12.703

AI25BTECH11001 - ABHISEK MOHAPATRA

Question: Let $\mathbf{V} = \mathbf{p}(x) = a_0 + a_1 x + a_2 x^2 : a_i \epsilon R$. Define $\mathbf{T} : V \to V$ by

$$\mathbf{T}(\mathbf{p}) = (\mathbf{p}(0) - \mathbf{p}(1)) + (\mathbf{p}(0) + \mathbf{p}(1))x + \mathbf{p}(0)x^2. \tag{1}$$

Then the sum of eigenvalues of $\mathbf{T}$ equals.

Solution: Given,

$$\mathbf{T}(\mathbf{p}) = (\mathbf{p}(0) - \mathbf{p}(1)) + (\mathbf{p}(0) + \mathbf{p}(1))x + \mathbf{p}(0)x^2. \tag{2}$$

Or,

$$\mathbf{T}\mathbf{p} = \begin{pmatrix} (\mathbf{p}(0) - \mathbf{p}(1)) \\ (\mathbf{p}(0) + \mathbf{p}(1))x \\ \mathbf{p}(0)x^2 \end{pmatrix}. \tag{3}$$

where $\mathbf{p} = \begin{pmatrix} a_0 \\ a_1 x \\ a_2 x^2 \end{pmatrix}$.

So, sum of the eigenvalues of T is $trace(\mathbf{T})$.

a solution can seen is

$$\begin{pmatrix} 0 & -\frac{1}{x} & -\frac{1}{x^2} \\ 2x & 1 & \frac{1}{x} \\ x^2 & 0 & 0 \end{pmatrix} \mathbf{p} = \begin{pmatrix} (\mathbf{p}(0) - \mathbf{p}(1)) \\ (\mathbf{p}(0) + \mathbf{p}(1))x \\ \mathbf{p}(0)x^2 \end{pmatrix}. \tag{4}$$

So the sum of eigenvalues = 1.