Practical - 1

Topic: limits and continuity

$$\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$$

$$= \lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \times \frac{\sqrt{a+2x} +}{\sqrt{a+2x} +} \right.$$

$$= \lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(a-x).(\sqrt{a+2x} + \sqrt{3x})}$$

$$= \frac{1}{3} \frac{\sqrt{3a+a}}{\sqrt{a+2a}} + \frac{2\sqrt{a}}{\sqrt{3a}}.$$

$$= \frac{1}{3} \times \frac{\sqrt{4a}}{\sqrt{3a}} + \frac{2\sqrt{a}}{\sqrt{3a}}.$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}.$$

$$= \frac{2}{3\sqrt{3}}$$

Ex:

2) $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$= \lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$= \lim_{y\to 0} \frac{a+y-a}{y\sqrt{a+y}\left(\sqrt{a+y}+\sqrt{a}\right)}$$

$$= \frac{1}{\sqrt{a+0}\left(\sqrt{a+0}+\sqrt{a}\right)}$$

$$= \frac{1}{\sqrt{a}\left(\sqrt{a}+\sqrt{a}\right)}$$

$$= \frac{1}{\sqrt{a}\left(2\sqrt{a}\right)}$$

$$= \frac{1}{2a}$$

3) $$\lim_{x\to \pi/6} \frac{\cos x-\sqrt{3}\sin x}{\pi-6x}.$$

By substituting $x-\frac{\pi}{6}=h.$

$$x = h+\frac{\pi}{6}$$

where $h\to 0.$

$$\lim_{h\to 0} \frac{\cos(h+\pi/6) - \sqrt{3}\sin(h+\pi/6)}{\pi - 6(h+\pi/6)}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \cdot \sin\pi/6 - \sqrt{3}\sin h \cdot \cos\pi/6 + \cos h \cdot \sin\pi/6}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0} \frac{\left(\cos h\frac{\sqrt{3}}{2} + \frac{\sin h}{2}\right) - \sqrt{3}\left(\frac{1}{2}\cos h - \frac{\sqrt{3}}{2}\sin h\right)}{6h}$$

$$\lim_{h\to 0} \frac{\frac{\sqrt{3}\cos h}{2} + \frac{\sin h}{2} - \frac{\sqrt{3}\cos h}{2} + \frac{3\sin h}{2}}{6h}$$

$$\lim_{h\to 0} \frac{4\sin h}{2(6h)}$$

$$\lim_{h\to 0} \frac{4\sin h}{12h} = \frac{1}{3}$$

4) $\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}}\right]$

By subtracting nr. and dr. b.t.s.

$$\lim_{x\to\infty} \left[\frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty} \left[\left(\frac{x^2+5-x^2+3}{x^2+3-x^2-1}\right) \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty} \frac{8}{2} \frac{(\sqrt{x^2+3} + \sqrt{x^2+1})}{(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2(1+3/x^2)} + \sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+5/x^2)} + \sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,

$= 4.$

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5(i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x < \frac{\pi}{2}$ $\Bigg\}$ at $x = \pi/2$.

$= \frac{\cos x}{\pi - 2x}$ for $\pi/2 < x < \pi$.

$f(\pi/2) = \frac{\sin 2(\pi/2)}{\sqrt{1-\cos^2(\pi/2)}}$ $\therefore f(\pi/2) = 0$.

f at $x = \frac{\pi}{2}$ defined.

(ii) $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\cos x}{\pi - 2x}$.

By substituting method,

$x - \frac{\pi}{2} = h$,

$x = h + \pi/2$.

where $h \to 0$.

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(2h + \frac{\pi}{2})}$

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$.

$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$.

$\lim_{h \to 0} \cancel{\frac{-\sin h}{-2h}}$

$= \frac{1}{2}$

$\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x \to \pi/2^-} \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2} - \frac{2 \sin x \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2^-} \frac{2 \sin x \cdot \cos x}{\sqrt{2} \sin x}$

$\lim_{x \to \pi/2} - \frac{2 \cos x}{\sqrt{2}}$

$\frac{R}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

LHL $\neq$ RHL.

$\therefore$ f is not contineous at $x = \pi/2$

$$f(x) = \begin{cases} \frac{x^2 - 9}{x-3} & 0 < x < 3 \\ x + 3 & 3 \le x < 6 \\ \frac{x^2-9}{x+3} & 6 \le x < 9 \end{cases}$$

at $x=3$ and $x=6$.

at $x = 3$.

$f(3) = \frac{x^2-9}{x-3} = 0$.

f at $x=3$ defined.

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$.

$f(3) = x+3 = 3+3 = 6.$

f is defined at $x=3$.

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6.$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3}.$

$\therefore$ LHL = RHL.

f is continous at $x=3$.

for $x=6$.

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3.$

2) $\lim_{x \to 6^+} \frac{x^2-9}{x+3}$

$\lim_{x \to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x \to 6^+} (x-3) = 6-3 = 3.$

$\lim_{x \to 6^+} x+3 = 3+6 = 9.$

$\therefore$ LHL $\neq$ RHL.

Function is not contenous.

6(i) $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & x<0 \\ k & x=0 \end{cases}$ at $x=0$.

Soln: f is continous at $x=0$.

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$\lim_{x \to 0} f(x) = f(0).$

$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = k.$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k.$

$2 \lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$

$2(2)^2 = k$

$k = 8.$

(ii) $f(x) = \left\{ \begin{array}{ll} (\sec^2 x)^{\cot^2 x} & x \neq 0 \\ = k & x = 0 \end{array} \right\}$ at $x = 0$

Soln: f is continous at $x = 0$.

