Numerical Methods: Term Paper

Background Cosmology with Time-Dependent Dark Energy Density

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April, 2024

Abstract

We live in an accelerated expanding universe. In standard cosmology, this effect has been taken into account by introduction of cosmological constant or dark energy. Recent experiments have found that this component density may not be a constant but varying with time. In this term paper, I have presented the behaviour of the mean matter, radiation and dark energy densities as a function of redshift. To conclude, this model also produces, in chronological order, the radiation-dominated, matter-dominated and then dark energy-dominated eras, and weakning of dark energy density with time. It can be seen that the effect of dark energy is less prominent compared to that in the constant dark energy model as it also weakens in the past. However, during the time range of its variation, such a behaviour has novel effects on the behaviour of scale factor.

1 Introduction

The evidence that we live in an accelerating expanding universe was obtained by [1, 2]. Dark energy or cosmological constant was thought to be the cause of this accelerated expansion. Dark energy density has been anticipated to be a constant. However, recent experiments have deducted from their analysis that it might be varying with time[3]. They have suggested that the equation of state parameter depends on the scale factor as $\omega(a) \approx -0.45 - 1.79(1-a) = 1.79a - 2.24$.

In this report, I will be presenting the effect of such dark energy density on the background cosmology in the Friedmann universe. In section 2, I discuss the governing equations. In section 3, I discuss how the equations are solved and in section 4, I discuss the conclusion of the study.

2 Background Cosmology

2.1 Friedmann Equation

The components in our universe are taken to be matter(dark and baryonic), radiation and dark energy. In the background cosmology, we assume that these components extend over the whole space with respective mean densities $\rho_m(a)$, $\rho_\gamma(a)$ and the new correction $\rho_\Lambda(a)$. Using the condition of homogeneity and isotropy of the universe, we can write the metric, also called FLRW metric of the universe as,

$$ds^2 = -c^2dt^2 + a^2(t)\left(dr^2 + r^2 d\Omega^2\right), \tag{2.1}$$

$k = 0$ since Ω_K is still very small. Using Einstein's equations $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, we get

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\sum_i \rho_i. \tag{2.2}$$

This is also called as Friedmann Equation.

2.2 Energy-Momentum Conservation

Suppose the dark energy is a perfect fluid not interacting with other components except through gravity then the dynamics of dark energy density is given by energy-momentum conservation equation $\Delta_\mu T^{\mu\nu} = 0$,

$$\frac{d\rho_\Lambda}{dt} = -3\frac{\dot{a}}{a}\left(\frac{P_\Lambda}{c^2} + \rho_\Lambda\right). \tag{2.3}$$

Equation of state of dark energy is $P_\Lambda = c^2\omega(a)\rho_\Lambda$. So we get,

$$\frac{d\rho_\Lambda}{dt} = -3\frac{\dot{a}}{a}\left(\omega(a) + 1\right)\rho_\Lambda,$$

$$\frac{d\rho_\Lambda}{da} = -3\left(\omega(a) + 1\right)\frac{\rho_\Lambda}{a}. \tag{2.4}$$

3 Numerical Solution of the System

First we solve equation 2.4. In order to solve it, I have used the *solve_ivp* function of the *scipy.integrate* module. The initial condition is a guess work to match the final dark energy density

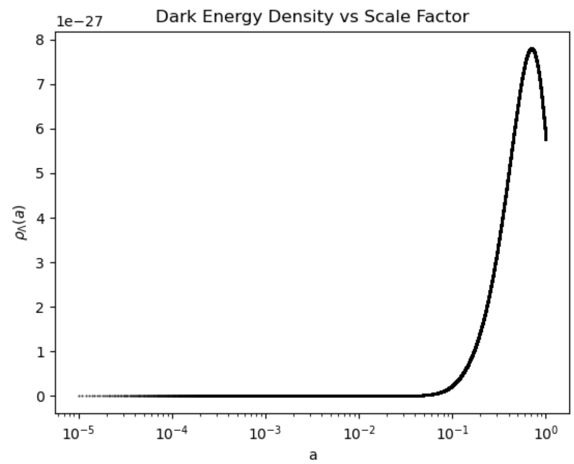

Figure 1: The plot shows the behaviour of Dark Energy density as a function of scale factor.

observed from experiments like [4]. To further the analysis, we can look at the comparison between the energy densities of matter, radiation and dark energy. Figure 2 shows this comparison. In figure 3, I have plotted the behaviour of scale factor for the two cases by solving coupled differential equations 2.2 and 2.4. The range in which they do not match is the range in which we see dark energy density variation.

4 Conclusion

As from figure 2, we can see that the proposed model follows chronologically the order of radiation-dominated, matter-dominated and dark energy-dominated eras. The surprise however is that the dark energy-density increases as universe expands. and attains a constant value after matter-domination ends. Surprisingly, the density weakens as we go in the past as well. This means such a time-dependent dark energy would have less prominent effects in the early universe compared to that in the constant dark energy density model.

References

[1] Adam G. Riess et al. "Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant". In: *The Astrophysical Journal* (1998).

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[3] DESI Collaboration. "DESI 2024 VI: Cosmological Constraints from the Measurements of Baryon Acoustic Oscillations". In: *Prepared for Submission to JCAP* (2024).

[4] N. Aghanim et al. "Planck 2018 results VI. Cosmological parameters". In: *Astronomy and Astrophysics* (2018).

