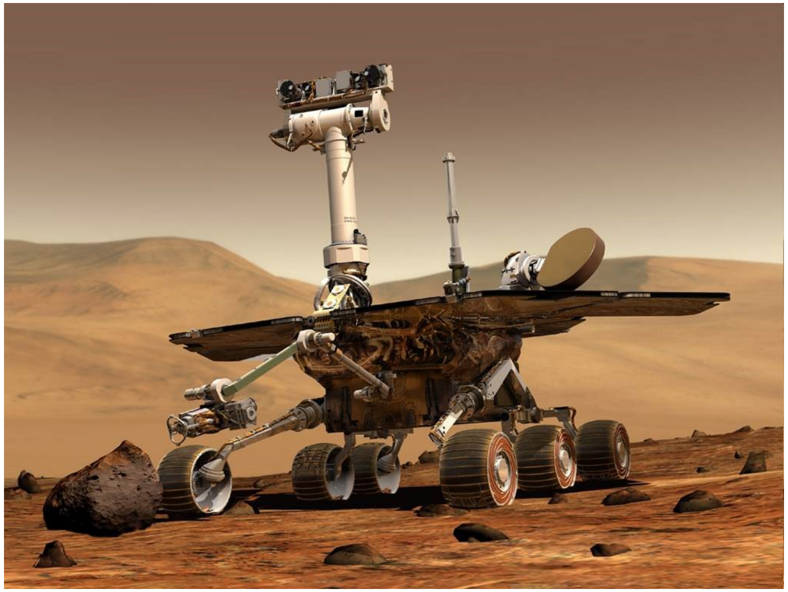
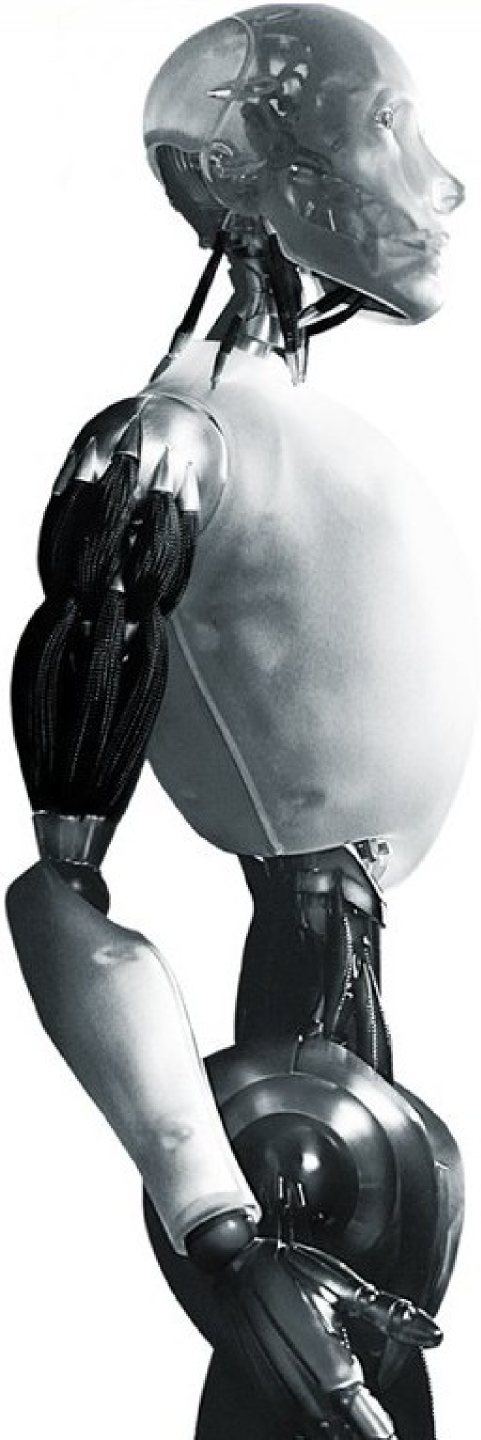
Disclaimer

These slides are intended as presentation aids for the lecture. They contain information that would otherwise be to difficult or time-consuming to reproduce on the board. But they are incomplete, not self-explanatory, and are not always used in the order they appear in this presentation. As a result, these slides should not be used as a script for this course. I recommend you take notes during class, maybe on the slides themselves. It has been shown that taking notes improves learning success.

What is the Difference?





Robotics

Mobile Robotics

TU Berlin

Oliver Brock

Reading for this set of slides

- Planning Algorithms (Steve LaValle)
  - 6 Combinatorial Motion Planning (6.1 – 6.3)
  - 8 Feedback Motion Planning (8.1, 8.2)
- Please refer to the slides for potential fields and vehicle kinematics

Please note that this set of slides is intended as support for the lecture, not as a stand-alone script. If you want to study for this course, please use these slides in conjunction with the indicated chapters in the text books. The textbooks are available online or in the TUB library (many copies that can be checked out for the entire semester. There are also some aspects of the lectures that will not be covered in the text books but can still be part of the homework or exam. For those It is important that you attend class or ask somebody about what was covered in class.

Nature-Made Mobility

Human-Made Mobility

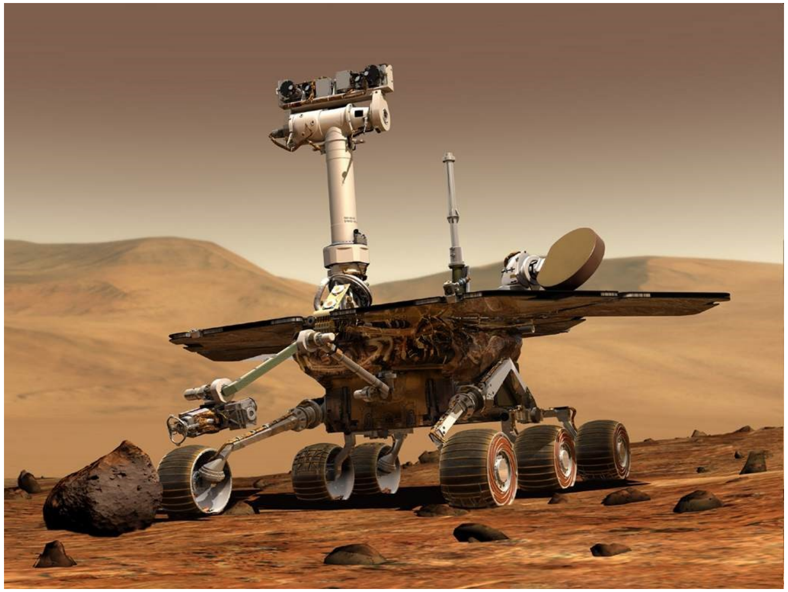
Segway Human Transporter

How does it move?

Big Dog

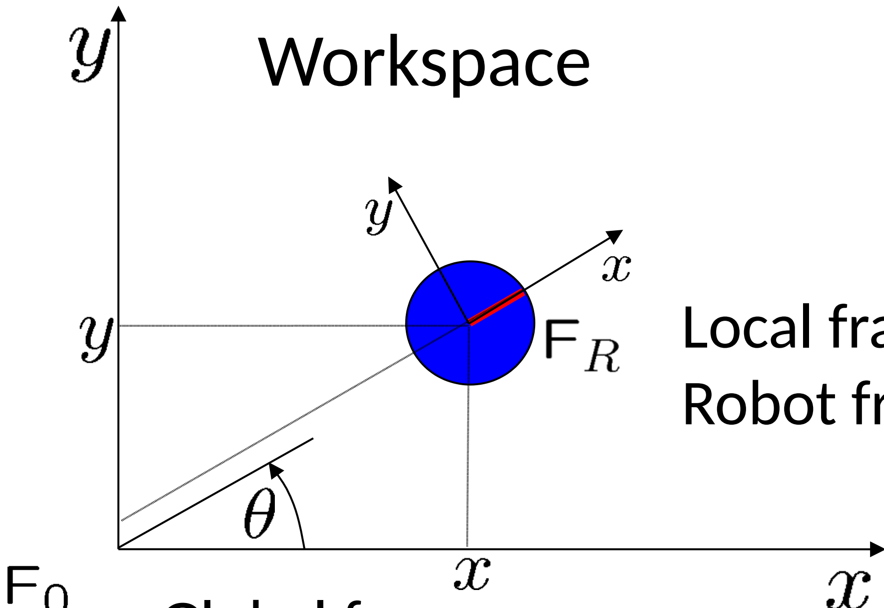
Play video

What is the Fundamental Difference?



Representation

Workspace

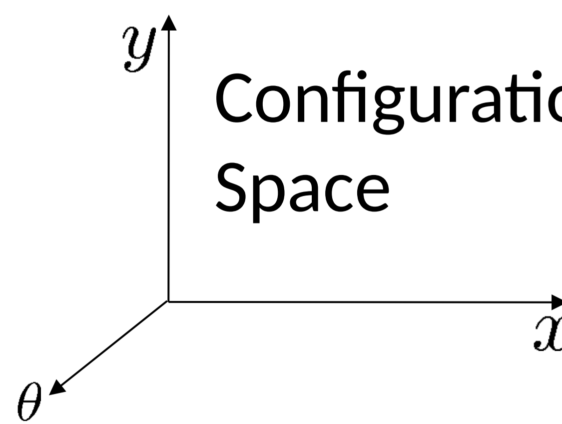


Local frame
Robot frame

Global frame
World frame
Reference frame

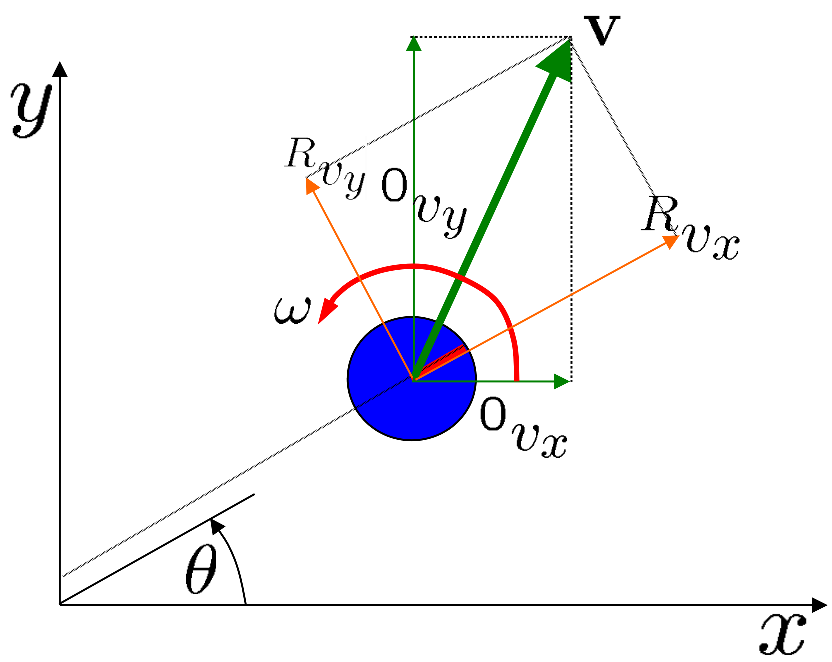
$$q = (\overbrace{x, y}^{\text{position}}, \theta)$$

orientation
or heading



Configuration
Space

Representation cont.



$$q = (x, y, \theta)$$

$$\dot{q} = (\dot{x}, \dot{y}, \dot{\theta})$$

$$\dot{q} = (v_x, v_y, \omega)$$

$$\dot{q} = (\mathbf{v}, \omega)$$

Estimating the Postition

- How can we estimate the behavior of the robot based on the command we send?
- Time: $\hat{s} = t \cdot v_{\mathsf{desired}}$
- Error: $s - \hat{s} = t \cdot (v_{\mathsf{actual}} - v_{\mathsf{desired}})$
- Error can be large!
- Error accumulates with time!

Encoders for Odometry

Odometer measures how far we go…

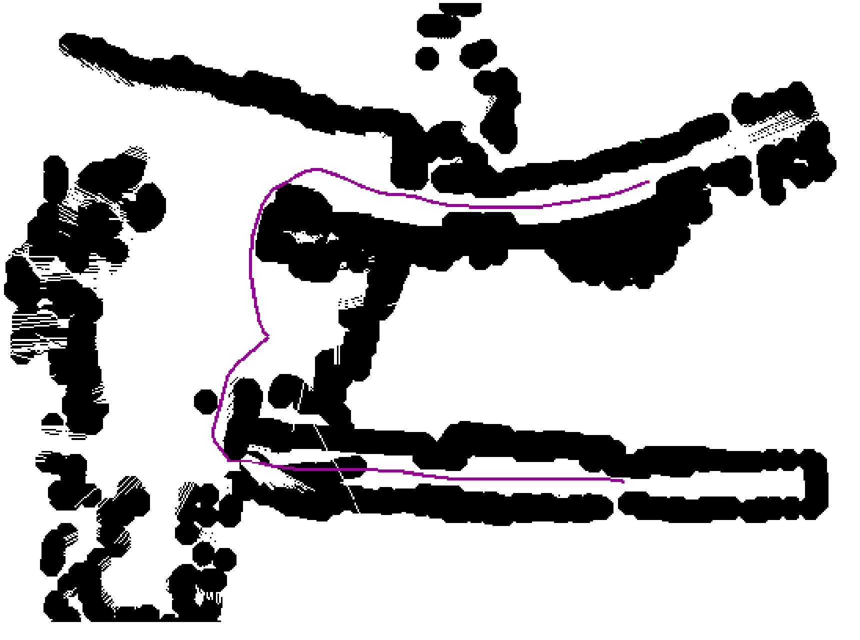
We can use odometry to estimate the robot's configuration based on the motor commands we sent.

Encoders!

Dead Reckoning

Error Sources

- Controller
- Mapping to motor command
- Performance of motor command
  - slippage!
  - actuation limits



Motion Constraints

nonholonomic

Nonholonomic equality constraint:

$$-\sin\theta\,\dot{x} + \cos\theta\,\dot{y} = 0$$

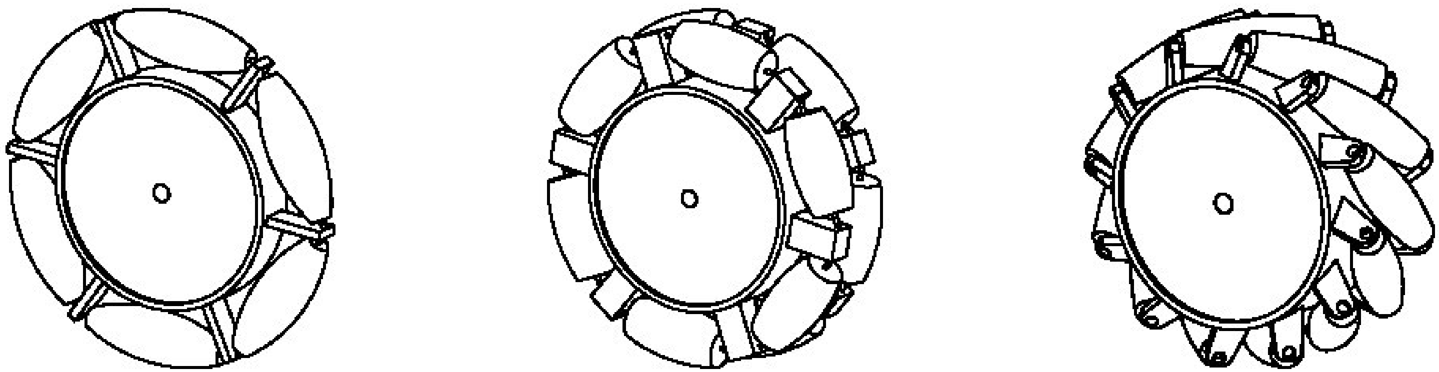
$$\overset{\theta=0^\circ}{\Rightarrow}\ 0\,\dot{x} + 1\,\dot{y} = 0$$

$$\overset{\theta=45^\circ}{\Rightarrow}\ -\frac{1}{\sqrt{2}}\,\dot{x} + \frac{1}{\sqrt{2}}\,\dot{y} = 0$$

$$\overset{\theta=90^\circ}{\Rightarrow}\ -1\,\dot{x} + 0\,\dot{y} = 0$$

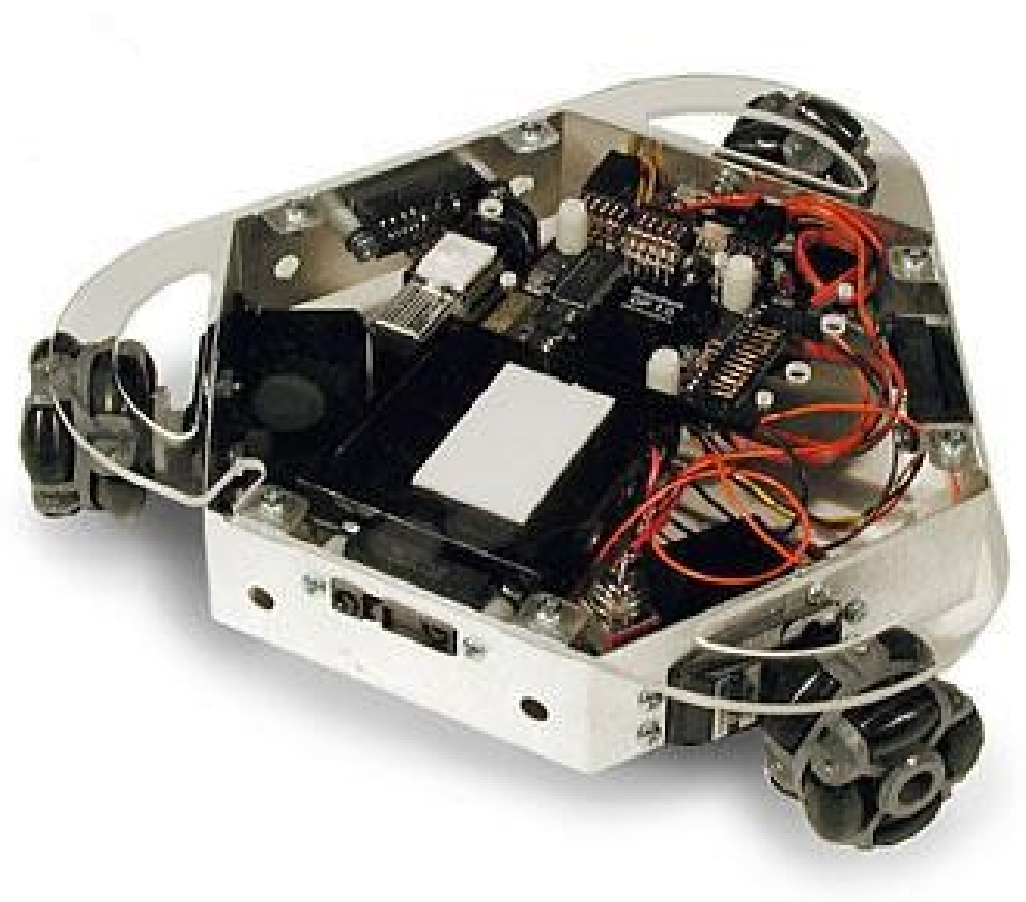
holonomic

Wheels

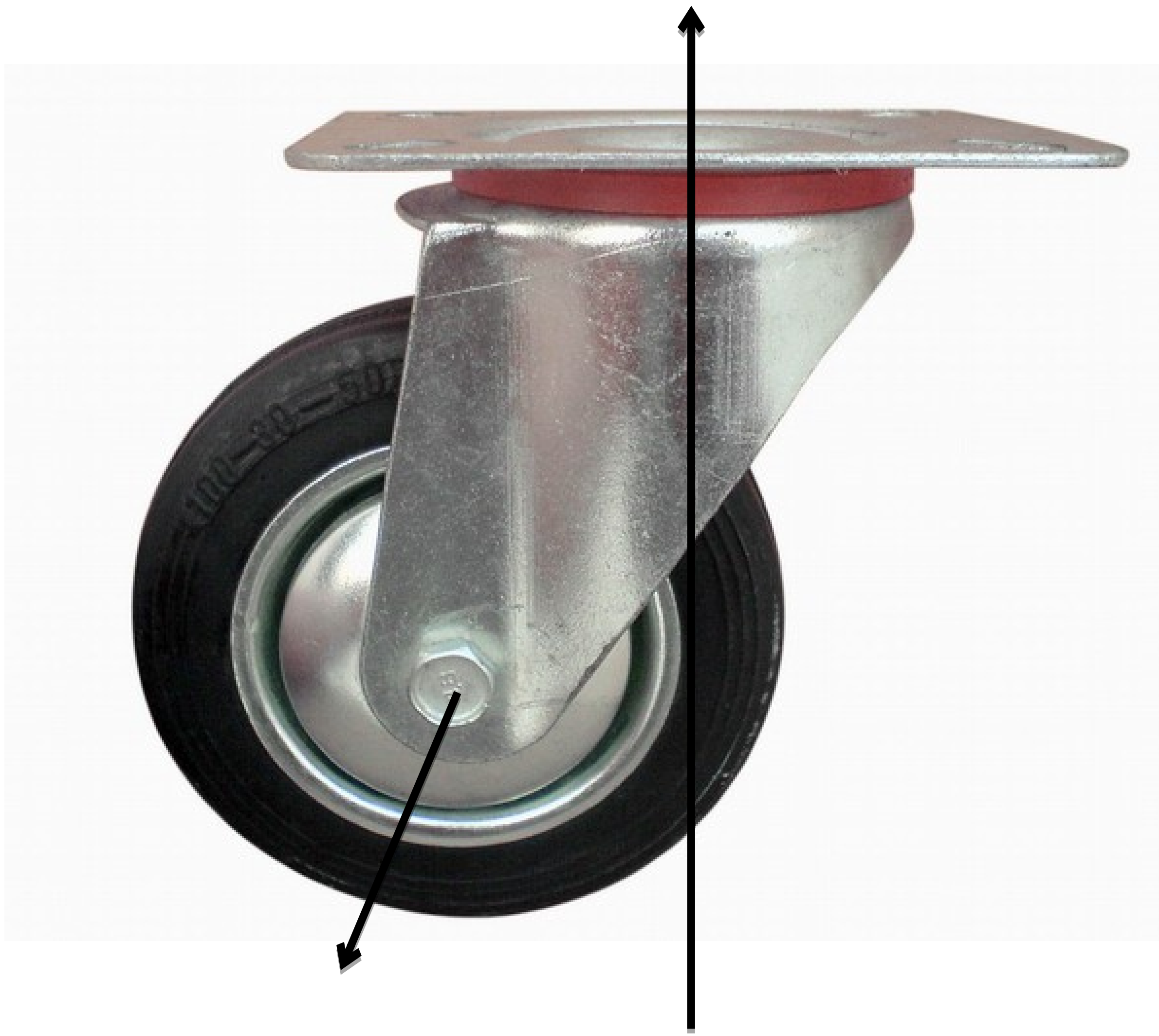


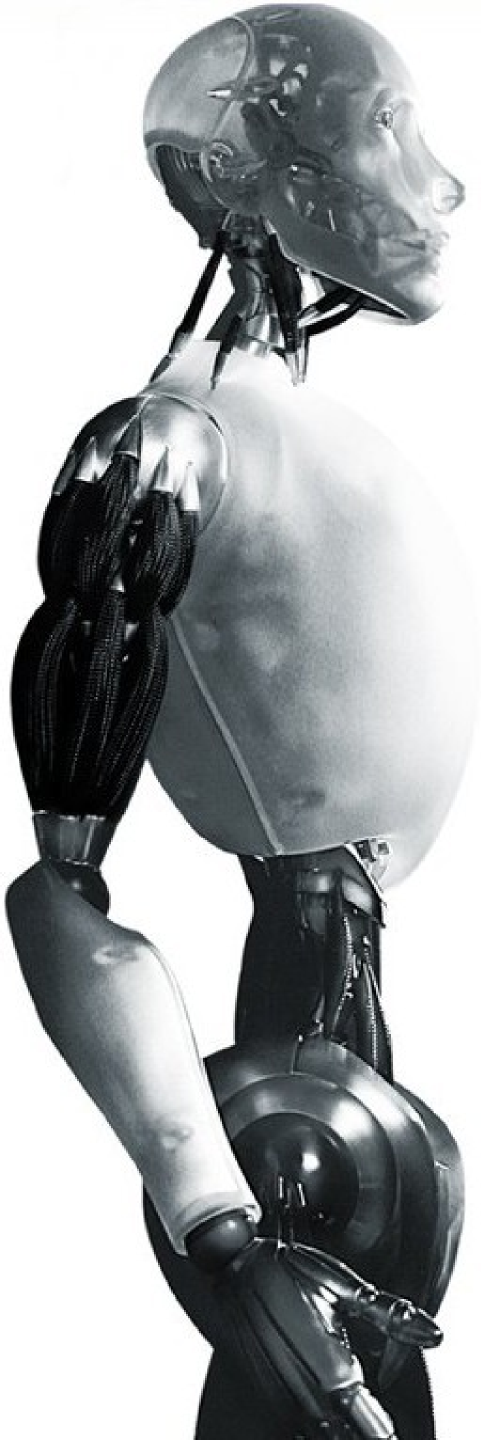
Universal Double Universal Swedish

Getting Rid of Nonholonomicity



Caster Wheel





Robotics

Drive Systems

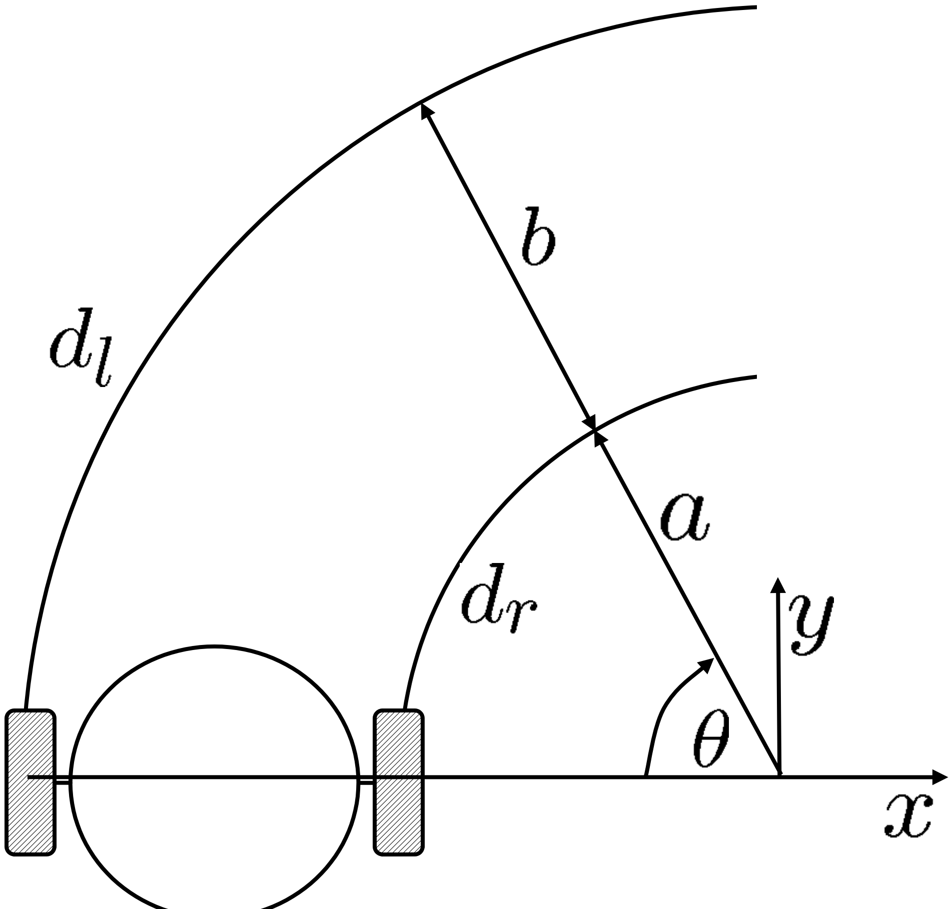
TU Berlin

Oliver Brock

Drive Systems we'll cover…

- Differential Drive (Amigobot)
- Tricycle
- Synchro-Drive
- Ackerman Steering
- Holonomic Delta Robot

Differential Drive



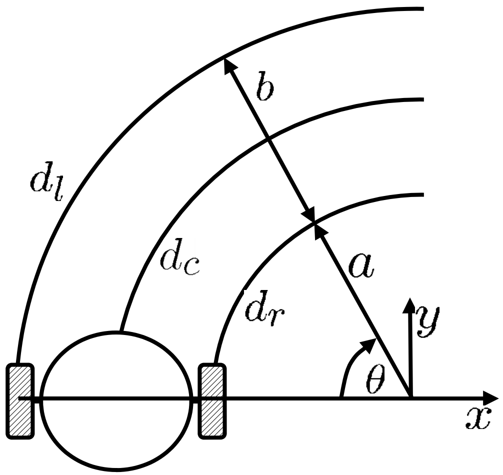
Circumference of a circle:

$$c = 2\pi r$$

$$2\pi = \frac{c}{r}$$

$$\theta = \frac{d_r}{a} = \frac{d_l}{a+b}$$

Differential Drive cont.



