Semi-Supervised Learning (SSL): Transductive SVM & Co-Training

Different Types of Learning

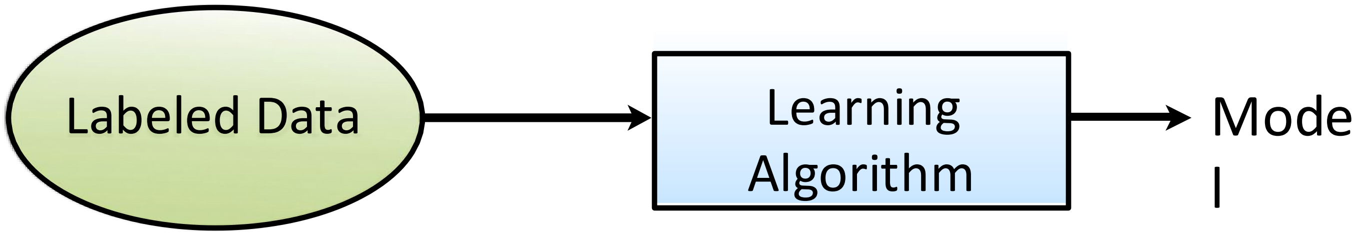
Supervised learning : learn from **labeled** data

Unsupervised learning : learn from **unlabeled** data

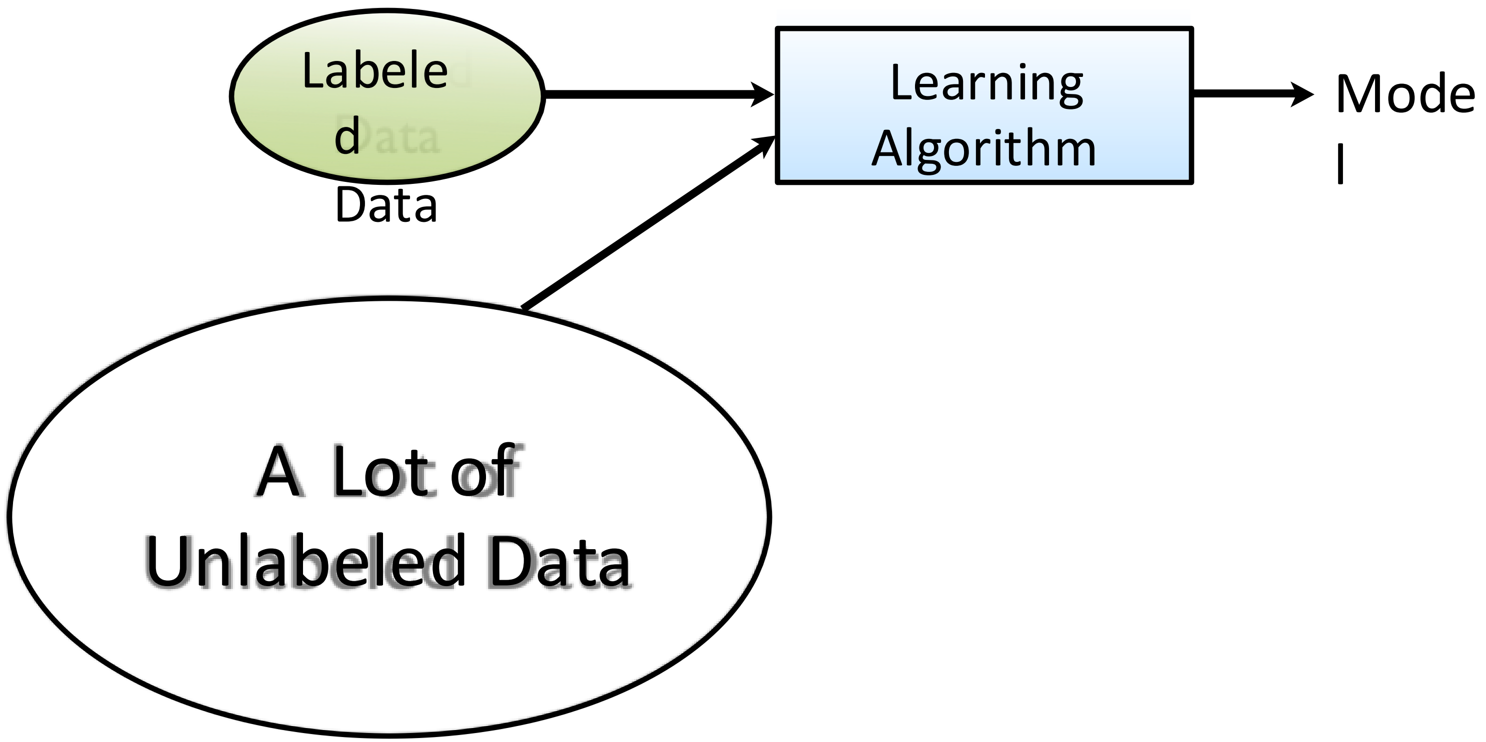
Semi-supervised learning : learn from both **labeled** and **unlabeled** data

usage	supervised learning	semi-supervised learning	unsupervised learning
$\{(x, y)\}$ labeled data	yes	yes	no
$\{x\}$ unlabeled data	no	yes	yes

Supervised Learning



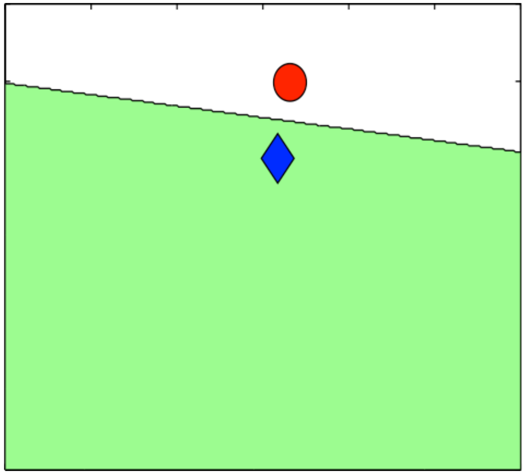
Semi-Supervised Learning (SSL)



Why SSL?

- Labeling is expensive and difficult
  - Human annotation is slow
  - Labels require experts
  - Imagine the cost/time of tagging millions of images!
- Unlabeled data
  - Cheap and sufficient

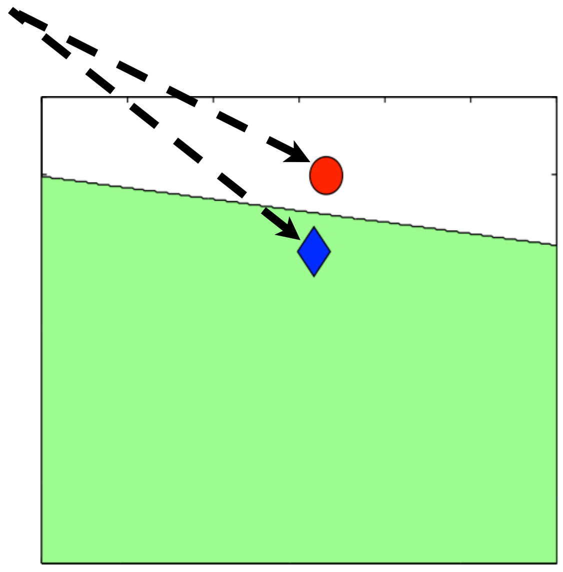
How can unlabeled data be helpful?