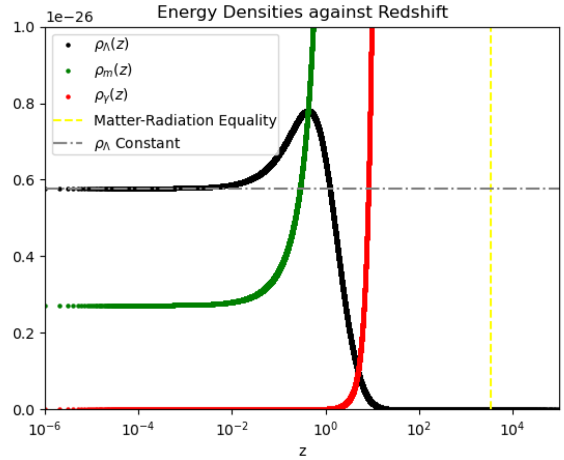

Figure 2: This plot shows the old constant dark energy density hypothesis in grey, new time-dependent dark energy density hypothesis in black, the matter density and radiation density as a function of redshift(unit of densities is kg/m^3). The yellow vertical line represents the redshift of matter-radiation equality at $z = 3400$, expectation is that the red and green line cross each other at this redshift.

A Observed values of Cosmological Parameters

Suppose that ρ_m and ρ_γ have been independently evolving over the history of the universe then it is well known that their dependence on scale factor is as follows,

$$\rho_m(a) = \frac{\rho_{m0}}{a^3}, \quad \rho_\gamma(a) = \frac{\rho_{\gamma 0}}{a^4} \tag{A.1}$$

where the subscript 0 is for quantities calculated today. Rewriting the equations, we have a coupled system of differential equations given by,

$$\dot{a} = a\sqrt{\frac{8\pi G}{3}\left(\frac{\rho_{m0}}{a^3} + \frac{\rho_{\gamma 0}}{a^4} + \rho_\Lambda\right)}, \quad \dot{\rho_\Lambda} = -3\left(\omega(a)+1\right)\rho_\Lambda\sqrt{\frac{8\pi G}{3}\left(\frac{\rho_{m0}}{a^3} + \frac{\rho_{\gamma 0}}{a^4} + \rho_\Lambda\right)}. \tag{A.2}$$

Using critical density $\rho_c = 3H_0^2/8\pi G$, we can write the above equations as,

$$\dot{a} = aH_0\sqrt{\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\gamma 0}}{a^4} + \frac{\rho_\Lambda}{\rho_c}}, \quad \dot{\rho_\Lambda} = -3H_0\left(\omega(a)+1\right)\rho_\Lambda\sqrt{\frac{\Omega_{m0}}{a^3} + \frac{\Omega_{\gamma 0}}{a^4} + \frac{\rho_\Lambda}{\rho_c}}. \tag{A.3}$$

Assuming $H_0 = 67.4$ km/s/Mpc, we get $\rho_c = 8.5 \times 10^{-27}$ kg/m^3 and from [4] $\Omega_{m0} = 0.32$, $\Omega_{\Lambda 0} = 0.68$ and $\Omega_{\gamma 0} = 9.23 \times 10^{-5}$. This implies $\rho_{\Lambda 0} = 5.78 \times 10^{-27}$ kg/m^3.

B Analytical Solution of Dark Energy Density

The governing equation is equation 2.4,

$$\frac{d\rho_\Lambda}{\rho_\Lambda} = -3\left(\omega(a)+1\right)\frac{da}{a} = (-5.37 + \frac{3.72}{a})da \tag{B.1}$$

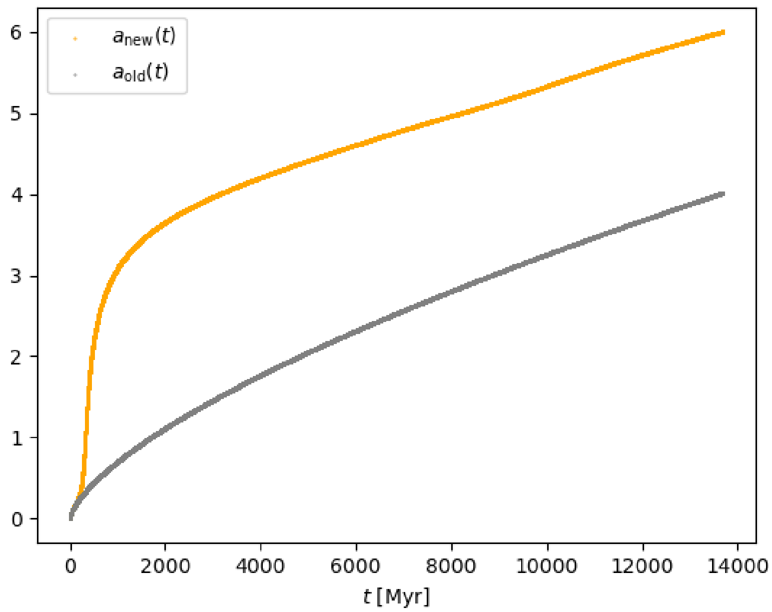

Figure 3: This plot shows the comparison of the scale factors from the two constant(old) and time-dependent(new) hypotheses. It can be seen that new scale factor shoots higher in given amount of time in the changing dark energy density range.

$$\int_{\rho_\Lambda}^{\rho_{\Lambda 0}} \frac{d\rho_\Lambda}{\rho_\Lambda} = \int_a^1 (-5.37 + \frac{3.72}{a}) da,$$

where $\rho_{\Lambda 0}$ is dark energy density today. Today the scale factor is 1.

$$\boxed{\rho_\Lambda(a) = \rho_{\Lambda 0} \times e^{3.72 \ln a - 5.37(a-1)}} \tag{B.2}$$

This equation tells us that the energy density has peak in between the asymptotes and goes to zero as $a \to 0$ and $a \to \infty$.