\\
num_1 \ < \ \ldots \ < \ num_n\\
\Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n\\
\Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}_n
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle ne_1\, ne_2\, \textbf{inc}\ t\rangle \vdash [num_1 = pat_1, \ldots, num_n = pat_n] : \textbf{vector}\,\langle ne_3\, ne_4\, \textbf{inc}\ t\rangle \triangleright [num_1 = pat'_1, \ldots, num_n = pat'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{ne_1 \leq num_1, ne_2 \geq ne_4\} \uplus \Sigma}
$$

$$
\frac{\begin{array}{l}
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \triangleright pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \triangleright pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n\\
E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_1 \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_1 \quad \ldots \quad E^{\mathrm{D}}, (\textbf{nums}, \textbf{vectors}) \vdash u_n \lessapprox t : t', \Sigma^{\mathrm{N}\prime}_n\\
ne_4 \equiv \textbf{length}\,(pat_1 \ldots pat_n)\\
\textbf{disjoint doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n)\\
num_1 \ > \ \ldots \ > \ num_n\\
\Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n\\
\Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}_n
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \textbf{vector}\,\langle ne_1\, ne_2\, \textbf{dec}\ t\rangle \vdash [num_1 = pat_1, \ldots, num_n = pat_n] : \textbf{vector}\,\langle ne_3\, ne_4\, \textbf{dec}\ t\rangle \triangleright [num_1 = pat'_1, \ldots, num_n = pat'_n], (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{ne_1 \geq num_1, ne_2 \geq ne_4\} \uplus}
$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne''_1\; ne'''_1\; order\; t\rangle \vdash pat_1 : \mathbf{vector}\,\langle ne''_1\; ne'_1\; order\; u_1\rangle \rhd pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne''_n\; ne'''_n\; order\; t\rangle \vdash pat_1 : \mathbf{vector}\,\langle ne''_n\; ne'_n\; order\; u_1\rangle \rhd pat'_n, E\\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_1 \precapprox t : t', \Sigma^{\mathrm{N}\prime}{}_1 \quad \ldots \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{vectors}) \vdash u_n \precapprox t : t', \Sigma^{\mathrm{N}\prime}{}_n\\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots, E^{\mathrm{T}}{}_n)\\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}}{}_n\\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus \ldots \uplus \Sigma^{\mathrm{N}\prime}{}_n\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle \vdash pat_1 : \ldots : pat_n : \mathbf{vector}\,\langle ne_1\; ne_4\; order\; t\rangle \rhd pat'_1 : \ldots : pat'_n, (E^{\mathrm{T}}{}_1 \uplus \ldots \uplus E^{\mathrm{T}}{}_n), \{ne'_1 + \ldots + ne'_n \leq ne_2\} \uplus \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}}$$

$$\frac{\begin{array}{l}E, t_1 \vdash pat_1 : u_1 \rhd pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad \ldots. \quad E, t_n \vdash pat_n : u_n \rhd pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n\\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, \ldots., E^{\mathrm{T}}{}_n)\end{array}}{E, (t_1, \ldots., t_n) \vdash (pat_1, \ldots., pat_n) : (u_1, \ldots., u_n) \rhd (pat'_1, \ldots., pat'_n), (E^{\mathrm{T}}{}_1 \uplus \ldots. \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}}{}_1 \uplus \ldots. \uplus \Sigma^{\mathrm{N}}{}_n} \quad \text{CHECK\_PAT\_TUP}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_1 : u_1 \rhd pat'_1, E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash pat_n : u_n \rhd pat'_n, E^{\mathrm{T}}{}_n, \Sigma^{\mathrm{N}}{}_n\\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n)\\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \precapprox t : t', \Sigma^{\mathrm{N}\prime}{}_1 \quad .. \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \precapprox t : t', \Sigma^{\mathrm{N}\prime}{}_n\\ \mathbf{disjoint\ doms}\,(E^{\mathrm{T}}{}_1, .., E^{\mathrm{T}}{}_n)\\ \Sigma^{\mathrm{N}} \equiv \Sigma^{\mathrm{N}}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}}{}_n\\ \Sigma^{\mathrm{N}\prime} \equiv \Sigma^{\mathrm{N}\prime}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}\prime}{}_n\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{list}\,\langle t\rangle \vdash [||pat_1, .., pat_n||] : \mathbf{list}\,\langle t\rangle \rhd [||pat'_1, .., pat'_n||], (E^{\mathrm{T}}{}_1 \uplus .. \uplus E^{\mathrm{T}}{}_n), \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}} \quad \text{CHECK\_PAT\_LIST}$$

$\boxed{E, t, widening \vdash exp : t' \rhd exp', I, E^{\mathrm{T}}}$ Typing expressions, collecting nexp constraints, effects, and new bindings

$$\frac{\begin{array}{l}E^{\mathrm{T}}(id) \rhd \{tid_0 \mapsto kinf_0, .., tid_n \mapsto kinf_n\}, \{\,\}, \mathbf{Ctor}, \mathbf{unit} \to x\langle t\_args\rangle\; \mathbf{pure}\\ u \equiv x\langle t\_args\rangle[t\_arg_0/tid_0 .. t\_arg_n/tid_n]\\ E^{\mathrm{D}}, widening \vdash u \precapprox t : t', \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : x \rhd id, \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, \{\,\}} \quad \text{CHECK\_EXP\_UNARYCTOR}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(id) \rhd \{\,\}, \{\,\}, tag, u\\ E^{\mathrm{D}}, widening, t \vdash id : u \rhd t', exp, \Sigma^{\mathrm{N}}, effect\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : u \rhd id, \langle \Sigma^{\mathrm{N}}, effect\rangle, \{\,\}} \quad \text{CHECK\_EXP\_LOCALVAR}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(id) \rhd \{tid_1 \mapsto kinf_1, .., tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u'\\ u \equiv u'[t\_arg_1/tid_1 .. t\_arg_n/tid_n]\\ E^{\mathrm{D}}, widening, t \vdash id : u \rhd t', exp, \Sigma^{\mathrm{N}\prime}, effect\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash id : u \rhd id, \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, effect\rangle, \{\,\}} \quad \text{CHECK\_EXP\_OTHERVAR}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \{\mathit{tid}_0 \mapsto \mathit{kinf}_0, \,..\,, \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \{\,\}, \mathbf{Ctor}, t'' \to x\langle \mathit{t\_args}\rangle\ \mathbf{pure} \\ t' \to u\ \mathbf{pure} \equiv t'' \to x\langle \mathit{t\_args}\rangle\ \mathbf{pure}[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \lessapprox t : t', \Sigma^{\mathrm{N}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{exp} : u' \triangleright \mathit{exp}, \langle \Sigma^{\mathrm{N}'}, \mathit{effect}\rangle, E^{\mathrm{T}'} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{id}(\mathit{exp}) : t \triangleright \mathit{id}(\mathit{exp}'), \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}, \mathit{effect}\rangle, \{\,\}} \quad \textsc{check\_exp\_ctor}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \{\mathit{tid}_0 \mapsto \mathit{kinf}_0, \,..\,, \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u \\ u[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \equiv u_i \to u_j\ \mathit{effect} \\ u_i \equiv (\mathbf{implicit}\ \langle \mathit{ne}\rangle, t_0, \,..\,, t_m) \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_0, \,..\,, t_m), \mathit{widening} \vdash (\mathit{exp}_0, \,..\,, \mathit{exp}_m) : u_i' \triangleright (\mathit{exp}_0', \,..\,, \mathit{exp}_m'), I, E^{\mathrm{T}'} \\ E^{\mathrm{D}}, \mathit{widening}, t \vdash \mathit{id}(\mathit{annot}, \mathit{exp}_0', \,..\,, \mathit{exp}_m') : u_j \triangleright u_j', \mathit{exp}'', \Sigma^{\mathrm{N}'}, \mathit{effect}' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{id}(\mathit{exp}_0, \,..\,, \mathit{exp}_m) : u_j \triangleright \mathit{exp}'', I \uplus \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus \langle \Sigma^{\mathrm{N}'}, \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appImplicit}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \{\mathit{tid}_0 \mapsto \mathit{kinf}_0, \,..\,, \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u \\ u[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \equiv u_i \to u_j\ \mathit{effect} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, \mathit{widening} \vdash \mathit{exp} : u_i' \triangleright \mathit{exp}', I, E^{\mathrm{T}'} \\ E^{\mathrm{D}}, \mathit{widening}, t \vdash \mathit{id}(\mathit{exp}') : u_j \triangleright u_j', \mathit{exp}'', \Sigma^{\mathrm{N}'}, \mathit{effect}' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{id}(\mathit{exp}) : u_j \triangleright \mathit{exp}'', I \uplus \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus \langle \Sigma^{\mathrm{N}'}, \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_app}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{overload}\,\{\mathit{tid}_0 \mapsto \mathit{kinf}_0, \,..\,, \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u : \mathit{tinf}_1 \ldots \mathit{tinf}_n \\ u[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \equiv u_i \to u_j\ \mathit{effect} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, \mathit{widening} \vdash \mathit{exp} : u_i' \triangleright \mathit{exp}', I, E^{\mathrm{T}'} \\ \sigma_{\mathbf{full}(u_i', t)}(\mathit{tinf}_1 \ldots \mathit{tinf}_n) \triangleright \mathit{tinf} \\ \langle (\{\mathit{id} \mapsto \mathit{tinf}\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{id}(\mathit{exp}) : t' \triangleright \mathit{exp}'', I', E^{\mathrm{T}''} \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{id}(\mathit{exp}) : u_j \triangleright \mathit{exp}'', I \uplus I' \uplus \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus \langle \Sigma^{\mathrm{N}'}, \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_appOverload}$$

