AI1110: Assignment 5

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Abstract—**This document contains the solution to Question of Chapter 16 (Probability) in the NCERT Class 11 Textbook.**

Probability 16.3 Example 8. A coin is tossed three times, consider the following events. A: 'No head appears', B: 'Exactly one head appears' and C: 'Atleast two heads appear'. Do they form a set of mutually exclusive and exhaustive events?

Solution. Let $X_i \in \{0, 1\}, i = 1, 2, 3$ represent a coin toss, or, the Bernoulli random variable. Then the outcome of the game is If

$$\Pr(X_i = 1) = p, \tag{1}$$
$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k}, \quad k = 0, \ldots, n \tag{2}$$

X is known as a Binomial random variable. For the given problem, $n = 3, p = \frac{1}{2}$

$$X = X_1 + X_2 + X_3 \tag{3}$$

Now let us calculate the probabilities of each events

Event	X
A	$X = 0$
B	$X = 1$
C	$X \geq 2$

TABLE I: Events

$$\Pr(A) = \Pr(X = 0) \tag{4}$$
$$= {}^3C_0 \left(\frac{1}{2}\right)^3 \tag{5}$$
$$= \frac{1}{8} \tag{6}$$
$$\Pr(B) = \Pr(X = 1) \tag{7}$$
$$= {}^3C_1 \left(\frac{1}{2}\right)^3 \tag{8}$$
$$= \frac{3}{8} \tag{9}$$

$$\Pr(C) = \Pr(X \geq 2) \tag{10}$$
$$= {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_3 \left(\frac{1}{2}\right)^3 \tag{11}$$
$$= \frac{1}{2} \tag{12}$$

The sum of probabilities

$$\Pr(A) + \Pr(B) + \Pr(C) = \frac{1}{8} + \frac{3}{8} + \frac{1}{2} \tag{13}$$
$$= 1 \tag{14}$$

The probabilities where both events happen at same time

$$\Pr(AB) = \Pr(\{X = 0\}\{X = 1\}) \tag{15}$$
$$= 0 \tag{16}$$
$$\Pr(BC) = \Pr(\{X = 1\}\{X \geq 2\}) \tag{17}$$
$$= 0 \tag{18}$$
$$\Pr(AC) = \Pr(\{X = 0\}\{X \geq 2\}) \tag{19}$$
$$= 0 \tag{20}$$

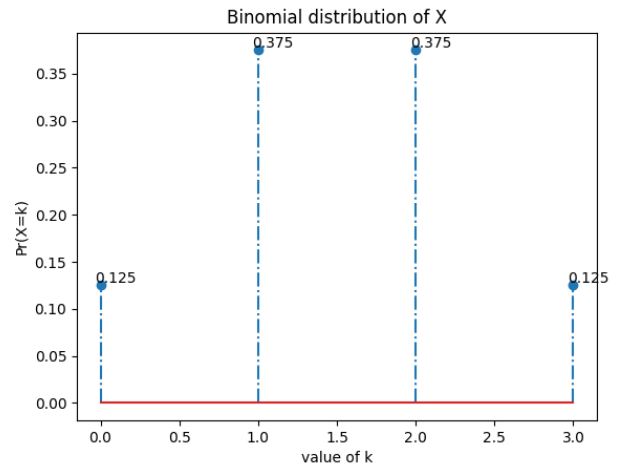by results (14),(16),(18),(20) we can prove that the events are mutually exculsive and exhaustive.

Fig. 1: PMF