$\sqrt{M}$-Best Candidates Based Soft-Demapper for Rotated M-QAM Constellation

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***Abstract*—In the second generation terrestrial digital video broadcasting (DVB-T2) systems, one of the important features is the rotated quadrature amplitude modulation (QAM) with cyclic Q delay exploiting signal space diversity to improve detection performance. However, the complexity of soft-demapper is increased, because it needs to calculate the Euclidean distances between the received signal and all candidate points, and then to find the minimums for log-likelihood ratio (LLR) calculation. In this paper, we propose a reduced complexity soft-demapper that selects only $\sqrt{M}$-best candidates for LLR calculation of either even or odd bits. The candidate set is formed by selecting the closest points to the image made by projecting the received signal onto the constellation grid lines. In total, it only calculates the Euclidean distances for $2\sqrt{M}$ points, which hence can reduce the complexity significantly. In spite of the reduced complexity, the $\sqrt{M}$-best candidate set is optimal because it always includes the minimum-distance point in the original full set. Finally, our results show that the proposed soft-demapper achieves the same bit error rate (BER) performance as the Max-Log based full search one, but the complexity is reduced by 67.5% and 85.9%, respectively for 64-QAM and 256-QAM cases.**

***Index Terms*—DVB-T2, rotated constellation, soft-demapper, LLR calculation, $\sqrt{M}$-best candidates.**

I. Introduction

Recently, the rotated constellation based signal space diversity has been considered as a promising diversity technique and attracted a lot of research interest [1]. In the second generation terrestrial digital video broadcasting (DVB-T2) standard, the quadrature amplitude modulation (QAM) rotation with cyclic quadrature (Q) component delay has been adopted [2]. It was shown to perform better than the classic non-rotated QAM, especially in the channels with deep fades and erasure events [3], [4]. However, the performance gain comes at the expense of increased complexity of soft-demapper in the receiver [5]. For each bit, it requires to find two closest constellation points (with the bit value being 0 or 1) to the received signal, and the log-likelihood ratio (LLR) can be obtained by calculating the difference of their Euclidian distances. The difficulty arises in how to quickly find the closest points in the faded constellation after the in-phase (I) and Q components are distorted by different fading. Since the faded constellation does not have a regular shape, it might have to calculate the 2-dimensional (2D) Euclidian distances from the received signal to all the constellation points and then find the corresponding minimum ones for each bit [5]. This scheme is called full search (more exactly, Max-Log based full search as described later), and we consider it as a benchmark in our study.

Most previous works focused on how to reduce the number of constellation point candidates for computation efficiency. In [6], the full constellation is decomposed into four overlapped sub-regions, corresponding to the four quadrants where the received signal may locate. According to the sign of received signal, only the points in one sub-region are selected for LLR calculation. In [7], a similar subset approach was considered. For each quadrant case, several L shape subsets with different size can be used, which provided different tradeoff between performance and complexity. If based on the smaller subquadrant, the subset can be somehow further reduced, which was investigated recently in [8]. In [9], the full constellation set is divided into sectors limited by parallel lines. For each bit, only two sectors closest to the received signal are selected for LLR calculation. However, considering that the selected (full or reduced) set can be commonly used by all the bits in the previous algorithms, the advantage of complexity reduction in [9] is not that much. In [10], a geometrical approach was proposed, which tried to directly find the best candidate without searching. With the aid of geometrical transformations, it first determined the closest lines to the received signal for each bit. Then, the closest point in the closest line was considered as the one with minimum distance. However, after constellation rotation and independent fading of I and Q components, the point found by this geometrical approach is not always the right one. In [11], the authors considered a different approach using decorrelation with interference cancellation, which can transform the soft-demapping into one-dimensional domain for complexity reduction.

To the best of our knowledge, all the previous works can not be guaranteed to achieve optimal performance as the original Max-Log based full search scheme in [5]. In addition, most of them can not be directly applied in the fading channels with erasure event, such as [6] and [10], and therefore still need full search for erasure channels. By now, there is no idea whether the performance of full search scheme can be achieved with lower complexity schemes. This issue will be investigated in this paper.

