Assignment 9

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Outline

Question Statement

Question: The joint p.d.f of X and Y is given by:

$$f_{XY}(x,y) = \begin{cases} 2(1-x) & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0 & \text{else} \end{cases} \tag{1}$$

Find the probability density function of z=xy

Solution

Solution: f denotes p.d.f. and F denotes c.d.f.

Approach

first we will find c.d.f. of z=xy then will differentiate it to get its p.d.f.

$$F_Z(z) = P(Z \leq z) \tag{2}$$
$$F_Z(z) = 1, z > 1 \tag{3}$$
$$f_Z(z) = 0, z > 1 \tag{4}$$

$$F_Z(z) = \int_{x=0}^{x=z}\int_{y=0}^{y=1} f_{XY}(x,y)\,dx\,dy + \int_{x=z}^{x=1}\int_{y=0}^{y=z/x} f_{XY}(x,y)\,dx\,dy, 0 < z \leq 1 \tag{5}$$

$$\Rightarrow F_Z(z) = \int_{x=0}^{x=z}\int_{y=0}^{y=1} 2(1-x)\,dx\,dy + \int_{x=z}^{x=1}\int_{y=1}^{y=z/x} 2(1-x)\,dx\,dy \tag{6}$$

$$\Rightarrow F_Z(z) = 2z - z^2 - 2z\log_e z - 2z + 2z^2 \tag{7}$$

$$\Rightarrow F_Z(z) = z^2 - 2z\log_e z \tag{8}$$

$$f_Z(z) = 2z - 2 - 2\log_e z \tag{9}$$

Answer

Joint p.d.f. of $z = xy$ is as follows:

$$f_Z(z) = \begin{cases} 2z - 2 - 2\log_e z & , 0 < z \leq 1 \\ 0, & z > 1 \end{cases} \tag{10}$$