The Sunyeav-Zel'dovich Effect

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An effect predicted more than three decades ago, the S-Z effect is coming into its own now as a probe of cosmological conditions, thanks to two wonderful scientists Rashid Sunyaev and Yakov Zeldovich, who predicted this effect to describe anisotropies in the CMB. This term paper describes the SZ Effect and the underlying physics of inverse Compton effect. It highlights the Thermal and Kinetic SZ effect and discusses how it is used in estimation of cosmological parameters, such as the Hubble constant, angular diameter distances, and peculiar velocities.

CONTENTS

I. COSMIC MICROWAVE BACKGROUND RADIATION

CMBR is a thermal blackbody background radiation with a monopole temperature 2.725485 ± 0.00057 K, discovered by American radio astronomers Arno Penzias and Robert Wilson in 1965. Soon after the discovery of the cosmic background radiation Zel′dovich and Sunyaev proposed that hot gas in galaxy clusters should cast a faint shadow because of the interaction between energetic electrons and the radiation photons. It is one of the small distortion of the CMB spectrum provoked by the scattering of the CMB photons by hot electrons in galaxy clusters. The underlying mechanism for this effect is inverse compton scattering where a low energy CMB photon is scattered by high energy moving electron of hot gas. The scattered photon gains energy and its frequency shifts towards higher value. Sunyaev Zel'dovich Effect (SZE) is now routinely observed, and it has become an important tool for studying the history of the universe.

II. INVERSE COMPTON EFFECT

The underlying principle for the Sunyaev-Zel'dovich Effect is the Inverse Compton Effect. The effect is observed when photons from the cosmic microwave background (CMB) move through the hot gas surrounding a galaxy cluster. The CMB photons are scattered to higher energies by the electrons in this gas ($\Delta\lambda \leq 0 \implies \nu$ increases), showing this phenomenon. This results in decrement in the Rayleigh-Jeans region and increment in the Wien's region. Observations of the Sunyaev–Zel'dovich effect provide a nearly redshift-independent means of detecting galaxy clusters.

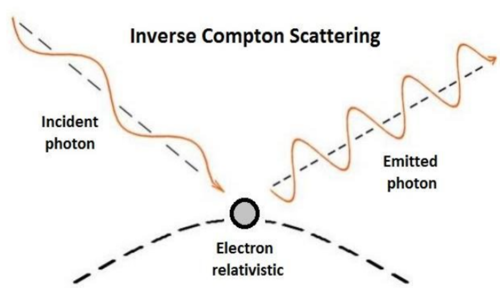

FIG. 1 *Schematic diagram showing Inverse Compton Effect.*

III. REDSHIFTING

The length scales are related to the cosmic scale factor $a(t)$ as $\vec{r}(t) = a(t)\vec{r_0}$ where, t_0 is the today's time. Thus, $a(t_0) = 1$. Now a photon emitted with a wavelength λ_{em} at time t_{em} observed at $t_{obs} = t_0$ with:

$$\lambda_{em} = \frac{a(t_{obs})}{a(t_{em})}\lambda_{em} = \frac{1}{a(t_{em})}\lambda_{em} \tag{1}$$

Thus, redshift is related to the scale factor via:

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1 = \frac{1}{a(t_{em})} - 1 \tag{2}$$

The redshift depends on the wavelengths as $\lambda_{obs} = (1+z)\,\lambda_{em}$; frequencies as $\nu_{obs} = \frac{\nu_{em}}{(1+z)}$; and temperatures by Wien's law as $T_{obs} = \frac{T_{em}}{(1+z)}$. Recall that SZ fractional intensity change $\frac{\Delta I_\nu}{\Delta I_{\nu 0}} = -yg(x)$ i.e. it only depends on the dimensionless ratio x evaluated at emission.

$$x = \frac{h\nu_{em}}{kT_{rad,em}} = \frac{h\nu_{obs}(1+z)}{kT_{rad,obs}(1+z)} = \frac{h\nu_{obs}}{kT_{rad,obs}} \tag{3}$$

Thus, from equation 3, it is clear that SZ fractional specific intensity change is redshift independent! Same goes for the integrated intensity change $\Delta I/I = 4y$. This also means that the (1) shape of the distortion is redshift independent, (2) max, min, and null in $\Delta I_\nu / I_\nu$ is always the same, (3) robust signature of and test of (thermal) SZ effect.