In this paper, we propose a new $\sqrt{M}$-best candidates based soft-demapper for rotated QAM. It utilizes the mapping feature that the original constellation points in the same row or column belong to the same subset for a specific even or odd bit (i.e., the subset with the bit value being 0 or 1). Therefore, the basic

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idea is to only select the best candidate in each row and column for further comparison and finding the minimum Euclidean distance. By this means, in the M-QAM constellation, we only need $\sqrt{M}$-best candidates for LLR calculation of all the even bits, so do the odd bits, which provides significant complexity reduction. Moreover, since we further search the minimum distance point from the $\sqrt{M}$-best candidate set, the final result is exactly same as searching from the full set. The performance and complexity of the proposed soft-demapper are compared with other schemes by simulations. The results show that our scheme can achieve the same bit error rate (BER) as the Max-Log based full search one, but with lower complexity. For 64-QAM and 256-QAM, its complexity is respectively 67.5% and 85.9% lower than that of the full search scheme.

The rest of this paper is organized as follows. Section II introduces the system model and conventional soft-demappers. In Section III, we propose the $\sqrt{M}$-best candidates based soft-demapper. Its complexity and performance are evaluated in Section IV. Finally, conclusions are drawn in Section V.

II. Background

A. System Model

We consider a transmission where n bits $\{b_0, b_1, \cdots, b_{n-1}\}$ are mapped on a M-QAM symbol $s = s_I + js_Q$ ($M = 2^n$). An example of 16-QAM with Gray mapping is illustrated in Fig. 1(a), where s_I conveys the even bits $\{b_0, b_2, \cdots, b_{n-2}\}$, and s_Q conveys the odd bits $\{b_1, b_3, \cdots, b_{n-1}\}$ [2]. With a rotation angle α, the rotated QAM symbol can be written as

$$\begin{aligned}\tilde{s} &= e^{j\alpha}s = \tilde{s}_I + j\tilde{s}_Q \\ &= (s_I \cos\alpha - s_Q \sin\alpha) + j(s_I \sin\alpha + s_Q \cos\alpha),\end{aligned} \tag{1}$$

where the corresponding rotated constellation is shown in Fig. 1(b). To further obtain diversity, the Q part is cyclically delayed by one cell (subcarrier) within a forward error correction (FEC) block. After cell interleaving, the I and Q parts which belong to the same symbol are transmitted on two different cells with independent fading.

At the receiver, the OFDM demodulation and phase compensation are performed in each cell. After that, the I and Q parts which belong to the same symbol are collected from the corresponding cells again. The received signal is equivalently written as

$$r = r_I + jr_Q = h_I\tilde{s}_I + jh_Q\tilde{s}_Q + w \tag{2}$$

where h_I and h_Q denote the amplitudes of the fading channel on the two cells, and w is the noise with power σ^2. In Figs. 1(c) and 1(d), the received signal and the corresponding faded constellations are shown under two different fading cases.

B. Soft-Demapping and LLR Calculation

To perform soft-demapping, we need to calculate the LLR of r for each bit b_i, which is defined as

$$LLR(b_i) = \log \frac{\sum_{\tilde{s}\in\mathbb{S}^1_{b_i}} \exp\left(-\frac{D(\tilde{s})}{2\sigma^2}\right)}{\sum_{\tilde{s}\in\mathbb{S}^0_{b_i}} \exp\left(-\frac{D(\tilde{s})}{2\sigma^2}\right)}, \tag{3}$$

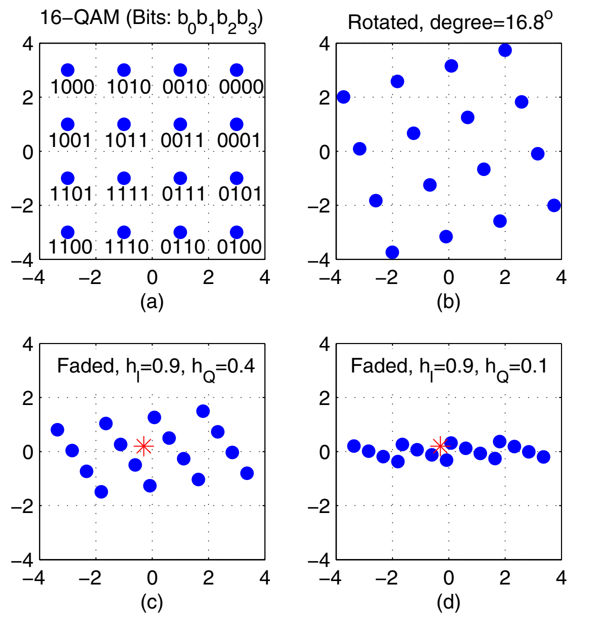

Fig. 1: (a) The original 16-QAM constellation; (b) The rotated 16-QAM constellation; (c) The faded constellation with $h_I = 0.9$, $h_Q = 0.4$; (d) The faded constellation with $h_I = 0.9$, $h_Q = 0.1$.