$\lim_{x \to 0} f(x) = f(0).$

$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = k.$

$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k.$

$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$

$2 \lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2$

$k = 8.$

(iii) $f(x) = \left\{ \begin{array}{ll} \dfrac{\sqrt{3} - \tan x}{\pi - 3x} & x \neq \dfrac{\pi}{3} \\ = k & x = \pi/3 \end{array} \right\}$ at $x = \pi/3$

$x - \dfrac{\pi}{3} = h$

$x = h + \dfrac{\pi}{3}$

where $h \to 0$.

$f(\pi/3 + h) = \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim\limits_{h \to 0} \dfrac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim\limits_{h \to 0} \dfrac{\sqrt{3} - \dfrac{\tan \pi/3 + \tan h}{1 - \tan \pi/3 \cdot \tan h}}{\pi - \pi - 3h}$

$\lim\limits_{h \to 0} \dfrac{\dfrac{\sqrt{3}(1 - \tan \pi/3 \tan h) - (\tan \pi/3 + \tan h)}{1 - \tan \pi/3 \tan h}}{-3h}$

$\lim\limits_{h \to 0} \dfrac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3} \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$

$\lim\limits_{h \to 0} \dfrac{\left(\dfrac{\sqrt{3} - 3\tan h - \sqrt{3} - \tan h}{1 - \sqrt{3}\tan h}\right)}{-3h}$

$\lim\limits_{h \to 0} \dfrac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$

$$\lim_{h \to 0} \frac{4 \tan h}{3h(1-\sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h \to 0} \frac{\tan h}{h} \lim_{h \to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= \frac{4}{3} \frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}\left(\frac{1}{1}\right) = \frac{4}{3}.$$

7(i) $f(x) = \left.\begin{matrix} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0 \\ 9 & x = 0 \end{matrix}\right\}$ a

$$f(x) = \frac{1-\cos 3x}{x \tan x}$$

$$\lim_{x \to 0} \frac{2\sin^2 x\, 3/2\, x}{x \tan x}$$

$$\lim_{x \to 0} \frac{\dfrac{2\sin^2 3x/2}{x^2} \times x^2}{x \cdot \dfrac{\tan x}{x^2} \times x^2}$$

$$= 2 \lim_{x \to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$$

$$= \cancel{2 \times \frac{9}{4}}$$

$$= \frac{9}{2}$$

$\lim_{x \to 0} f(x) = \frac{9}{2}$ $\quad 9 = f(0).$

$\therefore$ f is not cont. at $x = 0$.

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i8.

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \tan x} & x \neq 0. \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim\limits_{x \to 0} f(x) = f(0)$.

f has removable discontinuity at $x=0$

7(ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ x \neq 0 \end{cases}$ at $x=0$.

$$\lim_{x \to 0} \frac{(e^{3x}-1)\sin(\pi x/180)}{x^2}$$

$$\lim_{x \to 0} \frac{(e^{3x}-1)\sin(\pi x/180)}{x^2}.$$

$$\lim_{x \to 0} \frac{3 \cdot e^{3x}-1}{3x} \quad \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{3x} \quad \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3 \log e \frac{\pi}{180} = \frac{\pi}{60} = f(0).$$

f is cont. at $x=0$.

8) $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, $x \neq 0$ is cont. at $x = 0$.

Given f is cont. at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$\lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$\log e + 2 \lim_{x \to 0} \left(\frac{\sin^2 x/2}{x} \right)$

Mult. with 2 in nr. and dr.

$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$

9) $f(x) = \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is cont. at $x = \pi/2$

$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1 + \sin x}}{\sqrt{2} + \sqrt{1 + \sin x}}$

$\lim_{x \to \pi/2} \frac{1 - \sin x}{(1 - \sin x)(1 + \sin x)(\sqrt{2} + \sqrt{1 + \sin x})}$

$\lim_{x \to \pi/2} \frac{1}{(1 - \sin x)(\sqrt{2} + \sqrt{1 + \sin x})}$

$= \frac{1}{2(\sqrt{2} + \sqrt{2})} = \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$= \frac{1}{\ }$

Practical

Topic! Derivative.

Q1) Show that the foll function defined from R to R are differentiable

i) $\cot x$ 2) $\operatorname{cosec} x$ 3) $\sec x$

Q2) If $f(x) = 4x+1$ $x \le 2$
$= x^2+5$ $x > 0$ at $x=2$ then

find f is differentiable or not

Q3) If $f(x) = 4x+7$ $x<3$
$= x^2+3x+1$ $x \ge 3$ at $x=3$ then

find f is differentiable or not.

Q4) If $f(x) = 8x-5$ $x \le 2$
$= 3x^2+3x+x$ $x \ge 3$ at $x=3$ then

find f is differentiable or not

1) $\cot x$

$$f(x) = \cot x$$
$$= \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$
$$= \lim_{x \to a} \frac{\cot x - \cot a}{x-a}$$
$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x-a}$$
$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \cdot \tan a}$$

put $x-a=h$ $\quad x = a+h$ at $x \to a$, $h \to 0$.

$$Df(h) = \lim_{x \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$
$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1+\tan A \cdot \tan B}$

$\tan A - \tan B = \tan(A-B)(1+\tan A \cdot \tan B)$

$$= \lim_{h \to 0} \frac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$Df(a) = -\cos^2 a$.

$\therefore$ f is differentiable $\forall a \in R$.

2) $\operatorname{cosec} x$.

$f(x) = \operatorname{cosec} x$.

