

# AI1110: Assignment 5

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**Abstract**—This document contains the solution to Question of Chapter 16 (Probability) in the NCERT Class 11 Textbook.

**Probability 16.3 Example 8.** A coin is tossed three times, consider the following events. A: ‘No head appears’, B: ‘Exactly one head appears’ and C: ‘Atleast two heads appear’. Do they form a set of mutually exclusive and exhaustive events?

**Solution.** Let  $X_i \in \{0, 1\}$ ,  $i = 1, 2, 3$  represent a coin toss, or, the Bernoulli random variable. Then the outcome of the game is If

$$\Pr(X_i = 1) = p, \quad (1)$$

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k}, \quad k = 0, \dots, n \quad (2)$$

$X$  is known as a Binomial random variable. For the given problem,  $n = 3, p = \frac{1}{2}$

$$X = X_1 + X_2 + X_3 \quad (3)$$

Now let us calculate the probabilities of each events

Event	X
A	$X = 0$
B	$X = 1$
C	$X \geq 2$

TABLE I: Events

$$\Pr(A) = \Pr(X = 0) \quad (4)$$

$$= {}^3C_0 \left(\frac{1}{2}\right)^3 \quad (5)$$

$$= \frac{1}{8} \quad (6)$$

$$\Pr(B) = \Pr(X = 1) \quad (7)$$

$$= {}^3C_1 \left(\frac{1}{2}\right)^3 \quad (8)$$

$$= \frac{3}{8} \quad (9)$$

$$\Pr(C) = \Pr(X \geq 2) \quad (10)$$

$$= {}^3C_2 \left(\frac{1}{2}\right)^3 + {}^3C_3 \left(\frac{1}{2}\right)^3 \quad (11)$$

$$= \frac{1}{2} \quad (12)$$

The sum of probabilities

$$\Pr(A) + \Pr(B) + \Pr(C) = \frac{1}{8} + \frac{3}{8} + \frac{1}{2} \quad (13)$$

$$= 1 \quad (14)$$

The probabilities where both events happen at same time

$$\Pr(AB) = \Pr(\{X = 0\} \{X = 1\}) \quad (15)$$

$$= 0 \quad (16)$$

$$\Pr(BC) = \Pr(\{X = 1\} \{X \geq 2\}) \quad (17)$$

$$= 0 \quad (18)$$

$$\Pr(AC) = \Pr(\{X = 0\} \{X \geq 2\}) \quad (19)$$

$$= 0 \quad (20)$$

by results (14),(16),(18),(20) we can prove that the events are mutually exculsive and exhaustive.