

Assignment 1

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Question: Determine z transform of $\left(\frac{1}{2}\right)^n u[n]$ with its region of convergence.

Solution:

$$\mathcal{Z}\left\{\left(\frac{1}{2}\right)^n u[n]\right\} = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^n u[n] z^{-n} \quad (0.1)$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n u[n] z^{-n} \quad (0.2)$$

$$= \frac{1}{1 - \frac{1}{2}z^{-1}}, |z| > \frac{1}{2} \quad (0.3)$$

Two signals can have same z transform but different Region Of Convergences.

Pole: $|z| = \frac{1}{2}$

$$\text{case 1 : } |z| < \frac{1}{2}, (\text{no ROC}) \quad (0.4)$$

$$\text{case 2 : } \frac{1}{2} < |z| \quad (0.5)$$

$$(0.6)$$

causality : If ROC is the exterior of outermost pole then the system is said to be Causal.

Here the signal is casual.

wget <https://github.com/Abhipank/Digital-Signal-Processing/blob/main/CODES/assign1.py>

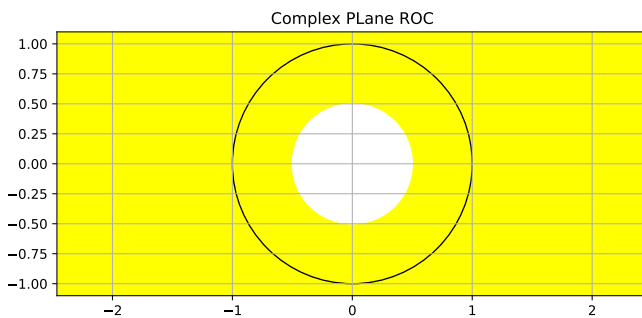


Fig. 0: Region Of Convergence

ROC includes the unit circle ,hence the system is stable.