where $\mathbb{S}^1_{b_i}$ and $\mathbb{S}^0_{b_i}$ are the sets of rotated constellation points with b_i being 1 and 0, respectively, and

$$D(\tilde{s}) = (r_I - h_I\tilde{s}_I)^2 + (r_Q - h_Q\tilde{s}_Q)^2 \tag{4}$$

is the Euclidean distance between the received signal r and the constellation points $\tilde{s}$ (after fading). In Fig. 2, the partition of $\mathbb{S}^0_{b_i}$ and $\mathbb{S}^1_{b_i}$ for LLR calculation of each bit is shown in the 16-QAM case. The exact LLR calculation in (3) has a very huge complexity and it is usually simplified by applying the Max-Log approximation, as recommended in the implementation guideline of DVB-T2 [5]. With Max-Log approximation, the LLR becomes

$$LLR(b_i) \approx \frac{1}{2\sigma^2}[\min_{\tilde{s}\in\mathbb{S}^0_{b_i}} D(\tilde{s}) - \min_{\tilde{s}\in\mathbb{S}^1_{b_i}} D(\tilde{s})], \tag{5}$$

which means to find the minimum Euclidean distance between the received signal and the constellation points in two different sets $\mathbb{S}^0_{b_i}$ and $\mathbb{S}^1_{b_i}$. As depicted in Figs. 1 and 2, after independent fading of the I and Q parts, the faded constellations do not have a regular shape anymore. In case if erasure event happens, i.e., h_I or h_Q is 0, all the constellation points will be located in one axis. Therefore, the soft-demapping becomes very challenging. One straightforward method is to calculate the 2D Euclidean distances between the received signal and all M constellation points, and then find the minimum ones for LLR calculation [5]. However, the complexity of the Max-Log based full search scheme is still too high for implementation in the real systems, especially for the high modulation order cases.

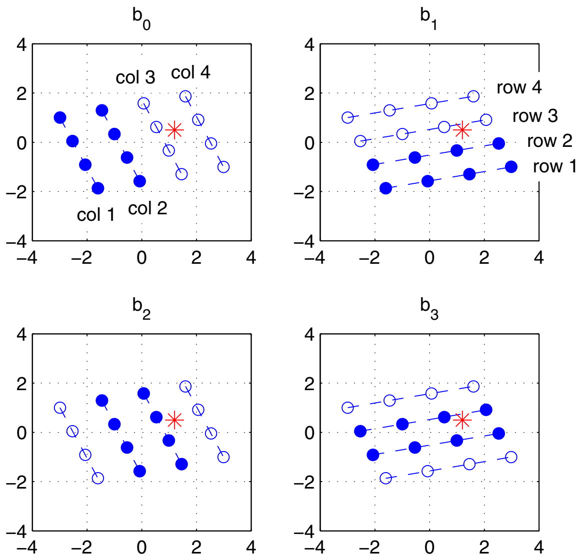

Fig. 2: The set partition for LLR calculation of each bit in the rotated 16-QAM case (• for $\mathbb{S}^1_{b_i}$, and ∘ for $\mathbb{S}^0_{b_i}$).

III. Proposed $\sqrt{M}$-Best Soft-Demapper

In this section, we will show that the constellation mapping features can be utilized to simplify the LLR calculation, and we propose a low complexity soft-demapper which achieves the same performance as the Max-Log based full search one.

