

AI1110 Assignment 11

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Outline

- 1 Abstract
- 2 Question
- 3 Theory
- 4 Solution

Abstract

- This document contains the solution to Question of Chapter 8 in the Papoulis Textbook.

Ex 8.18

The random variable x_i are i.i.d and $\mathcal{N}(0, \sigma)$ We observe that $x_1^2 + x_2^2 + \dots x_{10}^2 = 4$.Find the 0.95 confidence interval of σ .

Theory

confidence interval

The 95% confidence interval is a range of values that you can be 95% certain contains the true mean of the population.

chi squared distribution

the chi-squared distribution (also chi-square distribution) with k degrees of freedom is the distribution of a sum of the squares of k independent standard normal random variables.

Solution

The RVs x_i/σ are $\mathcal{N}(0, 1)$ hence, the sum $z = (x_1^2 + x_2^2 + \dots x_{10}^2) / \sigma^2$ has a $\chi^2(10)$ This yields:

$$\Pr(\chi_{0.025}^2(10) < z < \chi_{0.975}^2(10)) = 0.95 \quad (1)$$

$$\chi_{0.025}^2(10) = 3.25, \chi_{0.975}^2(10) = 20.48 \quad (2)$$

$$3.25 < \frac{4}{\sigma^2} < 20.48 \quad (3)$$

$$0.442 < \sigma < 1.109 \quad (4)$$

Output of Code

```
jammy@jammy-Inspiron-3593:~/Desktop/Assignment11$ python3 assignment10.py  
20.483177350807388  
3.2469727802368413
```

Figure 1: chi square critical values