Assignment 6

Due: Nov 22nd, 2024

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In questions 1 and 2, we will create a program that calculates the factorial of natural number n and stores the result as $x = fac(n)$. Here, function $fac(n)$ is the function that gives the factorial of n:

$$fac(n) = \begin{cases} 1, & \text{if } n = 0 \\ n * fac(n-1), & \text{if } n > 0 \end{cases}$$

1. Answer the following questions.
   a. What are the pre- and post- conditions needed for this program?
   b. Create a loop invariant p by replacing the named constant n by a variable y in the postcondition (what can be the range of variable y?) and create the corresponding loop condition B.
   c. Find a bound expression t for this program. Show your work.

2. Create a full proof outline under the total correctness for this program. Don't forget to initialize variables x and y before the loop starts, find reasonable values for them. Justify each logical implications used in the proof outline.

3. Find an optimized precondition p which contains no conditional expressions and create a full proof outline for the following minimal proof outline. You may assume that $0 \le i < size(b), 0 \le j < size(b), 0 \le k < size(b)$ so you don't need to worry about the array-index-out-of-bounds error.

$$\{p\}\, b[i] \coloneqq b[j]; b[j] \coloneqq b[k] \{b[i] > b[k]\}$$

4. Create a full proof outline for the following minimal proof outline. You may assume that $0 \le j < size(b), 0 \le k < size(b)$, so you don't need to worry about the array-index-out-of-bounds error.

$$\{k < b[k] < b[j]\}\ \ b\big[b[k]\big] \coloneqq b[j]\ \{b[k] \neq b[j]\}$$

5. Draw an evaluation graph for the configuration $\langle S, \sigma \rangle$ where $S \equiv [x \coloneqq 1 \parallel x \coloneqq -1]; y \coloneqq y + x$.

6. Draw an evaluation graph for the configuration $\langle W, \{x = 0, y = 1, n = 2\} \rangle$ where $W \equiv \textbf{while } x < n \textbf{ do } [x \coloneqq x + 1 \parallel y \coloneqq y * 2] \textbf{ od}$.

7. Given the following two threads written in provable triples. Answer the following questions and justify your answers.

$$S_1 \equiv \{x = 0\}\, y \coloneqq x + 2\, \{y = 2\}$$
$$S_2 \equiv \{x < 0\}\, z \coloneqq 0\, \{z > x\}$$

   a. Are these two threads disjoint?
   b. Do they have disjoint conditions?

8. Answer the following questions about the two following threads written in proof outlines. Here, T_1, T_2 and T_3 are statements. Remember, we only need to check if atomic statements cause interference.

$$S_1^* \equiv \{p_1\}\ \textbf{if}\ B_1\ \textbf{then}\ \{p_2\} < T_1 >\ \textbf{else}\ \{p_3\}\ \textbf{skip fi}\ \{p_4\}$$
$$S_2^* \equiv \{q_1\} < T_2 >;\ \{\textbf{inv}\ q_2\}\ \textbf{while}\ B_2\ \textbf{do}\ \{q_3\}\ < T_3 >\ \textbf{od}\ \{q_4\}$$

a. List the interference freedom checks to decide whether S_1^* interferes with S_2^*.
b. List the interference freedom checks to decide whether S_2^* interferes with S_1^*.