In the uncapacited facility location problem,

(a) w_{ij} corresponds to the dual constraint $v_j + w_{ij} \leqslant c_{ij}$, and x_{ij} corresponds to the primal constraint $x_{ij} \geqslant y_i$.

(b) w_{ij} corresponds to the primal constraint $x_{ij} \geqslant y_j$, and x_{ij} corresponds to the dual constraint $v_i - w_{ij} \leqslant c_{ij}$.

(c) w_{ij} corresponds to the primal constraint $x_{ij} \leqslant y_i$, and x_{ij} corresponds to the dual constraint $v_j - w_{ij} \leqslant c_{ij}$.

(d) w_{ij} corresponds to the dual constraint $v_j + w_{ij} \geqslant c_{ij}$, and x_{ij} corresponds to the primal constraint $x_{ij} \leqslant y_i$.

c.

Three coins are tossed simultaneously. What is the probability that the third coin would show a head given that the first and second coin have shown a head and a tail respectively?

(a) $\frac{1}{4}$

(b) $\frac{3}{4}$

(c) $\frac{1}{8}$

(d) $\frac{1}{2}$

d.

In the weighted MAXSAT problem, the input consists of:

▷ n Boolean variables $x_1, x_2, \ldots, x_n$,

▷ m clauses $C_1, C_2, \ldots, C_m$ (each of which consists of a disjunction (that is, an "or") of some number of the variables and their negations and

▷ a non-negative weight w_j for each clause C_j.

The objective of the problem is to find an assignment of true/false to the x_i that maximizes the weight of the satisfied clauses.

Which of the following is true?

(a) Setting each x_i to true independently with probability $\frac{1}{2}$ gives a randomized $\frac{1}{2}$ -approximation algorithm.

(b) Pr[clause C_j satisfied] $= \left(1 - \left(\frac{1}{2}\right)^{l_j}\right) \leqslant \frac{1}{2}$, where l_j is the number of literals in clause j

(c) $\sum_{j=1}^{m} w_j$ Pr[clause C_j satisfied] $\leqslant \frac{1}{2} \sum_{j=1}^{m} w_j \leqslant \frac{1}{2}$OPT

(d) If there is an $(\frac{7}{8} - \varepsilon)$-approximation algorithm for MAX E3SAT for any constant $\varepsilon > 0$, then $P = NP$.

a.

Which of the following captures the Linear programming relaxation for MAXSAT? Assume standard notations discussed in the lecture.

(a) maximize $\sum_{j=1}^{m} w_j z_j$
subject to $\sum_{i \in P_j} y_i + \sum_{i \in N_j} (1 - y_i) \geqslant z_j, \quad \forall C_j = \bigvee_{i \in P_j} x_i \vee \bigvee_{i \in N_j} \overline{x_i},$
$y_i \in \{0, 1\}, \quad i = 1, \ldots, n,$
$0 \leqslant z_j \leqslant 1, \quad j = 1, \ldots, m$

(b) maximize $\sum_{j=1}^{m} w_j z_j$
subject to $\sum_{i \in P_j} y_i + \sum_{i \in N_j} (1 - y_i) \geqslant z_j, \quad \forall C_j = \bigvee_{i \in P_j} x_i \vee \bigvee_{i \in N_j} \overline{x_i},$
$0 \leqslant y_i \leqslant 1, \quad i = 1, \ldots, n$
$0 \leqslant z_j \leqslant 1, \quad j = 1, \ldots, m$

(c) maximize $\sum_{j=1}^{m} w_j z_j$
subject to $\sum_{i \in P_j} (1 - y_i) + \sum_{i \in N_j} y_i \geqslant z_j, \quad \forall C_j = \bigvee_{i \in P_j} x_i \vee \bigvee_{i \in N_j} \overline{x_i},$
$y_i \in \{0, 1\}, \quad i = 1, \ldots, n,$
$0 \leqslant z_j \leqslant 1, \quad j = 1, \ldots, m$

(d) maximize $\sum_{j=1}^{m} w_j z_j$
subject to $\sum_{i \in P_j} (1 - y_i) + \sum_{i \in N_j} y_i \leqslant z_j, \quad \forall C_j = \bigvee_{i \in P_j} x_i \vee \bigvee_{i \in N_j} \overline{x_i},$
$y_i \in \{0, 1\}, \quad i = 1, \ldots, n,$
$0 \leqslant z_j \leqslant 1, \quad j = 1, \ldots, m$

b.

