

Assignment 5

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Question Statement

Suppose that the reliability of HIV test is specified as follows:
Of people having HIV, 90% of the test detect the disease but 10% go undetected. Of people free of HIV, 99% of the test are judged HIV -ve but 1% are diagnosed as showing HIV +ve. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test and pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV ?

Solution

Let E be the event that the person selected is actually having HIV and A be the event that the person's HIV test is diagnosed as +ve. We need to find $P(E|A)$ i.e. probability that person is actually having HIV provided he has been diagnosed as HIV +ve

Table

Events	X(random variable)
E	1
E'	0
A	1
A'	0

Also E' denotes the event that person selected is actually not having HIV.

$$P_E(1) = 0.1\% = 0.1/100 = 0.001 \quad (1)$$

$$P_E(0) = 1 - P_E(1) = 1 - 0.001 = 0.999 \quad (2)$$

$$P_A(1) = P_E(1)P_{A|E}(1|1) + P_E(0)P_{A|E}(1|0) \quad (3)$$

$$P_{A|E}(1|1) = 1\% = 0.01 \quad (4)$$

$$P_{A|E}(1|0) = 90\% = 0.9 \quad (5)$$

Approach

Using Bayes Theorem

$$P_{E|A}(1|1) = P_{EA}(1 \cap 1)/P_A(1) = \quad (6)$$

$$P_E(1)P_{A|E}(1|1)/(P_E(1)P_{A|E}(1|1) + P_E(0)P_{A|E}(1|0)) \quad (7)$$

$$\Rightarrow P_{E|A}(1|1) = 0.001 \times 0.01 / (0.001 \times 0.01 + 0.999 \times 0.9) \quad (7)$$

$$\Rightarrow P_{E|A}(1|1) = 0.0011 \quad (8)$$

Answer

Probability that the person which was reported as HIV +ve is actually having the disease = 0.083