$$\theta = \frac{d_r}{a} = \frac{d_l}{a+b}$$

$$(a+b)\, d_r = a\, d_l$$

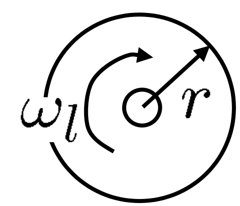
$$a = b\, \frac{d_r}{d_l - d_r}$$

$$\theta = \frac{d_l - d_r}{b}$$

$$d_c = \frac{d_l + d_r}{2}$$

$$\boxed{\begin{aligned} \omega &= \frac{v_l - v_r}{b} \\ v &= \frac{v_l + v_r}{2} \end{aligned}}$$

$$v_l = \omega_l\, r$$



Differential Drive cont. II

$$\omega = \frac{v_l - v_r}{b} \Rightarrow v_r = v_l - \omega b$$

$$v = \frac{v_l + v_r}{2} \Rightarrow v_r = 2v - v_l$$

$$v_l - \omega b = 2v - v_l$$

$$\boxed{\begin{aligned} v_l &= v + \frac{\omega b}{2} \\ v_r &= v - \frac{\omega b}{2} \end{aligned}}$$

Differential Drive Summary

$$v_l = v + \frac{\omega b}{2} \qquad \omega_l = \frac{v_l}{r} \qquad v = \frac{v_l + v_r}{2}$$

$$v_r = v - \frac{\omega b}{2} \qquad \omega_r = \frac{v_r}{r} \qquad \omega = \frac{v_l - v_r}{b}$$

Kinematic Equations of Motion

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = r \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{b} & -\frac{1}{b} \end{bmatrix} \begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{r} \begin{bmatrix} 1 & \frac{b}{2} \\ 1 & -\frac{b}{2} \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

Differential Drive Example

$$\begin{pmatrix} \omega_l \\ \omega_r \end{pmatrix} = \frac{1}{r} \begin{bmatrix} 1 & \frac{b}{2} \\ 1 & -\frac{b}{2} \end{bmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

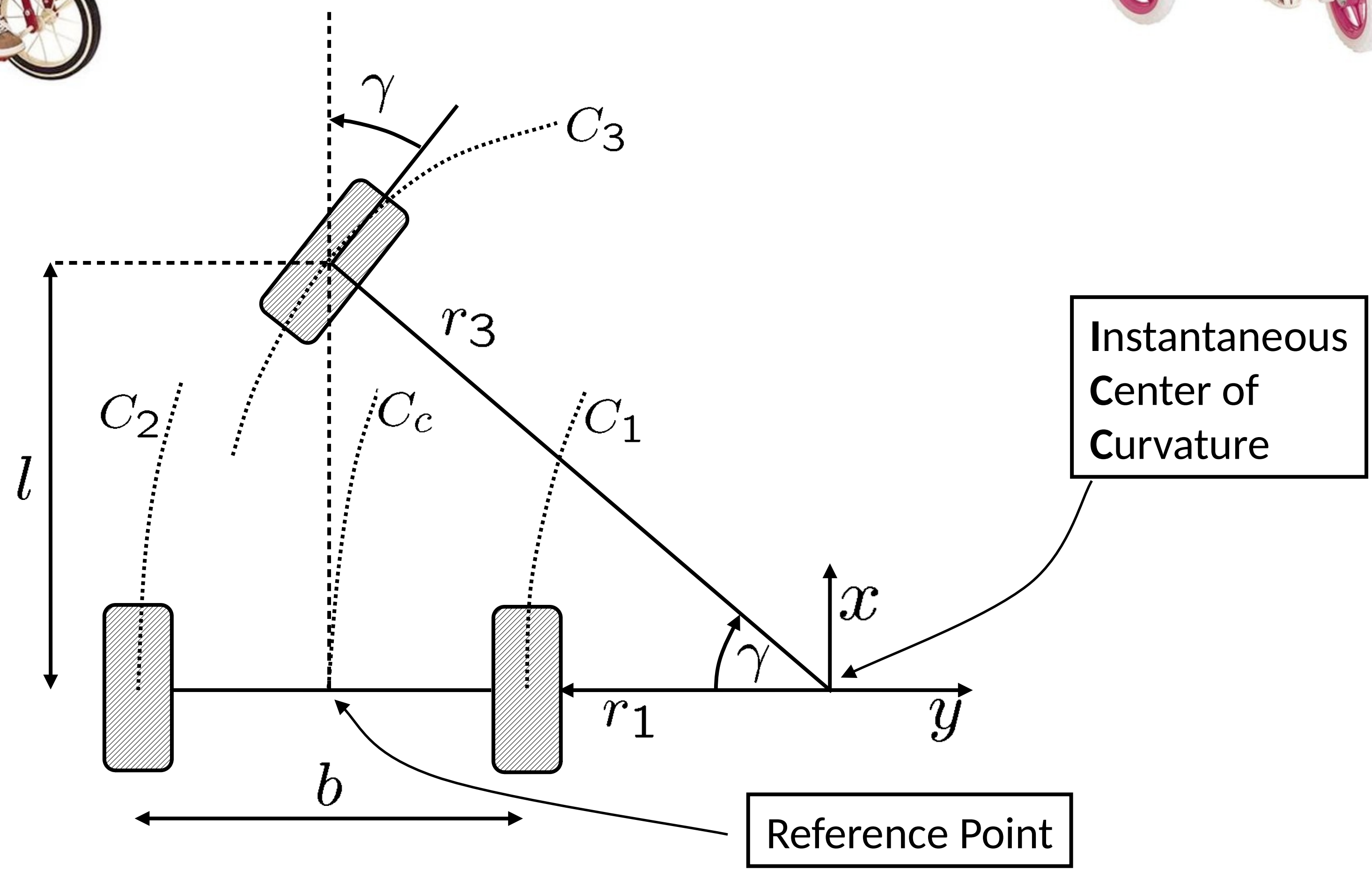
v=0, $\omega \neq 0$?   v$\neq$0, ω=0 ?

Wheel radius: 0.1m
Wheel base: 0.4m
Desired velocity: 0.5m/s
Desired turning velocity: 0.3rad/s

$$10 \begin{bmatrix} 1 & 0.2 \\ 1 & -0.2 \end{bmatrix} \begin{pmatrix} 0.5 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 5.6 \\ 4.4 \end{pmatrix}$$

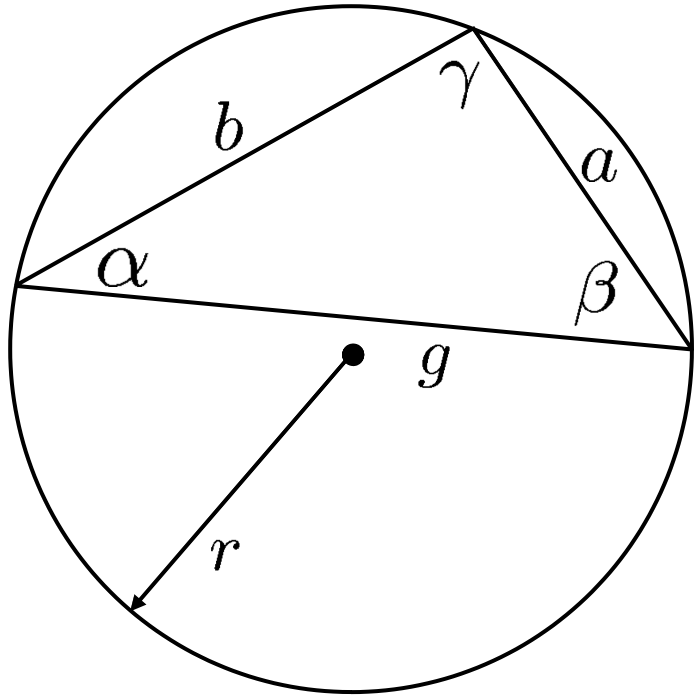
Tricycle

Tricycle



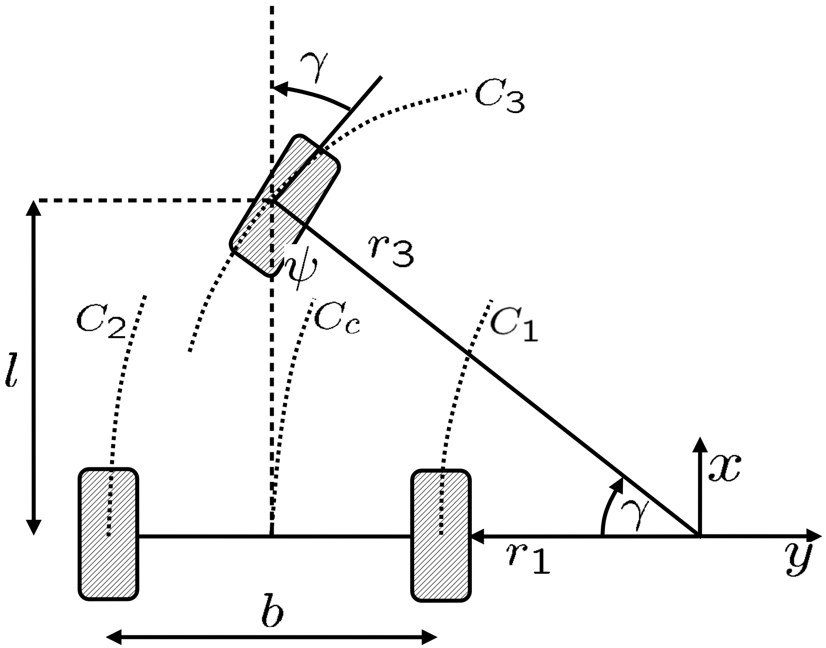
γ
C_3
r_3
C_2
C_c
C_1
l
x
γ
r_1
y
b
Instantaneous Center of Curvature
Reference Point

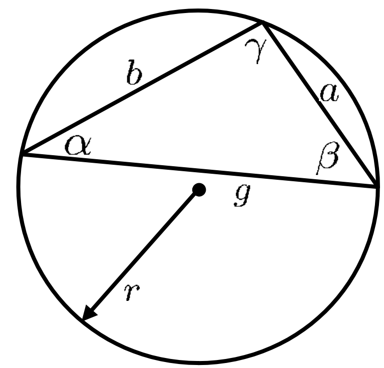
Sidebar: Law of Sines



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{g}{\sin\gamma} = 2r$$

Tricycle cont.





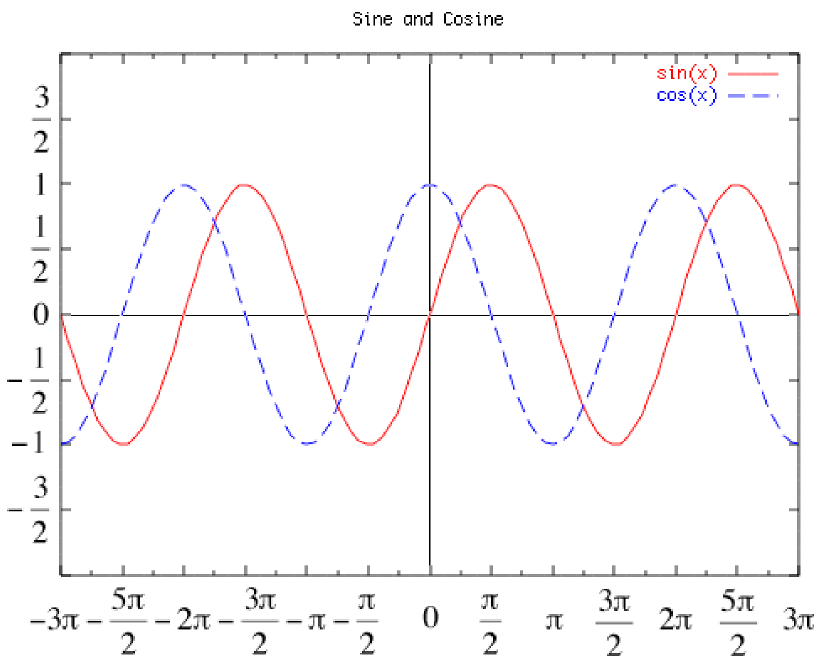
$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{g}{\sin\gamma} = 2r$$

$$\psi = \pi - \gamma - \frac{\pi}{2} = \frac{\pi}{2} - \gamma$$

$$\sin(\tfrac{\pi}{2} - \gamma) = -\sin(\gamma - \tfrac{\pi}{2}) = \cos\gamma$$

$$\frac{\sin\frac{\pi}{2}}{r_3} = \frac{\sin\gamma}{l} = \frac{\sin(\frac{\pi}{2} - \gamma)}{r_1 + \frac{b}{2}} = \frac{\cos\gamma}{r_1 + \frac{b}{2}}$$

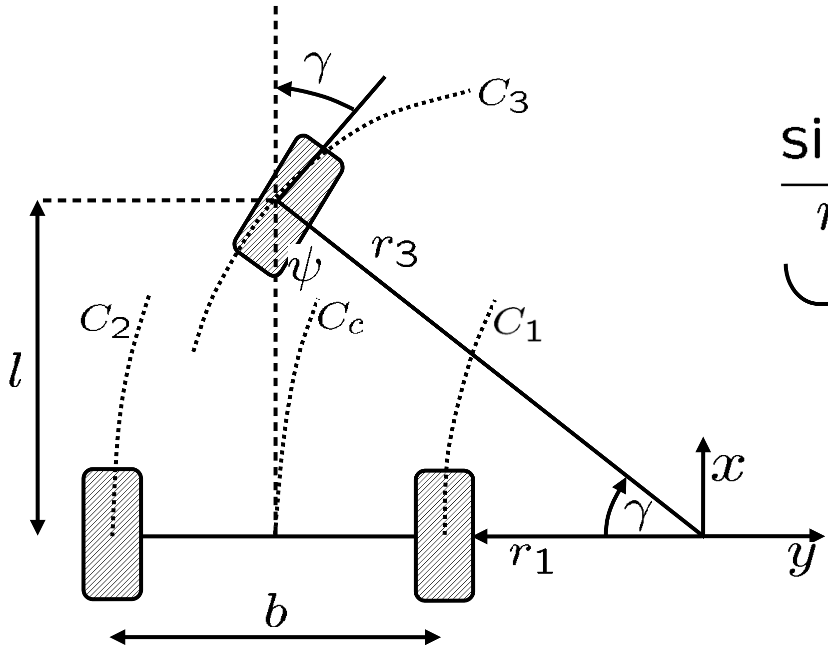
Sidebar: sin/cos identities



$$\sin\theta = -\sin(-\theta) = -cos(\theta + \frac{\pi}{2}) = \cos(\theta - \frac{\pi}{2})$$

$$\cos\theta = \cos(-\theta) = \sin(\theta + \frac{\pi}{2}) = -\sin(\theta - \frac{\pi}{2})$$

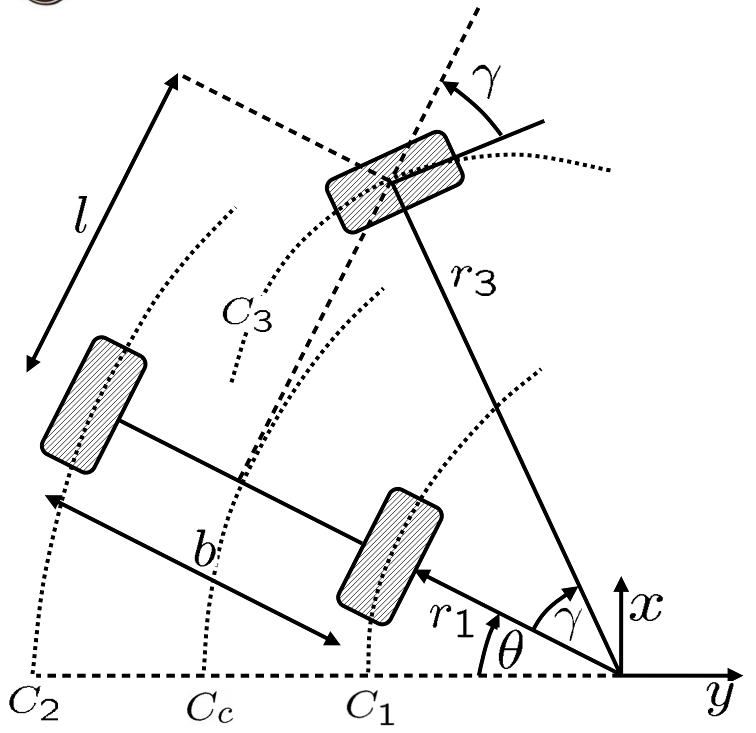
Tricycle cont. II



$$\underbrace{\frac{\sin\frac{\pi}{2}}{r_3} = \frac{\sin\gamma}{l}}_{r_3 = \frac{l}{\sin\gamma}} = \frac{\sin(\frac{\pi}{2}-\gamma)}{r_1+\frac{b}{2}} = \frac{\cos\gamma}{r_1+\frac{b}{2}}$$

$$\frac{\sin\gamma}{l} = \frac{\cos\gamma}{r_1+\frac{b}{2}} \Rightarrow r_1 = \frac{\cos\gamma}{\sin\gamma}l - \frac{b}{2}$$

Tricycle cont. III



$$r_1 = \frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}$$

$$r_3 = \frac{l}{\sin\gamma}$$

$$\theta = \frac{C_1}{r_1} \;\Rightarrow\; C_1 = \theta\, r_1 \,, \quad C_2 = \theta(r_1 + b)$$

$$\theta = \frac{C_3}{r_3}$$

Tricycle cont. III

$$r_1 = \frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}$$

$$r_3 = \frac{l}{\sin\gamma}$$

$$\theta = \frac{C_3}{r_3}$$

$$C_1 = \theta r_1$$

$$C_2 = \theta(r_1 + b)$$

$$C_1 = \frac{C_3}{r_3}\left(\frac{\cos\gamma}{\sin\gamma} l - \frac{b}{2}\right)$$

$$C_1 = C_3\left(\cos\gamma - \frac{b}{2l}\sin\gamma\right)$$

$$C_2 = \frac{C_3}{r_3}\left(\frac{\cos\gamma}{\sin\gamma} l + \frac{b}{2}\right)$$

$$C_2 = C_3\left(\cos\gamma + \frac{b}{2l}\sin\gamma\right)$$

Tricycle cont. IV

$$C_1 = C_3\left(\cos\gamma - \frac{b}{2l}\sin\gamma\right)$$

$$C_2 = C_3\left(\cos\gamma + \frac{b}{2l}\sin\gamma\right)$$

From Differential Drive

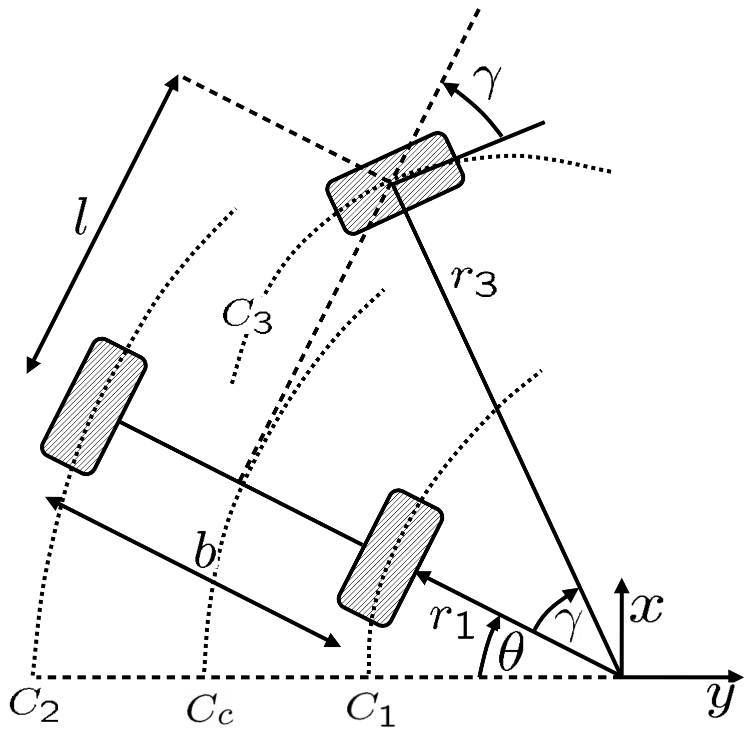
$$\begin{cases} \theta = \frac{C_2 - C_1}{b} \\ r_1 = b\frac{C_1}{C_2 - C_1} \end{cases}$$

$$\theta = \frac{C_3}{l}\sin\gamma$$

$$r_1 = l\,\frac{\cos\gamma}{\sin\gamma} - \frac{b}{2}$$

$$r_c = l\,\frac{\cos\gamma}{\sin\gamma}$$

Tricycle Summary



$$\theta = \frac{C_3}{l} \sin \gamma$$

$$r_1 = l \, \frac{\cos \gamma}{\sin \gamma} - \frac{b}{2}$$

$$r_2 = r_1 + b$$

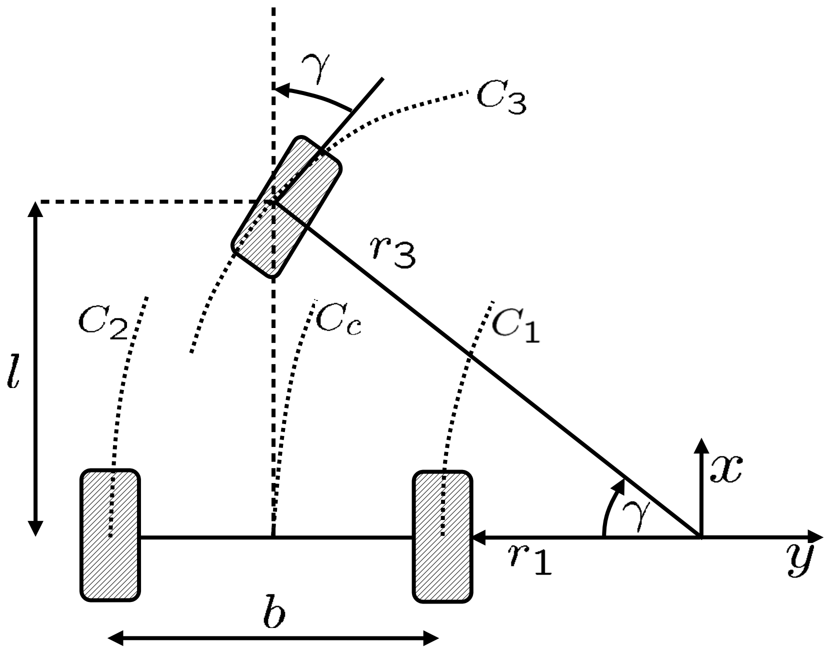
$$r_3 = \frac{l}{\sin \gamma}$$

$$r_c = l \, \frac{\cos \gamma}{\sin \gamma}$$

$$\omega = \frac{v_3}{l} \sin \gamma$$

$$v = \frac{v_1 + v_2}{2}$$

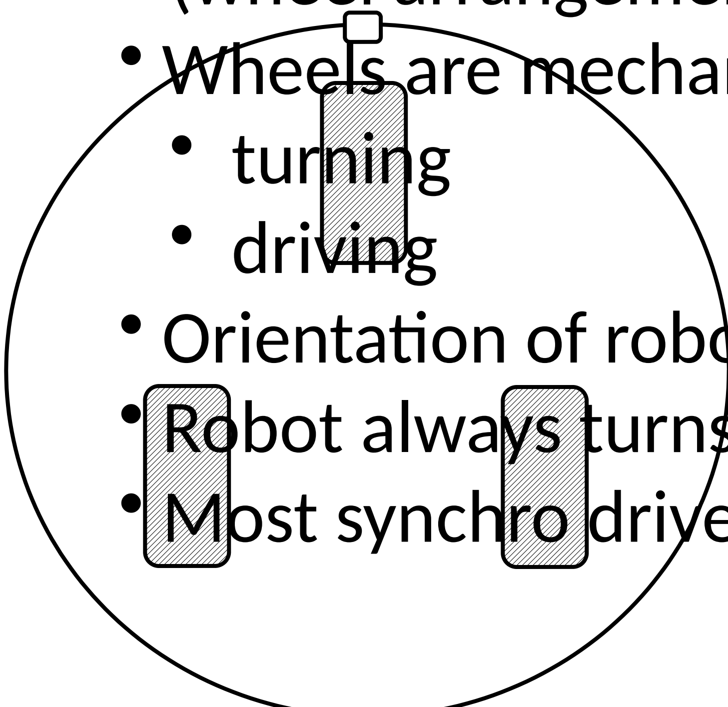
Tricycle in Local Frame

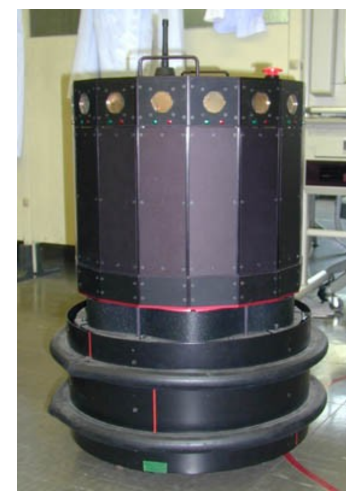


$$
\begin{aligned}
v_x(t) &= v_3(t)\cos\gamma(t) \\
v_y(t) &= 0 \\
\dot{\theta}(t) &= \frac{v_3(t)}{l}\sin\gamma(t)
\end{aligned}
$$

Synchro Drive

- Motivation: direct drive robots and tricycles are not very stable (wheel arrangement)
- Wheels are mechanically synchronized
  - turning
  - driving
- Orientation of robot is fixed
- Robot always turns about its center
- Most synchro drive robots have turret

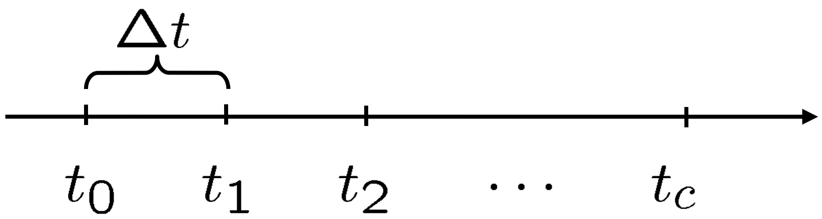




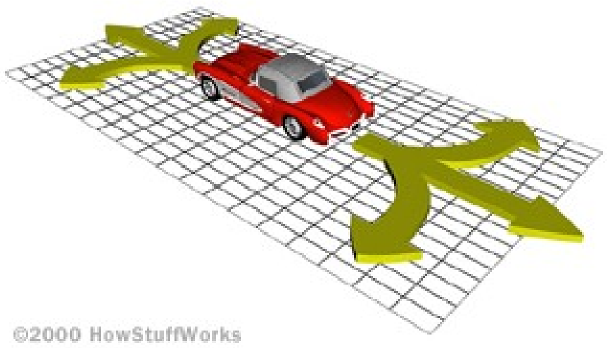
Synchro Drive cont.

$$
\begin{aligned}
x(t_c) &= x(t_0) + \int_{t_0}^{t_c} v(t) \cdot \cos\theta(t)\, dt \\
y(t_c) &= y(t_0) + \int_{t_0}^{t_c} v(t) \cdot \sin\theta(t)\, dt \\
v(t_c) &= v(t_0) + \int_{t_0}^{t_c} \dot{v}(t) dt \\
\theta(t_c) &= \theta(t_0) + \int_{t_0}^{t_c} \dot{\theta}(t) dt
\end{aligned}
$$

