

Assignment 4

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Question: If A, B, C are three events associated with a random experiment, prove that

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \quad (1)$$

Solution: Consider

$$E = B \cup C \quad (2)$$

$$P(A \cup B \cup C) = P(A \cup E) = P(A) + P(E) - P(A \cap E) \quad (3)$$

$$P(E) = P(B \cup C) = P(B) + P(C) - P(B \cap C) \quad (4)$$

$$A \cap E = A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad (5)$$

$$P(A \cap E) = P[(A \cap B) \cup (A \cap C)] = P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)] \quad (6)$$

$$P(A \cap E) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \quad (7)$$

using equation(3) and equation(4) and equation(7)

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \quad (8)$$

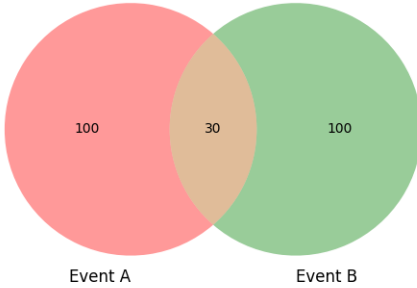


Fig. 1. By this figure generated by python code, we can verify equation (3) intuitively

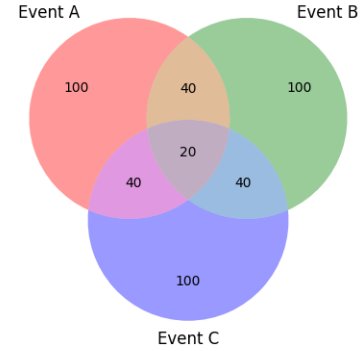


Fig. 2. By this figure generated by python code, we can verify equation (8) intuitively