Confidence-Based Adaptive k-Nearest Neighbors

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The Problem With Standard k-NN

- Static Hyperparameter: Uses one fixed k for all data points.
- Rigidity: Cannot adapt to varying local densities or noise.
- Overfitting: Small k is too sensitive to local outliers.
- Oversmoothing: Large k blurs class boundaries and minority groups.
- "One Size Fits All": Ignores that different regions need different neighborhood sizes.
- Noise Vulnerability: Blindly accepts "fake neighbors" in high dimensions.

Related Works

- Balsubramani et al. [1]: Established mathematical theory for adaptive stopping rules.
- Alexandre et al. [2]: Used local data curvature to adjust neighborhood size.
- Shiliang & Rongqing [3]: Assigned k based on the nearest training neighbor's optimal value.
- Wang et al. [4]: Improved accuracy by adapting the distance metric itself.
- Murrugarra-Llerena et al. [5]: Applied iterative graphs where difficult points get higher k.

Our Solution

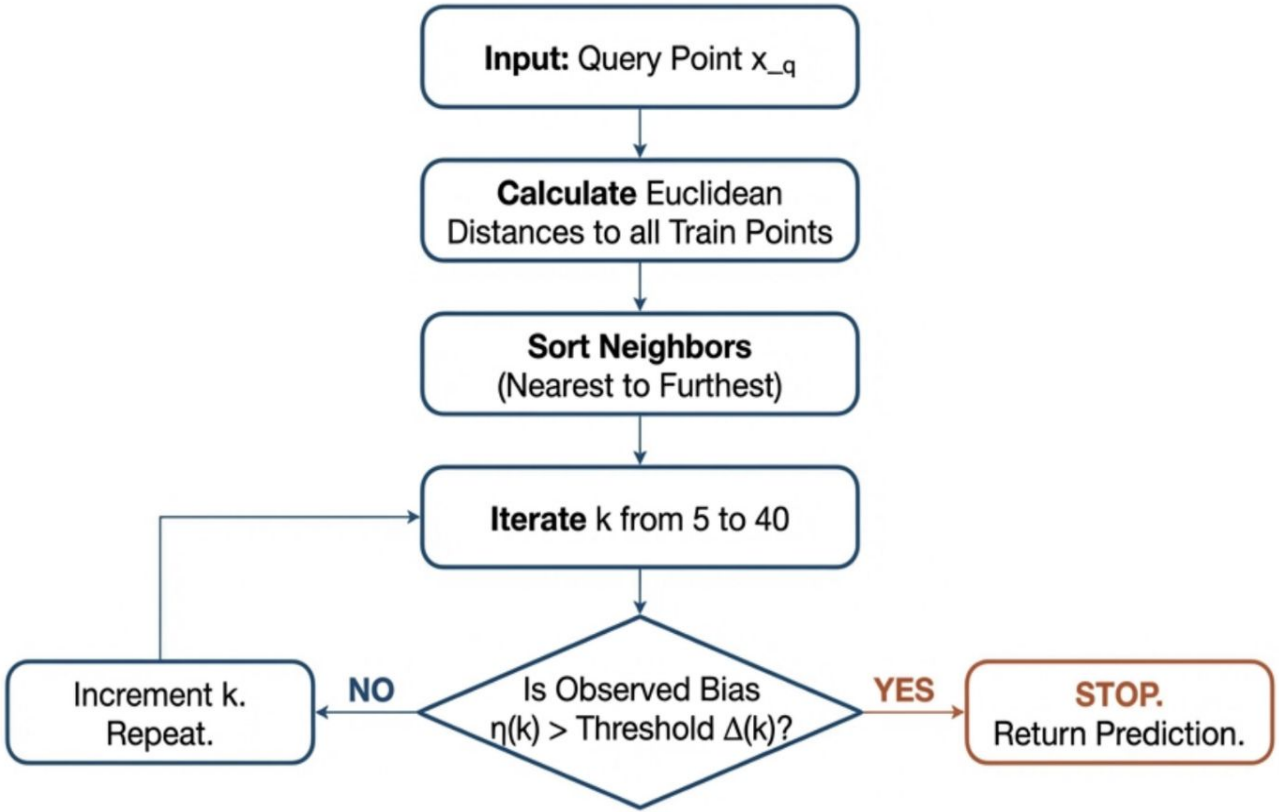
- Core Philosophy
  - Treats the classification of each test point as a unique statistical problem
  - Replaces "Fixed Neighbor Count" with "Fixed Confidence Level".
- Mechanism
  - Dynamically selects the optimal k for each query point during inference
  - Expansion Rule: The algorithm expands the neighborhood radius until the purity of the majority class exceeds a calculated statistical threshold.
- Key Advantages
  - Noise Tolerance: Automatically expands k in noisy regions to average out errors.
  - Precision: Stops at small k for clean, distinct data points to preserve boundaries.

