CS536 - Science of Programming

Assignment - 6

1)

a) Pre - Condition :-

$n \geq 0$

Post - Condition :-

$x = fac(n)$

b) loop invarient :-

$p \equiv x = fac(y) \wedge 0 \leq y \leq n$

loop condition :-

$B \equiv y \neq n$

c) Bound Expression :- we will be running the loop by initializing $y=0$ and then we will increase the value of y in each and every iteration. so, that means y will always increase starting from 0 and it will never be negative. so n-y will be a good loop bound expression.

$\boxed{n - y \geq 0}$

2) <u>Full Proof Outline</u> :-

$\{n \geq 0\}$

$x := 1; \{n \geq 0 \wedge x = 1\}\ y := 0; \{n > 0 \wedge x = 1 \wedge y = 0\}$

$\{\text{inv } p \equiv x = fac(y) \wedge 0 \leq y \leq n\}\{bd\ n - y\}$

while $y \neq n$ do

$\{x = fac(y) \wedge 0 \leq y \leq n \wedge y \neq n \wedge n - y = t_0\}$

$\{x * (y+1) = fac(y+1) \wedge 0 \leq (y+1) \leq n \wedge$

$n - (y+1) < t_0\}\ x := x * (y+1);$

$\{x = fac(y+1) \wedge 0 \leq (y+1) \leq n \wedge n - (y+1)$

$< t_0\}\ y := y + 1$

$\{x = fac(y) \wedge 0 \leq y \leq n \wedge n - y < t_0\}$

od

$\{x = fac(y) \wedge 0 \leq y \leq n \wedge y = n\}$

$\{x = fac(n)\}$

3) Full proof outline for the given minimal proof outline :-

Given :-

$\{P\}\ b[i] := b[j];\ b[j] := b[k]\ \{b[i] > b[k]\}$, $0 \le i < size(b)$, $0 \le j < size(b)$, $0 \le k < size(b)$.

Full proof outline :-

$\{P\}\ b[i] := b[j];\ \{q_0\}\ b[j] := [k]\ \{b[i] > b[k]\}$

Here,

$q_0 \equiv (b[i] > b[k])\ [b[k]/b[j]]$

$\equiv (b[i])\ [b[k]/b[j]] > (b[k])\ [b[k]/b[j]]$

$\equiv (\text{if } i = j \text{ then } b[k] \text{ else } b[i] \text{ fi}) > (\text{if } k = j \text{ then } b[k] \text{ else } b[k] \text{ fi})$

$\mapsto (\text{if } i = j \text{ then } b[k] \text{ else } b[i] \text{ fi}) > b[k]$

$\mapsto \text{if } i = j \text{ then } F \text{ else } b[i] > b[k] \text{ fi}$

$\mapsto i \neq j \wedge b[i] > b[k]$

$P \equiv (i \neq j \wedge b[i] > b[k])\ [b[j]/b[i]]$

$\equiv i \neq j \wedge (b[i])\ [b[j]/b[i]] > (b[k])\ [b[j]/b[i]]$

$\equiv i \neq j \wedge b[j] > (\text{if } k = i \text{ then } b[j] \text{ else } b[k] \text{ fi})$

$\mapsto i \neq j \wedge (\text{if } k = i \text{ then } F \text{ else } b[j] > b[k] \text{ fi})$

$\mapsto i \neq j \wedge k \neq i \wedge b[j] > b[k]$

Full proof outline :-

$\{i \neq j \wedge k \neq i \wedge b[j] > b[k]\}$ $b[i] := b[j];$ $\{b[i] >$ $b[k]\}$ $b[j] := b[k]$ $\{i \neq j \wedge b[i] > b[k]\}$.

4) Full proof outline for the given minimal proof outline :-

Given :-

$\{k < b[k] < b[j]\}$ $b[b[k]] := b[j]$ $\{b[k] \neq b[j]\}$, $0 \leq j < size(b)$, $0 \leq k < size(b)$

Full proof outline :-

$\{k < b[k] < b[j]\}$ $\{q_0\}$ $b[b[k]] := b[j]$

$\{b[k] \neq b[j]\}$

Here,

$q_0 \equiv b[k] \neq b[j][b[j] | b[b[k]]]$

$\equiv b[k][b[j] / b[b[k]]] \neq b[j][b[j] / b[b[k]]]$

$$\equiv (\text{if } k = b[k] \text{ then } b[j] \text{ else } b[k] \neq (\text{if } j = b[k] \text{ then } b[j] \text{ else } b[j]))$$

$$\mapsto (\text{if } k = b[k] \text{ then } b[j] \text{ else } b[k]) \neq b[j])$$

$$\mapsto \text{if } k = b[k] \text{ then } b[j] \neq b[j] \text{ else } b[k] \neq b[j]$$

$$\mapsto \text{if } k = b[k] \text{ then } F \text{ else } b[k] \neq b[j]$$

$$\mapsto \neg(k = b[k]) \wedge b[k] \neq b[j]$$

5) Given configuration,
$\langle S, \sigma\rangle$ where $S \equiv [x := 1 \,||\, x := -1]; y := y + x$

