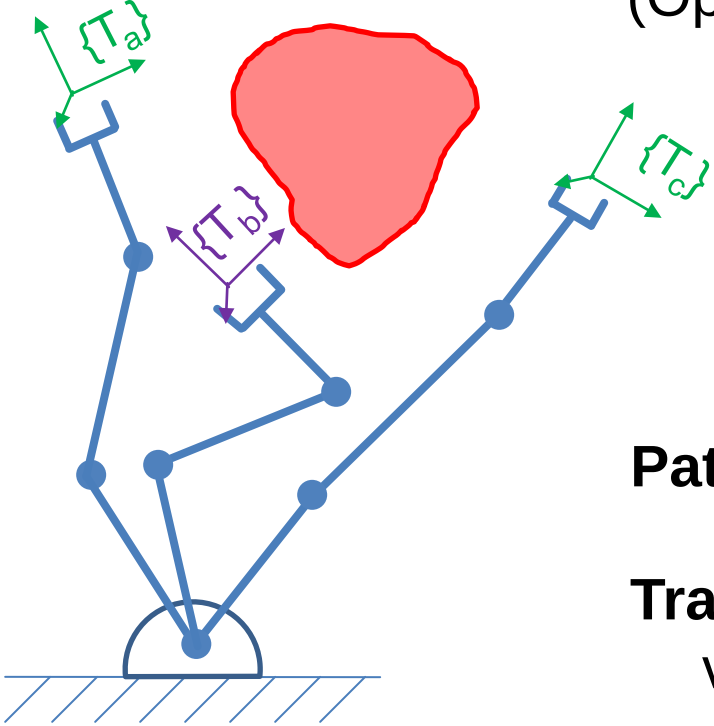
Trajectory Generation

Technische Universität Berlin

The Problem

Move the manipulator from an initial position $\{T_a\}$ to a desired final position $\{T_c\}$



(Optional: through some via point $\{T_b\}$)

Path points: initial, final and via points

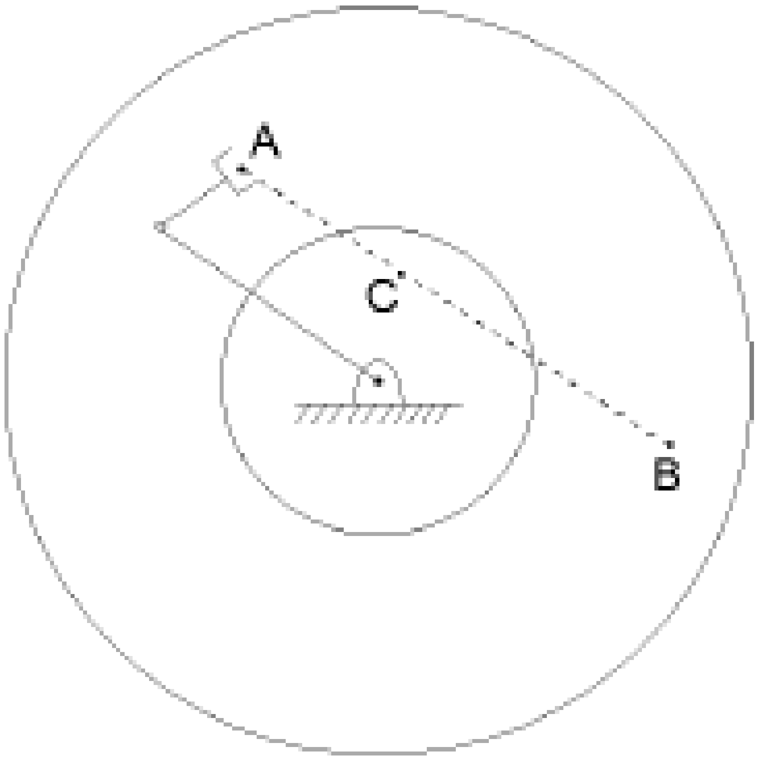
Trajectory: time history of position, velocity and acceleration for each DOF

State Spaces

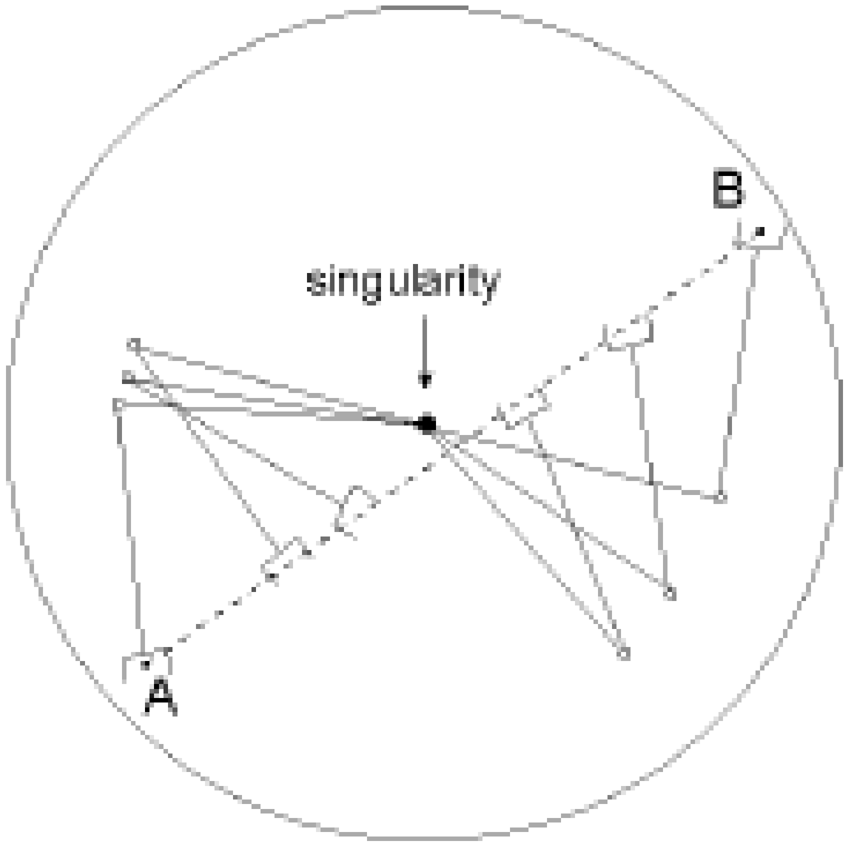
- Joint/Configuration Space
  - No problems with kinematic singularities
  - Less calculations
  - Cannot track shapes (e.g., a straight line)

