CS 536 - Science of Programming
Assignment-1

1)
a) $e_1 = e_2$ does not logically imply $e_1 \equiv e_2$

Here, "$=$" represents semantic equality
"$\equiv$" represents syntactic equality

Let us consider an example,

$e_1 = 10 + 20 \quad e_2 = 30$

$$10 + 20 = 30$$

Here, $10 + 20 = 30$ is semantically equal

$$10 + 20 \not\equiv 30$$

Here, $10 + 20 \equiv 30$ is not syntactically equal

So, $e_1 = e_2$ does not logically imply $e_1 \equiv e_2$

b) $e_1 \neq e_2$ logically implies $e_1 \not\equiv e_2$

Let us consider an example,

$e_1 = 2 + 2 \quad e_2 = 5$

$$2 + 2 \neq 5$$

Here, $2 + 2 \neq 5$ is not semantically equal

$$2 + 2 \not\equiv 5$$

Here, $2 + 2 \not\equiv 5$ is not syntactically equal

So, $e_1 \neq e_2$ logically implies $e_1 \not\equiv e_2$

2) Now, we will use truth tables to prove the following equivalences

a) $(p \vee q) \wedge q \Leftrightarrow q$

Truth Table for the above is :

p	q	$(p \vee q)$	$(p \vee q) \wedge q$
T	T	T	T
T	F	T	F
F	T	T	T
F	F	F	F

so, $(p \vee q) \wedge q \Leftrightarrow q$ is proved from the above logic table.

b) $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$

Truth Table for the above is :

p	q	$(p \leftrightarrow q)$	$\neg(p \leftrightarrow q)$	$\neg p$	$\neg p \leftrightarrow q$
T	T	T	F	F	F
T	F	F	T	F	T
F	T	F	T	T	T
F	F	T	F	T	F

so, $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$ is proved from the above logic table

c) $\neg p \wedge (p \vee q) \rightarrow q \Leftrightarrow T$

p	q	$\neg p$	$(p \vee q)$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q) \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T

So, $\neg P \wedge (P \vee q) \to q \Leftrightarrow T$ is proved from the above logic table

3)

a) $(P \to q) \vee (P \to r) \Leftrightarrow P \to q \vee r$

Take LHS first

LHS $\Leftrightarrow (P \to q) \vee (P \to r)$

$\Leftrightarrow (\neg P \vee q) \vee (\neg P \vee r)$ Implication rule

$\Leftrightarrow \neg P \vee (q \vee r)$ simplify

$\Leftrightarrow P \to (q \vee r)$ Implication rule

LHS = RHS

So, $(P \to q) \vee (P \to r) \Leftrightarrow P \to q \vee r$

b) $(P \vee q) \wedge \neg q \Leftrightarrow \neg (P \to q)$

LHS $\Leftrightarrow (P \vee q) \wedge \neg q$

$\Leftrightarrow (P \wedge \neg q) \vee (q \wedge \neg q)$ Distributive rule

$\Leftrightarrow (P \wedge \neg q) \vee F$ Contradiction rule

$\Leftrightarrow (P \wedge \neg q) \to$ 1 Identity rule

RHS $\Leftrightarrow \neg (P \to q)$

$\Leftrightarrow \neg (\neg P \vee q)$ Implication rule

$\Leftrightarrow \neg (\neg P) \wedge \neg q$ Double negation rule

$\Leftrightarrow (P \wedge \neg q) \to$ 2

1 $\Leftrightarrow$ 2 $\therefore$ LHS = RHS

So, $(P \vee q) \wedge \neg q \Leftrightarrow \neg (P \to q)$

c) $(P \to q) \wedge (\neg P \to q) \Leftrightarrow q$

LHS $\Leftrightarrow (P \to q) \wedge (\neg P \to q)$

$\Leftrightarrow (\neg P \vee q) \wedge (\neg(\neg P) \vee q)$ Implication rule

$\Leftrightarrow (\neg P \vee q) \wedge (P \vee q)$ Double negation rule

$\Leftrightarrow (\neg P \wedge P) \vee (\neg P \wedge q) \vee (q \wedge P) \wedge (q \wedge q)$ Distributive rule

$\Leftrightarrow F \vee (\neg p \wedge q) \vee (q \wedge p) \vee (q \wedge q)$ contradiction rule

$\Leftrightarrow (\neg p \wedge q) \vee (q \wedge p) \vee (q \wedge q)$ Identity rule

$\Leftrightarrow (\neg p \wedge q) \vee (p \wedge q) \vee (q \wedge q)$ commutative rule