$$\frac{\begin{array}{l} E^{\mathrm{T}}(\mathit{id}) \triangleright \{\mathit{tid}_0 \mapsto \mathit{kinf}_0, \,..\,, \mathit{tid}_n \mapsto \mathit{kinf}_n\}, \Sigma^{\mathrm{N}}, \mathit{tag}, u \\ u[\mathit{t\_arg}_0/\mathit{tid}_0 \,..\, \mathit{t\_arg}_n/\mathit{tid}_n] \equiv u_i \to u_j\ \mathit{effect} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, \mathit{widening} \vdash (\mathit{exp}_1, \mathit{exp}_2) : u_i' \triangleright (\mathit{exp}_1', \mathit{exp}_2'), I, E^{\mathrm{T}'} \\ E^{\mathrm{D}}, \mathit{widening}, t \vdash \mathit{exp}_1'\ \mathit{id}\ \mathit{exp}_2' : u_j \triangleright u_j', \mathit{exp}, \Sigma^{\mathrm{N}'}, \mathit{effect}' \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, \mathit{widening} \vdash \mathit{exp}_1\ \mathit{id}\ \mathit{exp}_2 : t \triangleright \mathit{exp}, I \uplus \langle \Sigma^{\mathrm{N}}, \mathit{effect}\rangle \uplus \langle \Sigma^{\mathrm{N}'}, \mathit{effect}'\rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_infix\_app}$$

$$\frac{\begin{array}{l}
E^{\mathrm{T}}(id) \rhd \mathbf{overload}\,\{ tid_0 \mapsto kinf_0, \,..\,, tid_n \mapsto kinf_n\}, \Sigma^{\mathrm{N}}, tag, u : tinf_1 \dots tinf_n \\
u[t_arg_0/tid_0 \,..\, t_arg_n/tid_n] \equiv u_i \to u_j\ effect \\
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u_i, widening \vdash (exp_1, exp_2) : u'_i \rhd (exp'_1, exp'_2), I, E^{\mathrm{T}\prime} \\
\sigma_{\mathbf{full}(u'_i, t)}(tinf_1 \dots tinf_n) \rhd tinf \\
\langle(\{id \mapsto tinf\} \uplus E^{\mathrm{T}}), E^{\mathrm{D}}\rangle, t, widening \vdash exp_1\ id\ exp_2 : t' \rhd exp, I', E^{\mathrm{T}\prime\prime}
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash exp_1\ id\ exp_2 : t \rhd exp, I \uplus I \uplus \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus \langle \Sigma^{\mathrm{N}\prime}, effect'\rangle, E^{\mathrm{T}}} \quad \textsc{check_exp_infix_appOverload}$$

$$\frac{\begin{array}{l}
E^{\mathrm{R}}(\,\overline{id_i}^{\,i}\,) \rhd x\langle t_args\rangle, \overline{t_i}^{\,i} \\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i, widening \vdash exp_i : u_i \rhd exp'_i, \langle {\Sigma^{\mathrm{N}}}_i, effect_i\rangle, E^{\mathrm{T}}}^{\,i} \\
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u_i \gtrapprox t_i : t'_i, {\Sigma^{\mathrm{N}\prime}}_i}^{\,i} \\
\Sigma^{\mathrm{N}} \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i} \\
\Sigma^{\mathrm{N}\prime} \equiv \uplus \overline{{\Sigma^{\mathrm{N}\prime}}_i}^{\,i}
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash \{\overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t_args\rangle \rhd \{\overline{id_i = exp'_i}^{\,i}\,;^?\}, \uplus\langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}\prime}, \uplus\,\overline{effect_i}^{\,i}\rangle, \{\,\}} \quad \textsc{check_exp_record}$$

$$\frac{\begin{array}{l}
\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash exp : x\langle t_args\rangle \rhd exp', I, E^{\mathrm{T}} \\
E^{\mathrm{R}}(x\langle t_args\rangle) \rhd \overline{id'_n : t'_n}^{\,n} \\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_i, widening \vdash exp_i : u_i \rhd exp'_i, I_i, E^{\mathrm{T}}}^{\,i} \\
\overline{id_i : t_i}^{\,i} \subset \overline{id'_n : t'_n}^{\,n} \\
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle, widening \vdash u_i \gtrapprox t_i : t''_i, {\Sigma^{\mathrm{N}\prime}}_i}^{\,i}
\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash \{exp\ \mathbf{with}\ \overline{id_i = exp_i}^{\,i}\,;^?\} : x\langle t_args\rangle \rhd \{exp'\ \mathbf{with}\ \overline{id_i = exp'_i}^{\,i}\,\}, I \uplus \overline{I_i}^{\,i}, E^{\mathrm{T}}} \quad \textsc{check_exp_recup}$$

$$\frac{\begin{array}{l}
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \rhd exp'_1, I_1, E^{\mathrm{T}\prime} \quad \dots \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash exp_n : u_n \rhd exp'_n, I_n, E^{\mathrm{T}\prime} \\
E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \gtrapprox t : t', {\Sigma^{\mathrm{N}}}_1 \quad \dots \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \gtrapprox t : t', {\Sigma^{\mathrm{N}}}_n \\
\mathbf{length}\,(exp_1 \dots exp_n) \equiv ne \\
\Sigma^{\mathrm{N}} \equiv \{ne = ne_2\} \uplus {\Sigma^{\mathrm{N}}}_1 \uplus \dots \uplus {\Sigma^{\mathrm{N}}}_n
\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp_1, \dots, exp_n] : \mathbf{vector}\,\langle ne_1\ num\ order\ t\rangle \rhd [exp'_1, \dots, exp'_n], \langle \Sigma^{\mathrm{N}}, \mathbf{pure}\rangle \uplus I_1 \uplus \dots \uplus I_n, E^{\mathrm{T}}} \quad \textsc{check_exp_vector}$$