$Df(a) = \lim \quad f(x) - f(a)$

Formula

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$= \lim_{h\to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot\sin\left(\frac{a-a-h}{2}\right)}{h\times \sin a\cdot \sin(a+h)}$$

$$= \lim_{h\to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a\cdot\sin(a+h/2)}$$

$$= \frac{-\frac{1}{2}\times 2\cos(2a/2)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$\frac{-2\sin\left(\frac{a+a+h}{2}\right)\cdot\sin\left(\frac{a-a-h}{2}\right)}{\cos a\cdot\cos(a+h)\times -h/2}$$

$$= -\frac{1}{2}\times -2\,\frac{\sin a}{\cos a\times\cos a}$$

$$= \tan a\cdot\sec a.$$

Q2) Soln:

LHD

$$Df(2^-) = \lim_{x\to2^-}\frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to2^-}\frac{4x+1-(4\times2+1)}{x-2}$$

$$= \lim_{x\to2^-}\frac{4x+1-9}{x-2}$$

$$= \lim_{x\to2^-}\frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4.$$

RHD:

$$Df(2^+) = \lim_{x\to2^+}\frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to2^+}\frac{x^2-4}{x-2}$$

$$= \lim_{x\to2^+}\frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4.$$

$Df(2^+) = 4$ RHD = LHD.

f is differenciable at $x=2$

Q3:

Soln: RHD:

$$Df(3^+) = \lim_{x\to3^+}\frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to3^+}\frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3^-} \frac{4(x-3)}{(x-3)}$$

$Df(3^-) = 4.$

RHD $\neq$ LHD

f is not differentiable at $x = 3$.

Q4) Soln:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD: $Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$= 3 \times 2 + 2 = 8.$

$Df(2^+) = 8$

LHD: $Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 11}$$

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$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)} \quad Df(2^-) = 8.$$

LHD = RHD f is differentiable at $x=3$.

A12
1/02/202

Practical-3

Topic: Application of derivative.

1) Find the intervals in which function is increasing or decreasing.

(i) $f(x) = x^3 - 5x - 11$

(ii) $f(x) = x^2 - 4x$

(iii) $f(x) = 2x^3 + x^2 - 20x + 4$.

(iv) $f(x) = x^3 - 27x + 5$

(v) $f(x) = 69 - 24x - 9x^2 + 2x^3$

2) Find the intervals in which function is concave upwards.

i) $y = 3x^2 - 2x^3$

ii) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

iii) $y = x^3 - 27x + 5$

iv) $y = 2x^3 + x^2 - 20x + 4$.

Solution:

Q1)

i) $f(x) = x^3 - 5x - 11$

$\therefore f'(x) = 3x^2 - 5$

$\therefore f'(x)$ is increasing iff $f'(x) > 0$.

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/2}) \cup (\sqrt{5/2}, \infty)$

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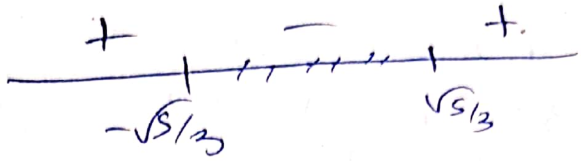
and f is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 5 < 0$

$\therefore 3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$



$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$\because f(x)$ is increasing iff $f'(x) > 0$.

$\therefore 2x - 4 > 0$

$\therefore 2(x-2) > 0$

$\therefore x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 \cancel{>} 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$.

and f is decreasing iff $f'(x) < 0$.

$\therefore 2x - 4 < 0$

$2(x-2) < 0$

$x - 2 < 0$.

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$.

$\therefore f'(x) = 6x^2 + 2x - 20$.

$\because f$ is increasing iff $f'(x) > 0$.

$\therefore 2(3x^2 + x - 10) > 0$.

$\therefore 3x^2 + 6x - 5x - 10 > 0$.

$(x+2)(3x-5) > 0$.

$x \in (-\infty, -2) \cup (5/3, \infty)$

OR...

and f is decreasing iff $f'(x) < 0$.

$\therefore 6x^2 + 2x - 20 < 0$.

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$.

$\therefore 3x^2 + 6x - 5x - 10 < 0$.

$\therefore 3x(x+2) - 5(x+2) < 0$.

$\therefore (x+2)(3x-5) < 0$.

$+ \quad -2 \quad - \quad 5/3 \quad +$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$.

$\therefore 3(x^2 - 9) > 0$.

$\therefore (x-3)(x+3) > 0$.

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$.

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$.

$\therefore (x-3)(x+3) < 0$.

$+ \quad -3 \quad - \quad 3 \quad +$

$\therefore x \in (-3, 3)$

3) $f(x)=2x^3-9x^2-24x+69$

$f'(x)=6x^2-18x-24<0$.

∴ f is increasing iff $f'(x)>0$.

∴ $6x^2-18x-24>0$.

∴ $6(x^2-3x-4)>0$.

∴ $x^2-4x+x-4>0$.

∴ $x(x-4)+1(x-4)>0$.

∴ $(x-4)(x+1)>0$.

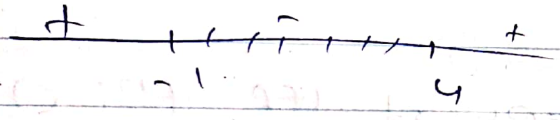
∴ $x\in(-\infty,-1)\cup(4,\infty)$

and f is decreasing iff $f'(x)<0$.

∴ $6x^2-18x-24<0$.

∴ $6(x^2-3x-4)<0$.

$(x-4)(x+1)<0$.



∴ $x\in(-1,4)$

2)

1) $y=3x^2-2x^3$

∴ $f(x)=3x^2-2x^3$

∴ $f'(x)=6x-6x^2$.

$f''(x)=6-12x$

f is concave upward iff $f''(x)>0$.

∴ $(6-12x)>0$.

