A Multi-Dimensional Stochastic-Weighted Framework for Persona-Based Vendor Scoring (VPS) *

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Abstract

The reconciliation of invoices and purchase orders is a critical, error-prone process. The existence of a comprehensive taxonomy of 82 distinct discrepancy types provides a structured foundation for quantitative analysis. This paper moves beyond simple error logging to propose a state-of-the-art mathematical model, the **Vendor Persona Score (VPS)**. This model quantifies vendor reliability by aggregating discrepancy data through a multi-dimensional framework that is sensitive to specific **buyer personas**. We introduce concepts of **Persona-Dependent Severity Weighting** (to differentiate error impact based on buyer-specific risk profiles), **Time-Decay Frequency** (to prioritize recent behavior), and an **Aggregated Risk Postulate (ARP)**. The final VPS is a normalized, continuous score from 0 to 100, enabling robust, data-driven, and context-aware comparison of vendors.

*This model is derived from the foundational taxonomy of 82 discrepancy types presented in "A Comprehensive Taxonomy of Invoice and Purchase Order Discrepancies".

Contents

1 Introduction

The procurement-to-pay (P2P) cycle is fundamental to corporate operations, yet it is fraught with potential for error. The foundational research identifying 82 distinct discrepancy types provides a granular vocabulary for these issues. However, organizations lack a method to synthesize this raw data into an actionable, comparative metric. A "good" vendor is not necessarily one with zero errors, but one with a low frequency of high-severity errors.

Crucially, the "severity" of an error is not universal; it is contingent upon the buyer's unique risk profile. A compliance-focused pharmaceutical buyer, for example, views regulatory discrepancies with far more severity than a margin-focused retail buyer, who may be more sensitive to unit price variations.

Currently, assessing vendor performance is often subjective or based on simple, unweighted error counts. This paper addresses this gap by proposing a dynamic mathematical model to compute a **Vendor Persona Score (VPS)**. This score provides a single, normalized metric of vendor reliability, $\mathcal{S}(v, T, p)$, which is a function of the vendor (v), time (T), and the specific buyer persona (p).

2 Model Formulation

We define a vendor's "persona" as a quantitative measure of their historical reliability, evaluated through the lens of a specific buyer persona. The model is constructed in four sequential layers.

2.1 Layer 1: The Persona-Based Discrepancy Severity Matrix ($\boldsymbol{\Omega}_p$)

First, we posit that the severity of a discrepancy is not a global constant but is dependent on the buyer's "persona" (p). Let P be the set of all buyer personas (e.g., p_c=Compliance-Focused, p_m=Margin-Sensitive, p_o=Operational-Speed-Focused).

Let $\mathcal{D} = \{d_1, d_2, \ldots, d_{82}\}$ be the set of 82 discrepancy types. We define a **Persona-Based Severity Weighting function**, Ω, that maps each discrepancy type d_i and persona p to a positive real number $\omega_{i,p}$.

$$\omega_{i,p} = \Omega(d_i, p) \quad \text{where } \omega_{i,p} \in \mathbb{R}^+ \tag{1}$$

This allows the model to reflect unique risk profiles:

- For a **"Compliance-Focused" persona** (p_c), the weight ω_{i,p_c} would be exceptionally high for "Compliance & Regulatory Issues" and "Vendor Mismatches".
- For a **"Margin-Sensitive" persona** (p_m), the weight ω_{i,p_m} would be higher for "Unit price variance", "Missing discounts", and "Unauthorized Freight Charges".

For each persona p, this generates a constant **Severity Vector** $\boldsymbol{\Omega}_p = [\omega_{1,p}, \omega_{2,p}, \ldots, \omega_{82,p}]$.

2.2 Layer 2: The Time-Decay Frequency Vector ($\mathbf{\Phi}(v, T)$)

Second, an error's relevance diminishes over time. This layer is *independent* of the buyer persona, as it only calculates the historical frequency of errors for a given vendor v.

Let $E_i(v)$ be the set of all timestamps $\{t_1, t_2, \ldots, t_n\}$ corresponding to each occurrence of discrepancy d_i by vendor v. Let T be the current time (evaluation time). We define the **Time-Decay Weighted Frequency** Φ_i for discrepancy d_i using an exponential decay function:

$$\Phi_i(v, T) = \sum_{j \in E_i(v)} \exp(-\lambda(T - t_j)) \tag{2}$$

Here, λ (lambda) is the **decay-rate hyperparameter**. This calculation is performed for all 82 discrepancy types to generate the vendor's **Discrepancy Vector** at time T: $\mathbf{\Phi}(v, T) = [\Phi_1(v, T), \Phi_2(v, T), \ldots, \Phi_{82}(v, T)]$.

