Assignment 13

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Outline

Question Statement

Question: if a state e_j is accessible from a persistent state e_i,then e_i is also accessible from e_j and moreover e_j is persistent.

Solution

Solution: suppose a state e_j is accessible from a persistent state e_i,but e_i is not accessible from e_j.Thus system goes from e_i to e_j in a certain number of steps with positive probability ${p_i}_j^m = a$ and after that it does not return to e_i.consequently starting from e_i the probability of the system not returning to e_i is at least a or the probability of the system eventually returning to e_i cannot exceed $1 - a$.Thus $f_{11} \leq 1 - a$ .But $1 - a$ is strictly less than 1,contradicting the assumption that e_i is persistent .Hence e_i must be accessible from e_j,that is,${p_j}_i^r = b > 0$ for some r.
from above ,we have

$$
\begin{aligned}
{p_i}_j^{n+m} &\geq {p_i}_k^m {p_k}_j^n && (1)\\
\Rightarrow {p_i}_j^{n+m+r} &\geq {p_i}_j^m {p_j}_i^{n+r} \geq {p_i}_j^m {p_j}_j^n {p_j}_i^r = ab{p_i}_j^n && (2)
\end{aligned}
$$

Similarly,

$$
{p_j}_j^{n+m+r} \geq {p_j}_i^r {p_i}_i^n {p_i}_j^m = ab{p_i}_i^n \qquad (3)
$$

Result

Thus the two series $\sum p_{i}{}_{i}^{n}$ and $\sum p_{j}{}_{j}^{n}$ converge or diverge together .But $\sum$ $\mathrm{p}_{i}{}_{i}^{n} = \infty$,since e_i is persistent and it follows now that e_j is also persistent .This completes the proof.