A. $\sqrt{M}$-Best Soft-Demapper

In the original QAM constellation in Fig. 1(a), an important feature exists for the constellation mapping. For any even bit b_i, the points in the same column always belong to the same set ($\mathbb{S}^0_{b_i}$ or $\mathbb{S}^1_{b_i}$). While for the odd bit, the points in the same row always belong to the same set. After the constellation is rotated and faded, this mapping feature is still valid, as show in Fig. 2. Thus, in each row and column, if the best candidate (the one with minimum distance to the received signal) can be predetermined, it is enough to select only the best candidate for distance comparison with other candidates, instead of selecting all the points. Based on this mapping feature, we can have the following proposition. [1]

Proposition 1. *For the LLR calculation of the even bits, there exists a reduced candidate set of size $\sqrt{M}$, which is composed of the best candidate in each column, and it is equivalent to the full candidate set in finding the optimal constellation point with minimum distance. So do the odd bits, but the candidate set is composed of the best candidate in each row.*

Proof: Please refer to Appendix A. ■

In other words, for either even or odd bits, we only need to select $\sqrt{M}$-best candidates for distance comparison, and it is as valid as the full search case in terms of finding the point with minimum distance. However, with the candidate set of reduced size, the whole LLR calculation can be significantly simplified.

The proposed $\sqrt{M}$-best candidates based soft-demapper can be performed in the following two steps:

1) Select $\sqrt{M}$-best candidates for the even bits, $\sqrt{M}$-best candidates for the odd bits, and calculate the Euclidean distances to the received signal;
2) Calculate the LLR for each bit.

Now we describe how to select $\sqrt{M}$-best candidates in Step 1. It is assumed that the row index has an increased order from bottom to top, and column index from left to right, as shown in Fig. 3. Consequently, the index of each constellation point can be jointed determined by the row index and column index, and denoted by $\tilde{s}$(row index, column index). Take the even bit case as an example, the straightforward method to find the best candidate in each column is projecting the received signal to the column, and then selecting the one closest to the projection line. As shown in Fig. 3, the parallel columns can be expressed as

$$y = -\frac{h_Q \cos\alpha}{h_I \sin\alpha}x + (a_Q + \frac{h_Q \cos\alpha}{h_I \sin\alpha}a_I) \tag{6}$$

where (a_I, a_Q) is the coordinate of one reference point located in the column. The perpendicular line which passes through the received signal r can be expressed as

$$y = \frac{h_I \sin\alpha}{h_Q \cos\alpha}x + (r_Q - \frac{h_I \sin\alpha}{h_Q \cos\alpha}r_I). \tag{7}$$

Hereafter, we use $\beta_{IC} = h_I \cos\alpha$, $\beta_{IS} = h_I \sin\alpha$, $\beta_{QC} = h_Q \cos\alpha$, $\beta_{QS} = h_Q \sin\alpha$ for expression simplicity. The cross point (c_I, c_Q) of two lines (6) and (7) can be calculated as

$$\begin{aligned} c_I &= \frac{r_I\beta_{IS}^2 + a_I\beta_{QC}^2 - (r_Q - a_Q)(\beta_{IS}\beta_{QC})}{\beta_{IS}^2 + \beta_{QC}^2}, \\ c_Q &= \frac{r_Q\beta_{QC}^2 + a_Q\beta_{IS}^2 - (r_I - a_I)(\beta_{IS}\beta_{QC})}{\beta_{IS}^2 + \beta_{QC}^2}, \end{aligned} \tag{8}$$

and the distance between the reference point (a_I, a_Q) and the cross point (c_I, c_Q) can be calculated as

$$d^e_{ac} = \frac{(r_I - a_I)\beta_{IS} + (r_Q - a_Q)\beta_{QC}}{\sqrt{\beta_{IS}^2 + \beta_{QC}^2}}. \tag{9}$$

Assume that the original distance between two neighbor points in the same column is d, it changes to $d_1 = d\sqrt{\beta_{IS}^2 + \beta_{QC}^2}$ after fading distortion. Thus, the normalized distance between the reference point and the cross point in each column is

$$k^e_{ac} = \frac{(r_I - a_I)\beta_{IS} + (r_Q - a_Q)\beta_{QC}}{d(\beta_{IS}^2 + \beta_{QC}^2)} \tag{10}$$

which can be used for finding the closest best candidate index with additional rounding processing. Similarly, for the odd bit

[1] Even though the constellation is rotated and faded, we keep using row and column to describe the corresponding lines as in the original constellation (as shown in Fig. 2).