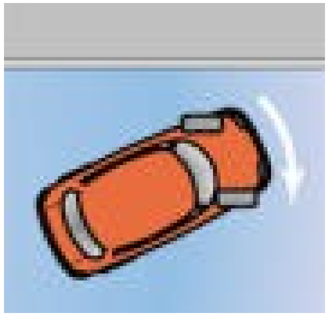
Synchro Drive cont. II

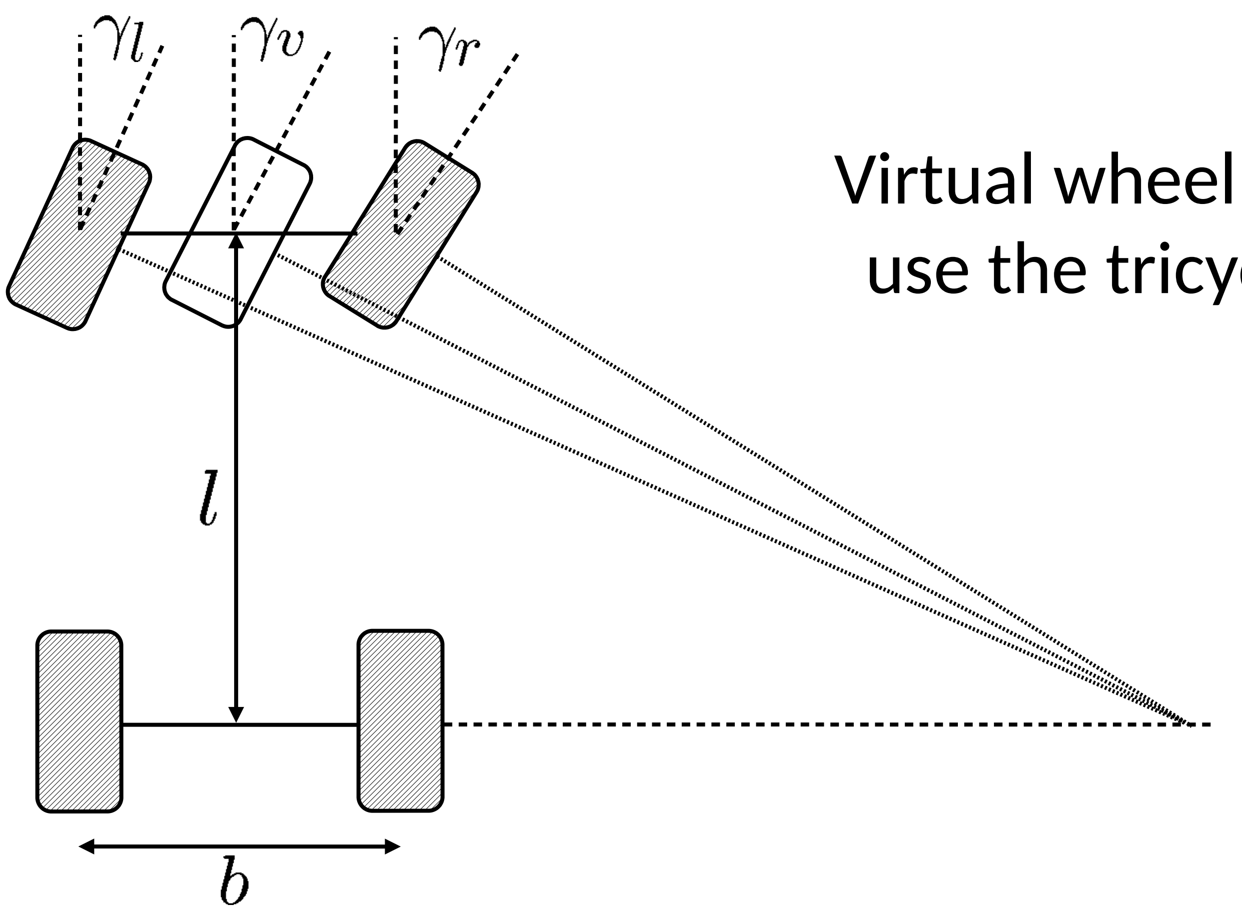


$$
\begin{aligned}
x(t_c) &= x(t_0) + \sum_{t_0}^{t_c} v(t) \cdot \cos\theta(t)\, \Delta t \\
y(t_c) &= y(t_0) + \sum_{t_0}^{t_c} v(t) \cdot \sin\theta(t)\, \Delta t \\
v(t_c) &= v(t_0) + \sum_{t_0}^{t_c} \dot{v}(t) \Delta t \\
\theta(t_c) &= \theta(t_0) + \sum_{t_0}^{t_c} \dot{\theta}(t) \Delta t
\end{aligned}
$$



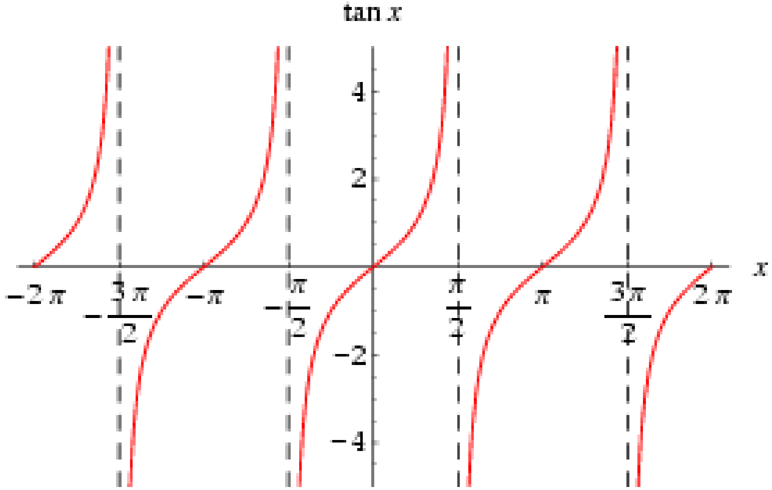
Ackerman Steering



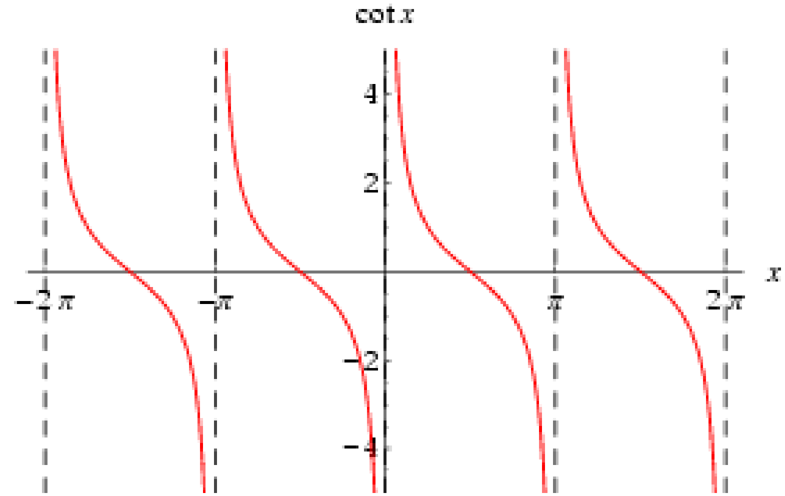


Virtual wheel allows us use the tricycle case!

Sidebar: Cotangent



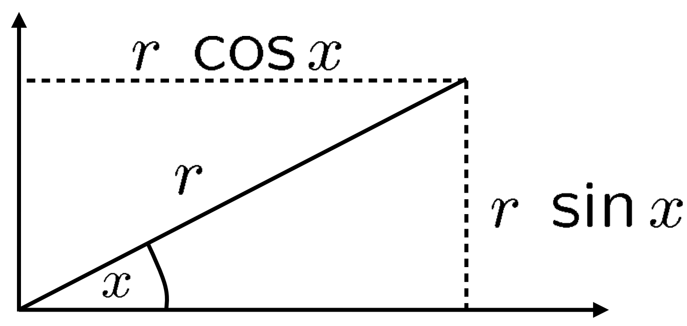
$$\tan x \equiv \frac{\sin x}{\cos x}$$

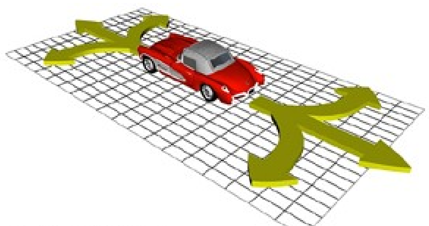


$$\cot x \equiv \frac{1}{\tan x}$$

$$\tan x = \frac{\sin x}{\cos x} = \frac{r\ \sin x}{r\ \cos x}$$

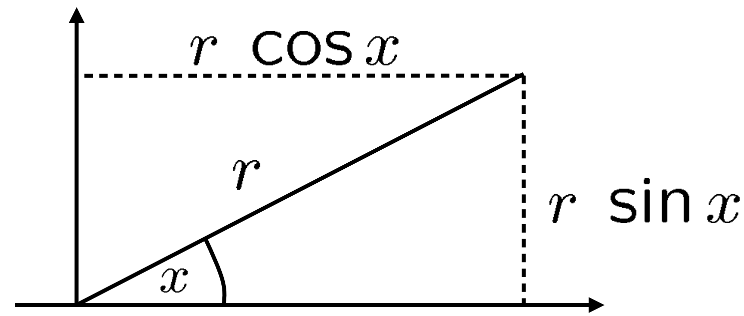
$$\cot x = \frac{\cos x}{\sin x} = \frac{r\ \cos x}{r\ \sin x}$$

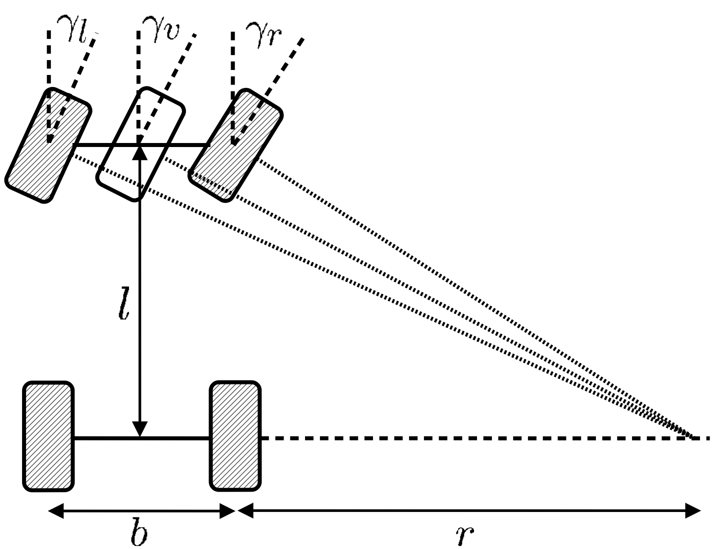




Ackerman Steering cont.

$$\cot x = \frac{\cos x}{\sin x} = \frac{r\ \cos x}{r\ \sin x}$$





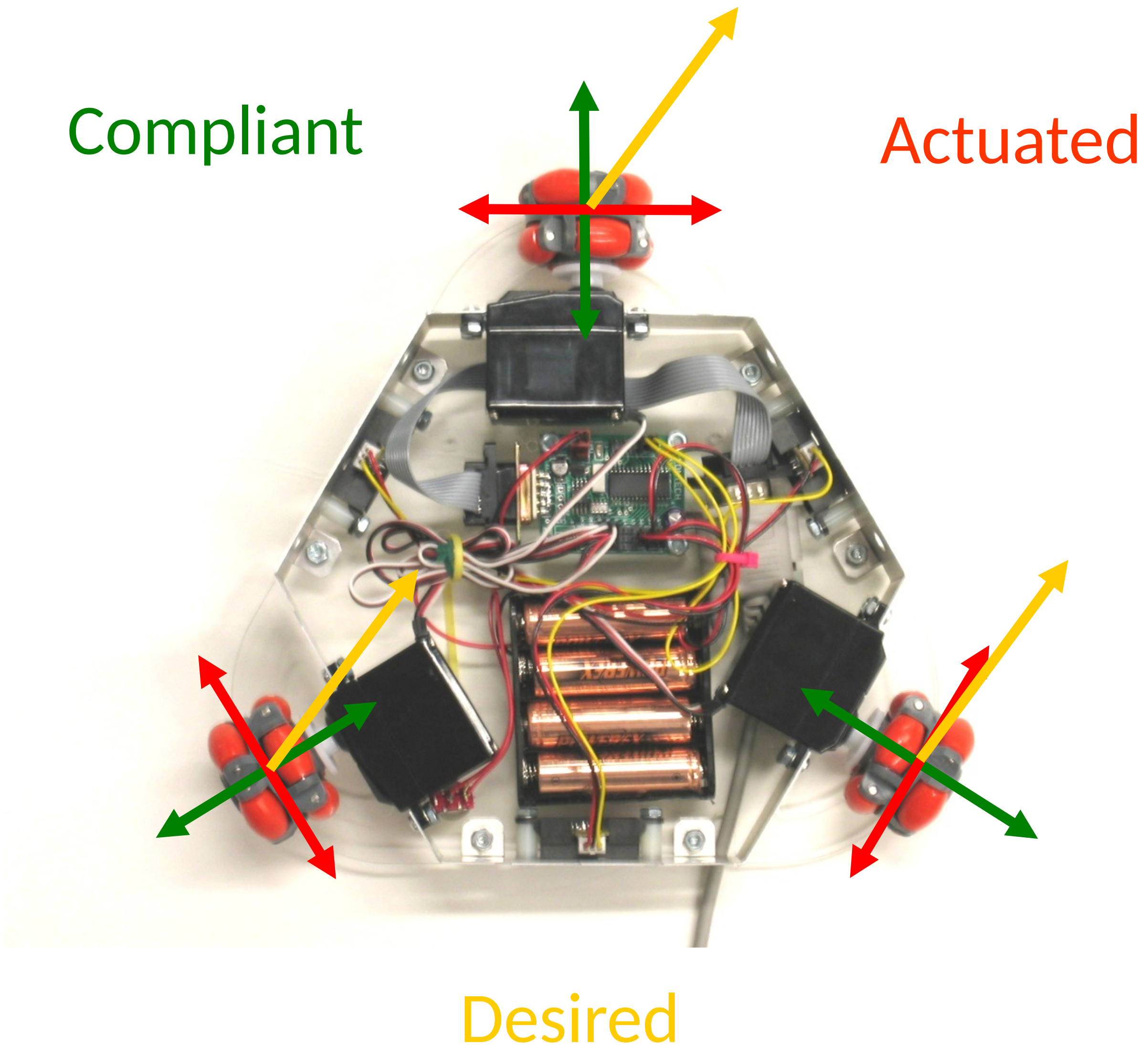
$$\cot \gamma_v = \frac{r + \frac{b}{2}}{l} = \frac{r}{l} + \frac{b}{2l}$$

$$= \cot \gamma_r + \frac{b}{2l}$$

$$= \cot \gamma_l - \frac{b}{2l}$$

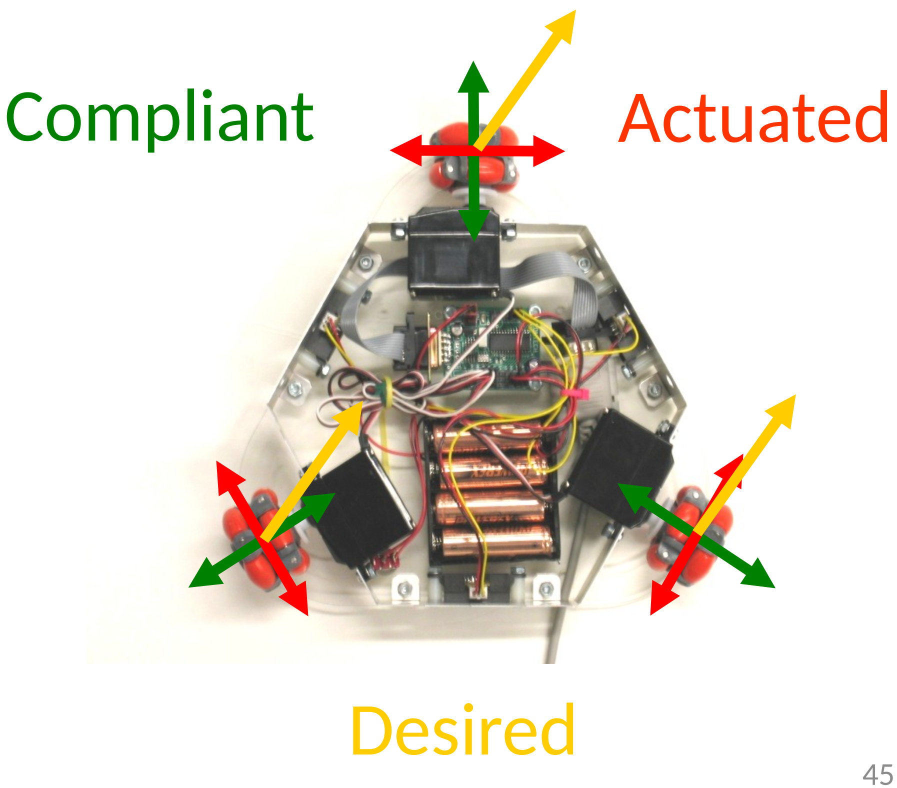
Now we can apply the tricycle case!

Achieving Holonomic Motion



We need:

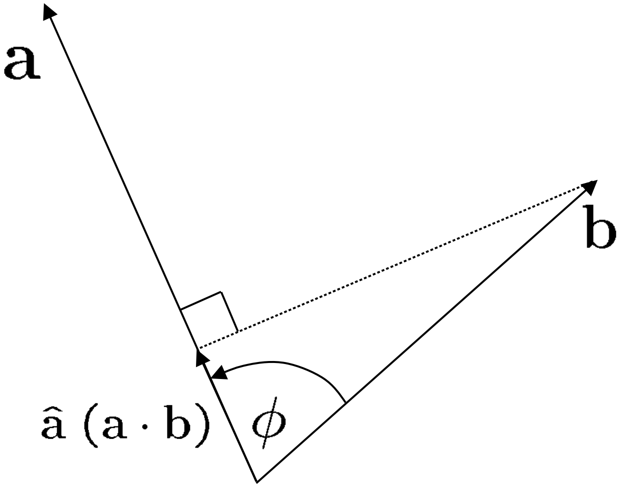
- Dot product
- Rolling Wheels



Sidebar: Dot Product

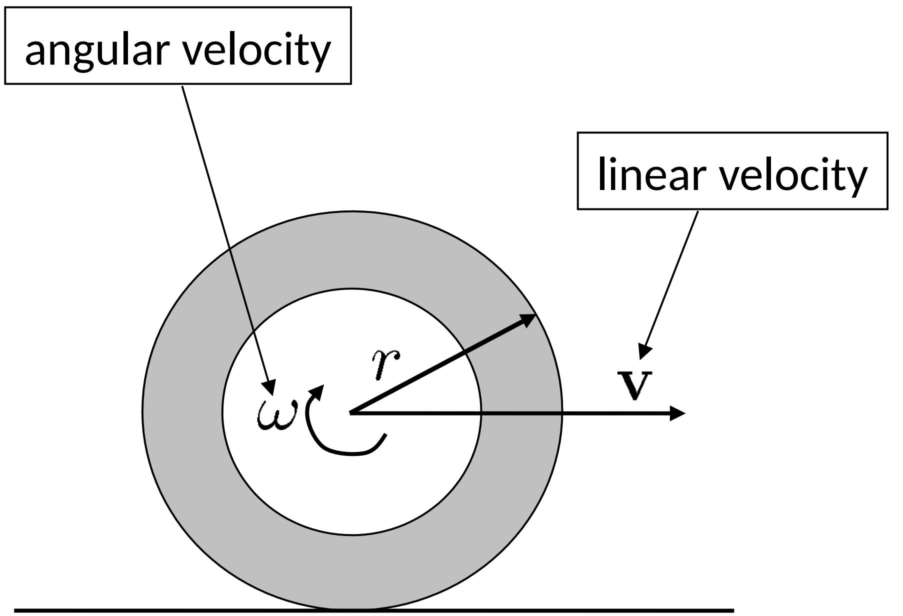
$$\mathbf{a} \cdot \mathbf{b} = \|\mathbf{a}\| \, \|\mathbf{b}\| \cos\phi$$

$$= \sum_{i=1}^{n} a_i b_i$$



$$\hat{\mathbf{a}} = \frac{\mathbf{a}}{\|\mathbf{a}\|}$$

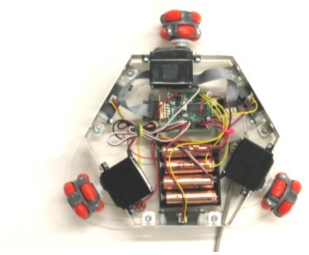
Sidebar: Rolling Wheels



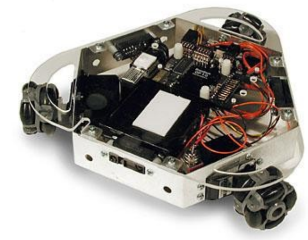
For a circle:

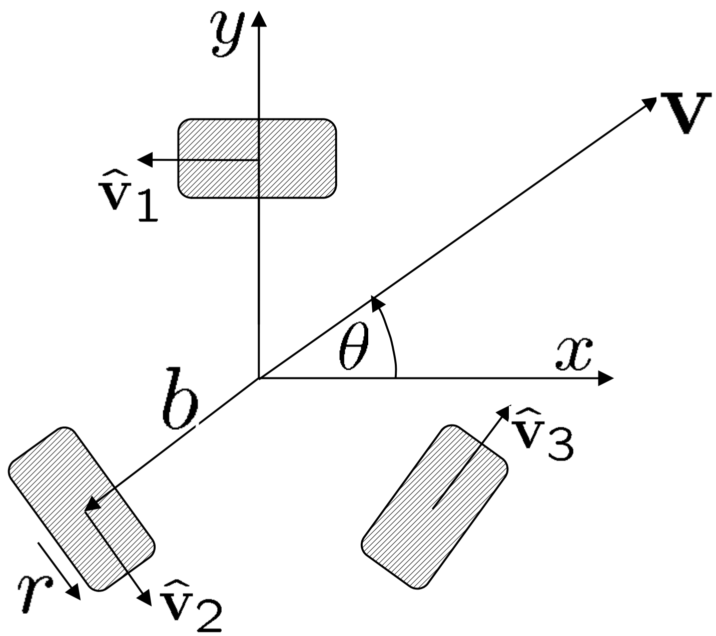
$$c = 2\pi r$$

$$\mathbf{v} = \omega \; r$$



Omniwheel





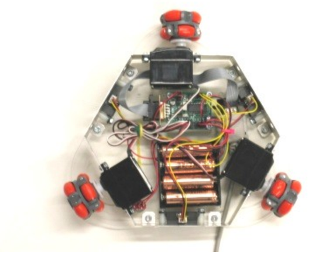
component of $\mathbf{v}$ along $\hat{\mathbf{v}}_1$: $\hat{\mathbf{v}}_1 \cdot \mathbf{v}$

expressed as angular velocity of the wheel: $\dfrac{\hat{\mathbf{v}} \cdot \mathbf{v}}{r}$

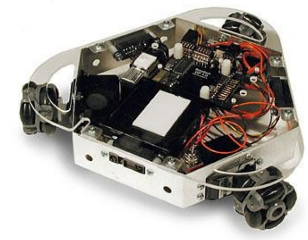
contribution of wheels to $\dot{\theta}$: $\dfrac{b\dot{\theta}}{r}$

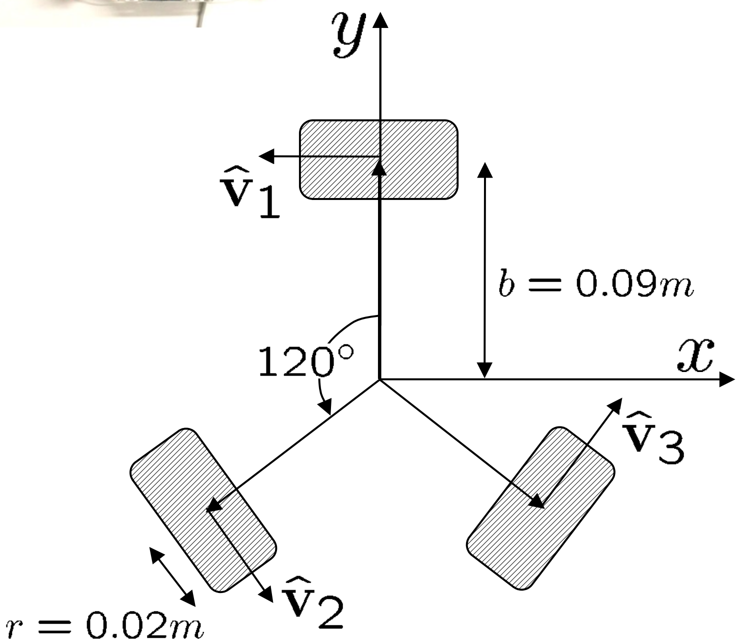
$$\begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix} \overset{?}{\Rightarrow} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$$

$$\begin{aligned} \omega_1 &= \left(\hat{\mathbf{v}}_1 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\ \omega_2 &= \left(\hat{\mathbf{v}}_2 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\ \omega_3 &= \left(\hat{\mathbf{v}}_3 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \end{aligned}$$



Delta Robot

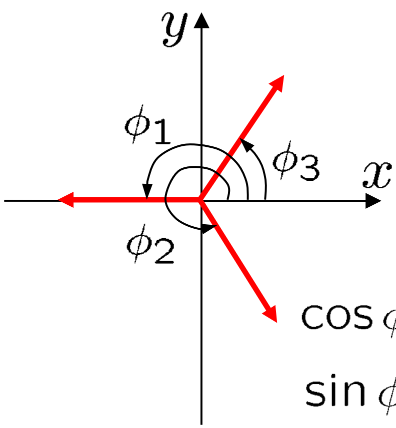




$$
\begin{aligned}
\omega_1 &= \left(\hat{\mathbf{v}}_1 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\
\omega_2 &= \left(\hat{\mathbf{v}}_2 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r \\
\omega_3 &= \left(\hat{\mathbf{v}}_3 \cdot \mathbf{v} + b\,\dot{\theta}\right)/r
\end{aligned}
$$

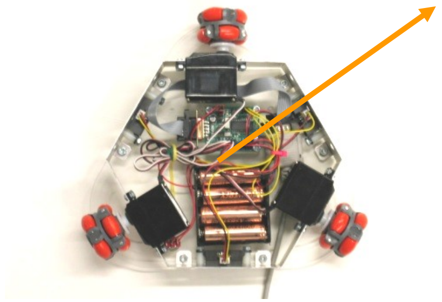
$$
\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \frac{1}{r} \begin{bmatrix} \cos\phi_1 & \sin\phi_1 & b \\ \cos\phi_2 & \sin\phi_2 & b \\ \cos\phi_3 & \sin\phi_3 & b \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix}
$$

$$
\begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} \approx 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} v_x \\ v_y \\ \dot{\theta} \end{pmatrix}
$$

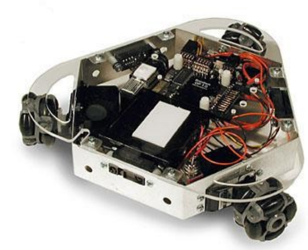


$$\cos\phi_3 = 0.5$$

$$\sin\phi_3 = \frac{\sqrt{3}}{2} \approx 0.866$$



Example



$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \|\mathbf{v}\|$$

$$= \begin{pmatrix} \cos 45^\circ \\ \sin 45^\circ \end{pmatrix} \sqrt{2} \cdot 0.05$$

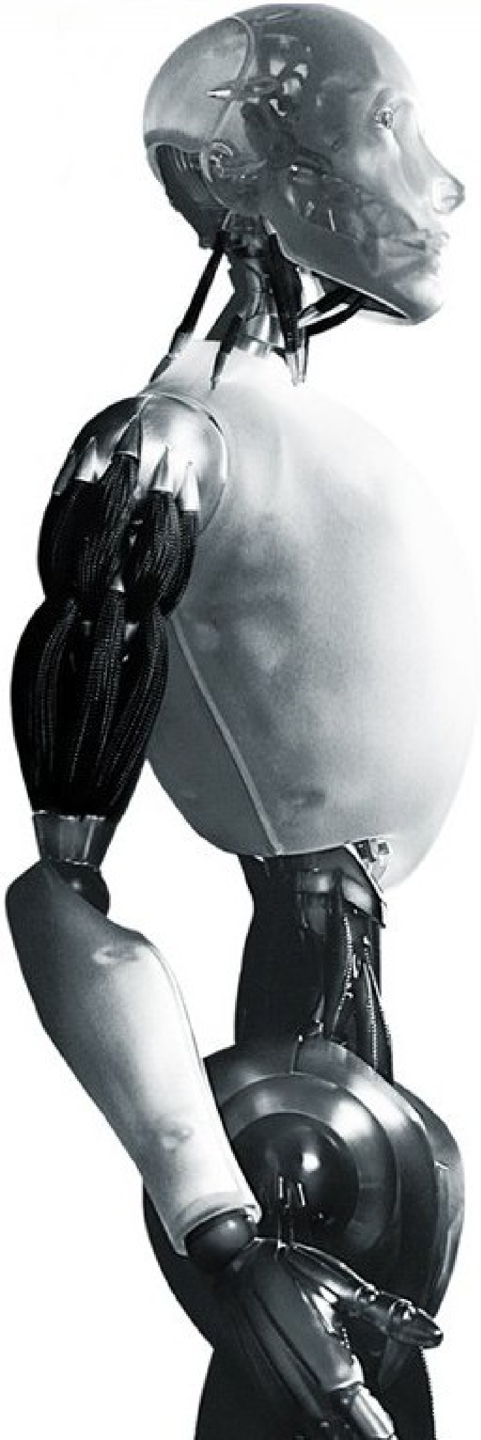
$$= \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} \sqrt{2} \cdot 0.05$$

$$= \begin{pmatrix} 0.05 \\ 0.05 \end{pmatrix}$$

wheelbase = 0.09m
radius of wheels = 0.02m
desired velocity = $\sqrt{2}$* 0.05 m/s
desired heading = 45 degrees
desired angular velocity = 0.5 rad/s

$$\begin{pmatrix} -0.25 \\ 1.135 \\ 5.665 \end{pmatrix} = 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} 0.05 \\ 0.05 \\ 0.5 \end{pmatrix}$$

$$\begin{pmatrix} -4.75 \\ 3.165 \\ 1.165 \end{pmatrix} = 50 \begin{bmatrix} -1 & 0 & 0.09 \\ 0.5 & -0.866 & 0.09 \\ 0.5 & 0.866 & 0.09 \end{bmatrix} \begin{pmatrix} 0.05 \\ 0.05 \\ -0.5 \end{pmatrix}$$



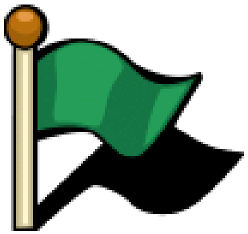
Robotics

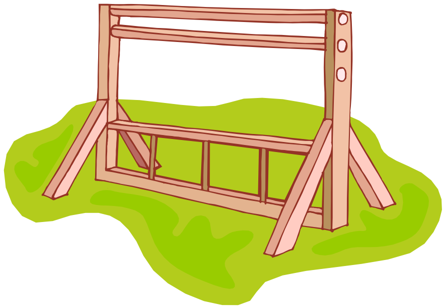
Generating Motion for Mobile Robots

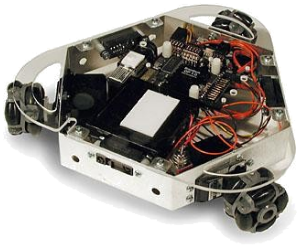
TU Berlin

Oliver Brock

How do we get there?







