The Simple SNR Estimation Algorithms for MPSK Signals

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Abstract: The data-aimed signal-to-noise ratio (SNR) estimation algorithm (ADML), decision-directed SNR estimation algorithm (DDML) and a new blind SNR estimation algorithm for MPSK signals over an additive White Gauss noise channel (AWGN) have been presented in this paper by using maximum-likelihood (ML) principle. The ADML has perfect performance and is as good as the performance lower bounds. The performance of DD algorithm depends directly on the accuracy of decision. so it's good for high SNR, but suffers when the SNR is low and, especially, less than 0dB. The new blind SNR estimation algorithm has good performance and low computational complexity for M=2(BPSK), but the performance degrades when M is increasing. The brief performance analysis, computer simulation and performance comparing with other SNR estimation algorithms are also given in this paper.

Index Terms: Maximum-likelihood principle, SNR estimation, MPSK signals

1. Introduction

The SNR estimation plays an important role in many modern wireless communication systems. It can help us adopt adaptive demodulation algorithms to enhance the performance and provide the channel quality information required by the handoff, power control, and channel assignment algorithms. There are some papers having investigated the methods of SNR estimation. Four SNR estimation algorithms for QPSK have been compared in [1]. A NDA estimation method for QPSK signals in AWGN and a data-aided (DA) estimation algorithm for QPSK signals in slow fading channel are discussed in [2] by considering the real and imaginary components and using the way of $|\text{average}|^2/\text{variance}$ respectively. And B.Li etc. provided a low bias alternative algorithm in low SNR for BPSK by using ML principle to estimate the signal's amplitude firstly in [3]. Summer and Wilson studied the SNR estimation for BPSK signals through considering the relationship between the ratio of two-order statistics, namely $z=E(r_k^2)/[E(|r_k|)]^2$, and the SNR, and brought a non-data-aided (NDA) estimation algorithm for BPSK signals in AWGN channels in [4]; on the basis of Summer's work, Ramesh etc. presented an estimation algorithm for BPSK signals in generalized Nakagami fading channels [5].

In this paper, we also start from the maximum likelihood module estimation of received signals and propose ADML, DDML and a new blind SNR estimation algorithm for MPSK signals. Like deriving of DD phase estimation algorithm in [6], the data-aimed maximum-likelihood (ADML) algorithm is deduced firstly and then the DDML is derived from it. The new blind SNR estimation algorithm is presented by firstly removing the data modulation.

2. SNR estimation for MPSK signals

In the receiver, the baud rate sampling signals after matched filter are the target signals. Assuming the signals are performed over an additive white Gaussian noise channel and the synchronization is perfect. Let $A=\{a^i\}$ be the constellation containing M constellation signals in which a^i are assumed to be independent and are all equally likely to be transmitted. So in the k-th symbol interval, the signals can be represented as

$$r_k = a_k + n_k, \quad k=1,2,\ldots,L \tag{1}$$

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Where $a_k \in A$ is the transmitted constellation signal; n_k is the complex independent identically distributed (i.i.d) zero-mean Gaussian random variables with independent real and imaginary parts having variance σ^2 and L denotes the length of observing data for SNR estimation. We can get from (1) that $E|r_k|^2 = E|a_k|^2 + E|n_k|^2 = E|a_k|^2 + 2\sigma^2$.

Then the SNR needed to be estimated is given by:

$$SNR = \frac{E|a_k|^2}{E|n_k|^2} = \frac{E|a_k|^2}{E|r_k|^2 - E|a_k|^2} = \lambda \tag{2}$$

For the MPSK signals, a_k are randomly get the values of $a^i = Ae^{j\frac{i\times 2\pi}{M}}$, i=0…M-1 at equal probability. So (2) can be rewritten as

$$SNR = \frac{A^2}{E|r_k|^2 - A^2} \tag{3}$$

Considering equation (3), we only need to estimate the value of module A. So the problem of SNR estimation is converted to the problem of parameter estimation for module of signals.

A. ADML and DDML

According to the rule of ML, we get

$$p(r_k \mid A) = \frac{1}{2\pi\sigma^2} e^{-\frac{|r_k - a_k|^2}{2\sigma^2}} \tag{4}$$

When the transmitted data are known and a_k equal to one of the constellation values a^i, the log-likelihood function is

$$\begin{aligned} \ln p(r \mid A) &= \sum_{k=0}^{L-1} \ln p(r_k \mid A) \\ &= C + \sum_{k=0}^{L-1} \frac{1}{2\sigma^2} |r_k - a_k|^2 \\ &= C + \sum_{k=0}^{L-1} \frac{1}{2\sigma^2} \left(|r_k|^2 + A^2 - 2A\, real(r_k e^{j\frac{i\times 2\pi}{M}}) \right) \end{aligned} \tag{5}$$

