CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 206	**GRAPH THEORY**	**BSC**	**3**	**1**	**0**	**4**

Preamble: This course introduces fundamental concepts in Graph Theory, including properties and characterisation of graph/trees and graph theoretic algorithms, which are widely used in Mathematical modelling and has got applications across Computer Science and other branches in Engineering.

Prerequisite: The topics covered under the course Discrete Mathematical Structures (MAT 203)

Course Outcomes: After the completion of the course the student will be able to

CO 1	Explain vertices and their properties, types of paths, classification of graphs and trees & their properties. **(Cognitive Knowledge Level: Understand)**
CO 2	Demonstrate the fundamental theorems on Eulerian and Hamiltonian graphs. **(Cognitive Knowledge Level: Understand)**
CO 3	Illustrate the working of Prim's and Kruskal's algorithms for finding minimum cost spanning tree and Dijkstra's and Floyd-Warshall algorithms for finding shortest paths. **(Cognitive Knowledge Level: Apply)**
CO 4	Explain planar graphs, their properties and an application for planar graphs. **(Cognitive Knowledge Level: Apply)**
CO 5	Illustrate how one can represent a graph in a computer. **(Cognitive Knowledge Level: Apply)**
CO 6	Explain the Vertex Color problem in graphs and illustrate an example application for vertex coloring. **(Cognitive Knowledge Level: Apply)**

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	√	√	√							√		√
CO 2	√	√	√	√						√		√
CO 3	√	√	√	√						√		√
CO 4	√	√	√	√						√		√
CO 5	√	√	√							√		√
CO 6	√	√	√			√				√		√

Abstract POs defined by National Board of Accreditation			
PO#	**Broad PO**	**PO#**	**Broad PO**
PO1	Engineering Knowledge	**PO7**	Environment and Sustainability
PO2	Problem Analysis	**PO8**	Ethics
PO3	Design/Development of solutions	**PO9**	Individual and team work
PO4	Conduct investigations of complex problems	**PO10**	Communication
PO5	Modern tool usage	**PO11**	Project Management and Finance
PO6	The Engineer and Society	**PO12**	Life long learning

Assessment Pattern

Bloom's Category	**Continuous Assessment Tests (%)**		**End Semester Examination (%)**
	1	**2**	
Remember	30	30	30
Understand	30	30	30
Apply	40	40	40
Analyse			
Evaluate			
Create			

Mark Distribution

Total Marks	CIE Marks	ESE Marks	ESE Duration
150	50	100	3 hours

Continuous Internal Evaluation Pattern:

Attendance : 10 marks

Continuous Assessment Tests : 25 marks

Continuous Assessment Assignment : 15 marks

Internal Examination Pattern:

Each of the two internal examinations has to be conducted out of 50 marks

First Internal Examination shall be preferably conducted after completing the first half of the syllabus and the Second Internal Examination shall be preferably conducted after completing remaining part of the syllabus.

There will be two parts: Part A and Part B. Part A contains 5 questions (preferably, 2 questions each from the completed modules and 1 question from the partly covered module), having 3 marks for each question adding up to 15 marks for part A. Students should answer all questions from Part A. Part B contains 7 questions (preferably, 3 questions each from the completed modules and 1 question from the partly covered module), each with 7 marks. Out of the 7 questions in Part B, a student should answer any 5.

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer anyone. Each question can have maximum 2 sub-divisions and carries 14 marks.

Syllabus

Module 1

Introduction to Graphs : Introduction- Basic definition – Application of graphs – finite, infinite and bipartite graphs – Incidence and Degree – Isolated vertex, pendant vertex and Null graph. Paths and circuits – Isomorphism, sub graphs, walks, paths and circuits, connected graphs, disconnected graphs and components.

Module 2

Eulerian and Hamiltonian graphs : Euler graphs, Operations on graphs, Hamiltonian paths and circuits, Travelling salesman problem. Directed graphs – types of digraphs, Digraphs and binary relation, Directed paths, Fleury's algorithm.