Without Unlabeled Data

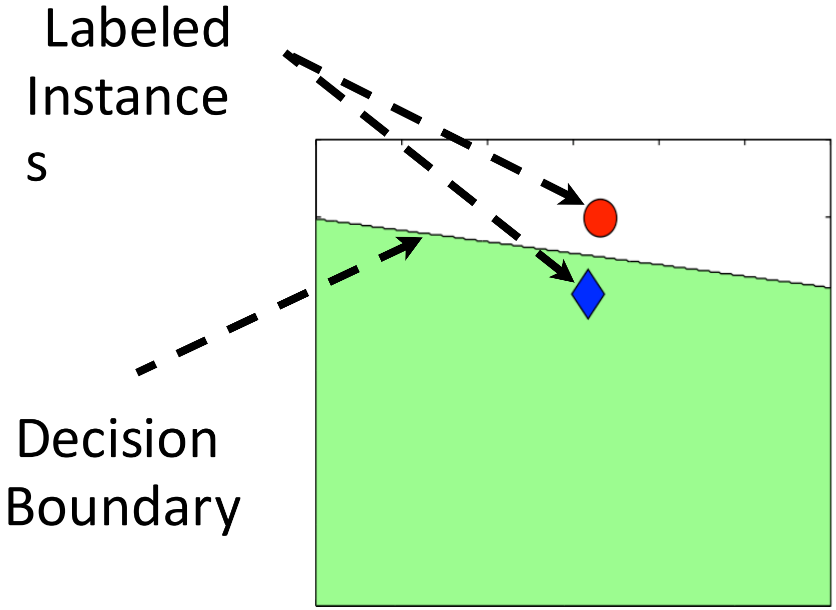
How can unlabeled data be helpful?

Labeled Instance s



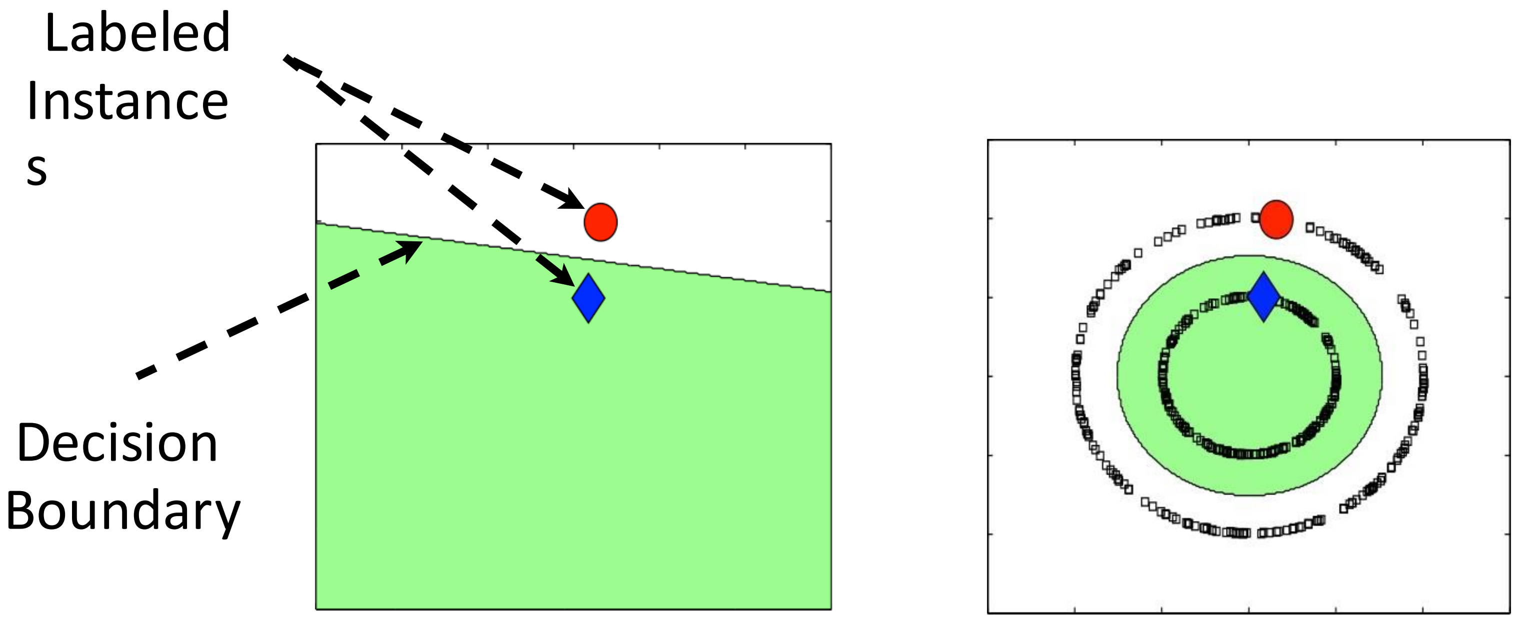
Without Unlabeled Data

How can unlabeled data be helpful?



Without Unlabeled Data

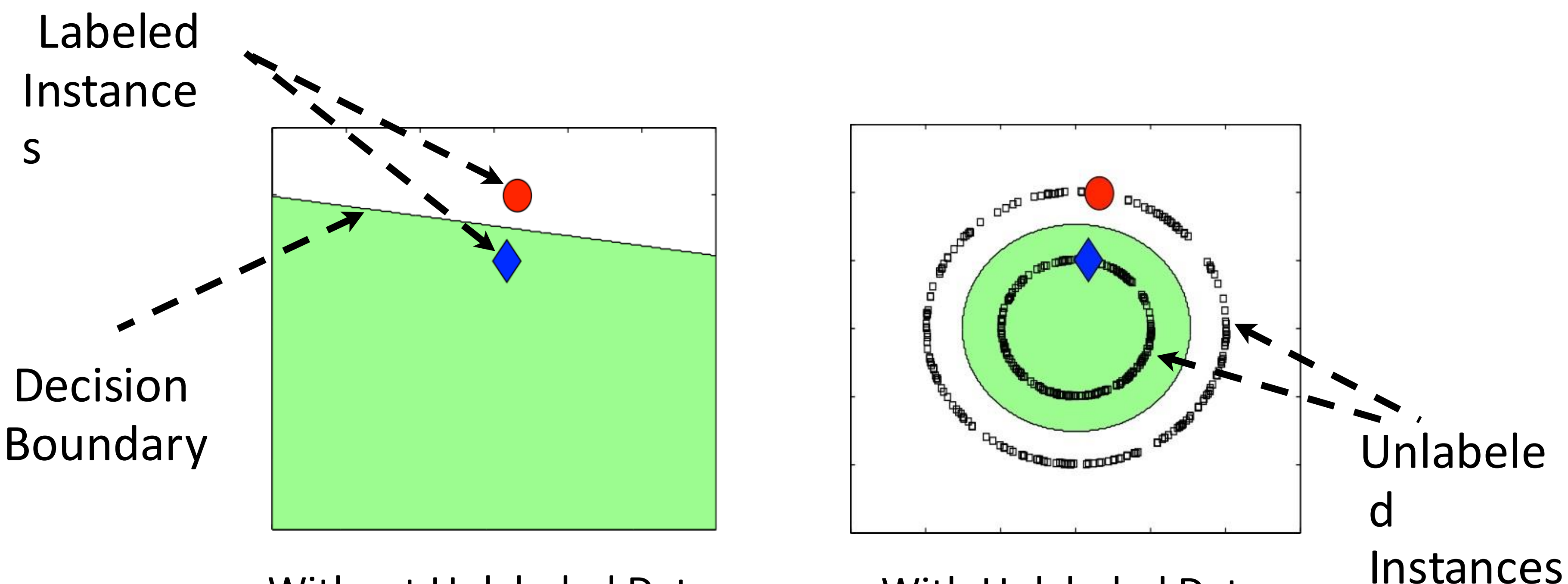
How can unlabeled data be helpful?