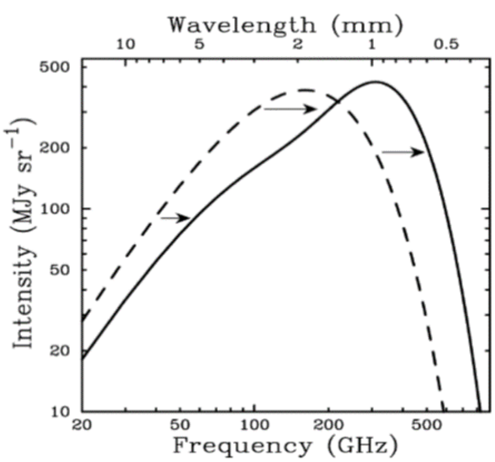

FIG. 2 *Shift in the CMB spectrum frequency due to SZ Effect.*

IV. THERMAL SUNYAEV ZEL'DOVICH EFFECT

We assume a sparse astrophysical plasma where no photon absorption or creation is happening hence conserving photon number, and we only have a redistribution of their energies. Let:

$N(E)$: number density of photons

Δ: photon energy change due to single scattering.

$<\Delta>$: average change of photon energy per photon electron collision.

t_c: time between collisions.

Thus, the 1-dimensional energy current is: AN/t_c. By applying the law of conservation of energy:

$$t_c \frac{dN}{dt} = -\frac{d(AN)}{dE} \tag{4}$$

Let's assume that the electrons in the ICM at temperature T follows maxwellian distribution, i.e.

$$p(\nu)d\nu = \sqrt{\frac{2}{\pi}} \left(\frac{m_e}{k_b T}\right)^{3/2} \nu^2 e^{\left(\frac{-m\nu^2}{2k_b T}\right)} d\nu \tag{5}$$

After using the Fokker-Plank equation and adding the quantum corrections by considering photons as bosons and the fact that bosons like to clump if photon occupation numbers (n) are not small i.e. stimulated transitions which ensures the final state is more probable if it is already occupied, the equations 4 becomes,

$$t_c \frac{dN}{dt} = \frac{-d(AN(1+n))}{dE} + \frac{d^2(BN)}{2dE^2}(1+n) - BN\frac{d^2(1+n)}{2dE^2} \tag{6}$$

Changing $N(E)$ to photon phase-space occupation number $n(\omega) : N \propto \omega^2 n(\omega)$. Now, we can rewrite the equation 6 to get the Komapaneets equation:

$$\frac{\partial n}{\partial t} = \left(\frac{\sigma_T n_e \hbar}{m_e c}\right) \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \left(n(1+n) + \frac{k_b T}{\hbar} \frac{\partial n}{\partial \omega}\right)\right] \tag{7}$$

The underlying assumptions are (1) Each photon scatters only once, low optical depth, (2) Electron gas is hotter than the CMB photon, first term in equation 7 drops out. Integrating equation 7, we get:

$$\Delta n = \int \left(\frac{\sigma_T n_e \hbar}{m_e c}\right) \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left[\omega^4 \frac{k_b T}{\hbar} \frac{\partial n}{\partial \omega}\right] dt \tag{8}$$

We can define a length element dl along the gas system such that: $dl = cdt$, them the comptonization parameter y becomes,

$$y = \int n_e \sigma_T \frac{k_b T_e}{m_e c^2} dl \tag{9}$$

The optical depth is: $\tau_e = \int n_e \sigma_T dl$. The frequency version of the same is:

$$\Delta n = \frac{y}{\nu^2} \frac{\partial}{\partial \nu} \left(\nu^4 \frac{\partial n}{d\nu}\right) \tag{10}$$

The CMB photons follow Planck black-body distribution:

$$I_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{k_b T}} - 1} \tag{11}$$

Thus, we get:

$$\frac{\partial I_\nu}{I_\nu} = \frac{\partial n}{n} = \frac{\partial T}{T} = -y\frac{xe^x}{e^x - 1}\left(4 - coth\left(\frac{x}{2s}\right)\right) \tag{12}$$

where, $x = \hbar\omega/k_b T_{rad}$, the normalized frequency. For a galaxy cluster with parameters: $T_e \approx 5 - 10$ MK $n_e \approx 10^{-3} cm^{-3}$, $L \approx 1$ Mpc. The equation 12 becomes:

$$\frac{\partial I_\nu}{I_\nu} = \frac{\partial T}{T} = -2y \approx 10^{-6} \tag{13}$$

So the spectral distortion due to thermal SZE is of order 1μK.

