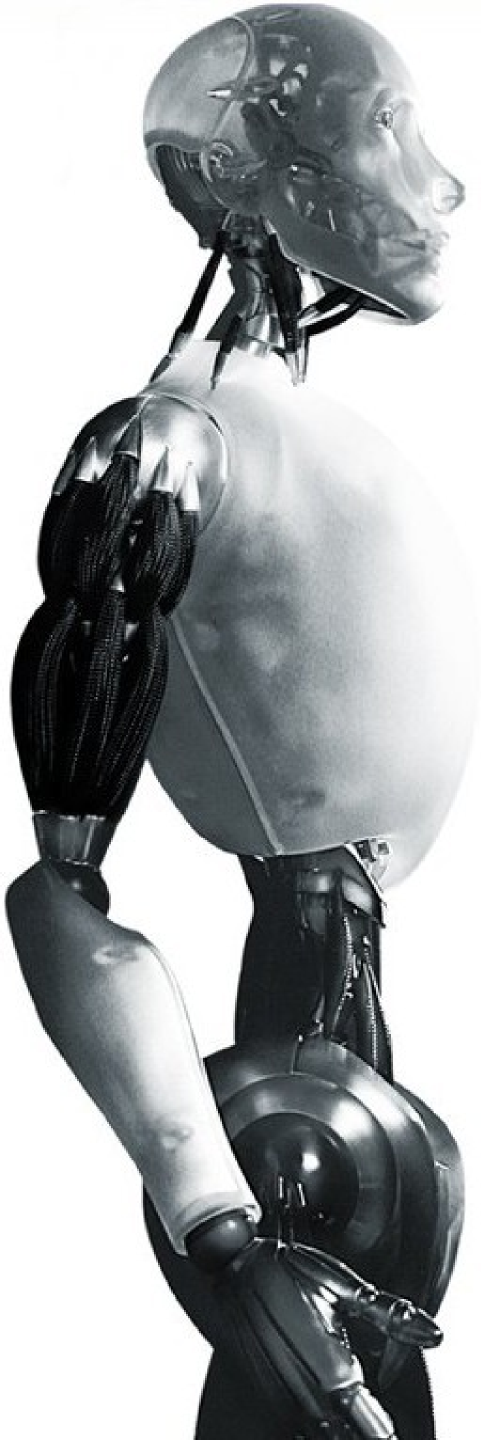


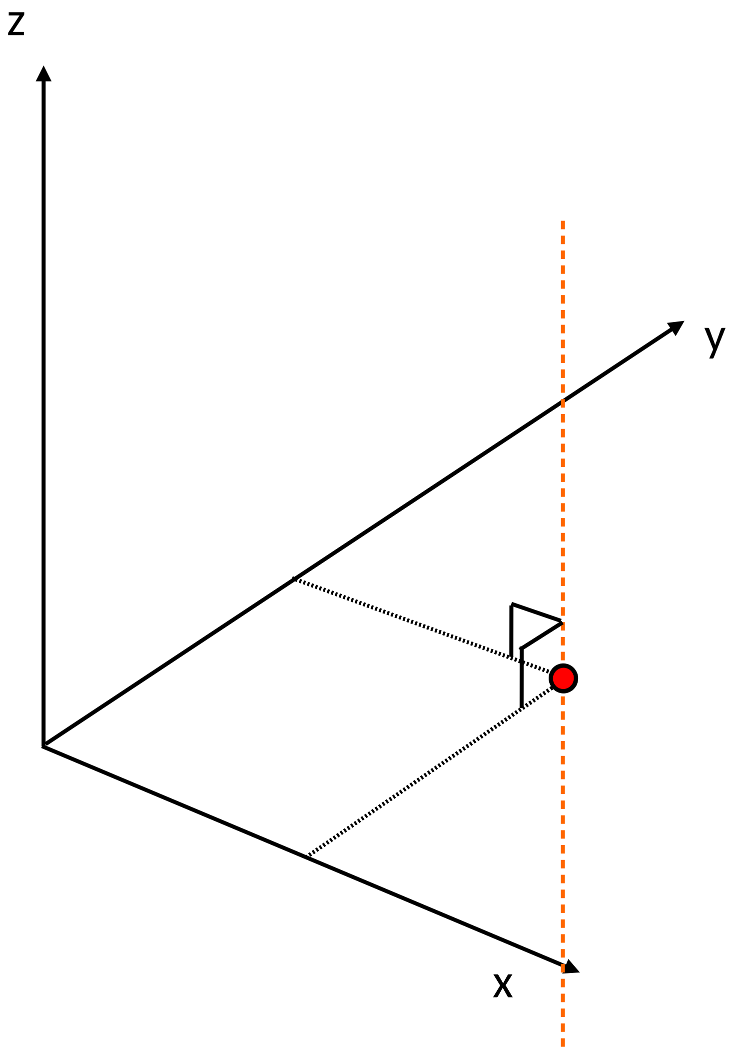
Robotics

Matrix Inverses

TU Berlin

Oliver Brock

Nullspace Intuition



point in x/y plane

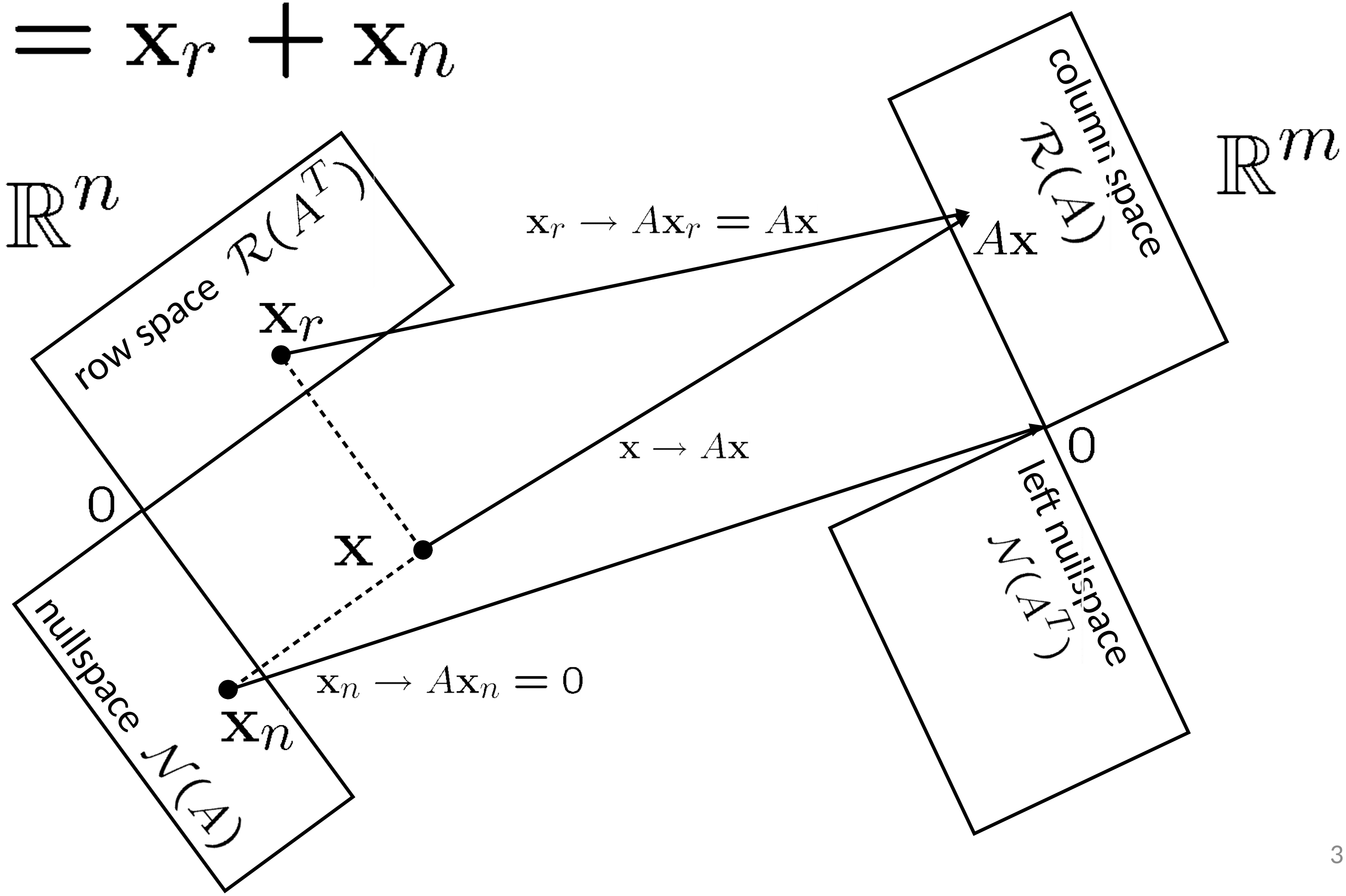
adding z component
does not change point
with respect to x/y plane

orthogonality

z-direction is nullspace
with respect to x/y plane

Fundamental Theorem of Linear Algebra

$$\mathbf{x} = \mathbf{x}_r + \mathbf{x}_n$$



Two-Sided Inverse

$$r = m = n$$

$$A^{-1}A = I = AA^{-1}$$

Left Inverse

$$r = n < m$$

$$\underbrace{(A^{\mathsf{T}}A)^{-1}_{n\times n}\,A^{\mathsf{T}}_{n\times m}}_{\text{left inverse}\,(n\times m)}\,A_{m\times n} = I_{n\times n}$$

Right Inverse

$$r = m < n$$

$$A_{m\times n}\ \underbrace{A^{T}_{n\times m}\,(AA^{T})^{-1}_{m\times m}}_{\text{right inverse}(n\times m)} = I_{m\times m}$$

Generalized or Pseudo Inverse

$$r < m, n$$

inverse of square matrix A:

$$\left\{A^{-1} \mid A = A \cdot A^{-1} \cdot A\right\}$$

uniquely defined

inverse for rectangular matrix A:

$$\left\{A^{+} \mid A = A \cdot A^{+} \cdot A\right\}$$

infinitely many!