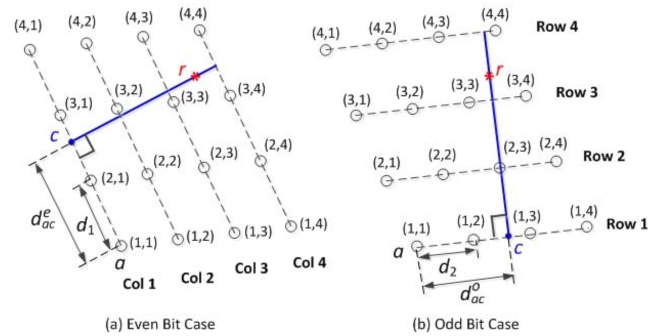

Fig. 3: Best candidate selection for the even and odd bits.

case, the normalized distance between the reference point and the cross point in each row can be calculated as

$$k_{ac}^{o} = \frac{(r_I - a_I)\beta_{IC} + (r_Q - a_Q)\beta_{QS}}{d(\beta_{IC}^2 + \beta_{QS}^2)}. \tag{11}$$

In Fig. 3, an example of finding the best candidate in the first row and column is shown. The detailed candidate selection method is described in Algorithm 1. After identifying the index of the best candidate, the Euclidean distance is calculated and saved.

Algorithm 1 Candidate Selection and Distance Calculation

Calculate β_{IC}, β_{IS}, β_{QC}, β_{QS}, β_{IC}^2, β_{IS}^2, β_{QC}^2, β_{QS}^2;
Calculate $\frac{1}{d(\beta_{IC}^2+\beta_{QS}^2)}$, $\frac{1}{d(\beta_{IS}^2+\beta_{QC}^2)}$;
for $i = 1 : \sqrt{M}$ **do**
  Calculate $a_I = h_I \tilde{s}_I(i,i)$, $a_Q = h_Q \tilde{s}_Q(i,i)$;
  % For Even Bits Case
  Calculate k_{ac}^{e} according to (10);
  Find the index of best candidate, $k^* = round(k_{ac}^{e} + i)$; if $k^* < 1$, $k^* = 1$; if $k^* > \sqrt{M}$, $k^* = \sqrt{M}$;
  Calculate $D_i^e = D\big(\tilde{s}(k^*, i)\big)$ according to (4);
  % For Odd Bits Case
  Calculate k_{ac}^{o} according to (11);
  Find the index of best candidate, $k^* = round(k_{ac}^{o} + i)$; if $k^* < 1$, $k^* = 1$; if $k^* > \sqrt{M}$, $k^* = \sqrt{M}$;
  Calculate $D_i^o = D\big(\tilde{s}(i, k^*)\big)$ according to (4).
end for

After obtaining the $\sqrt{M}$-best candidate set and corresponding Euclidean distance set, the LLR calculation can be simply performed with the aid of a lookup table (a table to indicate the corresponding subset for each specific bit). The details of the LLR calculation are described in Algorithm 2.

Algorithm 2 LLR Calculation

Given two Euclidean distance sets from Algorithm 1, $\mathbb{D}^e = \{D_1^e, \cdots, D_{\sqrt{M}}^e\}$, $\mathbb{D}^o = \{D_1^o, \cdots, D_{\sqrt{M}}^o\}$;
for $i = 0 : 2 : n-2$ **do**
  Divide $\mathbb{D}^e$ into 2 subsets, $\mathbb{D}_{b_i=0}^e$ and $\mathbb{D}_{b_i=1}^e$;
  Find $D_{b_i=0}^{min} = \min_{d \in \mathbb{D}_{b_i=0}^e} d$;
  Find $D_{b_i=1}^{min} = \min_{d \in \mathbb{D}_{b_i=1}^e} d$;
  Calculate $LLR(b_i) = \frac{1}{2\sigma^2}(D_{b_i=0}^{min} - D_{b_i=1}^{min})$;
end for
for $i = 1 : 2 : n-1$ **do**
  Divide $\mathbb{D}^o$ into 2 subsets, $\mathbb{D}_{b_i=0}^o$ and $\mathbb{D}_{b_i=1}^o$;
  Find $D_{b_i=0}^{min} = \min_{d \in \mathbb{D}_{b_i=0}^o} d$;
  Find $D_{b_i=1}^{min} = \min_{d \in \mathbb{D}_{b_i=1}^o} d$;
  Calculate $LLR(b_i) = \frac{1}{2\sigma^2}(D_{b_i=0}^{min} - D_{b_i=1}^{min})$.
end for