∴ $12(6/12-x)>0$

$x-1/2>0$

$x>1/2$

$\therefore f''(x) > 0.$

$x \in (1/2, \infty).$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7.$

$f'(x) = 4x^3 - 18x^2 + 24x + 5.$

$f''(x) = 12x^2 - 36x + 24.$

f is concave upward if $f''(x) > 0$

$\therefore (6 - 12x) > 0.$

$\therefore 12(6/12 - x) > 0.$

$(x-2)(x-1) > 0.$

$+ \quad 1 \quad - \quad 2 \quad +$ $\qquad x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27.$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0.$

$\therefore 6x > 0.$

$\therefore x > 0$

$x \in (0, \infty)$

4) $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18\cancel{x}\cancel{-24}.$

f is concave upward iff $f''(x) > 0.$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0 \quad \therefore x > 3/2 \quad \therefore x \in (3/2, \infty)$

$y = 2x^3 + x^2 - 20x + 4$

5) $f(x) = 2x^3 + x^2 - 20x + 4.$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upword iff $f''(x) > 0$

$\therefore f''(x) > 0.$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0.$

$\therefore x < -1/6.$

$\therefore f''(x) \ngtr 0.$

$\therefore$ There exist interval. $(-1/6, \infty)$

Practical – 4

Topic : Application of derivative and newton's method.

Q1) Find maximum and minimum value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

ii) $f(x) = 3 - 5x^3 + 3x^5$

iii) $f(x) = x^3 - 3x^2 + 1$ $[-1/2, 4)$

iv) $f(x) = 2x^3 - 3x^2 - 12x + 1$ $[-2, 3]$

Q2) Find the root of the following equation by newton's (Take 4 iteration only) correct upto 4 decimal.

i) $f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

ii) $f(x) = x^3 - 4x - 9$ in $[2, 3]$

iii) $f(x) = x^3 - 1.8x^2 - 10x - 10x + 17$ in $[1, 2]$

Q1

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$.

$\therefore 2x - 32/x^3 = 0$.

$\therefore 2x = 32/x^3$

$\therefore x^4 = 32/2$

$\therefore x^4 = 16$

$\therefore x = \pm 2$.

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + \frac{96}{2^4}$

$= 2 + 96/16$.

$= 2 + 6$

$= 8 > 0$.

$\therefore f$ has minimum value at $x = 2$.

$\therefore f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$.

$= 4 + 4$

$= 8$.

$\therefore f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$.

$\therefore f$ has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$, and $x = -2$.

(iii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0.$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0 \therefore$ f has max value

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and the minimum value 1 at $x = 1$

ii) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

Consider, $f'(x) = 0$.

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x-2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$= -6 < 0 \therefore$ f ha...

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$\therefore f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

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$\therefore$ f has minimum value at $x=2$.

$\therefore f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ f has max val. 1 at $x=0$ and f has minimum value -3 at $x=2$.

(2) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$.

$\therefore 6(x^2 - x - 2) = 0$.

$\therefore x^2 - x - 2 = 0$.

$\therefore x^2 + x - 2x - 2 = 0$.

$x(x+1) - 2(x+1) = 0$.

$x = 2$ or $x = -1$.

$\therefore f''(x) = 12x - 6$.

$\therefore f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$.

$\therefore f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$f''(-1) = 12(-1) - 6$

$= 12 - 6$

$= -18 < 0$

f has maximum values at $x = -1$.

$\therefore f(-1) = 2(-1)^3 - 3(-1) - 12(1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$.

$\therefore$ f has maximum valu 8 at $x = -1$ and f has maxi. value -19 at $x = 2$.

$$x^3 - 3x^2 - 55x + 9.5$$

Q2)

(1) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By newton's method,

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = \underline{0.1727}$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= \underline{-0.0829}$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.0895 - 1.0362 - 55$

$= -55.9467.$

$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$

$= 0.1712$

$f'(x_2) = -55.9393$

$= 0.1712.$

$\therefore$ The root of the equation is 0.1712.

) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4.$

$f(2) = 2^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9.$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6.$

let $x_0 = 3$ be the initial approximation.

$\therefore$ By newton's method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$

$= 20.5528 - 10.9568 - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7392 - 0.596/18.5096$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 19.8386 - 10.8284$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 21.9851 - 4$

$= 17.9851$

$= 2.7071 = \dfrac{0.0102}{17.9851}$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.7158 - 10.806 - 9 = -0.0901$

$f'(3) = 3(2.7015)^2 - 4 = 21.8943 - 4 = 17.8943$

$x_4 = 2.7015 + 0.0901/17.8943 = 2.7015 + 0.0050$

$= 2.7065.$

(3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $[1,2]$

$f'(x) = 3x^2 - 3.6x - 10$.

$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$.

$= 1 - 1.8 - 10 + 17$

$= 6.2$.

$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$.

$= 8 - 7.2 - 20 + 17 = -2.2$.

let $x_0 = 2$ be initial approximation by newton's method.

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 2 - 2.2 / 5.2$.