Without Unlabeled Data

With Unlabeled Data

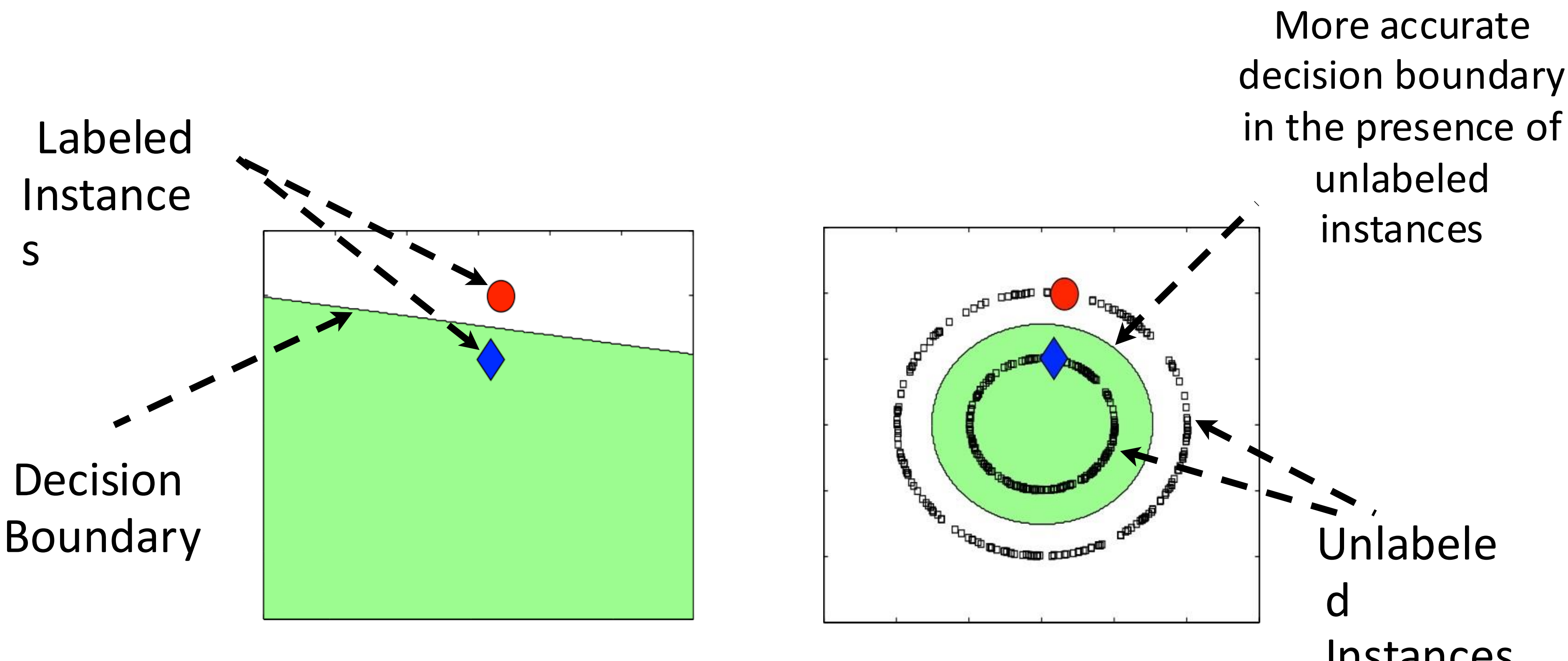
How can unlabeled data be helpful?



Without Unlabeled Data

With Unlabeled Data

How can unlabeled data be helpful?

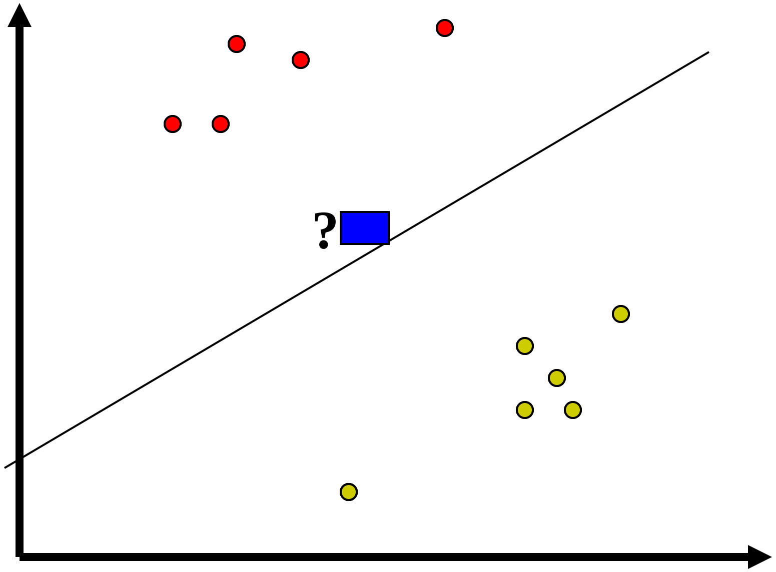


Without Unlabeled Data

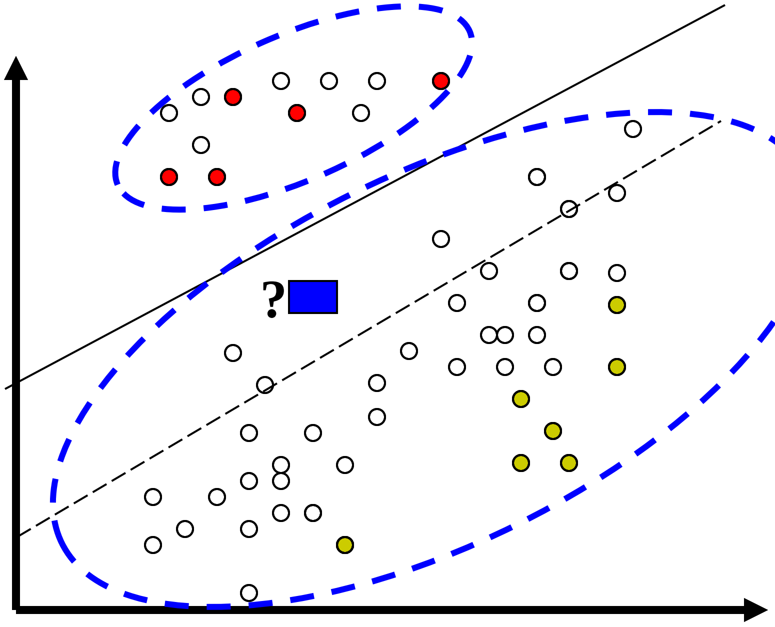
With Unlabeled Data

Example from [Belkin et al., JMLR 2006]

Clustering Assumption



Clustering Assumption



- Points with same label are connected through high density regions, thereby defining a cluster

- Clusters are separated through **low-density regions**

Inductive vs Transductive Learning

- Inductive learning
  - Induce a decision function that works well for ***all the possible examples***.