A. Thermal SZE as a Probe of Galaxy Cluster

In each line of sight SZ measures Comptonization parameter in a cluster:

$$y = \sigma_T \int \frac{n_e k T_e}{m_e c^2} ds = \frac{\sigma_T}{m_e c^2} \int P_e ds \approx \frac{\sigma_T k T_e}{m_e c^2} \int n_e \ ds \tag{14}$$

which gives the direct measurement of the projected pressure in column and if T_e known, a measure of electron column density. Now, SZ flux is measured by:

$$\int cos\ \theta\ y\ d\Omega \approx \int y\ \Omega = \frac{\int y\ dA}{D_A^2} \tag{15}$$

where $D_A(z)$ is the (angular diameter) distance

$$\int y\ \Omega \approx \frac{\sigma_T k T_e}{m_e c^2} \int n_e\ ds\ dA \propto M_{gas} \tag{16}$$

The above equation 16 displays how SZ flux gives intracluster cluster gas mass M_{gas}!

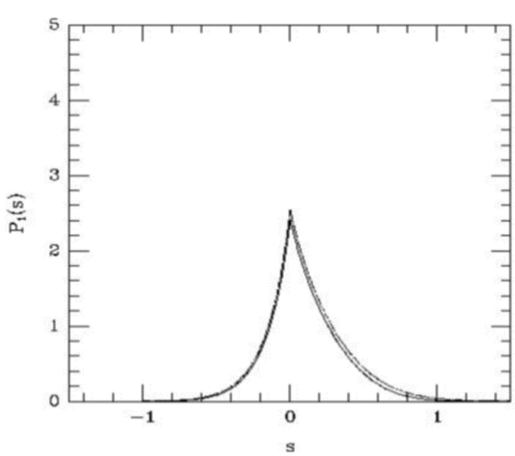

FIG. 3 *The probability distribution of scattered photon frequencies is slightly higher on up-scattering tail than down-scattering tail, showing the slight increase in frequency on average*

V. KINETIC SUNYAEV ZEL'DOVICH EFFECT

We have considered till now e^- velocities in ICM which are distributed isotropically, thus canceling the Doppler effect. But, when a cluster is moving along the line of sight with velocity $v_{cluster}$, the bulk motion adds uniform Doppler shift to the usual thermal SZ effect which has a net non-zero peculiar-motion then we will observe a net non zero Doppler effect. This is called Kinematic Sunyaev Zel′dovich Effect. At lower frequencies (Rayleigh-Jeans), this gives:

$$\frac{\Delta T}{T} = -\tau_e \left(\frac{v_{cluster}}{c}\right) \tag{17}$$

So, it retains black-body shape but it changes temperature in non-relativistic limit. It is much smaller than thermal SZE. Kinetic SZE is observed after 218 GHz for zero Thermal SZE component. The large scale density perturbations causes the bulk cluster motion/flows.

A. Kinematic SZE to calculate Peculiar velocities

The kinematic SZ effect can be used to determine the peculiar velocity of a cluster. This makes the kinematic SZ effect a unique and powerful cosmological tool. The peculiar velocity can directly be derived from the kinematic SZ effect after separating it from the thermal SZ effect. The maximum intensity of the kinematic effect is observed at a frequency of 218 GHz and at this frequency, radial velocity components of clusters might be measured.

