

12.703

AI25BTECH11001 - ABHISEK MOHAPATRA

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**Question:** Let  $\mathbf{V} = \mathbf{p}(x) = a_0 + a_1x + a_2x^2 : a_i \in R$ . Define  $\mathbf{T} : V \rightarrow V$  by

$$\mathbf{T}(\mathbf{p}) = (\mathbf{p}(0) - \mathbf{p}(1)) + (\mathbf{p}(0) + \mathbf{p}(1))x + \mathbf{p}(0)x^2. \quad (0.1)$$

Then the sum of eigenvalues of  $\mathbf{T}$  equals.

**Solution:** Given,

$$\mathbf{T}(\mathbf{p}) = (\mathbf{p}(0) - \mathbf{p}(1)) + (\mathbf{p}(0) + \mathbf{p}(1))x + \mathbf{p}(0)x^2. \quad (0.2)$$

Or,

$$\mathbf{T}\mathbf{p} = \begin{pmatrix} (\mathbf{p}(0) - \mathbf{p}(1)) \\ (\mathbf{p}(0) + \mathbf{p}(1))x \\ \mathbf{p}(0)x^2 \end{pmatrix}. \quad (0.3)$$

where  $\mathbf{p} = \begin{pmatrix} a_0 \\ a_1x \\ a_2x^2 \end{pmatrix}$ .

So, sum of the eigenvalues of  $\mathbf{T}$  is  $\text{trace}(\mathbf{T})$ .

a solution can seen is

$$\begin{pmatrix} 0 & -\frac{1}{x} & -\frac{1}{x^2} \\ 2x & 1 & \frac{1}{x} \\ x^2 & 0 & 0 \end{pmatrix} \mathbf{p} = \begin{pmatrix} (\mathbf{p}(0) - \mathbf{p}(1)) \\ (\mathbf{p}(0) + \mathbf{p}(1))x \\ \mathbf{p}(0)x^2 \end{pmatrix}. \quad (0.4)$$

So the sum of eigenvalues = 1.