ICSE class 12 paper 2018

1 Question 20(a)

Find the line of regression of y on x from the following table. Hence, estimate the y value when x=6.

x	1	2	3	4	5
y	7	6	5	4	3

Solution.
Given observations

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} ....., \begin{pmatrix} x_n \\ y_n \end{pmatrix} \tag{1.0.1}$$

Best fit a straight line to it , e_i are the corresponding residual errors,coeffients $a_0$and a_1

$$\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_{n-1} \\ y_n \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & x_1 \\ 1 & x_2 \\ . & \\ . & \\ 1 & x_{n-1} \\ 1 & x_n \end{pmatrix} \tag{1.0.2}$$

$$\mathbf{A} = \begin{pmatrix} a_1 \\ a_0 \end{pmatrix}, \mathbf{E} = \begin{pmatrix} e_1 \\ e_2 \\ . \\ . \\ e_{n-1} \\ e_n \end{pmatrix} \tag{1.0.3}$$

$$\mathbf{E} = \mathbf{Y} - \mathbf{X}\mathbf{A} \tag{1.0.4}$$

SSE is sum of square of errors , substitue E from above equation and get A

$$SSE = \left\|\mathbf{E}^\top \mathbf{E}\right\| \tag{1.0.5}$$
$$= (\mathbf{Y} - \mathbf{X}\mathbf{A})^\top (\mathbf{Y} - \mathbf{X}\mathbf{A}) \tag{1.0.6}$$
$$= \left(\mathbf{Y}^\top - \mathbf{A}^\top \mathbf{X}^\top\right)(\mathbf{Y} - \mathbf{X}\mathbf{A}) \tag{1.0.7}$$
$$= \left(\mathbf{Y}^\top \mathbf{Y} - 2\mathbf{A}^\top \mathbf{X}^\top \mathbf{Y} + \mathbf{A}^\top \mathbf{X}^\top \mathbf{X}\mathbf{A}\right) \tag{1.0.8}$$

for minimizing use gradient wrt A

$$\nabla SSE = \left(\nabla \mathbf{Y}^\top \mathbf{Y} - 2\nabla \mathbf{A}^\top \mathbf{X}^\top \mathbf{Y} + \nabla \mathbf{A}^\top \mathbf{X}^\top \mathbf{X}\mathbf{A}\right) \tag{1.0.9}$$
$$= 2\left(\mathbf{X}^\top \mathbf{X}\mathbf{A} - \mathbf{X}^\top \mathbf{Y}\right) \tag{1.0.10}$$

We now set this to zero at the optimum

$$\left(\mathbf{X}^\top \mathbf{X}\mathbf{A} - \mathbf{X}^\top \mathbf{Y}\right) = 0 \tag{1.0.11}$$
$$\mathbf{A} = \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{Y} \tag{1.0.12}$$

If we want to give line form after finding A

$$\mathbf{n}^\top \mathbf{x} = c, \mathbf{n} = \begin{pmatrix} \mathbf{A}^\top \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ 1 \end{pmatrix}, c = \mathbf{A}^\top \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.0.13}$$

For this problem

$$\begin{pmatrix} x \\ y \end{pmatrix} : \begin{pmatrix} 1 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 3 \end{pmatrix} \tag{1.0.14}$$

$$\mathbf{Y} = \begin{pmatrix} 7 \\ 6 \\ 5 \\ 4 \\ 3 \end{pmatrix}, \mathbf{X} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \\ 1 & 4 \\ 1 & 5 \end{pmatrix} \tag{1.0.15}$$

Substitue these Y and X and get A

$$A = \begin{pmatrix} -1 \\ 8 \end{pmatrix} \tag{1.0.16}$$

After using the line form stated before

$$y = 8 - x \tag{1.0.17}$$
$$x + y = 8 \tag{1.0.18}$$

When $x = 6$ then y must be 2 from the line of regression.

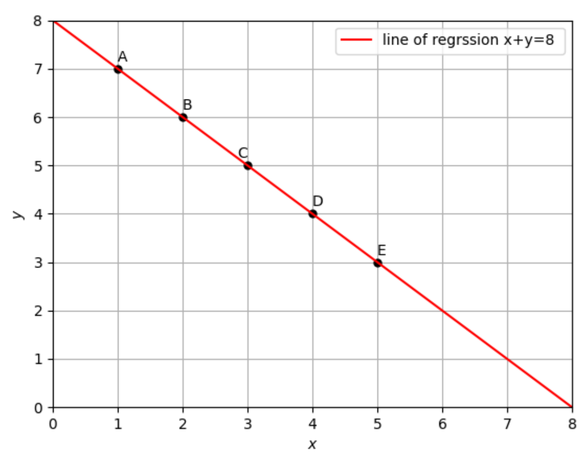

Fig. 0. plot of all points