Methodology

- The Logic:
  - Stop increasing k when Observed Bias > Dynamic Threshold.
- Observed Bias:
  - The percentage of the majority class in the current neighborhood (e.g., 4 out of 5 neighbors = 80%).
- Dynamic Threshold: A calculated bar we must clear to be "confident."
  - The algorithm expands the neighborhood radius until the purity of the majority class exceeds a calculated limit.
  - As k gets larger, the bar gets lower.
- Parameters used:
  - Strictness (C) = 0.7
  - Base Prior = 1/3 (random guessing for 3 classes).

$$\Delta(k) = \text{Base Prior} + C \cdot \sqrt{\frac{\ln(n)}{k}}$$

Methodology: Algorithm Implementation



Fallback: If loop finishes (k=40) without condition met, predict using majority vote at k_max.

Dataset & Preprocessing

- UCI Wine Dataset: 178 samples across 3 distinct cultivars (classes).
- Features: 13 continuous chemical attributes
    - Examples: total phenols, flavonoids, magnesium
- Z-Score Normalization: Standardized all features to prevent magnitude bias.
- Equal Weighting: Ensures high-range features like Proline do not dominate distance.

Experimental Design

- Validation Protocol: 70/30 train-test split for all trials.
- Reproducibility: Used fixed random seed 97 for direct comparability.
- Five Testing Conditions:
    - 1. Baseline: Clean, standardized data.
    - 2. Gaussian Noise: Simulated broken sensor readings.
    - 3. Irrelevant Features: Added noise columns to test dimensionality.
    - 4. Feature Outliers: Massive geometric distortions in specific features.
    - 5. Class Imbalance: Artificial scarcity in minority classes.

Results: Baseline (Clean Data)

- Condition: Standard Wine dataset.
- Accuracy: 96.30% vs. 92.59% baseline.
- Winner: Adaptive (+3.70%).
- Behavior: Expanded k at boundaries for robust consensus.
- Efficiency: Stopped at small k for clear points.

Algorithm	Accuracy
Standard k-NN ($k = 5$)	92.59%
Adaptive k-NN	**96.30%**

Table 1: Baseline Performance on Clean Data

$$\begin{bmatrix} 18 & 0 & 0 \\ 1 & 18 & 2 \\ 0 & 1 & 14 \end{bmatrix}$$

(a) Standard k-NN

$$\begin{bmatrix} 18 & 0 & 0 \\ 1 & 19 & 1 \\ 0 & 0 & 15 \end{bmatrix}$$

(b) Adaptive k-NN

Results: Noisy Features

- Gaussian Noise: Injected random noise (sigma=3) to simulate broken sensors.
- Performance Gap: Adaptive k-NN (68.52%) outperformed Standard k-NN (62.96%).
- Key Finding: Achieved a +5.56% improvement by expanding the search radius.
- Mechanism: Increased neighborhood size to average out and cancel high levels of random noise.

