

Tutorial 3

Design and analysis of algorithms [DAA]

Que 1 Write linear search pseudocode to search an element in a sorted array with minimum comparisons.

Ans: 1

```
boolean linear_Search(int a[], int e, int n)
{
    int flag = 0;
    for(int i = 0; i < n; i++)
    {
        if(a[i] == e)
            flag = 1;
        else
            flag = 0;
    }
    if(flag == 1)
        return true;
    else
        return false;
}
```

Que 2 Write pseudocode for iterative and recursive insertion sort. Insertion sort is called online sorting. Why? What about other sorting algorithms that has been discussed in lectures?

Ans: 2 Iterative -

```
void insertionSort(int a[], int n)
{
    for(int i = 1; i < n; i++)
    {
        int value = a[i];
        int j = i;
```

```

while(j > 0 && arr[j-1] > value)
{
    arr[j] = arr[j-1];
    j--;
}
arr[j] = value;
}
}

```

Recursive -

```

void insertionSort(int arr[], int i, int n)
{
    int value = arr[i];
    int j = i;
    while(j > 0 && arr[j-1] > value)
    {
        arr[j] = arr[j-1];
        j--;
    }
    arr[j] = value;
    if(i+1 <= n)
    {
        insertionSort(arr, i+1, n);
    }
}

```

Insertion sort is called an online sorting algorithm because insertion sort considers an input element per iteration and produces a partial solution without considering future elements.

Ques 3 Complexity of all sorting algorithms that has been discussed in lectures

	Best	Average	Worst
<u>Ans: 3</u> Bubble sort -	$O(n^2)$	$O(n^2)$	$O(n^2)$
Selection sort -	$O(n^2)$	$O(n^2)$	$O(n^2)$
Insertion sort -	$O(n)$	$O(n^2)$	$O(n^2)$
Heap Sort -	$O(n \log(n))$	$O(n \log n)$	$O(n \log n)$
Quick Sort -	$O(n \log n)$	$O(n \log n)$	$O(n^2)$
Merge Sort -	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Count Sort -			

Ques 4 Divide all the sorting algorithms into inplace / stable / online sort

Ans: 4

Sorting algorithm	In-place	Stable	Online
Bubble sort	Yes	Yes	No
Selection sort	Yes	No	No
Insertion sort	Yes	Yes	Yes
Quick sort	Yes	No	No
Merge sort	No	Yes	No
Heap sort	Yes	No	No

Ques 5 Write recursive/iterative pseudo code for binary search.
What is the Time and Space complexity of Linear and Binary Search?

Ans: 5 Iterative —

```
int binarySearch(int a[], int x)
{
    int low = 0, high = a.length - 1;
    while (low <= high)
    {
        int mid = (low + high) / 2;
        if (x == a[mid])
            return mid;
        else if (x < a[mid])
            high = mid - 1;
        else
            low = mid + 1;
    }
    return -1;
}
```

Recursive —

```
int binarySearch(int a[], int low, int high, int x)
{
    if (low > high)
        return -1;
    int mid = (low + high) / 2;
    if (x == a[mid])
        return mid;
    else if (x < a[mid])
        return binarySearch(a, low, mid - 1, x);
    else
        return binarySearch(a, mid + 1, high, x);
}
```

Time complexity \rightarrow Iterative - $O(\log N)$
Recursive - $O(\log N)$

Space complexity \rightarrow Iterative - $O(1)$
Recursive - $O(\log N)$

Ques 6 Write recurrence relation for binary recursive search.

Recurrence relation - $T(n) = T(n/2) + 1$

Derivation -

$$1^{\text{st}} \text{ step} - T(n) = T(n/2) + 1$$

$$2^{\text{nd}} \text{ step} - T(n/2) = T(n/4) + 1 \dots \dots [T(n/4) = T(n/2^2)]$$

$$3^{\text{rd}} \text{ step} - T(n/4) = T(n/8) + 1 \dots \dots [T(n/8) = T(n/2^3)]$$

$$\vdots$$
$$k^{\text{th}} \text{ step} = T(n/2^{k-1}) = T(n/2^k) + 1 \text{ (K times)}$$

Adding all equations

$$T(n) = T(n/2^k) + k$$

$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log n = k$$

$$k = \log n$$

$$T(n) = T(1) + \log n$$

$$T(n) = O(\log n)$$

Que 7 Find two indexes such that $A[i] + A[j] = k$ in minimum time complexity

vector<int> find(arr[], k, n)

{

vector<int> sol;

for i = 0 to n-1

for j = 0 to n

if $arr[i] + arr[j] = k$

sol.pushback(i)

sol.pushback(j)

return sol.

