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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

Fourth Semester B.Tech (Minor) Degree Examination June 2023 (2021 Admission)

Pages: 4

Course Code: CST284

Course Name: Mathematics for Machine Learning

Max. Marks: 100 Duration: 3 Hours

PART A

(Answer all questions; each question carries 3 marks) Marks

1 Let V = {(x, y): x ≥ 0, y ≥ 0} with standard operations. Is it a vector space? Justify your answer. 3

2 Let x = $x = \begin{bmatrix}1\\2\\3\end{bmatrix}, y = \begin{bmatrix}3\\2\\9\end{bmatrix}$ *and* $z = \begin{bmatrix}5\\2\\-1\end{bmatrix}$. Is the set {x, y, z} linearly independent? 3

3 Let x = $\begin{bmatrix}0\\6\\4\end{bmatrix}$, u = $\begin{bmatrix}2\\-1\\1\end{bmatrix}$, v = $\begin{bmatrix}-1\\2\\4\end{bmatrix}$. and W = span {u, v}. Note that u.v = 0. Find a vector a in W and a vector b that is orthogonal to W, such that x = a + b 3

4 One of the eigen value of the matrix A= $\begin{bmatrix}1 & 2 & 4\\3 & 0 & 6\\1 & 1 & p\end{bmatrix}$ is three. Find the sum of other two eigen value 3

5 Find the maximum and minimum values of $f(x) = x^4 - 3x^3 - 1$ on [−2.2]. 3

6 Compute the gradient of the function f(x,y,z)=$x^2e^{yz^2}$ 3

7 Find the mean and variance of the random variable X whose probability density function is $f(x) = \begin{cases}\frac{3}{4}(1-x)(x-3 & 1 \leq x \leq 3\\0 & otherwise\end{cases}$ 3

8 I roll two dice and observe two numbers X and Y. If Z=X−Y, find the range and PMF of Z. 3

9 Explain the principle of the gradient descent algorithm with a diagram. 3

10 An aeroplane can carry a maximum of 250 passengers. A profit of Rs. 1500 is made on each executive class ticket and a profit of Rs. 900 is made on each economy class ticket. The airline reserves at least 30 seats for executive class. However at least 4 times as 3

many passengers prefer to travel by economy class than by executive class. Formulate LPP in order to maximize the profit for the airline.

PART B

(Answer one full question from each module, each question carries 14 marks)

Module -1

11 a) Consider the matrix $\begin{bmatrix} 1 & 1 \\ -2 & h \end{bmatrix}$ and vector b=$\begin{pmatrix} k \\ 1 \end{pmatrix}$. Find all possible values of h and k so that the matrix equation Ax = b has: (a) no solution. (b) exactly one solution. (c) infinitely many solutions. 10

b) Find the inverse of $\begin{bmatrix} 4 & 3 \\ 6 & 5 \end{bmatrix}$ 4

12 a) (i) Define $T: P_2 \to \mathbb{R}^3$ by: $T(p) = \begin{bmatrix} p(-1) \\ p(0) \\ p(1) \end{bmatrix}$. Find the image under T of 2

$p(t) = 5 + 3t$

b) (ii) Show that T is a linear transformation 3

c) For each of the following matrices, find the characteristic equation, the eigenvalues and a basis for each eigenspace 9

$A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 3 & -1 \\ 1 & 3 \end{bmatrix}$

$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Module -2

13 a) For the space $\mathbb{R}^4$, let $w_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $w_2 = \begin{bmatrix} 3 \\ 3 \\ -1 \\ -1 \end{bmatrix}$, y = $\begin{bmatrix} 6 \\ 0 \\ 2 \\ 0 \end{bmatrix}$ and let W=span$\{w_1, w_2\}$. 4

Find a basis for W consisting of two orthogonal vectors

b) Find bases for the four fundamental subspaces associated with the matrix 10

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 2 & -6 & 4 \\ -3 & 9 & 1 \end{bmatrix}$$

14 a) Compute the singular value decomposition of 8

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

b) Diagonalize the following matrix. if possible 6

$$\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

Module -3

15 a) Consider the function $f(x,y) = x^3 - xy + y^3$. Find local maximum, local minimum of f 8

b) Compute the Jacobian of the coordinate transformation $x = u^2 - u^4, y = uv$ 6

16 a) Find the Maclaurin series for $tan^{-1}x^2$ 7

b) Write out the first five terms of the Taylor series for $\sqrt{x}$ centered at x = 1. 7

Module -4

17 a) An engineering college has made a study of the grade-point averages of graduating engineers, denoted by the random variable Y. It is desired to study these as a function of high school grade-point averages, denoted by the random variable X. The joint probability distribution is shown, where the grade point averages have been combined into live categories for each variable 8

		X				
		2.0	2.5	3.0	3.5	4.0
	2.0	0.05	0	0	0	0
	2.5	0. 10	0.04	0	0.01	0
Y	3.0	0.02	0.10	0.05	0.10	0.01
	3.5	0	0	0.10	0.20	0.10
	4.0	0	0	0.05	0.02	0.05

Find the marginal distributions for X and Y

b) Find E(X) and E(Y) 4

c) Find $P(X \geq 3, Y \geq 3)$ 2

18 a) The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker? 7

b) Suppose A, B, and C are mutually independent events with probabilities P(A) = 0.5, P(B) = 0.8, and P(C) = 0.3. Find the probability that at least one of these events occurs 7

Module -5

19 a) Use Method of Steepest Descent to find the minimum of $f(x,y) = 4x^2 - 4xy + 2y^2$ with initial point $x_0 = (2,3)$ 8

b) Find the point on the curve $y^2 = 2x$ which is at a minimum distance from the point(1,4) 6

20 a) Compare and contrast different gradient descent algorithm 7

b) Define Quadratic programming. Discuss the advantages of quadratic programming 7

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