

# Random Numbers

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**Abstract**—This manual provides a simple introduction to the generation of random numbers

## 1 UNIFORM RANDOM NUMBERS

Let  $U$  be a uniform random variable between 0 and 1.

1.1 Generate  $10^6$  samples of  $U$  using a C program and save into a file called uni.dat .

**Solution:** Download the following files and execute the C program.

```
wget https://github.com/karna-rash/A11110-Assignment/blob/master/codes/1.1.c
```

1.2 Load the uni.dat file into python and plot the empirical CDF of  $U$  using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \quad (1.1)$$

**Solution:** The following code plots Fig. 1.2

```
wget https://github.com/karna-rash/A11110-Assignment/blob/master/codes/1.2.py
```

1.3 Find a theoretical expression for  $F_U(x)$ . **Solution:** Since it is uniform distribution then pdf is given by:

$$p_U(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad (1.2)$$

$$F_U(x) = \int_{-\infty}^x p_U(x) dx \quad (1.3)$$

$$= \int_0^x 1 dx \quad (1.4)$$

$$= x \quad (1.5)$$

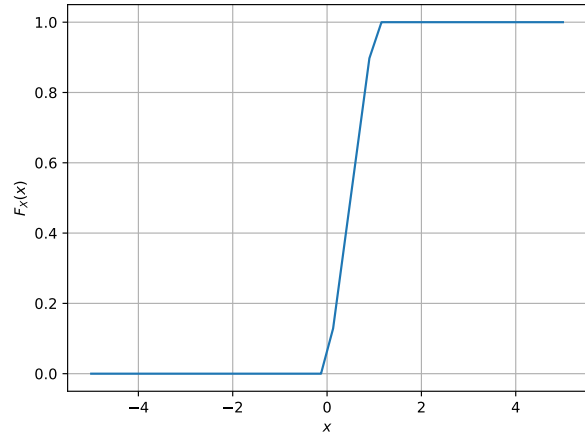


Fig. 1.2: The CDF of  $U$

1.4 The mean of  $U$  is defined as

$$E[U] = \frac{1}{N} \sum_{i=1}^N U_i \quad (1.6)$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \quad (1.7)$$

Write a C program to find the mean and variance of  $U$ .

**Solution:**Download and execute the code

```
wget https://github.com/karna-rash/A11110-Assignment/blob/master/codes/1.3-1.4.c
```

1.5 Verify your result theoretically given that

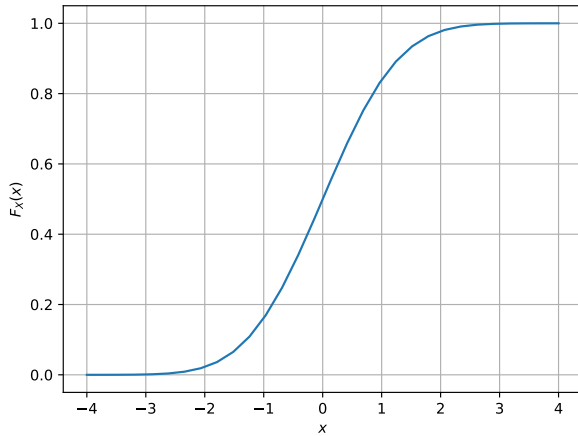
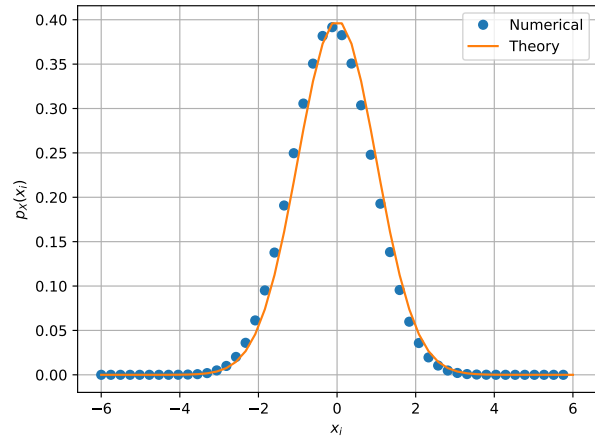
$$E[U^k] = \int_{-\infty}^{\infty} x^k dF_U(x) \quad (1.8)$$