2.3 Layer 3: The Aggregated Risk Postulate (ARP)

Third, we combine the *persona-specific* severity of each error ($\omega_{i,p}$) with the vendor's dynamic, time-weighted frequency of committing that error (Φ_i). This yields a raw **Aggregated Risk Score (ARP)**, $R(v, T, p)$, for vendor v at time T, relative to persona p.

This is postulated as the dot product of the Persona Severity Vector and the Discrepancy Vector:

$$R(v, T, p) = \mathbf{\Omega}_p \cdot \mathbf{\Phi}(v, T) = \sum_{i=1}^{82} \omega_{i,p} \cdot \Phi_i(v, T) \tag{3}$$

This score, $R(v, T, p)$, represents the total weighted risk a vendor poses *to a specific buyer persona* at time T. This score is unbounded ($R \in [0, \infty)$).

2.4 Layer 4: The Normalized Vendor Persona Score (VPS) Theorem

Finally, to create an intuitive and comparable score, we map the unbounded raw risk score $R(v, T, p)$ to a bounded interval, [0, 100].

We propose the **Vendor Persona Scoring Theorem**, which defines the score $\mathcal{S}(v, T, p)$ using a negative exponential function.

$$\mathcal{S}(v, T, p) = 100 \cdot \exp(-\kappa \cdot R(v, T, p)) \tag{4}$$

This ensures that a risk score of 0 yields a perfect score of 100, and as risk R approaches infinity, the score $\mathcal{S}$ asymptotically approaches 0. The **sensitivity hyperparameter** κ (kappa) can also be made persona-dependent (κ_p) to reflect different risk tolerances, though for simplicity we present it as a global constant here.

3 Analysis of Model Components (WHY and HOW)

This section provides a detailed justification for each major formula in the model.

3.1 The Persona-Based Severity Weight ($\omega_{i,p}$)

$$\omega_{i,p} = \Omega(d_i, p) \tag{5}$$

- **WHY (Purpose):** This formula assigns a numerical value to the business impact of an error, *relative to a specific buyer's priorities.* It establishes that the "cost" of an error is subjective. A "Margin-Sensitive" buyer may tolerate a "Compliance" error that a "Pharmaceutical" buyer would find unacceptable, and vice-versa.

- **HOW (Derivation):** This is an **expert-driven parameterization**. For each persona p, a set of weights $\mathbf{\Omega}_p$ is derived by a consensus of stakeholders (e.g., procurement, finance, legal) representing that persona. They would classify each d_i into risk buckets and assign numerical weights that reflect their specific risk profile.

3.2 The Time-Decay Frequency (Φ_i)

$$\Phi_i(v, T) = \sum_{j \in E_i(v)} \exp(-\lambda(T - t_j)) \tag{6}$$

- **WHY (Purpose):** This calculates a "weighted count" of an error, valuing recent performance over past performance. It answers: "How much of a problem is this error *right now*?" This component remains independent of persona, as it is a factual representation of the vendor's actions.

- **HOW (Derivation):** This is derived from the **continuous compounding decay model**. For each error instance j, its "value" decays from 1 (at time t_j) towards 0. We **sum** the decayed values of all historical occurrences to get the total "present-day" frequency Φ_i.

3.3 The Aggregated Risk Postulate (R)

$$R(v, T, p) = \sum_{i=1}^{82} \omega_{i,p} \cdot \Phi_i(v, T) \tag{7}$$

- **WHY (Purpose):** This is the core aggregation step. It "rolls up" all 82 error frequencies into a **single, comprehensive risk number** that is **custom-tailored to the buyer's persona**. It combines the *persona-specific potential impact* ($\omega_{i,p}$) with the *observed frequency* (Φ_i).

- **HOW (Derivation):** This is a **weighted sum**, geometrically represented as a **dot product** of two vectors: $\mathbf{\Omega}_p$ (persona severity) and $\mathbf{\Phi}$ (vendor frequency). This is a standard and robust method for combining two vectors into a single scalar.

