OPTION PRICING MODELS
Midterm Report

Team E-20

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INTRODUCTION

The Binomial and Black-Scholes models are two primary methods for pricing options in financial markets. Both models have their unique advantages, disadvantages, and applications. This report will provide a qualitative comparison of these models, discuss their general workings, and illustrate their effectiveness using a case study where both methods were applied to the same assets.

BLACK-SCHOLES MODEL

General Working

The Black-Scholes model, developed by Fischer Black, Myron Scholes, and Robert Merton in 1973, is used to estimate the theoretical value of European-style options. It assumes that the price of the underlying asset follows a lognormal distribution and evolves over time according to a geometric Brownian motion with constant drift and volatility. The model calculates the price of a call option using the following formula:

$$C = SN(d_1) - Ke^{-rt}N(d_2)$$

Where:

- $d_1 = \frac{\ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})t}{\sigma\sqrt{t}}$
- $d_2 = d_1 - \sigma\sqrt{t}$
- C = Call option price
- S = Current stock price
- K = Strike price
- r = Risk-free interest rate
- t = Time to maturity
- N = Cumulative standard normal distribution

Advantages:

- Widely Used and Accepted: Provides a standardized framework for option pricing.
- Efficient Calculation: Once the inputs are known, the model provides a quick and efficient way to calculate the option .
- Foundation for Other Models**: Forms the basis for more complex derivative pricing models.

Disadvantages:

- Assumptions: Assumes constant volatility and risk-free rate, no dividends, and no early exercise (only applicable to European options).
- Market Realities: Does not account for market frictions such as taxes, transaction costs, and liquidity constraints.
- Volatility Skew: Fails to accurately predict implied volatilities, leading to discrepancies known as the volatility smile or skew.

BINOMIAL MODEL

General Working

The Binomial option pricing model, developed by Cox, Ross, and Rubinstein in 1979, uses a discrete-time approach to model the underlying asset's price over multiple periods. It constructs a binomial tree, where each node represents a possible price of the underlying asset at a given point in time. At each node, the model considers two possible moves: up or down.

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}}$$

Where u and d are the factors by which the price can move up or down, and Δt is the length of each time step.

Advantages:

- Flexibility: Can handle American options, dividends, and varying volatility and interest rates.
- Intuitive and Visual: The tree structure provides a clear and intuitive way to understand how the option value evolves over time.
- Arbitrage-Free: By iterating over small time steps, it ensures that the pricing is consistent with the absence of arbitrage opportunities.

Disadvantages:

- Computational Complexity: Requires more computational power, especially for options with long maturities or frequent time steps.
- Simplistic Assumptions: Assumes discrete time steps, which may not capture the continuous nature of financial markets accurately.

Case Study: Applying Both Models to the Same Assets

Consider an option on a stock currently priced at $100, with a strike price of $100, volatility of 30%, a risk-free interest rate of 5%, and time to maturity of one year. The option does not pay dividends.

Black-Scholes Model Application:

Using the Black-Scholes formula, we calculate the call option price as follows:

$$d_1 = \frac{\ln(\frac{100}{100}) + (0.05 + \frac{0.3^2}{2}) \times 1}{0.3\sqrt{1}} = \frac{0.05+0.045}{0.3} = 0.3167$$

$$d_2 = 0.3167 - 0.3 = 0.0167$$

$$C = 100 \times N(0.3167) - 100 \times e^{-0.05\times 1} \times N(0.0167)$$

Assuming the cumulative normal distribution values:

$N(0.3167) \approx 0.624$

$N(0.0167) \approx 0.507$

$$C = 100 \times 0.624 - 100 \times 0.9512 \times 0.507 \approx 62.4 - 48.2 = 14.2$$

The Black-Scholes price of the call option is approximately $14.20.

Binomial Model Application:

- $u = e^{\sigma\sqrt{\Delta t}} = e^{0.3\sqrt{0.5}} = e^{0.3\times 0.7071} \approx 1.2311$
- $d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.3\sqrt{0.5}} = e^{-0.3\times 0.7071} \approx 0.8123$

Stock Prices at Each Node

1. **Initial Price:**
   - $S = 100$
2. **After First Step:**
   - Up: $S_u = S \times u = 100 \times 1.2311 = 123.11$
   - Down: $S_d = S \times d = 100 \times 0.8123 = 81.23$
3. **After Second Step:**
   - Up-Up: $S_{uu} = S_u \times u = 123.11 \times 1.2311 = 151.29$
   - Up-Down (or Down-Up): $S_{ud} = S_{du} = S = 100$
   - Down-Down: $S_{dd} = S_d \times d = 81.23 \times 0.8123 = 65.96$

Option Values at Maturity

1. **At T = 1 Year (End of Second Step):**
   - $V_{uu} = \max(0, S_{uu} - K) = \max(0, 151.29 - 100) = 51.29$
   - $V_{ud} = V_{du} = \max(0, S_{ud} - K) = \max(0, 100 - 100) = 0$
   - $V_{dd} = \max(0, S_{dd} - K) = \max(0, 65.96 - 100) = 0$

Option Values at T = 0.5 Years (First Step)

1. **Calculate the present value of the expected option values using risk-neutral probabilities:**
   - $V_u = e^{-r\Delta t}\,(p \times V_{uu} + (1-p) \times V_{ud})$
     - $V_u = e^{-0.05\times0.5}\,(0.5081 \times 51.29 + 0.4919 \times 0)$
     - $V_u = e^{-0.025}\,(0.5081 \times 51.29) \approx 0.9753\,(26.07) \approx 25.42$
   - $V_d = e^{-r\Delta t}\,(p \times V_{du} + (1-p) \times V_{dd})$
     - $V_d = e^{-0.05\times0.5}\,(0.5081 \times 0 + 0.4919 \times 0) = 0$

2. **Initial Option Value:**
   - $V_0 = e^{-r\Delta t}\,(p \times V_u + (1-p) \times V_d)$
     - $V_0 = e^{-0.05\times0.5}\,(0.5081 \times 25.42 + 0.4919 \times 0) \approx 0.9753\,(12.91) \approx 12.60$

Comparison and Conclusion

Effectiveness:

- Accuracy: The Black-Scholes model provides a higher theoretical price due to its assumption of continuous price changes and constant volatility. However, it may be less accurate for American options or in volatile markets.
- Flexibility: The Binomial model is more flexible, handling early exercise, varying volatility, and dividends, making it suitable for a broader range of options.

Application:

- Black-Scholes: Best suited for European options on non-dividend-paying stocks with stable volatility.
- Binomial: More appropriate for American options, dividend-paying stocks, and scenarios with changing market conditions.

In practice, the choice of model depends on the specific characteristics of the option being priced and the market conditions. Both models provide valuable insights, but the binomial model's flexibility often makes it the preferred choice for complex options.
By understanding the strengths and limitations of each model, investors and traders can make more informed decisions and better manage their risk exposures.

THANK YOU