Suppose the integrality gap of a particular LP relaxation is $\mathcal{O}(\log n)$. What does this imply?

(a) All approximation algorithms using any arbitrary LP relaxation will have an approximation guarantee of $\Omega(\log n)$ at best.

(b) All approximation algorithms using this particular LP relaxation will have an approximation guarantee of $\Omega(\log n)$ at best, but there can be other LP relaxations, which can possibly guarantee an $o(\log n)$ approximation factor.

(c) All approximation algorithms using any arbitrary LP relaxation will have an approximation guarantee of $\omega(\log n)$ at best.

(d) All approximation algorithms using this particular LP relaxation will have an approximation guarantee of $\omega(\log n)$ at best, but there can be other LP relaxations, which can possibly guarantee an $\mathcal{O}(\log n)$ approximation factor.

b.

Which of the following is true regarding combinatorial algorithms?

(a) Combinatorial algorithms use a much faster polynomial-time Integer Linear Programming solver instead of LP solver.

(b) Deterministic Rounding and Randomized Rounding methods are considered to be combinatorial algorithms

(c) Primal-dual algorithms do not require us to solve Linear Programs, and hence can be considered to be combinatorial algorithms

(d) Combinatorial Algorithms are theoretically much less desirable than algorithms based on LP-solver.

c.

Let y be the dual-LP variables of the dual of the LP relaxation of the General Steiner tree/forest problem on an input graph $G(V, E)$ and pairs of vertices $(s_1, t_1), \ldots, (s_k, t_k)$, discussed in class. What are the exact dual variables?

(a) y_S, for each subset $S \subseteq V$ that contains either s_i or t_i but not both, for some i.

(b) y_Z, for each subset $Z \subseteq E$ that contains at least one edge with endpoint either s_i or t_i but not both, for some i.

(c) y_v for each vertex $v \in V$.

(d) y_e for each edge $e \in E$.

a.

Recall the 2-factor primal-dual algorithm discussed in class for the generalized Steiner tree/forest problem. The algorithm first computes F, a subset of edges, using primal-dual method, which connects s_i and t_i for all i. Then the algorithm improves F to F' by removing edges one by one (in reverse order in which they were added), whenever it is possible to remove an edge and still keep it to be a feasible solution. Which of the following is true regarding F and F'?

(a) F and F' must be forests.

(b) F can have cycles, but F' must be a forest.

(c) F must be a forest, but F' can have cycles.

(d) F and F' can both have cycles.

a.

Consider the Generalized Steiner Forest problem where $(s_1, t_1), (s_2, t_2), \ldots, (s_k, t_k)$ are the pairs of terminals. Recall the following lemma discussed in class:

Lemma 1. *For any $\mathcal{C}$ in any iteration of the algorithm,*

$$\sum_{C \in \mathcal{C}} |\delta(C) \cap F'| \leqslant 2|\mathcal{C}|$$

What is $\mathcal{C}$ here?

(a) The final set of connected components C after the algorithm terminated.

(b) Set of connected components C such that for all $i \in \{1, 2, \ldots, k\}$, $|C \cap \{s_i, t_i\}| = 1$.

(c) Set of connected components C such that there exists $i \in \{1, 2, \ldots, k\}$, $|C \cap \{s_i, t_i\}| = 1$.

(d) Set of connected components C such that $|C \cap \{s_1, t_1, s_2, t_2, \ldots, s_k, t_k\}| = k$.

c.

Recall the proof of **Lemma 1** which states

Lemma 1. *For any $\mathcal{C}$ in any iteration of the algorithm,*

$$\sum_{C \in \mathcal{C}} |\delta(C) \cap F'| \leqslant 2|\mathcal{C}|$$

The proof contained an argument to *'contract'* a subset C of vertices. What does it mean to contract C.

(a) Delete the entire subset C, and for each pair of nodes $u, v \notin C$ where u, v both had a neighbour in C, connect u, v by an edge.

(b) Replace the entire subset C by a vertex v, and for each node $u \notin C$ which had a neighbour in C, connect u, v by an edge.

(c) Delete all edges with both endpoints lying in C.

(d) Delete all vertices in C.

b.

Which of the following is <u>**NOT**</u> a metric $d : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}_{\geqslant 0}$?

(a) $d((x, y), (x', y')) = |x - x'| + |y - y'|$

(b) $d((x, y), (x', y')) = \max(|x - x'|, |y - y'|)$

(c) $d((x, y), (x', y')) = \min(|x - x'|, |y - y'|)$

(d) $d((x, y), (x', y')) = \sqrt{(x - x')^2 + (y - y')^2}$

c.

What is the weight of the minimum generalized Steiner forest on $(s_1, t_1) = (2, 4)$, and $(s_2.t_2) = (3, 5)$ on the graph shown below (Figure 1)?

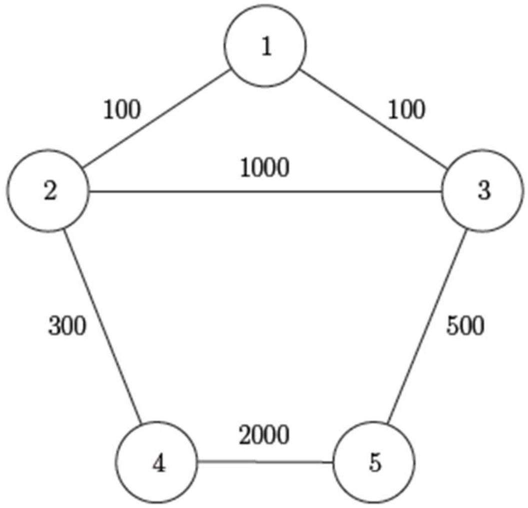

Figure 1: Problem 8

(a) 14
(b) 800
(c) 1000
(d) 1800

b.

For an undirected weighted Graph $G(V, E, (w_e)_{e \in E})$, for a cut $(S, V \setminus S)$, where $S \subseteq V$, which of the following is the correct definition of the semi-metric $d : V \times V \to \mathbb{R}_{\geqslant 0}$ associated with the cut $(S, V \setminus S)$?

(a)

$$d_{uv} = \begin{cases} w_{uv} & \text{if } uv \in E \text{ and } u, v \in S \\ 0 & \text{otherwise} \end{cases}$$

(b)

$$d_{uv} = \begin{cases} 1 & \text{if } u, v \in S \text{ or } u, v \in V \setminus S \\ 0 & \text{otherwise} \end{cases}$$

(c)

$$d_{uv} = \begin{cases} 1 & \text{if } |S \cap \{u, v\}| = 1 \\ 0 & \text{otherwise} \end{cases}$$

(d)

$$d_{uv} = \begin{cases} 1 & \text{if } uv \in E \text{ and } |S \cap \{u, v\}| = 1 \\ 0 & \text{otherwise} \end{cases}$$

d.

Which of the following is an **INCORRECT** step in the 2-factor approximation algorithm for the multiway cut problem separating $s_1, s_2, \ldots, s_k$? Assume that the graph is $G(V, E)$ and the weight of edge $e \in E$ is w_e.

(a) For each s_i, take the graph G_i which is constructed by adding a separate dummy node t_i.

(b) In G_i, t_i connects to every other node with edges, each of weight $2 \sum_{e \in E} w_e$.

(c) Compute the minimum weight $s_i - t_i$ cut in the graph G_i, let the set of edges be F_i.

(d) Output the set of edges which appear exactly once in $F_1, F_2, \ldots, F_k$ combined.

d.