Random Numbers

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Contents

***Abstract*—This manual provides a simple introduction to the generation of random numbers**

1 Uniform Random Numbers

Let U be a uniform random variable between 0 and 1.

1.1 Generate 10^6 samples of U using a C program and save into a file called uni.dat .
Solution: Download the following files and execute the C program.

```
wget https://github.com/karna-rash/A11110-
    Assignment/blob/master/codes/1.1.c
gcc -o 1.1 1.1.c
./1.1
```

1.2 Load the uni.dat file into python and plot the empirical CDF of U using the samples in uni.dat. The CDF is defined as

$$F_U(x) = \Pr(U \leq x) \tag{1.1}$$

Solution: The following code plots Fig. 1.2

```
wget https://github.com/karna-rash/A11110-
    Assignment/blob/master/codes/1.2.py
python3 1.2.py
```

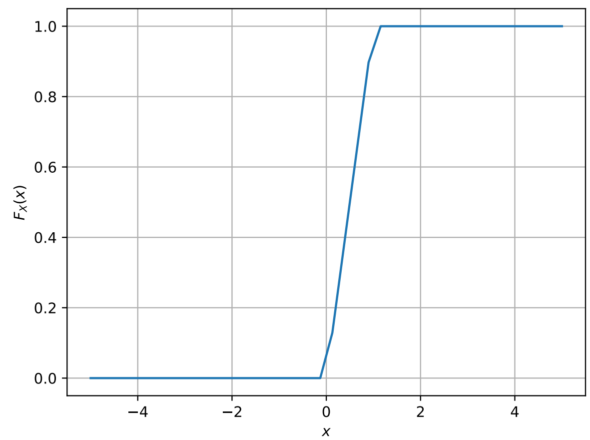

Fig. 1.2: The CDF of U

1.3 Find a theoretical expression for $F_U(x)$. **Solution:** Since it is uniform distribution then pdf is given by:

$$p_U(x) = \begin{cases} 1 & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases} \tag{1.2}$$

$$F_U(x) = \int_{-\infty}^{x} p_U(x)dx \tag{1.3}$$
$$= \int_{0}^{x} 1.dx \tag{1.4}$$
$$= x \tag{1.5}$$

1.4 The mean of U is defined as

$$E[U] = \frac{1}{N}\sum_{i=1}^{N} U_i \tag{1.6}$$

and its variance as

$$\text{var}[U] = E[U - E[U]]^2 \tag{1.7}$$

Write a C program to find the mean and variance of U.
Solution: Download and excecute the code

```
wget https://github.com/karna-rash/A11110-
    Assignment/blob/master/codes/1.3-1.4.c
gcc -o 1.3-1.4 1.3-1.4.c
./1.3-1.4
```

1.5 Verify your result theoretically given that

$$E\left[U^k\right] = \int_{-\infty}^{\infty} x^k dF_U(x) \tag{1.8}$$

Solution: Mean and variance are given by :

$$E[U] = \int_0^1 x.dx = 0.5 \tag{1.9}$$

$$Var[U] = E\left[U^2\right] - E[U]^2 = \int_0^1 x^2.dx - 0 \tag{1.10}$$

$$= \frac{1}{3} \tag{1.11}$$

2 Central Limit Theorem

2.1 Generate 10^6 samples of the random variable

$$X = \sum_{i=1}^{12} U_i - 6 \tag{2.1}$$

using a C program, where $U_i, i = 1, 2, \ldots, 12$ are a set of independent uniform random variables between 0 and 1 and save in a file called gau.dat **Soltion:**Download and exceute the code

```
wget https://github.com/karna-rash/A11110-
     Assignment/blob/master/codes/2.1.c
gcc -o 2.1 2.1.c
./2.1
wget https://github.com/karna-rash/A11110-
     Assignment/blob/master/codes/2.1.py
python3 2.1.py
```

2.2 Load gau.dat in python and plot the empirical CDF of X using the samples in gau.dat. What properties does a CDF have?
Solution: The CDF of X is plotted in Fig. 2.2
Properties:
a) it is symmetric about $(0, 0.5)$
b) its tends to one as we approch infinity

```
wget https://github.com/karna-rash/A11110-
     Assignment/blob/master/codes/2.2.py
python3 2.2.py
```

2.3 Load gau.dat in python and plot the empirical PDF of X using the samples in gau.dat. The PDF of X is defined as

$$p_X(x) = \frac{d}{dx} F_X(x) \tag{2.2}$$

What properties does the PDF have?
Solution: The PDF of X is plotted in Fig. 2.3 using the code below
Properties:
a) it is symmetric about $x = 0$