Module 3

Trees and Graph Algorithms : Trees – properties, pendant vertex, Distance and centres in a tree - Rooted and binary trees, counting trees, spanning trees, Prim's algorithm and Kruskal's algorithm, Dijkstra's shortest path algorithm, Floyd-Warshall shortest path algorithm.

Module 4

Connectivity and Planar Graphs : Vertex Connectivity, Edge Connectivity, Cut set and Cut Vertices, Fundamental circuits, Planar graphs, Kuratowski's theorem (proof not required), Different representations of planar graphs, Euler's theorem, Geometric dual.

Module 5

Graph Representations and Vertex Colouring : Matrix representation of graphs- Adjacency matrix, Incidence Matrix, Circuit Matrix, Path Matrix. Coloring- Chromatic number, Chromatic polynomial, Matchings, Coverings, Four color problem and Five color problem. Greedy colouring algorithm.

Text book:

1. Narsingh Deo, Graph theory, PHI,1979

Reference Books:

1. R. Diestel, *Graph Theory,* free online edition, 2016: diestel-graph-theory.com/basic.html.
2. Douglas B. West, Introduction to Graph Theory, Prentice Hall India Ltd.,2001
3. Robin J. Wilson, Introduction to Graph Theory, Longman Group Ltd.,2010
4. J.A. Bondy and U.S.R. Murty. Graph theory with Applications

Sample Course Level Assessment Questions.

Course Outcome 1 (CO1):

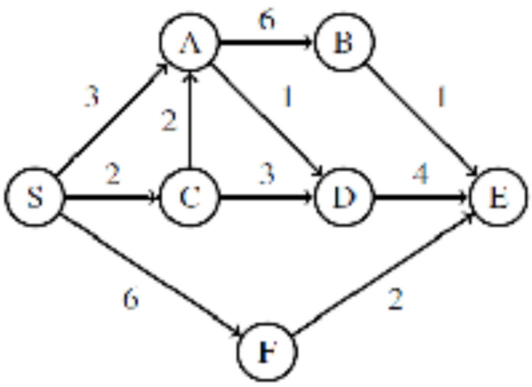
1. Differentiate a walk, path and circuit in a graph.
2. Is it possible to construct a graph with 12 vertices such that two of the vertices have degree 3 and the remaining vertices have degree 4? Justify
3. Prove that a simple graph with n vertices must be connected, if it has more than $\frac{(n-1)(n-2)}{2}$ edges.
4. Prove the statement: If a graph (connected or disconnected) has exactly two odd degree, then there must be a path joining these two vertices.

Course Outcome 2 (CO2):

1. Define Hamiltonian circuit and Euler graph. Give one example for each.
2. Define directed graphs. Differentiate between symmetric digraphs and asymmetric digraphs.
3. Prove that a connected graph G is an Euler graph if all vertices of G are of even degree.
4. Prove that a graph G of n vertices always has a Hamiltonian path if the sum of the degrees of every pair of vertices Vi, Vj in G satisfies the condition $d(Vi) + d(Vj) = n - 1$

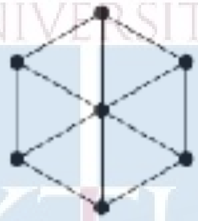
Course Outcome 3 (CO3):

1. Discuss the centre of a tree with suitable example.
2. Define binary tree. Then prove that number of pendant vertices in a binary tree is $\frac{(n+1)}{2}$
3. Prove that a tree with n vertices has $n - 1$ edges.
4. Explain Floyd Warshall algorithm.
5. Run Dijkstra's algorithm on the following directed graph, starting at vertex ***S***.



Course Outcome 4 (CO4):

1. Define edge connectivity, vertex connectivity and separable graphs. Give an example for each.
2. Prove that a connected graph with ***n*** vertices and e edges has $e - n + 2$ edges.
3. Prove the statement: Every cut set in a connected graph G must also contain at least one branch of every spanning tree of G.
4. Draw the geometrical dual ***(G*)*** of the graph given below, also check whether ***G*** and **G*** are self-duals or not, substantiate your answer clearly.