$= 2 - 0.4230 = 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 1.6592 + 0.0204/7.7143$

$= 1.6592 + 0.0026$

$= 1.6618$

$f(x_3) = (1.6618)^3 - 1.8\,(1.6618)^2 - 10(1.6618) + 17$

$= 4.5892 - 4.9708 - 16.618 + 17$

$= 0.0004.$

$f'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$

$= 8.2847 - 5.9824 - 10$

$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$

$= 1.618 + \dfrac{0.0004}{7.6977}$

$= 1.6618$

AK
06/02/2020

Practicals

Integration

Q1) Solve the foll integration.

i) $\frac{dx}{\sqrt{x^2+2x-3}}$

ii) $\int (4e^{3x}+1)\,dx$

~~iii) ∫(4e~~ iii) $\int (2x^2-3\sin x+5\sqrt{x})\,dx$.

iv) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

v) $\int 7\sin(2t^4)\,dt$

vi) $\int \sqrt{x}\,(x^2-1)\,dx$.

vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$.

ix) $\int e^{\cos^2 x}\sin 2x\,dx$.

x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$

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i) $\int \frac{1}{x^2+2x-3}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}}$

$a^2+2ab+b^2 = (a+b)^2$

$= \int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$

Substitute.

put $x-1=t$

$dx = \frac{1}{t} \times dt$ where $t=1$ $t=x+1$

$\int \frac{1}{\sqrt{t^2-4}}\,dt$

Using:

$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln(|x+\sqrt{x^2-a^2}|)$

$= \ln(|t+\sqrt{t^2-4}|)$

$t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln|x+1+\sqrt{x^2+2x-3}|+c$

2) $\int (4e^{3x}+1)\,dx$

$\int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x$

$= \frac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + c$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + c$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\,dx$

Split the dn.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

5) $\int t^7 \times \sin(2t^4)dt.$

put $u = 2t^4.$

$du = 2 \times 4t^3.$

$= \int t^7 \times \sin(2t^4) \times \frac{1}{2 \times 4t^3} du.$

$= \int t^4 \sin(2t^4) \times \frac{1}{2 \times 4} du$

$= \int t^4 \sin(2t^4 \times \frac{1}{8} du = \frac{t^4 \times \sin(2t^4)}{8} du$

subsitute t^4 with $u/2$

$= \int \frac{u/2 \times \sin(u)}{8} du.$

$= \int \frac{u \times \sin(u)}{2} / 8 \, du$

$= \int \frac{u \times \sin(u)}{16} du$

$= \frac{1}{16} \int u \times \sin u \, dx.$

$\int u dv = uv - \int v du.$

when $u = u.$

$dv = \sin u \times du.$ $\quad v = -\cos(u) \, dv.$

$du = 1 du$

$= \frac{1}{16} \times (u \times (-\cos(u)) + \sin(u))$

Return the substitution $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C.$

vi) $\int \sqrt{x}(x^2-1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - x^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$I_1 = \dfrac{x^{5/2+1}}{5/2+1} = \dfrac{x^{7/2}}{7/2} = \dfrac{2x^{7/2}}{7} = \dfrac{2\sqrt{x^7}}{7}$

$I_2 = \dfrac{x^{1/2+1}}{1/2+1} = \dfrac{x^{3/2}}{3/2} = \dfrac{2x^{3/2}}{3/2} = \dfrac{2\sqrt{x^3}}{3}$

$= \dfrac{2x^3\sqrt{x}}{7} + \dfrac{2\sqrt{x^3}}{3} + C.$

vii) $\int \dfrac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx.$

$= \int \dfrac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$dt = \cos x$

$= \int \dfrac{\cos(x)}{\sin(x)^{3/2}} \times \dfrac{1}{\cos(x)\,dt}$

$= \dfrac{1}{\sin x^{3/2}}\,dt.$

$= \dfrac{1}{t^{2/3}}\,dt.$

$I = \int \dfrac{1}{t^{2/3}}\,dt = \dfrac{-1}{(2/3-1)\,t^{2/3-1}}$

$= \dfrac{-1}{-1/3\,t^{2/3-1}}$

$= 3\sqrt[3]{t}.$

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$\frac{x^2-2x}{x^3-3x^2+1} dx$

put $x^3-3x^2+1 = dt$

$$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3x2x} dt.$$

$$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x} dt.$$

$$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3} dt.$$

$$= \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$$

$$= \frac{1}{3} \int \frac{1}{t} dt. \quad \int \frac{1}{x} dx = (\ln|x|)$$

$$= \frac{1}{3} \times \ln|t| + c$$

$$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + c.$$

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Practical-6.

$$y = \sqrt{4-x^2}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}}$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1+\frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}$$

$$= \int_{-2}^{2} \sqrt{\frac{4}{4-x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$= 2\left[\sin^{-1}(x/2)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(\frac{-\pi}{2}\right)\right]$$

$$= 2\pi$$

3) $y = x^{3/2} \quad x \in [0, 4]$

$\frac{dy}{dx} = x^{3/2} \quad x \in [0, 4.]$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

$L = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx.$

$= \int_0^4 \sqrt{1 + \frac{9}{4} x} \, dx.$

$= \frac{1}{2} \int_0^4 \sqrt{4 + 9x} \, dx.$

$= \frac{1}{2} \left[(4 + 9x)^{3/2} \right.$

$= \frac{-1}{27} \left[(4 + 0)^{7/2} - (4 + 31)^{7/3} \right]$

$= \frac{1}{27} \left(40^{3/2} - 8 \right)$ units

4) $x = 3 \sin t \quad y = 3 \cos t$

$\frac{dx}{dt} = 3 \cos t \quad dy/dt = -3 \sin t.$

$l = \int_0^{2\pi} \sqrt{\left(dy/dt\right)^2 + \left(\frac{dx}{dt}\right)^2} \, dt.$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3 \sin t)^2} \, dt.$

$= \int_0^{2\pi} 3 \, dt.$

$= 3 \left[\pi \right]_0^{2\pi}$

$= 3 [2\pi - 0]$

$= 6\pi$ units.