- Operational/Cartesian Space
  - Can track shapes
  - *BUT*: singularities, more expensive at run time, …

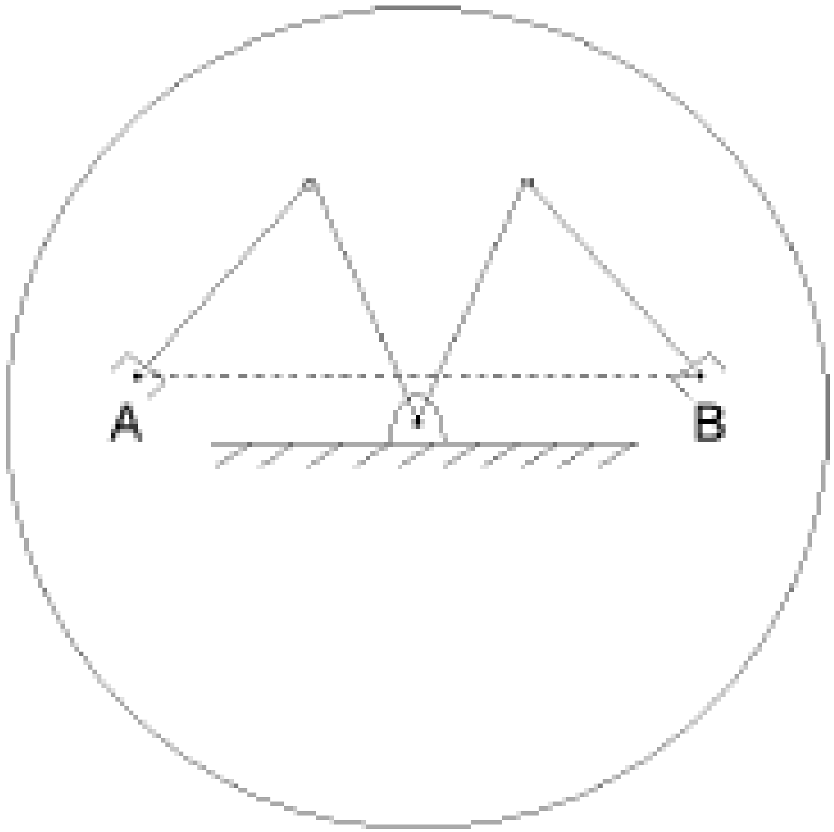
Unreachable Intermediate Points



Kinematic Singularities

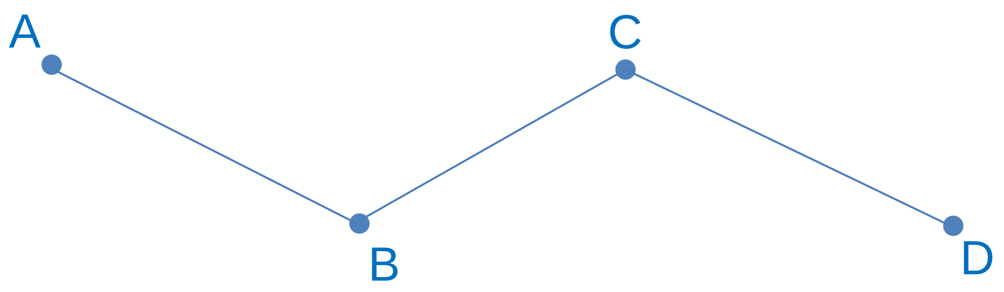


Different Joint Space Solutions

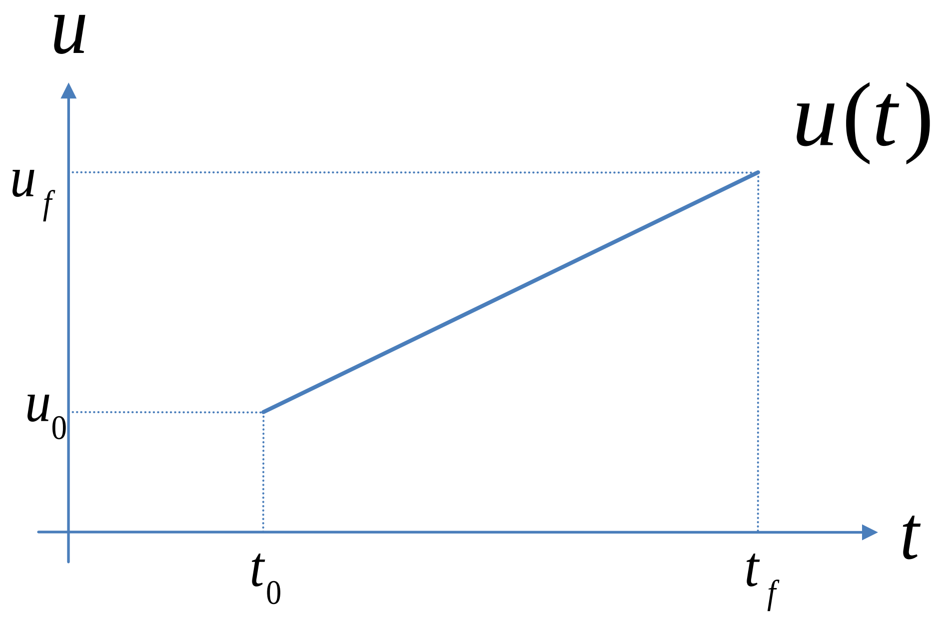


Candidate Curves

Linear



Linear Interpolation



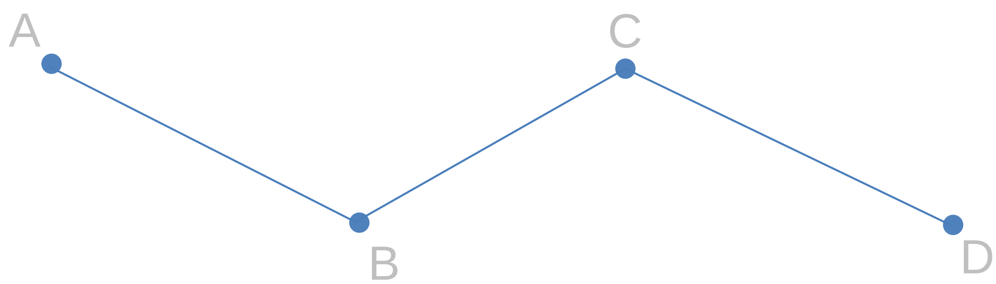
$$u(t) = a_0 + a_1 t$$

Two conditions: $u(0) = u_0 \qquad u(t_f) = u_f$

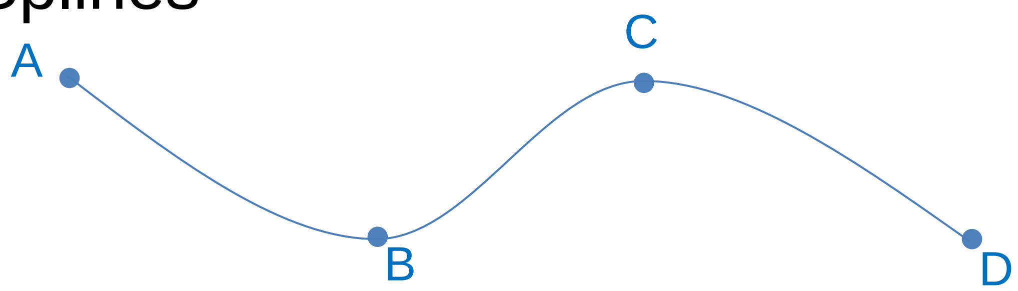
→ No control over velocities: discontinuities at beginning and end of motion require infinite acceleration!