<u>Evaluation graph</u>:-

$$\langle [x := 1 \,||\, x := -1]; y := y+x, \sigma\rangle$$

$$\swarrow \qquad\qquad \searrow$$

$$\langle [E \,||\, x := -1]; y := y+x, \sigma[x \mapsto 1]\rangle \qquad \langle [x := 1 \,||\, E]; y := y+x, \sigma[x \mapsto -1]\rangle$$

$$\downarrow \qquad\qquad \downarrow$$

$$\langle [E \,||\, E]; y := y+x, \sigma[x \mapsto -1]\rangle \qquad \langle [E \,||\, E]; y := y+x, \sigma[x \mapsto 1]\rangle$$

$$\downarrow \qquad\qquad \downarrow$$

$$\langle E, \sigma[x \mapsto -1][y \mapsto \sigma(y) - 1]\rangle \qquad \langle E, \sigma[x \mapsto 1][y \mapsto \sigma(y) + 1]\rangle$$

6) Given configuration,

$\langle W, \{x=0, y=1, n=2\}\rangle$ where $W \equiv$ while $x<n$ do $[x:=x+1 \,||\, y:=y*2]$ od

<u>Evaluation graph</u> :-

$$\langle \text{while } x<n \text{ do } [x:=x+1 \,||\, y:=y*2] \text{ od}, \{x=0, y=1, n=2\}\rangle$$

$$\downarrow$$

$$\langle [x:=x+1 \,||\, y:=y*2]; W, \{x=0, y=1, n=2\}\rangle$$

$$\swarrow \qquad\qquad \searrow$$

$$\langle [E \,||\, y:=y*2]; W, \{x=1, y=1, n=2\}\rangle \qquad \langle [x:=x+1 \,||\, E]; W, \{x=0, y=2, n=2\}\rangle$$

$$\searrow \qquad\qquad \swarrow$$

$$\langle W, \{x=1, y=2, n=2\}\rangle$$

$$\downarrow$$

$$\langle [x:=x+1 \,||\, y:=y*2]; W, \{x=1, y=2, n=2\}\rangle$$

$$\swarrow \qquad\qquad \searrow$$

$$\langle [E \,||\, y:=y*2]; W, \{x=2, y=2, n=2\}\rangle \qquad \langle [x:=x+1 \,||\, E]; W, \{x=1, y=4, n=2\}\rangle$$

$$\langle W, \{x=2, y=4, n=2\}\rangle$$

$$\downarrow$$

$$\langle E, \{x=2, y=4, n=2\}\rangle$$

7) Given threads,

$$S_1 \equiv \{x=0\}\ y:=x+2\ \{y=2\}$$
$$S_2 \equiv \{x<0\}\ z:=0\ \{z>x\}$$

i	j	change(S_i)	vars(S_j)	free(P_j, q_j)	S_i int S_j	S_i int cond$_j$
1	2	y	z	x, z	No	No
2	1	z	y	x, y	No	No

a) For two threads to be disjoint they must satisfy below condition

$$\text{change}(S_i) \cap \text{vars}(S_j) = \text{change}(S_j) \cap \text{vars}(S_i) = \phi$$

Here, change(S_1) = y, vars(S_2) = z | change(S_2) = z, vars(S_1) = y so,

$$\{y\} \cap \{z\} = \phi$$

$$\{z\} \cap \{y\} = \phi$$

so, the condition is satisfied. Hence, they are disjoint.

b) To determine whether they have disjoint conditions we need to satisfy below condition.

$change(S_1) \cap free(P_2, q_2) = \phi$ &

$change(S_2) \cap free(P_1, q_1) = \phi$.

Here,

$change(S_1) = y$	$change(S_2) = z$
$free(P_2, q_2) = x, z$	$free(P_1, q_1) = x, y$

so, $\{y\} \cap \{x, z\} = \phi$

&

$$\{z\} \cap \{x, y\} = \phi$$

So, the above two threads have disjoint conditions

8)
a) The interference freedom checks to decide whether S_1^* interferes with S_2^*

1) $\{P_2 \wedge q_1\} \langle T_1 \rangle \{q_1\}$
2) $\{P_2 \wedge inv\, q_2\} \langle T_1 \rangle \{inv\, q_2\}$
3) $\{P_2 \wedge q_3\} \langle T_1 \rangle \{q_3\}$
4) $\{P_2 \wedge q_4\} \langle T_1 \rangle \{q_4\}$

b) The interference freedom checks to decide whether S_2^* inteferes with S_1^*.

1) $\{q_1 \wedge P_1\} \langle T_2 \rangle \{P_1\}$
$\{q_1 \wedge P_1\} \langle T_3 \rangle \{P_1\}$

2) $\{q_1 \wedge P_2\} \langle T_2 \rangle \{P_2\}$
$\{q_1 \wedge P_2\} \langle T_3 \rangle \{P_2\}$

3) $\{q_1 \wedge P_3\} \langle T_2 \rangle \{P_3\}$

$\{q_1 \wedge P_3\} \langle T_3 \rangle \{P_3\}$

4) $\{q_2 \wedge P_4\} \langle T_2 \rangle \{P_4\}$

$\{q_2 \wedge P_4\} \langle T_3 \rangle \{P_4\}$