# TECHNOTHLON 2020 JUNIORS THEME - TOY STORY

## **SECTION A**

Andy's birthday party has just been moved up by a week. All the toys in Andy's room are worried about being replaced by fancy new toys. Sheriff Woody, Andy's favourite toy, and the other's are waiting for the party to get over and meet the new toys.

# **QUESTION 1**

At the party, Andy's mother hosts a game. Since everyone didn't get Andy a present, she removes the name tags from all the gifts, so all 121 of Andy's friends either lie or tell the truth about getting Andy a gift. Now, everyone shakes hands with each other. Now, Andy's mom asks each person "How many truth-tellers did you shake hands with?" Each child gave a different answer ranging from 0 to 121. Andy must answer the number of liars in the party to get a surprise gift.

a) 1 b) 121 c) 61 d) 62

The surprise gift turns out to be the brand new Buzz Lightyear. All the toys are quickly impressed by Buzz's exciting features, but what they really want to know is how smart and witty Buzz is. So Woody, Mr. Potato Head, and Buzz decide to play an interesting game of Chess to test Buzz's intelligence.

## **QUESTION 2**

The rules of this game are as follows.

- 1). One who wins continues to play with the resting player.
- 2). No matches end with a tie.

The results are as follows:

- 1). Mr. Potato Head played 13 matches.
- 2). Woody played 19 matches.
- 3). Buzz played 22 matches.

Who lost the 8th match?

| a) Mr. Potato | b) Buzz | c) Woody | d) Not enough |
|---------------|---------|----------|---------------|
| Head          | ·       | , ,      | Information   |

Woody starts getting jealous of the attention Buzz has been given by other toys. He decides to trap Buzz behind a desk. However, he fails and all the other toys are furious at him. But before they can do anything, Andy grabs Woody and takes him to a restaurant- Pizza Planet. Little does Woody know that Buzz also got in the car, and the two get into a huge fight, which leads them stranded. Somehow they find their way to the Pizza Planet, but end up entering the arcade section.

#### **QUESTION 3**

Trying to get to Andy, Woody and Buzz get pushed over by some running kids into a competitive crane game. Andy's neighbor Sid and another kid named John begin to play the game, the rules of which are as follows. Sid and John's virtual characters in the game are standing on a 5x10 floor (5x10 means 50 1x1 square pieces arranged in a rectangle). The two players, which are Sid's and John's virtual characters take turns cutting the bar along the division lines (lines formed between the squares) of the floor. A cut should be straight and must divide one piece in two pieces. The player who is left with a single tile wins the game and is allowed to operate the crane to pick a toy. Sid takes the first turn in the game. If both the players play optimally, who has the winning strategy?

a) Sid b) John c) Undeterminable d) Both have the same probability of winning

After playing the game for a few times, Sid gets to pick a toy from the game, and gets lucky, picking both Woody and Buzz at the same time. At Sid's house, Woody and Buzz find lots and lots of other toys, who have been destroyed by Sid over time. So Woody and Buzz convince these toys to help them escape from Sid's home.

## **QUESTION 4**

The toys tell Buzz and Woody that Sid's door is closed by Mr. Lock, which can only be opened if they answer the riddle asked by the lock correctly. The lock works on the concept of **mad** arithmetic. There are no carries in **mad** arithmetic. For each pair of **mad** digits, to Add, take the larger, but to Multiply, take the smaller. For example:

Addition and multiplication are associative and commutative and multiplication distributes over addition.

A prime in a normal world is a number that can be factored only as 1 and number itself. But instead of number 1(multiplicative identity), we need a different number in the mad world.

Are there infinitely many primes in the mad world?

Find the number of mad primes in the range from 1 to 100, both inclusive?