Algorithm	**Accuracy**
Standard k-NN ($k = 5$)	62.96%
Adaptive k-NN	**68.52%**

Table 2: Performance with Gaussian Noise

$$\begin{bmatrix} 14 & 1 & 3 \\ 3 & 16 & 2 \\ 1 & 10 & 4 \end{bmatrix}$$

(a) Standard k-NN

$$\begin{bmatrix} 15 & 0 & 3 \\ 1 & 17 & 3 \\ 0 & 10 & 5 \end{bmatrix}$$

(b) Adaptive k-NN

Results: Irrelevant Features (Dimensionality)

- Irrelevant Features (Exp 3): Appended 8 columns of pure random noise.
- Performance Gap: Adaptive k-NN (98.15%) vs. Standard k-NN (90.74%).
- Key Finding: Largest performance gain of the study at +7.41% improvement.
- Mechanism: Confidence-based statistical check rejected "fake" neighbors caused by noise dimensions.
- Reliability: Only made predictions when a local majority was statistically significant.

Algorithm	Accuracy
Standard k-NN ($k = 5$)	90.74%
Adaptive k-NN	**98.15%**

Table 3: Performance with Irrelevant Features

$$\begin{bmatrix} 18 & 0 & 0 \\ 2 & 17 & 2 \\ 0 & 1 & 14 \end{bmatrix}$$

(a) Standard k-NN

$$\begin{bmatrix} 18 & 0 & 0 \\ 1 & 20 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

(b) Adaptive k-NN

Results: Feature Outliers

- Accuracy: Adaptive (96.30%) beat Standard (94.44%).
- Winner: Adaptive (+1.85%).
- Mechanism: Expanded k to dilute outlier influence.
- Purity Check: Monitored class purity to trigger neighborhood growth.

Algorithm	Accuracy
Standard k-NN ($k = 5$)	94.44%
Adaptive k-NN	**96.30%**

Table 4: Performance with Feature Outliers

$$\begin{bmatrix} 18 & 0 & 0 \\ 2 & 18 & 1 \\ 0 & 0 & 15 \end{bmatrix}$$

(a) Standard k-NN

$$\begin{bmatrix} 18 & 0 & 0 \\ 1 & 19 & 1 \\ 0 & 0 & 15 \end{bmatrix}$$

(b) Adaptive k-NN

Results: High Class Imbalance

- Condition: Removed 75% of Class 1 samples.
- Accuracy: Standard (100%) beat Adaptive (94.74%).
- Winner: Standard k-NN (+5.26%).
- Failure Point: Adaptive was overly cautious in sparse regions.
- Trade-off: Seeking high confidence pulled in incorrect neighbors.