- Transductive learning
  - Find a decision function that works well for ***the given test examples***
  - Problem setting
    - Given labeled data and unlabeled data
    - Find set of labels that best fit unlabeled data
    - Note that we do not extend to unseen examples

SSL Methods

- Non-graph based SSL methods
  - Transductive SVM
  - Co-training
  - Active learning
  - MixMatch

- Graph based SSL methods
  - Label propagation
  - Belief propagation
  - Graph neural networks

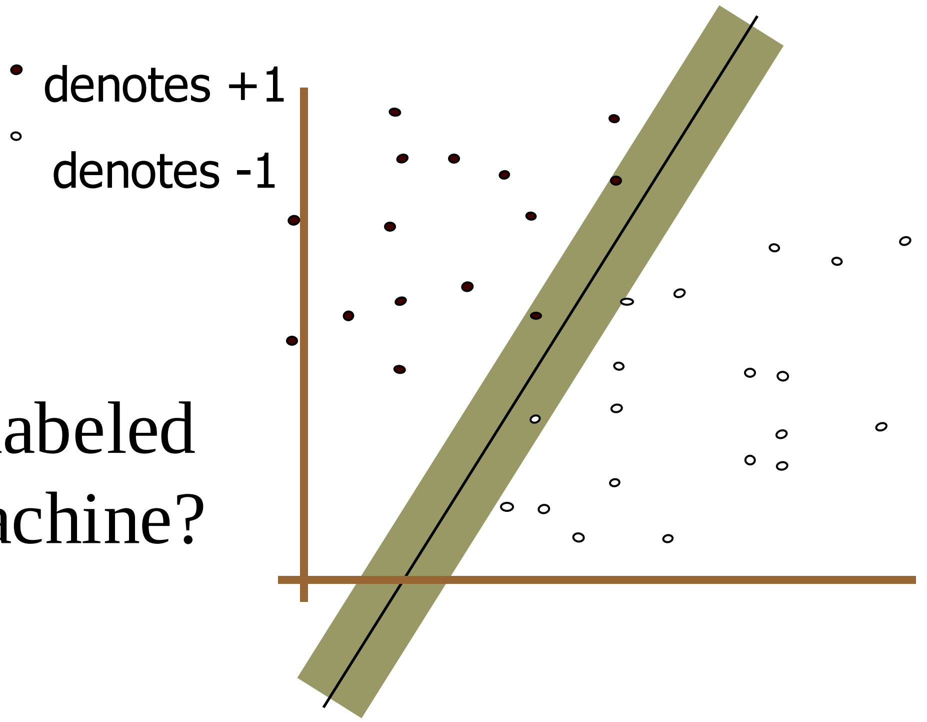
Transductive Support Vector Machine (TSVM)

Cluster Assumption vs. Maximum Margin

- Support Vector Machine (SVM)
  - Maximum margin classifier

→ low density around decision boundary

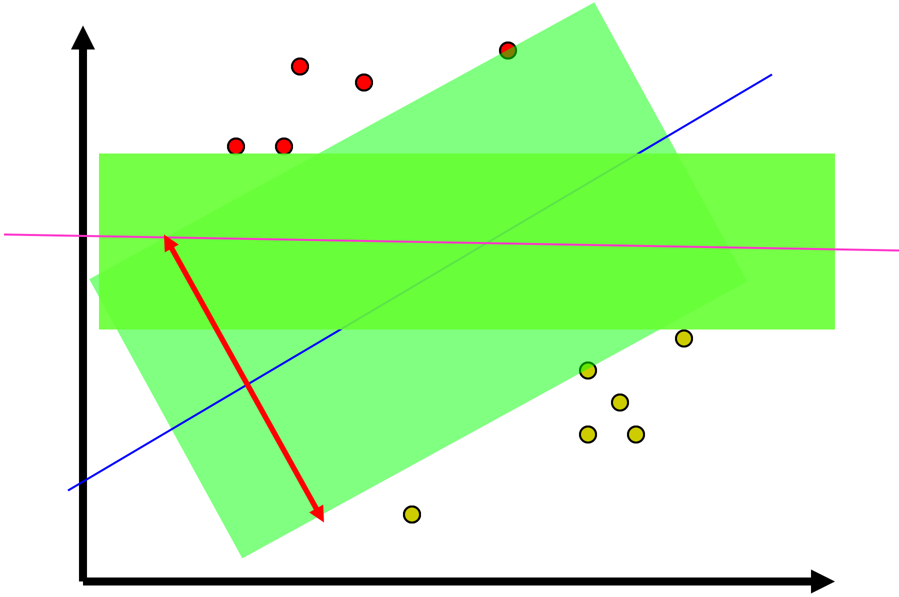
→ Cluster assumption



- What about using the unlabeled data in support vector machine?

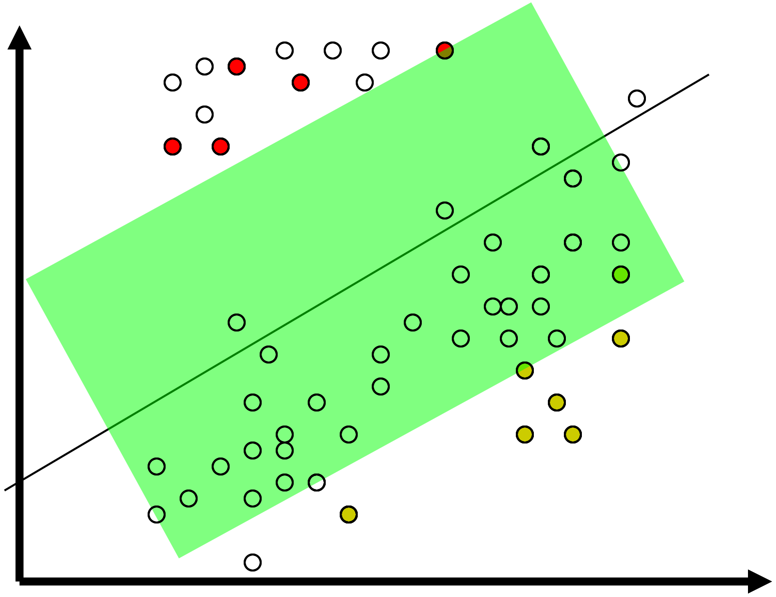
Transductive SVM

- Decision boundary given a small number of labeled examples
- Support vector machine
  - Maximum margin classifier



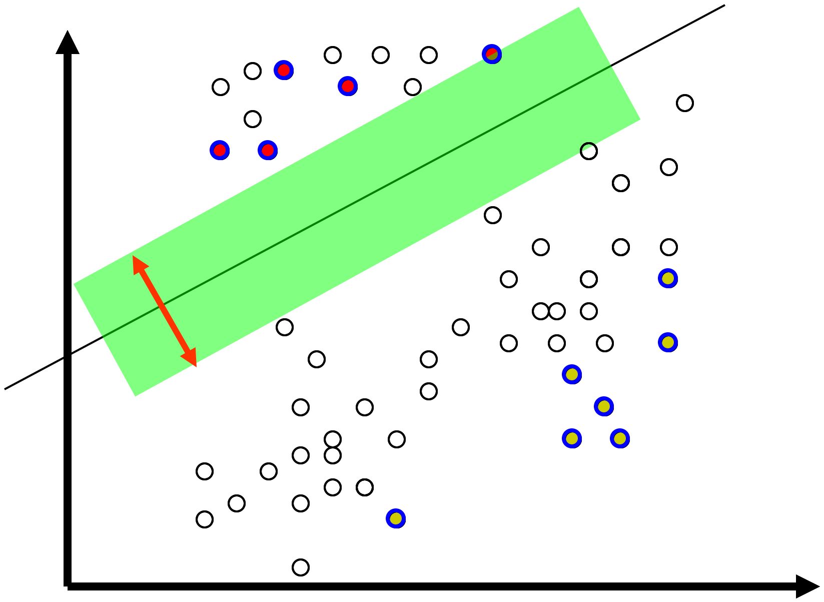
Transductive SVM

- Decision boundary given a small number of labeled examples
- How to change decision boundary given both labeled and unlabeled examples ?