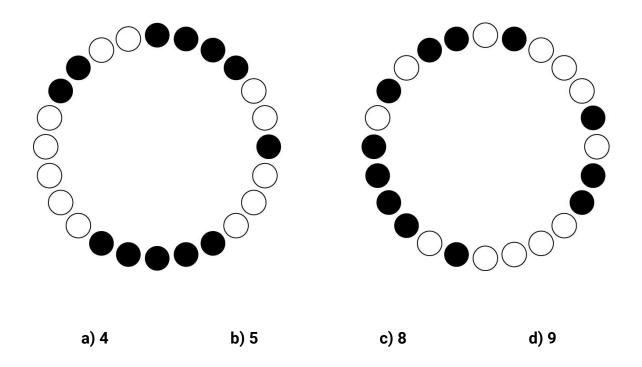
Woody and Buzz escape, only to find Sid's dog, Buster chasing them. They reach the door, only to find another lock. The lock asks them the following riddle.

### **QUESTION 5**

The white and black coins make two patterns as shown in the figures. To open the lock Woody and Buzz must convert pattern on the left to the pattern on the right by

making the following move. In one move they can pick up any coins from the pattern and place them just the next empty slot clockwise. What is the minimum number of moves for the transformation from pattern on the left to the pattern on the right? (Remember you cannot rotate the pattern)

(These are white and black pieces)



# **SECTION B (Multiple Correct type)**

Woody and Buzz make it to Andy's home safely, and Buzz's friendship strengthens with the rest of the toys, including Woody.

It's a bright Friday morning, and Andy is leaving for summer camp. Looking forward to going to the camp with his favorite toy Woody, he grabs Woody in a hurry, ripping his arm slightly. Andy leaves Woody on the shelf and goes camping, but Woody falls off from the shelf and out the window, landing in the yard where Andy's mom, Mrs. Davis, is having a yard sale!

## **QUESTION 6**

The yard sale consists of many old broken toys and other items. The greedy toy collector Al McWhiggin spots Woody and asks Mrs. Davis about buying it. Mrs. Davis refuses to say Woody is Andy's favorite toy. After much persuasion from Al, she agrees to sell Woody only if Al completes her demands. She tells Al that he must pay her in only denominations of 97, 98, and 99. Moreover, the amount Al must pay her, let's say N, should be such that any integral sum greater than N can be paid by using denominations of 97, 98, and 99 in a single exchange. How much money must Al pay to buy Woody? (Mark the correct options regarding N)

a) N is 4646 b) N has exactly 3 c) N is even d) The 2nd digit of prime factors N from left is 6

Recollecting a television commercial, Andy's toys figure out the buyer was Al McWhiggin, owner of the shop Al's Toy Barn. Buzz, Hamm the piggy bank, Mr. Potato Head, Slinky Dog, and Rex decide to go on a mission to save Andy from Al. Meanwhile, Al brings Andy to his building, which is right in front of his shop. Al drops Andy in his home and leaves, and Andy sees some other toys involved in an exciting game of chess. They are the cowgirl Jessie, the horse Bullseye and Stinky Pete.

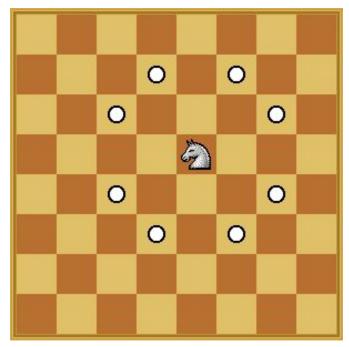
#### **OUESTION 7**

The game of chess is a bit different than the traditional one. The modified game of chess consist of just a knight and an 8\*8 chessboard is played between the two players, Jessie and Stinky Pete. The rules of the game are:

1)Standard chess rules are applied for the moves of the knight

- 2)Both players take turns alternatively to move the knight to a new square
- 3)The first player who can't move the knight to the new square losses the game (i.e. one cannot return to a square which has already been used)

The knight may be placed randomly on any square of the chessboard initially. Jessie makes the first move.



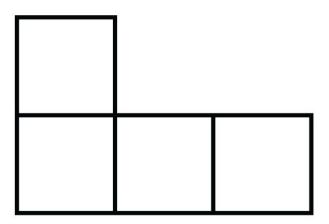
Which of the following options is correct?

- A. Jessie wins if initially, the knight is at a black square
- B. Stinky Pete wins if initially, the knight is at a black square
- C. Jessie wins if initially, the knight is at a white square
- D. Stinky Pete wins if initially, the knight is at a white square

The other toys in Al's apartment immediately recognize Woody and tell him all about his history and how they were a "gang" and the part of a famous TV show. They convince Woody to join them, and Al will sell them to a museum in Japan where they will be adored by all children. Meanwhile, Buzz and the team are walking tirelessly towards Al's shop.

