

# AI1110: Assignment 2

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Given the observations

I. QUESTION 20(A)

Find the line of regression of  $y$  on  $x$  from the following table.

x	1	2	3	4	5
y	7	6	5	4	3

Hence, estimate the  $y$  value when  $x=6$ .

**Solution.**

Given observations  $\left(\begin{smallmatrix} x_1 \\ y_1 \end{smallmatrix}\right), \left(\begin{smallmatrix} x_2 \\ y_2 \end{smallmatrix}\right), \dots, \left(\begin{smallmatrix} x_n \\ y_n \end{smallmatrix}\right)$  best fit a straight line to it

$$Y = a_0 + a_1 X \quad (1)$$

The residual error

$$E_i = (y_i - (a_0 + a_1 x_i)) \quad (2)$$

Sum of squares of errors should be minimum

$$S_r = \sum_{i=1}^n E_i^2 \quad (3)$$

$$\frac{\partial S_r}{\partial a_0} = 0, \frac{\partial S_r}{\partial a_1} = 0, \quad (4)$$

we will get two equations after solving

$$na_0 + a_1 \sum_{i=1}^n x_i = \sum_{i=1}^n y_i \quad (5)$$

$$a_0 \sum_{i=1}^n x_i + a_1 \sum_{i=1}^n x_i^2 = \sum_{i=1}^n x_i y_i \quad (6)$$

Finally we will get

$$a_1 = \frac{n \sum_{i=1}^n x_i y_i - \sum_{i=1}^n x_i \sum_{i=1}^n y_i}{n \sum_{i=1}^n x_i^2 - (\sum_{i=1}^n x_i)^2} \quad (7)$$

$$a_0 = \left( \frac{\sum_{i=1}^n y_i}{n} \right) - a_1 \left( \frac{\sum_{i=1}^n x_i}{n} \right) \quad (8)$$

$$\left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right) : \mathbf{A}\left(\begin{smallmatrix} 1 \\ 7 \end{smallmatrix}\right), \mathbf{B}\left(\begin{smallmatrix} 2 \\ 6 \end{smallmatrix}\right), \mathbf{C}\left(\begin{smallmatrix} 3 \\ 5 \end{smallmatrix}\right), \mathbf{D}\left(\begin{smallmatrix} 4 \\ 4 \end{smallmatrix}\right), \mathbf{E}\left(\begin{smallmatrix} 5 \\ 3 \end{smallmatrix}\right) \quad (9)$$

x	y	xy	$x^2$
1	7	7	1
2	6	12	4
3	5	15	9
4	4	16	16
5	3	15	25
$\sum x = 15$	$\sum y = 25$	$\sum xy = 65$	$\sum x^2 = 55$

Mean values and coefficient  $a_0, a_1$  :

$$\bar{x} = 3, \bar{y} = 5, a_1 = -1, a_0 = 8 \quad (10)$$

The line of regression:

$$Y = 8 - X \quad (11)$$

$$X + Y = 8 \quad (12)$$

When  $x = 6$  then  $y$  must be 2 from the line of regression.

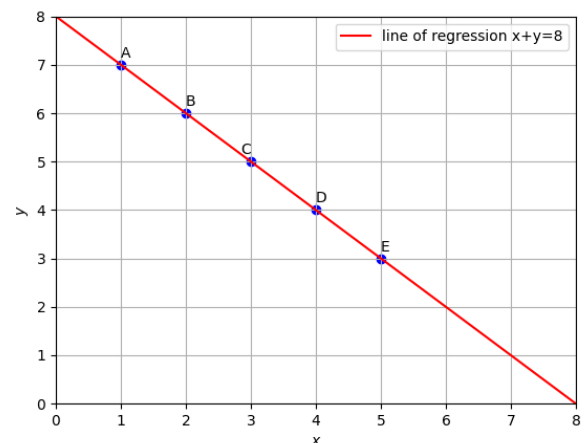


Fig. 0: plot of all points