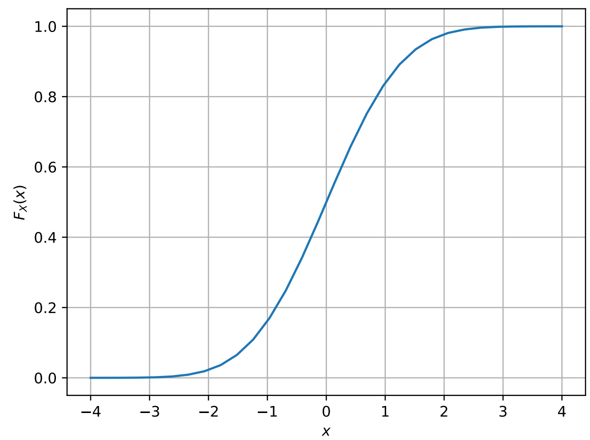

Fig. 2.2: The CDF of X

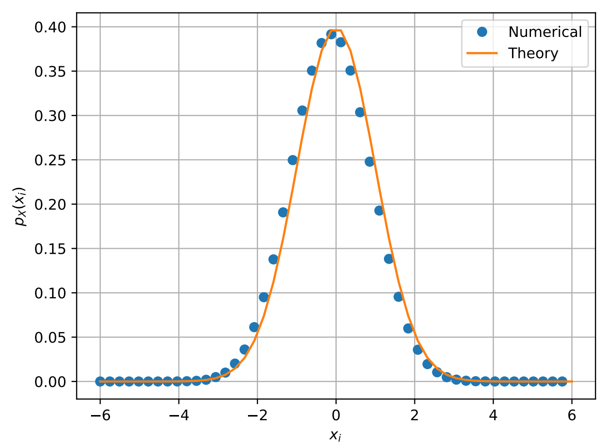

Fig. 2.3: The PDF of X

b) its tends to zero as we approch infinity

```
wget https://github.com/karna-rash/A11110-
     Assignment/blob/master/codes/2.3.py
python3 2.3.py
```

2.4 Find the mean and variance of X by writing a C program. **Solution:**

```
wget https://github.com/karna-rash/A11110-
     Assignment/blob/master/codes/gau_mean.
     c
gcc -o gau_mean gau_mean.c
./gau_mean
```

2.5 Given that

$$p_X(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right), -\infty < x < \infty, \tag{2.3}$$

repeat the above exercise theoretically.
Solution:

a) Mean :

$$E(X) = \int_{-\infty}^{\infty} x p_X(x) dx \quad (2.4)$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x exp\left(-\frac{x^2}{2}\right) dx \quad (2.5)$$
$$= -\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\left(exp\left(-\frac{x^2}{2}\right)\right) \quad (2.6)$$
$$= 0 \quad (2.7)$$

b) variance

$$E\left(X^2\right) = \int_{-\infty}^{\infty} x^2 p_X(x) dx \quad (2.8)$$
$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} x^2 exp\left(-\frac{x^2}{2}\right) dx \quad (2.9)$$
$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} exp\left(-\frac{x^2}{2}\right) dx \quad (2.10)$$
$$= \frac{\sqrt{2\pi}}{\sqrt{2\pi}} \quad (2.11)$$
$$= 1 \quad (2.12)$$

3 From Uniform to Other

3.1 Generate samples of

$$V = -2\ln(1-U) \quad (3.1)$$

and plot its CDF. **Solution:** Refer the figure 3.1

```
wget https://github.com/karna-rash/A11110-
    Assignment/blob/master/codes/3.1.c
gcc -o 3.1 3.1.c
./3.1
wget https://github.com/karna-rash/A11110-
    Assignment/blob/master/codes/3.1.py
python3 3.1.py
```

3.2 Find a theoretical expression for $F_V(x)$. **Soltion:** Let $V = G(U)$ then

$$F_V(x) = F_U(G^{-1}(x)) \quad (3.2)$$
$$= F_U(1 - e^{-\frac{x}{2}}) \quad (3.3)$$
$$\quad (3.4)$$

$$F_V(x) = \begin{cases} G^{-1}(x) & 0 \le 1 - e-\frac{x}{2} \le 1 \\ 0 & 1 - e^{-\frac{x}{2}} \ge 0 \end{cases} \quad (3.5)$$

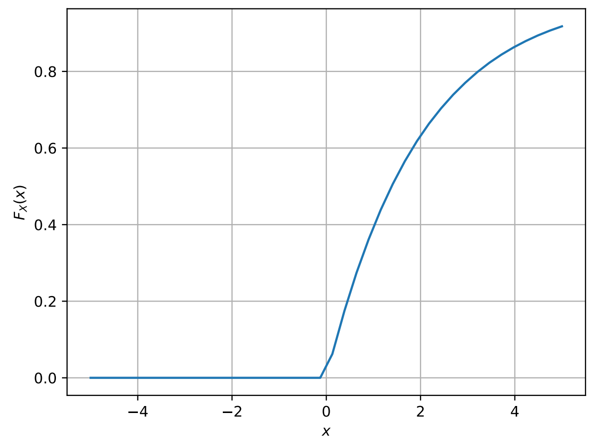

Fig. 3.1: The PDF of X

$$F_V(x) = \begin{cases} 1 - e^{-\frac{x}{2}} & 0 \le x \le \infty \\ 0 & x \le 0 \end{cases} \quad (3.6)$$