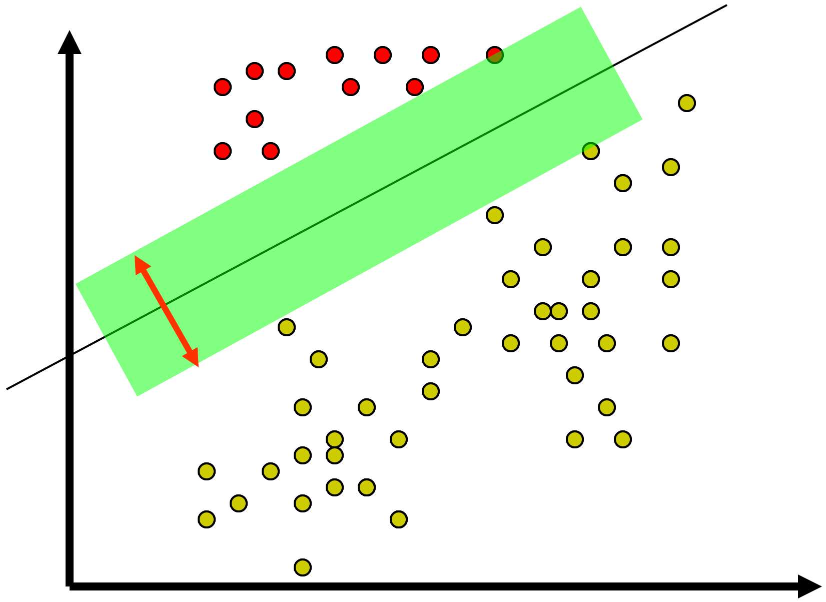
Transductive SVM

- Decision boundary given a small number of labeled examples
- Move the decision boundary to place with low density
  - Maximum margin



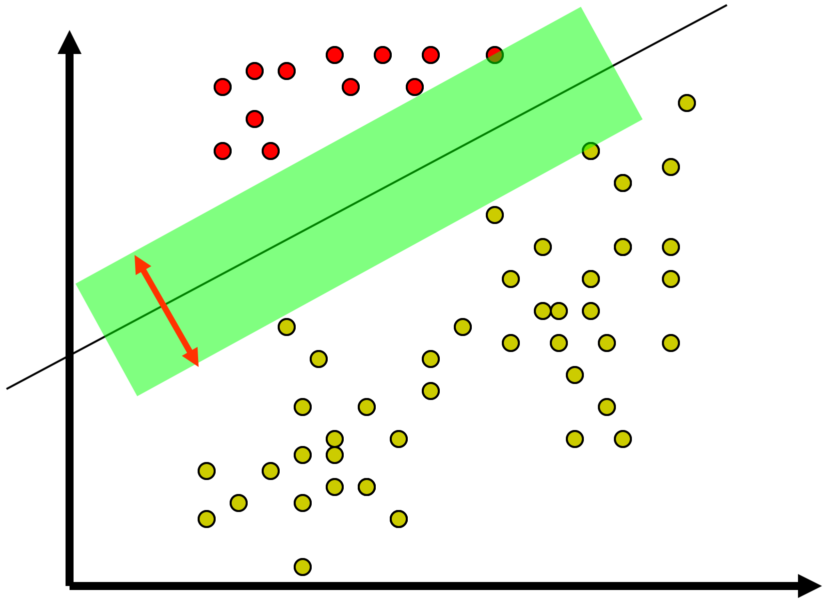
Transductive SVM

- Decision boundary given a small number of labeled examples
- Move the decision boundary to place with low density
- Classification results



Transductive SVM

- Decision boundary given a small number of labeled examples
- Move the decision boundary to place with low density
- Classification results
- How to formulate this idea?



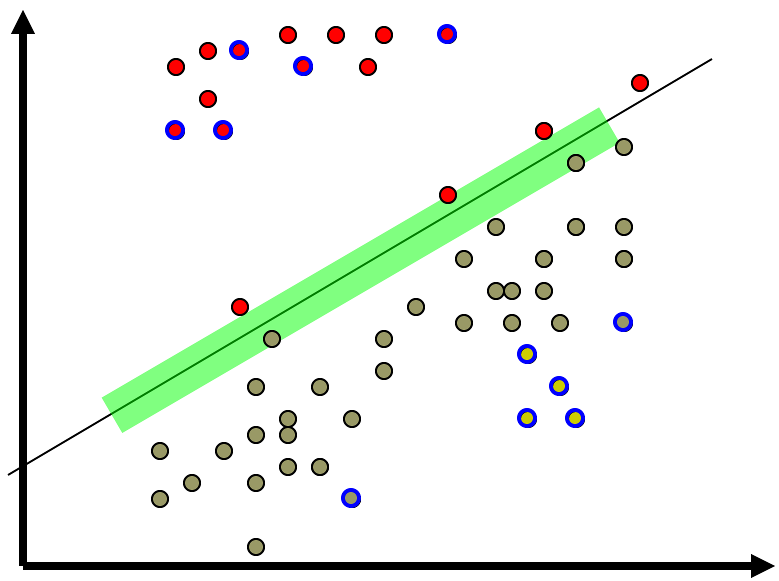
Transductive SVM: Formulation

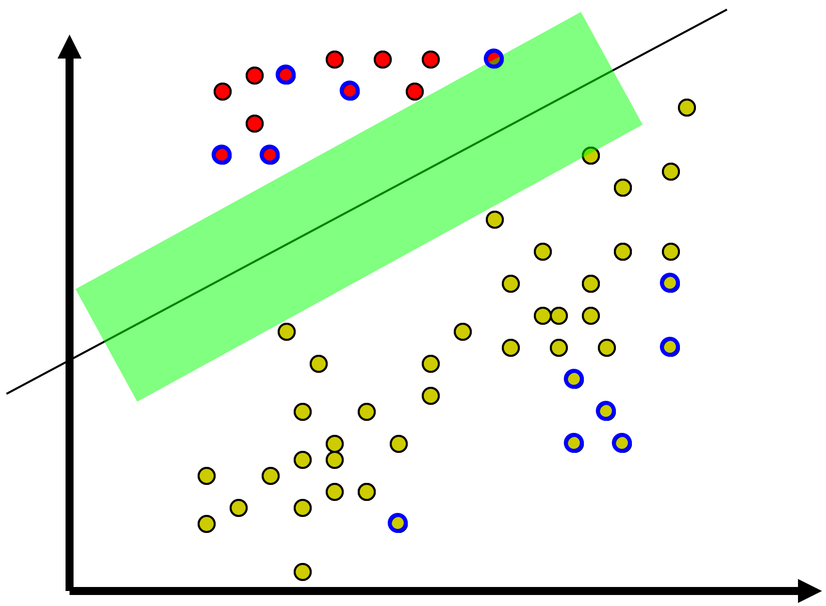
- Labeled data L: $L = \{(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)\}$
- Unlabeled data D: $D = \{(x_{n+1}), (x_{n+2}), ..., (x_{n+m})\}$
- Maximum margin principle for ***mixture of labeled and unlabeled data***
  - **Step I**: Given label assignment of each unlabeled data, compute its maximum margin
  - **Step II**: Given the maximum margin of unlabeled data, update the label assignment

Tranductive SVM

Different label assignment for unlabeled data

→ different maximum margin





Transductive SVM: Formulation

Original SVM

$$\{\vec{w}^*, b^*\} = \underset{\vec{w}, b}{\operatorname{argmin}}\ \vec{w} \cdot \vec{w}$$

$$\left.\begin{array}{l} y_1\left(\vec{w} \cdot \vec{x}_1 + b\right) \ge 1 \\ y_2\left(\vec{w} \cdot \vec{x}_2 + b\right) \ge 1 \\ \dots\dots \\ y_n\left(\vec{w} \cdot \vec{x}_n + b\right) \ge 1 \end{array}\right\} \text{labeled examples}$$

