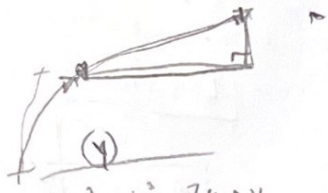
SC01 – 2D Motion

Period 1 Name Allan Zhou

1. A spit-wad is shot from the ground into the air. At a height of 14.1 m, its velocity is observed to be $\vec{v} = 17.6\hat{\imath} + 6.1\hat{\jmath}$ in meters per second.
   a. To what maximum height does the spit-wad rise?
   b. What total horizontal distance does the spit-wad travel?
   c. What is the magnitude and the direction of the spit-wad's velocity just before it hits the ground?



$V_f^2 = V_0^2 + 2a\Delta y$

$(6.1)^2 = (V_0)^2 + 2(-9.8)(14.1)$

$V_0 = 17.708 \text{ m/s}$

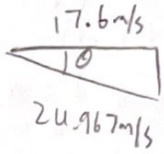
$t = 1.8069 \text{ sec}$

$H_{max} = \frac{1}{2} g t^2 = 15.998 \text{ m}$

$d = s \cdot t$

$= (17.6 \text{ m/s})(1.8069 \times 2)$

$= 63.994 \text{ m}$



$\theta = \tan^{-1}\left(\frac{-17.708}{17.6}\right) =$

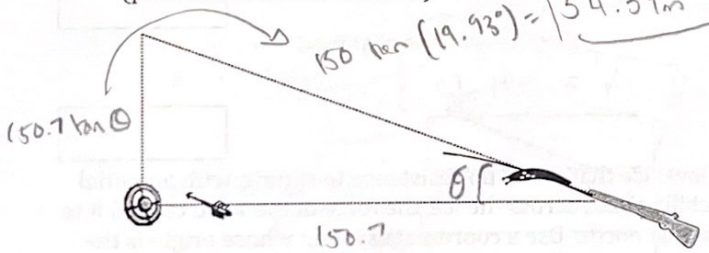
24.97 m/s @ −45.175°

a.

b.

c.

2. A rifle that shoots slugs at 48 m/s is to be aimed at a target of salt that is 150.7 m away and level with the rifle. How high (in meters) above the target must the rifle barrel be pointed so that the slug hits the target? (picture not drawn to scale!)



	x	y
		$V_0 = V\sin\theta$
		V_f
	$S = V\cos\theta$	Δy
	t	$a = -g$
		t

$\Delta y = V_0 t + \frac{1}{2} a t^2$ $\quad V\sin\theta \cdot t + (-4.9)(t)^2 = 0$

$V\sin\theta = 4.9t$

$\Delta x = s \cdot t = 150.7 = (V\cos\theta)(t) = 150.7$

$\theta = 19.93°$

$48\sin\theta = 4.9\left(\frac{150.7}{48\cos\theta}\right)$

$\frac{48^2}{2} \cdot \sin 2\theta = 4.9(150.7)$

3. A Cessna 172, diving at an angle of 53.0 degrees with the vertical, releases a bag of flour at an altitude of 830 m. The flour hits the ground 8.00 s after being released.
   a. What is the speed of the Cessna?
   b. How far did the flour bag travel horizontally during its flight?
   c. What are the horizontal and vertical components of its velocity just before striking the ground and exploding in a plume of white starch?

x	y
d=	$\Delta y = -830m$
$s = V_x \cos\theta$	V_y
$t = 8s$	$V_F = V_0 + at = V_0 - 8g$
	$a = -g$
	$t = 8s$

$\Delta y = \left(\frac{V_0 + V_F}{2}\right)t = V_0 t + \frac{1}{2}gt^2$

$-830 = \left(\frac{V_0 + V_0 - 8g}{2}\right)8$

$V_{0y} = -64.55 m/s$

$V_0 = V_{0y} / \sin 37° = 107.259 m/s$

$d = s \cdot t$
$= (107.259 \cos\theta)(8)$
$= 685.286m$

$V_{Fy} = V_0 + at$
$= V_0 \sin\theta - g8$
$= 142.95 m/s$

a.

b.

c. $\langle 85.6, 142.95 \rangle$

4. The velocity of a disturbed marmot in meters per second is $v(t) = 7 - 4t$, where t is in seconds.
   a. Find the position function and use it to find the displacement of the troubled marmot during the interval $t = 2s$ to $t = 6s$.

$\Delta x = \int_2^6 v(t)\,dt$

$= 7t - 2t^2 \Big|_2^6$

$= -26.67m$

   b. What is the average velocity for this interval? (Be careful! Many of you missed this the first (and second) (and third) (and fourth) (and fifth) times you saw it!)