$$\frac{\begin{array}{l}
E, \mathbf{vector}\,\langle ne\ ne'\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_1\ ne'_1\ \mathbf{inc}\ u\rangle \rhd exp'_1, I_1, E^{\mathrm{T}} \\
E, \mathbf{range}\,\langle ne_2\ ne'_2\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_3\ ne'_3\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}
\end{array}}{E, t, widening \vdash exp_1[exp_2] : u \rhd exp'_1[exp'_2], I_1 \uplus I_2 \uplus \langle\{ne_1 \le ne_3, ne_3 + ne'_3 \le ne_1 + ne'_1\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check_exp_vectorgetInc}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; order\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_1\; ne_1'\; \mathbf{dec}\; u\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_2\; ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_3\; ne_3'\rangle \rhd exp_2', I_2, E^{\mathrm{T}} \end{array}}{E, t, widening \vdash exp_1[exp_2] : u \rhd exp_1'[exp_2'], I_1 \uplus I_2 \uplus \langle\{ne_1 \geq ne_3, ne_3 + (-ne_3') \leq ne_1 + (-ne_1')\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \;\textsc{check\_exp\_vectorgetDec}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne_1'\; \mathbf{inc}\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_2\; ne_2'\; \mathbf{inc}\; u\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\; ne_3'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_4\; ne_4'\rangle \rhd exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\; ne_5'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_3 : \mathbf{range}\,\langle ne_6\; ne_6'\rangle \rhd exp_3', I_3, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle, widening \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\; ne_7'\; \mathbf{inc}\; u\rangle \rhd exp_1'[exp_2' : exp_3'], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \geq ne_4, ne \leq ne_4', ne' \leq ne_4 + ne_6', ne_4 \leq ne_2, ne_4 + ne_6' \leq}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne_1'\; \mathbf{dec}\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : \mathbf{vector}\,\langle ne_2\; ne_2'\; \mathbf{dec}\; u\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_3\; ne_3'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{range}\,\langle ne_4\; ne_4'\rangle \rhd exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_5\; ne_5'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_3 : \mathbf{range}\,\langle ne_6\; ne_6'\rangle \rhd exp_3', I_3, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle, widening \vdash exp_1[exp_2..exp_3] : \mathbf{vector}\,\langle ne_7\; ne_7'\; \mathbf{dec}\; u\rangle \rhd exp_1'[exp_2' : exp_3'], I_1 \uplus I_2 \uplus I_3 \uplus \langle\{ne \leq ne_4, ne \geq ne_4', ne' \leq ne_6' + (-ne_4), ne_4' \geq ne_2, ne_6' + (-}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; u\rangle \rhd exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_1'\; ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{range}\,\langle ne_3\; ne_4\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, t, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_2 : u \rhd exp_2', I_2, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{inc}\; t\rangle, widening \vdash [exp\; \mathbf{with}\; exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; u\rangle \rhd [exp'\; \mathbf{with}\; exp_1' = exp_2'], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \leq ne_3, ne_2 \geq ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \;\textsc{check\_exp}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; u\rangle \rhd exp', I, E^{\mathrm{T}} \\ E, \mathbf{range}\,\langle ne_1'\; ne_2'\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{range}\,\langle ne_3\; ne_4\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, t, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_2 : u \rhd exp_2', I_2, E^{\mathrm{T}} \end{array}}{E, \mathbf{vector}\,\langle ne\; ne'\; \mathbf{dec}\; t\rangle, widening \vdash [exp\; \mathbf{with}\; exp_1 = exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; u\rangle \rhd [exp'\; \mathbf{with}\; exp_1' = exp_2'], I \uplus I_1 \uplus I_2 \uplus \langle\{ne_1 \geq ne_3, ne_2 \geq ne_4\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \;\textsc{check\_ex}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \rhd exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \rhd exp_2', I_2, E^{\mathrm{T}} \\ E, \mathbf{vector}\,\langle ne_9\; ne_{10}\; \mathbf{inc}\; t\rangle, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : \mathbf{vector}\,\langle ne_{11}\; ne_{12}\; \mathbf{inc}\; u\rangle \rhd exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_3 \leq ne_5, ne_3 + ne_4 \leq ne_7, ne_{12} = ne_8 + (-ne_6), ne_6 + \mathbf{one} \leq ne_8\}, \mathbf{pure}\rangle \end{array}}{E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle, widening \vdash [exp\; \mathbf{with}\; exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \rhd [exp'\; \mathbf{with}\; exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}} \;\textsc{check\_exp\_vecran}$$

$$\frac{\begin{array}{l} E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \rhd exp', I, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \rhd exp_1', I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \rhd exp_2', I_2, E^{\mathrm{T}} \\ E, u, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : u' \rhd exp_3', I_3, E^{\mathrm{T}} \\ I_4 \equiv \langle\{ne_3 \leq ne_5, ne_3 + ne_4 \leq ne_7\}, \mathbf{pure}\rangle \end{array}}{E, \mathbf{vector}\,\langle ne_1\; ne_2\; order\; t\rangle, widening \vdash [exp\; \mathbf{with}\; exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; u\rangle \rhd [exp'\; \mathbf{with}\; exp_1' : exp_2' = exp_3'], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}} \;\textsc{check\_exp\_vecran}$$

$$\dfrac{\begin{array}{l}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \rhd exp', I, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \rhd exp'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}\\ E, \mathbf{vector}\,\langle ne_9\ ne_{10}\ \mathbf{dec}\ t\rangle, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : \mathbf{vector}\,\langle ne_{11}\ ne_{12}\ \mathbf{dec}\ u\rangle \rhd exp'_3, I_3, E^{\mathrm{T}}\\ I_4 \equiv \langle\{ne_5 \le ne_3, ne_3 + (-ne_4) \le ne_6 + (-ne_8), ne_8 + \mathbf{one} \le ne_6\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\ \mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \rhd [exp'\ \mathbf{with}\ exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_VECRAN}$$

$$\dfrac{\begin{array}{l}E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \rhd exp', I, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle ne_5\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_1 : \mathbf{atom}\,\langle ne_6\rangle \rhd exp'_1, I_1, E^{\mathrm{T}}\\ E, \mathbf{atom}\,\langle ne_7\rangle, (\mathbf{none}, \mathbf{vectors}) \vdash exp_2 : \mathbf{atom}\,\langle ne_8\rangle \rhd exp'_2, I_2, E^{\mathrm{T}}\\ E, u, (\mathbf{nums}, \mathbf{vectors}) \vdash exp_3 : u' \rhd exp'_3, I_3, E^{\mathrm{T}}\\ I_4 \equiv \langle\{ne_5 \le ne_3, ne_3 + (-ne_4) \le ne_6 + (-ne_8), ne_8 + \mathbf{one} \le ne_6\}, \mathbf{pure}\rangle\end{array}}{E, \mathbf{vector}\,\langle ne_1\ ne_2\ order\ t\rangle, widening \vdash [exp\ \mathbf{with}\ exp_1 : exp_2 = exp_3] : \mathbf{vector}\,\langle ne_3\ ne_4\ \mathbf{dec}\ u\rangle \rhd [exp'\ \mathbf{with}\ exp'_1 : exp'_2 = exp'_3], I \uplus I_1 \uplus I_2 \uplus I_3 \uplus I_4, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_VECRAN}$$

$$\dfrac{\begin{array}{l}E^{\mathrm{R}}(x\langle t\_args\rangle) \rhd \overline{id_i : t_i}^{\,i}\ id : u\ \overline{id'_j : t'_j}^{\,j}\\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t'', widening \vdash exp : x\langle t\_args\rangle \rhd exp', I, E^{\mathrm{T}}\\ E^{\mathrm{D}}, widening, t \vdash exp'.id : u \rhd t', exp'_1, \Sigma^{\mathrm{N}'}, effect\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, widening \vdash exp.id : u \rhd exp'_1, I \uplus \langle \Sigma^{\mathrm{N}'}, effect\rangle, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_FIELD}$$

