CSE578: Assignment #1

Due on Thursday, February 4, 2016

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Problem 1

Implement the DLT (Direct Linear Transformation) based calibration that we discussed in the class. You could use C/C++ or Matlab for this. 7. However, you are expected to implement it yourselves and not use an existing implementation.

Procedure:

- A matrix is computed by appending two rows from each point in the matrix. Hence we get a matrix of dimension 2n x 12 from n points.
- Single value decomposition of A is computed.

$$A = UDV^T$$

- Column of V corresponding to smallest eigen value is picked.

$$x_i = P\,x_i$$

$$\begin{bmatrix} u_i \\ v_i \\ w_i \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} \\ P_{21} & P_{22} & P_{23} & P_{24} \\ P_{31} & P_{32} & P_{33} & P_{34} \end{bmatrix} \begin{bmatrix} U_i \\ V_i \\ W_i \\ T_i \end{bmatrix}$$

$$x = \begin{bmatrix} A^T \\ B^T \\ C^T \end{bmatrix} x_i$$

$$x_i = \frac{u_i}{w_i} = \frac{A^T x_i}{C^T x_i} \Rightarrow x_i C^T x_i - A^T x_i = 0$$

$$y_i = \frac{v_i}{w_i} = \frac{B^T x_i}{C^T x_i} \Rightarrow y_i C^T x_i - B^T x_i = 0$$

$$P = \begin{bmatrix} A \\ B \\ C \end{bmatrix}_{12\times 1}$$

$$a_{x_i}^T P = 0$$

$$a_{y_i}^T P = 0$$

where,

$$a_{x_i}^T = \left(-X_i^T, 0^T, x_i X_i^T\right)_{1\times 12}$$

$$a_{y_i}^T = \left(0^T, -X_i^T, y_i X_i^T\right)_{1\times 12}$$

Problem 1 continued on next page...

$$\begin{bmatrix} a^T x_1 \\ a^T y_1 \\ \vdots \\ a^T x_i \\ a^T y_i \\ \vdots \\ a^T x_n \\ a^T y_n \end{bmatrix}$$

$$P = M_{2n\times 12} \cdot P_{12\times 1} = 0_{2n\times 1}$$

$$M \times P = w$$

Therefore, find P such that it maximizes

$$\Omega = w^T w$$

$$\Rightarrow \hat{P} =_P w^T w =_P P^T M^T M P$$
$$||P||_2 = \sum_{i,j} P_{ij}^2 = 1.$$

Equivalent to finding the null space of M. Solution $\rightarrow$ Eigenvector to the smallest eigenvalue of M,

$$M = U_{2n\times 12} \cdot S_{12\times 12} \cdot V^T_{12\times 12}$$

$$\begin{aligned} \Omega &= P^T M^T M P \\ &= P^T V S U^T U S V^T P \\ &= P^T V S^2 V^T P \\ &= P^T \left(\sum s^2 v_i v_i^T \right) P \end{aligned}$$

$$\hat{P} = \begin{pmatrix} \hat{A} \\ \hat{B} \\ \hat{C} \end{pmatrix} = V_{12}$$

No solution exists if all x_i's on the same plane. Therefore, reduced rank of M,

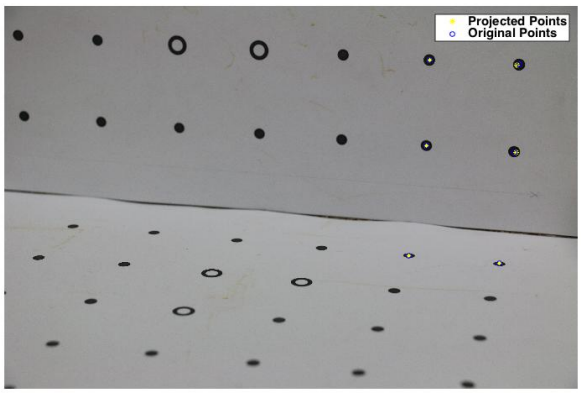
$$\hat{P} = \hat{K}\hat{R}(I_3| - \hat{X}_0) = (\hat{H}_\infty|\hat{h})$$

$$\hat{X}_0 = -\hat{H}_\infty^{-1}\hat{h}$$
$$\hat{H}_\infty = \hat{K}\hat{R}$$

where $\hat{R}$ is rotation matrix and $\hat{K}$ is upper triangular matrix.
Fundamental Camera Matrix P by DLT:

$$\begin{bmatrix} 0.0043 & -0.0008 & 0.0003 & -0.9062 \\ 0.0002 & 0.0043 & -0.0012 & -0.4228 \\ -0.0000 & -0.0000 & -0.0000 & -0.0002 \end{bmatrix}$$