A binary variables for label of each example

Transductive SVM

$$\{\vec{w}^*, b^*\} = \boxed{\underset{y_{n+1}, \dots, y_{n+m}}{\operatorname{argmin}}}\ \underset{\vec{w}, b}{\operatorname{argmin}}\ \vec{w} \cdot \vec{w}$$

$$\left.\begin{array}{l} y_1\left(\vec{w} \cdot \vec{x}_1 + b\right) \ge 1 \\ y_2\left(\vec{w} \cdot \vec{x}_2 + b\right) \ge 1 \\ \dots\dots \\ y_n\left(\vec{w} \cdot \vec{x}_n + b\right) \ge 1 \end{array}\right\} \text{labeled examples}$$

$$\boxed{\left.\begin{array}{l} y_{n+1}\left(\vec{w} \cdot \vec{x}_{n+1} + b\right) \ge 1 \\ \dots\dots \\ y_{n+m}\left(\vec{w} \cdot \vec{x}_{n+m} + b\right) \ge 1 \end{array}\right\} \text{unlabeled examples}}$$

Constraints for unlabeled data

Introducing Slack Variables

$$\{\vec{w}^*, b^*\} = \underset{y_{n+1},\ldots,y_{n+m}}{\operatorname{argmin}} \; \underset{\vec{w},b}{\operatorname{argmin}} \; \vec{w}\cdot\vec{w} + \boxed{\sum_{i=1}^{n} \xi_i} + \boxed{\sum_{i=1}^{n} \eta_i}$$

$$\left.\begin{array}{l} y_1\left(\vec{w}\cdot\vec{x}_1 + b\right) \geq 1 - \xi_1 \\ y_2\left(\vec{w}\cdot\vec{x}_2 + b\right) \geq 1 - \xi_2 \\ \ldots. \\ y_n\left(\vec{w}\cdot\vec{x}_n + b\right) \geq 1 - \xi_n \end{array}\right\} \text{labeled examples} \qquad \left.\begin{array}{l} y_{n+1}\left(\vec{w}\cdot\vec{x}_{n+1} + b\right) \geq 1 + \eta_1 \\ \ldots. \\ y_{n+m}\left(\vec{w}\cdot\vec{x}_{n+m} + b\right) \geq 1 + \eta_m \end{array}\right\} \text{unlabeled examples}$$

- No longer convex optimization problem
- How to optimize transductive SVM?
  - Alternating optimization

Alternating Optimization

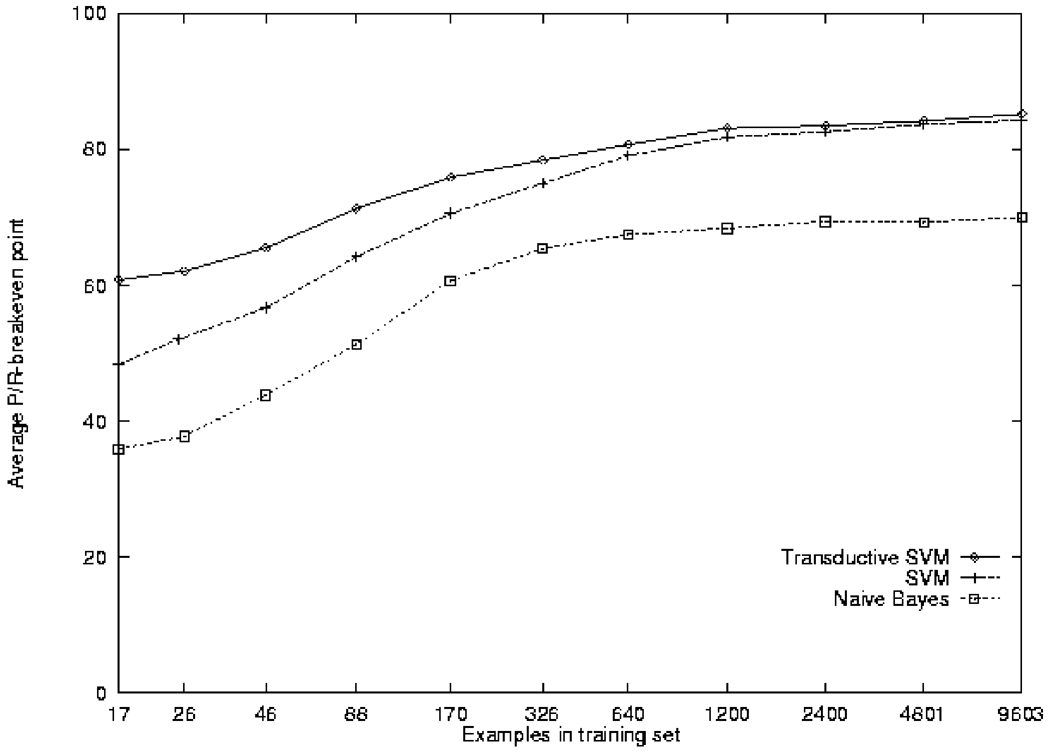
$$\{\vec{w}^*, b^*\} = \underset{y_{n+1},\ldots,y_{n+m}}{\operatorname{argmin}} \ \underset{\vec{w},b}{\operatorname{argmin}} \ \vec{w}\cdot\vec{w} + \sum_{i=1}^{n}\xi_i + \sum_{i=1}^{n}\eta_i$$

$$\left.\begin{aligned} &y_1\left(\vec{w}\cdot\vec{x}_1 + b\right) \geq 1 - \xi_1 \\ &y_2\left(\vec{w}\cdot\vec{x}_2 + b\right) \geq 1 - \xi_2 \\ &\ldots. \\ &y_n\left(\vec{w}\cdot\vec{x}_n + b\right) \geq 1 - \xi_n \end{aligned}\right\} \text{labeled examples}$$

$$\left.\begin{aligned} &y_{n+1}\left(\vec{w}\cdot\vec{x}_{n+1} + b\right) \geq 1 - \eta_1 \\ &\ldots. \\ &y_{n+m}\left(\vec{w}\cdot\vec{x}_{n+m} + b\right) \geq 1 - \eta_m \end{aligned}\right\} \text{unlabeled examples}$$

- Step 1: fix $y_{n+1},\ldots, y_{n+m}$, learn weights **w=(**$\vec{w},b$**)**
- Step 2: fix weights **w**, predict $y_{n+1},\ldots, y_{n+m}$

Text Classification Results (Joachims 99)



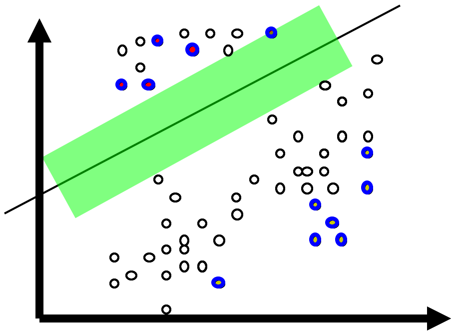
10 categories from
the Reuter collection