Candidate Curves

Linear

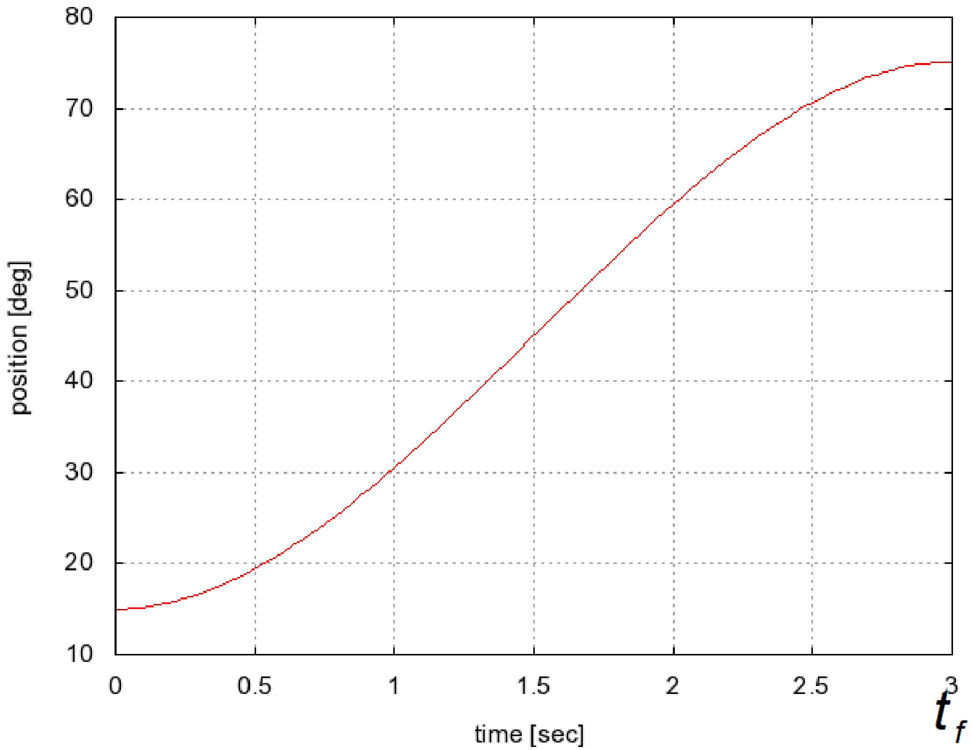


Polynomials/Splines



Cubic Splines

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

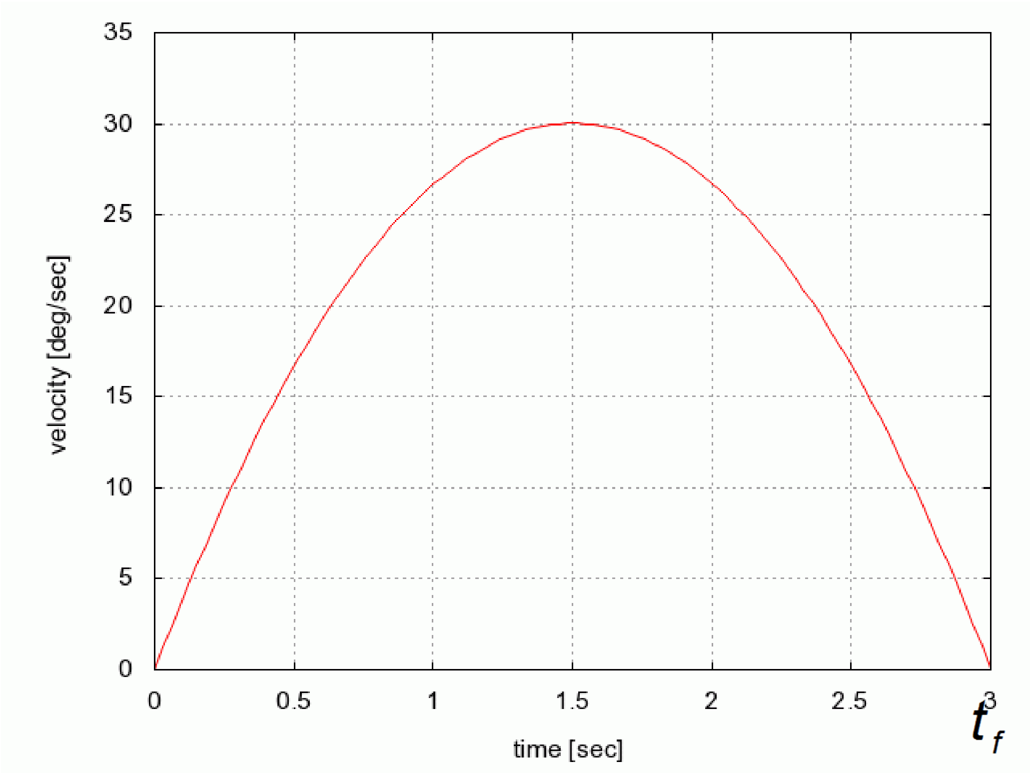


Initial Conditions:

$$u(0) = u_0 \qquad u(t_f) = u_f$$

Cubic Splines