3.4 The Normalized VPS ($\mathcal{S}$)

$$\mathcal{S}(v, T, p) = 100 \cdot \exp(-\kappa \cdot R(v, T, p)) \tag{8}$$

- **WHY (Purpose):** This achieves **normalization and usability**. The raw risk score $R(v, T, p)$ is hard to interpret. This formula transforms it into an intuitive "report card" score from 0 to 100. It allows a buyer to ask: "Given my company's specific risk profile (persona p), what is the score for this vendor?"
- **HOW (Derivation):** This is an **exponential mapping function**.
  1. We start with the properties we need: (1) If $R = 0$, Score=100. (2) If $R > 0$, Score < 100. (3) If $R \to \infty$, Score $\to 0$.
  2. The function $f(R) = \exp(-R)$ satisfies properties 2 and 3.
  3. To satisfy property 1, we multiply by 100: $f(R) = 100 \cdot \exp(-R)$.
  4. To control sensitivity, we introduce κ, giving $100 \cdot \exp(-\kappa R)$. This gives a smooth, non-linear penalty.

4 Sample Working with Dummy Data

To demonstrate the model, we simulate a microcosm with 2 vendors (v_A, v_B), 2 buyer personas (p_c = Compliance-Focused, p_m = Margin-Sensitive), and 3 discrepancy types (d_1, d_2, d_3).

4.1 Step 0: Define Inputs

- **Evaluation Time:** $T = 100$ (e.g., Day 100)
- **Hyperparameters:** $\lambda = 0.05$ (decay rate), $\kappa = 0.1$ (sensitivity)
- **Discrepancy Types:**
  - d_1: "Missing discounts"
  - d_2: "E-invoicing non-compliance"
  - d_3: "Typing errors"
- **Persona Severity Vectors ($\mathbf{\Omega}_p = [\omega_1, \omega_2, \omega_3]$):**
  - $\mathbf{\Omega}_{p_c}$ (Compliance): $[2.0, 10.0, 1.0]$ (High penalty for d_2)
  - $\mathbf{\Omega}_{p_m}$ (Margin): $[10.0, 2.0, 1.0]$ (High penalty for d_1)
- **Vendor Error History ($E_i(v)$) (Timestamps of errors):**
  - **Vendor A (v_A):**
    * $E_1(v_A)$ (Discount): $\{90, 80, 70\}$ (3 recent-ish errors)

    * $E_2(v_A)$ (Compliance): $\{20\}$ (1 old error)
    * $E_3(v_A)$ (Typing): $\{95, 90, 85, 80\}$ (Many recent errors)
  - **Vendor B (v_B):**
    * $E_1(v_B)$ (Discount): $\{10\}$ (1 very old error)
    * $E_2(v_B)$ (Compliance): $\{95, 85, 75\}$ (3 recent-ish errors)
    * $E_3(v_B)$ (Typing): $\{50\}$ (1 old error)

4.2 Step 1: Calculate $\mathbf{\Phi}(v, T)$ (Time-Decay Frequencies)

We use $\Phi_i(v, T) = \sum \exp(-0.05 \cdot (100 - t_j))$.

- **For Vendor A (v_A):**
  - $\Phi_1(v_A) = e^{-0.05(10)} + e^{-0.05(20)} + e^{-0.05(30)} = 0.607 + 0.368 + 0.223 = \mathbf{1.198}$
  - $\Phi_2(v_A) = e^{-0.05(80)} = e^{-4.0} = \mathbf{0.018}$
  - $\Phi_3(v_A) = e^{-0.05(5)} + e^{-0.05(10)} + e^{-0.05(15)} + e^{-0.05(20)} = 0.779 + 0.607 + 0.472 + 0.368 = \mathbf{2.226}$

  $\mathbf{\Phi}(v_A) = [1.198, 0.018, 2.226]$

- **For Vendor B (v_B):**
  - $\Phi_1(v_B) = e^{-0.05(90)} = e^{-4.5} = \mathbf{0.011}$
  - $\Phi_2(v_B) = e^{-0.05(5)} + e^{-0.05(15)} + e^{-0.05(25)} = 0.779 + 0.472 + 0.287 = \mathbf{1.538}$
  - $\Phi_3(v_B) = e^{-0.05(50)} = e^{-2.5} = \mathbf{0.082}$

  $\mathbf{\Phi}(v_B) = [0.011, 1.538, 0.082]$

4.3 Step 2: Calculate $R(v, T, p)$ (Aggregated Risk)

We use $R(v, T, p) = \boldsymbol{\Omega}_p \cdot \boldsymbol{\Phi}(v, T)$.