Problem 1 continued on next page...




```
clear all;
im = imread('IMG_5455.JPG');
szx = size(im, 1);
szy = size(im, 2);

x = [4.917905405405405e+03;4.868608108108108e+03;4.726878378378378e+03;4.061364864864866e
    +03;4.030554054054055e+03;3.864175675675676e+03];
y = [5.581756756756758e+02;1.396229729729730e+03;2.456121621621622e+03;5.150405405405404e
    +02;1.334608108108108e+03;2.382175675675676e+03];

X = [0,0,0,36,36,36];
Y = [72, 36, 0, 72, 36, 0];
Z = [0,0,36,0,0,36];

XI = [x';y';ones(6,1)'];
XW = [X;Y;Z;ones(6,1)'];

P = fdlt(XW,XI)
projected = P*XW;

projected(1, :) = projected(1, :)./projected(3, :);
projected(2, :) = projected(2, :)./projected(3, :);
imshow(im);
hold on;
plot(projected(1,:), projected(2,:), 'y*');
plot(x,y,'r*');

figure, imshow(im);
hold on;
plot(projected(1,:), projected(2,:), 'r*');

figure, imshow(im);
hold on;
plot(x, y, 'y*');
```

```
function P = fdlt(xw, xi)

    x = xi(1,:);
    y = xi(2,:);
    z = xi(3,:);
    n = size(xw,2);
    A = zeros(2*n,12);
    for i = 1 : n
        A(2*i-1:2*i, :) = [-z(i)*xw(:,i)' zeros(1,4) x(i)*xw(:,i)';
                           zeros(1,4) -z(i)*xw(:,i)' y(i)*xw(:,i)'];
```

```
    end

 for i = 1 : 3
     A(i, :) = A(i, :)/norm(A(i, :));

    [u,s,v] = svd(A);
    P = reshape(v(:, 12), [4, 3])';
end
```

Problem 2

Implement the RANSAC based 8. variant of the calibration that we discussed in the class. Note that these two algorithms use a set of known correspondences between real-world points and image points.

RANSAC is a resampling technique that generates candidate solutions by using the minimum number observations (data points) required to estimate the underlying model parameters. As pointed out by Fischler and Bolles, unlike conventional sampling techniques that use as much of the data as possible to obtain an initial solution and then proceed to prune outliers, RANSAC uses the smallest set possible and proceeds to enlarge this set with consistent data points.

The number of iterations, N, is chosen high enough to ensure that the probability p (usually set to 0.99) that at least one of the sets of random samples does not include an outlier. Let u represent the probability that any selected data point is an inlier and $v = 1u$ the probability of observing an outlier. N iterations of the minimum number of points denoted m are required, where