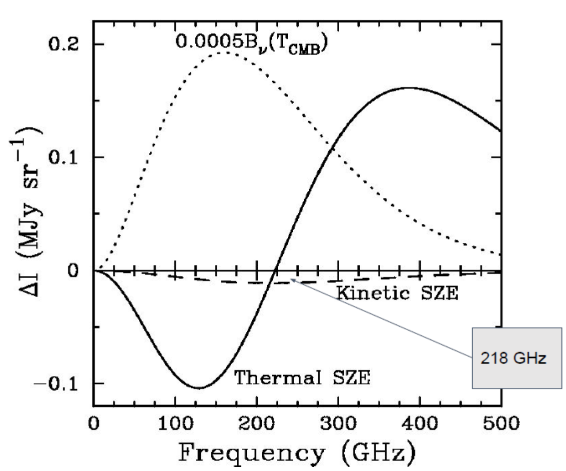

FIG. 4 *We can see that in Rayleigh-Jeans region there is a decrement in the intensity and in Wien regions there is an increment in the intensity with no change in intensity at* 218 *GHz.*

VI. COSMOLOGICAL SIGNIFICANCE

A. Calculating Absolute Angular Diameter Distance

If we combine thermal SZE observation with Bremsstrahlung radiation we can find the absolute angular diameter distance of the cluster by assuming a β density model of gas in the cluster (ICM).

$$\text{3D gas model: } n_e = n_{e0}\left[1+\left(\frac{R}{r_c}\right)^2\right]^{-\frac{3\beta}{2}} \tag{18}$$

This would give projected X-ray Surface Brightness to be:

$$I(R) = I_0\left[1+\left(\frac{R}{r_c}\right)^2\right]^{-\frac{3\beta+1}{2}} \tag{19}$$

Using the Ray-leigh Jeans low energy approximation, we calculate the projected temperature decrements profile:

$$\Delta T_{SZE} = \frac{\Delta}{T_0\left[1+\left(\frac{R}{r_c}\right)^2\right]^{-\frac{3\beta+1}{2}}} \tag{20}$$

where $R/r_c = \theta/\theta_c$. Now, with this information, we can calculate the absolute angular diameter distance D_A related by $dl = D_A\ d\theta$, where dl is the projected length from the center of the electron gas and $d\theta$ is the angular size which we measure. Now, for Thermal SZE the change in Temperature would be:

$$\Delta T_{SZE} \sim \int dl\ n_e\ T_e \tag{21}$$

and the Bremsstrahlung radiation:

$$S_x \sim \int dl\ n_e^2\ \Lambda_{eH} \tag{22}$$

where Λ_{eH} is the X-ray cooling function. So, substituting dl from the above two equations we get:

$$D_A \sim \frac{\Delta (T_0)^2\ \Lambda_{eH0}}{S_{x0}\ T_{e0}^2}\frac{1}{\theta_c} \tag{23}$$

Now, D_A can be related to the Hubble constant by:

$$D_A(z) = \frac{1}{(1+z)}\frac{c}{H_0}\int_0^z \frac{dz'}{E(z')} \tag{24}$$

B. Determining Hubble's constant

We can use SZE to get an estimate of the Hubble constant in a way that is completely independent of all other estimates of H_0. Consider, a cluster that has a characteristic radius, number density and temperature of R, n, and T respectively. The magnitude of the thermal S-Z effect depends on nTR. The temperature comes directly from the X-ray spectrum, and the bremsstrahlung surface brightness depends on $n^2T^{1/2}R$. Measurement of the S-Z effect, the spectrum, and the luminosity therefore give us an independent determination of n, T, and R. Now, suppose that the redshift (z) and apparent angular size of the cluster have been measured (for spherical systems), then from R and the angular size of the cluster the angular diameter distance D_A is estimated.

VII. CONCLUSIONS

The concept of Sunyaev-Zel'dovich Effect was explained by inverse Compton scattering of CMB photons from hot energetic electrons of ICM gas. The net result is the decrement of the intensity below 218 GHz and increment in the intensity level above 218 GHz. There is a spectral distortion of order a few μK for thermal SZE where low energy photons are up-scattered, explained by equation of $\Delta I/I$. Kinetic SZE is observed due to change of reference frame which can be used to find the cluster's peculiar velocity. Thermal SZE in combination with Bremsstrahlung emission can be used to determine the absolute angular diameter distance which on comparison with formula for co-moving distance can give us Hubble constant. Apart from this, we can calculate cluster mass fraction, Ω_m, Ω_Λ, etc.

VIII. ACKNOWLEDGMENTS

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