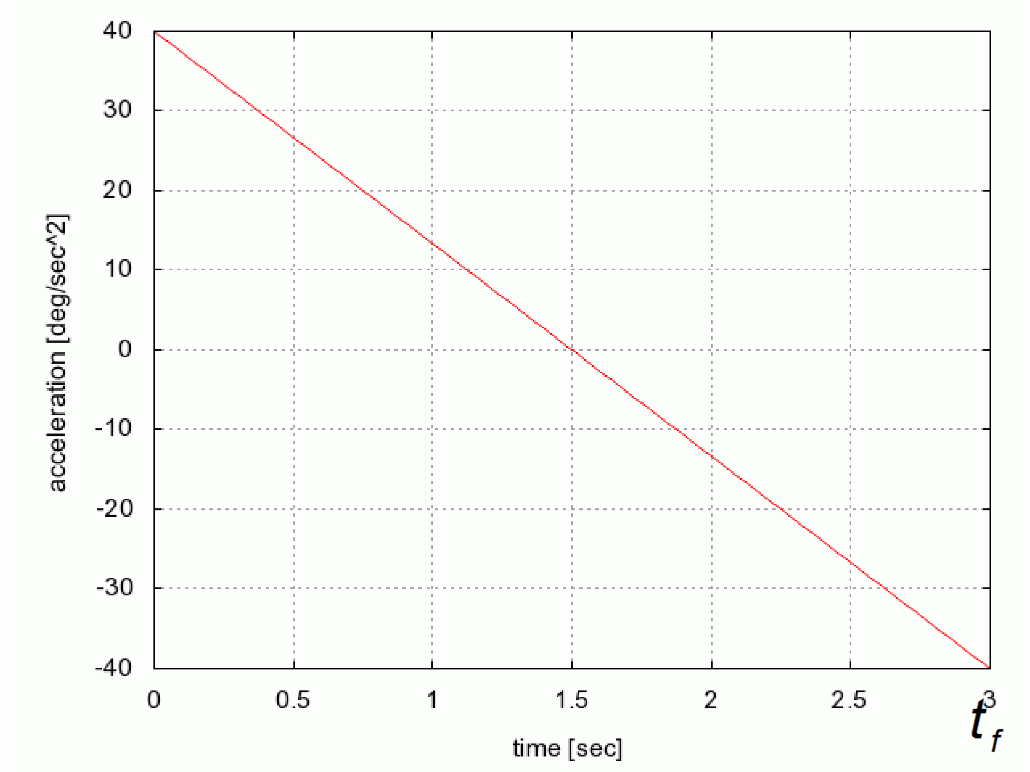
$$\dot{u}(t) = a_1 + 2a_2 t + 3a_3 t^2$$



Initial Conditions: $\dot{u}(0) = 0$ $\dot{u}(t_f) = 0$

Cubic Splines

$$\ddot{u}(t) = 2a_2 + 6a_3 t$$



→ No control over acceleration

(use higher order polynomials (Quintics, Septics, ...))