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'', widening \vdash exp : u \rhd exp', I, E^{\mathrm{T}}\\ \overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, u \vdash pat_i : u'_i \rhd pat'_i, E^{\mathrm{T}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}\\ \overline{\langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_i), E^{\mathrm{D}}\rangle, t, widening \vdash exp_i : u''_i \rhd exp'_i, I_i, E^{\mathrm{T}}{}'_i}^{\,i}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash \mathbf{switch}\ exp\{\ \overline{\mathbf{case}\ pat_i \to exp_i}^{\,i}\ \} : u \rhd \mathbf{switch}\ exp'\{\ \overline{\mathbf{case}\ pat'_i \to exp'_i}^{\,i}\ \}, I \uplus\ \overline{I_i \uplus \langle \Sigma^{\mathrm{N}}{}_i, \mathbf{pure}\rangle}^{\,i}, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_CASE}$$

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t'', widening \vdash exp : u \rhd exp', I, E^{\mathrm{T}}\\ E^{\mathrm{D}} \vdash typ \rightsquigarrow t'\\ E^{\mathrm{D}}, widening, t' \vdash exp' : u \rhd u', exp'', \Sigma^{\mathrm{N}}, effect\\ E^{\mathrm{D}}, widening, t \vdash exp'' : t' \rhd u'', exp''', \Sigma^{\mathrm{N}'}, effect'\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash (typ)exp : t \rhd exp''', I \uplus \langle \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}'}, effect \uplus effect'\rangle, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_TYPED}$$

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash letbind \rhd letbind', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}, effect, \{\,\}\\ \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}}\rangle, t, widening \vdash exp : u \rhd exp', I_2, E^{\mathrm{T}}{}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t, widening \vdash letbind\ \mathbf{in}\ exp : t \rhd letbind'\ \mathbf{in}\ exp', \langle \Sigma^{\mathrm{N}}, effect\rangle \uplus I_2, E^{\mathrm{T}}}\ \text{CHECK\_EXP\_LET}$$

$$\frac{E, t_1, \mathit{widening} \vdash \mathit{exp}_1 : u_1 \rhd \mathit{exp}'_1, I_1, E^{\mathrm{T}}{}_1 \quad .... \quad E, t_n, \mathit{widening} \vdash \mathit{exp}_n : u_n \rhd \mathit{exp}'_n, I_n, E^{\mathrm{T}}{}_n}{E, (t_1, \, ... \, , t_n), \mathit{widening} \vdash (\mathit{exp}_1, \, ... \, , \mathit{exp}_n) : (u_1, \, ... \, , u_n) \rhd (\mathit{exp}'_1, \, ... \, , \mathit{exp}'_n), I_1 \uplus .... \uplus I_n, E^{\mathrm{T}}} \quad \textsc{check\_exp\_tup}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash \mathit{exp}_1 : u_1 \rhd \mathit{exp}'_1, I_1, E^{\mathrm{T}}{}_1 \quad .. \quad \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t, (\mathbf{nums}, \mathbf{none}) \vdash \mathit{exp}_n : u_n \rhd \mathit{exp}'_n, I_n, E^{\mathrm{T}}{}_n \\ E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_1 \lessapprox t : t', \Sigma^{\mathrm{N}}{}_1 \quad .. \quad E^{\mathrm{D}}, (\mathbf{nums}, \mathbf{none}) \vdash u_n \lessapprox t : t', \Sigma^{\mathrm{N}}{}_n\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{list}\, \langle t \rangle, \mathit{widening} \vdash [||\mathit{exp}_1, \, .. \, , \mathit{exp}_n||] : \mathbf{list}\, \langle u \rangle \rhd [||\mathit{exp}'_1, \, .. \, , \mathit{exp}'_n||], \langle \Sigma^{\mathrm{N}}{}_1 \uplus .. \uplus \Sigma^{\mathrm{N}}{}_n, \mathbf{pure} \rangle \uplus I_1 \uplus .. \uplus I_n, E^{\mathrm{T}}} \quad \textsc{check\_exp\_list}$$

$$\frac{\begin{array}{l}E, \mathbf{bit}, \mathit{widening} \vdash \mathit{exp}_1 : \mathbf{bit} \rhd \mathit{exp}'_1, I_1, E^{\mathrm{T}\prime} \\ E, t, \mathit{widening} \vdash \mathit{exp}_2 : u_1 \rhd \mathit{exp}'_2, I_2, E^{\mathrm{T}}{}_2 \\ E, t, \mathit{widening} \vdash \mathit{exp}_3 : u_2 \rhd \mathit{exp}'_3, I_3, E^{\mathrm{T}}{}_3 \\ E^{\mathrm{D}}, \mathit{widening} \vdash u_1 \lessapprox t : t', \Sigma^{\mathrm{N}}{}_1 \\ E^{\mathrm{D}}, \mathit{widening} \vdash u_2 \lessapprox t : t', \Sigma^{\mathrm{N}}{}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, t, \mathit{widening} \vdash \mathbf{if}\ \mathit{exp}_1\ \mathbf{then}\ \mathit{exp}_2\ \mathbf{else}\ \mathit{exp}_3 : u \rhd \mathbf{if}\ \mathit{exp}'_1\ \mathbf{then}\ \mathit{exp}'_2\ \mathbf{else}\ \mathit{exp}'_3, \langle \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2, \mathbf{pure} \rangle \uplus I_1 \uplus I_2 \uplus I_3, (E^{\mathrm{T}}{}_2 \cap E^{\mathrm{T}}{}_3)} \quad \textsc{check\_exp\_if}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\, \langle \mathit{ne}_1\ \mathit{ne}_2 \rangle, \mathit{widening} \vdash \mathit{exp}_1 : \mathbf{range}\, \langle \mathit{ne}_7\ \mathit{ne}_8 \rangle \rhd \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\, \langle \mathit{ne}_3\ \mathit{ne}_4 \rangle, \mathit{widening} \vdash \mathit{exp}_2 : \mathbf{range}\, \langle \mathit{ne}_9\ \mathit{ne}_{10} \rangle \rhd \mathit{exp}'_2, I_2, E^{\mathrm{T}} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{range}\, \langle \mathit{ne}_5\ \mathit{ne}_6 \rangle, \mathit{widening} \vdash \mathit{exp}_3 : \mathbf{range}\, \langle \mathit{ne}_{11}\ \mathit{ne}_{12} \rangle \rhd \mathit{exp}'_3, I_3, E^{\mathrm{T}} \\ \langle (E^{\mathrm{T}} \uplus \{ \mathit{id} \mapsto \mathbf{range}\, \langle \mathit{ne}_1\ \mathit{ne}_4 \rangle \}), E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp}_4 : t \rhd \mathit{exp}'_4, I_4, E^{\mathrm{T}\prime}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{foreach}\, (\mathit{id}\ \mathbf{from}\ \mathit{exp}_1\ \mathbf{to}\ \mathit{exp}_2\ \mathbf{by}\ \mathit{exp}_3) \mathit{exp}_4 : t \rhd \mathbf{foreach}\, (\mathit{id}\ \mathbf{from}\ \mathit{exp}'_1\ \mathbf{to}\ \mathit{exp}'_2\ \mathbf{by}\ \mathit{exp}'_3) \mathit{exp}'_4, I_1 \uplus I_2 \uplus I_3 \uplus I_4 \uplus \langle \{ \mathit{ne}_1 \leq \mathit{ne}_3 + \mathit{ne}_4 \}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{c}$$

