AI1110 Assignment 7

SADINENI ABHINAY-CS21BTECH11055

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Outline

Abstract

- This document contains the solution to Question of Chapter 13 (Probability) in the NCERT Class 12 Textbook.

Question

Probability ex 13.5 q8.

Suppose X has a binomial distribution $B\left(6, \frac{1}{2}\right)$ .Show that $X = 3$ is most likely outcome.
($hint$: $\Pr(X = 3)$ is the max among all $\Pr(x_i), x_i = 0, 1, 2, 3, 4, 5, 6$)

Theory

Binomial Distribution

the binomial distribution with parameters n and p is the discrete probability distribution of the number of successes in a sequence of n independent experiments, each asking a yes–no question, and each with its own Boolean-valued outcome: success (with probability p) or failure (with probability $q = 1–p$)

The Expression is given by:

$$\sum_{i=0}^{n} \Pr(X = i) = \sum_{i=0}^{n} {}^{n}C_i (\mathrm{p})^i (1 - \mathrm{p})^{n-i} \tag{1}$$

$$\Pr(X = i) = {}^{n}C_i (\mathrm{p})^i (1 - \mathrm{p})^{n-i} \tag{2}$$

Generalized problem

Suppose X has a binomial distribution $B(n, p)$ .Find the most likely outcome.

Solution

Given X has binomial distribution $\mathrm{B}(n, p)$ as we know

$$\Pr(X = i) = {}^{n}C_{i}(\mathrm{p})^{i}(1-\mathrm{p})^{n-i} \tag{3}$$

Now to find maximum probability the following conditions

$$\frac{\Pr(X = k)}{\Pr(X = (k+1))} \geq 1 \tag{4}$$

$$\frac{\Pr(X = k)}{\Pr(X = (k-1))} \geq 1 \tag{5}$$

Solving equation (4)

$$\frac{\Pr(X=k)}{\Pr(X=(k-1))} = \frac{{}^{n}C_{k}}{{}^{n}C_{k-1}} \cdot \frac{p^{k}(1-p)^{n-k}}{p^{k-1}(1-p)^{n+1-k}} = \frac{n+1-k}{k} \cdot \frac{p}{1-p} \tag{6}$$

$$\implies \frac{n+1-k}{k} \cdot \frac{p}{1-p} \geq 1 \tag{7}$$

$$\implies k \leq (n+1)\,p \tag{8}$$

Solving equation (5)

$$\frac{\Pr(X=k)}{\Pr(X=(k+1))} = \frac{{}^{n}C_{k}}{{}^{n}C_{k+1}} \cdot \frac{p^{k}(1-p)^{n-k}}{p^{k-1}(1-p)^{n-1-k}} = \frac{k+1}{n-k} \cdot \frac{1-p}{p} \tag{9}$$

$$\implies \frac{k+1}{n-k} \cdot \frac{1-p}{p} \geq 1 \tag{10}$$

$$\implies k \geq (n+1)\,p - 1 \tag{11}$$

Result

Result

Combining Results from two conditions

$$(n+1)\,p - 1 \leq k \leq (n+1)\,p \tag{12}$$

k must be a integer in between these two values ,X=k is the most likely outcome

Note: if $(n+1)$p is integer then k can have both values ie. $(n+1)$p-1 and $(n+1)$p

Answer

For this given question ,$n = 6, p = \frac{1}{2}$ then

$$(n+1)\,p = 3.5 \tag{13}$$

$$\implies 2.5 \leq k \leq 3.5 \tag{14}$$

since k is a integer the value of k is 3
$\therefore X = 3$ is the most likely outcome.

PMF

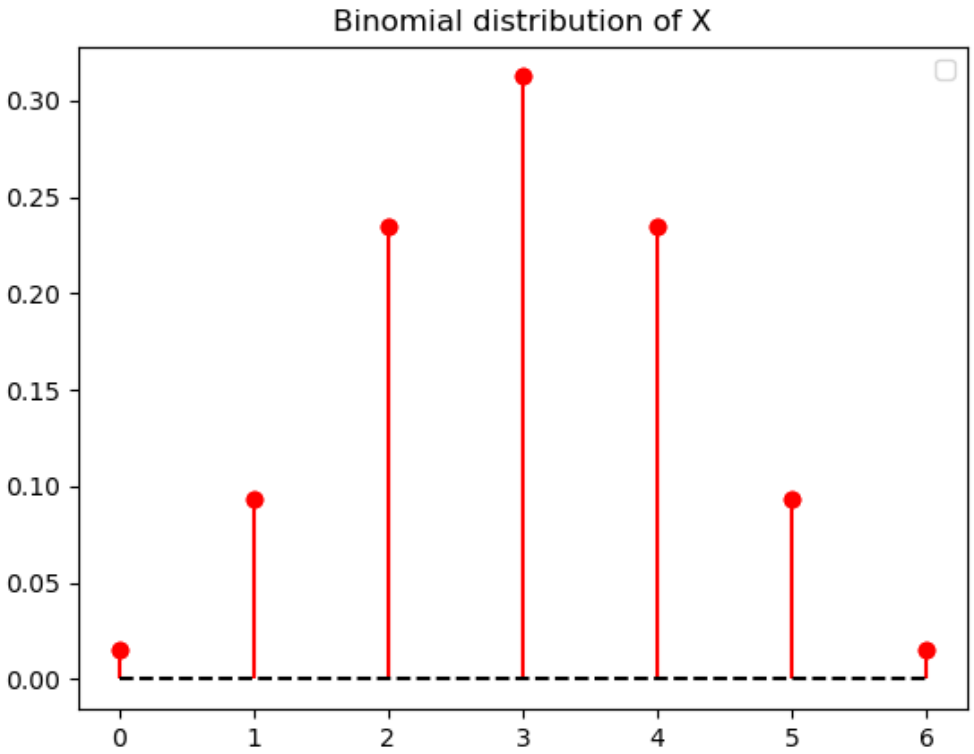

Figure 1: PMF