## **QUESTION 8**

Buzz's team reaches the final intersection, and the intersection sign will reveal the direction of Al's toy store. The sign is in the shape of an nxn chessboard with the four corner squares of dimension 1x1 removed. Buzz and his team must figure out the exact value(s) of n for which they can cover the intersection signboard with L-tetrominoes so that the sign reveals the wanted direction.



Which of the following can be the value of n?

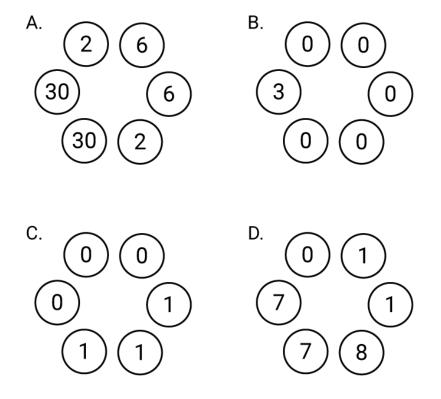
a) 7

- b) 8
- c) 10
- d) 12

Buzz and his team see Al driving from his store to his apartment, and follow him to his building.

## **QUESTION 9**

In the building, there is a lift with a numeric keyboard attached to it. Around the lift are six panels placed in a circle showing numbers, and their respective control rods. The lift works only when the numbers on all the six panels are equal. In each turn, the characters (Buzz and his team) can increase any three consecutive numbers around by 1. Which group/groups of the given six numbers (as shown in the figures), when entered on the keyboard can be made equal using the above technique (increasing 3 consecutive numbers by one) so that the lift works?



Team Buzz reaches Al's apartment in time and rescues Woody. He tells how much Andy misses him, convincing him to come back. The toys jump from the window to another building, ending up in a lab.

## **QUESTION 10**

The lab is owned by a Dr. Browne, who is studying the different mutations of the COVID virus, in hopes of making a vaccine. So far, he has learned that-

- The COVID virus has ten variations, namely COVID-1, COVID-2,...., COVID-10
- A virus-cell may only exist in one particular variation at any instant.
- Every second, all virus cells of exactly one random variation out of the ten, multiply according to a specific pathway-
  - Each virus-cell belonging to that randomly selected variation divides into 10 cells(one of each variation)

- Eg. If at t=0, one cell is in the COVID-3 variation, and all other nine variations have zero cells at t=0, and COVID-3 has been selected randomly to multiply at that instant,
- Then at t=1, COVID-3 will have zero active cells, and all other nine variations will have one cell each.

Dr. Browne wanted to time how long it takes for the collection of virus cells to reach a particular configuration.

So, Dr. Browne finally began his experiment, taking exactly one virus-cell of each variation and keeping a close eye on the changing configurations at each second.

After a while, he finally got the configuration he desired -

- Variations COVID-3,6,9 had 224 cells each
- Variation COVID-10 had 112 cells
- Variation COVID-8 had 222 cells
- Variation COVID-4 had 223 cells
- Variation COVID-1 had 168 cells
- Variation COVID-2 had 216 cells
- Variation COVID-5 had 208 cells
- Variation COVID-7 had 196 cells

But to his horror, he realized that he forgot to start his stopwatch for the experiment.

Judging from the configuration only, which of the following statements are true?

- a) The time in the clock is t = 8
- b) COVID-3 never multiplies during the experiment and COVID-7 multiplies the most times.
- c) COVID-1 and COVID-9 have the same population for the first 6 seconds
- d) The population of COVID-10 increases fourfold in the last 3 seconds

#### **SECTION C**

The toys get back to Andy's room safely before he comes back from his trip, and he is very excited to find the new toys as well.

Many years have happily passed and Woody, Buzz, Jessie, Mr. Potato head were all cherished by Andy. But now Andy has grown up and is going to college. Getting bored all alone in Andy's room, they decide to play a game.