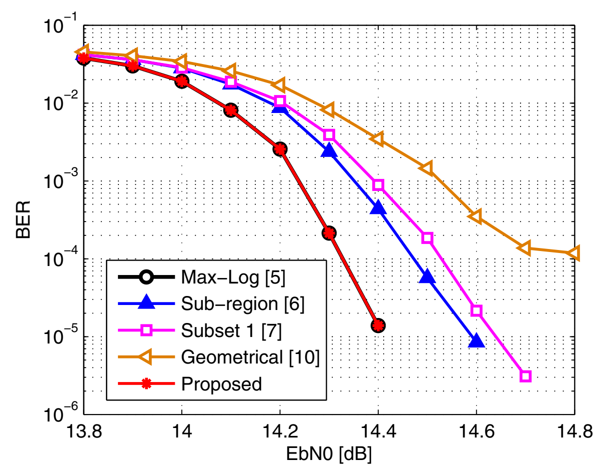

Fig. 4: BER performance of different soft-demappers for the case of 64-QAM and code rate $\frac{3}{4}$.

IV. Numerical and Simulation Results

In this section, we evaluate the performance and complexity of our proposed soft-demapper. For comparison we consider four other schemes, the Max-Log based full search [5], sub-region [6], subset [7], and geometrical [10]. For the subset based scheme in [7], the subset 1 is considered.

We performed a set of simulations of the DVB-T2 systems in a memoryless Rayleigh channel with 10% erasures [5]. The considered FEC frame has a size of 64800 bits, with different combination of the modulation (64-QAM or 256-QAM) and various code rates. In addition, ideal channel estimation is assumed.

In Fig. 4, the BER performance of different soft-demapper schemes is compared for the case of 64-QAM and code rate $\frac{3}{4}$. It is observed that our proposed soft-demapper can achieve exactly the same performance as the full search one. However, the other schemes always have performance degradation when compared with the full search one. At 10^{-4} BER, compared with the full search scheme, the performance degradation of the sub-region, subset and geometrical schemes is 0.15dB, 0.2dB, and 0.5dB, respectively. In Fig. 5, the performance for the case of 256-QAM and code rate $\frac{3}{4}$ is illustrated. Our soft-demapper has the same BER performance as the full search scheme. The performance gap between other schemes and the full search one is reduced compared to the 64-QAM case. The reason is that the rotation angle in the 256-QAM case is very small, and therefore the performance loss of other schemes is

TABLE I: Complexity of Different Soft-Demapper Schemes

Scheme	No. Euc. Dist. Calc.	MUL	ADD	RECIP	SQRT	COMP
Max-Log Full Search [5]	M	$4M+n$	$3M+n$			$n(M-2)$
Sub-region [6]	$M_0=(\frac{\sqrt{M}}{2}+1)^2$	$4M_0+n$	$3M_0+n$			$n(M_0-2)+2$
Subset 1 [7]	$M_1=\frac{3M}{4}$	$4M_1+n$	$3M_1+n$			$n(M_1-2)+2$
Geometrical [10]	$2\log_2 M$	$11n+48$	$9n+24$	6	2	$2+2n$
Proposed $\sqrt{M}$-Best	$2\sqrt{M}$	$8\sqrt{M}+n+18$	$8\sqrt{M}+n+7$	2		$(n+6)\sqrt{M}-2n$

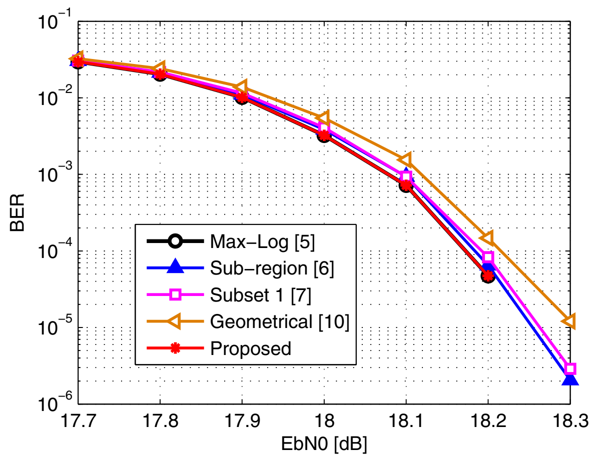

Fig. 5: BER performance of different soft-demappers for the case of 256-QAM and code rate $\frac{3}{4}$.