The symbol C in (5) denotes some terms having no relation to A. Setting to zero the derivative of (6) and solving for A' yields the location for the minimum of the log-likelihood function

$$\hat{A}_{ADML} = \sum_{k=0}^{L-1} real(r_k e^{j\frac{i\times 2\pi}{M}}) \tag{6}$$

Combining (3) with (6), we can get the estimation value of SNR, this algorithm is called ADML. When the transmitted data is unknown, we can substitute a_k with the decision value in (6). So we get

$$\hat{A}_{DDML} = \sum_{k=0}^{L-1} real(r_k e^{j\frac{\hat{i}\times 2\pi}{M}}) \tag{7}$$

We call this algorithm DDML. The rule of one hard decision scheme can simply be explained as follows: 1. Make the mean value of two adjacent constellation signals' angles be the boundary of every decision areas. 2. When the received signals fall in one of the decision areas, the decision value is the angle which the area correspond to, namely the angle of the constellation signal included in this decision area. Of course, we can change the decision scheme to improve the decision accuracy and then the performance of DDML SNR estimation will be improved.

B. A new blind SNR estimation algorithm

We can see that DDML need decision and the performance of DDML decreases rapidly in low SNR range for more and more wrong decisions. It is obvious that we can avoid the decision process by taking M power to remove the data modulation. We define

$$r_k' = r_k^M = A^M + n_k' = A' + n_k' \tag{8}$$

and we assume that the term of noise n_k' is also the complex independent identically distributed (i.i.d) zero-mean Gaussian random variables with independent real and imaginary parts having variance σ_1^2. Then

$$p(r_k' \mid A') = \frac{1}{\sqrt{2\pi\sigma_1^2}} e^{-\frac{|r_k' - A'|^2}{2\sigma_1^2}} \tag{9}$$

For Equ.8, the module A' is a certain quantity. So the log-likelihood function is

$$\begin{aligned} \ln p(r' \mid A') &= \sum_{k=0}^{L-1} \ln p(r_k' \mid A') \\ &= C' + \sum_{k=0}^{L-1} \frac{1}{2\sigma_1^2} |r_k' - A'|^2 \\ &= C' + \sum_{k=0}^{L-1} \frac{1}{2\sigma_1^2} \left(|r_k'|^2 + A'^2 - 2A'\, real(r_k') \right) \end{aligned} \tag{10}$$

The symbol C' in (10) denotes the terms having no relation to A'. Seeking the partial derivative of (10) and setting it zero, then

$$A' = \frac{1}{L}\sum_{k=0}^{L-1} real(r_k') \qquad (11)$$

Let $A = \sqrt[M]{A'}$. Substituting the value of A into equ3, we can get the SNR estimation value.

3. PERFORMANCE ANALYSIS AND COMPUTER SIMULATION

A. The performance of ADML and DDML

The performance of ADML is perfect and it is deduced according to ML principle, so we can take it as the performance bounds for SNR estimation. The DDML reveals the essence of some existing algorithm with estimating the SNR by $|average|^2$/variance. For BPSK and QPSK, all the algorithms of $|average|^2$/variance need decision process and they are hard to handle the high-order MPSK (M>4) signals. The performance of DDML is the best in this kind of algorithms which need hard decision, and furthermore it is easy to handle the high-order MPSK signals.

The DDML presented in this paper for QPSK has been compared with other two algorithms in [1] and [2]. The algorithm called algorithm1 is the best one among four presented algorithms in [1] and the algorithm presented in [2] is called algorithm2.

In the computer simulation, the standard deviation (SD) is defined as

$$SD = sqrt\left(E\left[(\hat{SNR} - SNR)^2 \right] \right) \qquad (12)$$

where the function *sqrt*( ) denotes the operation of square root and the symbol $\hat{SNR}$ is the estimation value of SNR.

From Fig.1-4, the performance of ADML is perfect and the curve superposes that of CRLB_DA well. And DDML works well at high SNR but is bias and works poorly at low SNR (lower than 5dB). We find that the performance of DDML is better than that of the algorithm1 and identical to that of algorithm2. However the DDML can handle the high-order MPSK signals easily, whereas algorithm2 can't.