5) $x = \frac{1}{6} y^3 + \frac{1}{2y}$

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$

$I = \int_0^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy.$

$\int_0^2 \sqrt{1 + \frac{(y^4 - 1)}{4y^4}}\, dy.$

$= \int_0^2 \sqrt{\frac{(y^4 - 1) + y \times y^4 \times 1}{4y^4}}\, dy.$

$\int_0^2 \frac{y^4 + 1}{2y^2}\, dy.$

$= \frac{1}{2} \int_1^2 y^2\, dy + \frac{1}{2} \int_1^2 y^{-2}\, dy.$

$= \frac{1}{2} \left[\frac{y^3}{3} - \frac{y^{-1}}{1} \right]_1^2$

$= \frac{1}{2} \left(\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1 \right)$

$= \frac{1}{2} \left[7/3 + 1/2 \right]$

$= \frac{1}{2} \left[17/6 \right]$

$= 17/12$ units

$e^{x^2} dx$ with $n=4$.

$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y_0	y_1	y_2	y_3	y_4

$\int e^{x^2} dx = \frac{h}{3}[(y_0+y_4)+4(y_1+y_3)+2(y_2)]$

$= \frac{0.5}{3}[(1+54.5982)+4(1.284+9.4877)+2.3855]$

$= \frac{0.5}{3}[55.5982+43.0866+5.436]$

$= \int_0^2 e^{x^2} dx = 17.3535.$

$\int x^2 dx \quad n=4.$

$h = \frac{4-0}{4} = 1.$

x	0	1	2	3	4
y	0	1	4	9	16.
	y_0	y_1	y_2	y_3	y_4

$\int x^2 dx = \frac{h}{3}[(y_0+y_4)+4(y_1+y_3)+2y_2]$

$= \frac{1}{3}[0+16+4(1+9)+2\times4].$

$= \frac{1}{3}[16+4(10)+8] = \frac{64}{3}$

$= \int_0^4 \cancel{x^3 dx} = \cancel{17.3535} = 21.3333.$

$\cancel{\int_0^4 x^2 dx \quad n = 4.}$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6.$

$h = \dfrac{\frac{\pi}{3} - 0}{6} = \dfrac{\pi}{18}.$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$ $\left(\frac{\pi}{3}\right)$
y	0	1.4167	0.6585	0.7071	0.8017	0.8752	0.9306
	y_0	y_1	y_2	y_3	y_4	y_5	y_6.

$\int \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{\pi}{54}\left[1.3473 + 4(1.999) + 2(1.3865)\right]$

$= \frac{\pi}{54} \times 12.1163.$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049.$

Practical-7.

Differential equation

Q)

1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$p(x) = \frac{1}{x}$ $\quad Q = (x) = \frac{e^x}{x}$

$I.F = e^{\int 1/x} dx.$

$= e^{\int 1/x dx}.$

$= e^{ln} x.$

$IF = x.$

$y(I.F) = \int Q(x)(I.F) dx + c$

$= \int \frac{e^x}{x} . x . dx + c.$

$= \int \int e^{x} dx + 1$

$xy = e^x + c.$

Q2) $e^x \frac{dy}{dx} + 2e^x y = 1.$

$\frac{dy}{dx} + 2\frac{e^x}{e^x} y = \frac{1}{e^x}$ ($\div$ by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}.$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$\int p(x) dx.$

$IF = e^{\int 2x}$

$= e^{2x}.$

$y = (I.F) = \int Q(x)(I.F)dx + c.$

$y \cdot e^{2x} \int e^{-x} + 2x \, dx + c$

$= \int e^{x} dx + c.$

$y \cdot e^{2x} = e^x + c.$

(ii) $x \frac{dy}{dx} = \frac{\cos x}{x} - 2y.$

$y(I.F) = \int Q(x)(I.F)\,dx + c.$

$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c.$

$= \int \cos x + c$

$= x^2 y = \sin x + c$

v) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on b.t.s)

$p(x) = 3/x \quad Q(x) = \sin x/x^3$

$= e^{\int p(x)\,dx}$

$= e^{\int 3/x\,dx}$

$= e^{3\ln x}$

$= e^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)\,dx + c.$

$= \int \frac{\sin x}{x^3} \cdot x^3\,dx + c.$

$= \int \sin x\,dx + c.$

$x^3 y = -\cos x + c.$

$e^{2x}\frac{dy}{dx} + \cancel{2e^{2x}}y = 2x.$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \qquad Q(x) = 2x e^{-2x} = 2x e^{-2x}$

$F = e^{\int P(x)\,dx}$

$= e^{\int 2\,dx}$

$$y(I.F) = \int Q(x)(I.F)\,dx + c$$
$$= \int 2x\,dx + c$$
$$ye^{2x} = x^2 + c$$

(vi) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$$
$$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$$
$$\therefore \log|\tan x| = -\log|\tan y| + c$$
$$\log|\tan x \cdot \tan y| = c$$
$$\tan x \cdot \tan y = e^c$$

vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Diff b.t.s

$$x-y+1 = v$$
$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$
$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$
$$1 - \frac{dv}{dx} = \sin^2 v$$
$$\frac{dv}{dx} = 1 - \sin^2 v$$
$$= \cos^2 v$$
$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

put $2x+3y = v$.

$$2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$= \cancel{\int \frac{v+1}{v}\, dx} + \int \frac{1}{v+1}\, dv = 3x$$

$$v + \log(|6|) = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$\log|2x+3y+1| + c$$

$$\ldots = x - \log|2x+3y+\ldots$$

Topic: Qulari's method.

$\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$ find

$\rightarrow$ $f(x) = y + e^{x} - 2$ $x_0 = 0$ $y(0) = 2$ h

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2725	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2. $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find $y(1) =$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.468	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

Q3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0)$ $h=0.2$ find $y(1)=?$ 56

$x_0 = 0$ $y(0)=1$ $h=0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
	0	1	0	1
0	0.2	1	0.4472	1.0894
1	0.4	1.0894	0.6059	1.2105
2	0.6	1.2105	0.7040	1.3513
3	0.8	1.3513	0.7696	1.5051
4	1	1.5051		
5				

$= y(1) = 1.5051$

1) $\frac{dy}{dx} = 3x^2+1$ $y(1)=2$ find $y(2)$ $h=0.5$

$h=0.25$

$y_0=2$ $x_0=1$ $h=0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
			4	4
0	1	2		
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

i.e. $y_0 = 2$ $x_0 = 1$ $h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.6426	299.9960
4	2	299.9960		

$y(2) = 299.9960.$

5.) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

x	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = 3.6.$

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Practical - 9

Limits and partial order derivative

(1) Evaluate the foll limits.

