CS536 Assignment 1

Due: Sep. 10th, 2024

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In each question, quantified variables have domain $\mathbb{Z}$ unless otherwise specified.

1. Let e_1 and e_2 be two expressions.
   a. In general, does $e_1 = e_2$ logically imply $e_1 \equiv e_2$? If yes, briefly justify it (in a sentence or two); if no, give a counterexample.
   b. In general, does $e_1 \neq e_2$ logically imply $e_1 \not\equiv e_2$? If yes, briefly justify it (in a sentence or two); if no, give a counterexample.

2. Use truth tables to prove the following logical equivalences.
   a. $(p \vee q) \wedge q \Leftrightarrow q$
   b. $\neg(p \leftrightarrow q) \Leftrightarrow \neg p \leftrightarrow q$
   c. $\neg p \wedge (p \vee q) \to q \Leftrightarrow T$

3. Give a formal proof of truth for each of the following logical equivalences. Remember to clarify which rule(s) you use in each step.
   a. $(p \to q) \vee (p \to r) \Leftrightarrow p \to q \vee r$
   b. $(p \vee q) \wedge \neg q \Leftrightarrow \neg(p \to q)$
   c. $(p \to q) \wedge (\neg p \to q) \Leftrightarrow q$

4. Give a formal proof of truth for each of the following logical implications. Remember to clarify which rule(s) you use in each step.
   a. $\neg(p \wedge q) \wedge p \Rightarrow \neg q$
   b. $p \wedge q \vee q \wedge r \Rightarrow p \vee q \vee r$
   c. $(p \to q) \wedge (\neg p \to r) \Rightarrow q \vee r$ Hint: Try to conjunct $(p \vee \neg p)$ to the left-hand side

5. Find all states σ (containing only bindings for p, q and r) such that $\sigma \vDash p \leftrightarrow q \leftrightarrow r$. Briefly explain how you find each state.

6. Decide true or false for each of the following statements. Briefly explain your answers.
   a. $\{b = 5, i = 0, x = 6\}$ is proper for predicate $x > b[i]$.
   b. $\{x = 4, y = -1\}$ is proper for expression $x \,/\, sqrt(y)$.
   c. $\{x = 5, y = 2\} \vDash T$.
   d. Let $\sigma = \{p = T,\ b = (2, 0, 4)\}$, then $\sigma \vDash p \leftrightarrow b\big[b[1]\big] = 2$.
   e. If $a \equiv b$, then $\textbf{if } x \geq 0 \textbf{ then } b[0] \textbf{ else } a[1][3] \textbf{ fi}$ is not a legal expression.

7. Give a formal proof of truth for each of the following logical equivalences. Remember to clarify which rule(s) you use in each step.
   a. $\neg \forall x \geq 1. x^2 > x \Leftrightarrow \exists x. x \geq 1 \wedge x^2 \leq x$
   b. $\neg \exists x. \exists y. x > y \wedge x < y \Leftrightarrow \forall x. \forall y. x \leq y \vee x \geq y$
   c. $\neg\Big(\big(\exists x. \exists y. Q(x,y)\big) \wedge \forall x. \forall y. Q(y,x)\Big) \Leftrightarrow \big(\forall x. \forall y. \neg Q(x,y)\big) \vee \exists x. \exists y. \neg Q(y,x)$ Here, $Q(x,y)$ is a predicate function.

8. Define the following predicate functions.
    a. Define predicate function $isGreater(b, m, x)$ which returns $True$ if and only if positive m is not larger than the length of array b, and integer x is greater than each of the first m numbers in b. For example, $isGreater\big((2,4,1,6),3,5\big)$ returns $True$, $isGreater\big((2,4,1,6),3,4\big)$ returns $False$.

    b. Define predicate function $hasGreater(a, b)$ which returns $True$ if and only if every integer in array b is greater than some integer in array a. For example, $hasGreater\big((1,5,2,8),(3,2,5)\big)$ returns $True$, $hasGreater\big((1,5,2,8),(3,1,5)\big)$ returns $False$.

    c. Define predicate function $Extends(a, b)$ which returns $True$ if and only if array b is an extension of array a; in other words, we have $a[0] = b[0]$, $a[1] = b[1]$, $a[2] = b[2]$ … for all elements in a. For example, $Extends\big((1,4,2),(1,4,2,9,3)\big)$ returns $True$.

9. Let u and v some be variables and α and β be some values (u and v might be the same variable, α and β might be the same value). When does $\sigma[u \mapsto \alpha][v \mapsto \beta] = \sigma[v \mapsto \beta][u \mapsto \alpha]$ and when does $\sigma[u \mapsto \alpha][v \mapsto \beta] \equiv \sigma[v \mapsto \beta][u \mapsto \alpha]$? Discuss the four different cases depending on whether $u \equiv v$ and whether $\alpha = \beta$.

		$\sigma[u \mapsto \alpha][v \mapsto \beta] = \sigma[v \mapsto \beta][u \mapsto \alpha]$?	$\sigma[u \mapsto \alpha][v \mapsto \beta] \equiv \sigma[v \mapsto \beta][u \mapsto \alpha]$?
$u \equiv v$	$\alpha = \beta$		
$u \equiv v$	$\alpha \neq \beta$		
$u \not\equiv v$	$\alpha = \beta$		
$u \not\equiv v$	$\alpha \neq \beta$		

10. Consider state $\sigma = \{x = 2, y = 5\}$. Answer the following questions and show your work.
    a. Find state $\sigma[x \mapsto \sigma(y)][y \mapsto \sigma(x)]$.
    b. Let $\tau = \sigma[x \mapsto 3]$, and $\gamma = \tau[y \mapsto \tau(x) * 4]$. Find γ.