#### **QUESTION 11**

Rex the dinosaur opens up Hamm the piggy bank and finds 2020 coins. Rex and the others thought of dividing the coins amongst themselves, with the other toys getting 2000 coins and Rex getting 20 coins. But Rex wanted all the coins to himself, so he told the other toys:

"I will divide the treasure into 2 groups with a positive integer number of coins. Then, I choose a random group and divide it into 2 other groups with a positive integer number of coins. I will continue this until there are 2020 groups in total. At any time of this process, If you can find 1000 groups with a total of exactly 2000 coins, then you can take this. But if you cannot, you get nothing."

The toys (other than Rex) definitely can take 1000 groups like that after a-th divide times whatever how Rex divides. Find the sum of digits for the smallest such a.

Andy, having outgrown his toys, keeps only Woody in his college backpack and decides to keep the rest in the attic, keeping them in a garbage bag. However, Andy's mom mistakes it for garbage and throws it out. However, the toys somehow get out and climb into the donation box. Woody couldn't leave his friends behind and sneaks into the donation box. The toys reach Sunnyside. Woody tries to take the other toys back to Andy, but they refuse and Woody leaves alone. Andy's toys are happily welcomed by the other toys at Sunnyside, and their head- Lotso the bear.

## **QUESTION 12**

The daycare toys show the new toys their lockers, which are in a long rectangular array, with three rows of N lockers each, where N is some positive integer between 400 and 450. The lockers in the top row were originally numbered from 1 to N, the middle row N+1 to 2N, and the bottom row 2N+1 to 3N, all from left to right. Now the kids at the daycare come and start playing with all the toys. Andy's toys are subjected to a very rough playtime with the toddlers, while Lotso and his other friends are safe in the other room, being played delicately with the older kids.

The toddlers also messed up the locker numbering system. Originally Mr. Potato Head's, Bullseye's, and Rex's lockers were located one in each row. When Andy's toys return to their lockers, they see that Mr. Potato Head's, Bullseye's and Rex's lockers have been changed and that each of them now has the locker number that used to belong to one of the others (from the three toys mentioned). What were their

locker numbers, assuming that all are three-digit numbers? Give your answer as the sum of digits of all the locker numbers.

So, while Lotso and his friends were safe in the other room, Andy's toys had a bad and painful afternoon with the toddlers. Buzz goes to Lotso to talk to him about moving his friends to the other room, but Lotso's friend Big Baby resets Buzz to his default settings, and traps Andy's toys in their lockers, with Buzz keeping an eye on them. Woody gets to know from some other toys that Sunnyside is a bad place, and decides to rescue his friends. He reaches Sunnyside in the day and makes a plan with the other toys on how to escape.

## **QUESTION 13**

The toys use Rex and Hamm fighting as a distraction, and Jessie is able to trap Buzz in a container. Woody and Slinky the dog must reach the Clapping Monkey Guard in the surveillance room and switch off all the cameras. But they don't know which passage in the ceiling reaches the surveillance room. So, the toys question the trapped Buzz about the passage number. The passages are numbered from 11 to 1111 (both inclusive), but the problem is the toys don't know whether Buzz is lying or telling the truth. However, Mrs. Potato Head has a special deductive power, and she can tell whether Buzz is lying or not. This is how the conversation goes:

1.q-is the number less than 500?Buzz lies2.q-is the number a perfect square?Buzz lies3.q-is the number a perfect cube?Buzz says the truth4-is the second digit 1

Mrs. Potato Head couldn't make out whether Buzz lied or told the truth. Woody and Slinky think they have got the answer. So, what number should Mrs. Potato Head guess which will lead Woody and Slinky to the right passage?

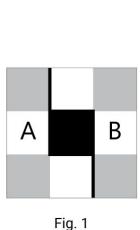
Woody and Slinky reach the Monkey Guard, subdue him and get the keys to unlock the door. Meanwhile, the other toys reset Buzz to his normal state.

#### **QUESTION 14**

The toys meet at the playground and must reach the garbage chute in order to escape. But the toys have a shortcut to the garbage chute, which works as follows. The playground is made up of 23x23 sand tiles and below one of these tiles is an underground tunnel that opens directly below the garbage chute. Under each square is a message which indicates the minimum number of steps needed to reach the square with the correct path to the chute from that square. Each step takes one from a square to another square sharing a common side. What is the minimum number of squares the toys must dig up (in the worst-case scenario) in order to bring up the underground path?

