Assignment 12

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Outline

Question Statement

Question: Given a WSS process x(t),we form the sum $\hat{x}(t) = \sum c_n e^{-jnw_0 t}$.Show that $E[|x(t) - \hat{x}(t)|^2] = 0$ for $0 < t < T$.

Solution

Solution:

$$\hat{x}(t) = \sum c_n e^{-jnw_0 t}, c_n = 1/T \times \int_{t=0}^{t=T} x(t) e^{-jnw_0 t} \tag{1}$$

$$E[|x(t) - \hat{x}(t)|^2] \tag{2}$$

$$\Rightarrow E[(x(t) - \hat{x}(t)(\overline{x}(t) - \overline{\hat{x}}(t)] \tag{3}$$

$$\Rightarrow E[|x(t)|^2] + E[|\hat{x}(t)|^2] - E[\hat{x}(t)\overline{x}(t)] - E[x(t)\overline{\hat{x}}(t)] \tag{4}$$

From equation(1)

$$E[x(t)] = E[\hat{x}(t)] \tag{5}$$

$$\Rightarrow E[|x(t)|^2] = E[|\hat{x}(t)|^2] = c_n^2 \tag{6}$$

$$E[\hat{x}(t)\overline{x}(t)] = E[x(t)\overline{\hat{x}}(t)] = E[|x(t)|^2] = c_n^2 \tag{7}$$

Plugging the values in the equation(4),we get:

$$E[|x(t) - \hat{x}(t)|^2] \tag{8}$$

$$\Rightarrow E[(x(t) - \hat{x}(t)(\overline{x}(t) - \overline{\hat{x}}(t)] \tag{9}$$

$$\Rightarrow E[|x(t)|^2] + E[|\hat{x}(t)|^2] - E[\hat{x}(t)\overline{x}(t)] - E[x(t)\overline{\hat{x}}(t)] \tag{10}$$

$$\Rightarrow c_n^2 + c_n 2 - c_n^2 - c_n^2 = 0 \tag{11}$$

Result

$E[|x(t) - \hat{x}(t)|^2] = 0$

Hence Proved.