Assumptions:

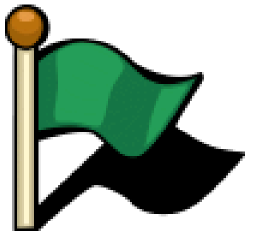
- Perfect knowledge of the world
- Perfect motion execution

How realistic is this?

NOT!

But let’s just assume…

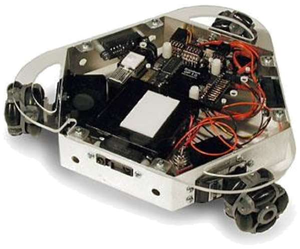
Heading



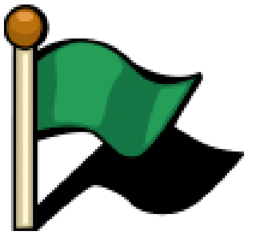
robot position

$$\mathbf{v} = {}^0\mathbf{g} - {}^0\mathbf{p}$$

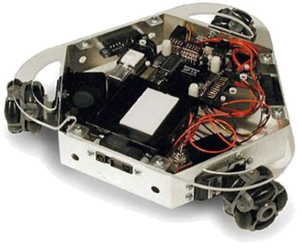
goal position



Problems with Heading



$$\mathbf{v} = {}^{0}\mathbf{g} - {}^{0}\mathbf{p}$$

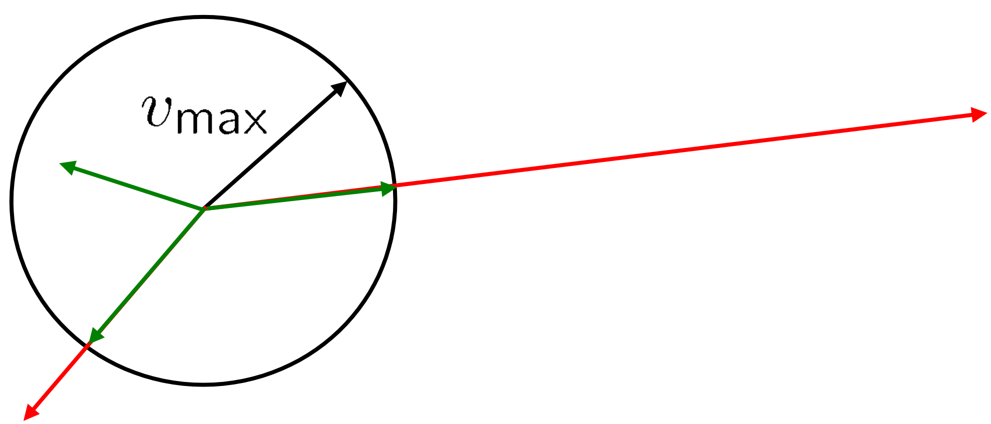


Problems:

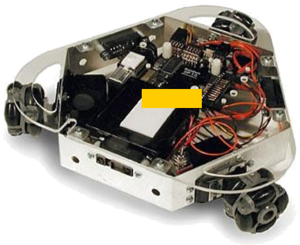
- Obstacles
- Unlimited Velocity

Saturating Velocity

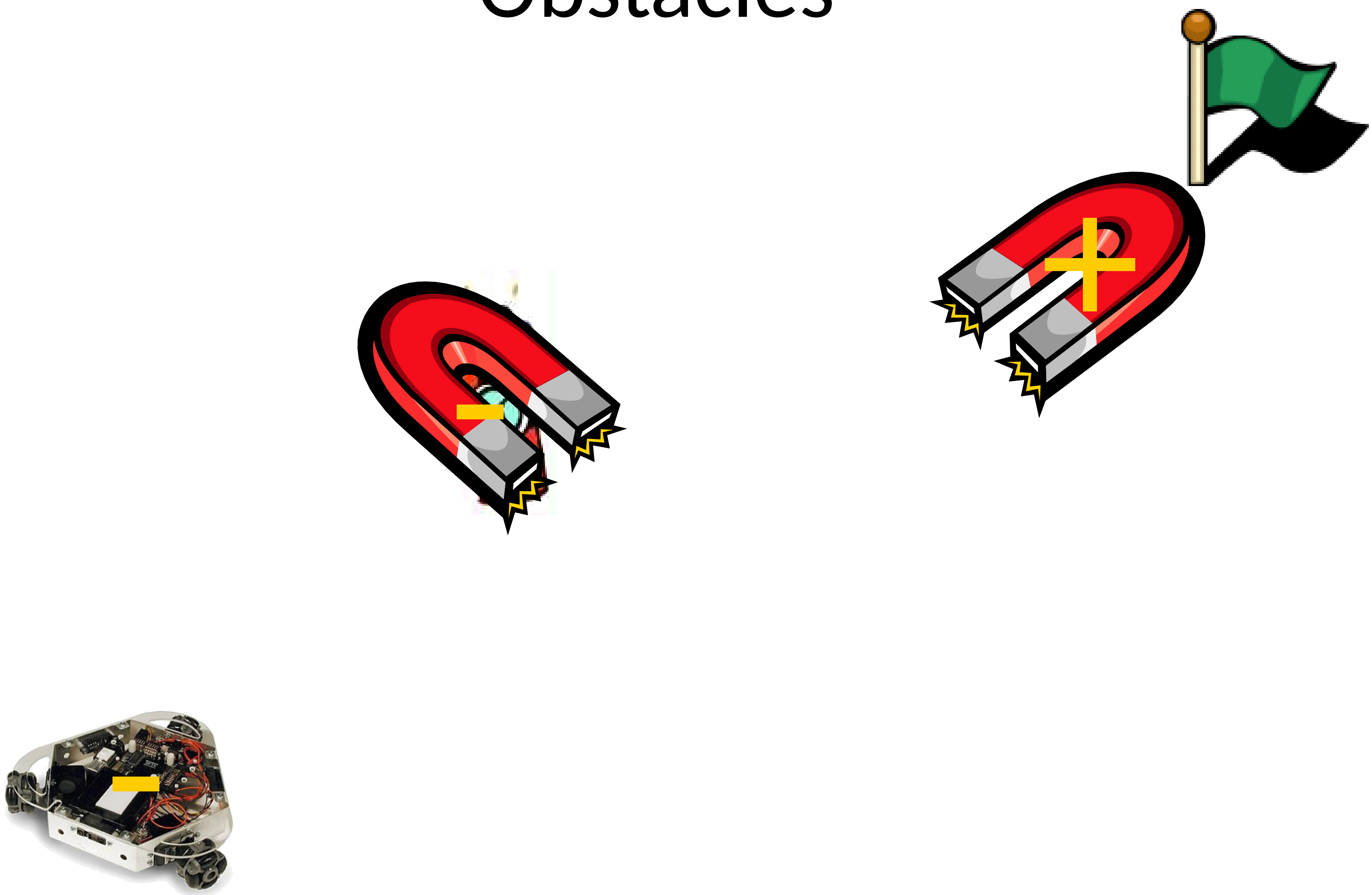
$$\mathbf{v} = \begin{cases} \mathbf{v} & \text{if } \|\mathbf{v}\| < v_{\text{max}} \\ v_{\text{max}} \dfrac{\mathbf{v}}{\|\mathbf{v}\|} & \text{otherwise} \end{cases}$$



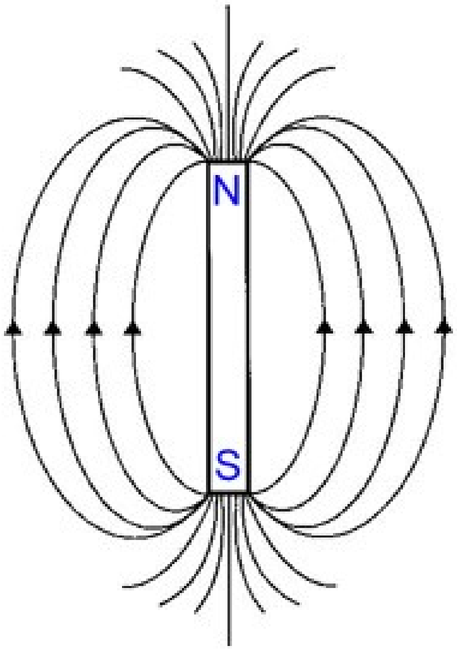
Physical Model



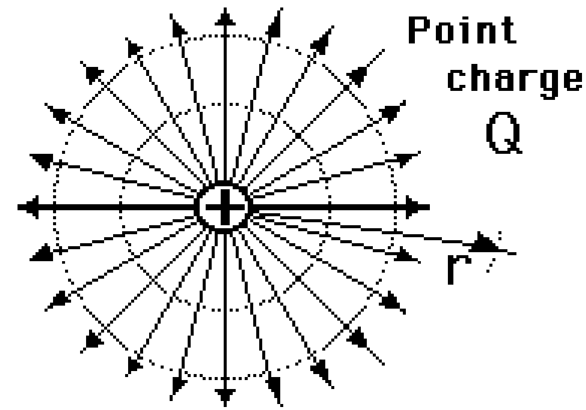
Obstacles



Electric Charges are better…

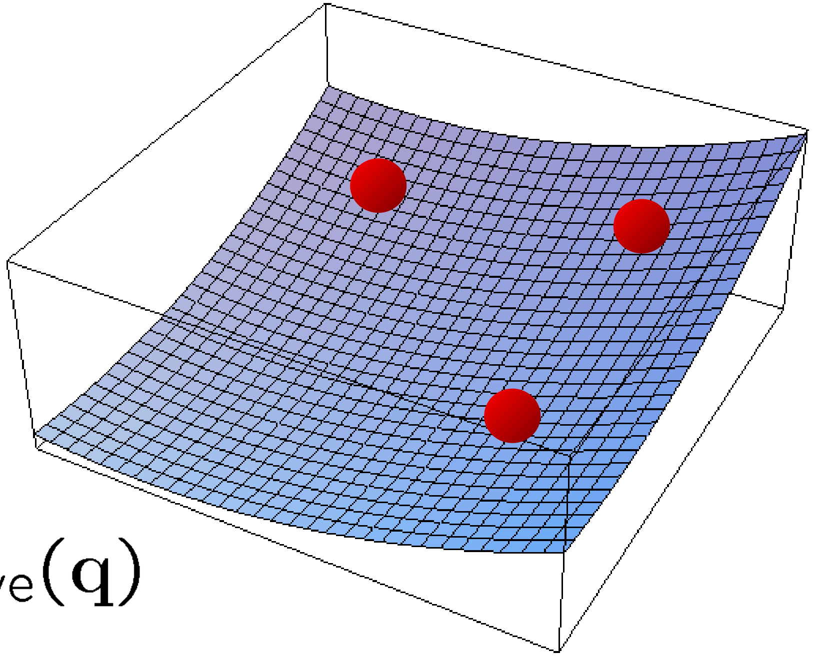


depends on
direction



$$\mathbf{F} = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

Attractive Potential



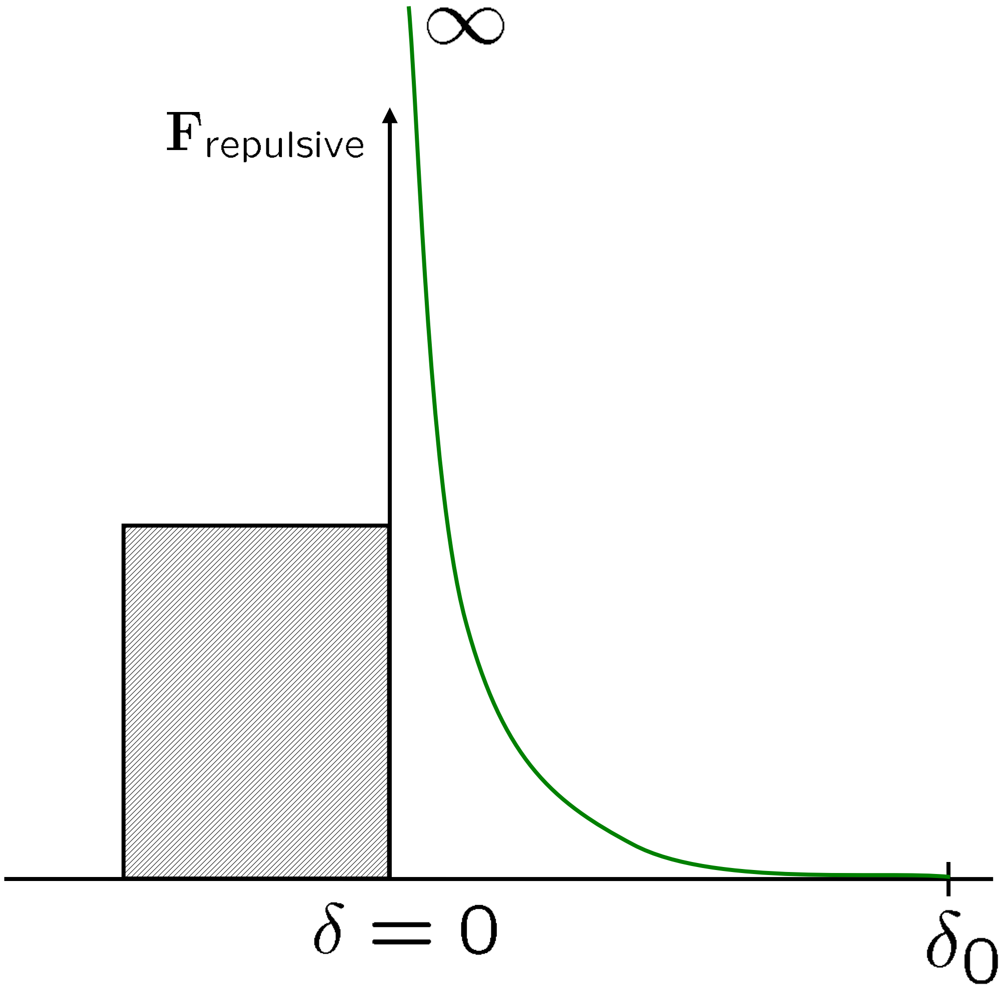
$$U_{\text{attractive}}(\mathbf{q}) = \frac{1}{2} k\, \delta^2_{\text{goal}}(\mathbf{q})$$

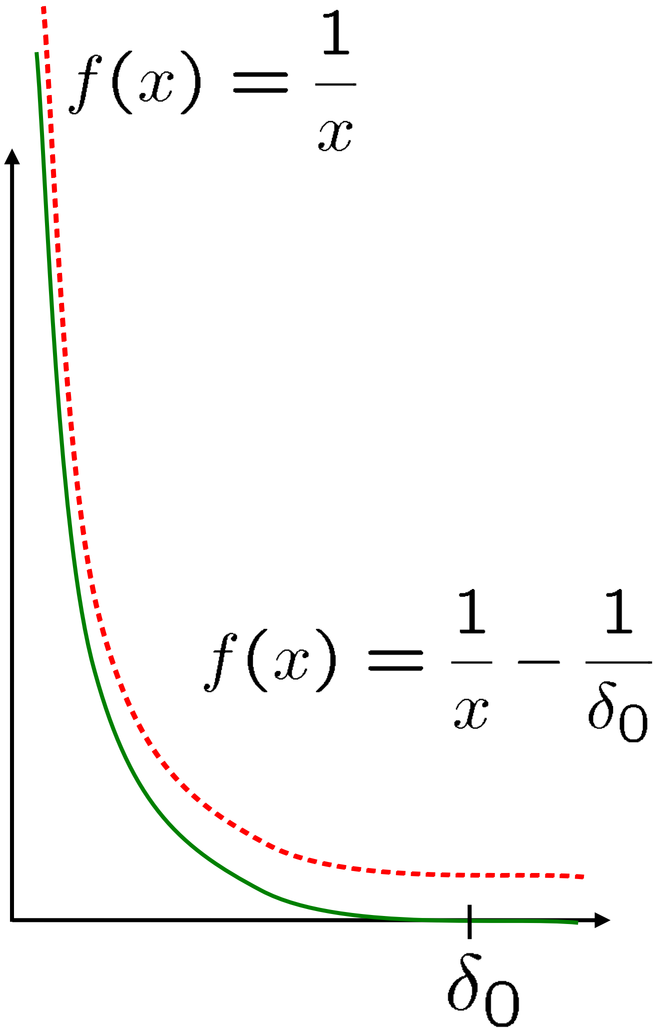
$$\begin{aligned} \mathbf{F}_{\text{attractive}}(\mathbf{q}) &= -\nabla U_{\text{attractive}}(\mathbf{q}) \\ &= -k\, \delta_{\text{goal}}(\mathbf{q}) \end{aligned}$$

$$\mathbf{F}_{\text{charge}} = \frac{k \cdot q_1 \cdot q_2}{r^2}$$

NOT physically motivated!

Designing Repulsive Potential



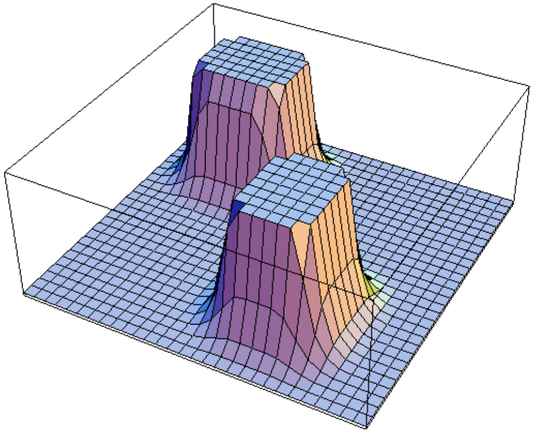


Repulsive Potential

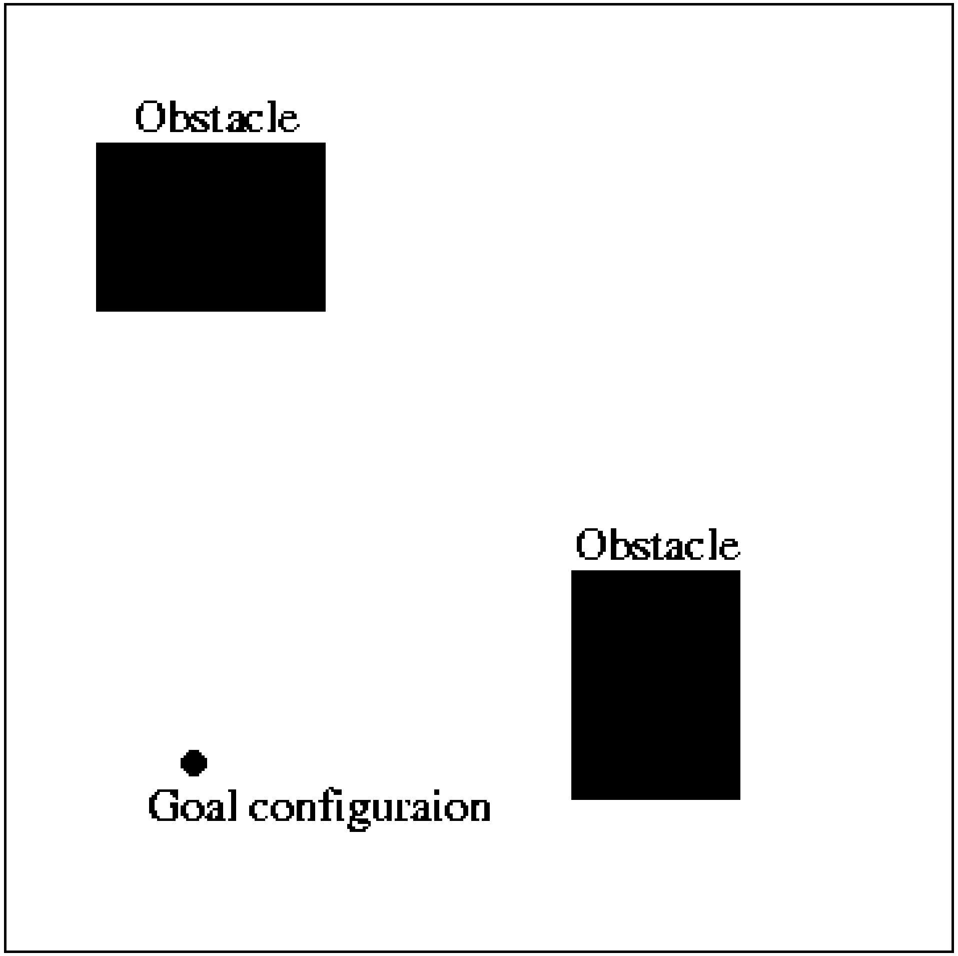
$$\mathbf{F}_{\text{repulsive}}(\mathrm{q}) = -\nabla U_{\text{repulsive}}(\mathbf{q})$$

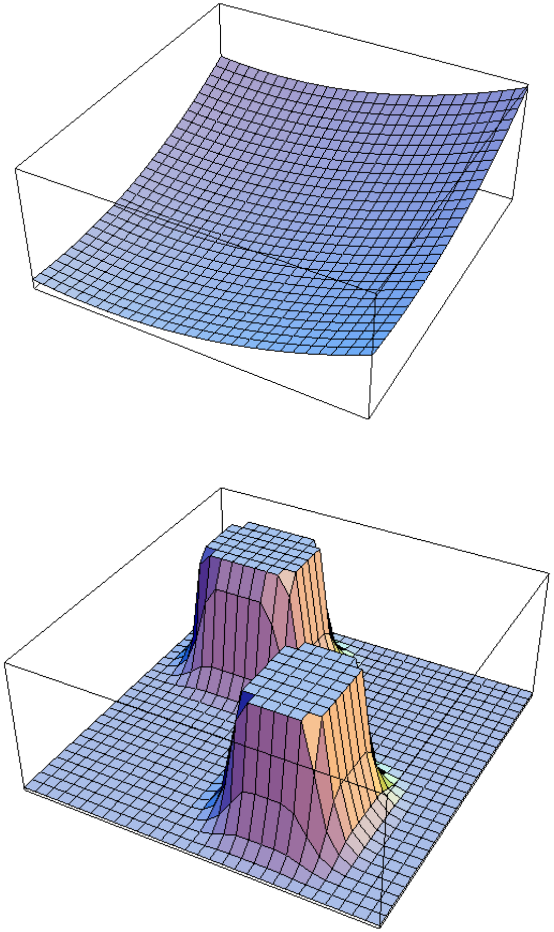
$$= \begin{cases} -k\left(\frac{1}{\delta_{\text{obstacle}}(\mathbf{q})} - \frac{1}{\delta_0}\right) & \text{if } \delta_{\text{obstacle}}(\mathbf{q}) < \delta_0 \\ 0 & \text{otherwise} \end{cases}$$

$$U_{\text{repulsive}}(\mathrm{q}) = \begin{cases} \frac{1}{2}k\left(\frac{1}{\delta_{\text{obstacle}}(\mathbf{q})} - \frac{1}{\delta_0}\right)^2 & \text{if } \delta_{\text{obstacle}}(\mathbf{q}) < \delta_0 \\ 0 & \text{otherwise} \end{cases}$$



Let’s put it together

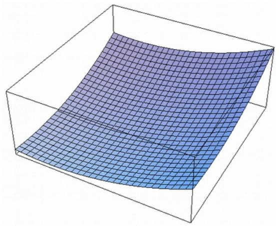




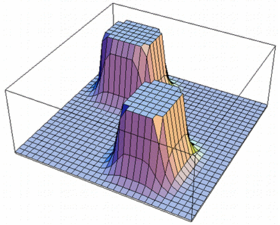
Artificial Potential Function

$$U(\mathbf{q}) = U_{\text{attractive}}(\mathbf{q}) + U_{\text{repulsive}}(\mathbf{q})$$

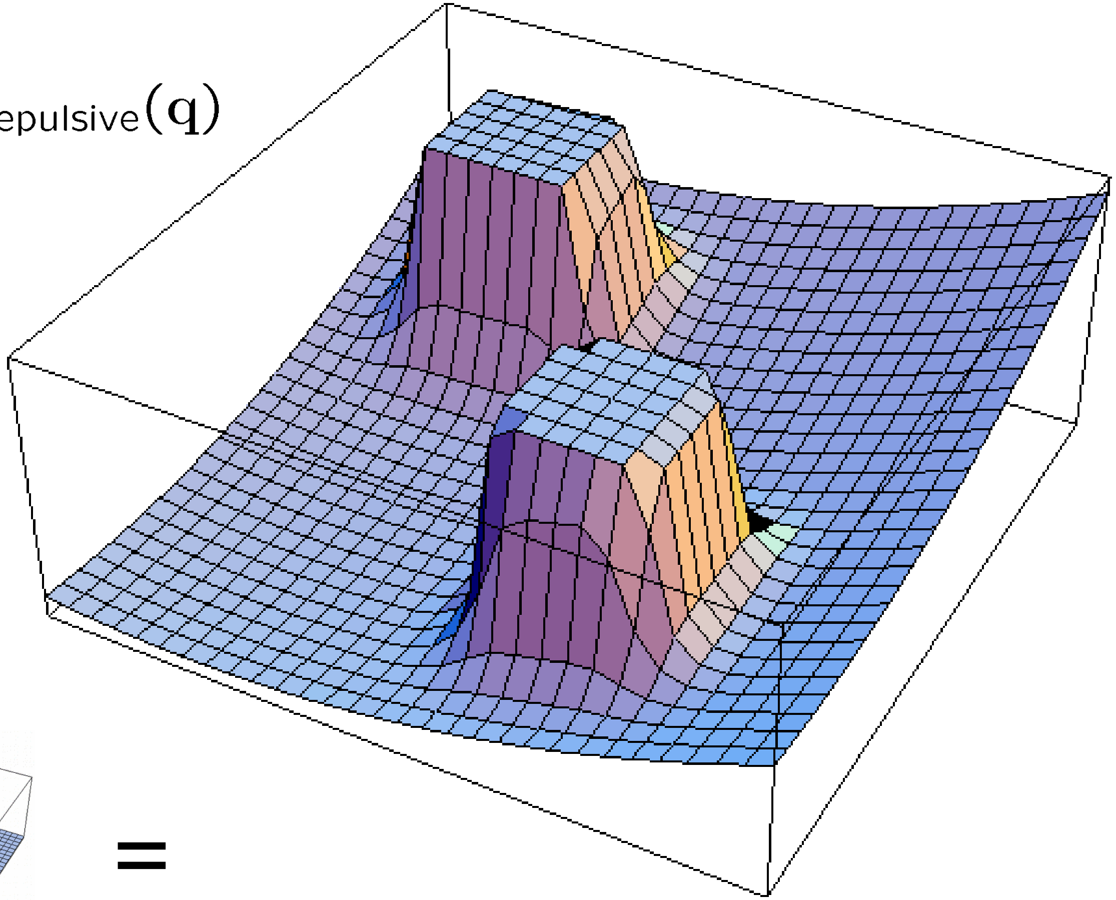
$$\mathbf{F}(\mathbf{q}) = -\nabla U(\mathbf{q})$$