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Ques 8 Which sorting is best for practical uses? Explain

Ans Quicksort is the fastest general-purpose sort. In most practical situations quicksort is a method of choice. If stability is important and space is available, merge sort might be best. In some performance-critical applications, the focus may be on just sorting numbers, so it is reasonable to avoid the costs of using references and sort primitive types instead.

Ques 9 What do you mean by number of inversions in an array? Count the number of inversions in Array $arr[] = \{7, 21, 31, 8, 10, 1, 20, 6, 4, 5\}$ using merge sort.

Ans: 9 Inversion count for an array indicates how far or close the array is from being sorted. If the array is already sorted then the inversion count is 0, but if the array is sorted in reverse order, the inversion count is the maximum.

Pair of inversions in array $arr[] = \{7, 1\} \{7, 6\} \{7, 4\} \{7, 5\} \{21, 8\} \{21, 10\}$
 $\{21, 1\} \{21, 20\} \{21, 6\} \{21, 4\} \{21, 5\}$
 $\{31, 8\} \{31, 10\} \{31, 1\} \{31, 20\} \{31, 6\}$
 $\{31, 4\} \{31, 5\} \{8, 1\} \{8, 6\} \{8, 4\} \{8, 5\}$
 $\{10, 1\} \{10, 6\} \{10, 4\} \{10, 5\} \{20, 6\} \{20, 4\}$
 $\{20, 5\} \{6, 4\} \{6, 5\}$

Count = 31

Que 10 In which cases Quick sort will give the best and the worst case time complexity?

Ans: 10 Best case - The best case occurs when the partition process always picks the middle element as pivot. Following is the recurrence relation for best case.

$$T(n) = 2T(n/2) + O(n).$$

Worst case - The worst case occurs when the partition process always picks greatest or smallest element as pivot. In above partition strategy is considered where last element is always picked as pivot, the worst case would occur when the array is already sorted in increasing or decreasing order.

Que 11 Write Recurrence relation of Merge and Quick sort in best and worst case? What are the similarities and differences between complexities of two algorithms and why?

Ans: 11 Quick Sort \rightarrow

Recurrence relation - Best case - $T(n) = 2T(n/2) + O(n)$

Worst case - $T(n) = T(n-1) + O(n)$

Merge Sort \rightarrow

Recurrence relation - Best case - $2T(n/2) + O(n)$

Worst case - $2T(n/2) + O(n)$

Time complexity -

	Best Case	Worst Case
Quick sort	$O(n \log n)$	$O(n^2)$
Merge sort.	$O(n \log n)$	$O(n \log n)$

Ques 12 Selection sort is not stable by default but can you write a version of stable selection sort.

Ans

```
void stableSelectionSort(int a[], int n)
```

```
{
    for (int i = 0; i < n - 1; i++)
    {
        int min = i;
        for (int j = i + 1; j < n; j++)
        {
            if (a[min] > a[j])
                min = j;
        }
        int key = a[min];
        while (min > i)
        {
            a[min] = a[min - 1];
            min--;
        }
        a[i] = key;
    }
}
```

Ques 13 Bubble sort scans whole array even when array is sorted. Can you modify the bubble sort so that it doesn't scan the whole array once it is sorted.

Ans

```
void bubbleSort(int a[], int n)
```

```
{
    int flag, temp;
    for (int i = 0; i < n - 1; i++)
    {
        flag = 0;
        for (j = 0; j < n - i - 1; j++)
        {
```

```

flag = 1;
if (arr[j] > arr[j+1])
{
    temp = arr[j];
    arr[j] = arr[j+1];
    arr[j+1] = temp;
}
}
if (flag == 0)
    break;
}
}

```

Ques 14 Your computer has a RAM (Physical memory) of 2 GB and you are given an array of 4 GB of sorting. Which algorithm you are going to use for this purpose and Why? Also explain the concept of External and Internal Sorting.

Ans: 14 As the size of given array exceeds the size of RAM ~~therefore~~ ^{therefore} we will use K-way merge sort as sorting technique as it takes a part of array & sort it, whole array is not loaded into main memory altogether.

External sorting - This algo loads a part of array and sort it whole array is not loaded into the RAM.
 especially used to sort array of large size.
 eg - k-way merge sort.

Internal sorting - This algo needs whole array to reside in RAM during execution
 eg - bubble sort.