$$1 - p = (1 - u^m)^N$$

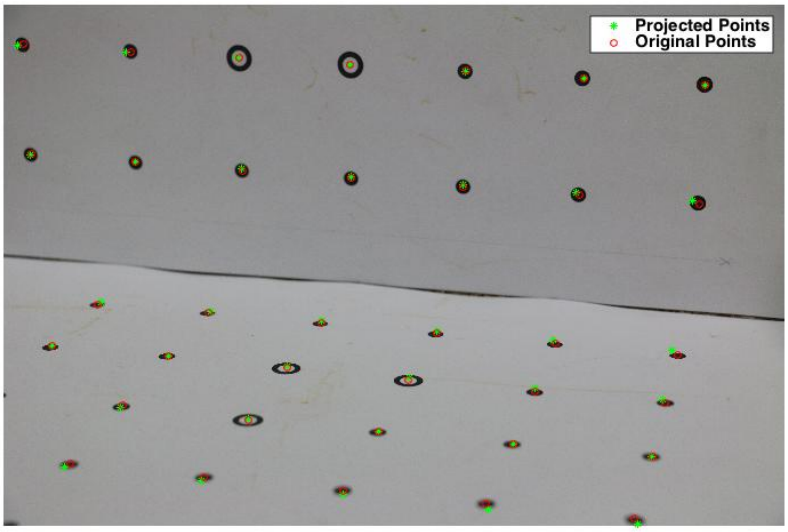
and thus with some manipulation,

$$N = \frac{log(1-p)}{log(1-(1-v)^m)}$$

Considering 0.75 to be the probablity of a point being an outlier, we get $N = 20$.
Fundamental Camera Matrix P by RANSAC:

$$\begin{bmatrix} 0.0040 & 0.0005 & 0.0018 & -0.9113 \\ 0.0002 & 0.0043 & -0.0008 & -0.4116 \\ -0.0000 & 0.0000 & -0.0000 & -0.0002 \end{bmatrix}$$

Problem 2 continued on next page...




```
close all;

clc;
im = imread('IMG_5455.JPG');

% imshow(im);
% [x, y] = ginput();

x = [4.916500e+03 4.068500e+03 3.244500e+03 2.428500e+03 1.652500e+03 8.925000e+02 1.405000e
    +02 4.876500e+03...
    4.036500e+03 3.220500e+03 2.436500e+03 1.676500e+03 9.245000e+02 1.965000e+02 4.724500e
    +03 3.868500e+03...
    3.036500e+03 2.228500e+03 1.436500e+03 6.605000e+02 4.636500e+03 3.724500e+03 2.836500e
    +03 1.988500e+03...
    1.156500e+03 3.405000e+02 4.548500e+03 3.572500e+03 2.628500e+03 1.716500e+03 8.365000e
    +02 4.420500e+03...
    3.380500e+03 2.380500e+03 1.404500e+03 4.685000e+02];
y = [5.645000e+02 5.165000e+02 4.605000e+02 4.205000e+02 3.725000e+02 3.325000e+02 2.765000e
    +02 1.396500e+03...
    1.332500e+03 1.276500e+03 1.212500e+03 1.164500e+03 1.100500e+03 1.044500e+03 2.452500e
    +03 2.372500e+03...
    2.300500e+03 2.228500e+03 2.156500e+03 2.100500e+03 2.788500e+03 2.708500e+03 2.628500e
    +03 2.540500e+03...
    2.460500e+03 2.388500e+03 3.164500e+03 3.076500e+03 2.988500e+03 2.908500e+03 2.804500e
    +03 3.596500e+03...
    3.492500e+03 3.404500e+03 3.308500e+03 3.212500e+03];

X = [0:36:216 0:36:216 0:36:180 0:36:180 0:36:144 0:36:144];
Y = [72*ones(7,1)' 36*ones(7,1)' zeros(22,1)'];
Z = [zeros(14, 1)' 36*ones(6,1)' 72*ones(6,1)' 108*ones(5,1)' 144*ones(5,1)'];

N = 5;

theta = 1;

max = 0;
XI = [x;y;ones(36,1)'];
XW = [X;Y;Z;ones(36,1)'];

for i = 1 : 150
```

Problem 2 continued on next page...

```
    r = [randi([1 14], 1, 3) randi([15 36], 1, 3)];
    P = fdlt(XW(:, r),XI(:, r));
     projected = P * XW;
     projected_x = projected(1, :)./projected(3, :);
     projected_y = projected(2, :)./projected(3, :);
     error_x = (abs(projected_x - x));
     error_y = (abs(projected_y - y));
     error = error_x + error_y;
      t = error < theta;
      number_of_inliers = sum(t);
     if (number_of_inliers > max)
         max = number_of_inliers;
         solution = P;
      end
end

projected = solution * XW;
projected(1, :) = projected(1, :)./projected(3, :);
projected(2, :) = projected(2, :)./projected(3, :);
imshow(im);
hold on;
plot(projected(1,:), projected(2,:), 'y*');
plot(x,y,'r*');
```