$$\frac{\begin{array}{l}E, t, \mathit{widening} \vdash \mathit{exp}_1 : u \rhd \mathit{exp}'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{list}\, \langle t \rangle, \mathit{widening} \vdash \mathit{exp}_2 : \mathbf{list}\, \langle u \rangle \rhd \mathit{exp}'_2, I_2, E^{\mathrm{T}}\end{array}}{E, \mathbf{list}\, \langle t \rangle, \mathit{widening} \vdash \mathit{exp}_1 :: \mathit{exp}_2 : \mathbf{list}\, \langle u \rangle \rhd \mathit{exp}'_1 :: \mathit{exp}'_2, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \textsc{check\_exp\_cons}$$

$$\frac{\mathit{widening}, t \vdash \mathit{lit} : u \Rightarrow \mathit{exp}, \Sigma^{\mathrm{N}}}{E, t, \mathit{widening} \vdash \mathit{lit} : u \rhd \mathit{exp}, \langle \Sigma^{\mathrm{N}}, \mathbf{pure} \rangle, E^{\mathrm{T}}} \quad \textsc{check\_exp\_lit}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I, E^{\mathrm{T}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \{ \mathit{exp} \} : \mathbf{unit} \rhd \{ \mathit{exp}' \}, I, E^{\mathrm{T}}} \quad \textsc{check\_exp\_blockbase}$$

$$\frac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I_1, E^{\mathrm{T}}{}_1 \\ \langle (E^{\mathrm{T}} \uplus E^{\mathrm{T}}{}_1), E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \{ \overline{\mathit{exp}_i}^{\,i} \} : \mathbf{unit} \rhd \{ \overline{\mathit{exp}'_i}^{\,i} \}, I_2, E^{\mathrm{T}}{}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \{ \mathit{exp};\ \overline{\mathit{exp}_i}^{\,i} \} : \mathbf{unit} \rhd \{ \mathit{exp}';\ \overline{\mathit{exp}'_i}^{\,i} \}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \textsc{check\_exp\_blockrec}$$

$$\frac{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \rhd \mathit{exp}', I, E^{\mathrm{T}}{}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}} \rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\, \{ \mathit{exp} \} : \mathbf{unit} \rhd \{ \mathit{exp}' \}, I, E^{\mathrm{T}}} \quad \textsc{check\_exp\_nondetbase}$$

$$\frac{\begin{array}{c}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathit{exp} : \mathbf{unit} \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\,\{\,\overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \triangleright \{\,\overline{\mathit{exp}'_i}^{\,i}\,\}, I_2, {E^{\mathrm{T}}}_2\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathbf{unit}, \mathit{widening} \vdash \mathbf{nondet}\,\{\mathit{exp};\,\overline{\mathit{exp}_i}^{\,i}\,\} : \mathbf{unit} \triangleright \{\mathit{exp}';\,\overline{\mathit{exp}'_i}^{\,i}\,\}, I_1 \uplus I_2, E^{\mathrm{T}}} \quad \text{CHECK\_EXP\_NONDETREC}$$

$$\frac{\begin{array}{c}E, t, \mathit{widening} \vdash \mathit{exp} : u \triangleright \mathit{exp}', I_1, {E^{\mathrm{T}}}_1 \\ E, \mathit{widening} \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I_2, {E^{\mathrm{T}}}_2\end{array}}{E, \mathbf{unit}, \mathit{widening} \vdash \mathit{lexp} := \mathit{exp} : \mathbf{unit} \triangleright \mathit{lexp}' := \mathit{exp}', I \uplus I_2, {E^{\mathrm{T}}}_2} \quad \text{CHECK\_EXP\_ASSIGN}$$

$\boxed{E, \mathit{widening} \vdash \mathit{lexp} : t \triangleright \mathit{lexp}', I, E^{\mathrm{T}}}$ Check the left hand side of an assignment

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, \langle\{\,\}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_WREG}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_WLOCL}$$

$$\frac{E^{\mathrm{T}}(\mathit{id}) \triangleright t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_VAR}$$

$$\frac{\mathit{id} \notin \mathbf{dom}\,(E^{\mathrm{T}})}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash \mathit{id} : t \triangleright \mathit{id}, I_\epsilon, \{\mathit{id} \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \text{CHECK\_LEXP\_WNEW}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{register}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \precapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_WREGCAST}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(\mathit{id}) \triangleright \mathbf{reg}\,\langle t\rangle \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \precapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_WLOCLCAST}$$

$$\frac{\begin{array}{l}E^{\mathrm{T}}(\mathit{id}) \triangleright t \\ E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u \\ E^{\mathrm{D}}, \mathit{widening} \vdash u \precapprox t : u, \Sigma^{\mathrm{N}}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, \mathit{widening} \vdash (\mathit{typ})\mathit{id} : t \triangleright \mathit{id}, \langle\Sigma^{\mathrm{N}}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \text{CHECK\_LEXP\_VARCAST}$$

$$\frac{\begin{array}{c} id \notin \mathbf{dom}\,(E^{\mathrm{T}}) \\ E^{\mathrm{D}} \vdash typ \rightsquigarrow t \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, widening \vdash (typ)id : t \triangleright id, I_{\epsilon}, \{id \mapsto \mathbf{reg}\,\langle t\rangle\}} \quad \textsc{check\_lexp\_wnewCast}$$

$$\frac{\begin{array}{c} E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, (t_1, .., t_n, t) \to t'\,\{\,\overline{base\_effect_i}^{\,i}, \mathbf{wmem}, \overline{base\_effect'_j}^{\,j}\,\} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_1, .., t_n), widening \vdash exp : u_1 \triangleright exp', I, E^{\mathrm{T}}{}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, widening \vdash id(exp) : t \triangleright id(exp'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wmem}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wmem}$$

$$\frac{\begin{array}{c} E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, (t_1, .., t_n, t) \to t'\,\{\,\overline{base\_effect_i}^{\,i}, \mathbf{wreg}, \overline{base\_effect'_j}^{\,j}\,\} \\ \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, (t_1, .., t_n), widening \vdash exp : u_1 \triangleright exp', I, E^{\mathrm{T}}{}_1 \end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, widening \vdash id(exp) : t \triangleright id(exp'), I \uplus \langle \Sigma^{\mathrm{N}}, \{\mathbf{wreg}\}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wregCall}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{inc}\; t\rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle \{ne_1 \leq ne, ne_1 + ne_2 \geq ne\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitInc}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp : u \triangleright exp', I_1, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_1\; ne_2\; \mathbf{dec}\; t\rangle \triangleright lexp', I_2, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp] : t \triangleright lexp'[exp'], I_1 \uplus I_2 \uplus \langle \{ne \leq ne_1, ne_1 + (-ne_2) \leq ne\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wbitDec}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne_1\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; t\rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2 + (-ne_1)\; \mathbf{inc}\; t\rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle \{ne_3 \leq ne_1, ne_3 + ne_4 \leq ne_2 + (-ne_1)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wslic}$$

