Maximum Likelihood Estimation for SNR of PSK and QAM

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***Abstract*—Signal-to-noise ratio (SNR) is an important parameter of channel quality. The performance of adaptive communications in time-varying channels is affected significantly by the SNR estimation accuracy.**

In this paper, the new methods of SNR estimation for PSK and QAM are proposed. The methods are based on maximum likelihood estimation (MLE) using the probability density function (pdf) of received signal envelope, and are used in unknown signal power conditions without the carrier synchronization. The proposed methods are shown to estimate SNR efficiently by numerical results of normalized mean square error (NMSE) of SNR.

I. Introduction

The channel state information (CSI) estimation becomes more and more important in many wireless communication systems, such as adaptive Modulation and Coding (AMC), handoff, power control, or cognitive radio applications. The performance of these systems is optimized by the CSI. SNR is one of the key parameters in CSI. Since the SNR estimation error causes serious performance degradations, the exact estimation of SNR is required.

SNR estimation can be classified into data-aided (DA) and non-data-aided (NDA) estimators. SNR is estimated with the knowledge of transmitted symbols in DA and without the knowledge in NDA. DA estimators for the feedback channels in orthogonal frequency division multiplexing (OFDM) have been studied in [1], [2]. NDA estimators are presented in [3]–[6] for sinusoidal wave, PSK or FSK by the log-likelihood approach. For QAM, moments of received signal envelope are introduced in NDA [7]–[9]. However, the estimated SNR does not have satisfactory accuracy in high SNR. In [10], the decision-directed ML (DDML) SNR estimation and iterative blind SNR estimation algorithms are proposed. The performance of DDML estimation deteriorates in low SNR, where the decision results include errors.

In this paper, the new methods are proposed to estimate SNR of PSK and QAM. The methods are based on maximum likelihood estimation using the pdf of received signal envelope, and do not require the carrier synchronization or signal power information. The pdf of received signal envelope conditioned on SNR is derived from the PSK and QAM signal constellations. The estimation results of the proposed methods are compared with those of the moment-based method [8] by computer simulations. Simulations results show that the proposed methods yield better performance than those of 2nd and 4th moment methods in PSK, and outperform M_2M_4methods in 16QAM and 64QAM in high SNR.

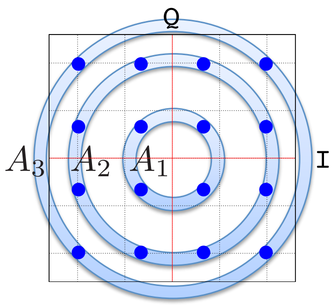


Fig. 1: amplitude levels

II. Probability Density Function and Log Likelihood

In this section, the SNR estimation method using log likelihood is presented for PSK, 16QAM and 64QAM. The log likelihood is calculated by the pdf of normalized received signal envelope. Let consider the additive white Gaussian noise (AWGN) channel. The received signal $r_n(n = 1, 2, \dots, N)$ can be written as

$$r_n = x_n + w_n, \tag{1}$$

where x_n is a transmitted signal and w_n is the AWGN with zero mean and variance σ^2, w_n is independently and identically distributed (i.i.d.), where $n(= 1, 2, \dots, N)$ is the time index in the observation interval. When the carrier is the sinusoidal wave, the pdf of envelope of received signal is the Nakagami-Rice distribution in a constant envelope modulation of PSK. The pdf for 16QAM and 64QAM includes the Nakagami-Rice distribution with parameters of modulated signal envelope and its probability. Fig. 1 shows 16QAM constellation. 16QAM has 3 amplitude levels. The amplitude of modulated signal envelope is written by A_q with its probability $p_q(q = 1, 2, \dots, Q)$. The number of amplitude levels, Q, depends on modulation.