**Solution:** Mean and variance are given by :

$$E[U] = \int_0^1 x dx = 0.5 \quad (1.9)$$

$$\text{Var}[U] = E[U^2] - E[U]^2 = \int_0^1 x^2 dx - 0 \quad (1.10)$$

$$= \frac{1}{3} \quad (1.11)$$

Fig. 2.2: The CDF of  $X$ Fig. 2.3: The PDF of  $X$ 

## 2 CENTRAL LIMIT THEOREM

2.1 Generate  $10^6$  samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \quad (2.1)$$

using a C program, where  $U_i, i = 1, 2, \dots, 12$  are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat **Soltion:**Download and execute the code

```
wget https://github.com/karna-rash/A11110-
Assignment/blob/master/codes/2.1.c
wget https://github.com/karna-rash/A11110-
Assignment/blob/master/codes/2.1.py
```

2.2 Load gau.dat in python and plot the empirical CDF of  $X$  using the samples in gau.dat. What properties does a CDF have?

**Solution:** The CDF of  $X$  is plotted in Fig. 2.2

```
wget https://github.com/karna-rash/A11110-
Assignment/blob/master/codes/2.2.py
```

2.3 Load gau.dat in python and plot the empirical PDF of  $X$  using the samples in gau.dat. The PDF of  $X$  is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \quad (2.2)$$

What properties does the PDF have?

**Solution:** The PDF of  $X$  is plotted in Fig. 2.3 using the code below

```
wget https://github.com/karna-rash/A11110-
Assignment/blob/master/codes/2.3.py
```

2.4 Find the mean and variance of  $X$  by writing a C program. **Solution:**

```
wget https://github.com/karna-rash/A11110-
Assignment/blob/master/codes/gau_mean.
c
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \quad (2.3)$$

repeat the above exercise theoretically.

**Solution:**

a) Mean :

$$E(X) = \int_{-\infty}^{\infty} x p_X(x) dx \quad (2.4)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x \exp\left(-\frac{x^2}{2}\right) dx \quad (2.5)$$

$$= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\left(\exp\left(-\frac{x^2}{2}\right)\right) \quad (2.6)$$

$$= 0 \quad (2.7)$$

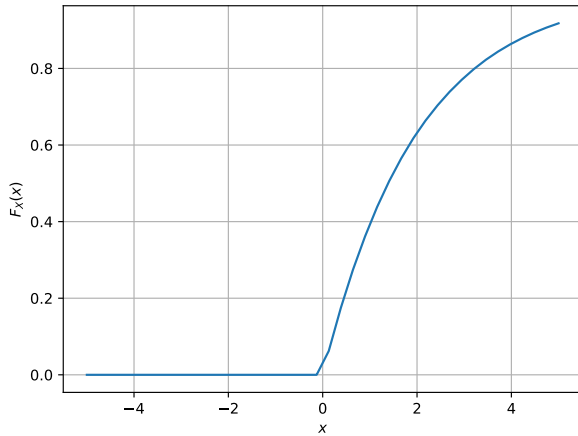


Fig. 3.1: The PDF of  $X$

$$F_V(x) = \begin{cases} 1 - e^{-\frac{x}{2}} & 0 \leq x \leq \infty \\ 0 & x \leq 0 \end{cases} \quad (3.6)$$

b) variance

$$E(X^2) = \int_{-\infty}^{\infty} x^2 p_X(x) dx \quad (2.8)$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 \exp\left(-\frac{x^2}{2}\right) dx \quad (2.9)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2}{2}\right) dx \quad (2.10)$$

$$= \frac{\sqrt{2\pi}}{\sqrt{2\pi}} \quad (2.11)$$

$$= 1 \quad (2.12)$$

### 3 FROM UNIFORM TO OTHER

3.1 Generate samples of

$$V = -2 \ln(1 - U) \quad (3.1)$$

and plot its CDF.

```
wget https://github.com/karna-rash/A11110-Assignment/blob/master/codes/3.1.c
wget https://github.com/karna-rash/A11110-Assignment/blob/master/codes/3.1.py
```

3.2 Find a theoretical expression for  $F_V(x)$ .

**Soltion:** Let  $V = G(U)$  then

$$F_V(x) = F_U(G^{-1}(x)) \quad (3.2)$$

$$= F_U(1 - e^{-\frac{x}{2}}) \quad (3.3)$$

$$(3.4)$$

$$F_V(x) = \begin{cases} G^{-1}(x) & 0 \leq 1 - e^{-\frac{x}{2}} \leq 1 \\ 0 & 1 - e^{-\frac{x}{2}} \geq 0 \end{cases} \quad (3.5)$$