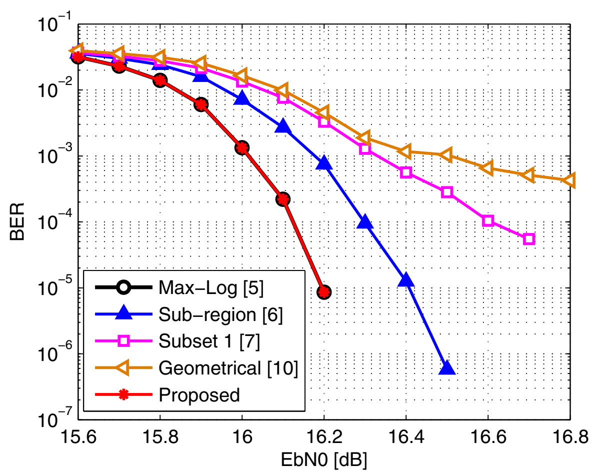

Fig. 6: BER performance of different soft-demappers for the case of 64-QAM and code rate $\frac{4}{5}$.

relatively smaller.

In Figs. 6 and 7, the BER performance of different schemes is compared for the less robust code rate specified in the DVB-T2 standard, $\frac{4}{5}$ and $\frac{5}{6}$. Again, our scheme achieves the same performance as the full search one. However, the performance degradation of other schemes becomes more serious when compared to the case of code rate $\frac{3}{4}$.

For complexity comparison of the various soft-demappers, we first compare the number of required Euclidean distance

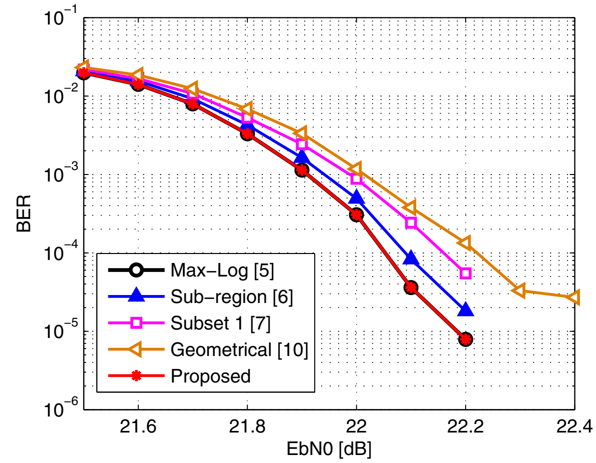

Fig. 7: BER performance of different soft-demappers for the case of 256-QAM and code rate $\frac{5}{6}$.

calculations per OFDM cell (M-QAM with n bits) in each scheme, since this is the dominated part. As shown in Table I, the full search, sub-region, and subset based schemes require $O(M)$ distance calculations. However, the complexity order of the geometrical scheme and our proposed one is lower, $O(\log_2 M)$ and $O(\sqrt{M})$, respectively. A more detailed complexity comparison is given in terms of five types of arithmetic operations: MUL (real multiplications), ADD (real additions or subtractions), RECIP (real reciprocals), SQRT (square roots), and COMP (comparisons or rounding). The number of the operations may be translated into the number of necessary DSP instructions according to the following rules: one MUL, ADD, RECIP, SQRT, and COMP is assumed to require 4, 1, 6, 10, and 1 DSP instructions, respectively [12].

In Fig. 8, the complexity of different schemes is shown in terms of number of DSP instructions. Compared to the full search scheme, the complexity of our scheme is 67.5% and 85.9% lower, respectively for 64-QAM and 256-QAM cases. In addition, its complexity is also lower than the sub-region and subset schemes. The geometrical scheme has comparable complexity as our scheme. However, its low complexity comes at the cost of serious performance degradation, as described before.