Cubic Splines

$$u(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Four equations: $u(0) = u_0 \quad u(t_f) = u_f \quad \dot{u}(0) = 0 \quad \dot{u}(t_f) = 0$

Four unknowns: a_0, a_1, a_2, a_3

$$\Rightarrow u(t) = \underbrace{u_0}_{a_0} + \underbrace{\left(\frac{3}{t_f^2}\right)(u_f - u_0)}_{a_2} t^2 - \underbrace{\left(\frac{2}{t_f^3}\right)(u_f - u_0)}_{a_3} t^3$$

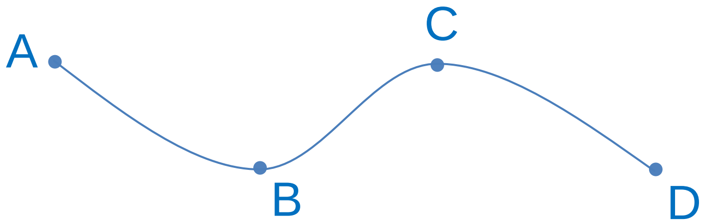
Including Via Points (1)

Concatenate cubic splines, e.g. for including one via point:

$$u_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$u_1(0) = u_0 \qquad u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0 \qquad \dot{u}_1(t_{via}) = \dot{u}_{via}$$



$$u_2(t) = b_0 + b_1\left(t - t_{\mathrm{via}}\right) + b_2\left(t - t_{\mathrm{via}}\right)^2 + b_3\left(t - t_{\mathrm{via}}\right)^3$$

$$u_2(t_{via}) = u_{via} \qquad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{via}) = \dot{u}_{via} \qquad \dot{u}_2(t_f) = \dot{u}_f$$

Including Via Points (2)

$$u_1(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

$$u_1(0) = u_0 \qquad u_1(t_{via}) = u_{via}$$

$$\dot{u}_1(0) = \dot{u}_0 \qquad \dot{u}_1(t_{via}) = \dot{u}_{via}$$

$$\Rightarrow a_0 = u_0 \qquad a_2 = \frac{3}{t_{via}^2}(u_{via} - u_0) - \frac{2}{t_{via}}\dot{u}_0 - \frac{1}{t_{via}}\dot{u}_{via}$$

$$a_1 = \dot{u}_0 \qquad a_3 = -\frac{2}{t_{via}^3}(u_{via} - u_0) + \frac{1}{t_{via}^2}(\dot{u}_0 + \dot{u}_{via})$$

Including Via Points (3)

$$u_2(t) = b_0 + b_1(t - t_{\text{via}}) + b_2(t - t_{\text{via}})^2 + b_3(t - t_{\text{via}})^3$$

$$u_2(t_{via}) = u_{via} \qquad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{via}) = \dot{u}_{via} \qquad \dot{u}_2(t_f) = \dot{u}_f$$

$$\Rightarrow b_0 = u_{via} \qquad b_2 = \frac{3}{(t_f - t_{via})^2}(u_f - u_{via}) - \frac{2}{t_f - t_{via}}\dot{u}_{via} - \frac{1}{t_f - t_{via}}\dot{u}_f$$

$$b_1 = \dot{u}_{via} \qquad b_3 = -\frac{2}{(t_f - t_{via})^3}(u_f - u_{via}) + \frac{1}{(t_f - t_{via})^2}(\dot{u}_{via} + \dot{u}_f)$$

How to choose the velocity at the via point?

How To Choose Velocities At Via Points

$$u_2(t) = b_0 + b_1\left(t - t_{\text{via}}\right) + b_2\left(t - t_{\text{via}}\right)^2 + b_3\left(t - t_{\text{via}}\right)^3$$

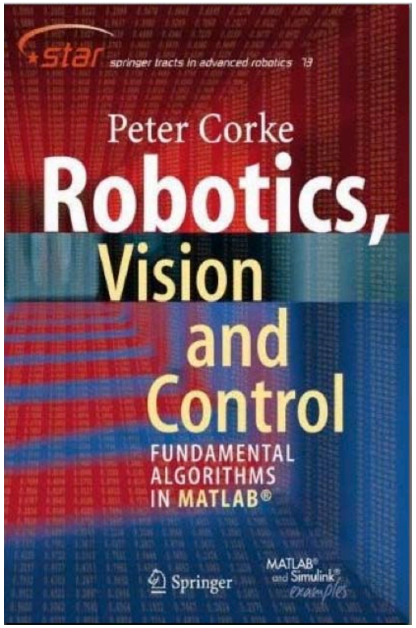
$$u_2(t_{via}) = u_{via} \qquad u_2(t_f) = u_f$$

$$\dot{u}_2(t_{via}) = \dot{u}_{via} \qquad \dot{u}_2(t_f) = 0$$

1. Let user specify
2. Use a heuristic (see assignment)
3. Alter the boundary conditions:
   Remove velocity constraints and
   force acceleration and velocity to be continuous

Recommendation For Reading

- You find a brief tutorial on how to solve equation systems with Matlab in this textbook:

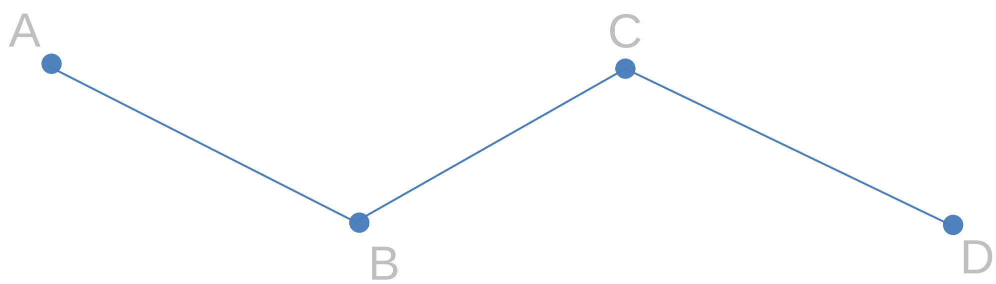


Peter Corke:
Robotics, Vision and Control

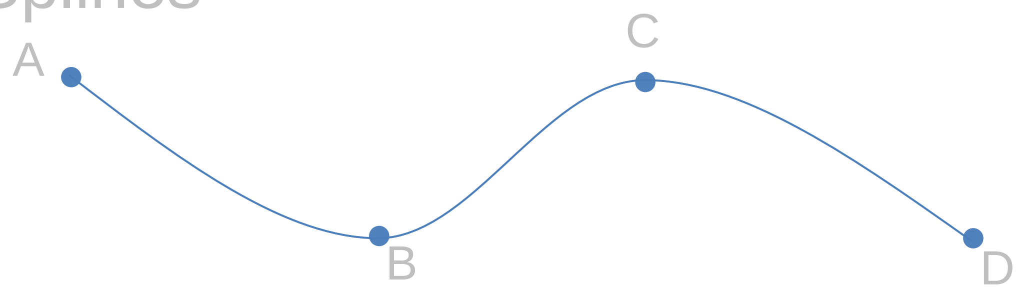
(All the topics of this class are also covered in the Craig textbook)

Candidate Curves

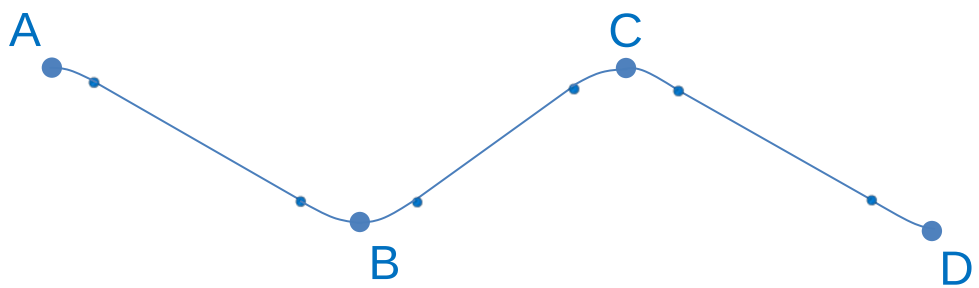
Linear



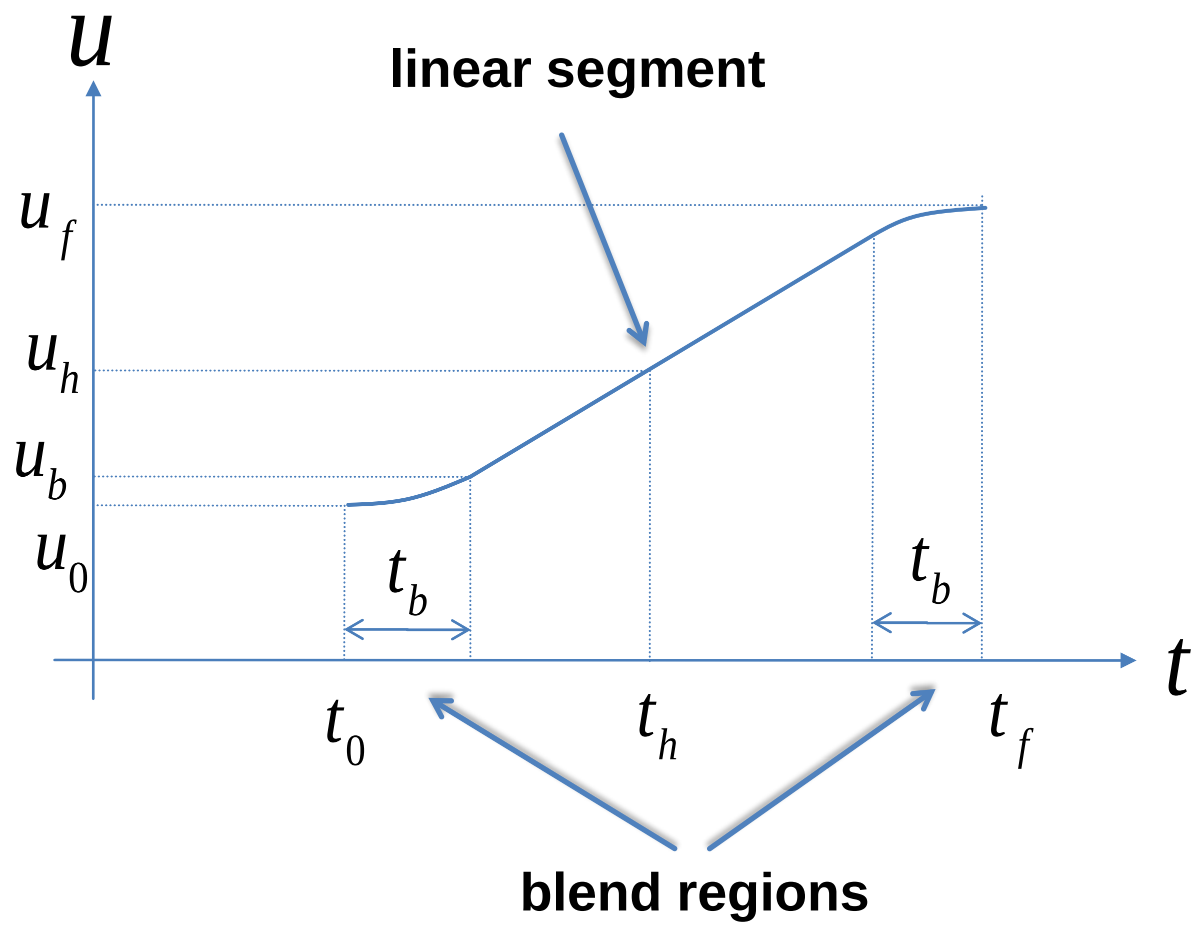
Polynomials/Splines



Linear with polynomial blends



Linear with Parabolic Blends (1)