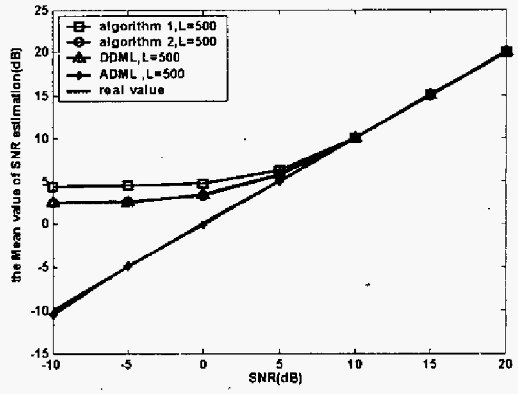

Fig.1 The mean of SNR estimation for QPSK

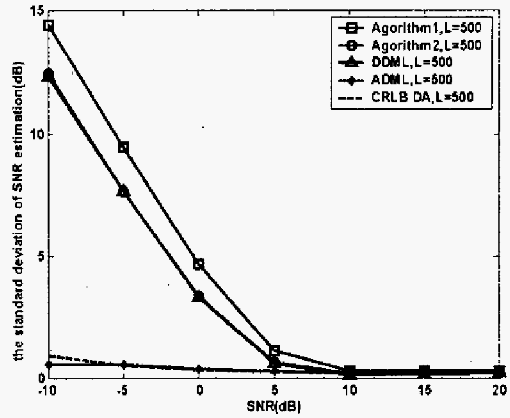

Fig.2 The standard deviation of SNR estimation for QPSK

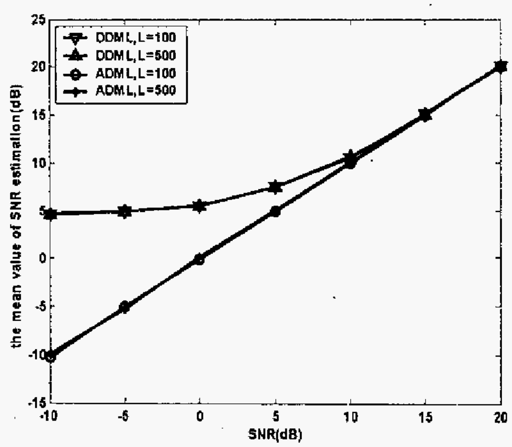

Fig.3 The mean of SNR estimation for 8PSK

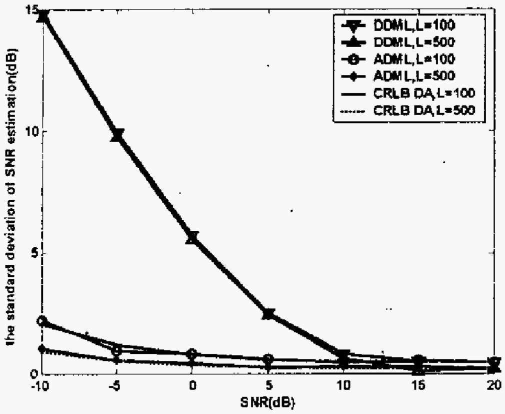

Fig.4 The standard deviation of SNR estimation for 8PSK

B. The performance of the new presented blind SNR estimation algorithm

The new blind SNR estimation algorithm is deduced according to ML principle, so it is optimal estimation. On the other hand, the M-th power process will introduce noise penalty. For example, the 4-th power method amplifies the noise 4 times when it remove data modulation (for QPSK), which introduces a (approximately) 12dB noise penalty. However the noise penalty introduced by the 2-th power process is relatively smaller, the performance of the new blind SNR estimation algorithm for BPSK is very good. We compare it with two good performance algorithms. One called algorithm3 is the iterative algorithm presented by [3] and another one called algorithm4 is an algorithm basing on two-order statistics presented by [4]. Both of them need no decision process and can estimate much lower SNR value accurately than that DD algorithms can do.

For algorithm3, we set the range of amplitude be [0.0001, 0.999] and the iterative times is 15. For algorithm4, the range of SNR estimation is [0-6dB] in [4], and we broaden it to [-5-12dB] by using the below 5th-order approximate polynomial which is almost the largest range that we can broaden.

$$\hat{\lambda} = 10^{4}(-0.412929\ 58452235 \times z^{5} + 2.66418532\ 072905 \times z^{4} - 6.86724072\ 350538 \times z^{3} + 8.84039993\ 634297 \times z^{2} - 5.68658561\ 155135 \times z + 1.46404579\ 5143920) \qquad (13)$$

From Fig.5-6, the performance of the new blind SNR estimation algorithm is much better than that of the other two algorithms. The new algorithm is not only working effectively at high SNR, but also having better performance at low SNR than that of the other two algorithms. When the number of observation symbols (L) is 5000, the mean is superposing upon the real SNR values by and large in the whole range of [-20,30dB], the standard deviation of -10dB is less than 0.4dB and the standard deviation of -20dB is less than 4dB. When L=1000, the mean has a little offset at SNR less than -15dB, the standard deviation of -10dB is less than 2.5dB and the standard deviation of -20dB is less than 5.5dB. And we also can see from Fig.5-6, the performance of the other two algorithms degrades rapidly when SNR is larger than about 20dB.