Algorithm	**Accuracy**
Standard k-NN ($k = 5$)	**100.00%**
Adaptive k-NN	94.74%

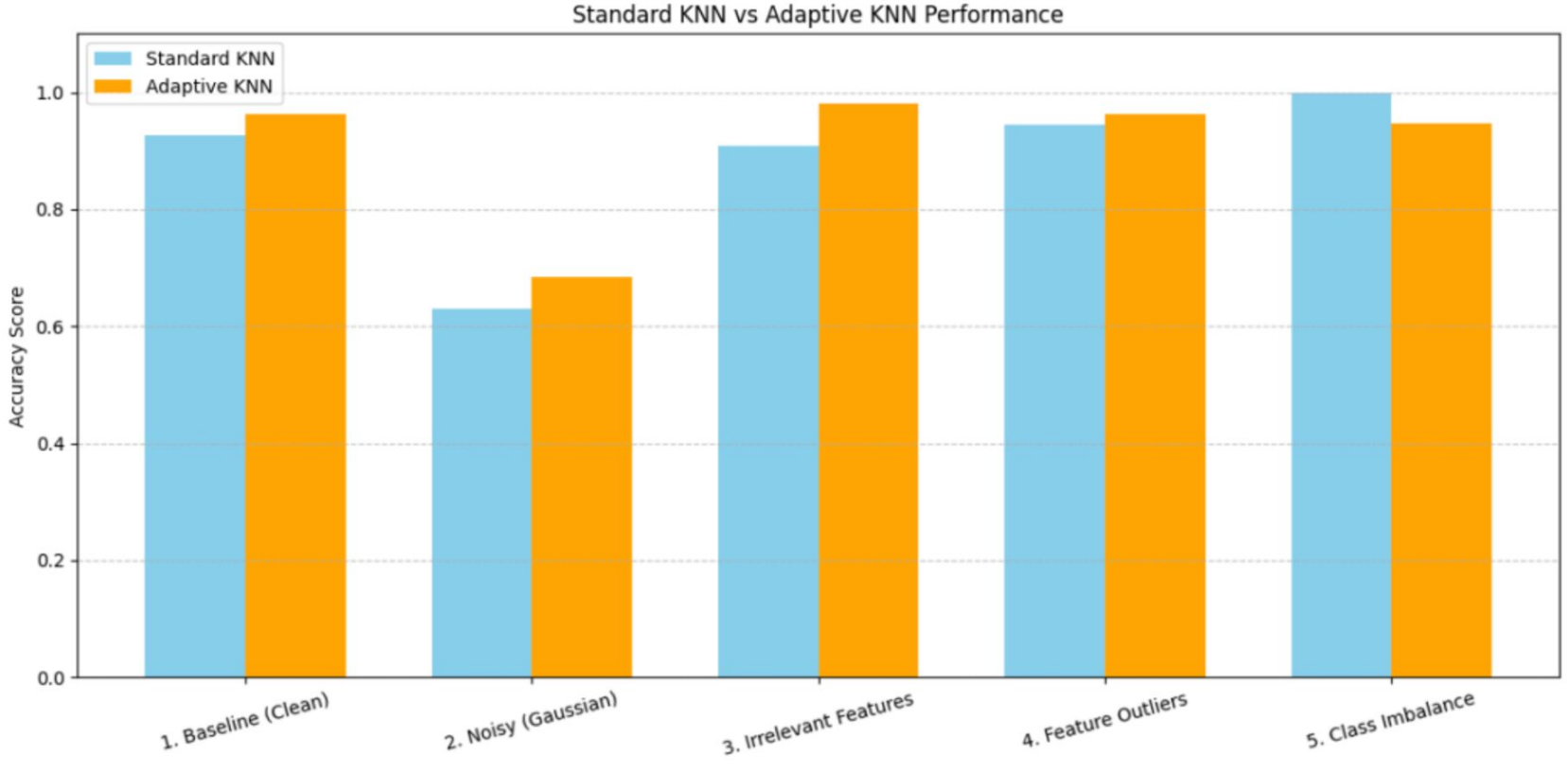
Table 5: Performance with Class Imbalance

$$\begin{bmatrix} 18 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

(a) Standard k-NN

$$\begin{bmatrix} 18 & 0 & 0 \\ 1 & 3 & 1 \\ 0 & 0 & 15 \end{bmatrix}$$

(b) Adaptive k-NN


Standard KNN vs Adaptive KNN Performance
Standard KNN
Adaptive KNN
Accuracy Score
1.0
0.8
0.6
0.4
0.2
0.0
1. Baseline (Clean)
2. Noisy (Gaussian)
3. Irrelevant Features
4. Feature Outliers
5. Class Imbalance

Conclusion

- Conclusion:
  - We successfully implemented a dynamic confidence rule.
  - The Adaptive algorithm is superior in "messy" environments (noise, outliers, extra dimensions).
  - It avoids overfitting local noise while preventing over smoothing in clean regions.
- Future Work:
  - Optimize the strictness parameter (C) dynamically during training.
  - Test on different distances (e.g., Manhattan & Minkowski distance)
  - Apply to larger datasets like MNIST or other real-world datasets to test trade-offs and performance.