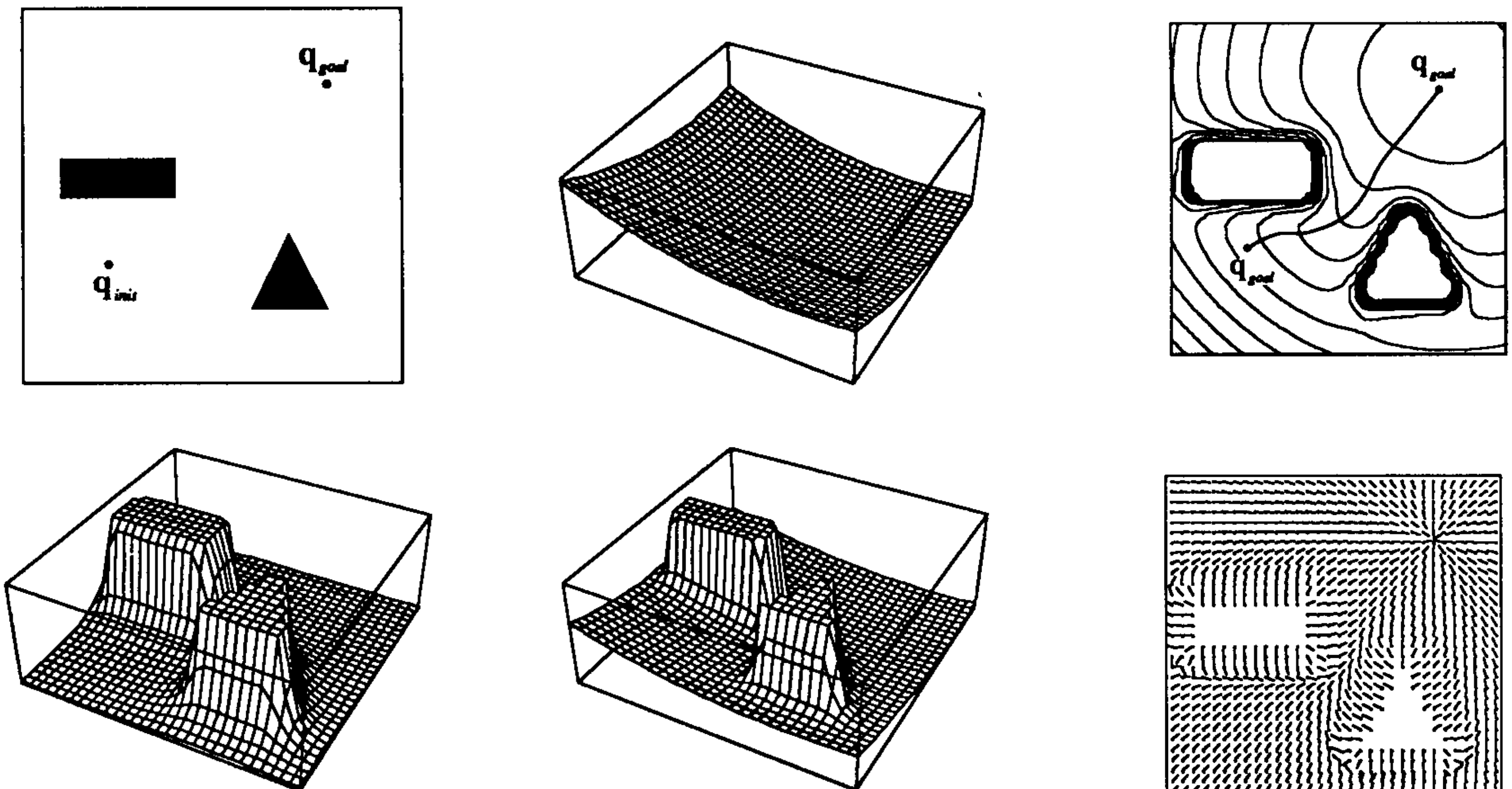
+



=

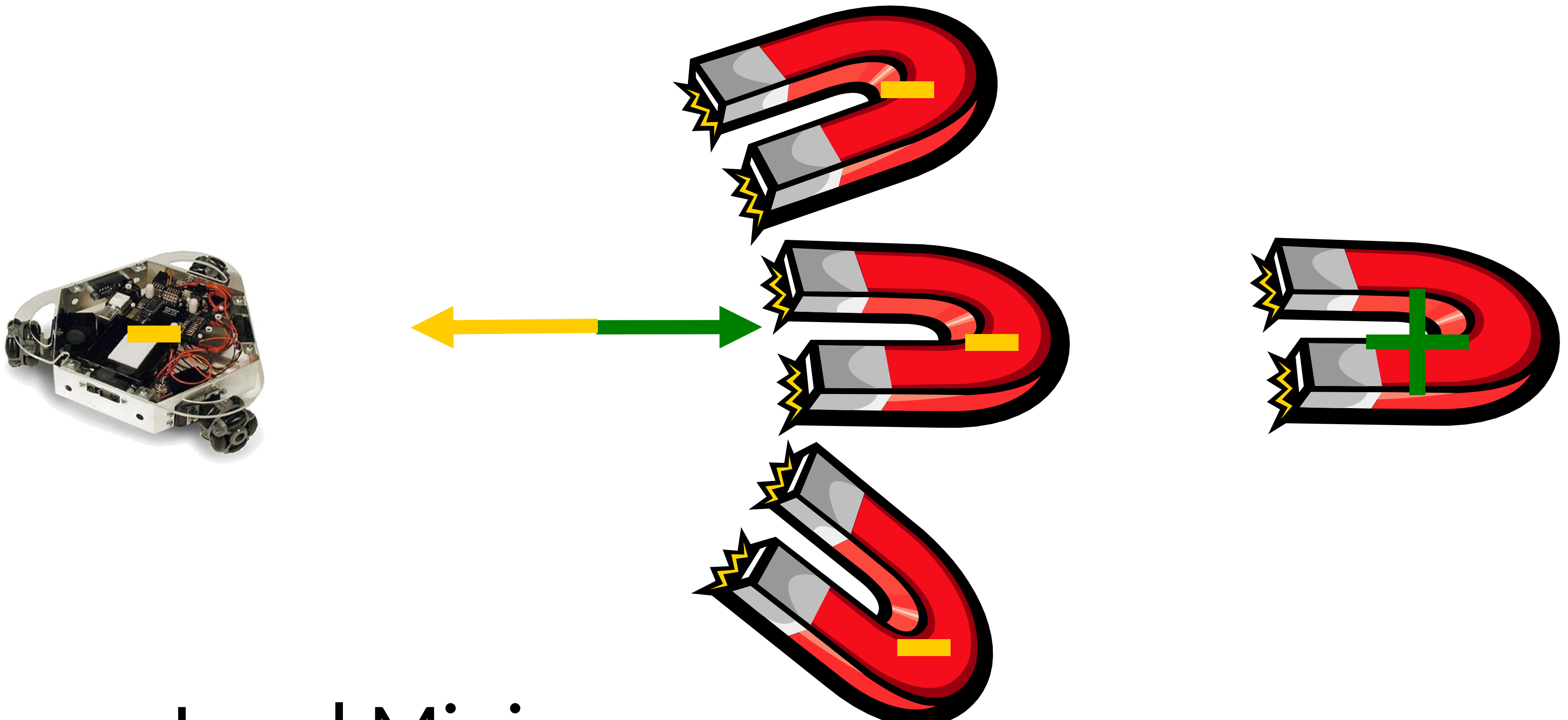


Potential Field Approach



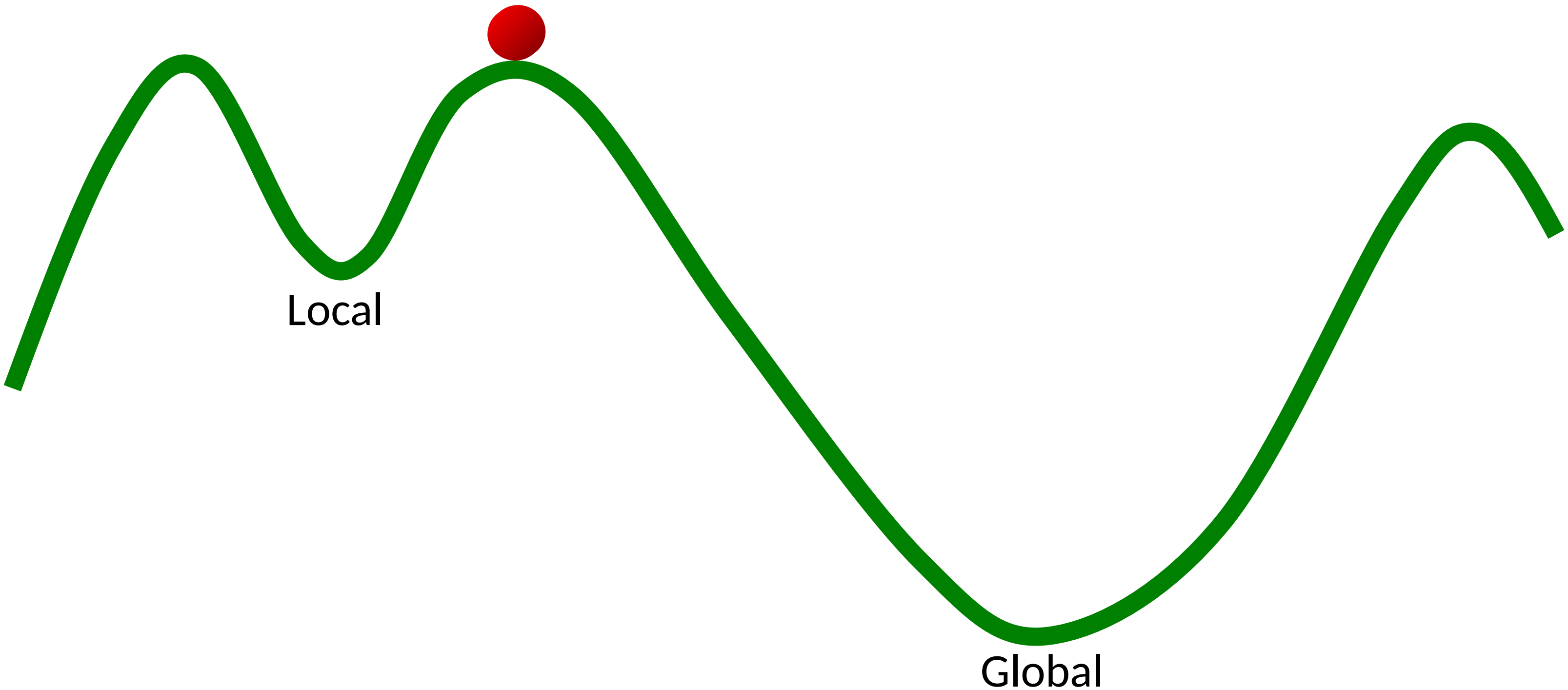
Jean-Claude Latombe

Getting Stuck

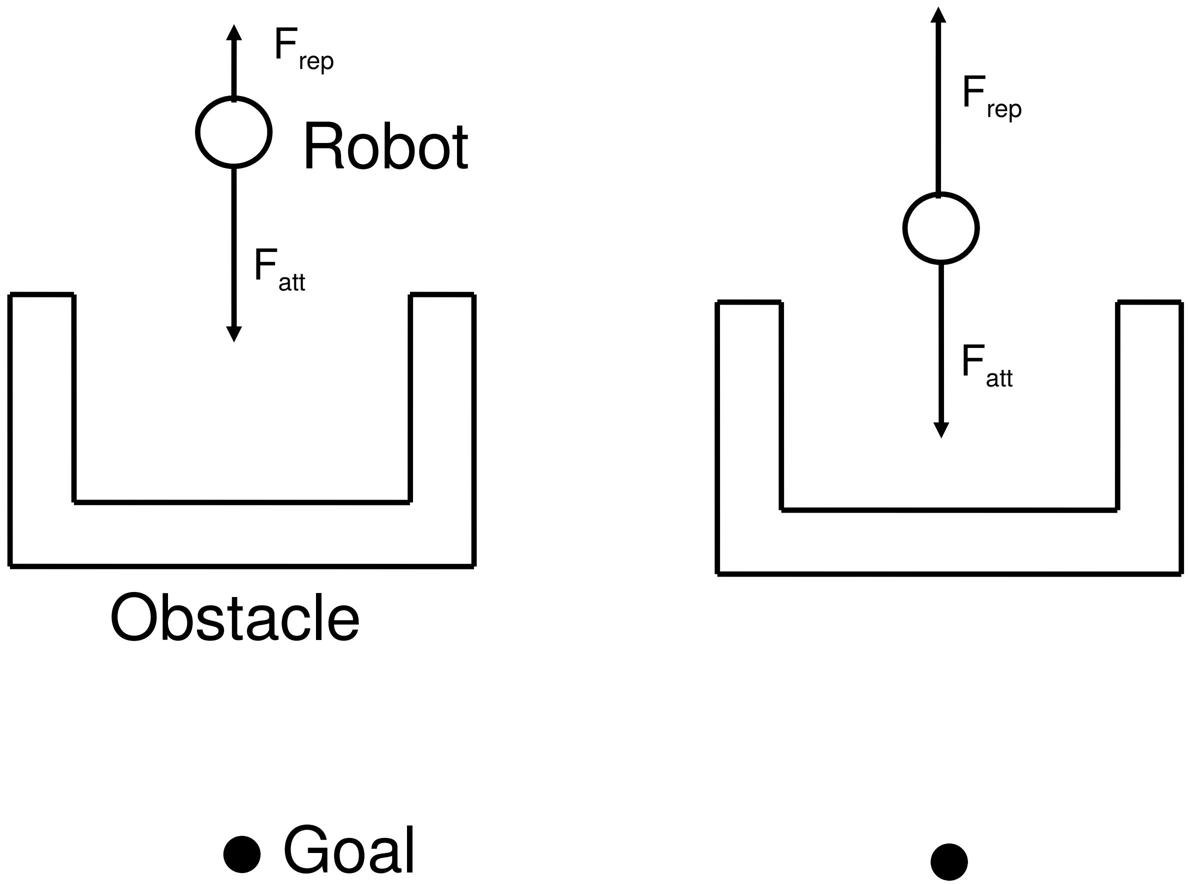


Local Minimum

Minima

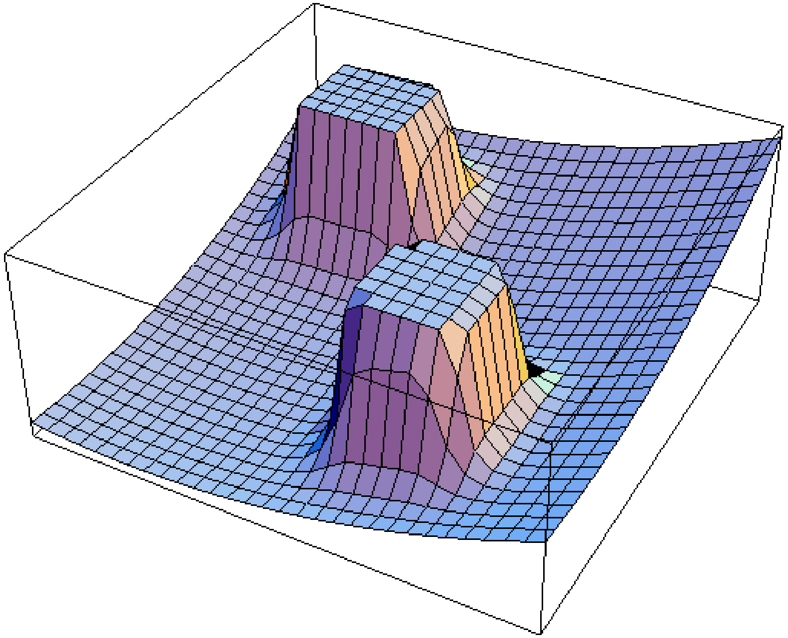


Local Minimum



Potential Field Approach

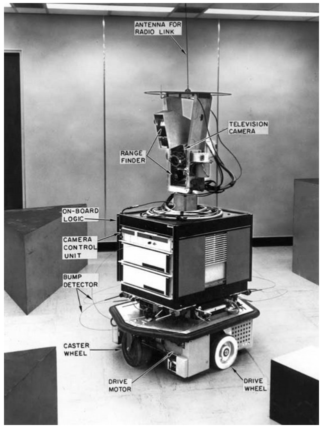
- Requirements:
  - Sensing
  - Odometry
- Pros:
  - Easy and efficient
- Cons
  - Local minima



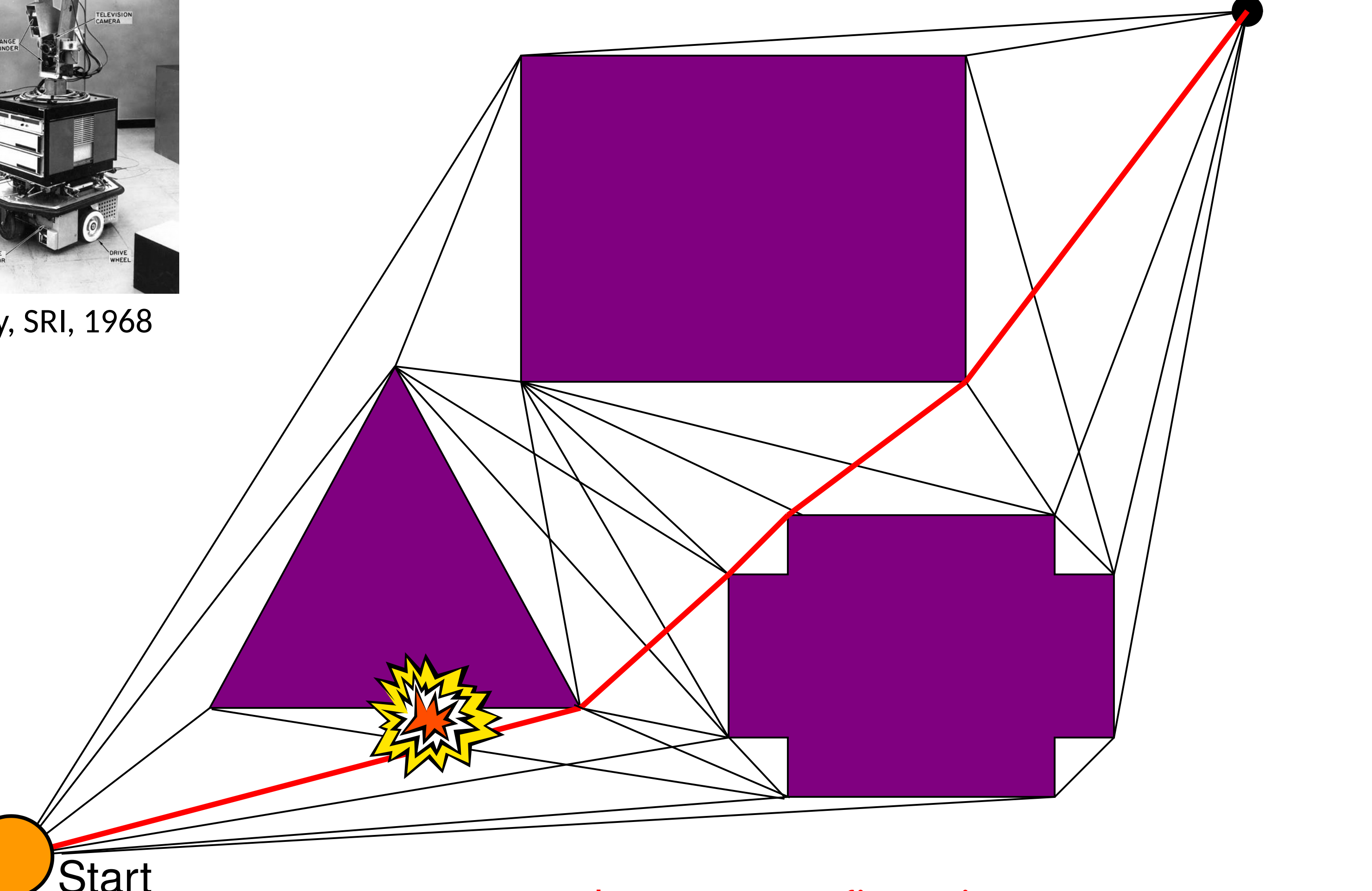
Overcoming Local Minima

- Why do they exist?
  - Definition of repulsive potential
  - Only uses local information
- How can we overcome them?
  - Use global information!

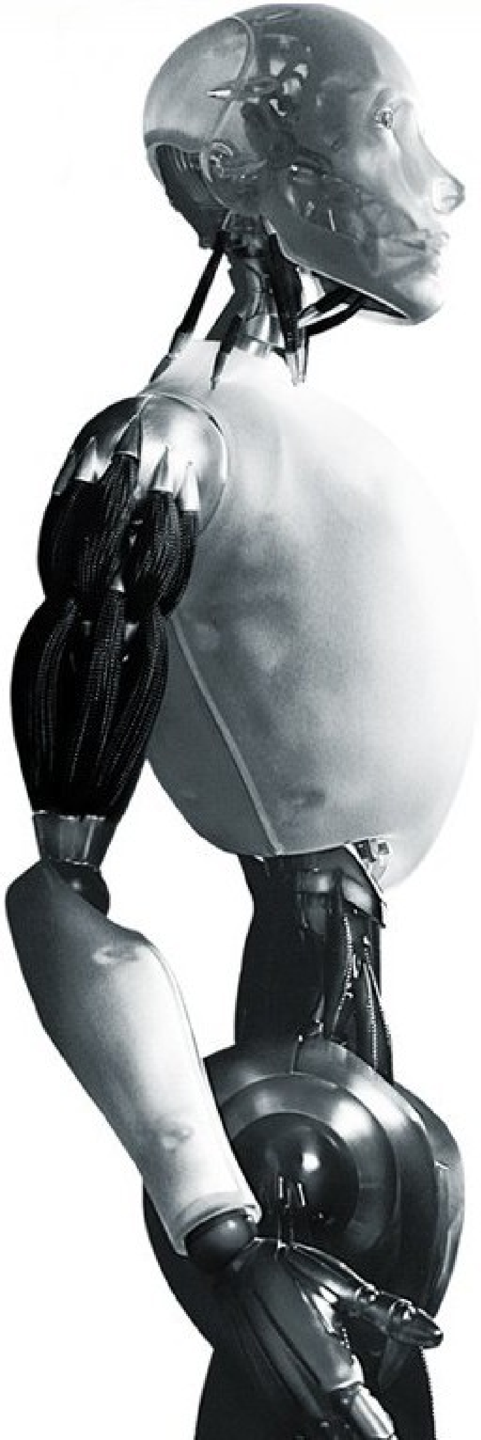
Visibility Map



Shakey, SRI, 1968



Workspace $\neq$ Configuration Space



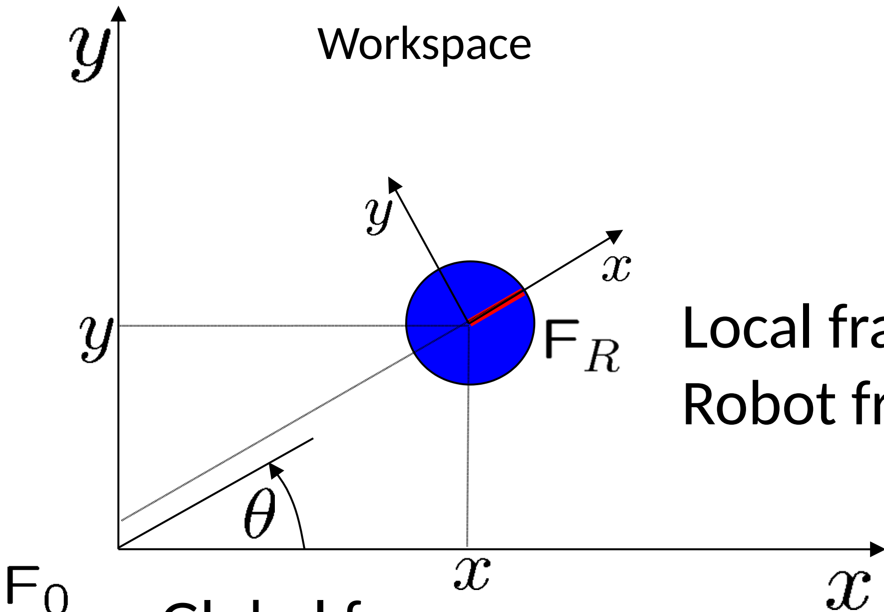
Robotics

Configuration Space Obstacles

TU Berlin

Oliver Brock

Review: Workspace / C-Space

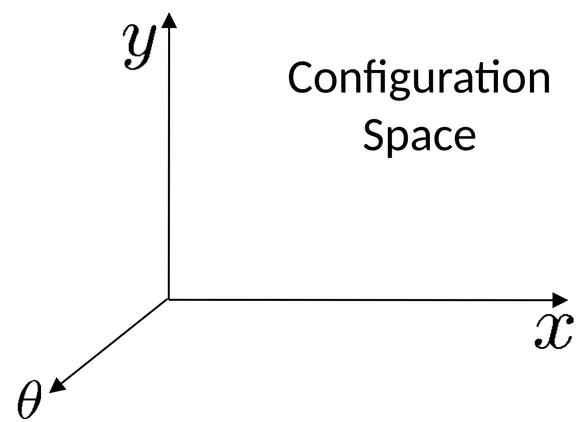


Local frame
Robot frame

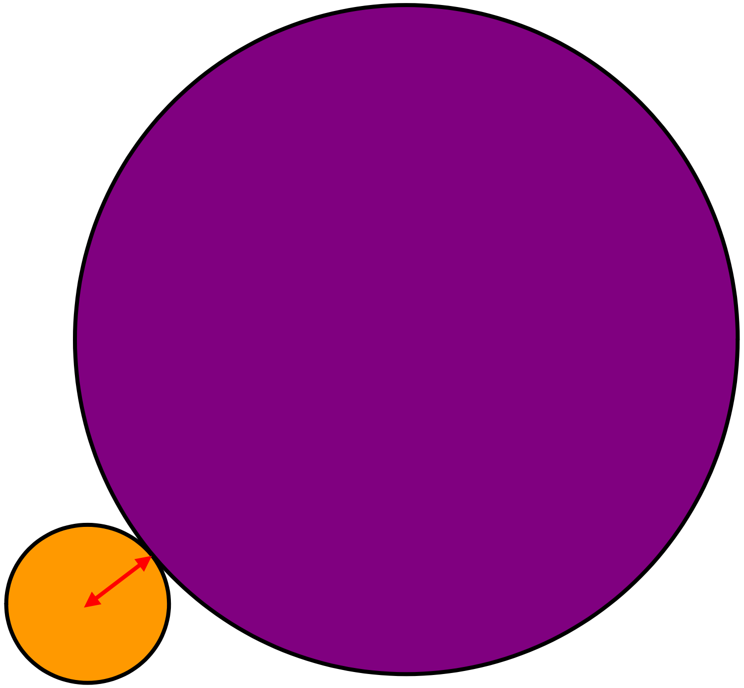
Global frame
World frame
Reference frame

