

# AI1110 Assignment 1

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April 11, 2022

## Question 1a

Solve the following inequation, write down the solution set and represent it on the real number line:

$$11x - 4 < 15x + 4 \leq 14 + 13x, x \in W$$

## Solution

$$-4 + 11x < 15x + 4 \leq 14 + 13x, x \in W \quad (1)$$

now consider each equation in this expression as a line, i.e.,  $L_1$ ,  $L_2$  and  $L_3$

$$L_1 \equiv 11x - y - 4 \quad (2)$$

$$L_2 \equiv 15x - y + 4 \quad (3)$$

$$L_3 \equiv 13x - y + 14 \quad (4)$$

In vector form,

$$L_1 \equiv \begin{pmatrix} 11 & -1 \end{pmatrix} \mathbf{x} = 4 \quad (5)$$

$$L_2 \equiv \begin{pmatrix} 15 & -1 \end{pmatrix} \mathbf{x} = -4 \quad (6)$$

$$L_3 \equiv \begin{pmatrix} 13 & -1 \end{pmatrix} \mathbf{x} = -14 \quad (7)$$

So we need to find the range of  $x$  at where the line  $L_2$  lies between line  $L_1$  and the line  $L_3$

We can obtain the intersection point of  $L_1$  and  $L_2$  by the following way,

$$\begin{pmatrix} 11 & -1 \\ 15 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 4 \\ -4 \end{pmatrix} \quad (8)$$

The augmented matrix for the above matrix equation is

$$\left( \begin{array}{cc|c} 11 & -1 & 4 \\ 15 & -1 & -4 \end{array} \right) \quad (9)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \left( \begin{array}{cc|c} -4 & 0 & 8 \\ 15 & -1 & -4 \end{array} \right) \quad (10)$$

$$\xrightarrow{R_1 \leftarrow R_1 / -4} \left( \begin{array}{cc|c} 1 & 0 & -2 \\ 15 & -1 & -4 \end{array} \right) \quad (11)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 15R_1} \left( \begin{array}{cc|c} 1 & 0 & -2 \\ 0 & -1 & 26 \end{array} \right) \quad (12)$$

$$\xrightarrow{R_2 \leftarrow R_2 / -1} \left( \begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -26 \end{array} \right) \quad (13)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} -4 \\ -26 \end{pmatrix} \quad (14)$$

Hence the point of intersection of lines  $L_1$  and  $L_2$  is  $\begin{pmatrix} -4 \\ -26 \end{pmatrix}$ .

Similarly we get the  $x$  value at intersection point of lines  $L_2$  and  $L_3$

$$\begin{pmatrix} 15 & -1 \\ 13 & -1 \end{pmatrix} \mathbf{x} = \begin{pmatrix} -4 \\ -14 \end{pmatrix} \quad (15)$$

The augmented matrix for the above matrix equation is

$$\left( \begin{array}{cc|c} 15 & -1 & -4 \\ 13 & -1 & -14 \end{array} \right) \quad (16)$$

$$\xrightarrow{R_1 \leftarrow R_1 - R_2} \left( \begin{array}{cc|c} 2 & 0 & 10 \\ 13 & -1 & -14 \end{array} \right) \quad (17)$$

$$\xrightarrow{R_1 \leftarrow R_1 / 2} \left( \begin{array}{cc|c} 1 & 0 & 5 \\ 13 & -1 & -14 \end{array} \right) \quad (18)$$

$$\xrightarrow{R_2 \leftarrow R_2 - 13R_1} \left( \begin{array}{cc|c} 1 & 0 & 4.67 \\ 0 & -1 & -79 \end{array} \right) \quad (19)$$

$$\xrightarrow{R_2 \leftarrow R_2 / -1} \left( \begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 79 \end{array} \right) \quad (20)$$

$$\Rightarrow \mathbf{x} = \begin{pmatrix} 5 \\ 79 \end{pmatrix} \quad (21)$$

Hence the point of intersection of lines  $L_2$  and  $L_3$  is  $\begin{pmatrix} 5 \\ 79 \end{pmatrix}$ .

Since  $L_1 < L_2 \leq L_3$ , this implies the corresponding  $x$ -coordinates follows,  $-2 < x \leq 5$  and  $x \in W$

Therefore the whole numbers in this range are,

$$\boxed{\{0, 1, 2, 3, 4, 5\}}$$

Here is the plot of corresponding points on the real number line

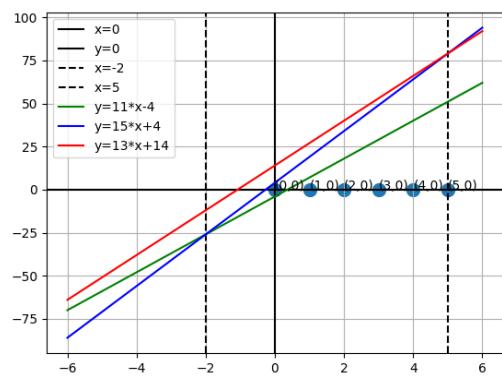


Figure 1: lines  $L_1$ ,  $L_2$  and  $L_3$

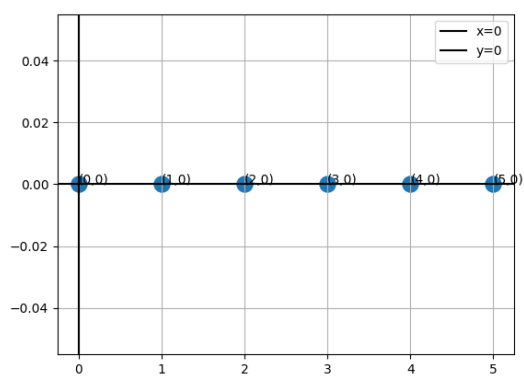


Figure 2: set of points that obey given expression on real number line