Sample Final

(Make sure you finish in 35-40 mins)

1. Which one of the following is syntactically equivalent to $(\forall x.\, x < y \to z > y)\ [x + z \,/\, z]$?
   a. $\forall x.\, x < y \to z > y$
   b. $\forall x.\, x < y \to (x + z) > y$
   c. $\forall u.\, u < y \to z > y$
   d. $\forall u.\, u < y \to (x + z) > y$

2. While calculating $sp(x \leq y,\ \textbf{if } x = z \textbf{ then } x \coloneqq z + 1 \textbf{ else } z \coloneqq y \textbf{ fi})$, what is the collection of variables that we need to age?
   a. $\{x, y, z\}$
   b. $\{x, y\}$
   c. $\{x, z\}$
   d. $\{x\}$

3. We know that $\tau \vDash sp(p, S)$, then which one of the following must be true?
   a. For some state $\sigma \vDash p$, it is the case that $M(S, \sigma) - \bot = \{\tau\}$.
   b. For some state $\sigma \vDash p$, it is the case that $\tau \notin M(S, \sigma) - \bot$.
   c. For some state $\sigma \vDash p$, it is the case that $\tau \in M(S, \sigma) - \bot$.
   d. For some state $\sigma \vDash p$, it is the case that $\bot \in M(S, \sigma)$.

4. Which one of the following is semantically equivalent to $(b[i] = 1)\ \big[1 \,/\, b[k]\big]$? You may assume that i and k are both valid indices of array b.
   a. $i = k \vee b[i] = 1$
   b. $i \neq k \vee b[i] = 1$
   c. $i = k \vee b[k] = 1$
   d. $i \neq k \vee b[k] = 1$

5. Under total correctness, create a full proof outline for the following minimal proof outline. Remember that each triple and each implication you use in the proof outline must be provable.

$$\{p\}\ \textbf{if } x < 0 \textbf{ then } x \coloneqq x \div y \textbf{ else } x \coloneqq sqrt(x) \textbf{ fi}\ \{b[x] > y\}$$

6. The following program calculates the sum $2^0 + 2^1 + 2^2 + \cdots + 2^k$, where $k > 0$. We want to store the sum in a variable x and show that $x = 2^{k+1} - 1$ in the postcondition. Create a full proof outline under total correctness of this program by fulfilling the missing conditions and statements.

$\{k > 0\}\ x \coloneqq 1; \{$ _____ $\}$ _____ ; $\{$_____ $\}$

$\{$**inv** _____$\}$ $\{$**bd** _____$\}$

while $i \neq k$ **do**

$\{$_____$\}$

$\{$_____$\}\ x \coloneqq x + 2^i;$

$\{$_____$\}\ i \coloneqq i + 1$

$\{$_____$\}$

od

$\{x = 2^{i+1} - 1 \wedge 0 \leq i \leq k \wedge$ _____$\}$

$\{x = 2^{k+1} - 1\}$

7. Given the following two threads written in proof outlines:

$$S_1 \equiv \{x = 0\}\ y \coloneqq x + 2\ \{y = 2\}$$
$$S_2 \equiv \{x < 0\}\ z \coloneqq 0\ \{z > x\}$$

Do these two threads interfere with each other? Show your work.