Problem 3

Use the image that is provided along with this assignment along with the 9. real-world measurements that are given to you to compute the cameras internal and external parameters using each of the above algorithms. Note that you need to manually estimate the image co- ordinates of the given world points. You may implement and use the 10. Harris Corner detector to help select these points. Describe your observations regarding the results.

$$A = RQ$$

R: Rotation matrix
Q:Upper triangular matrix

$$\hat{H}_{\infty}^{-1} = \hat{R}^T \cdot \hat{K}^{-1}$$

Normalize K

$$\hat{\hat{K}} = \frac{1}{\hat{K}_{33}} \hat{K}$$

$$k = \begin{bmatrix} -1.4574 & -0.0112 & -0.0726 \\ 0 & -1.4592 & 0.1060 \\ 0 & 0 & 0.0001 \end{bmatrix} * 10^4$$

$$R = \begin{bmatrix} -0.9587 & -0.0057 & -0.2845 \\ -0.0965 & -0.9341 & 0.3437 \\ -0.2677 & 0.3569 & 0.8950 \end{bmatrix}$$

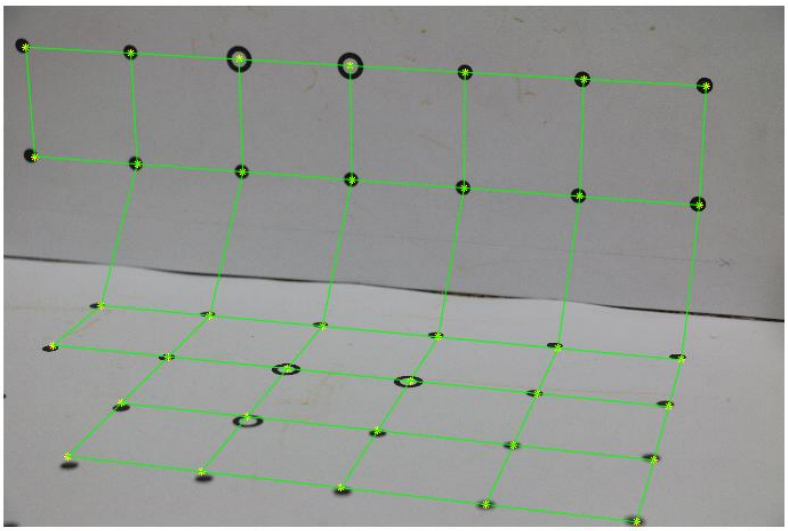
$$t = \begin{bmatrix} -55.3688 \\ 199.8165 \\ 615.0012 \end{bmatrix}$$

Problem 3 continued on next page...

```
[Q, W] = qr(inv(P(1:3, 1:3)));

% P = [KR -KRC]

% Rotation Matrix
R = inv(Q);

% Intrinsic Camera Matrix
K = inv(W);
K = K/K(3,3);

% Camera Center
C = - (inv(R) * inv(K) * P(:, 4));
```

