Week 1 Assignment

Which of the following is NOT true about an NP-Complete problem, $\mathcal{A}$?

(a) $\mathcal{A}$ admits a polynomial time verifier.

(b) There exists a deterministic polynomial-time algorithm for $\mathcal{A}$ that solves $\mathcal{A}$ optimally, assuming P$\neq$NP.

(c) All other problems in class NP can be polynomial-time reducible to $\mathcal{A}$.

(d) $\mathcal{A}$ belongs to the complexity class NP-Hard.

b.

Which of the following is an NP-Complete problem?

(a) 2-SAT

(b) Shortest path

(c) Longest Path

(d) Minimum spanning tree

c.

Which of the following is a characteristic of fixed parameter tractable (FPT) algorithm ?

(a) An FPT algorithm outputs an approximate solution with provable guarantee and runs in time $f(k) \cdot n^{\mathcal{O}(1)}$, where k is the given parameter.

(b) An FPT algorithm outputs a correct solution and works reasonably fast without provable guarantee.

(c) An FPT algorithm always outputs a correct solution and runs in time $f(k) \cdot n^{\mathcal{O}(1)}$, where k is the given parameter.

(d) An FPT algorithm outputs an optimal solution without provable guarantee.

c.

Assume that in the Integer Linear Programming (ILP) formulation of the Set cover problem, the integrality constraints are replaced with continuous decision variables. This approach is called as

(a) LP relaxation

(b) LP approximation

(c) LP rounding

(d) LP duality

a.

Let x^* and y^* denote the primal and dual optimal solution respectively. Consider the following statements:

A. Whenever $x_i^* > 0$, the corresponding dual constraint is tight for all $i \in [n]$.
B. Whenever $y_j^* > 0$, the corresponding primal constraint is tight for all $j \in [m]$.

Choose the correct option.

(a) Only Statement A is correct for any feasible solution
(b) Only Statement B is correct for any feasible solution
(c) Both A and B are correct
(d) None of A and B is correct

c.

For minimization problems like Set Cover, the optimal objective function value for the dual LP can <u>NEVER</u> be

(a) less than the optimal objective function value to its primal LP
(b) equal to the optimal objective function value to its primal ILP
(c) greater than the optimal objective function value to its primal ILP
(d) less than or equal to the optimal objective function value to its primal LP

c.

Consider the Set cover problem. We have an f approximation algorithm, $\mathcal{A}$ to solve Set cover, where f denotes the frequency of most frequent element. Let X be the value returned by $\mathcal{A}$. If OPT denotes the optimum solution for Set cover, then which of the following is true?

(a) X is at most f.OPT.
(b) X is at most OPT/f.
(c) X is at least f.OPT.
(d) X is at least OPT/f.

a.

Which of the following is <u>NOT</u> an NP-Complete problem?

(a) Knapsack
(b) Set Cover
(c) Integer Linear Programming
(d) Linear Programming

d.

Which of the following statements is true in the context of Primal-Dual Method for designing approximation algorithms?

(a) We maintain a primal infeasible solution and iteratively build a dual feasible solution.

(b) We maintain a dual feasible solution and iteratively make the primal assignment more feasible.

(c) Primal-Dual method cannot be used without a LP solver.

(d) Primal-Dual method are employed in problems like min cost max flow problem and bipartite matching, however, their only drawback is that they run slower than Dual Rounding.

b.

A **Vertex Cover** of a graph $G = (V, E)$ is a subset $V' \subseteq V$ such that every edge in E is incident to at least one vertex in V'. In other words, for every edge $(i, j) \in E$, at least one of the vertices i or j is in the vertex cover V'. Which of the following exactly depicts the Integer Linear Programming for the **Vertex Cover** Problem?

(a) $\max \sum_{i \in V} x_i$
subject to $x_i + x_j \geqslant 1 \quad \forall (i, j) \in E$
$0 \leqslant x_i \leqslant 1 \quad \forall i \in V$

(b) $\min \sum_{i \in V} x_i$
subject to $x_i + x_j = 1 \quad \forall (i, j) \in E$
$x_i \in \{0, 1\} \quad \forall i \in V$

(c) $\min \sum_{i \in V} x_i$
subject to $x_i + x_j \geqslant 1 \quad \forall (i, j) \in E$
$x_i \in \{0, 1\} \quad \forall i \in V$

(d) $\min \sum_{i \in V} x_i$
subject to $x_i + x_j \geqslant 1 \quad \forall (i, j) \in E$
$0 \leqslant x_i \leqslant 1 \quad \forall i \in V$

c.