TABLE I: Parameters q, h_q and p_q

ω	q	h_q	p_q
BPSK	1	1	1
16QAM	1	$\frac{1}{\sqrt{5}}$	$\frac{1}{4}$
	2	1	$\frac{1}{2}$
	3	$\frac{3}{\sqrt{5}}$	$\frac{1}{4}$
64QAM	1	$\frac{1}{\sqrt{21}}$	$\frac{1}{16}$
	2	$\sqrt{\frac{5}{21}}$	$\frac{1}{8}$
	3	$\frac{3}{\sqrt{21}}$	$\frac{1}{16}$
	4	$\sqrt{\frac{13}{21}}$	$\frac{1}{8}$
	5	$\sqrt{\frac{17}{21}}$	$\frac{1}{8}$
	6	$\frac{5}{\sqrt{21}}$	$\frac{3}{16}$
	7	$\sqrt{\frac{29}{21}}$	$\frac{1}{16}$
	8	$\sqrt{\frac{37}{21}}$	$\frac{1}{8}$
	9	$\frac{7}{\sqrt{21}}$	$\frac{1}{16}$

The received signal envelope $|r_n|$ is normalized by the second moment M_2 of r_n to give the normalized received signal envelope y_n.

$$M_2 = \mathrm{E}[|r_n|^2] \tag{2}$$
$$y_n = \frac{|r_n|}{\sqrt{M_2}} \tag{3}$$

The pdf of normalized received signal envelope is given by

$$f(y_n|\alpha,\sigma^2) = \sum_{q=1}^{Q} p_q \times \frac{M_2 y_n}{\sigma^2} I_0\left[\frac{A_q\sqrt{M_2}y_n}{\sigma^2}\right] \times \exp\left[-\frac{M_2 y_n^2 + A_q^2}{2\sigma^2}\right], \tag{4}$$

where I_0 is the 0th-order modified Bessel function of the first kind and α denote the modulation of PSK, 16QAM, or 64QAM.

Now define SNR γ, second moment M_2 and normalized envelope h_q as

$$\gamma = \frac{\mathrm{E}[A_q^2]}{2\sigma^2}, \tag{5}$$
$$M_2 = \mathrm{E}[|r_n|^2] \tag{6}$$
$$= \mathrm{E}[A_q^2] + 2\sigma^2, \tag{7}$$
$$h_q = \frac{A_q}{\sqrt{\mathrm{E}[A_q^2]}}. \tag{8}$$

Then substitution of (8), (5) and (6) into (4) yields the pdf $g(y_n|\alpha,\gamma)$ of normalized received signal envelope y_n conditioned on modulation α and SNR γ.

$$g(y_n|\omega,\gamma) = \sum_{q=1}^{Q} p_q \times 2(\gamma+1)y_n I_0\left[2h_q\sqrt{\gamma(\gamma+1)}y_n\right] \times \exp\left[-(\gamma+1)y_n^2 - h_q^2\gamma\right]. \tag{9}$$

In Table. I, the parameters for each modulation methods are listed. For example, the pdf $g(y_n|16\mathrm{QAM},\gamma)$ is expressed by

$$g(y_n|16\mathrm{QAM},\gamma) = \frac{1}{2}(\gamma+1)y_n \times \left(I_0\left[\frac{\sqrt{8\gamma(\gamma+1)}y_n}{\sqrt{5}}\right]\exp\left[-\frac{5(\gamma+1)y_n^2+\gamma}{5}\right] + 2I_0\left[2\sqrt{\gamma(\gamma+1)}y_n\right]\exp\left[-\left\{(\gamma+1)y_n^2+\gamma\right\}\right] + I_0\left[\frac{6\sqrt{\gamma(\gamma+1)}y_n}{\sqrt{5}}\right]\exp\left[-\frac{5(\gamma+1)y_n^2+9\gamma}{5}\right]\right). \tag{10}$$