Problem 4

Use the real-world measurements that are provided along with the estimated camera parameters to compute the image of a wireframe of the object. Note that you will be computing the location of image points as xi = P.Xi, and not use the image points. Overlay (draw) the wireframe over the actual image of the object using straight lines between the computed points xi. What do you observe about the overlay?



```
close all;
clear;
clc;
im = imread('IMG_5455.JPG');
% [x, y] = ginput();

x = [4.916500e+03 4.068500e+03 3.244500e+03 2.428500e+03 1.652500e+03 8.925000e+02 1.405000e
    +02 4.876500e+03 4.036500e+03 3.220500e+03 2.436500e+03 1.676500e+03 9.245000e+02
    1.965000e+02 4.724500e+03 3.868500e+03 3.036500e+03 2.228500e+03 1.436500e+03 6.605000e
    +02 4.636500e+03 3.724500e+03 2.836500e+03 1.988500e+03 1.156500e+03 3.405000e+02
    4.548500e+03 3.572500e+03 2.628500e+03 1.716500e+03 8.365000e+02 4.420500e+03 3.380500e
    +03 2.380500e+03 1.404500e+03 4.685000e+02];
y = [5.645000e+02 5.165000e+02 4.605000e+02 4.205000e+02 3.725000e+02 3.325000e+02 2.765000e
    +02 1.396500e+03 1.332500e+03 1.276500e+03 1.212500e+03 1.164500e+03 1.100500e+03
```

```
    1.044500e+03 2.452500e+03 2.372500e+03 2.300500e+03 2.228500e+03 2.156500e+03 2.100500e
    +03 2.788500e+03 2.708500e+03 2.628500e+03 2.540500e+03 2.460500e+03 2.388500e+03
    3.164500e+03 3.076500e+03 2.988500e+03 2.908500e+03 2.804500e+03 3.596500e+03 3.492500e
    +03 3.404500e+03 3.308500e+03 3.212500e+03];

X = [0:36:216 0:36:216 0:36:180 0:36:180 0:36:144 0:36:144];
Y = [72*ones(7,1)' 36*ones(7,1)' zeros(22,1)'];
Z = [zeros(14, 1)' 36*ones(6,1)' 72*ones(6,1)' 108*ones(5,1)' 144*ones(5,1)'];

N = 250;

theta = 1;

max = 0;
XI = [x;y;ones(36,1)'];
XW = [X;Y;Z;ones(36,1)'];

for i = 1 : N
     r = [randi([1 14], 1, 3) randi([15 36], 1, 3)];
     P = fdlt(XW(:, r),XI(:, r));
     projected = P * XW;
     projected_x = projected(1, :)./projected(3, :);
     projected_y = projected(2, :)./projected(3, :);
     error_x = (abs(projected_x - x));
     error_y = (abs(projected_y - y));
     error = error_x + error_y;
      t = error < theta;
      number_of_inliers = sum(t);
     if (number_of_inliers > max)
         max = number_of_inliers;
         solution = P;
      end
end

projected = solution * XW;
projected(1, :) = projected(1, :)./projected(3, :);
projected(2, :) = projected(2, :)./projected(3, :);

figure, imshow(im);
hold on;
plot(projected(1,:), projected(2,:), 'y*');
plot(x, y, 'r*');
hold on;

for i = 2:36
    if (i ~= 8 && i ~= 15 && i ~= 21 && i ~= 27 && i ~= 32)
    plot([projected(1, i), projected(1, i-1)], [projected(2, i), projected(2, i-1)], 'g');
    end
end

for i = 1:7
        plot([projected(1, i), projected(1, 7+i)], [projected(2, i), projected(2, 7+i)], 'g'
    );
        if (i <= 6)
        plot([projected(1, 7+i), projected(1, 14+i)], [projected(2, 7+i), projected(2, 14+i)
    ], 'g');
        plot([projected(1, 14+i), projected(1, 20+i)], [projected(2, 14+i), projected(2, 20+
    i)], 'g');
        end
        if (i <= 5)
        plot([projected(1, 20+i), projected(1, 26+i)], [projected(2, 20+i), projected(2, 26+
    i)], 'g');
```

Problem 4 continued on next page...

```
63          plot([projected(1, 26+i), projected(1, 31+i)], [projected(2, 26+i), projected(2, 31+
        i)], 'g');
64          end
65  end
```