$$q = (\overbrace{x, y}^{\text{position}}, \theta)$$

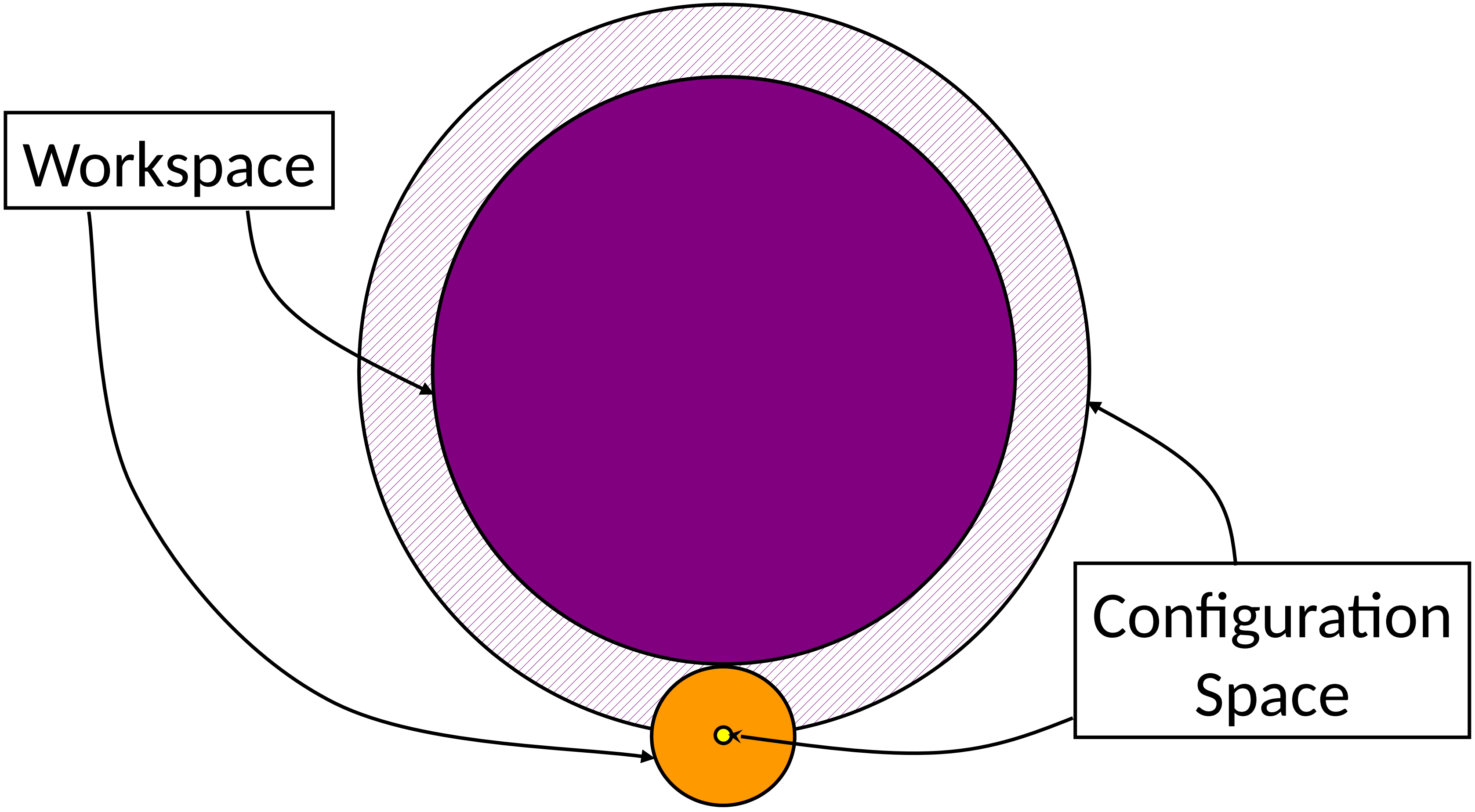
orientation
or heading



Computing C-Space: Growing Obstacles

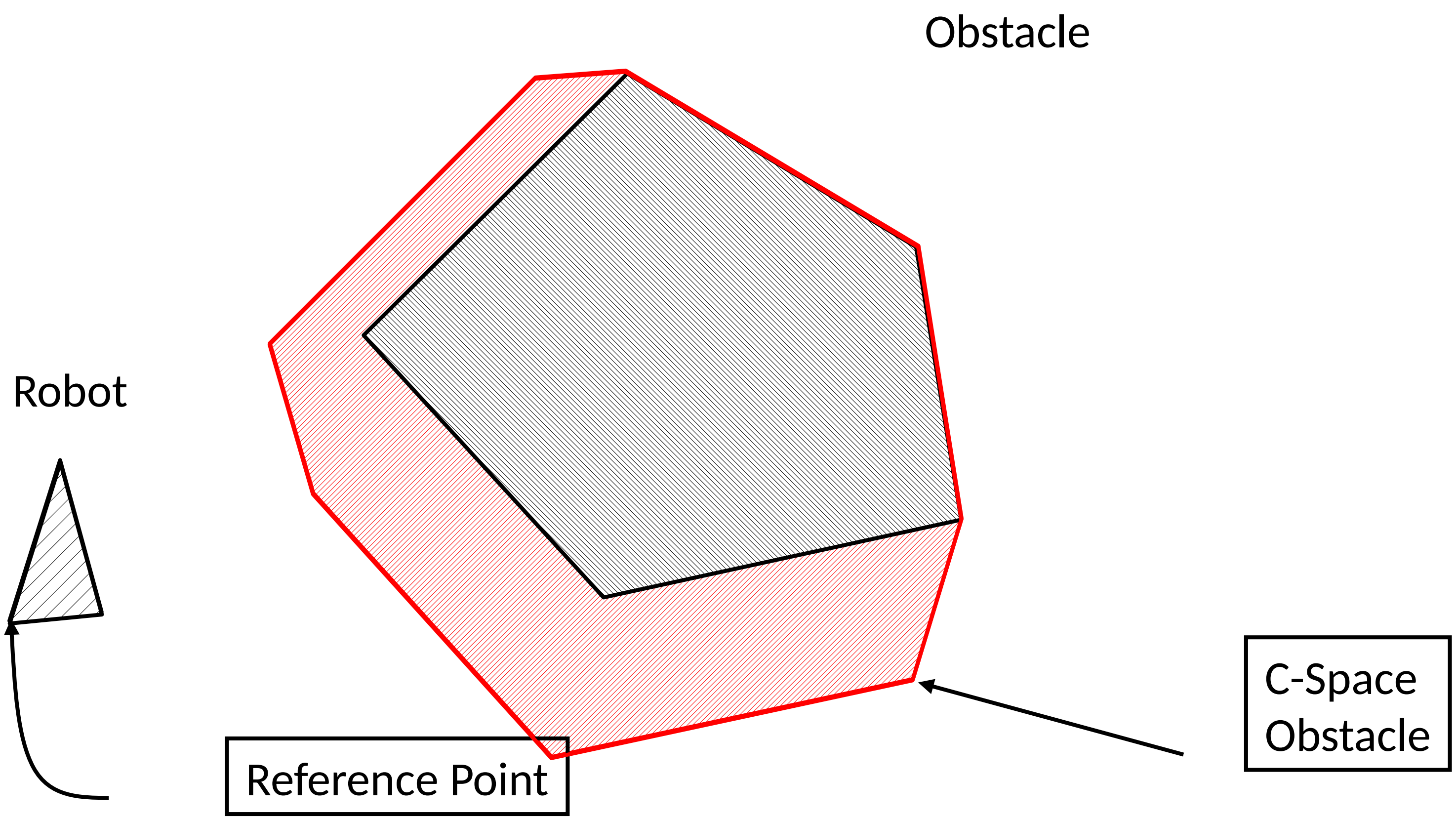


Sliding Along the Boundary



How about changing θ ?

Translational Case (Fixed θ)

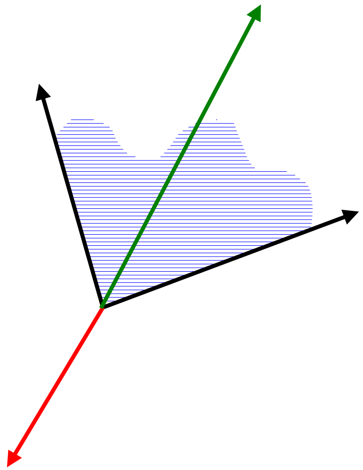


Sidebar: Linear Combination

$$a_1 \cdot \vec{x}_1 + a_2 \cdot \vec{x}_2 + \cdots + a_n \cdot \vec{x}_n$$

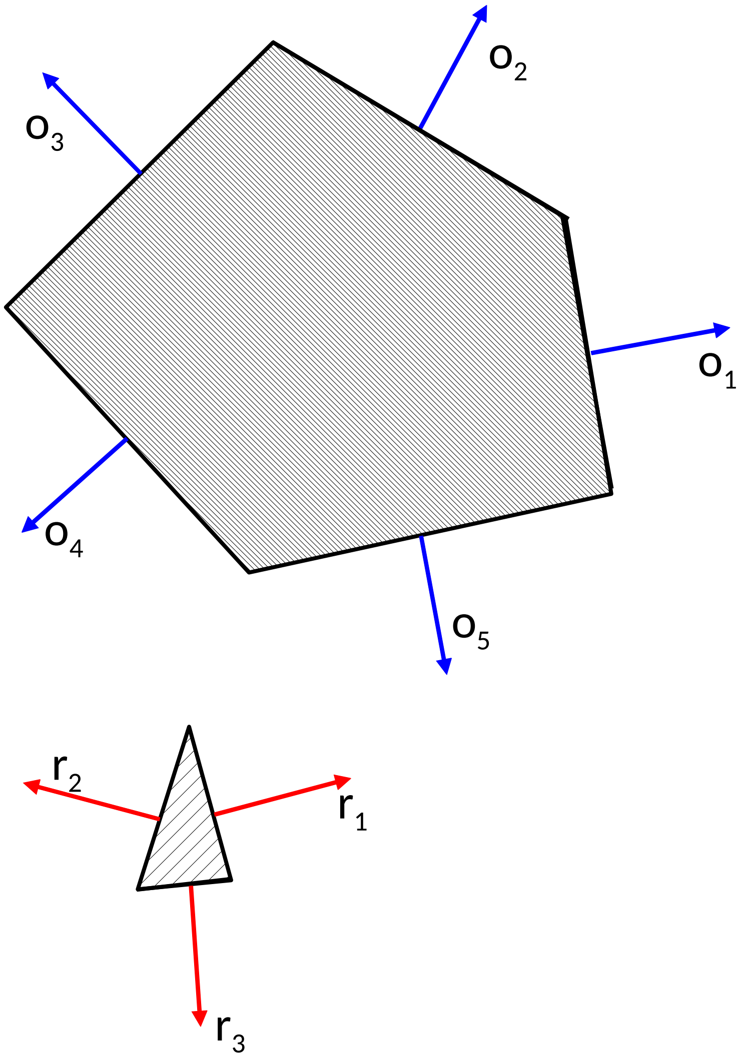
is called a *positive linear combination* if and only if

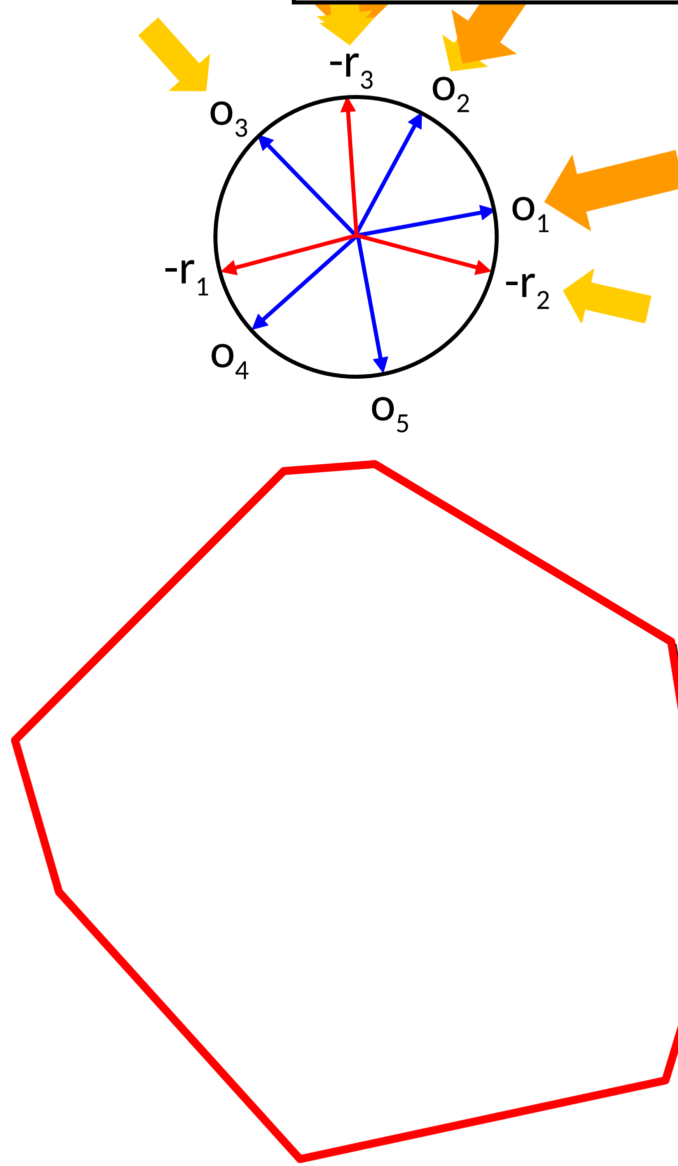
$$a_1, a_2, \cdots, a_n > 0$$



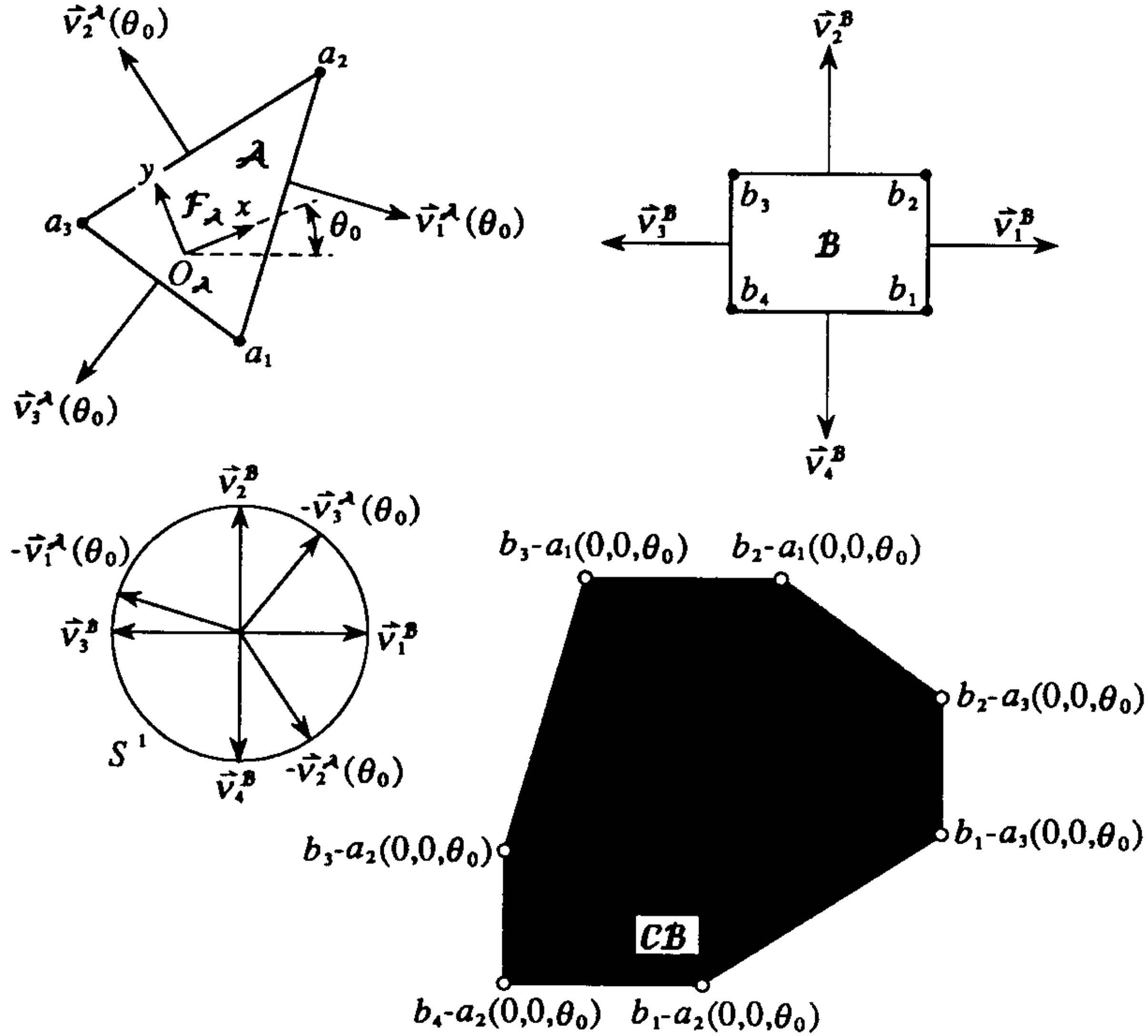
C-Obstacle Const

positive linear combination

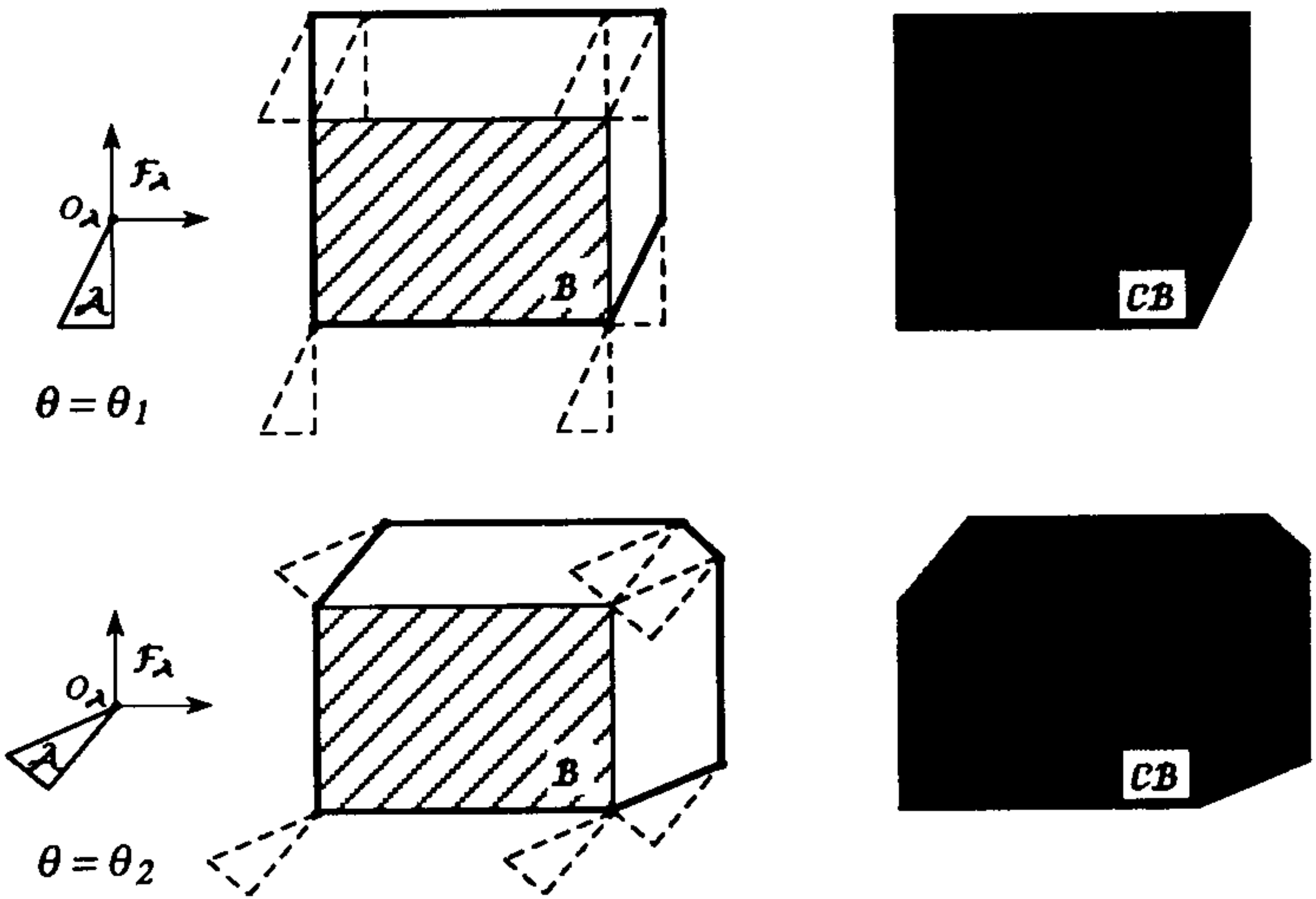




C-Obstacle Construction

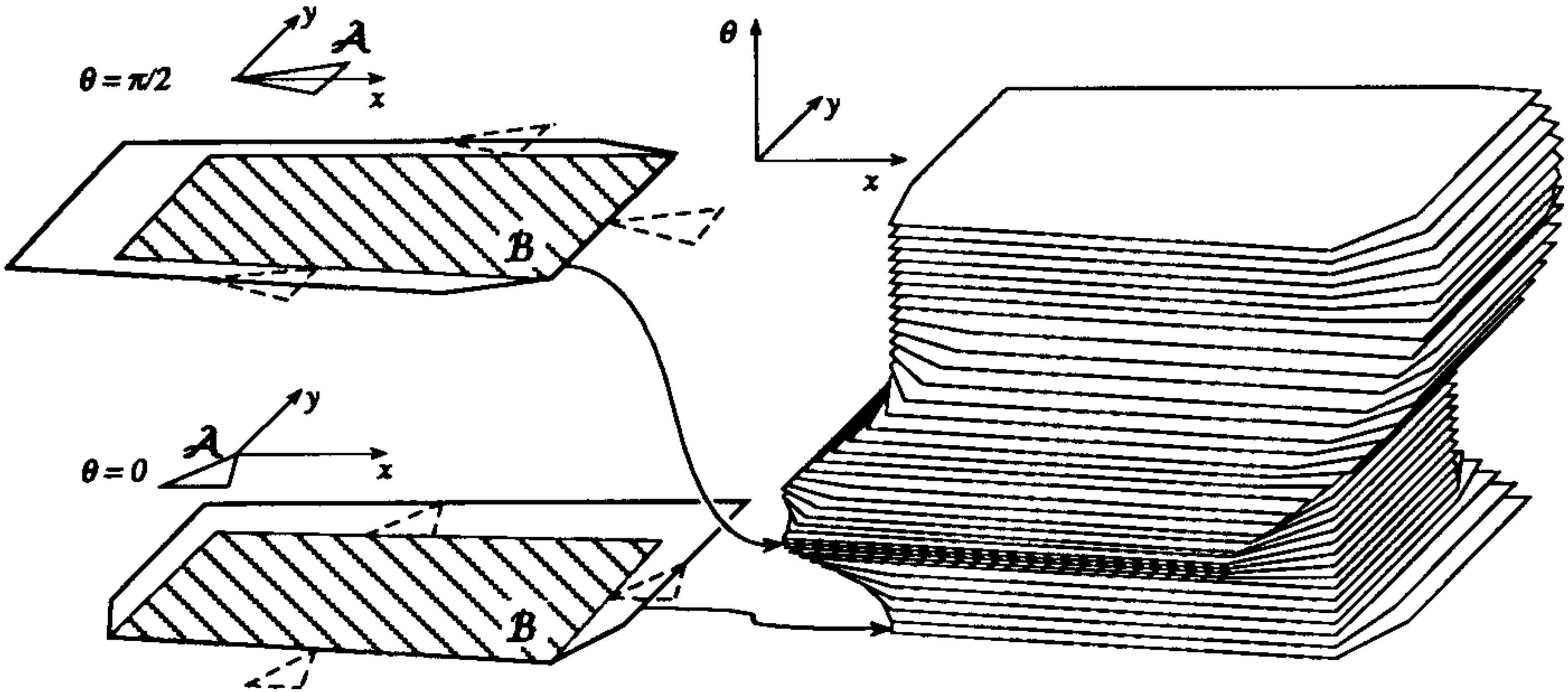


C-Obstacles for Varying θ



Jean-Claude Latombe

C-Obstacle in 3D

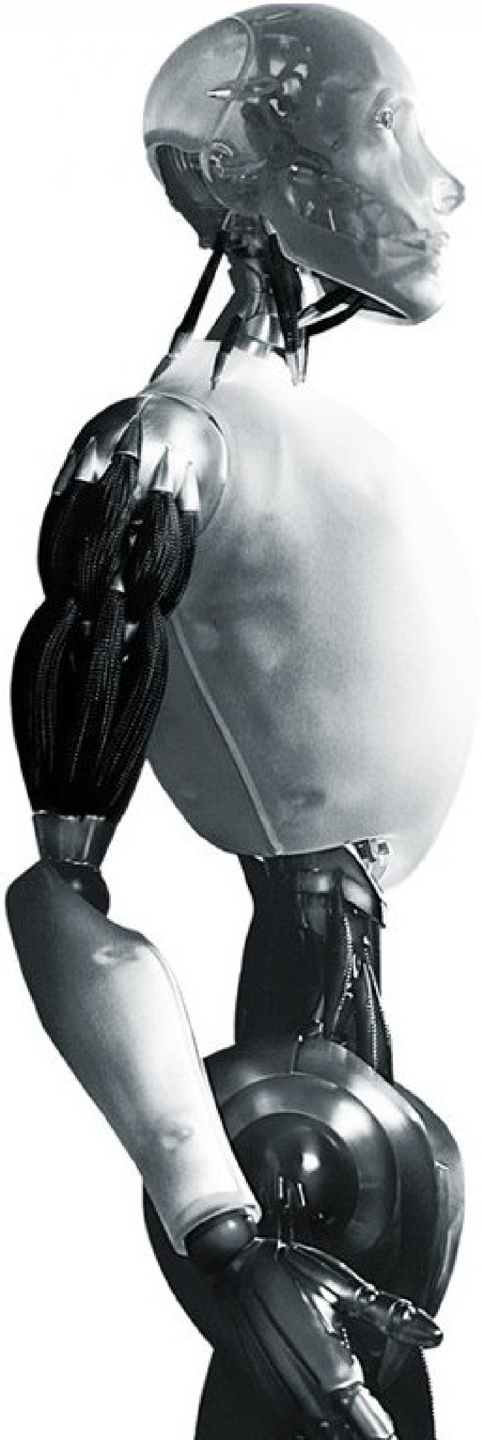


Jean-Claude Latombe

Okay, what next?

- We have computed C-space obstacles for polygonal robots in the plane.
- How can we actually compute a motion to the goal?
- Lesson from potential field approach:

Global Information



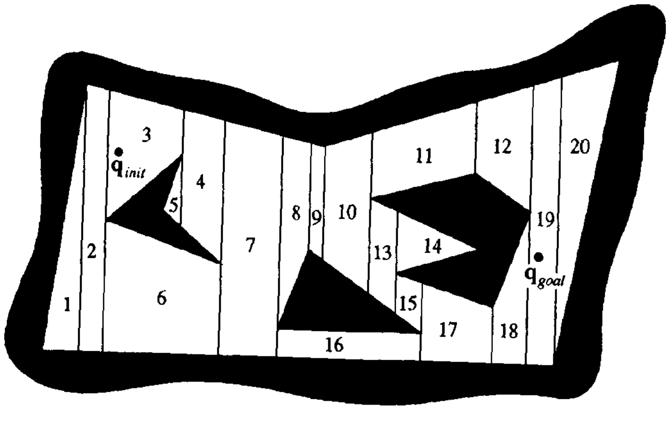
Robotics

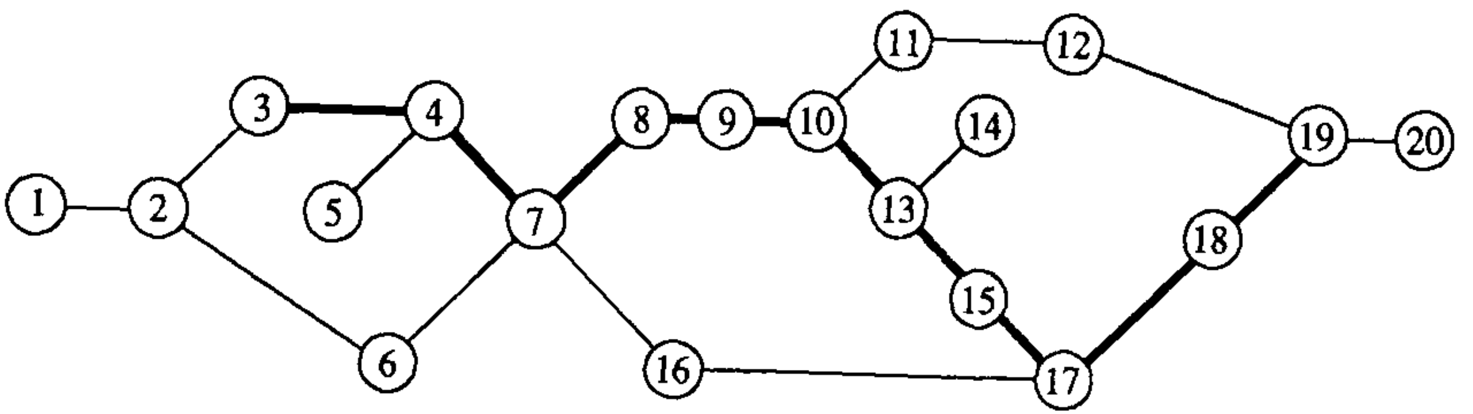
Motion Planning for Mobile Robots

TU Berlin

Oliver Brock

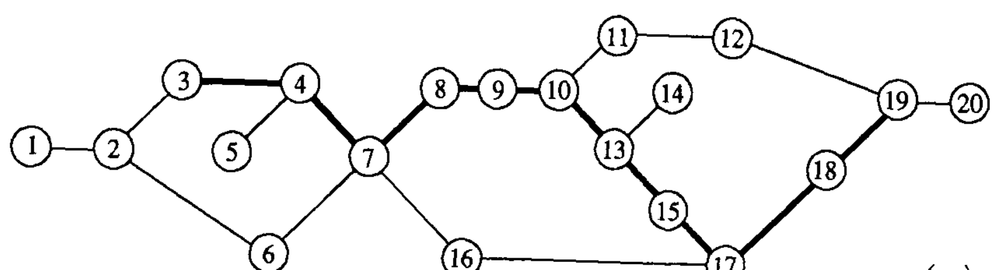
Exact Cell Decomposition cont. II





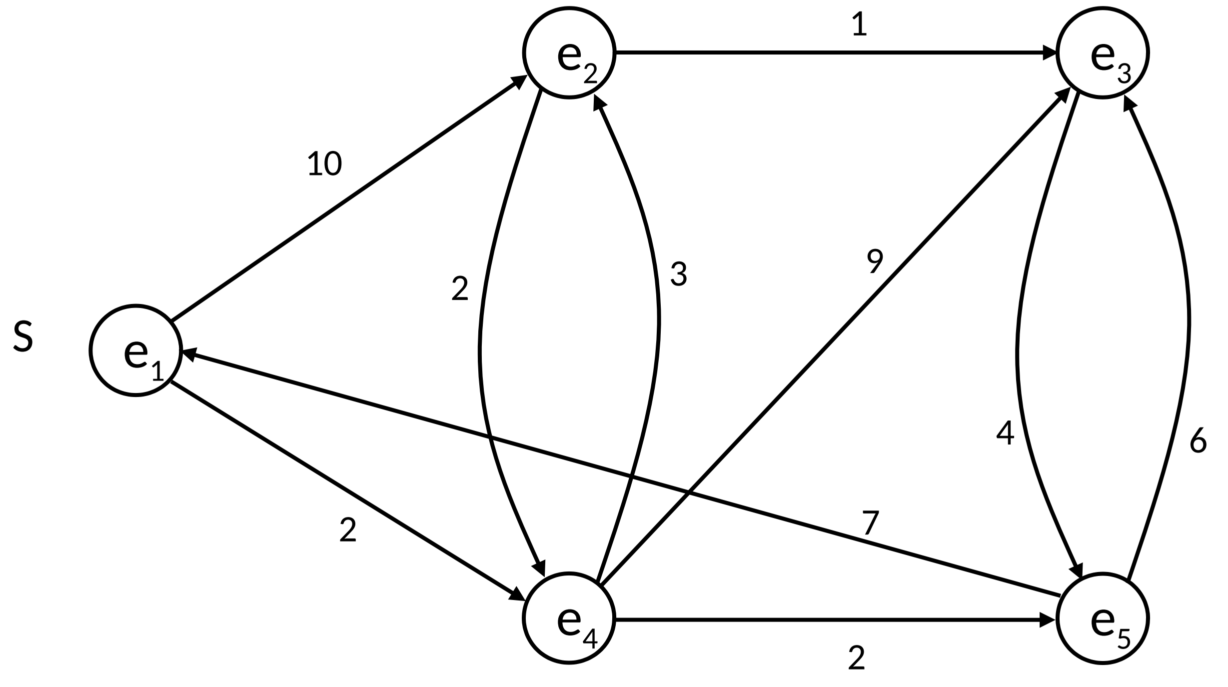
Sidebar: Dijkstra's Algorithm

- Single-source shortest-path problem
- Weighted, directed graph G(V,E)
- Given a node e $\in$ E, what is the shortest path to all other reachable nodes?