V. Conclusions

In this paper, we proposed a new $\sqrt{M}$-best soft-demapper that selects the closest points to the image of received signal projected onto the grid lines of QAM constellation. When

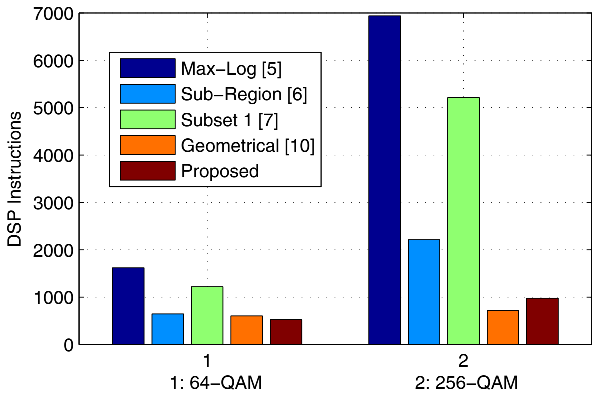

Fig. 8: Complexity comparison of different schemes in terms of number of required DSP instructions.

it is applied for rotated QAM constellation in DVB-T2, the complexity is comparable to the previous schemes whereas the performance is always better because the candidate selection is exactly equivalent to the Max-Log based full search scheme. In particular, the proposed algorithm is the best for 64-QAM case considering both performance and complexity.

Appendix A
Proof of Proposition 1

Proof: We first consider the even bit case. As described in Section III, for any even bit b_i, the points in the same column always belong to the same set. Thus, the set $\mathbb{S}_{b_i}^0$ and $\mathbb{S}_{b_i}^1$ can be expressed as

$$\mathbb{S}_{b_i}^0 = \cup_{k \in \mathbb{K}_{b_i=0}^{col}} \mathbb{S}_k^{col}, \ \mathbb{S}_{b_i}^1 = \cup_{k \in \mathbb{K}_{b_i=1}^{col}} \mathbb{S}_k^{col}, \ i = 0, 2, \cdots, n-2;$$

where k is the column index in the constellation, $\mathbb{S}_k^{col}$ is the set of the candidate points in the k-th column, $\mathbb{K}_{b_i=0}^{col}$ and $\mathbb{K}_{b_i=1}^{col}$ denote the set of column index in which the candidate points are with $b_i = 0$ and $b_i = 1$, respectively, and it satisfies

$$\mathbb{K}_{b_i=0}^{col} \cap \mathbb{K}_{b_i=1}^{col} = \emptyset, \ \mathbb{K}_{b_i=0}^{col} \cup \mathbb{K}_{b_i=1}^{col} = \{1, 2, \cdots, \sqrt{M}\}, \quad i = 0, 2, \cdots, n-2. \quad (12)$$

Accordingly, the main part in (5) can be rewritten as

$$\begin{aligned} \Delta D(b_i) &= \min_{\tilde{s} \in \mathbb{S}_{b_i}^0} D(\tilde{s}) - \min_{\tilde{s} \in \mathbb{S}_{b_i}^1} D(\tilde{s}) \\ &= \min_{k \in \mathbb{K}_{b_i=0}^{col}} \min_{\tilde{s} \in \mathbb{S}_k^{col}} D(\tilde{s}) - \min_{k \in \mathbb{K}_{b_i=1}^{col}} \min_{\tilde{s} \in \mathbb{S}_k^{col}} D(\tilde{s}) \\ &= \min_{k \in \mathbb{K}_{b_i=0}^{col}} D(\tilde{s}_{col,k}^*) - \min_{k \in \mathbb{K}_{b_i=1}^{col}} D(\tilde{s}_{col,k}^*) \end{aligned} \quad (13)$$

where $\tilde{s}_{col,k}^* = \arg\min_{\tilde{s} \in \mathbb{S}_k^{col}} D(\tilde{s})$ denotes the best candidate point with minimum distance to the received signal in the k-th column. Obviously, the set $\{\tilde{s}_{col,1}^*, \tilde{s}_{col,2}^*, \cdots, \tilde{s}_{col,\sqrt{M}}^*\}$ of size $\sqrt{M}$ is the optimally reduced subset of the full set, which gives same results for LLR calculation of the even bits.

Similarly, for LLR calculation of the odd bits, we can prove that the set $\{\tilde{s}_{row,1}^*, \tilde{s}_{row,2}^*, \cdots, \tilde{s}_{row,\sqrt{M}}^*\}$ is optimal as the full set, in which $\tilde{s}_{row,k}^*$ denotes the best candidate point with minimum distance to the received signal in the k-th row. ■

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