Course Outcome 5 (CO5):

1. Show that if ***A(G)*** is an incidence matrix of a connected graph ***G*** with ***n*** vertices, then rank of ***A(G)*** is ***n−1***.
2. Show that if ***B*** is a cycle matrix of a connected graph ***G*** with ***n*** vertices and ***m*** edges, then rank ***B = m−n+1***.
3. Derive the relations between the reduced incidence matrix, the fundamental cycle matrix, and the fundamental cut-set matrix of a graph ***G***.
4. Characterize simple, self-dual graphs in terms of their cycle and cut-set matrices.

Course Outcome 6 (CO6):

1. Show that an n vertex graph is a tree iff its chromatic polynomial is $Pn(\lambda) = \lambda(\lambda - 1)^{n-1}$
2. Prove the statement: "A covering g of a graph is minimal if g contains no path of length three or more."
3. Find the chromatic polynomial of the graph

Model Question paper

QP Code :

Total Pages: 4

Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

IV SEMESTER B.TECH DEGREE EXAMINATION, MONTH and YEAR

Course Code: MAT 206

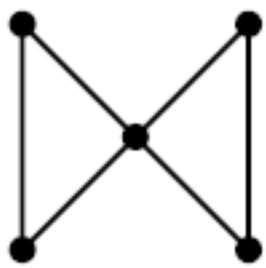
Course Name: GRAPH THEORY

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries3 marks. Marks

1	Construct a simple graph of 12 vertices with two of them having degree 1, three having degree 3 and the remaining seven having degree 10.	(3)
2	What is the largest number of vertices in a graph with 35 edges, if all vertices are of degree at least 3 ?	(3)
3	Define a Euler graph. Give an example of Eulerian graph which is not Hamiltonian	(3)
4	Give an example of a strongly connected simple digraph without a directed Hamiltonian path.	(3)
5	What is the sum of the degrees of any tree of ***n*** vertices?	(3)
6	How many spanning trees are there for the following graph	(3)



7 Show that in a simple connected planar graph ***G*** having ***V***-vertices, ***E***-edges, and no triangles $E <= 3V - 6$. (3)

8 Let G be the following disconnected planar graph. Draw its dual G*, and the dual of the dual (G*)*. (3)

G

9 Consider the circuit matrix ***B*** and incidence matrix ***A*** of a simple connected graph whose columns are arranged using the same order of edges. Prove that every row of ***B*** is orthogonal to every row of ***A***? (3)

10 A graph is *critical* if the removal of any one of its vertices (and the edges adjacent to that vertex) results in a graph with a lower chromatic number. Show that K_n is critical for all $n > 1$. (3)

PART B

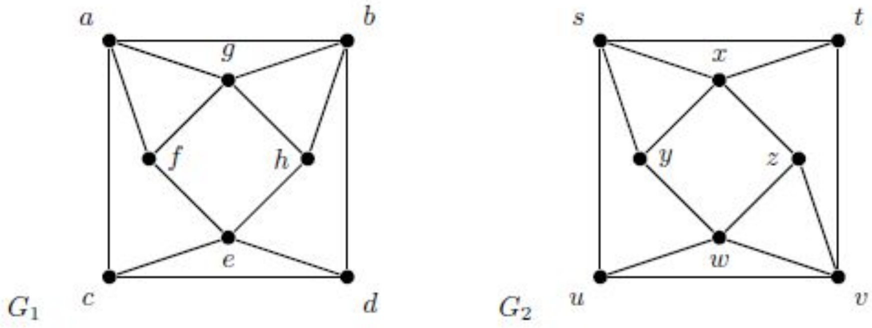
Answer any one Question from each module. Each question carries 14 Marks

11 a) Prove that for any simple graph with at least two vertices has two vertices of the same degree. (6)

b) Prove that in a complete graph with ***n*** vertices there are **(n-1)/2** edge disjoint Hamiltonian circuits and $n >= 3$ (8)

OR

12 a) Determine whether the following graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic or not. Give justification. (6)



b) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges (8)

13 a) Let S be a set of 5 elements. Construct a graph G whose vertices are subsets of S of size 2 and two such subsets are adjacent in G if they are disjoint. (8)

i. Draw the graph G.