If $a_1, a_2, \ldots, a_k$ are positive real numbers, which of the following statements is true according to the Arithmetic-Geometric Mean inequality?

(a) $\left(\prod_{i=1}^{k} a_i\right)$ is always less than or equal to $(\frac{1}{k} \sum_{i=1}^{k} a_i)^{1/k}$.

(b) $\left(\prod_{i=1}^{k} a_i\right)$ is always less than or equal to $(\frac{1}{k} \sum_{i=1}^{k} a_i)^{k}$.

(c) $\left(\prod_{i=1}^{k} a_i\right)^{1/k}$ is always equal to $\frac{1}{k} \sum_{i=1}^{k} a_i$.

(d) $\left(\prod_{i=1}^{k} a_i\right)^{1/k}$ is always greater than or equal to $\frac{1}{k} \sum_{i=1}^{k} a_i$.

b.

Consider the statements given below:

i *A convex combination of* $x_1, x_2, \cdots, x_n$ *is* $a_1.x_1 + a_2.x_2 + \cdots + a_n.x_n$ *where each* $a_i \in \mathbb{Z}_{>0}$ $\forall i \in [n]$, *and* $\sum_{i=1}^{n} a_i = 1$.

ii $\sum_{i=1}^{n} a_i.x_i \leqslant \max(x_1, x_2, \cdots, x_n)$

iii $\sum_{i=1}^{n} a_i.x_i \geqslant \min(x_1, x_2, \cdots, x_n)$

Which of the following is <u>INCORRECT</u> here?

(a) Statement (i)
(b) Statement (ii)
(c) Statement (iii)
(d) Statements (ii) and (iii)

a.

Let Φ be a CNF formula consisting of m clauses and n variables such that length of each clause is at most 3. Consider the algorithm given below.

Algorithm 1 Approx-Max3SAT(Φ)

```
for i = 1 to n do
    Flip a fair coin
    if Heads then
        x_i ← true
    else
        x_i ← false
    end if
end for
return x
```

Which of the following is **INCORRECT** about Algorithm 1?

(a) The running time of Approx-Max3SAT is $\mathcal{O}(n)$.

(b) The expected number of clauses satisfied is exactly $\frac{m}{2}$ for any instance of 3-SAT.

(c) For any instance of 3-SAT, there exists a truth assignment that satisfies at least $\frac{7}{8}$ of the clauses.

(d) Any instance of 3-SAT with at most 7 clauses is satisfiable.

b.

Consider the Linear Programming relaxation of weighted MAXSAT problem (see Question 4). Let (y^*, z^*) be an optimal solution to the LP relaxation. Independently setting each variable x_i to 1 with probability y_i^* provides a approximation guarantee of at least

(a) 1

(b) $\frac{7}{8}$

(c) 0.63

(d) $\frac{15}{16}$

c.

Consider the following statements given below.

i The process of taking a randomized algorithm and turning it into a deterministic algorithm is called Pruning.

ii The process of taking a randomized algorithm and turning it into a deterministic algorithm is called Derandomization.

iii The process of taking a randomized algorithm and turning it into a deterministic algorithm can be done with the help of a separation oracle.

iv The process of taking a randomized algorithm and turning it into a deterministic algorithm can be done by method of conditional expectation.

(a) Only i and iii is true

(b) Only i and iv are true

(c) Only ii and iii are true

(d) Only ii and iv are true

d.

Consider the Maximum Cut (MAXCUT) problem on an undirected graph $G = (V, E)$ with non-negative edge weights $w_{ij} \geqslant 0$ for each edge $(i, j) \in E$. A randomized approximation algorithm is applied, where each vertex $v \in V$ is placed into one of two sets U and $W = V - U$ independently. Let Z be the random variable representing the total weight of edges in the cut, and let OPT denote the optimal value of the MAXCUT instance. Which of the following statements is **NOT** true regarding this algorithm?

(a) The probability that any specific edge (i, j) is in the cut is $\frac{1}{4}$, since the endpoints are placed in different sets independently with probability $\frac{1}{2}$ each.

(b) The probability that any specific edge (i, j) is in the cut is $\frac{1}{2}$, since the endpoints are placed in different sets independently with probability $\frac{1}{2}$.

(c) $\mathbb{E}[Z] = \sum_{(i,j)\in E} w_{ij}\mathbb{E}[X_{ij}]$

(d) $\mathbb{E}[Z] \geqslant \frac{1}{2}\text{OPT}$

a.