$$\frac{\begin{array}{c} E, \mathbf{atom}\,\langle ne_1\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_1 : u_1 \triangleright exp'_1, I_1, E^{\mathrm{T}} \\ E, \mathbf{atom}\,\langle ne_2\rangle, (\mathbf{nums}, \mathbf{none}) \vdash exp_2 : u_2 \triangleright exp'_2, I_2, E^{\mathrm{T}} \\ E, (\mathbf{none}, \mathbf{vectors}) \vdash lexp : \mathbf{vector}\,\langle ne_3\; ne_4\; \mathbf{inc}\; t\rangle \triangleright lexp', I_3, E^{\mathrm{T}} \end{array}}{E, widening \vdash lexp[exp_1 : exp_2] : \mathbf{vector}\,\langle ne_1\; ne_2 + (-ne_1)\; \mathbf{inc}\; t\rangle \triangleright lexp'[exp'_1 : exp'_2], I_1 \uplus I_2 \uplus I_3 \uplus \langle \{ne_1 \leq ne_3, ne_3 + (-ne_4) \leq ne_1 + (-ne_2)\}, \mathbf{pure}\rangle, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_ws}$$

$$\frac{\begin{array}{c} E^{\mathrm{R}}(x\langle t\_args\rangle) \triangleright \overline{id_i : t_i}^{\,i}\; id : t\; \overline{id'_j : t'_j}^{\,j} \\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, widening \vdash lexp : x\langle t\_args\rangle \triangleright lexp', I, E^{\mathrm{T}} \end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, widening \vdash lexp.id : t \triangleright lexp'.id, I, E^{\mathrm{T}}} \quad \textsc{check\_lexp\_wrecord}$$

$\boxed{E \vdash letbind \triangleright letbind', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}, effect, E^{\mathrm{K}}}$ Build the environment for a let binding, collecting index constraints

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}{}_2, \Sigma^{\mathrm{N}}\\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1\\ \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_2, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t, (\mathbf{none}, \mathbf{none}) \vdash \mathit{exp} : u' \triangleright \mathit{exp}', \langle \Sigma^{\mathrm{N}}{}_2, \mathit{effect}\rangle, E^{\mathrm{T}}{}_2\\ \texttt{<<no parses (char 49): <E\_k u+ E\_k2,E\_a,E\_r,E\_e>,(none,none) |- u' < u,*** S\_N3 >>}\end{array}}{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{let}\ \mathit{typschm}\ \mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\ \mathit{typschm}\ \mathit{pat}' = \mathit{exp}', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}} \uplus \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2 \uplus \Sigma^{\mathrm{N}}{}_3, \mathit{effect}, E^{\mathrm{K}}{}_2} \quad \text{CHECK\_LETBIND\_VAL\_ANNOT}$$

$$\dfrac{\begin{array}{l}\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t \vdash \mathit{pat} : u \triangleright \mathit{pat}', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1\\ \texttt{<<no parses (char 26): <(E\_t u+ E\_t1),E\_d>,u |- e***xp : u' gives exp',<S\_N2,effect>,E\_t2 >>}\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{let}\ \mathit{pat} = \mathit{exp} \triangleright \mathbf{let}\ \mathit{pat}' = \mathit{exp}', E^{\mathrm{T}}{}_1, \Sigma^{\mathrm{N}}{}_1 \uplus \Sigma^{\mathrm{N}}{}_2, \mathit{effect}, \{\,\}} \quad \text{CHECK\_LETBIND\_VAL\_NOANNOT}$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{type\_def} \triangleright E}$ Check a type definition

$$\dfrac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ \mathit{id}\ \mathit{name\_scm\_opt} = \mathit{typschm} \triangleright \langle\{\,\}, \langle\{\,\}, \{\mathit{id} \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_ABBREV}$$

$$\dfrac{\begin{array}{l}E^{\mathrm{D}} \vdash \mathit{typ}_1 \leadsto t_1 \quad .. \quad E^{\mathrm{D}} \vdash \mathit{typ}_n \leadsto t_n\\ E^{\mathrm{R}} \equiv \{\{\mathit{id}_1 : t_1, .., \mathit{id}_n : t_n\} \mapsto x\}\end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ struct}\ \{\mathit{typ}_1\ \mathit{id}_1; ..; \mathit{typ}_n\ \mathit{id}_n\ ;^?\} \triangleright \langle\{\,\}, \langle\{x \mapsto K\_Typ\}, \{\,\}, E^{\mathrm{R}}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_RECORD}$$

$$\dfrac{\begin{array}{l}\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}\\ \langle E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \leadsto t_1 \quad .. \quad \langle E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \leadsto t_n\\ \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\} \equiv \uplus\, \overline{E^{\mathrm{K}}{}_i}^{\,i}\\ E^{\mathrm{R}}_1 \equiv \{\{\mathit{id}_1 : t_1, .., \mathit{id}_n : t_n\} \mapsto \{x'_1 \mapsto k_1, .., x'_m \mapsto k_m\}, \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}, \mathbf{None}, x\langle x'_1 .. x'_m\rangle\}\\ E^{\mathrm{K}}{}'_1 \equiv \{x \mapsto K\_Lam(k_1 .. k_m \to K\_Typ)\}\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ struct\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i} . \{\mathit{typ}_1\ \mathit{id}_1; ..; \mathit{typ}_n\ \mathit{id}_n\ ;^?\} \triangleright \langle\{\,\}, \langle E^{\mathrm{K}\prime}, \{\,\}, E^{\mathrm{R}}_1, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_QUANT\_RECORD}$$

$$\dfrac{\begin{array}{l}E^{\mathrm{T}} \equiv \{\mathit{id}_1 \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_1 \to x\ \mathbf{pure}, \ldots, \mathit{id}_n \mapsto \{\,\}, \{\,\}, \mathbf{Ctor}, t_n \to x\ \mathbf{pure}\}\\ E^{\mathrm{K}}{}_1 \equiv \{x \mapsto K\_Typ\}\\ \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_1 \leadsto t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}}{}_1, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ}_n \leadsto t_n\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ x\ \mathit{name\_scm\_opt} = \mathbf{const\ union}\ \{\mathit{typ}_1\ \mathit{id}_1; \ldots; \mathit{typ}_n\ \mathit{id}_n\ ;^?\} \triangleright \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}{}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_TD\_UNQUANT\_UNION}$$

$$\frac{\begin{array}{l}
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash quant_item_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\
\{x'_1 \mapsto k_1, \ldots, x'_m \mapsto k_m\} \equiv \uplus \overline{{E^{\mathrm{K}}}_i}^{i}\\
E^{\mathrm{K}\prime} \equiv \{x \mapsto K_Lam(k_1 \ldots k_m \to K_Typ)\} \uplus \overline{{E^{\mathrm{K}}}_i}^{i}\\
\langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_1 \leadsto t_1 \quad \ldots \quad \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash typ_n \leadsto t_n\\
t \equiv x\langle x'_1 \ldots x'_m\rangle\\
E^{\mathrm{T}} \equiv \{id_1 \mapsto E^{\mathrm{K}\prime}, \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}, \mathbf{Ctor}, t_1 \to t\ \mathbf{pure}, \ldots, id_n \mapsto E^{\mathrm{K}\prime}, \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}, \mathbf{Ctor}, t_n \to t\ \mathbf{pure}\}
\end{array}}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{typedef}\ id\ name_scm_opt = \mathbf{const\ union\ forall}\ \overline{quant_item_i}^{i} . \{typ_1\ id_1; \ldots; typ_n\ id_n\ ;^?\} \triangleright \langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, \{\}, \{\}, \{\}\rangle\rangle}\ \textsc{check_td_quant_union}$$

$$\frac{\begin{array}{l}
E^{\mathrm{T}} \equiv \{id_1 \mapsto x, \ldots, id_n \mapsto x\}\\
E^{\mathrm{E}} \equiv \{x \mapsto \{num_1 \mapsto id_1 \ldots num_n \mapsto id_n\}\}
\end{array}}{E^{\mathrm{D}} \vdash \mathbf{typedef}\ x\ name_scm_opt = \mathbf{enumerate}\ \{id_1; \ldots; id_n\ ;^?\} \triangleright \langle E^{\mathrm{T}}, \langle \{id \mapsto K_Typ\}, \{\}, \{\}, E^{\mathrm{E}}\rangle\rangle}\ \textsc{check_td_enumerate}$$

