

12.35

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Let \mathbf{A} be a 2×2 real matrix with eigenvalues 1 and -1, and corresponding eigenvectors $\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$. If $\mathbf{A}^{2021} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find $a + b + c + d$ (round off to 2 decimal places).

Solution: Required,

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = a + b + c + d \quad (1)$$

As per Question,

$$\mathbf{A} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -1 \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (2)$$

$$\mathbf{A} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (3)$$

$$\mathbf{A}^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (4)$$

So,

$$\mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (5)$$

$$\begin{pmatrix} 1 & 1 \end{pmatrix} \mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -2 \quad (6)$$