## **QUESTION 15**

The toys escape and reach the garbage truck. The inside of the truck is covered in an odd number of white and black squares, which resembles a chessboard. The ride to Andy's home is a long one, and Mr. Potato Head comes up with an interesting riddle. Suppose that the central square is first to cut out, so as to leave an even number of squares for the division. Now, it is obvious that a square three by three can only be divided in one way, as shown in Fig. 1. It will be seen that the pieces A and B are of the same size and shape and that any other way of cutting would only produce the same shaped pieces, so remember that these variations are not counted in different ways. The puzzle Mr.Potato Head proposes is to cut the 5x5 board (Fig. 2) into two pieces of the same size and shape in as many different ways as possible. One way of doing it is shown. How many different ways are there altogether? (A piece which when turned over resembles another piece is not considered to be of a different shape.)



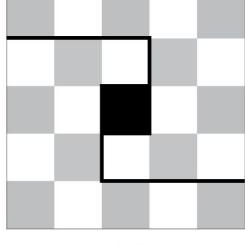


Fig. 2

#### **SECTION D**

The toys get off the garbage truck at Andy's home and reach his room just before he is about to leave for college. Andy realizes that it's time for a new kid to play with his toys. He donates his toys to Bonnie, and the toys as well as Bonnie were very happy.

One morning, Bonnie made a toy Forky with some waste materials in her class. Forky came to life but wasn't ready to admit he was a toy and kept jumping into the trash can. Bonnie and her parents go on a road trip, and she takes all her toys and Forky.

#### **QUESTION 16**

On the road trip, Bonnie gets hungry, and her mom decides to give her some peapods. However, these are magical peapods with k ( $k\ge2$ ) peas, with at least one good pea and at least one bad pea. The experiment starts at t=0. When the time changes from even to odd, initially the good peas double in number and then each bad pea eats some of the peas. When time changes from odd to even the number of good and bad peas get interchanged. For certain values of k, Bonnie observed that the number of good peas is zero at some t=x sec and the number of bad peas is positive and (at t=x sec) is the same as the number of bad peas at t=(x-1) sec. Find the sum of all k for k<25.

## Rules for eating:

- 1) One bad pea can eat only one pea.
- 2) A bad pea cannot eat a bad pea which just ate another bad pea.

- 3) The first priority for a bad pea to eat is a good pea, the second priority is a bad pea which just ate a good pea and at last, if there isn't any good pea or a bad pea (which just ate a good pea) available then a bad pea eats a bad pea(which did not eat any kind of pea).
- 4) If there does not exist any pea which does not satisfy the priority order of the bad pea, then the bad pea will not eat any pea.

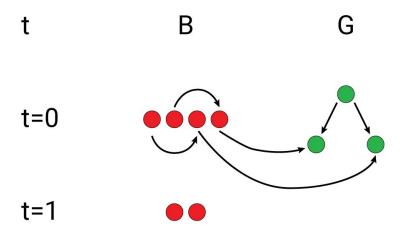
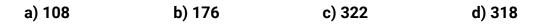


Fig: showing a case for  $\mathbf{k} = 5$ 

Example: For the first five seconds the given below cases are the only cases for  $\mathbf{k} = 3$ 

| Case 1 |   | Case 2 |   |   |   |
|--------|---|--------|---|---|---|
| t      | В | G      | t | В | G |
| 0      | 1 | 2      | 0 | 2 | 1 |
| 1      | 1 | 3      | 1 | 2 | 0 |
| 2      | 3 | 1      | 2 | 0 | 2 |
| 3      | 2 | 0      | 3 | 0 | 4 |
| 4      | 0 | 2      | 4 | 4 | 0 |
| 5      | 0 | 4      | 5 | 2 | 0 |

So for case 2,  $\mathbf{k}$ =3 satisfies the question's condition because at  $\mathbf{t}$ =1 number of good peas = 0 and the number of bad peas at  $\mathbf{t}$ =1 is the same as the number of bad peas at  $\mathbf{t}$ =0.