$$\begin{aligned} R(v_A, T, p_c) &= [2.0, 10.0, 1.0] \cdot [1.198, 0.018, 2.226] \\ &= (2.0 \cdot 1.198) + (10.0 \cdot 0.018) + (1.0 \cdot 2.226) \\ &= 2.396 + 0.180 + 2.226 = \mathbf{4.802} \end{aligned}$$

$$\begin{aligned} R(v_A, T, p_m) &= [10.0, 2.0, 1.0] \cdot [1.198, 0.018, 2.226] \\ &= (10.0 \cdot 1.198) + (2.0 \cdot 0.018) + (1.0 \cdot 2.226) \\ &= 11.980 + 0.036 + 2.226 = \mathbf{14.242} \end{aligned}$$

$$\begin{aligned} R(v_B, T, p_c) &= [2.0, 10.0, 1.0] \cdot [0.011, 1.538, 0.082] \\ &= (2.0 \cdot 0.011) + (10.0 \cdot 1.538) + (1.0 \cdot 0.082) \\ &= 0.022 + 15.380 + 0.082 = \mathbf{15.484} \end{aligned}$$

$$\begin{aligned} R(v_B, T, p_m) &= [10.0, 2.0, 1.0] \cdot [0.011, 1.538, 0.082] \\ &= (10.0 \cdot 0.011) + (2.0 \cdot 1.538) + (1.0 \cdot 0.082) \\ &= 0.110 + 3.076 + 0.082 = \mathbf{3.268} \end{aligned}$$

4.4 Step 3: Calculate $\mathcal{S}(v, T, p)$ (Final VPS)

We use $\mathcal{S}(v, T, p) = 100 \cdot \exp(-0.1 \cdot R)$.

- $\mathcal{S}(v_A, T, p_c) = 100 \cdot \exp(-0.1 \cdot 4.802) = 100 \cdot \exp(-0.480) = \mathbf{61.9}$
- $\mathcal{S}(v_A, T, p_m) = 100 \cdot \exp(-0.1 \cdot 14.242) = 100 \cdot \exp(-1.424) = \mathbf{24.1}$
- $\mathcal{S}(v_B, T, p_c) = 100 \cdot \exp(-0.1 \cdot 15.484) = 100 \cdot \exp(-1.548) = \mathbf{21.3}$
- $\mathcal{S}(v_B, T, p_m) = 100 \cdot \exp(-0.1 \cdot 3.268) = 100 \cdot \exp(-0.327) = \mathbf{72.1}$

4.5 Results and Analysis

The final scores, presented in the table below, demonstrate the model's utility.

Analysis: The model correctly differentiates the vendors based on the buyer's priorities.

- **Vendor A**, who frequently misses discounts, is rated very poorly by the Margin-Sensitive buyer (p_m, Score=24.1) but is passable to the Compliance-Focused buyer (p_c, Score=61.9), as their single compliance error was long ago.
- **Vendor B**, who has recent compliance issues, is rated very poorly by the Compliance-Focused buyer (p_c, Score=21.3) but is the *preferred vendor* for the Margin-Sensitive buyer (p_m, Score=72.1), who does not heavily penalize compliance errors.

Table 1: Final Vendor Persona Scores

Vendor	**Buyer Persona**	**Aggregated Risk (R)**	**Final Score ($\mathcal{S}$)**
Vendor A	p_c (Compliance-Focused)	4.802	**61.9**
Vendor A	p_m (Margin-Sensitive)	14.242	**24.1**
Vendor B	p_c (Compliance-Focused)	15.484	**21.3**
Vendor B	p_m (Margin-Sensitive)	3.268	**72.1**

This demonstrates the model's power to provide nuanced, context-aware vendor rankings.

5 Conclusion

The comprehensive taxonomy of 82 invoice-PO discrepancies provides the essential dataset for quantitative analysis. This paper has successfully translated that taxonomy into a novel, state-of-the-art **Vendor Persona Score (VPS)** model.

The key innovation of this framework is its **persona-based severity weighting** ($\boldsymbol{\Omega}_p$). By rejecting a "one-size-fits-all" risk model, the VPS provides a score that is directly relevant to a buyer's specific business context and risk tolerance.

By integrating this flexible severity weighting with time-decay and exponential normalization, this framework provides a robust, dynamic, and context-aware metric for vendor assessment. It moves procurement teams beyond simple error-counting, enabling them to make strategically sound, data-driven decisions that align with their unique operational and compliance profiles.