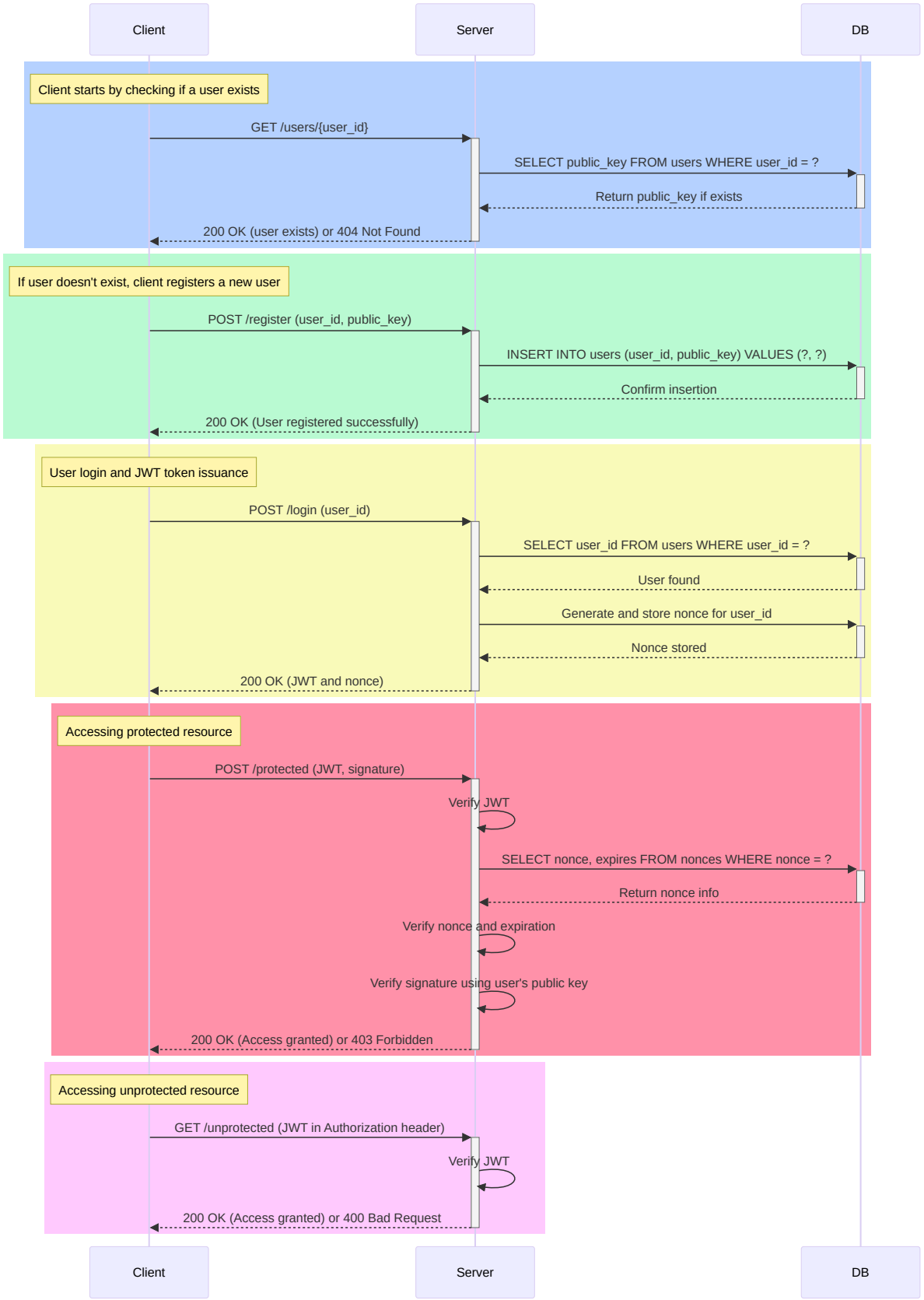
Design a secure login and token verification mechanism for a FastAPI backend using PGP or a similar key exchange mechanism to sign JWT tokens, ensuring that only a specific user can use a given token.

Link: https://github.com/AbhinavMir/signjwt

Summary: In the FastAPI application, secure user authentication is achieved through RSA key pairs and JWTs. Upon registration, a user's RSA public key is stored in our database. For login, a unique nonce and a JWT containing this nonce and user information are generated, signed with a secret key. Accessing protected endpoints requires users to submit this JWT along with a digital signature, which is verified using the user's stored public key to confirm identity and ensure message integrity.



Breakdown

1. RSA Key Generation

When a new user is registered, an RSA key pair is generated on the client's machine. RSA is an asymmetric cryptographic algorithm used for secure data transmission. The keys are stored on the client's local machine, and in our case, we save it in `keys` folder.

- **Key Generation**:

$$(e, d, n) \leftarrow RSAKeyGen(bitsize)$$

- e is the public exponent (usually 65537).
- d is the private exponent.
- n is the modulus, the product of two large prime numbers p and q.

The user's public key $K_{pub} = (e, n)$ is stored in the database, while the private key $K_{priv} = (d, n)$ is kept securely by the user.

2. User Registration

Upon registration, the user's public key and user ID are stored in the database. The server expects the client to send a JSON-formatted input of username and public key.

- **Database Operation**:

$$INSERTINTO_{users}(userid, publickey)VALUES(?, ?)$$

3. Login and Nonce Generation

During login, a nonce—a unique, random string used to ensure freshness of the request—is generated.

- **Nonce Generation**:

$$nonce \leftarrow SecureRandomString(16)$$

The nonce, along with its expiration time, is associated with the user and stored in the database.

- **JWT Generation**:

$$JWT = Encode(payload, SECRET\_KEY, algorithm)$$

  - `payload` includes the user ID, the expiration time, and the nonce.
  - The JWT is signed using the HMAC with SHA-256 algorithm, where `SECRET_KEY` is the key.

4. Accessing Protected Resources

When accessing protected resources, the user must provide a JWT and a digital signature proving possession of their private key. We do this with the following piece of code on the client's side.

```
message = f"{nonce}{jwt}".encode()
signature = private_key_obj.sign(
        message,
        padding.PSS(
            mgf=padding.MGF1(hashes.SHA256()),
            salt_length=padding.PSS.MAX_LENGTH
        ),
        hashes.SHA256()
    )
```

- **JWT Verification**:

$$\text{Verify}(JWT, SECRET\_KEY, algorithm)$$

- **Signature Verification**:
  - The user creates a signature using their private RSA key on a message that concatenates the nonce and the JWT.

$$signature \leftarrow RSASign(K_{priv}, \text{hash}(nonce||JWT))$$

  - The server verifies the signature using the user's public key.

$$RSASignVerify(K_{pub}, \text{hash}(nonce||JWT), signature)$$