Still not convinced that he is a toy, Forky decides to jump from the car, and seeing this Woody also jumps because he knows Bonnie loves Forky. Woody convinces Forky that Bonnie loves him, and they start walking towards the car. On their way, they enter a shop.

## **QUESTION 17**

While in the shop, Woody and Forky meet a doll Gabby, who doesn't have a voice box. She and her magical dummies try to kidnap Woody and steal his voice box. Let there be n dummies. The dummies have a special way of multiplying, i.e. the number of dummies (n) can be replaced by a\*b number of dummies if a+2b=n, where n, a and b are natural numbers. Woody knows that his voicebox will get stolen if the population of dummies ever becomes exactly 2020, which is the unluckiest number. The dummies know that after a sequence of such replacements, there are some numbers from which 2020 can't ever be reached, even if they keep multiplying forever, and hence Woody can escape. Find the sum of all initial numbers, from which 2020 can't be reached.

a) 28 b) 36 c) 27 d) 38

Woody escapes from the shop, and Gabby captures Forky. At the playground, Woody finds Bo Peep and her goats; they had been sent off to another child long back. Bo agrees to help Woody save Forky and get back to Bonnie. Meanwhile, Buzz also set out to find Woody and Forky but got lost in a carnival, which is between the shop where Forky is being held and where the car is parked.

#### **QUESTION 18**

At the carnival, Buzz is picked up by a man who owns a game stall at the carnival, and Buzz becomes a game prize. A kid comes up to play the game and says he wants Buzz as the prize. The carnival game is as follows- There are 128 coins lying in a line some with heads up and some with tails up. In one query, the kid can specify the position of the coin and get the face that is facing up as the answer. But there is a trick. Before every 8n+1th query the kid makes the host performs one of the 4 actions:

- 1. Reverses the sequences of coins
- 2. Flips all coins
- 3. Flips all coins and then reverses the sequence
- 4. Does nothing

The kid can ask as many queries as he wants but he has to minimize the number of queries and tell the current sequence of coins just after he gets the answer. Which among these is the minimum number of queries in which the kid can surely arrive at a solution?

a) 248

b) 208

c) 168

d) 128

While the kid was playing the game, Buzz with the help of two other toys who were at the stall as game prizes, named Ducky and Bunny, is able to escape and spots Bo with Woody in the carnival. They meet up and head towards the shop where Forky is trapped. They get inside the store and spot Forky with Gabby on a shelf.

# **QUESTION 19**

The shelf is in the form of a histogram with n columns of heights in increasing order .

There are pillars of height i, where 1<=i<=17, 2 pillars each for even height except for 4, 10 and 16 for which it has 3 pillars and 3 pillars each for odd height. In order to climb the histogram shaped shelf, Woody decided to use dominoes. What the largest number of non-overlapping dominos that Woody can arrange inside of this histogram so that he can reach the top and rescue Forky. (A domino is a 1×2 or 2×1 rectangle)

- A. 205
- B. 206
- C. 207
- D. 208

Woody goes inside the shelf and sees Forky with Gabby, while the other toys dismantle the dummies. Gabby decides to play one final game with Woody, and if he wins he can take Forky back.

#### **QUESTION 20**

There are n+1 cheats numbered from 0 to n (left to right). And given a number k. There is a marker on cheat number n, Gabby starts first and they take turns in the game according to the following rules:-

- 1) On their move, they can move the marker either 1,2 Or k steps to the left.
- 2) On each move, they can only move the marker if there is a cheat there. (I.e.they can't move it beyond the cheat 0)
- 3) The player who can't move the marker loses.

Both Woody and Gabby play optimally. They ask Forky to choose the values of n and k for them with a restriction that 1 <= 10 and 3 <= 10.

Obviously Forky wants Woody to win and also figures out that for certain values of n and k Woody will beat Gabby no matter how Gabby plays (Woody plays optimally). Find the total number of pairs (n, k) which Forky can choose to make Woody win under the given constraints so that Woody is able to escape with Forky, Buzz, Bo Peep, and other toys, and reach Bonnie just before their car is about to leave.

a) 58 b) 59 c) 70 d) 63