$\Leftrightarrow (\neg p \wedge q) \vee (p \wedge q) \vee q$ Idempotent rule

$\Leftrightarrow (\neg p \wedge q) \vee q$ Absorption rule

$\Leftrightarrow q$ Absorption rule

LHS $\Leftrightarrow$ RHS

So, $(p \rightarrow q) \wedge (\neg p \rightarrow q) \Leftrightarrow q$

4)

a) $\neg(p \wedge q) \wedge p \Rightarrow \neg q$

Take LHS

$\Rightarrow \neg(p \wedge q) \wedge p$

$= (\neg p \vee \neg q) \wedge p$ Demorgan's Rule

$= (\neg p \wedge p) \vee (\neg q \wedge p)$ Distributive Rule

$= F \vee (\neg q \wedge p)$ contradiction Rule

$= \neg q \wedge p$ Identity Rule

$= p \wedge \neg q$ commutative Rule

From the Rule of Simplication

$p \wedge \neg q \Rightarrow \neg q$

$\neg(p \wedge q) \wedge p \Rightarrow \neg q$

Hence, Proved

b) $p \wedge q \vee q \wedge r \Rightarrow p \vee q \vee r$

Take LHS

$\Rightarrow p \wedge q \vee q \wedge r$

$= (p \wedge q) \vee (q \wedge r)$ Associative Rule

$= (p \vee (q \wedge r)) \wedge (q \vee (q \wedge r))$ Distributive Rule

$= (p \vee q) \wedge (p \vee r) \wedge (q \vee q) \wedge (q \wedge r)$ Distributive Rule

$= (p \vee q) \wedge (p \vee r) \wedge q \wedge (q \vee r)$ Idempotent Rule

$= (p \vee q \vee r) \wedge q \wedge (q \vee r)$ Associative Rule

$= p \vee q \vee r$

~~[illegible]~~

$p \wedge q \vee q \wedge r \Rightarrow p \vee q \vee r$

$\therefore$ Hence Proved.

c) $(p \to q) \wedge (\neg p \to r) \Rightarrow q \vee r$

$(p \to q) \wedge (\neg p \to r) \Rightarrow$ LHS

$= ((p \to q) \wedge (\neg p \to r)) \wedge (p \vee \neg p)$ Tautology

$= (\neg p \vee q) \wedge (p \vee r) \wedge (p \vee \neg p)$. Implication Rule

$= (\neg p \vee q) \wedge (p \vee r)$ Simplifying

$= (q \vee \neg p) \wedge (p \vee r)$ Commutative Rule

$= (q \vee r) \vee (\neg p \wedge p)$ Distributive Rule

$= (q \vee r) \vee F = q \vee r$ Contradiction Rule & Identity Rule.

$\therefore (p \to q) \wedge (\neg p \to r) \Rightarrow q \vee r$.

$\therefore$ Hence, Proved

5) $\sigma \models p \leftrightarrow q \leftrightarrow r$

Truth Table:

p	q	r	$(p \leftrightarrow q)$	$((p \leftrightarrow q) \leftrightarrow r)$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
F	T	T	F	F
T	F	F	F	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	F

From the above truth table, the states such that $\sigma \models p \leftrightarrow q \leftrightarrow r$ are:-

1) $\sigma = \{p = T, q = T, r = T\}$
2) $\sigma = \{p = T, q = F, r = F\}$
3) $\sigma = \{p = F, q = T, r = F\}$
4) $\sigma = \{p = F, q = F, r = T\}$

6)

a) $\{b = 5, i = 0, x = 6\}$, $x > b[i]$

Here i is defined but b is not an array, it is an integer. So, we can't access $b[i]$. It is not valid.

So, $\{b = 5, i = 0, x = 6\}$ is not proper for predicate $x > b[i]$

Hence, False

b) $\{x=4, y=-1\}$, $x/sqrt(y)$

Here, the square root of a negative number is undefined.

so, $\{x=4, y=-1\}$ is not proper for $x/sqrt(y)$

$\therefore$ Hence, False

c) $\{x=5, y=2\} \models T$

The above given statement just states that the set satisfies a true condition.

$\therefore$ Hence, True

d) Let $\sigma = \{p=T, b=(2,0,4)\}$, $\sigma \models p \Leftrightarrow b[b[1]]=2$

$p \Leftrightarrow b[b[1]] = 2$

$T \Leftrightarrow b[0] = 2$ $\because b[1] = 0$

$T \Leftrightarrow 2 = 2$ $\because b[0] = 2$

$T \Leftrightarrow T$

$\therefore$ So, The state σ satisfies the condition

i.e. $p \Leftrightarrow b[b[1]] = 2$.

Hence, True

e) if $a \equiv b$, then if $x \geq 0$ then $b[0]$ else $a[1][3]$ fi is not a legal expression.

True, because $b[0]$ is a one dimensional array and $a[1][3]$ is a two dimensional array and they are not of same type. So, They can't and they are not of same type.

So, It is true that the above is not a legal expression.