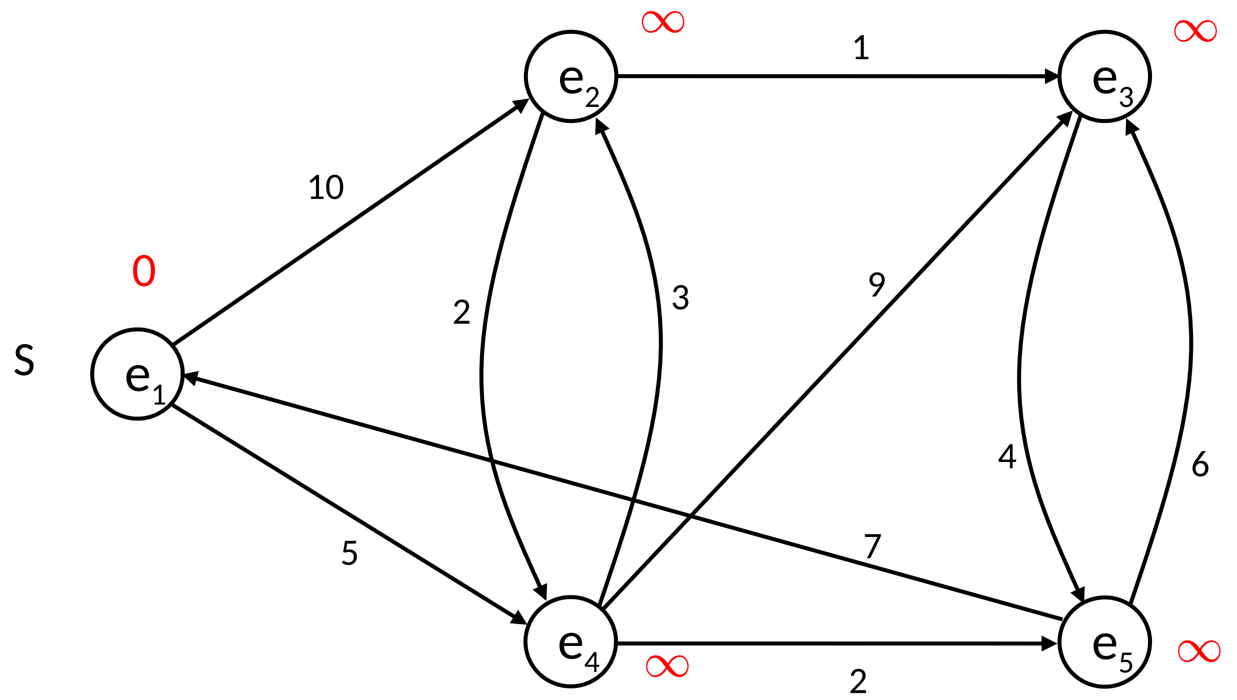


(c)

Sidebar: Dijkstra cont.



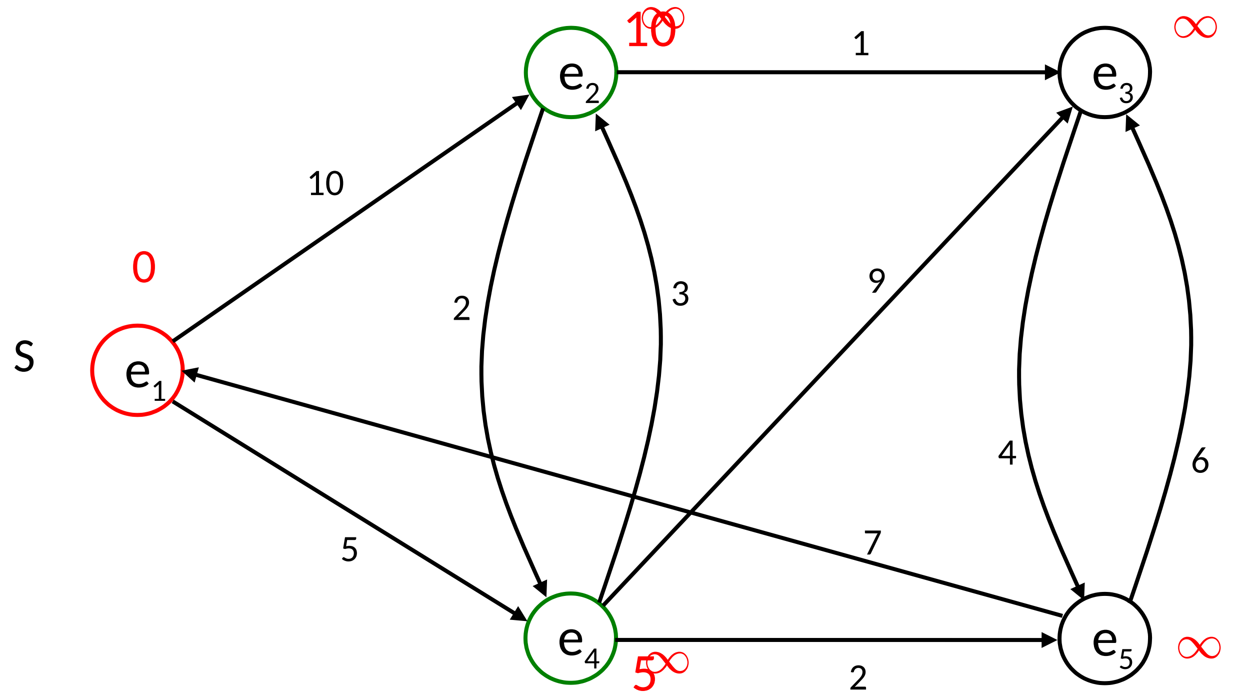
Sidebar: Dijkstra Initialization



Current nodes: $\{e_1=0, e_2=\infty, e_3=\infty, e_4=\infty, e_5=\infty\}$

Nodes completed: $\{\emptyset\}$

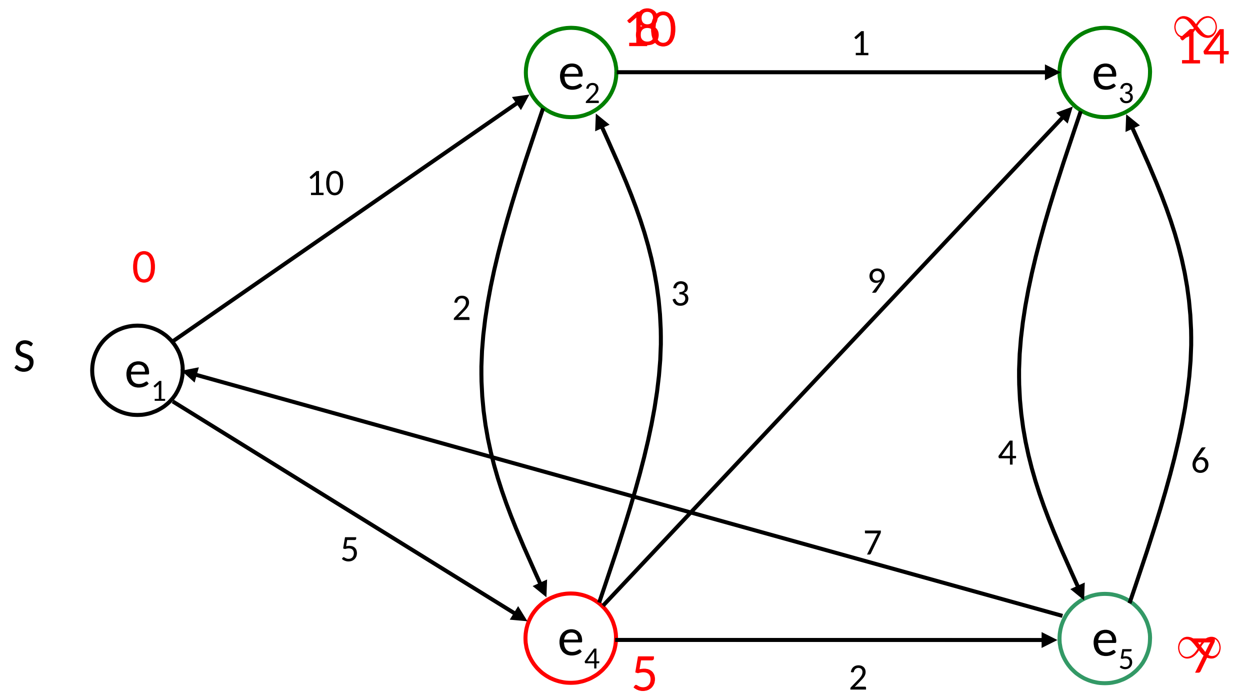
Sidebar: Dijkstra Relaxation 1



Current nodes: $\{e_1=0, e_2=\infty, e_3=\infty, e_4=\infty, e_5=\infty\}$ $\{e_4=5, e_2=10, e_3=\infty, e_5=\infty\}$

Nodes completed: $\{\emptyset\}$ $\{e_1\}$

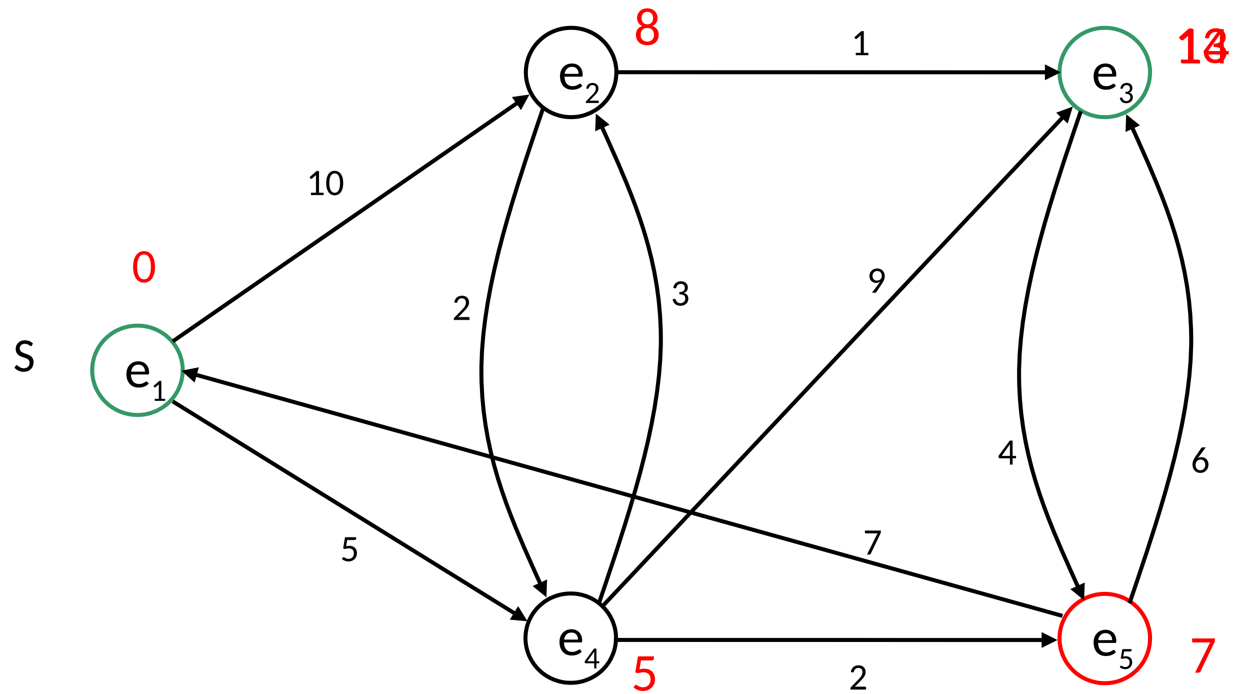
Sidebar: Dijkstra Relaxation 2



Current nodes: $\{e_4=5, e_2=10, e_3=\infty, e_5=\infty\}$ $\{e_5=7, e_2=8, e_3=14\}$

Nodes completed: $\{e_1\}$ $\{e_1, e_4\}$

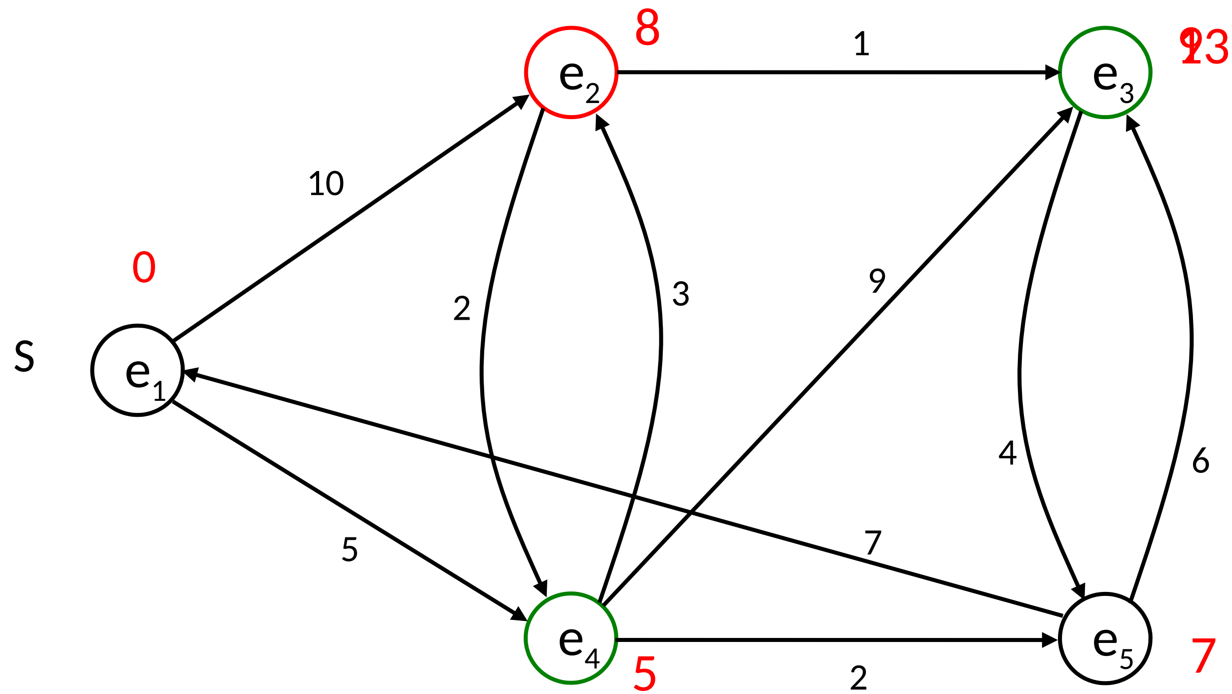
Sidebar: Dijkstra Relaxation 3



Current nodes: {e_2=8, e_3=13} {e_5=7, e_2=8, e_3=14}

Nodes completed: {e_1,e_4} {e_1,e_4,e_5}

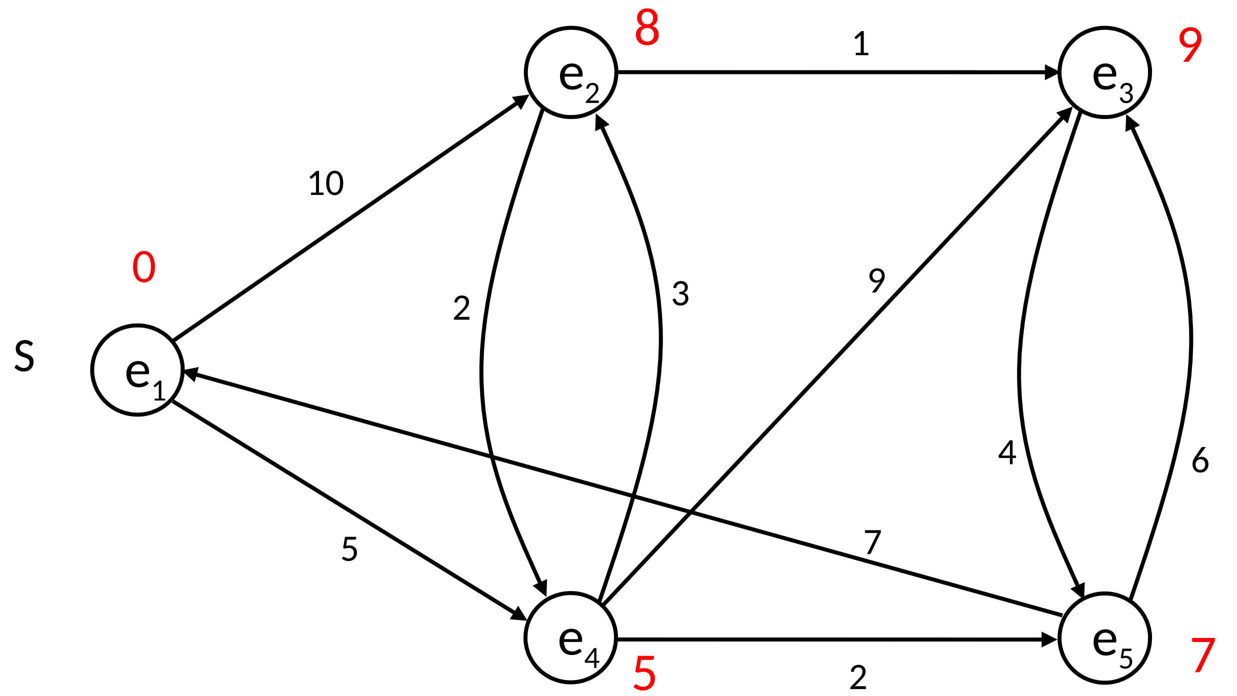
Sidebar: Dijkstra Relaxation 4



Current nodes: $\{e_3=9\}$ $\{e_2=8, e_3=13\}$

Nodes completed: $\{e_1, e_4, e_5\}$ $\{e_1, e_2, e_5, e_4\}$

Sidebar: Dijkstra Relaxation 5



Current nodes: {}{e_3=9}

Nodes completed: {e_1, e_2, e_4, e_5}

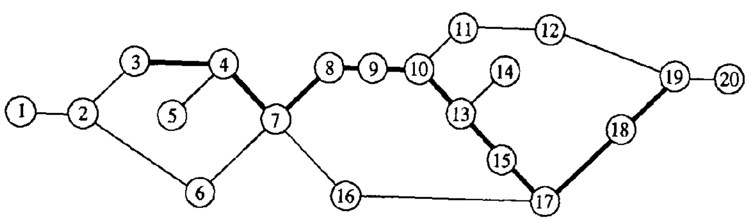
Sidebar: Dijkstra cont. III

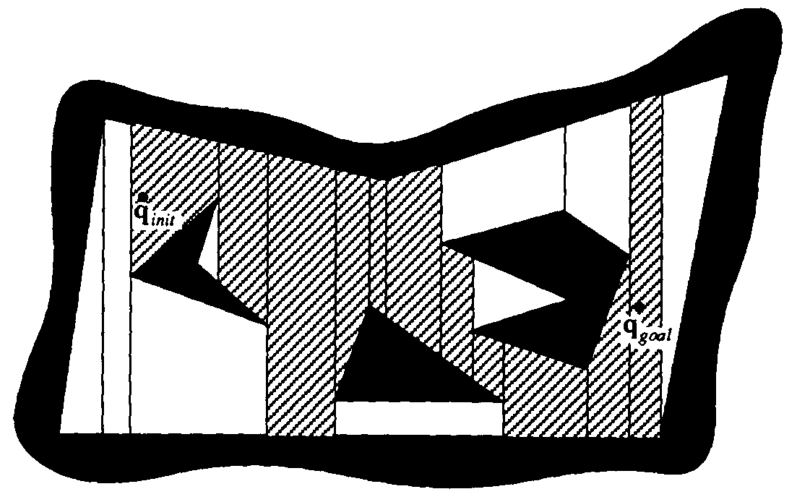
```
Dijkstra(G,w,s)
Initialize-Single-Source(G,s)
S ← ∅
Q ← V[G]
while Q ≠ 0
   do u ← Extract-Min(Q)
       S ← S ∪ {u}
       for each vertex v ∈ Adj[u]
           do Relax (u,v,w)
```

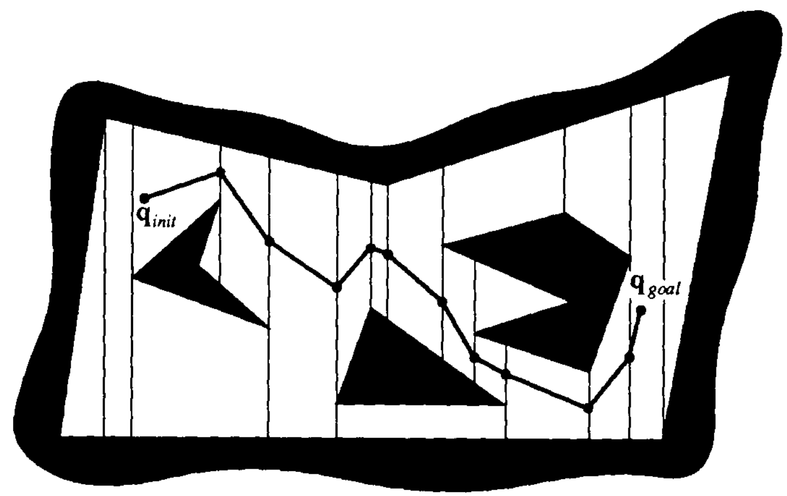
```
Intialize-Single-Source(G,s)
   for each vertex v ∈ V[G]
       do d[v]← ∞
           π[v] ← NIL
d[s] ← 0
```

```
Relax(u,v,w)
//Is it shorter to reach v via u?
    if d[v] > d[u] + w(u,v)
    then d[v] ← d[u] + w(u,v)
         π[v] ← u
```

Exact Cell Decomposition cont. III





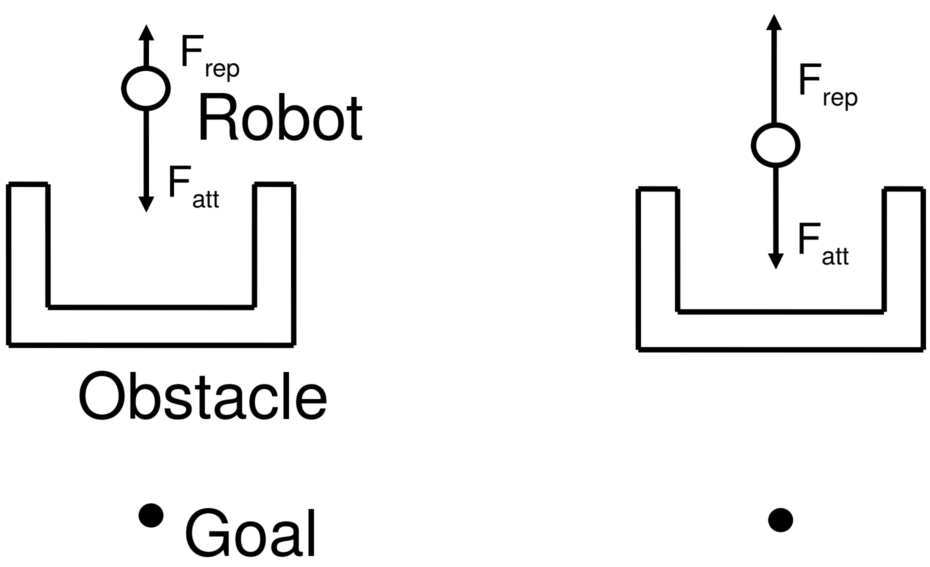


Adaptation of Dijkstra’s Algorithm

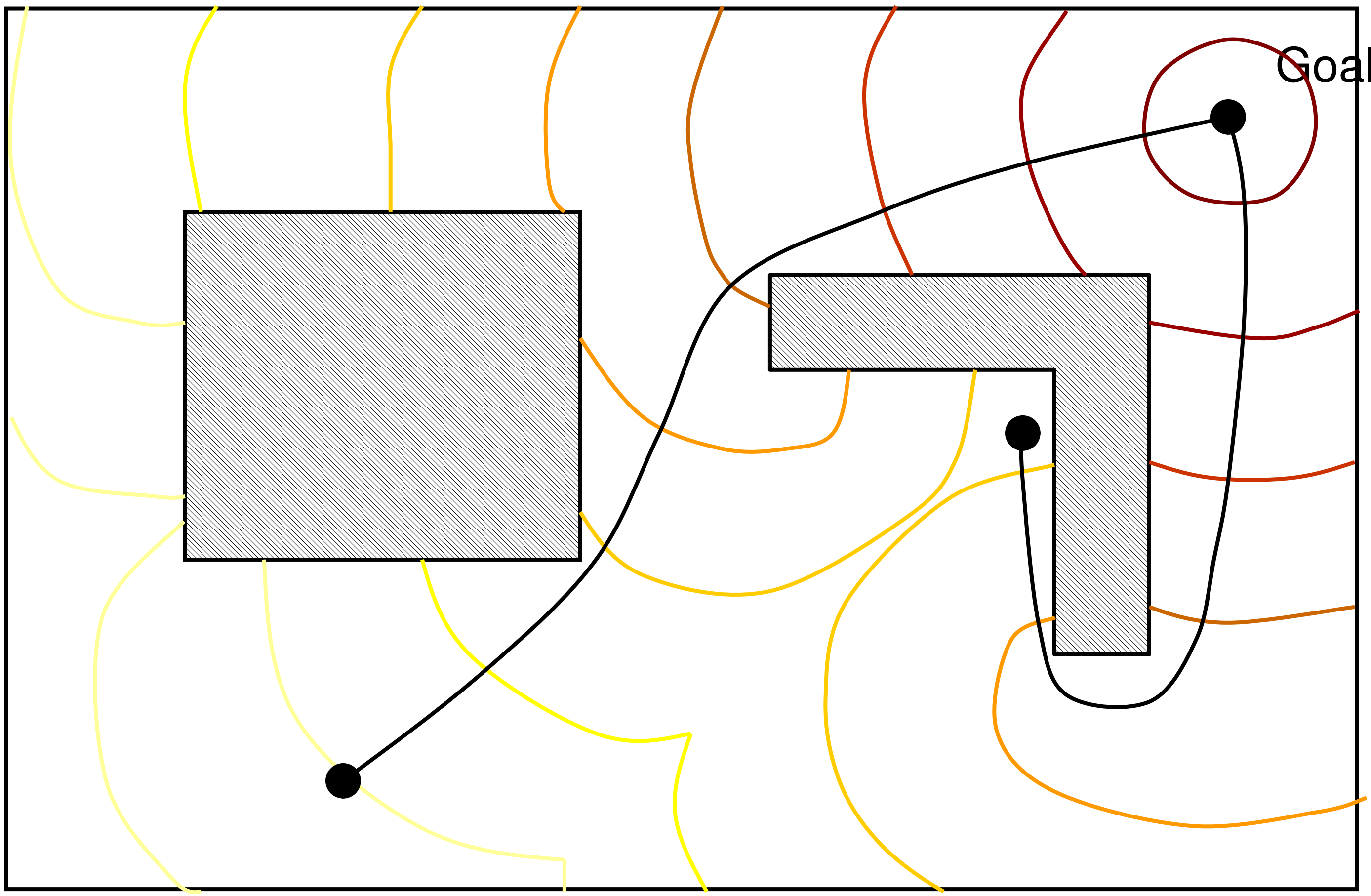
- For every undirected edge add two directed edges in opposite directions
- Determine weight of edge based on
  - distance
  - difficulty of passage
  - other properties of the space

