Assignment 10

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Outline

Question Statement

Question: The random variables X_k are independent with densities $f_k(x)$ and the random variable variable n is independent of X_k with $P(n = k) = p_k$.Show that

$$s = \sum X_k \Rightarrow F_s(s) = \sum p_k[f_1(s) \times f_2(s) \ldots\ldots \times f_n(s)] \tag{1}$$

Solution

Solution: f denotes p.d.f. and F denotes c.d.f.

Approach

we will use the concept of conditional probability and extension of bayes theorem in probability density distribution

We shall first determine the conditional probability

$$F_s(s|n = n) = P(s \leq s, n = n)/P(n = n) \tag{2}$$

The event $(s \leq s, n = n)$ consists of all outcomes such that n=n and $\sum X_k \leq s$.since the RV n is independent of RVs X_k,this yields

$$F_s(s) = P(\sum X_k \leq s)P(n = n)/p(n = n) = P(\sum X_k) \tag{3}$$

From the independence of RVs X_k,it follows that

$$F_s(s|n = n) = f_1(s) \times f_2(s) \times f_3(s)...... \times f_n(s) \tag{4}$$

Extending the concept of bayes theorem in probability density distribution concept,we get

$$F(x) = F(x|A_1)P(A_1) + ... + F(x|A_n)P(A_n) \tag{5}$$

setting $A_k = (n = k)$ in equation(5) ,we obtain

$$F_s(s) = \sum P(X_k)[f_1(s) \times f_2(s)...... \times f_n(s)] \tag{6}$$

$$\Rightarrow F_s(s) = \sum P(X_k)[f_1(s) \times f_2(s)..... \times f_n(s)] \tag{7}$$

Hence Proved