DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
SUBJECTCODE: 21MT2103RA
PROBABILITY STATISTICS AND QUEUING THEORY

Tutorial 5:
Demonstration of Expectation of discrete and continuous random variables
Demonstration of Expectation of a function of a random variable.

Date of the Session: //______________________________Time of the Session:_____ to ______

Learning outcomes:

- Understand Expectation of discrete and continuous random variables
- Understand Expectation of a function of a random variable

PRE-TUTORIAL

1. Choose the correct answer: The expectation of a random variable is related to
   (a) range of the random variable
   (b) maximum value that the random variable can take
   (c) mean value of the random variable.

Solution:

The answer is (c). The expectation of a random variable is related to the average or mean value of the random variable.

2. Write the formula for the expectation of a function of a discrete random variable.

Solution:

If X is a discrete random variable with PMF $P_X(x)$, and g(X) is a function of X, the expectation of g(X) is given by the following formula.

$$E[g(X)] = \sum_{x_k \in R_X} g(x_k) P_X(x_k)$$

IN-TUTORIAL:

1. Six men and five women apply for an executive position in a small company. Two
of the applicants are selected for an interview. Let X denote the number of women
in the interview pool. We have found the probability mass function of X.

$X = x$	0	1	2
$P(x)$	$\frac{2}{11}$	$\frac{5}{11}$	$\frac{4}{11}$

How many women do you expect in the interview pool?

Solution:

Expected number of women in the interview pool is

$$E(X) = \sum_x x P_X(x)$$

$$= \left[\left(0\times\frac{2}{11}\right)+\left(1\times\frac{5}{11}\right)+\left(2\times\frac{4}{11}\right)\right]$$

$$= \frac{13}{11}$$

2. Compute the expectation of X^2 where X is a random variable with the following probability density function:

$$f(x)=\begin{cases} 4x^3, if\ 0<x<1 \\ 0,\ otherwise \end{cases}$$

Solution:

The expectation of a function of a continuous random variable is computed using the following formula.

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)\cdot f(x)\,dx.$$

So, $E[X^2]$ is computed as follows:

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$
$$= \int_0^1 x^2 4x^3\,dx$$
$$= 4\left[\frac{x^6}{6}\right]_0^1$$
$$= \frac{4}{6}$$

POST-TUTORIAL

1. Determine the mean (i.e., E[X]) and E[X^2] of a discrete random variable X whose Cumulative Probability Distribution (CDF) function is given below:

$X = x$	1	2	3	4	5	6
$F_x(x)$	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1

Solution:

To compute expectation, we need the Probability Mass Function (PMF) of the discrete random variable. From the CDF, first calculate the probability distribution of the random variable. Then, compute E[X] and E[X^2] as follows:

X	$p(x)$
1	$F(1) = \frac{1}{6}$
2	$F(2) - F(1) = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$
3	$F(3) - F(2) = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$
4	$F(4) - F(3) = \frac{4}{6} - \frac{3}{6} = \frac{1}{6}$
5	$F(5) - F(4) = \frac{5}{6} - \frac{4}{6} = \frac{1}{6}$
6	$F(6) - F(5) = 1 - \frac{5}{6} = \frac{1}{6}$

The probability mass function is

$X = x$	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean of the random variable $X = E(X) = \sum_x x\, P_X(x)$

$$= \left(1\times\frac{1}{6}\right)+\left(2\times\frac{1}{6}\right)+\left(3\times\frac{1}{6}\right)+\left(4\times\frac{1}{6}\right)+\left(5\times\frac{1}{6}\right)+\left(6\times\frac{1}{6}\right)$$

$$= \frac{1}{6}\left(1+2+3+4+5+6\right)$$

$$= \frac{7}{2}$$

$$E\left(X^2\right) = \sum_x x^2\, P_X(x)$$

$$= \left(1^2\times\frac{1}{6}\right)+\left(2^2\times\frac{1}{6}\right)+\left(3^2\times\frac{1}{6}\right)+\left(4^2\times\frac{1}{6}\right)+\left(5^2\times\frac{1}{6}\right)+\left(6^2\times\frac{1}{6}\right)$$

$$= \frac{1}{6}\left(1^2+2^2+3^2+4^2+5^2+6^2\right)$$

$$= \frac{91}{6}$$