3299 test documents

1000 informative words
selected by MI criterion

Summary

- Based on maximum margin principle



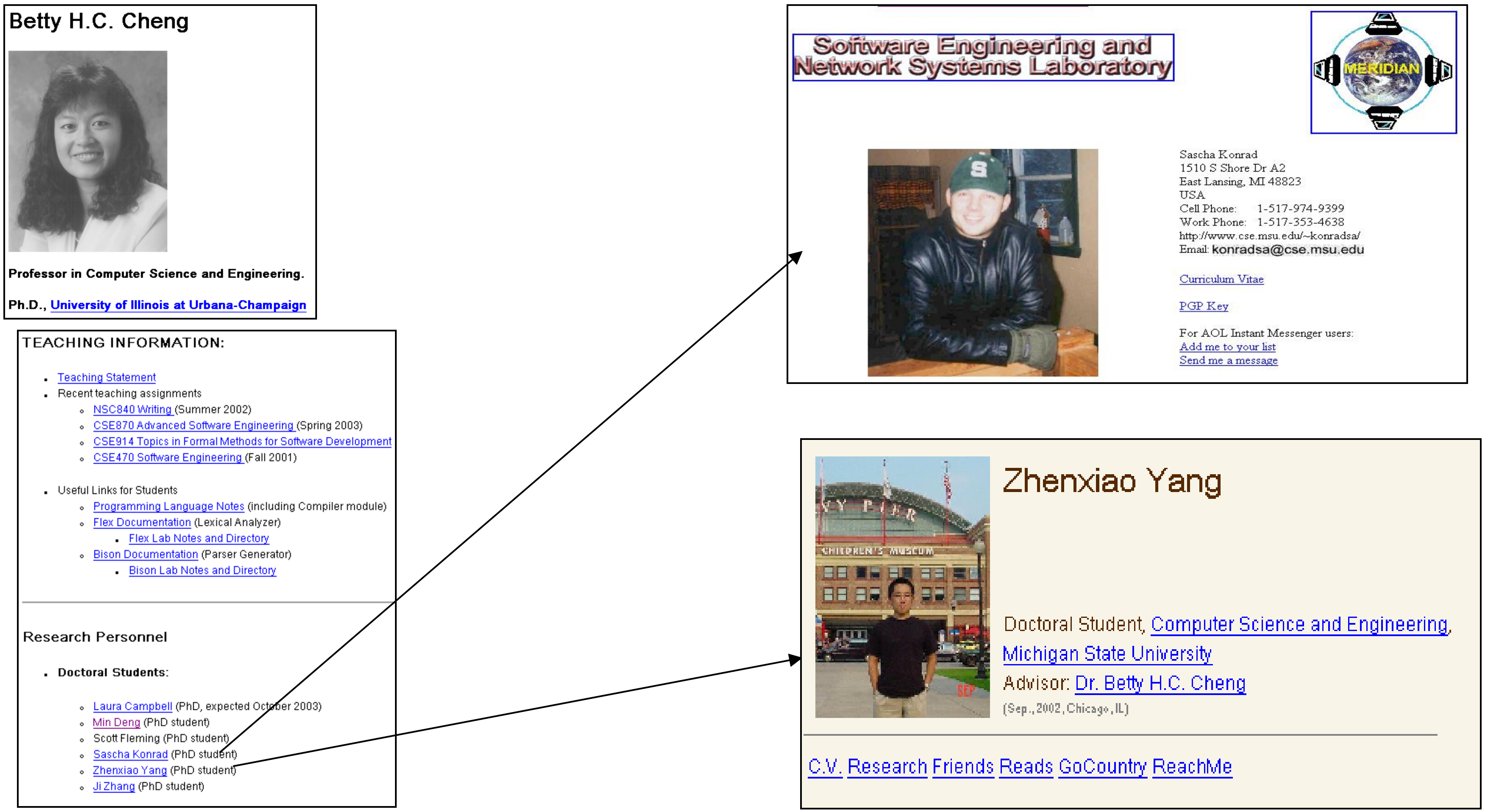
- Classification margin is decided by
  - Labeled data
  - Unlabeled data with assigned labels
- High computational cost
  - Variants: Low Density Separation (LDS), Semi-Supervised Support Vector Machine (S3VM)

Co-Training

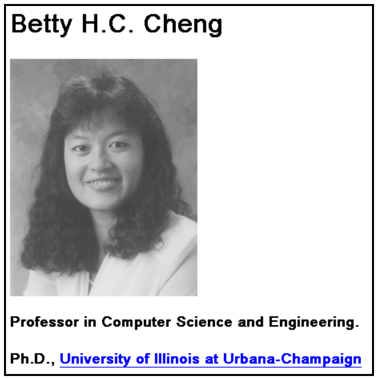
Co-training [Blum & Mitchell, 1998]

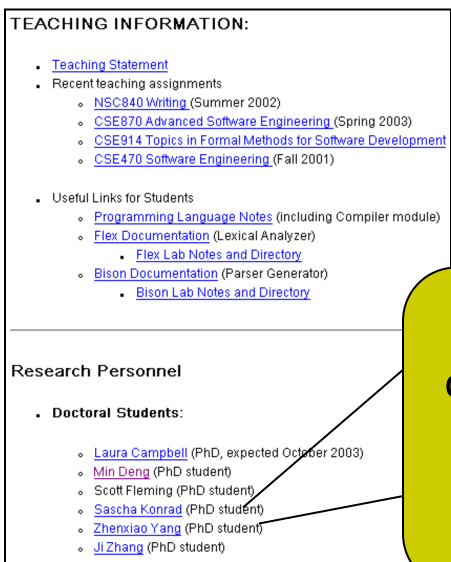
- Classify web pages into
  - category for students and category for professors
- Two views of web pages
  - Content
    - “I am currently the second year Ph.D. student …”
  - Hyperlinks
    - “My advisor is …”
    - “My students: …”

Co-training for Semi-Supervised Learning

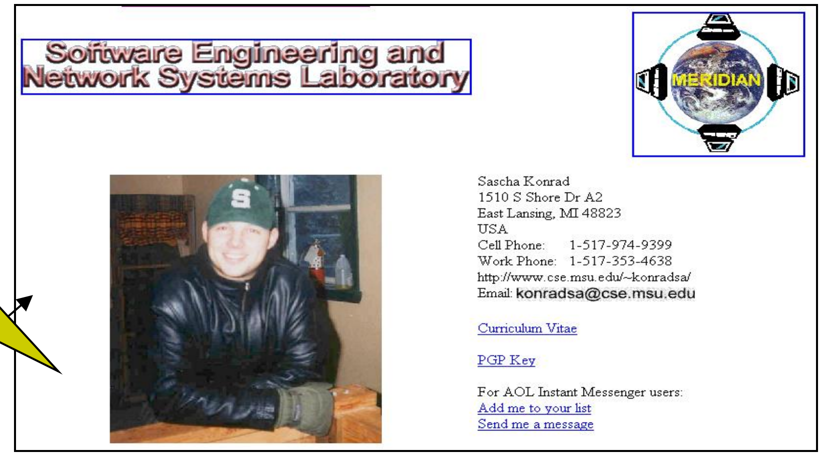


Co-training for Semi-Supervised Learning





It is easier to classify this web page using hyperlinks



It is easy to classify the type of this web page based on its content

Co-training

- Two representation for each web page

Content representation:

(doctoral, student, computer, university…)

Hyperlink representation:

Inlinks: Prof. Cheng

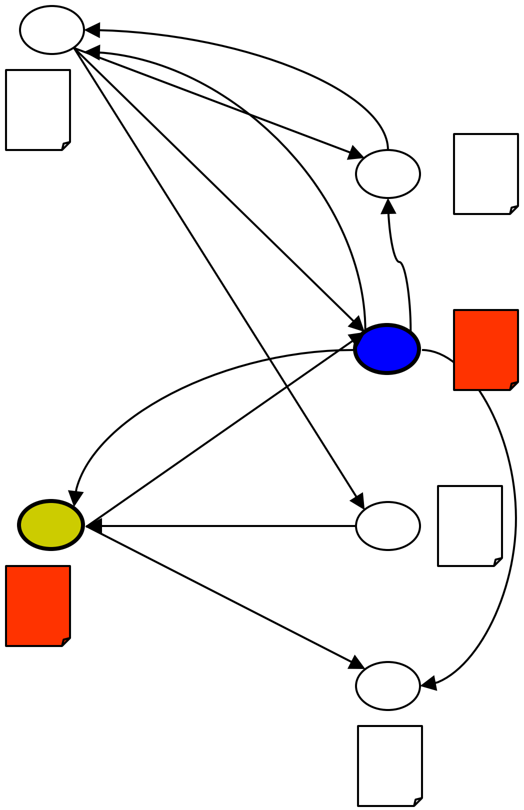
Oulinks: Prof. Cheng

Co-training: Classification Scheme

- Train a content-based classifier using labeled web pages
  - Apply the content-based classifier to classify unlabeled web pages
- Label the web pages that have been confidently classified
- Train a hyperlink based classifier using the web pages that are initially labeled and labeled by the content-based classifier
  - Apply the hyperlink-based classifier to classify unlabeled web pages
- Label the web pages that have been confidently classified