Linear with Parabolic Blends (2)

- Idea: blends with constant acceleration

$$u(t) = \tfrac{1}{2}\ddot{u}t^2 + u_0$$

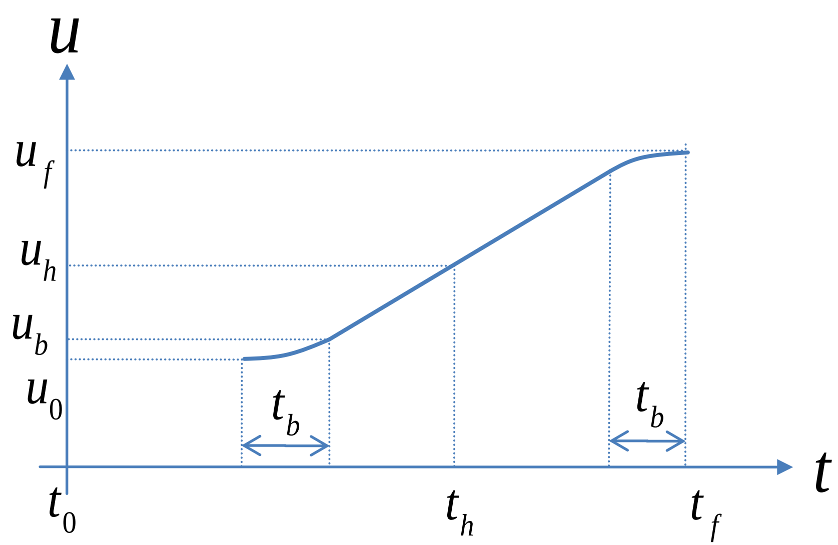
  - Velocity at transition t_b:

$$\ddot{u}t_b = \frac{u_h - u_b}{t_h - t_b}$$

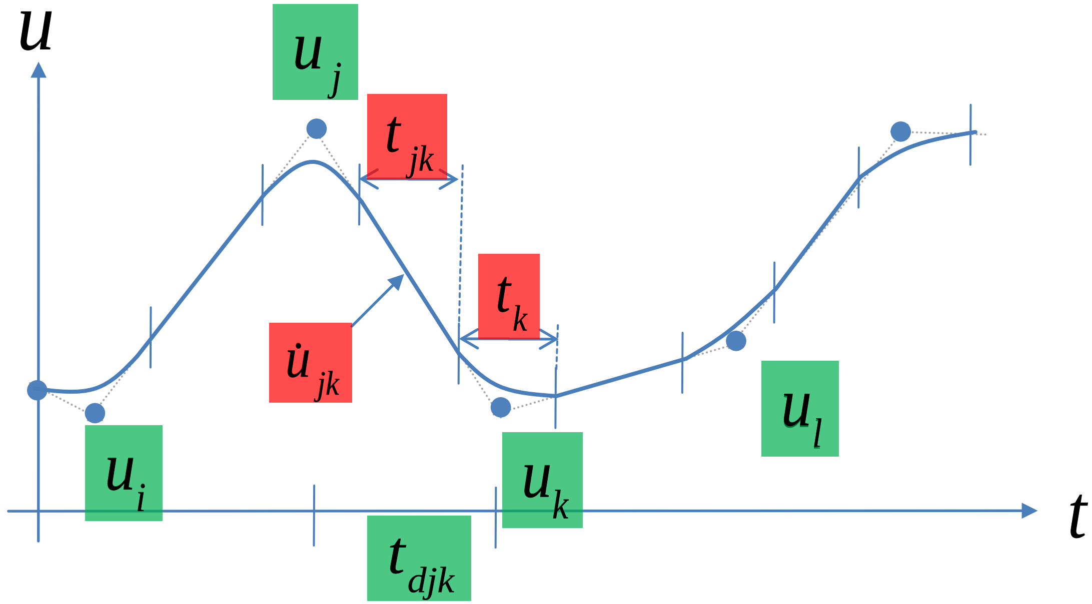
  - Distance u_b:

$$u_b = u_0 + \tfrac{1}{2}\ddot{u}t_b^2$$

  - Solve for unknowns



Optional: Including Via Points



Given:

$$u, t_{d**}, |\ddot{u}|$$

Calculate:

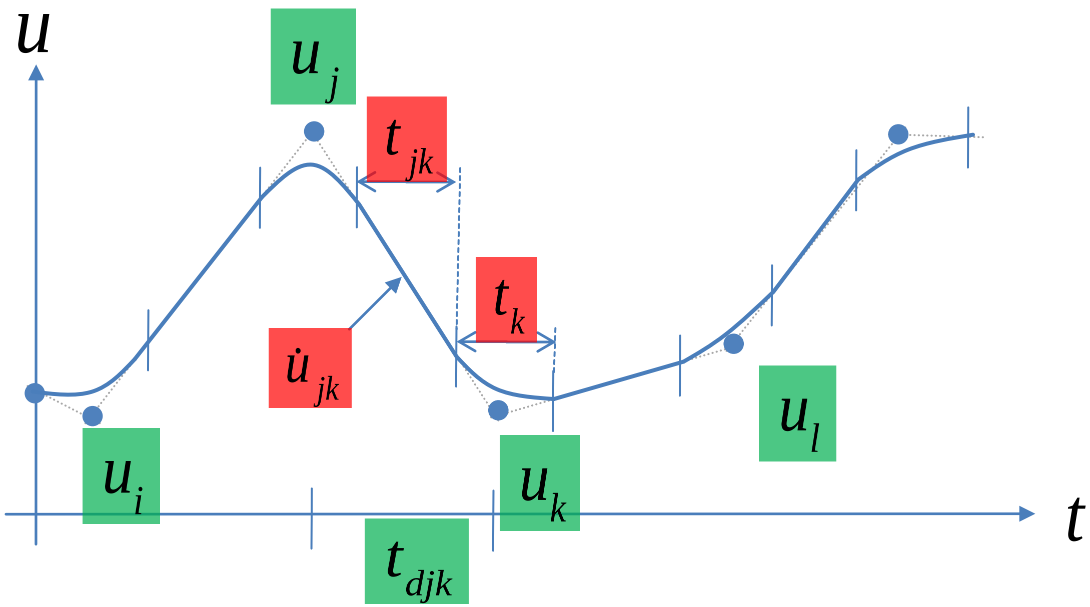
$$\dot{u}_{jk} = \frac{u_k - u_j}{t_{djk}}$$

$$t_k = \frac{\dot{u}_{kl} - \dot{u}_{jk}}{\ddot{u}_k}$$

$$\ddot{u}_k = SGN(\dot{u}_{kl} - \dot{u}_{jk})|\ddot{u}_k|$$

$$t_{jk} = t_{djk} - \tfrac{1}{2} t_j - \tfrac{1}{2} t_k$$

Optional: Including Via Points



Given:

$$u, t_{d**}, |\ddot{u}|$$

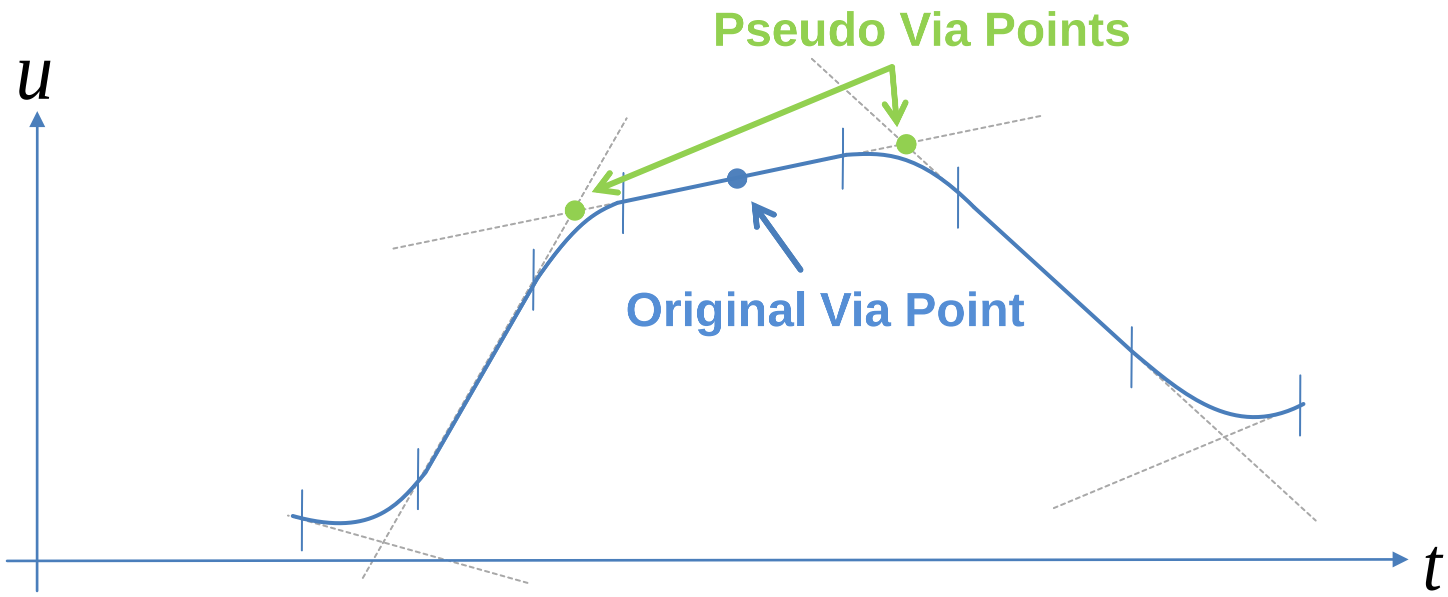
Calculate first segment (similar for last):

$$\dot{u}_{12} = \frac{u_2 - u_1}{t_{d12} - \frac{1}{2}t_1} \qquad t_1 = t_{d12} - \sqrt{t_{d12}^2 - \frac{2(u_2 - u_1)}{\ddot{u}_1}}$$

$$\ddot{u}_k = SGN(u_2 - u_1)|\ddot{u}_1| \qquad t_{12} = t_{d12} - t_1 - \frac{1}{2}t_2$$

Optional: How to pass *exactly* through a via point?

- Replace it by two pseudo via points



- Use high acceleration
- Repeat the via point (if we want to stop there)