Assignment 8

Abhishek Kumar

IIT Hyderabad

24May,2022

Outline

Question Statement

Question: Let X represent a binomial random variable with parameters n and p.Show that and Compute

(a) $E[X] = np$

(b) $E[X(X-1)] = n(n-1)p^2$

(c) $E[X(X-1)(X-2)] = n(n-1)(n-2)p^3$

(d) $E[X^2]$

(e) $E[X^3]$

Solution

Solution: Let $P(X)$ be probability of random Variable X and r denotes all values that X can take.

$$E[X] = \sum X \times P(X = r) \tag{1}$$

$$\Rightarrow E[X] = \sum r \times \binom{n}{r} \times p^r \times (1-p)^{n-r} \tag{2}$$

$$\tag{3}$$

Using Binomial coefficients property:

$$\binom{n}{r} = n/r \times \binom{n-1}{r-1} \tag{4}$$

$$\Rightarrow E[X] = \sum r \times n/r \times \binom{n-1}{r-1} \times p^r \times (1-p)^{n-r} \tag{5}$$

$$\Rightarrow E[X] = np \sum \binom{n-1}{r-1} \times p^{r-1} \times (1-p)^{((n-1)-(r-1))} \tag{6}$$

$$\Rightarrow E[X] = np(p+1-p)^{n-1} \tag{7}$$

$$\Rightarrow E[X] = np \tag{8}$$

$$E[X(X-1)] = \sum X(X-1) \times P(X=r) \tag{9}$$

$$\tag{10}$$

$$\Rightarrow E[X(X-1)] = \sum r(r-1) \times \binom{n}{r} \times p^r \times (1-p)^{n-r} \tag{11}$$

Using Binomial coefficients property:

$$\binom{n}{r} = n/r \times (n-1)/(r-1) \times \binom{n-1}{r-1} \tag{12}$$

$$\Rightarrow E[X(X-1)] = \sum r(r-1) \times n(n-1)/r(r-1) \times \binom{n-1}{r-1} p^r (1-p)^{n-r} \tag{13}$$

$$\Rightarrow E[X(X-1)] = n(n-1)p^2 \sum \binom{n-2}{r-2} \times p^{r-2} \times (1-p)^{((n-2)-(r-2))} \tag{14}$$

$$\Rightarrow E[X(X-1)] = n(n-1)p^2(p+1-p)^{n-2} \tag{15}$$

$$\Rightarrow E[X(X-1)] = n(n-1)p^2 \tag{16}$$

$$E[X(X-1)(X-2)] = \sum X(X-1)(X-2) \times P(X=r) \tag{17}$$

$$\Rightarrow E[X(X-1)(X-2)] = \sum r(r-1)(r-2) \times \binom{n}{r} \times p^r \times (1-p)^{n-r} \tag{18}$$

Using Binomial coefficients property:

$$\binom{n}{r} = n(n-1)(n-2)/(r)(r-1)(r-2) \times \binom{n-3}{r-3} \tag{19}$$

$$\Rightarrow E[X(X-1)(X-2)] = n(n-1)(n-2)p^3 \sum \binom{n-3}{r-3} p^{r-3}(1-p)^{(n-r)} \tag{20}$$

$$\Rightarrow E[X(X-1)(X-2)] = n(n-1)(n-2)p^3 \tag{21}$$

From equation 16

$$E[X(X-1)] = n(n-1)p^2 \tag{22}$$
$$\Rightarrow E[X^2] - E(X) = n(n-1)p^2 \tag{23}$$
$$\Rightarrow E[X^2] = n(n-1)p^2 + np \tag{24}$$

From equation 21

$$E[X(X-1)(X-2)] = n(n-1)(n-2)p^3 \tag{25}$$
$$\Rightarrow E[X^3] - 3 \times E[X^2] + 2 \times E[X] = n(n-1)p^2 \tag{26}$$
$$\Rightarrow E[X^3] = n(n-1)(n-2)p^3 + 3n(n-1)p^2 + np \tag{27}$$