

12.35

AI25BTECH11001 - ABHISEK MOHAPATRA

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**Question:** Let  $\mathbf{A}$  be a  $2 \times 2$  real matrix with eigenvalues 1 and -1, and corresponding eigenvectors  $\begin{pmatrix} \sqrt{3} \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ . If  $\mathbf{A}^{2021} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ , find  $a + b + c + d$  (round off to 2 decimal places).

**Solution:** Required,

$$(1 \quad 1) \mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = a + b + c + d \quad (0.1)$$

As per Question,

$$\mathbf{A} \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -1 \begin{pmatrix} 2 \\ 2 \end{pmatrix} \quad (0.2)$$

$$\mathbf{A} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (0.3)$$

$$\mathbf{A}^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (0.4)$$

So,

$$\mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (0.5)$$

$$(1 \quad 1) \mathbf{A}^{2021} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -1 (1 \quad 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix} = -2 \quad (0.6)$$