AI1110
Assignment 13

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CS21BTECH11059

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Outline

question 15.25

For the random walk model with two absorbing barriers , considering a slight generalization of the transition prabability matrix in (15-20) with $p_i j$ as in (15-19) for i$\geq$ $1. Inthatcasestatede_0$ and e_N atr absorbing states and $e_1, e_2, ..., e_N - 1$ are transient states $f_0, 0 = 1$, and $f_N, N = 1$ and $f_N, 0 = 0$

theory

for j = 0

$$f_i, 0 = q_i f_i - 1, 0 + r_i f_i, 0 + p_i f_i + 1, 0 fori \geq 1 \quad (1)$$

$$or(f_i + 1, 0 - f_i, 0)p_i = q_i(f_i 1, 0 - f_i - 1, 0) \quad (2)$$

Thus

$$f_i + 1, 0 - f_i, 0 = \frac{q_i}{p_i}(f_i 1, 0 - f_i - 1, 0) = \frac{q_i.q_i - 1...q_1}{p_i.p_i - 1...p_1}(f_1, 0 - 1) \quad (3)$$

$$= \sigma_i(f_1, 0 - 1) \quad (4)$$

$$(5)$$

where

second Part-1

where

$$\sigma_i = \frac{q_i.q_i - 1...q_1}{p_i.p_i - 1...p_1}(f_1, 0 - 1) \tag{6}$$

$$\tag{7}$$

where,

$$f_k, 0 - 1 = \sum_{i=0}^{k} -1(f_i + 1, 0 - f_i, 0) = \sum_{i=1}^{k} -1\sigma_i(f_1, 0 - 1) \tag{8}$$

With k = N we get $f_1, 0$ = -1/$\sum_{i=0}^{N} -1\sigma_i$ and hence starting from any transient state e_k the desired probability of absorption into state e_0 is given by

$$f_k, 0 = 1 - \frac{\sum_{i=0}^{k} -1\sigma_i}{\sum_{i=0}^{N} -1\sigma_i} k = 1, 2, ...N - 1 givenby \tag{9}$$

second part-2

In the special case of a uniform random walk p_i=p, q_i=q, r_i-0 so we have

$$f_k, 0 = 1 - \frac{1-(\frac{q}{p})^k}{1-(\frac{q}{p})^N} = \frac{(\frac{q}{p})^k - (\frac{q}{p})^N}{1-(\frac{q}{p})^N} \tag{10}$$

$$\frac{1-(\frac{p}{q})^N - k}{-1-(\frac{p}{q})^N}, k = 1, 2, 3...N - 1 \tag{11}$$