7)
a) $\neg \forall x \geq 1 . x^2 > x \Longleftrightarrow \exists x . x \geq 1 \land x^2 \leq x$

Take LHS

$= \neg \forall x \geq 1 . x^2 > x$

$= \exists x . \neg (x \geq 1 \Rightarrow x^2 > x)$ Negation of Universal Quantifier Rule

$= \exists x . (x \geq 1 \land \neg (x^2 > x))$ Negation of Implication rule

$= \exists x . x \geq 1 \land x^2 \leq x$ Negation of Inequality rule.

LHS = RHS

$\neg \forall x \geq 1 . x^2 > x \Longleftrightarrow \exists x . x \geq 1 \land x^2 \leq x$

Hence, Proved.

b) $\neg \exists x . \exists y . x > y \land x < y \Longleftrightarrow \forall x . \forall y . x \leq y \lor x \geq y$

Take LHS

$= \neg \exists x . \exists y . x > y \land x < y$

$= \forall x . \neg \exists y . x > y \land x < y$ Negation of Existential quantifier rule

$= \forall x . \forall y . \neg (x > y \land x < y)$ Negation of Existential Quantifier rule

$= \forall x . \forall y . (\neg (x > y) \lor \neg (x < y))$ DeMorgan's Rule

$= \forall x . \forall y . x \leq y \lor x \geq y$ Negation of Inequality

So, LHS = RHS

$\neg \exists x . \exists y . x > y \land x < y \Longleftrightarrow \forall x . \forall y . x \leq y \lor x \geq y$

Hence, Proved.

c) $\neg((\exists x . \exists y . Q(x,y)) \land \forall x . \forall y . Q(y,x)) \iff (\forall x . \forall y . \neg Q(x,y)) \lor \exists x . \exists y . \neg Q(y,x)$

Take LHS

$= \neg((\exists x . \exists y . Q(x,y)) \land \forall x . \forall y . Q(y,x))$

$= \neg(\exists x . \exists y . Q(x,y)) \lor \neg(\forall x . \forall y . Q(y,x))$ Demorgan's Rule

$= (\forall x \forall y \neg Q(x,y)) \lor \neg(\forall x . \forall y\, Q(y,x))$ Negation of existential Quantifier

$= (\forall x . \forall y \neg Q(x,y)) \lor \exists x . \exists y . \neg Q(y,x)$ Negation of Universal Quantifier

So, LHS = RHS

$\neg((\exists x . \exists y . Q(x,y)) \land \forall x . \forall y . Q(y,x)) \iff (\forall x . \forall y . \neg Q(x,y)) \lor \exists x . \exists y . \neg Q(y,x)$

Hence, Proved

8)

a) $isGreater(b,m,x) \equiv (m \leq length(b)) \land \forall i (0 \leq i < m \rightarrow x > b[i])$

b) $hasGreater(a,b) \equiv \forall j\, \exists i\, (b[j] > a[i])$

c) $Extends(a,b) \equiv (length(a) \leq length(b)) \land \forall i\, (0 \leq i < length(a) \rightarrow a[i] = b[i])$

9)

		$\sigma[u\mapsto\alpha][v\mapsto\beta]=\sigma[v\mapsto\beta][u\mapsto\alpha]$?	$\sigma[u\mapsto\alpha][v\mapsto\beta]\equiv\sigma[v\mapsto\beta][u\mapsto\alpha]$?
$u\equiv v$	$\alpha=\beta$	Yes, as they both are syntactically equal	Yes, because we are updating same variable with the same value two times on the both sides
$u\equiv v$	$\alpha\neq\beta$	NO, on the LHS $u\equiv v$ is bind with β and on the RHS $u\equiv v$ is with α	NO, as they are not equal semantically
$u\not\equiv v$	$\alpha=\beta$	Yes, because on LHS and RHS both u, v are bind with α or β since they are the same value	NO, they are not same because, since u and v are both different variables.
$u\not\equiv u$	$\alpha\neq\beta$	Yes, because on the both LHS and RHS, u is bind with α and the v is bind with β	NO, they are not same because, u and v are two both different variables

10) $\sigma=\{x=2, y=5\}$

a) $\sigma[x\mapsto\sigma(y)][y\mapsto\sigma(x)]$

$=\sigma[x\to5][y\to2]$

$=\{x=5, y=2\}$

The resulting state is $\sigma[x\mapsto\sigma(y)][y\mapsto\sigma(x)]=\{x=5, y=2\}$

b) Let $\tau=\sigma[x\mapsto3]$, $\gamma=\tau[y\mapsto\tau(x)*4]$

$\tau=\sigma[x\mapsto3]=\{x=3, y=5\}$

$$r = T[y \mapsto T(x) * 4]$$

$$r = T\{y \mapsto 3 * 4\}$$

$$r = T\{y \mapsto 12\}$$

$$r = \{x = 3, y = 12\}$$