Wilcoxon Signed-Rank Test

The **Signed-Rank Test**, specifically the **Wilcoxon Signed-Rank Test**, is a **non-parametric** statistical test used to compare **two related samples** or repeated measurements on a single sample to assess whether their population mean ranks differ. It is often used as a **non-parametric alternative to the paired t-test** when the data cannot be assumed to be normally distributed.

✅ **When to Use the Wilcoxon Signed-Rank Test:**

- You have **paired data** (e.g., before-and-after treatment for the same subject).
- The **differences** between pairs are **symmetrically distributed** (not necessarily normally distributed).
- The measurement scale is at least **ordinal**.

🔦 **Hypotheses:**

- **Null hypothesis (H_0):** The median difference between the paired observations is zero.
- **Alternative hypothesis (H_1):**
  - Two-sided: The median difference is not zero.
  - One-sided: The median difference is greater than or less than zero.

A nutritionist claims that the median daily protein intake of a group of athletes is **80 grams**. A random sample of 10 athletes yields the following daily protein intakes (in grams):

[85, 78, 82, 79, 90, 76, 88, 84, 83, 77]

Test at the 5% level whether the median intake differs from 80 grams using the **Wilcoxon Signed-Rank Test**.

Step 1: Calculate Differences from Hypothesized Median (80)

Athlete	Intake	Difference (Intake - 80)
1	85	+5
2	78	-2
3	82	+2
4	79	-1
5	90	+10
6	76	-4
7	88	+8
8	84	+4
9	83	+3
10	77	-3

No differences are zero, so we keep all 10.

Step 2: Rank Absolute Differences

Difference	Absolute	Rank	Sign
+1	1	1	+
-2	2	2.5	-
+2	2	2.5	+
-3	3	4	-
+3	3	4	+
-4	4	6	-
+4	4	6	+
+5	5	8	+
+8	8	9	+
+10	10	10	+

Step 3: Sum of Ranks

•**Positive ranks** = 1 + 2.5 + 4 + 6 + 8 + 9 + 10 = **40.5**

•**Negative ranks** = 2.5 + 4 + 6 = **12.5**

Step 4: Test Statistic (W)

•W = **smaller of the two rank sums** = **12.5**

Step 5: Critical Value

•For **n = 10**, two-tailed test at α = 0.05 → critical value = **8** (from Wilcoxon signed-rank table)

Step 6: Decision

•Since **W = 12.5 > 8**, we **fail to reject H_o**.

A researcher wants to know whether a new teaching method affects students' performance. She measures the scores of 8 students before and after using the new method. The scores are:

Student	Before	After
1	60	65
2	72	70
3	75	78
4	68	70
5	80	85
6	65	64
7	70	72
8	74	76

At 5% significance level, test whether the new teaching method has a significant effect on students’ performance using the Wilcoxon Signed-Rank Test.

Step 1: Compute the Differences (After - Before)

Student	Before	After	Difference
1	60	65	+5
2	72	70	-2
3	75	78	+3
4	68	70	+2
5	80	85	+5
6	65	64	-1
7	70	72	+2
8	74	76	+2

Step 2: Exclude zero differences (none in this case).

Step 3: Compute Absolute Differences and Rank Them

Student	Difference	Absolute	Rank	Sign
6	-1	1	1	-
2	-2	2	3.5	-
4	+2	2	3.5	+
7	+2	2	3.5	+
8	+2	2	3.5	+
3	+3	3	6	+
1	+5	5	7.5	+
5	+5	5	7.5	+

Step 4: Calculate Sum of Positive and Negative Ranks

- **Positive Ranks** = 3.5 + 3.5 + 3.5 + 6 + 7.5 + 7.5 = **31.5**
- **Negative Ranks** = 1 + 3.5 = **4.5**

Step 5: Test Statistic W

- W = **smaller** of the two rank sums = **4.5**

Step 6: Decision Rule

- For **n = 8**, the **critical value** at $\alpha = 0.05$ (two-tailed) from the Wilcoxon signed-rank table is **4**.

Step 7: Conclusion

- Since **W = 4.5 > 4**, we **fail to reject H_o**.
- **Conclusion**: There is **no statistically significant** difference in scores before and after the teaching method at the 5% level.

Before	After
58.5	60.0
60.3	54.9
61.7	58.1
69.0	62.1
64.0	58.5
62.6	59.9
56.7	54.4
63.6	60.2
68.2	62.3
59.4	58.7