# Assignment 7

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# Outline

# Question Statement

**Question:**Two players A and B play a series of match on a condition that A will win the series if he succeeds in winning m matches before B wins n matches.The probability of winning a match for A is $p$ and B is $q = 1 - p$.What is probability that A will win the series?

# Solution

**Solution:**Consider

| **Probability** | **Event** |
|---|---|
| $P_A$ | Probability that A wins |
| $P_B$ | probability that B wins |

Table 1

Clearly by the end of $(m + n - 1)$th match there must be a winner and $P_A + P_B = 1$.Question asks to find $P_A$.

A can win in the following mutually exclusive ways.

| $X_k$**(random variable)** | **A wins m matches in m+k matches** |
| --- | --- |

Table 3

where

$$k = 0, 1, 2, ...., n - 1 \tag{1}$$

$X_1, X_2, ..., X_{n-1}$ are mutually exclusive events.
if A and B are mutually exclusive events then $P(A + B) = P(A) + P(B)$.

$$\Rightarrow P_A = P(X_1 + X_2 + X_3 + ... + X_{n-1}) = P(X_1) + P(X_2) + .. + P(X_{n-1}) \tag{2}$$

## To find $P(X_k)$

For A to win $m$ matches in exactly $m+k$ matches, A must win the last game and and $(m-1)$ matches matches in any order among the first $(m-k+1)$ matches.
$P(X_k) = P$(A wins (m-1) matches among first (m+k-1) matches) $\times$ P(A wins the last game)

$$\Rightarrow P(X_k) = \binom{(m+k-1)}{(m-1)} \times p^{m-1} \times q^k \times p \tag{3}$$

$$\Rightarrow P(X_k) = ((m-k+1)! \times p^m \times q^k)/(m-1)! \tag{4}$$

$$P_A = \sum P(X_k) \tag{5}$$

$$\Rightarrow P_A = p^m(1 + (m/1) \times q + ..$$

$$... + (m(m+1)..(m+n-2)/1 \times 2... \times (n-1)) \times q^{n-1}) \tag{6}$$