$\boxed{E \vdash fundef \triangleright fundef', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}$ Check a function definition

$$\frac{\begin{array}{l}
E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ effect\\
\overline{E^{\mathrm{D}} \vdash quant_item_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{i}\\
\Sigma^{\mathrm{N}\prime\prime} \equiv \uplus \overline{{\Sigma^{\mathrm{N}}}_i}^{i}\\
E^{\mathrm{K}\prime} \equiv \overline{{E^{\mathrm{K}}}_i}^{i}\\
{E^{\mathrm{D}}}_1 \equiv \langle E^{\mathrm{K}\prime}, \{\}, \{\}, \{\}\rangle \uplus E^{\mathrm{D}}\\
{E^{\mathrm{D}}}_1 \vdash typ \leadsto u\\
\texttt{<<no parses (char 12): E_d1 |- u <*** t, S_N2 >>}\\
\overline{\langle E^{\mathrm{T}}, {E^{\mathrm{D}}}_1\rangle, t_1 \vdash pat_j : u_j \triangleright pat'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{j}\\
\texttt{<<no parses (char 29): </<(E_t u+ E_tj),E_d1>,u |- e***xpj : u' gives expj',<S_N''''j,effect'j>,E_t'j//j/> >>}\\
\Sigma^{\mathrm{N}\prime\prime\prime\prime\prime} \equiv {\Sigma^{\mathrm{N}}}_2 \uplus \overline{\Sigma^{\mathrm{N}\prime\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime}_j}^{j}\\
effect \equiv \uplus \overline{effect'_j}^{j}\\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime\prime})
\end{array}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\ rec\ forall}\ \overline{quant_item_i}^{i} . typ\ \mathbf{effect}\ effect\ \overline{id\ pat_j = exp_j}^{j} \triangleright \mathbf{function\ rec\ forall}\ \overline{quant_item_i}^{i} . typ\ \mathbf{effect}\ effect\ \overline{id\ pat'_j = exp'_j}^{j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}\ \textsc{check_fd_rec}$$

$E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \rightarrow t\ \mathit{effect}$

$E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow u$

<<no parses (char 11): E_d |- u <*** t, S_N2 >>

$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}', E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j}$

<<no parses (char 28): </<(E_t u+ E_tj),E_d>,u |- e***xpj : uj' gives expj',<S_N'''j,effect'j>,E_t'j//j/> >>

$\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j}$

$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}{}_2 \uplus \Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j})$

$$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$$

CHECK_FD_REC_FUNCTION2

$\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \rightsquigarrow E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}$

$\Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}$

$E^{\mathrm{K}\prime} \equiv E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}$

$\langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \rightsquigarrow t$

$\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j}$

$E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \rightarrow t\ \mathit{effect}\})$

<<no parses (char 44): </<(E_t' u+ E_tj),<E_k',E_a,E_r,E_e>>,t |- e***xpj : u'j gives expj', <S_N'''j,effect'j>,E_t'j//j/> >>

$\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j}$

$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j})$

$$\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\,.\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\,rec\,forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\,.\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}$$

C

$E^{\mathrm{D}} \vdash \mathit{typ} \rightsquigarrow t$

$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime}_j}^{\,j}$

$E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \{\,\}, \mathbf{Global}, t_1 \rightarrow t\ \mathit{effect}\})$

<<no parses (char 29): </<(E_t' u+ E_tj),E_d>,t |- e***xpj : uj' gives expj', <S_N'j,effect'j>,E_t'j//j/> >>

$\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j}$

$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j})$

$$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\,rec}\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}$$

CHECK_FD_REC_FUNCTION_NO_SPEC

$E^{\mathrm{T}}(id) \triangleright E^{\mathrm{K}\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}$

$\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto {E^{\mathrm{K}}}_i, {\Sigma^{\mathrm{N}}}_i}^{\,i}$

$\Sigma^{\mathrm{N}\prime\prime} \equiv \biguplus \overline{{\Sigma^{\mathrm{N}}}_i}^{\,i}$

$E^{\mathrm{K}\prime\prime} \equiv \overline{{E^{\mathrm{K}}}_i}^{\,i}$

$\langle E^{\mathrm{K}\prime\prime} \uplus E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \leadsto u$

<<no parses (char 35): <E_k'' u+ E_k, E_a,E_r,E_e> |- u <*** t, S_N2 >>

$\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}} \uplus E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j}$

<<no parses (char 57): </<(E_t u- id u+ E_tj),<E_k u+ E_k'',E_a,E_r,E_e>>,t |- e***xpj : uj' gives expj', <S_N'''j,effect'j>,E_t'j//j/> >>

$\Sigma^{\mathrm{N}\prime\prime\prime\prime} \equiv \biguplus \overline{\Sigma^{\mathrm{N}\prime\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime\prime}_j}^{\,j}$

$\mathit{effect} \equiv \biguplus \overline{\mathit{effect}'_j}^{\,j}$

$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \Sigma^{\mathrm{N}\prime\prime} \uplus \Sigma^{\mathrm{N}\prime\prime\prime\prime})$

$\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$ C

$E^{\mathrm{T}}(id) \triangleright \{\,\}, {\Sigma^{\mathrm{N}}}_1, \mathbf{Global}, t_1 \to t\ \mathit{effect}$

$E^{\mathrm{D}} \vdash \mathit{typ} \leadsto u$

<<no parses (char 11): E_d |- u <*** t, S_N2 >>

$\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, {E^{\mathrm{T}}}_j, \Sigma^{\mathrm{N}\prime}_j}^{\,j}$

<<no parses (char 36): </<(E_t u- id u+ E_tj),E_d>, u |- e***xpj : uj' gives expj', <S_N''j,effect'j>,E_t'j//j/> >>

$\mathit{effect} \equiv \biguplus \overline{\mathit{effect}'_j}^{\,j}$

$\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,({\Sigma^{\mathrm{N}}}_1 \uplus {\Sigma^{\mathrm{N}}}_2 \uplus \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j})$

