# Assignment 5

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**Question:**Suppose that the reliability of HIV test is specified as follows:
Of people having HIV, $90\%$ of the test detect the disease but $10\%$ go undetected.Of people free of HIV,$99\%$ of the test are judged HIV $-$ve but $1\%$ are diagnosed as showing HIV $+$ve.From a large population of which only $0.1\%$ have HIV,one person is selected at random,given the HIV test and pathologist reports him/her as HIV $+$ve.What is the probability that the person actually has HIV ?

**Solution:** Let $E$ be the event that the person selected is actually having HIV and $A$ be the event that the person's HIV test is diagnosed as $+$ve.We need to find $P(E|A)$ i.e. probability that person is actually having HIV provided he has been diagnosed as HIV $+$ve .

| Event | X(random variable) |
|---|---|
| E | 1 |
| $E'$ | 0 |
| A | 1 |
| $A'$ | 0 |

TABLE II
EVENTS

Also $E'$ denotes the event that person selected is actually not having HIV.

$$P_E(1) = 0.1\% = 0.1/100 = 0.001 \tag{1}$$
$$P_E(0) = 1 - P_E(1) = 1 - 0.001 = 0.999 \tag{2}$$
$$P_A(1) = P_E(1)P_{A|E}(1|1) + P_E(1)P_{A|E}(1|0) \tag{3}$$
$$P_{A|E}(1|1) = 1\% = 0.9 \tag{4}$$
$$P_{A|E}(1|0) = 90\% = 0.01 \tag{5}$$

Using Bayes Theorem,

$$P_{E|A}(1|1) = P_{EA}(1 \cap 1)/P_A(1) = P_E(1)P_{A|E}(1|1)/(P_E(1)P_{A|E}(1|1) + P_E(0)P_{A|E}(1|0)) \tag{6}$$
$$\Rightarrow P_{E|A}(1|1) = 0.001 \times 0.9/(0.001 \times 0.9 + 0.999 \times 0.01) \tag{7}$$
$$\Rightarrow P_{E|A}(1|1) = 0.083 \tag{8}$$