ii. How many edges must be added to G in order for G to have a Hamiltonian cycle?

b) Let G be a graph with exactly two connected components, both being Eulerian. What is the minimum number of edges that need to be added to G to obtain an Eulerian graph? (6)

OR

14 a) Show that a k-connected graph with no hamiltonian cycle has an independent set of size $k + 1$. (8)

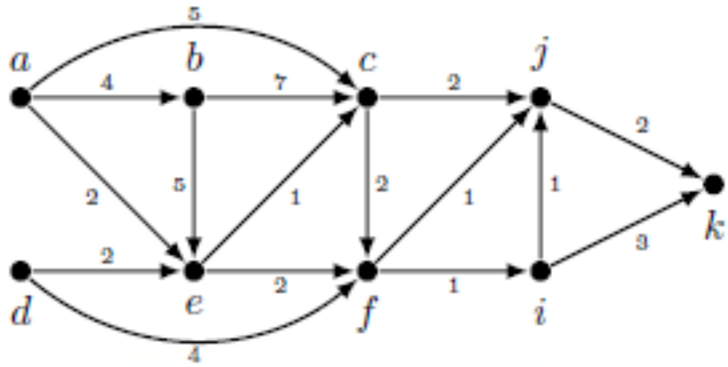
b) i. Let G be a graph that has exactly two connected components, both being Hamiltonian graphs. Find the minimum number of edges that one needs to add to G to obtain a Hamiltonian graph. (6)

ii. For which values of n the graph Q_n (hyper-cube on n vertices) is Eulerian.

15 a) A tree T has at least one vertex v of degree 4, and at least one vertex w of degree 3. Prove that T has at least 5 leaves. (5)

b) Write Dijkstra's shortest path algorithm. (9)

Consider the following weighted directed graph ***G***.



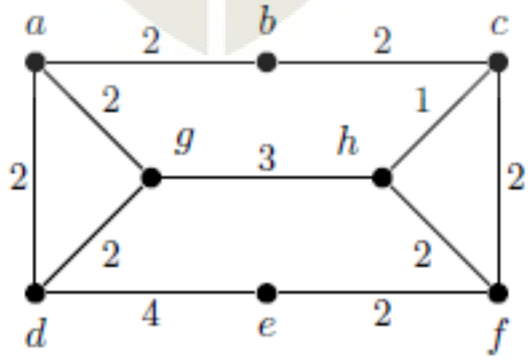
Find the shortest path between ***a*** and every other vertices in ***G*** using Dijkstra's shortest path algorithm.

OR

16 a) Define pendent vertices in a binary tree? Prove that the number of pendent vertices in a binary tree with ***n*** vertices is ***(n+1)/2.*** (5)

b) Write Prim's algorithm for finding minimum spanning tree. (9)

Find a minimum spanning tree in the following weighted graph, using Prim's algorithm.



Determine the number of minimum spanning trees for the given graph.

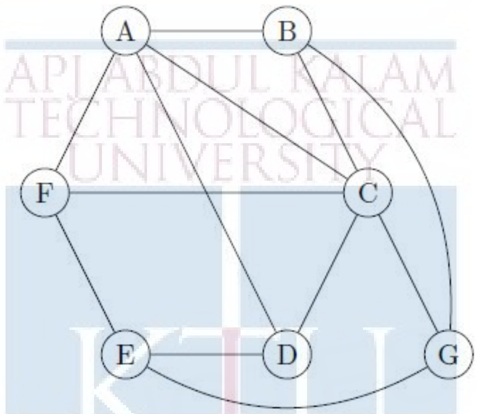
17 a) i. State and prove Euler's Theorem relating the number of faces, edges and vertices for a planar graph. (9)

ii. If ***G*** is a 5-regular simple graph and **|*V*| = 10**, prove that ***G*** is non-planar.

b) Let ***G*** be a connected graph and ***e*** an edge of ***G***. Show that ***e*** is a cut-edge if and only if ***e*** belongs to every spanning tree. (5)