(i) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$$

Applying limits

$$\frac{(-4)^2-3(-1)+(-1)^2-1}{(-4)(-1)+5}.$$

$$\frac{-64+3+1-1}{4+5} = \frac{-61}{9}$$

(ii) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Applying limit.

$$\frac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)}$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

$$\frac{1+(1)(1)}{(1)^2} = 2$$

Q2) Find f_x, f_y, for each of the following

i) $f(x,y) = xy\,e^{x^2+y^2}$

$$f(x) = \frac{\partial f}{\partial x}$$

$$= \frac{\partial (xye^{x^2+y^2})}{\partial x}$$

$$= y[x \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dx}(x)]$$

$$= y[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1)]$$

$$= y \cdot e^{x^2+y^2}[2x+1]$$

Now, $f(y) = \frac{\partial f}{\partial y}$

$$= \frac{\partial (xye^{x^2+y^2})}{\partial y}$$

$$= x \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$$

$$= x[y \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dy}(y)]$$

$$= x \cdot [2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}]$$

$$\cancel{= x \cdot e^{x^2+y^2}}[2y^2+1]$$

$f(x,y) = e^x \cos y$

$\therefore f(x) = e^x \cos y$

$f(y) = e^x(-\sin y)$

$= -e^x \sin y$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{f}{\partial x} = \frac{d(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$= x3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3) Using defination find values of fx, fy at (0,0) for $f(x,y) = \frac{2x}{1+y^2}$

$fx(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$fx(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h - 0}{2} \; 2$

$= 2$

$\therefore fx = 2$

Similarly

$fy(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{0-0}{}$

24) Find all second order partial derivatives of f also verify whether $f_{xy} = f_{yx}$.

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$\therefore f(x) = \frac{\partial f}{\partial x} = \partial\left(\frac{y^2 - xy}{x^2}\right)$

$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy) \cdot \frac{d}{dx}(x^2)}{(x^2)^2}$

$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$

$= \frac{-x^2y - 2xy^2 + 2x^2y}{x^4} = \frac{x(xy - 2y^2)}{x^4}$

$f(x) = \frac{xy - 2y^2}{x^3}$

$f(y) = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right)$

$= \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$

$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$

$= \frac{1}{x^2} \cdot 2y - \frac{1}{x}$

$f(y) = \frac{2y - x}{x^2}$

(2)

$$f(x\,x) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy - 2y^2)(3x^2)}{x^6}$$

$$= \frac{6x^2y^2 - 2x^3y}{x^6}$$

$$= \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$f(y\,y) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial y} = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(yx) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(2y/x^2 - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(2y/x^2 - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= -\frac{4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$\therefore f(x,y) = f(yx) = \frac{x - 4y}{x^4}$$

Hence verified.

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial y}$$

$$= 0 + 3(2y)(x^2) + 0$$

$$f(y) = 6x^2y$$

ii)

$$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial}{\partial x}\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)$$

$$= 6x+6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$$

$$= 6x+6y^2-2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$$

$$= 6x+6y^2-2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$$

$$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$$

$$= 6x^2(1) = 6x^2$$

$$f(xy) = \frac{\partial\left(3x^2+6xy^2-\frac{2x}{x^2+1}\right)}{\partial y}$$

$$= 0 + 6x(2y)$$

$$= 12xy$$

$$f(yx) = \frac{\partial fy}{\partial x}$$

$$= \frac{\partial(6x^2y)}{\partial x}$$

$$= 12xy$$

$$\therefore f(xy) = f(yx) = 12xy$$

Hence verified

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= \cos(xy)(x) + e^{x+y}(1)$

$= x\cos xy + e^{x+y}$

$f(xx) = \frac{\partial f_y}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

$f(yy) = \frac{\partial f_y}{\partial y} = \frac{\partial(x\cos xy + e^{x+y})}{\partial z}$

$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$f(yx) = \partial f \ldots = \partial(x\cos x\ldots + e^{x+y})$

Q5) Find the linearization of $f(x,y)$ at given

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$

$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$

$fx(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$fy(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)($

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= 2 + x-1 + (y-1)$

Practical - 10.

Q1 Find the derivative of the foll func at given points and in the direction of given vector.

i) $f(x,y) = x+2y-3$ $\quad a=(1,-1)$ $\quad u = 3i-j$

there, $u = 3i-j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(3/\sqrt{10}, -1/\sqrt{10}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1-2-3 = -4.$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1-\frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\frac{3}{\sqrt{10}}\right) + 2\left(-1-\frac{h}{\sqrt{10}}\right) - 3$

$= 1+\frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h\to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h\to 0} \frac{4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

(iii) $f(x) = y^2 - 4x + 1$ $\quad a = (3,4) \quad u = i + 5j$

Here, $u = i + 5j$ is not a unit vector

$|u| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$.

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}}(1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$.

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f^{*}\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= 25h^2 + 40h - 4h$

$$\frac{h\left(\frac{25h}{26}+\frac{36}{\sqrt{26}}\right)}{h}$$

$$\therefore D_u f(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

$2x+3y$ $a = (1,2)$, $u = (3i+4j)$

Here $a = 3i+4j$ is not a unit v.