Global Potential Functions

- Goal: avoid local minima
- Problem: requires global information
- Solution: **Navigation Function**

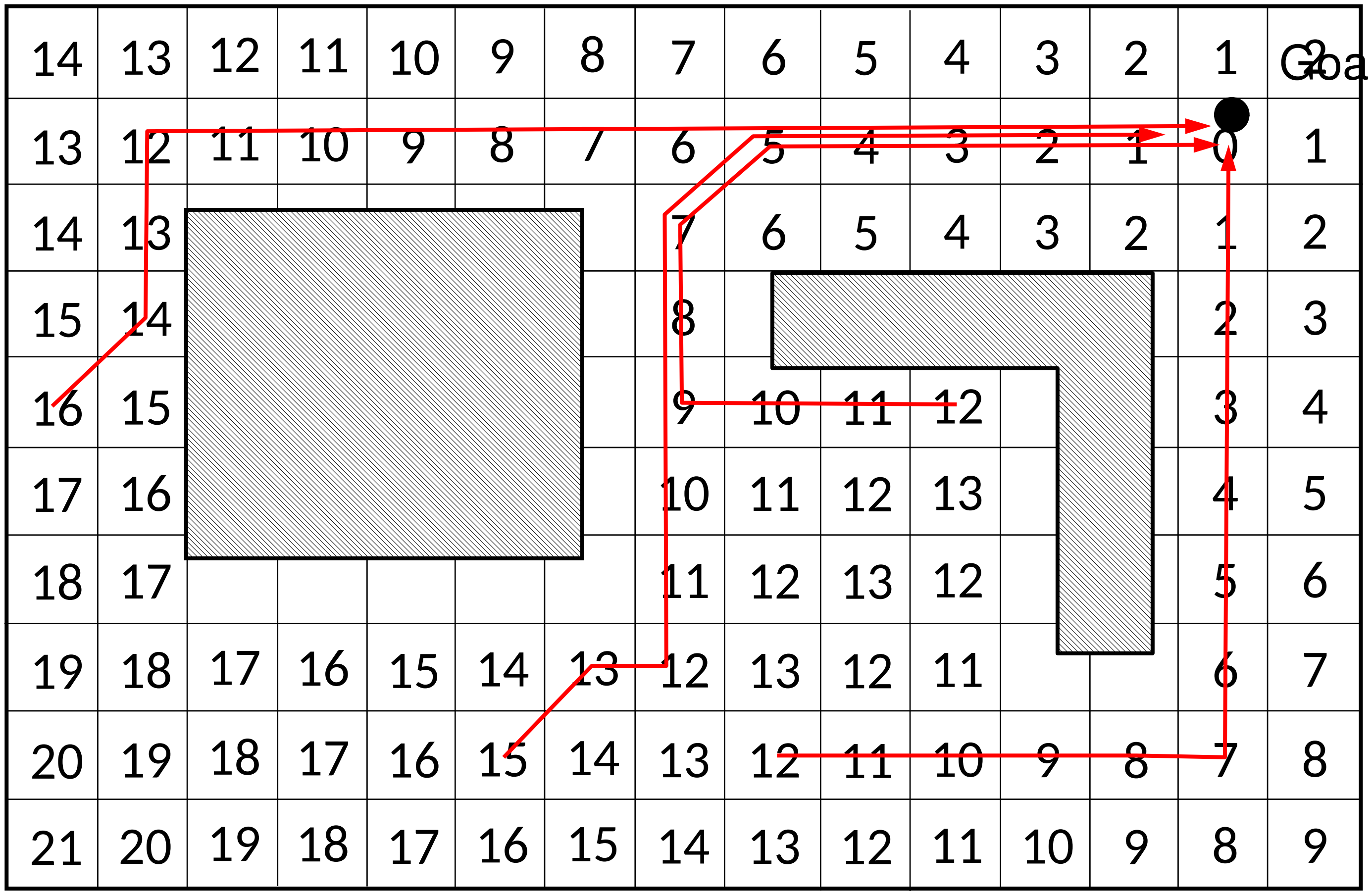


Navigation Function NF1

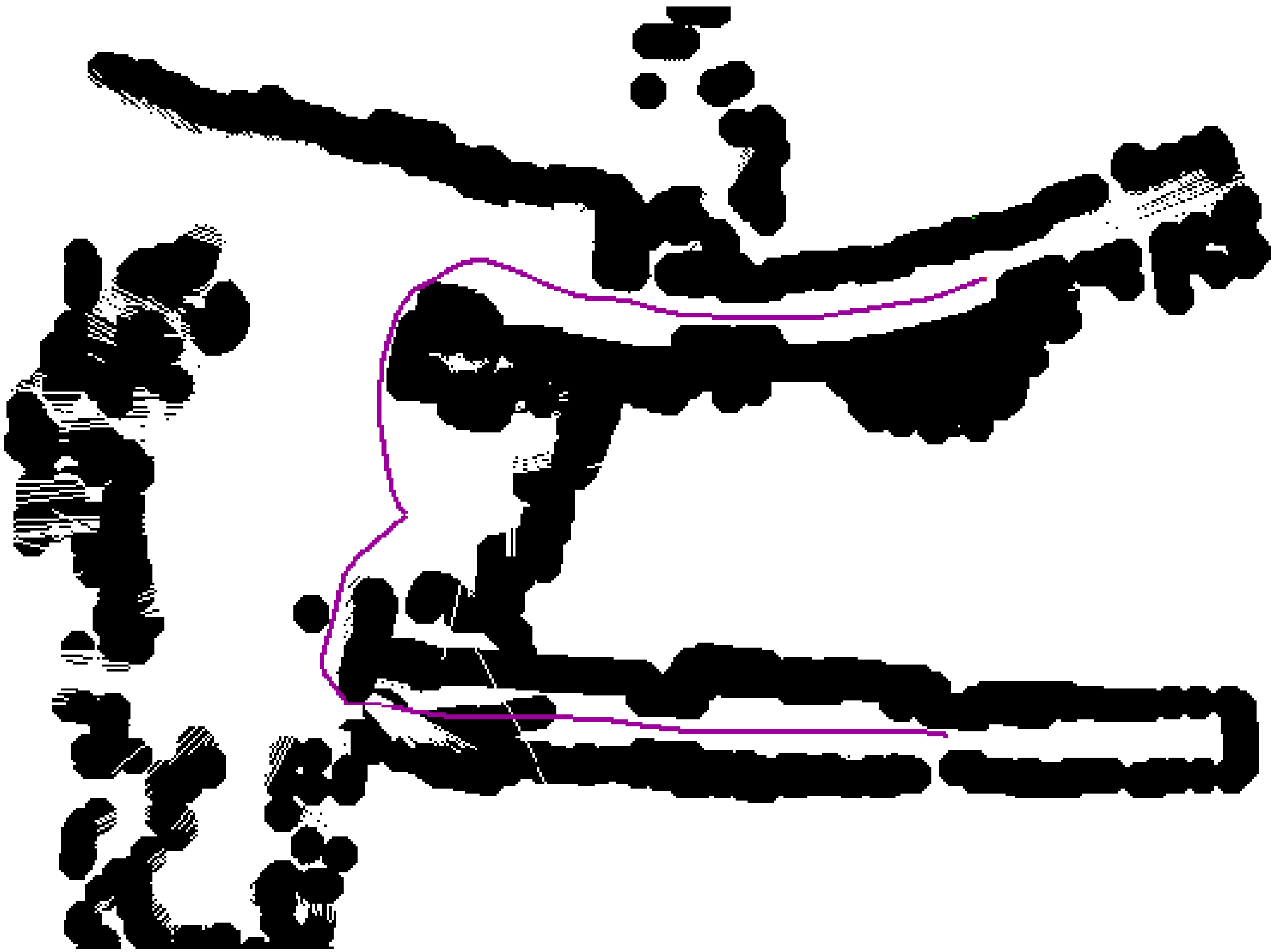


Wave Front Expansion

NF1 Real-World Scenario

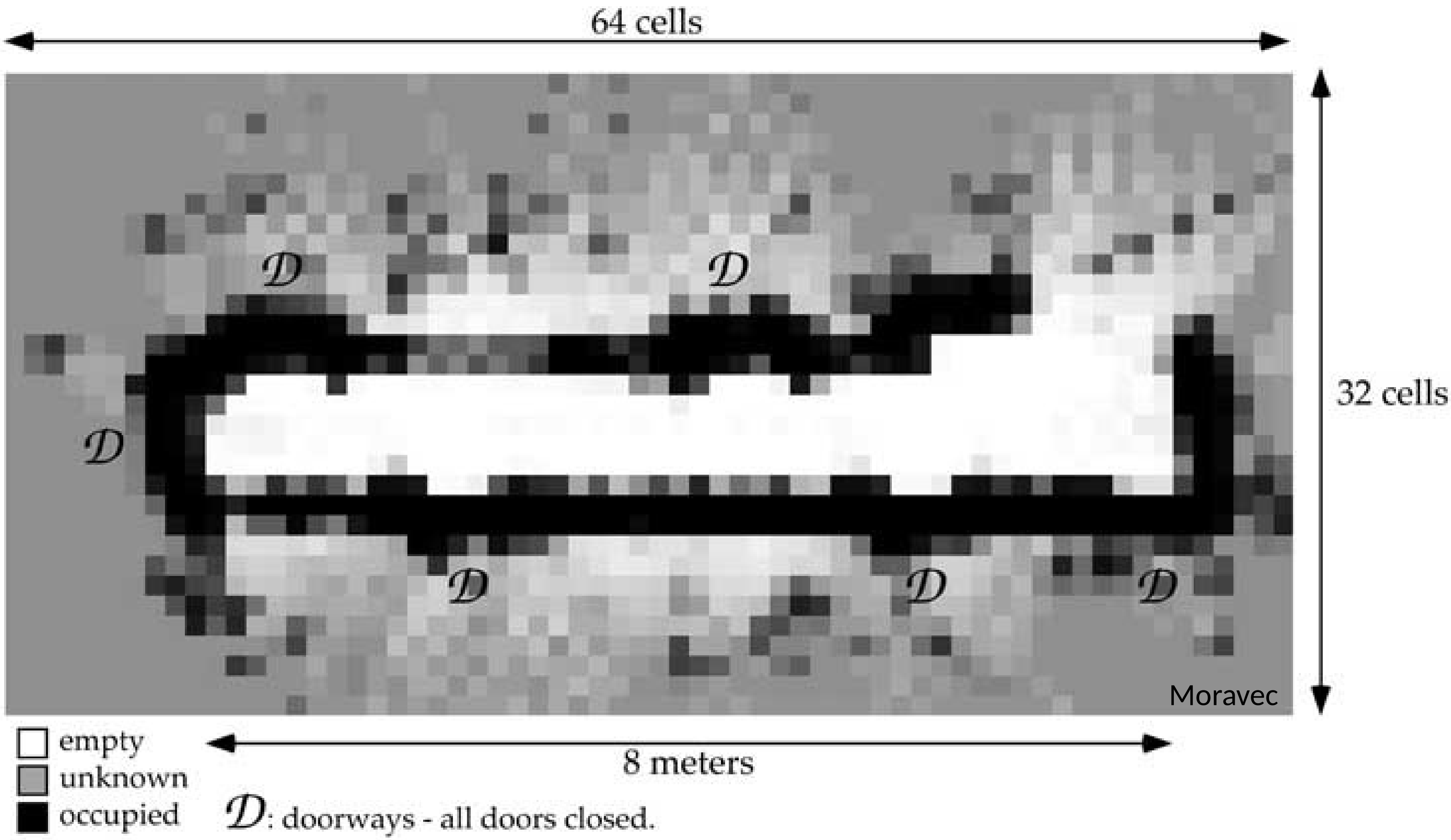


Binary Occupancy Grid

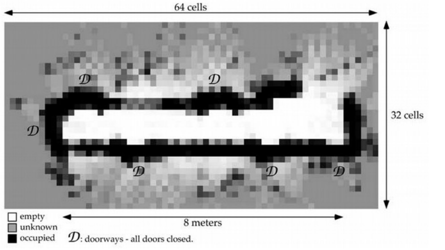


A little video…

General Occupancy Grid

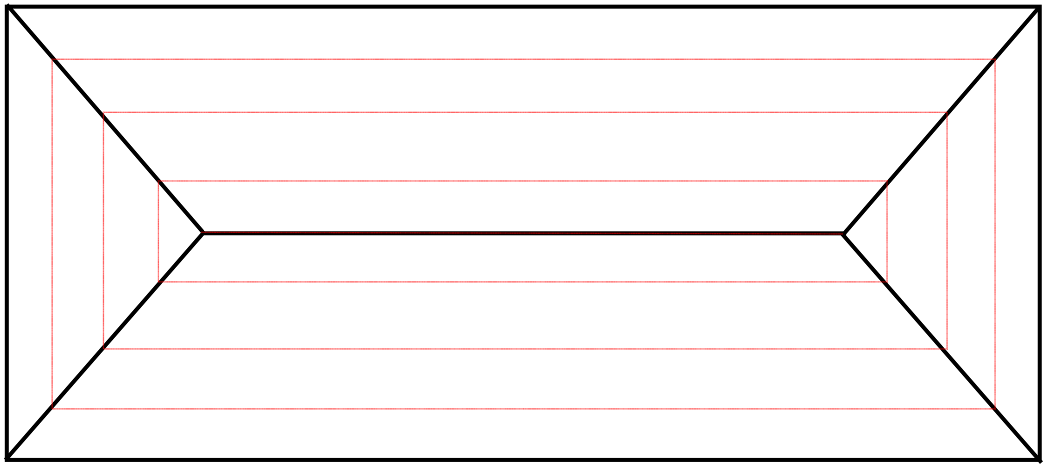


Occupancy Grids



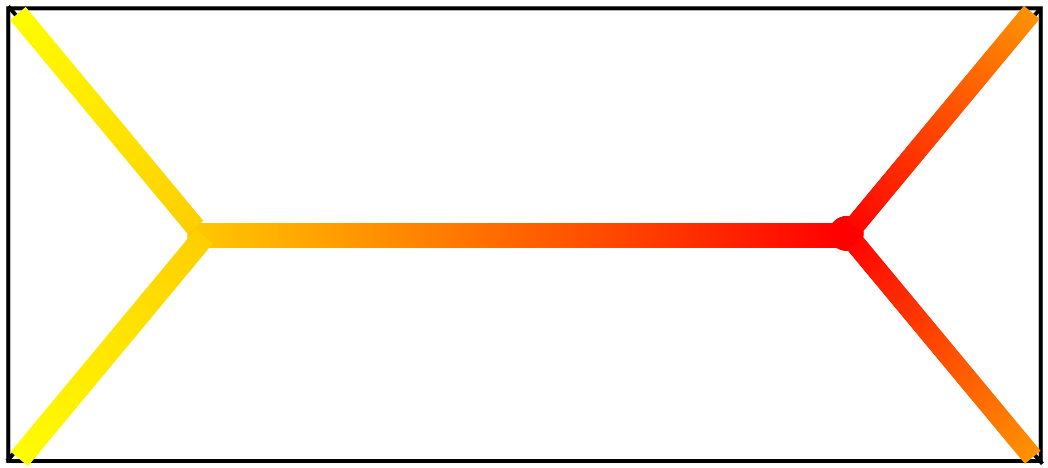
- Active update based on sensory information
  - binary: obstacle – no obstacle
  - [0,1]: probability of obstacle being present
- Automatic updated based on time
  - the probability of an obstacle can decrease if cell is unobserved
- Navigation function in that grid!

NF2 – Step 1



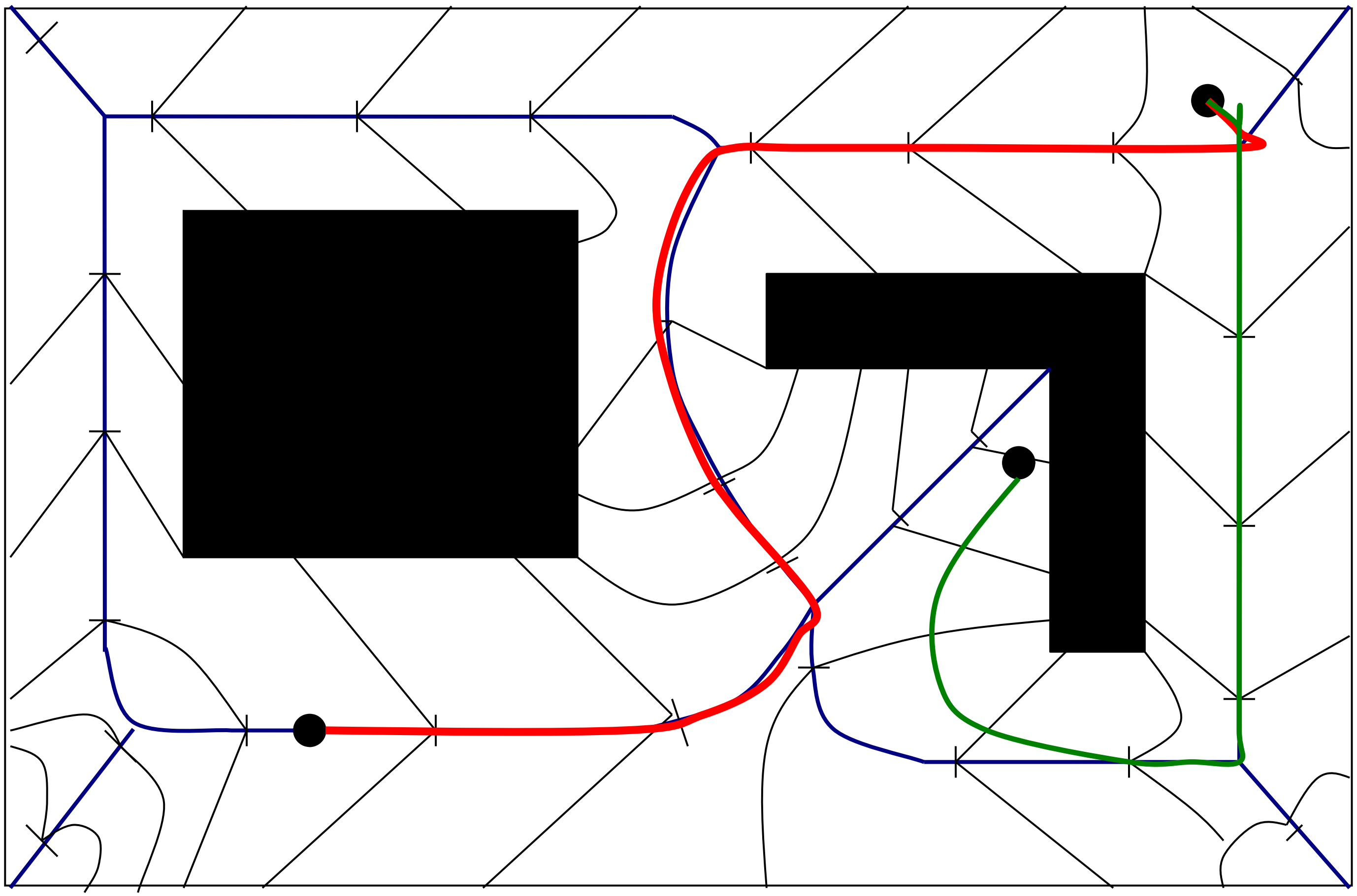
Compute medial axis with wave front expansion

NF2 – Step 2



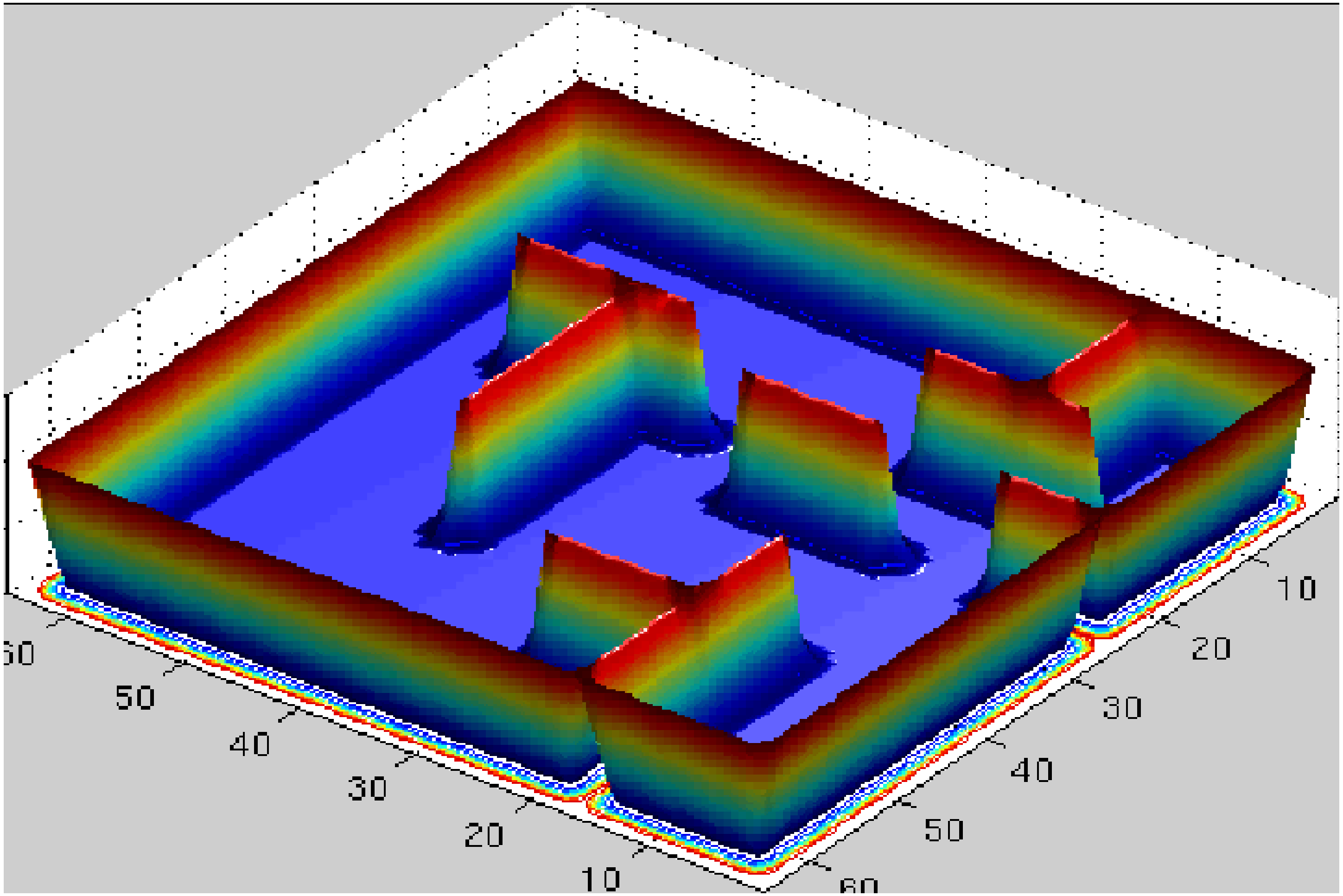
Compute wave front expansion along medial axis

NF2 – Step 3



Compute wave front expansion from medial axis

Harmonic Potentials

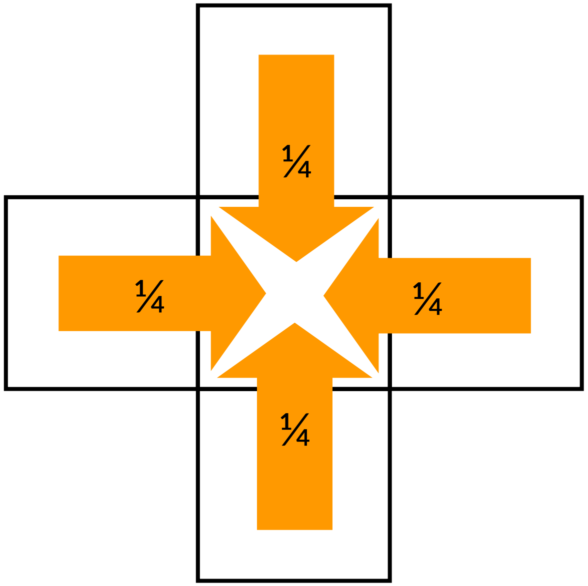


Courtesy of John Sweeney

Harmonic Potentials

- Harmonic functions
- Solutions to **Laplace's equation** (PDE)
- Intuition: heat transfer
- Numerical solutions: relaxation
- No local minima
- Require a lot of computation time
- Susceptible to numerical rounding error

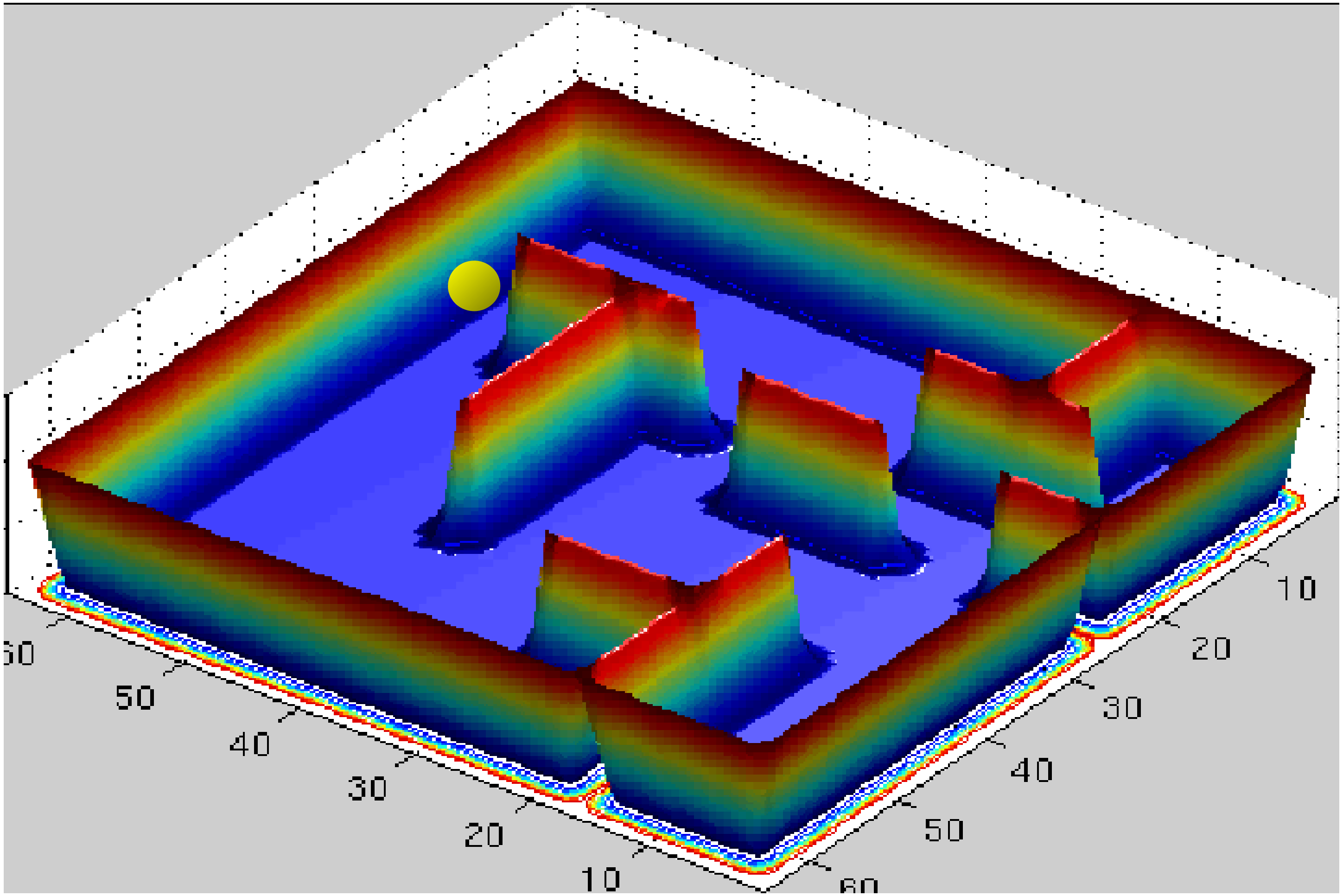
Example: Jacobi Iteration



the new value of a cell for the next iteration
is ¼ of the sum of its 4-neighbors

$$\mathsf{cell}_{(x,y,t+1)} := \frac{\mathsf{cell}_{(x-1,y,t)} + \mathsf{cell}_{(x+1,y,t)} + \mathsf{cell}_{(x,y-1,t)} + \mathsf{cell}_{(x,y+1,t)}}{4}$$

Harmonic Potentials



Courtesy of John Sweeney

Summary

- Local Methods
  - Potential Field Approach
  - Subject to local minima!
- Global Methods
  - Potential Field Approach with global navigation function
    - NF1, NF2, Harmonic Potential
  - Cell Decomposition
  - Visibility Method