OR

18 a) State Kuratowski's theorem, and use it to show that the graph G below is not planar. Draw G on the plane without edges crossing. Your drawing should use the labelling of the vertices given. (9)



b) Let ***G*** be a connected graph and ***e*** an edge of ***G***. Show that ***e*** belongs to a loop if and only if ***e*** belongs to no spanning tree. (5)

19 a) Define the circuit matrix $B(G)$ of a connected graph ***G*** with ***n*** vertices and ***e*** edges with an example. Prove that the rank of $B(G)$ is $e-n+1$ (7)

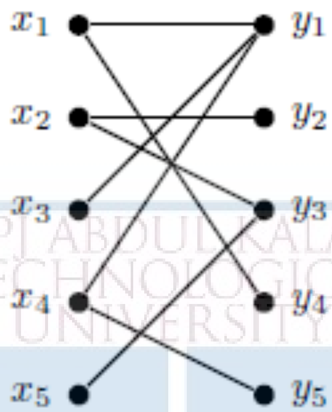
b) Give the definition of the chromatic polynomial $P_G(k)$. Directly from the definition, prove that the chromatic polynomials of W_n and C_n satisfy the identity $P_{W_n}(k) = k\, P_{C_{n-1}}(k-1)$. (7)

OR

20 a) Define the incidence matrix of a graph ***G*** with an example. Prove that the rank of an incidence matrix of a connected graph with ***n*** vertices is ***n-1.*** (4)

b) i. A graph ***G*** has chromatic polynomial $P_G(k) = k^4-4k^3+5k^2-2k$. How many vertices and edges does ***G*** have? Is ***G*** bipartite? Justify your answers.

ii. Find a maximum matching in the graph below and use Hall's theorem to show that it is indeed maximum. (10)



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Assignments

Assignment must include applications of the above theory in Computer Science.

Teaching Plan		
No	Topic	No. of Lectures
1	**Module-I (Introduction to Graphs)**	**(8)**
1.	Introduction- Basic definition – Application of graphs – finite and infinite graphs, bipartite graphs,	1
2.	Incidence and Degree – Isolated vertex, pendent vertex and Null graph	1
3.	Paths and circuits	1
4.	Isomorphism	1
5.	Sub graphs, walks	1
6.	Paths and circuits	1
7.	Connected graphs.	1
8.	Disconnected graphs and components	1
2	**Module-II (Eulerian and Hamiltonian graphs)**	**(8)**
1.	Euler graphs	1
2.	Operations on graphs	1
3.	Hamiltonian paths and circuits	1
4.	Hamiltonian paths circuits	1
5.	Travelling salesman problem	1
6.	Directed graphs – types of digraphs,	1
7.	Digraphs and binary relation, Directed paths	1
8.	Fleury's algorithm	1
3	**Module-III (Trees and Graph Algorithms)**	(11)
1.	Trees – properties	1
2.	Trees – properties	1
3.	Trees – properties, pendent vertex	1
4.	Distance and centres in a tree	1

5.	Rooted and binary tree	1
6.	Counting trees	1
7.	Spanning trees, Fundamental circuits	1
8.	Prim’s algorithm	1
9.	Kruskal’s algorithm	1
10.	Dijkstra’s shortest path algorithm	1
11.	Floyd-Warshall shortest path algorithm	1
4	**Module-IV (Connectivity and Planar Graphs)**	(9)
1.	Vertex Connectivity, Edge Connectivity	1
2.	Cut set and Cut Vertices	1
3.	Fundamental circuits	1
4.	Fundamental circuits	1
5.	Planar graphs	1
6.	Kuratowski’s theorem	1
7.	Different representations of planar graphs	1
8.	Euler's theorem	1
9.	Geometric dual	1
5	**Module-V (Graph Representations and Vertex Colouring)**	(9)
1.	Matrix representation of graphs- Adjacency matrix, Incidence Matrix	1
2.	Circuit Matrix, Path Matrix	1
3.	Colouring- chromatic number,	1
4.	Chromatic polynomial	1
5.	Matching	1
6.	Covering	1
7.	Four colour problem and five colour problem	1

8.	Four colour problem and five colour problem	1
9.	Greedy colouring algorithm.	1