$|\bar{u}| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$

$f(a+hu) = 2\left(1+\frac{3h}{5}\right) + 3\left(2+\frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h\to 0} \frac{18h + 8 - 8}{h}$

$= \frac{18}{5}$

Q2) Find gradiant vector for the foll

i) $f(x,y) = x^y + y^x \to a = (1,1)$

$dx = y \cdot x^{y-1} + y^x \log y$

$dy = x^y \log x + \log^{x-1}$

$f(x,y) = (f x, f y)$

$= (y x^{y-1} + y^2 \log y, x^3 \log x + x y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2 \quad a = (1,-1)$

$f x = \frac{1}{1+x^2} \cdot y^2$

$f y = 2y \tan^{-1} x$

$f(x,y) = (f x, f y)$

$= (y x^{y-1} + y^2 \log y, x^3 \log x + x y^{x-1})$

~~f(1,1)~~ $= f(x,y) = (f x, f y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2)\right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

Q3) Find the equation of tangent and nor... to each of the following using cu... at given points.

(i) $x^2 \cos y + e^{xy} = 2$, at $(1,0)$

$fx = \cos y \, 2x + e^{xy} y$.

$fy = x^2(-\sin y) + e^{xy}. x$

$(x_0, y_0) = (1, 0)$ $\therefore$ $x_0 = 1$, $y_0 ...$

eqn of tangent.

$fx(x - x_0) + fy(y - y_0) = 0$.

$fx(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$.

$fy(x_0, y_0) = (1)^2(-\sin 0) + e^0 . 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$.

$2x - 2 + y = 0$.

$2x + y - 2 = 0$. $\rightarrow$ It is the ~~equation~~ eq... tangent.

eqn of normal.

$= ax + by + c = 0$.

$= bx + ay + d = 0$.

$1(1) + 2(y) + d = 0$.

$1 + 2y + d = 0$.

$= 1 + 2(0) + d = 0$.

$x^2+y^2-2x+3y+2=0$ at $(2,-2)$

$f_x = 2x+0-2+0+0.$

$= 2x-2$

$f_y = 0+2y-0+3+0.$

$= 2y+3.$

$(x_0, y_0) = (2,-2)$ $\therefore$ $x_0 = 2$, $y_0 = -2.$

$f_x(x_0, y_0) = 2(2)-2 = 2$

$f_y(x_0, y_0) = 2(-2)+3 = -1.$

eqn of tangent.

$f_x(x-x_0) + f_y(y-y_0) = 0.$

$2(x-2) + (-1)(y+2) = 0$

$2x-2-y-2=0.$

$2x-2-4=0$ $\longrightarrow$ It is reqd. eqn of tangent.

of normal.

Q1) Find the eq^n of tangent and normal line to each of the following surface

i) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= -2z + 3$

$f_z = 0 - 2y + 0 + x$

$= -2y + x$

$f_x(x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$

$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$

Eq^n of tangent

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z - z_0) = 0$

$f_x(x - x_0) + f_y(y - y_0) + 0\,(z - 0) = 0$

$= 4(x-2) + 3(y-1) + 0\,(z-0) = 0$

$= 4x - 8 + 3y - 3 = 0$

$= 4x + 3y - 11 = 0$

This is the required eq^n of tangent. eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

$3xyz - x - y + z = -4$ at $(1,-1,2)$

$3xyz - x - y + 4 = 0$ at $(1,-1,2)$

$f_x = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$$f_z = 3xy - 0 - 0 + 1 + 0.$$
$$f_z = 3xy + 1$$

$(x_0, y_0, z_0) = (1, -1, 2)$. $x_0 = 1, y_0 = -1, z_0 = 2$

$f_z(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_x(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$f_y(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2.$

eqn of tangent

eq $-7(x-1) + 5(y+1) - 2(z-2) = 0.$

$-7x + 7 + 5y + 5 - 2z + 4 = 0.$

$-7x + 5y - 2z + 16 = 0 \rightarrow$ This is is reqd eqn of tangent

eqn of normal at $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Find the local maxima and minima for the following function

$$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$$

$f_x = 6x + 0 - 3y + 6 - 0$
$= 6x - 3y + 6.$

$f_y = 0 + 2y - 3x + 0 - 4.$
$= 2y - 3x - 4.$

$f_x = 0.$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0. \rightarrow (1).$

$2x - y = -2$

$2x - y = -2$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \rightarrow (2)$

Multiply eqn 1 with 2.

$4x - 2y = -4$

$2y - 3x = 4$

$x = 0.$

substitute value of x in eqn (1).

$2(0) - y = -2.$

$y = 2.$

$\therefore$ Critical points are $(0,2)$

$r = f_{xx} = 6.$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore f$ has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$0 + 4 - 0 + 0 - 8$

$= -4.$

(iii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$fx = 8x^3 + 6xy$

$fy = 3x^2 - 2y$

$fx = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$\therefore 4x^2 + 3y = 0 \rightarrow (1)$

$fy = 0$

$3x^2 - 2y = 0 \rightarrow (2)$

Multiply eqn (1) with 3

(2) with 4.

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$12y = 0$

$y = 0$ //

Substitute value of y in eqn (1).

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0,0)$

$r = fxx = 24x^2 + 6x$

$t = fyy = 0 - 2 = -2$

$s = fxy = 6x - 0 = 6x = 6(0) = 0$

at $(0,0)$

$r = 24(0) + 6(0) = 0$

$\therefore r = 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$ $\therefore 2x + 2 = 0$

$x = \frac{-2}{2}$ $\therefore x = -1$

$f_y = 0$ $-2y + 8 = 0$

$y = \frac{-8}{-2}$

$= 4$

$\therefore$ critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$