$\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}}, \Sigma^{\mathrm{N}}$ CHECK_FD_FUNCTION2

$$
\frac{
\begin{array}{l}
\overline{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{quant\_item}_i \leadsto E^{\mathrm{K}}{}_i, \Sigma^{\mathrm{N}}{}_i}^{\,i}\\
\Sigma^{\mathrm{N}\prime} \equiv \uplus\, \overline{\Sigma^{\mathrm{N}}{}_i}^{\,i}\\
E^{\mathrm{K}\prime\prime} \equiv E^{\mathrm{K}} \uplus \overline{E^{\mathrm{K}}{}_i}^{\,i}\\
\langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathit{typ} \leadsto t\\
\overline{\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}\prime\prime}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j}\\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto E^{\mathrm{K}\prime\prime}, \Sigma^{\mathrm{N}\prime}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\
\texttt{<<no parses (char 44): </<(E\_t u+ E\_tj),<E\_k'',E\_a,E\_r,E\_e>>,t |- e***xpj : uj' gives expj', <S\_N''j,effect'j>,E\_t'j//j/> >>}\\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j}\\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}\prime} \uplus \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j})
\end{array}
}{
\langle E^{\mathrm{T}}, \langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle\rangle \vdash \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function\ forall}\ \overline{\mathit{quant\_item}_i}^{\,i}\ .\ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}
}\quad \text{CHECK\_}
$$

$$
\frac{
\begin{array}{l}
E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t\\
\overline{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle, t_1 \vdash \mathit{pat}_j : u_j \triangleright \mathit{pat}'_j, E^{\mathrm{T}}{}_j, \Sigma^{\mathrm{N}\prime}_j}^{\,j}\\
E^{\mathrm{T}\prime} \equiv (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \{\,\}, \Sigma^{\mathrm{N}}, \mathbf{Global}, t_1 \to t\ \mathit{effect}\})\\
\texttt{<<no parses (char 28): </<(E\_t u+ E\_tj),E\_d>,t |- e***xpj : uj' gives exp', <S\_N'j,effect'j>,E\_t'j//j/> >>}\\
\mathit{effect} \equiv \uplus\, \overline{\mathit{effect}'_j}^{\,j}\\
\Sigma^{\mathrm{N}} \equiv \mathbf{resolve}\,(\uplus\, \overline{\Sigma^{\mathrm{N}\prime}_j \uplus \Sigma^{\mathrm{N}\prime\prime}_j}^{\,j})
\end{array}
}{
\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}_j = \mathit{exp}_j}^{\,j} \triangleright \mathbf{function}\ \ \mathit{typ}\ \mathbf{effect}\ \mathit{effect}\ \overline{\mathit{id}\ \mathit{pat}'_j = \mathit{exp}'_j}^{\,j}, E^{\mathrm{T}\prime}, \Sigma^{\mathrm{N}}
}\quad \text{CHECK\_FD\_FUNCTION\_NO\_SPEC2}
$$

$\boxed{E \vdash \mathit{val\_spec} \triangleright E^{\mathrm{T}}}$ Check a value specification

$$
\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val}\ \mathit{typschm}\ \mathit{id} \triangleright \{\mathit{id} \mapsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}, \mathbf{Global}, t\}}\quad \text{CHECK\_SPEC\_VAL\_SPEC}
$$

$$
\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{val\,extern}\ \mathit{typschm}\ \mathit{id} = \mathit{string} \triangleright \{\mathit{id} \mapsto E^{\mathrm{K}}{}_1, \Sigma^{\mathrm{N}}, \mathbf{Extern}\,, t\}}\quad \text{CHECK\_SPEC\_EXTERN}
$$

$\boxed{E^{\mathrm{D}} \vdash \mathit{default\_spec} \triangleright E^{\mathrm{T}}, E^{\mathrm{K}}{}_1}$ Check a default typing specification

$$
\frac{E^{\mathrm{K}} \vdash \mathit{base\_kind} \leadsto k}{\langle E^{\mathrm{K}}, E^{\mathrm{A}}, E^{\mathrm{R}}, E^{\mathrm{E}}\rangle \vdash \mathbf{default}\ \mathit{base\_kind}\ \mathit{'x} \triangleright \{\,\}, \{\mathit{'x} \mapsto k\ \mathbf{default}\}}\quad \text{CHECK\_DEFAULT\_KIND}
$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typschm} \leadsto t, {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}}{E^{\mathrm{D}} \vdash \mathbf{default}\, \mathit{typschm}\ \mathit{id} \triangleright \{\mathit{id} \mapsto {E^{\mathrm{K}}}_1, \Sigma^{\mathrm{N}}, \mathbf{Default}, t\}, \{\,\}} \quad \text{CHECK\_DEFAULT\_TYP}$$

$\boxed{E \vdash \mathit{def} \triangleright \mathit{def}', E'}$ Check a definition

$$\frac{E^{\mathrm{D}} \vdash \mathit{type\_def} \triangleright E}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{type\_def} \triangleright \mathit{type\_def}, \langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \uplus E} \quad \text{CHECK\_DEF\_TDEF}$$

$$\frac{E \vdash \mathit{fundef} \triangleright \mathit{fundef}', E^{\mathrm{T}}, \Sigma^{\mathrm{N}}}{E \vdash \mathit{fundef} \triangleright \mathit{fundef}', E \uplus \langle E^{\mathrm{T}}, \epsilon\rangle} \quad \text{CHECK\_DEF\_FDEF}$$

$$\frac{\begin{array}{l} E \vdash \mathit{letbind} \triangleright \mathit{letbind}', \{\mathit{id}_1 \mapsto t_1, \,..\,, \mathit{id}_n \mapsto t_n\}, \Sigma^{\mathrm{N}}, \mathbf{pure}, E^{\mathrm{K}} \\ {\Sigma^{\mathrm{N}}}_1 \equiv \mathbf{resolve}\,(\Sigma^{\mathrm{N}}) \end{array}}{E \vdash \mathit{letbind} \triangleright \mathit{letbind}', E \uplus \langle\{\mathit{id}_1 \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_1, \,..\,, \mathit{id}_n \mapsto E^{\mathrm{K}}, \Sigma^{\mathrm{N}}, \mathbf{None}, t_n\}, \epsilon\rangle} \quad \text{CHECK\_DEF\_VDEF}$$

$$\frac{E \vdash \mathit{val\_spec} \triangleright E^{\mathrm{T}}}{E \vdash \mathit{val\_spec} \triangleright \mathit{val\_spec}, E \uplus \langle E^{\mathrm{T}}, \epsilon\rangle} \quad \text{CHECK\_DEF\_VSPEC}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{default\_spec} \triangleright {E^{\mathrm{T}}}_1, {E^{\mathrm{K}}}_1}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathit{default\_spec} \triangleright \mathit{default\_spec}, \langle (E^{\mathrm{T}} \uplus {E^{\mathrm{T}}}_1), E^{\mathrm{D}} \uplus \langle {E^{\mathrm{K}}}_1, \{\,\}, \{\,\}, \{\,\}\rangle\rangle} \quad \text{CHECK\_DEF\_DEFAULT}$$

$$\frac{E^{\mathrm{D}} \vdash \mathit{typ} \leadsto t}{\langle E^{\mathrm{T}}, E^{\mathrm{D}}\rangle \vdash \mathbf{register}\, \mathit{typ}\ \mathit{id} \triangleright \mathbf{register}\, \mathit{typ}\ \mathit{id}, \langle (E^{\mathrm{T}} \uplus \{\mathit{id} \mapsto \mathbf{register}\, \langle t\rangle\}), E^{\mathrm{D}}\rangle} \quad \text{CHECK\_DEF\_REGISTER}$$

$\boxed{E \vdash \mathit{defs} \triangleright \mathit{defs}', E'}$ Check definitions, potentially given default environment of built-in library

$$\frac{\begin{array}{l} E \vdash \mathit{def} \triangleright \mathit{def}', E_1 \\ E \uplus E_1 \vdash \overline{\mathit{def}_i}^{\,i} \triangleright \overline{\mathit{def}'_i}^{\,i}, E_2 \end{array}}{E \vdash \mathit{def}\ \overline{\mathit{def}_i}^{\,i} \triangleright \mathit{def}'\ \overline{\mathit{def}'_i}^{\,i}, E_2} \quad \text{CHECK\_DEFS\_DEFS}$$

6 Sail operational semantics {TODO}