For QPSK, the 4-th power introduces 12dB noise penalty and the performance of the new algorithm isn't as good as that for BPSK signals. We also compare it with an algorithm which called algorithm5 and is derived from algorithm4. Algorithm5 can handle the QPSK signal by considering the real and image components respectively and using the method:

$$\hat{z} \approx \frac{\overline{r_{k\_I}^{\,2}}}{\overline{|r_{k\_I}|}^{2}} \approx \frac{\overline{r_{k\_Q}^{\,2}}}{\overline{|r_{k\_Q}|}^{2}} \approx \frac{\overline{r_{k\_I}^{\,2}} + \overline{r_{k\_Q}^{\,2}}}{\overline{|r_{k\_I}|}^{2} + \overline{|r_{k\_Q}|}^{2}} \qquad (14)$$

where the subscripts I and Q represent the real and imaginary components respectively.

From Fig.7-8, compared with the performance for QPSK, the performance of the new blind algorithm degrades obviously and it is worse than that of algorithm5 for low SNR. For example, the new blind algorithm's performance of L=5000 is almost equal to the algorithm5's performance of L=1000 in the range [-20,15dB]. On the other hand, the new blind algorithm can work very well at very high SNR (SNR>20dB), whereas the algorithm5 suffers. It is worth mention that if more observation symbols are used for estimation, the performance of new blind algorithm will be better. So the disadvantage that brought by 4-th power method can be alleviated in this way.

4. Conclusion

The analytical and simulation's results show that the ADML is as good as a referenced performance bounds, the DDML is the best performance among the algorithms need decision or the algorithms of |average|^2/variance and DDML can easily handle the high-order MPSK signals.

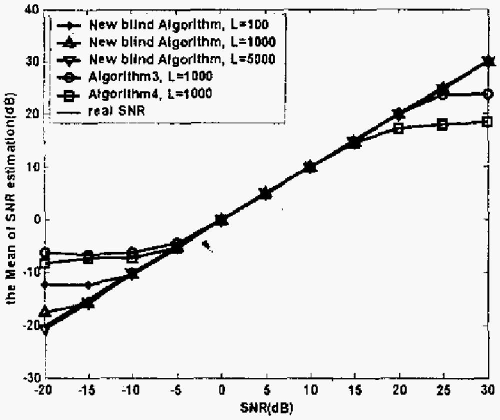

Fig.5 The mean of SNR estimation for BPSK

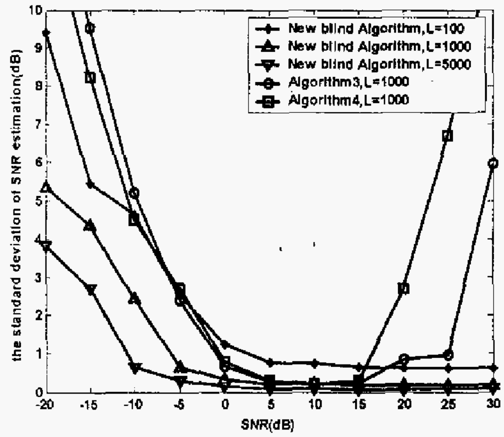

Fig.6 The standard deviation of SNR estimation for BPSK

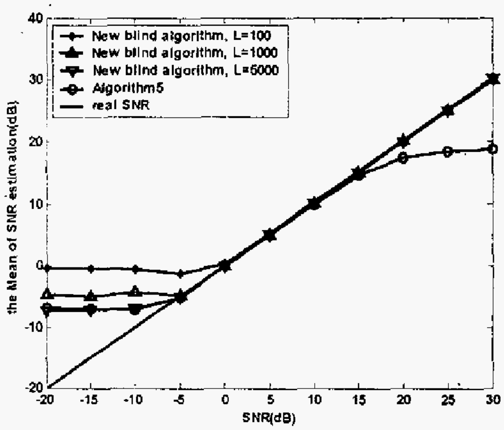

Fig.7 The mean of SNR estimation for QPSK

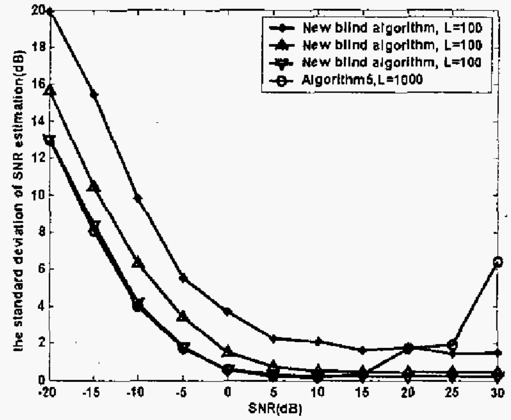

Fig.8 The standard deviation of SNR estimation for QPSK

and the new blind SNR estimation algorithm has excellent performance for BPSK signals, but with M increasing, the performance of the new blind algorithm become worse gradually for the M-th power operation introducing noise penalty. It can be alleviated by using more observation symbols.

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