Computing a Pseudoinverse

$$A = U\Sigma V^{\mathsf{T}}$$

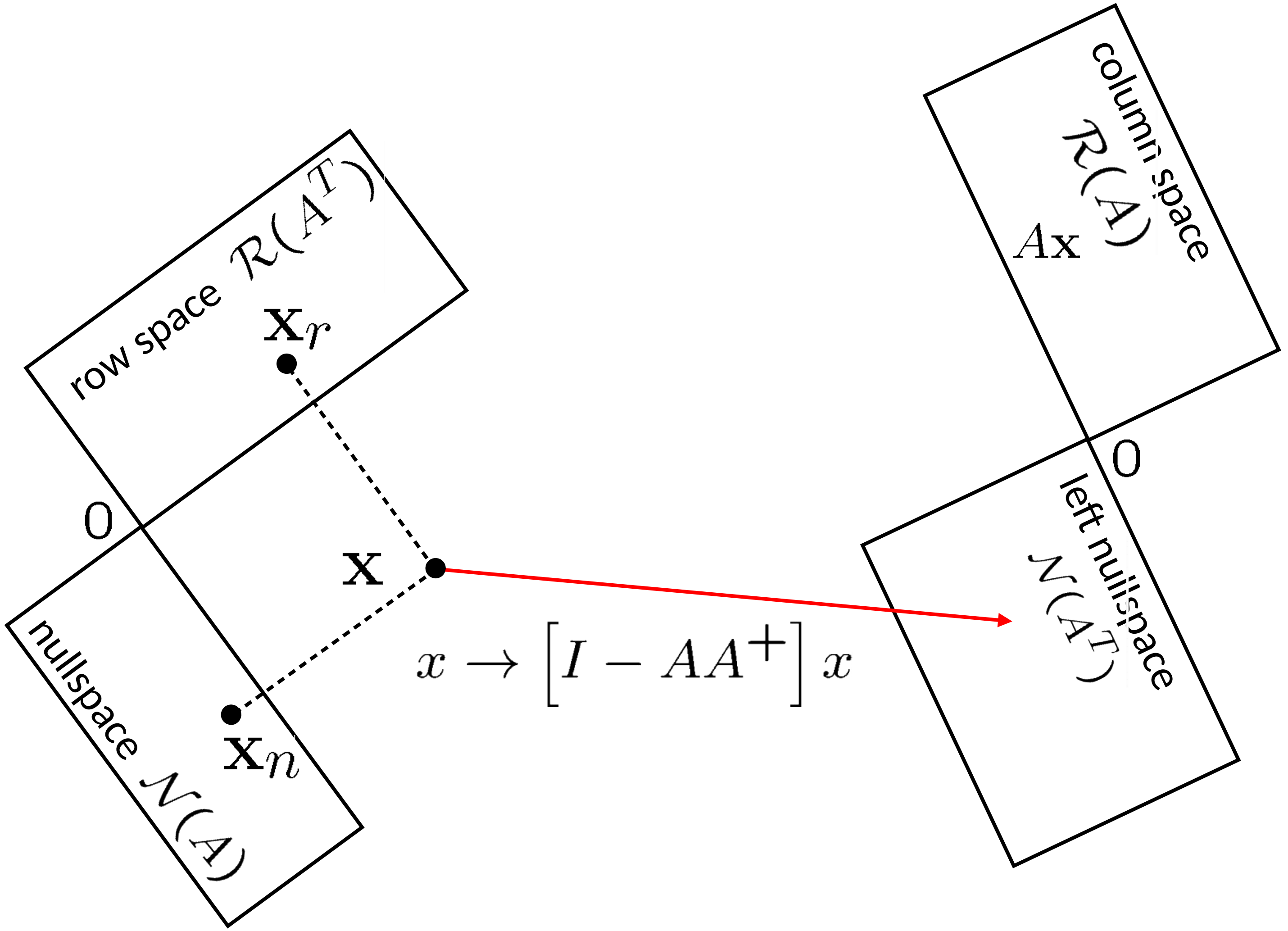
$$A^{+} = V\Sigma^{+}U^{\mathsf{T}}$$

Dynamically Consistent Pseudoinverse

$$J(\mathbf{q})A^{-1}(\mathbf{q})\left[I - J^T(\mathbf{q})J^{\#^T}(\mathbf{q})\right]\mathbf{\Gamma}_0 = 0.$$

$$\bar{J} = A^{-1} J^T \Lambda$$

Nullspace Mapping



for rectangular matrices m $\times$ n

Exploiting Redundancy

$$\tau = J^T(\mathbf{q})\ \mathbf{F} + \underbrace{\left[I - J^T(\mathbf{q})\ J^{\#^T}(\mathbf{q}) \right]}_{\text{maps into the left nullspace of } J^T}\ \tau_0$$

Dynamically Consistent Inverse

$$\bar{J} = M^{-1}(\mathbf{q})\ J^T(\mathbf{q})\ \Lambda(\mathbf{q})$$

minimizes instantaneous kinetic energy $\rightarrow$ “least motion”

Λ can be seen as pseudo kinetic energy matrix