Problem 5

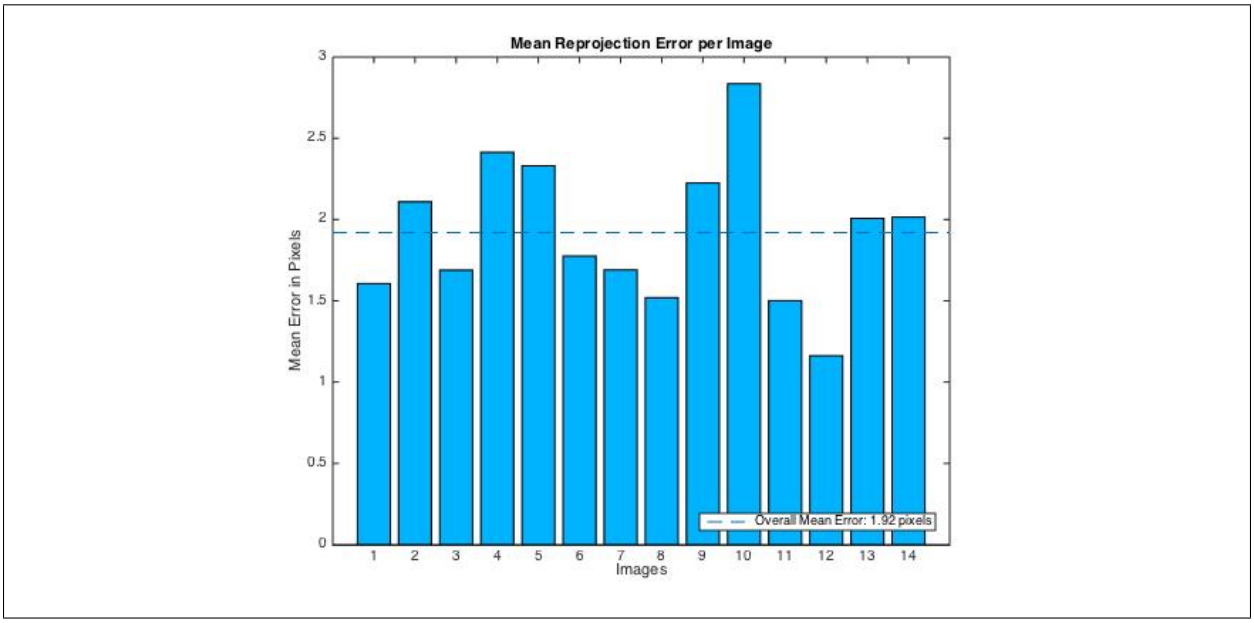
Repeat the calibration of the camera using Zhangs method using either the available OpenCV or Matlab implementation. How does your results compare with DLT based method?

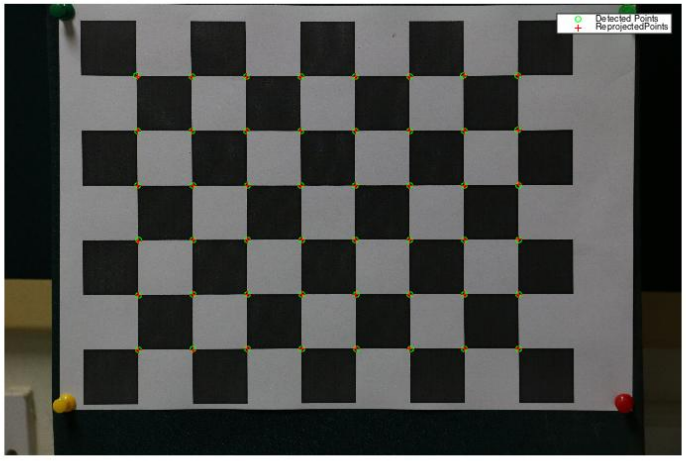
We use a checkboard pattern for camera calibration in this method. The pattern is placed on model plane (Z component in world coordinate is zero) as illustrated below. Several pictures are taken by changing the location and rotation angle. Camera parameters are estimated by Zhang's method as described his research paper.
Estimated camera matrix K':

$$1.0e+04 * \begin{bmatrix} 1.3478 & 0 & 0 \\ 0 & 1.3531 & 0 \\ 0.2750 & 0.1836 & 0.0001 \end{bmatrix}$$

Radial distortion parameters:

$$\begin{bmatrix} 0.2849 \\ -0.7455 \end{bmatrix}$$



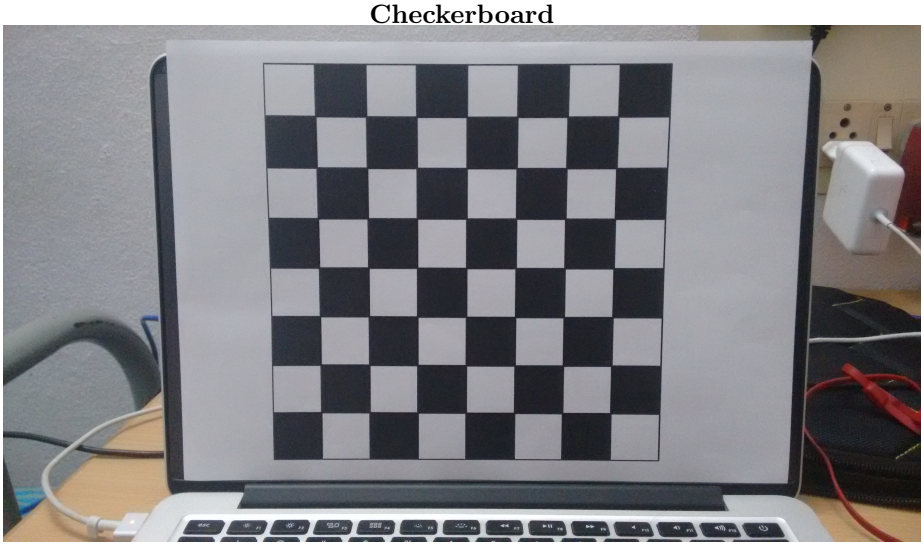



```
clear;
close all;
clc;

location = '/Users/abhinavmoudgil95/Documents/MATLAB/CV/assignment1/images/';
images = imageSet(location);
imageFileNames = images.ImageLocation;
[imagePoints, boardSize] = detectCheckerboardPoints(imageFileNames);
squareSizeInMM = 29;
worldPoints = generateCheckerboardPoints(boardSize,squareSizeInMM);
params = estimateCameraParameters(imagePoints,worldPoints);
showReprojectionErrors(params);

figure;
imshow(imageFileNames{1});
hold on;
plot(imagePoints(:,1,1), imagePoints(:,2,1),'go');
plot(params.ReprojectedPoints(:,1,1), params.ReprojectedPoints(:,2,1),'r+');
legend('Detected Points','ReprojectedPoints');
hold off;
```

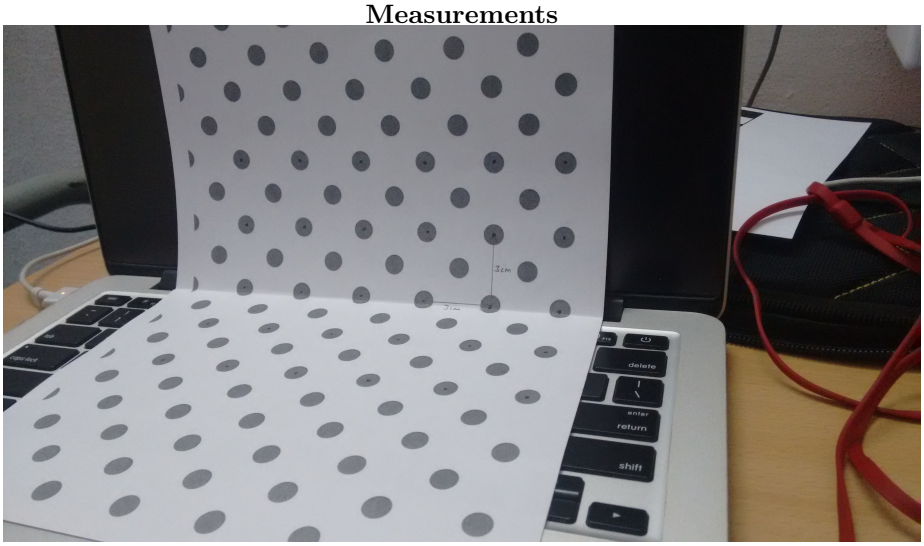
Problem 6

Repeat the 3 calibration methods using your own camera and your own calibration object for which you measure and determine the world co-ordinates. Use a printed checkerboard pattern for Zhangs method. Describe and comment on the results of each method.