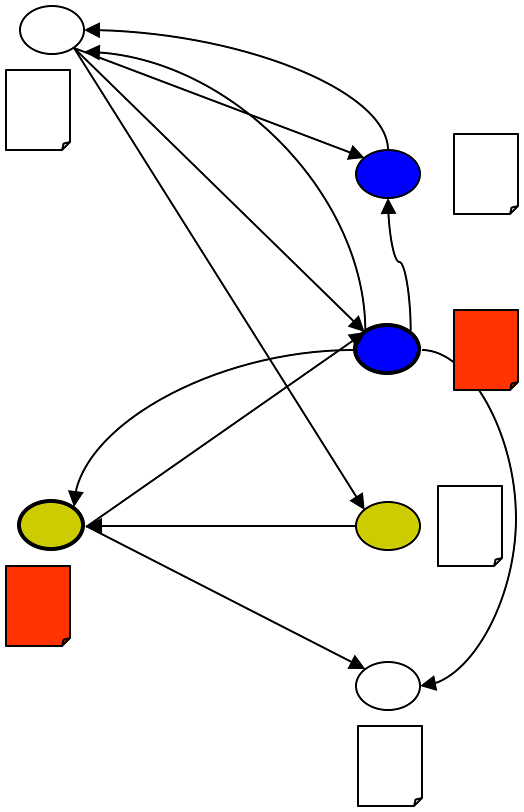
Co-training

- Train a content-based classifier using labeled examples



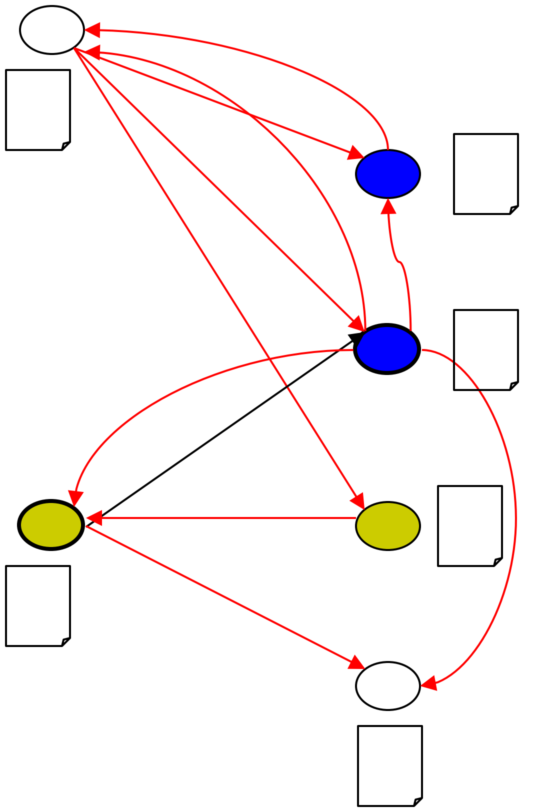
Co-training

- Train a content-based classifier using labeled examples
- Label the unlabeled examples that are confidently classified



Co-training

- Train a content-based classifier using labeled examples
- Label the unlabeled examples that are confidently classified
- Train a hyperlink-based classifier



Co-training

- Train a content-based classifier using labeled examples
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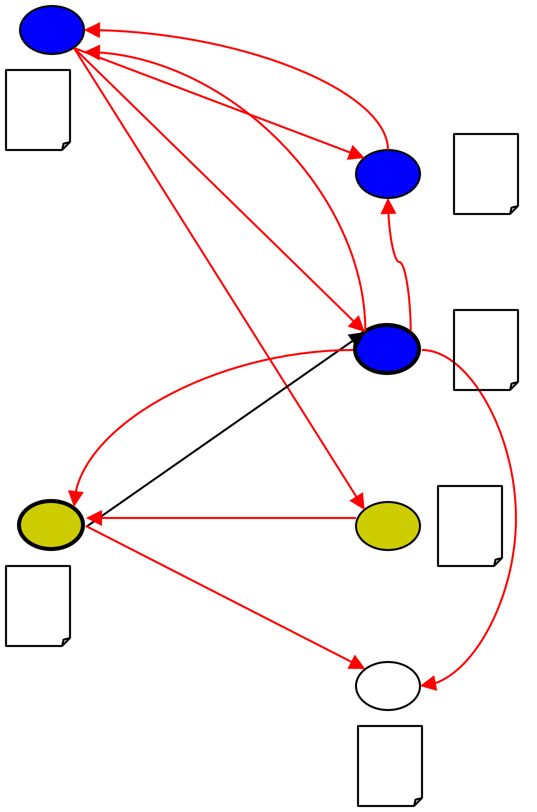

Co-training

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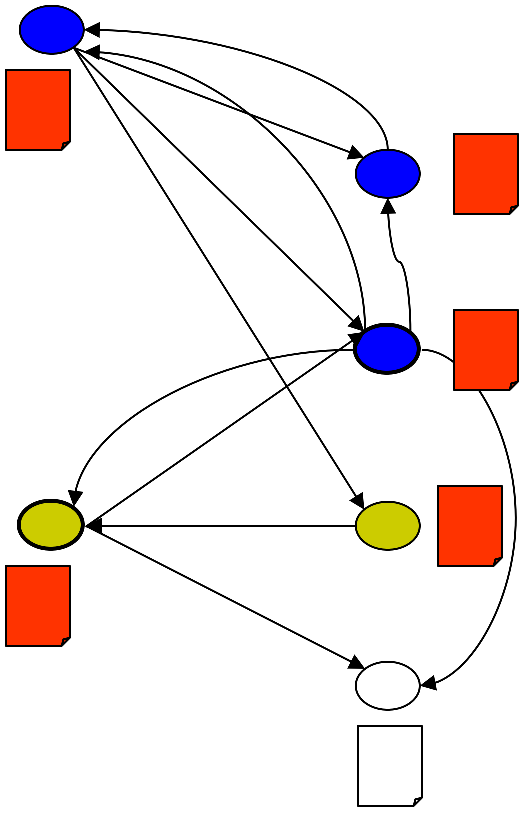


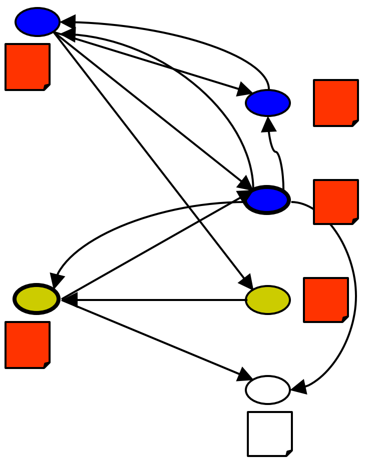
Text Classification Results

(Blum & Mitchell 98)

	Page-based classifier	Hyperlink-based classifier	Combined classifier
Supervised training	12.9	12.4	11.1
Co-training	6.2	11.6	5.0

Table 2: Error rate in percent for classifying web pages as course home pages. The top row shows errors when training on only the labeled examples. Bottom row shows errors when co-training, using both labeled and unlabeled examples.

Summary



- Assume two views of objects
  - Two sufficient representations
- Key idea
  - Augment training examples of one view by exploiting the classifier of the other view
- Extension to multiple view
- Challenge: find equivalent views