$\frac{\Delta x}{\Delta t} = \frac{-26.67m}{\text{not used}} = -6.67 m/s$

   c. What is the acceleration of the marmot?

$a = -4 m/s^2$

5. A frightened chinchilla runs onto a large area of level ice that offers no resistance to sliding, with an initial velocity of 8.0 m/s toward the east. As the chinchilla slides across the ice, the force of the wind caused it to have a constant acceleration of 1.4 m/s², directed due north. Use a coordinate system whose origin is the chinchilla's initial position on the ice with the positive x axis directed toward the east. In unit vector notation, what are the chinchilla's....
   a. velocity when it has slid for 3 seconds?

y: $V_f = V_0 + at = 0 + (1.4)3 = 4.2 m/s$

$V = 8\hat{i} + 4.2\hat{j}$

   b. position when it has slid for 3 seconds?

$\Delta x = d = s \cdot t = (8 m/s)(3 sec) = 24m$

$\Delta y = \cancel{V_0 t} + \frac{1}{2}at^2 = 6.3m$

$\rho = 24\hat{i} + 6.3\hat{j}$

6. You "kick" (probably not a kick you want to get in the way of) a rock horizontally off a 40.0 m high cliff into a pool of water. If you hear the sound of the splash 5.00 s later, what was the velocity of the rock when it hits the water? Assume the speed of sound in air to be 343 m/s.

$\Delta y = v_0 t + \frac{1}{2}at^2$

$-40 = \frac{1}{2}(-9.8)(t)^2$

$t = 2.85714 \text{ sec}$

$d_{xy} = (343 \text{ m/s})(5 - 2.85714)$

$= 735 \text{ m}$

$V_p = at + V_0 = 28 \text{ m/s}$

$d_x = 733.911 \text{ m}$

$= s_x t$

$= (2.857)(s)$

$s = 256.882$

257.25 m/s, 21 m/s, 735, 733.911

258.44 m/s, 6.22° S of E

256.882, (9.8)(2.857)

7. Add the following vectors and record your answer as magnitude and angle.
   - 16 m/s, 15 degrees E of S
   - 25 m/s, 40 degrees N of E
   - 61 m/s, 30 degrees W of S
   - $(16\hat{\imath} - 3\hat{\jmath})$m/s

55.908 @ 80.95° S of E

8. Find $\frac{dy}{dx}$ for each of the following equations.
   a. $y = 20x^6 + 3x^4 + 2x + 6$ — $120x^5 + 12x^3 + 2$
   b. $y = \sqrt{x} + 12$ — $x^{\frac{1}{2}}$ — $\frac{1}{2}x^{-\frac{1}{2}}$
   c. $y = 14x^6$ — $84x^5$

9. Evaluate the following indefinite integrals.
   a. $y = \int 4x\,dx$ — $2x^2 + C$
   b. $y = \int (8x^3 + 6x^2)\,dx$ — $2x^4 + 2x^3 + C$
   c. $y = \int 3x^{-2}\,dx$ — $-3x^{-1} + C$
   d. $y = \int \frac{4}{x^2}\,dx$ — x^{-2} — $-x^{-1} + C$

10. Evaluate the following definite integrals.
   a. $y = \int_2^8 (2x + 3)\,dx$ — $x^2 + 3x \Big|_2^8$ = 78
   b. $y = \int_{-4}^2 (3x^2)\,dx$ — $x^3 \Big|_{-4}^2$ — $8 - (-64)$ — 72
   a. $y = \int_1^8 (16x^3 + 9x^2)\,dx$ — $4x^4 + 3x^3 \Big|_1^8$ = 17913
   c. $y = \int_0^8 4\,dx$ — $4x \Big|_0^8$ — 32

11. The velocity of a Bolt as a function of time is given by the formula:

$12\frac{m}{s^4} - 3\,m/s$

$v(t) = at^3 - bt^2$, where $a = 12\ ^m/_{s^4}$ and $b = 3\ ^m/_{s^3}$

a. What is Usain's average velocity between 2.0 and 6.0 seconds?

$\frac{\Delta x}{\Delta t} = \frac{3t^4 - 3t^3\Big|_2^6}{4} = \frac{[3(6^4) - 6^3] - [3(2)^4 - 2^3]}{4}$

908 m/s

b. What is the acceleration of the Bolt at 4 seconds?

$a = 3at^2 - 2bt$

$3(12)(16)\frac{m}{s^2} - 2(3)(4)\frac{m}{s^2}$

$= 552\ m/s^2$