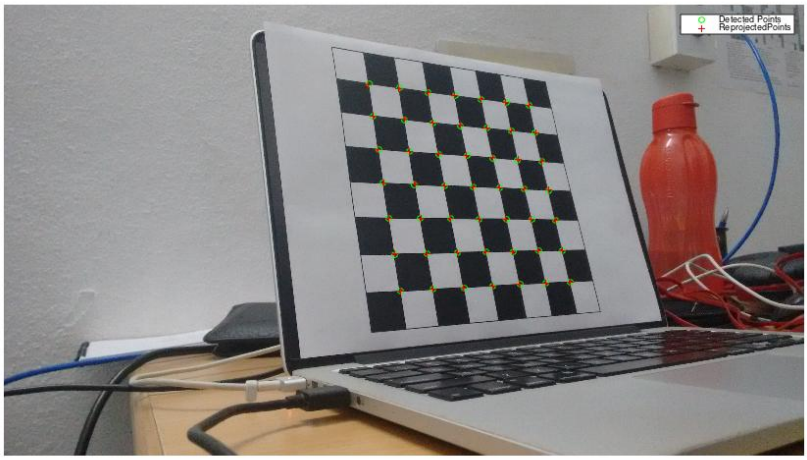
Checkerboard

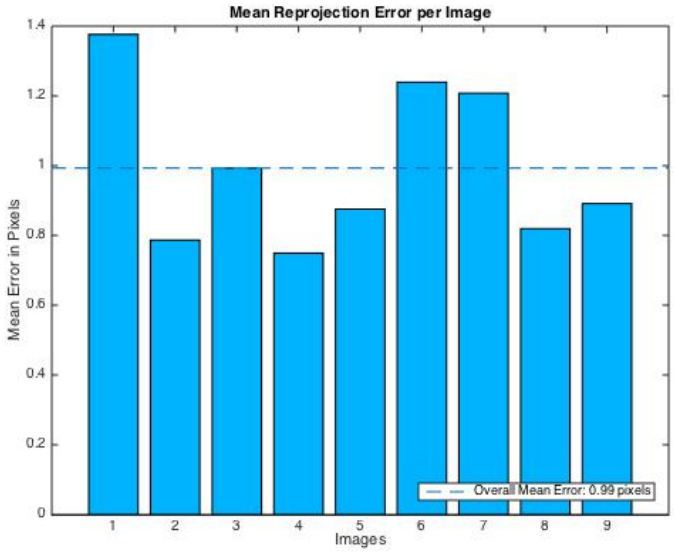


Measurements

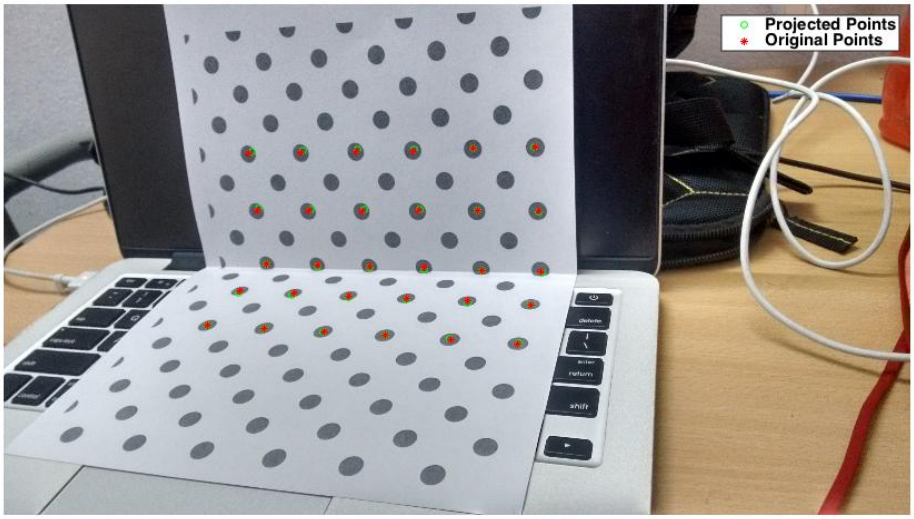


Problem 6 continued on next page...


Detected Points
ReprojectedPoints


Mean Reprojection Error per Image
Mean Error in Pixels
Images
Overall Mean Error: 0.99 pixels
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1
2
3
4
5
6
7
8
9


Projected Points
Original Points


Projected Points
Original Points