Which of the following algorithms solves a Linear Program in polynomial time where access to a polynomial-time separation oracle is enough (no explicit description is needed)?

(a) Ellipsoid Method

(b) Simplex Algorithm

(c) Gaussian Elimination

(d) Newton's Method

a.

Consider the classical Steiner tree problem on the graph $G(V, E)$, with terminal set $T \subseteq V$. What would the prize π_v for all vertices $v \in V$, so that the prize collecting Steiner tree problem also yields the same output on $G(V, E)$?

(a) $\pi_v = 0$ for all $v \in V$

(b) $\pi_v = \infty$ for all $v \in V$

(c) $\pi_v = 0$ for all $v \in T$ and $\pi_v = \infty$ for all $v \in V \setminus T$

(d) $\pi_v = \infty$ for all $v \in T$ and $\pi_v = 0$ for all $v \in V \setminus T$

d.

Consider the following algorithm for prize collecting Steiner tree problem on the graph $G(V, E)$:

Algorithm 1 Prize Collecting Steiner Tree Approx Algorithm

1: Solve relaxed LP. Let (x^*, y^*) be the optimal solution
2: $U \leftarrow \{v \in V \mid y_v^* \geqslant \alpha\}$ for some $\alpha \in (0, 1)$
3: Build a min-cost Steiner tree T on the terminal set $U \subseteq V$
4: **return** T

Assume that the following hold true for the Steiner tree T computed at step 3 (assume standard notation discussed in class):

$$\sum_{e \in E[T]} c_e \leqslant \frac{2}{\alpha} \sum_{e \in E[G]} c_e x_e^* \text{ and } \sum_{v \in V \setminus V[T]} \pi_v \leqslant \frac{1}{1-\alpha} \sum_{v \in V} \pi_v (1 - y_v^*)$$

What would be the best possible approximation factor guarantee for arbitrary $\alpha \in (0, 1)$?

(a) 1.5

(b) 2

(c) $\max\left\{\frac{2}{\alpha}, \frac{1}{1-\alpha}\right\}$

(d) $\min\left\{\frac{2}{\alpha}, \frac{1}{1-\alpha}\right\}$

c.

Consider algorithm 1 in Problem 3, what is the best approximation guarantee achievable if we repeat this algorithm $|V|$ times, for each value of α in $\{y_v^* \mid v \in V\}$?

(a) 2

(b) 3

(c) $\dfrac{\sqrt{e}}{\sqrt{e}-1}$

(d) $\dfrac{\sqrt{5}+1}{2}$

a.

Which of the following can be considered as the objective function (to be **minimized**) of the Prize Collecting Steiner Tree problem for the graph $G(V, E)$? Let c_e be the cost of edge $e \in E$ and let π_v be the prize for vertex $v \in V$. Let x_e be the indicator variable that takes the value 1 if edge $e \in E$ is a part of the final output (0 otherwise), and let y_v be the indicator variable that takes the value 1 if vertex $v \in V$ is a part of the final output (0 otherwise).

(a) $\sum\limits_{e \in E} c_e x_e + \sum\limits_{v \in V} \pi_v y_v$

(b) $\sum\limits_{e \in E} c_e x_e + \sum\limits_{v \in V} \pi_v (1 - y_v)$

(c) $\sum\limits_{e \in E} c_e (1 - x_e) + \sum\limits_{v \in V} \pi_v y_v$

(d) $\sum\limits_{e \in E} c_e (1 - x_e) + \sum\limits_{v \in V} \pi_v (1 - y_v)$

b.

Consider the following constraint of the LP relaxation of the prize collecting Steiner tree problem.

$$\forall i \in V, \forall S \subseteq V \setminus \{i\}, r \in S, \sum_{e \in \delta(S)} x_e \geqslant y_i$$

Here, for subsets S of V, what does $\delta(S)$ mean?

(a) Set of edges with both endpoints in S

(b) Set of edges with at most one endpoint in S

(c) Set of edges with exactly one endpoint in S

(d) Set of edges with no endpoint in S

c.

Which of the following is **INCORRECT** regarding the dual LP formulation of the uncapacitated facility location problem? Assume F to be the set of facilities, D to be the set of clients, f_i to be the cost to open the i-th facility and $c_{i,j}$ to be the cost of assigning the j-th client to the i-th facility. Moreover, let v_j, $w_{i,j}$ be the dual variables as discussed in class.

(a) Objective is to maximize $\sum_{j \in D} v_j$

(b) One constraint is $\sum_{j \in D} w_{i,j} \leqslant f_i, \forall i \in F$

(c) Another constraint is $v_j \leqslant c_{i,j} - w_{i,j}, \forall i \in F, \forall j \in D$

(d) v_j can be unbounded and take any real value, $\forall j \in D$, but $w_{i,j} \geqslant 0, \forall i \in F, \forall j \in D$

c.

Which best represents the 'max-flow min-cut theorem'?

(a) max-flow = min-cut

(b) max-flow < min-cut

(c) max-flow > min-cut

(d) The edges where flow reaches the maximum in any max-flow, form the edges of a min-cut

a.

In the uncapacitated facility location problem, where F is the set of facilities, and D is the set of clients, let (x^*, y^*) be an optimal relaxed primal LP solution, as discussed in class. What did we mean by the natural definition 'neighbours of j', $N(j)$, for all $j \in D$?

(a) $N(j) = \{j' \in D \mid \exists i \text{ such that } x^*_{i,j} = x^*_{i,j'}\}$

(b) $N(j) = \{j' \in D \mid \exists i \text{ such that } x^*_{i,j} + x^*_{i,j'} < 1\}$

(c) $N(j) = \{i \in F \mid x^*_{i,j} = 0\}$

(d) $N(j) = \{i \in F \mid x^*_{i,j} > 0\}$

d.

In the uncapacitated facility location problem, which of the following statements did we get only after assuming metric property? As usual, D be the set of clients, F be the set of facilities and $c_{i,j}$ be the cost of assigning client $j \in D$ to facility $i \in F$.

(a) Each client is assigned to exactly one facility

(b) For all clients $j, j' \in D$ and facilities $i, i' \in F$, we have $c_{i,j} \leqslant c_{i,j'} + c_{i',j'} + c_{i',j}$

(c) Each facility is assigned to at least one client

(d) Number of facilities is equal to the number of clients

b.