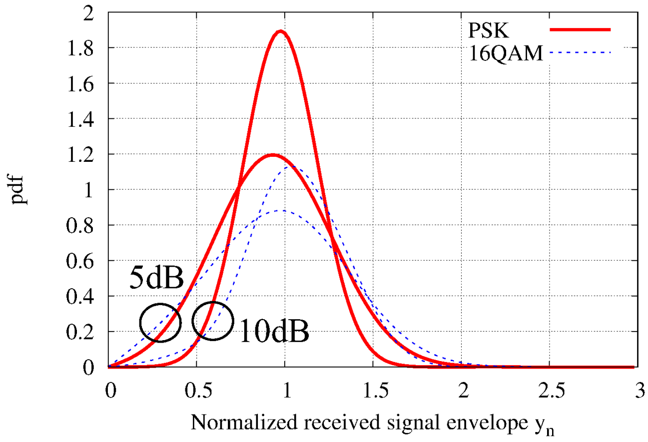


Fig. 2: the normalized pdf

Fig. 2 demonstrates the conditional pdf for PSK and 16QAM when SNR is 5dB and 10dB. From Fig. 2, it is shown that the pdf of 16QAM distributes in a wide range of received signal envelope compared with PSK. The true second moment is not known of the receiver, so the estimated second moment $\hat{M}_2$

$$\hat{M}_2 = \frac{1}{N}\sum_{n=1}^{N}|r_n|^2 \tag{11}$$

is employed instead of M_2.

The normalized received signal envelope y_n is the independent random variable which has the conditional pdf of (9). The likelihood function $L(\gamma)$ is written by

$$L(\gamma) = \prod_{n=1}^{N} g(y_n|\gamma). \tag{12}$$

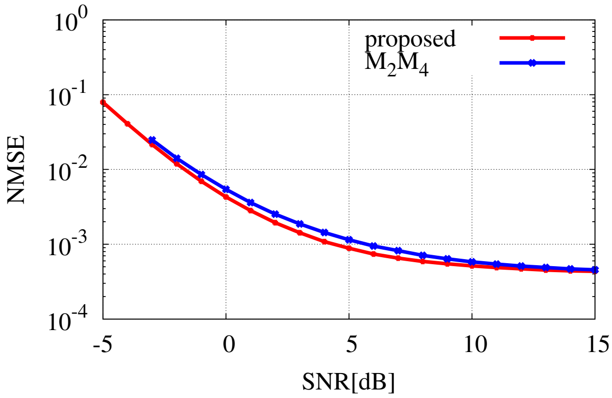
The logarithm of the both sides yields

$$LL(\gamma) = \sum_{n=1}^{N} \log g(y_n|\gamma). \tag{13}$$

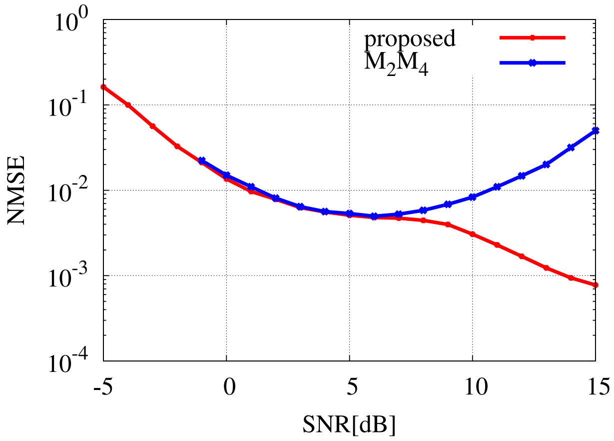
Equation (13) is the log likelihood function of γ. The maximum likelihood estimated $\hat{\gamma}$ is given by

$$\hat{\gamma} = \arg\max_{\gamma} LL(\gamma) \tag{14}$$

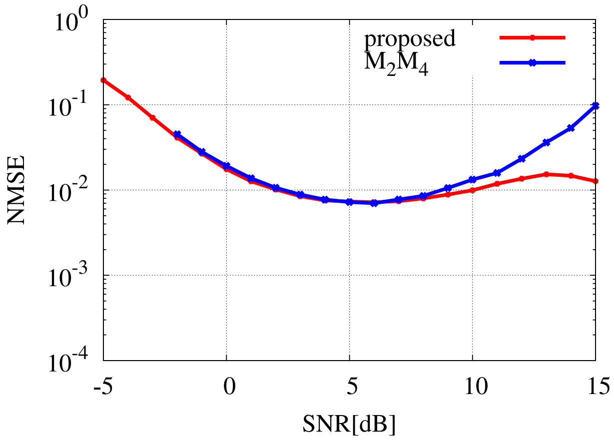
Therefore, SNR estimation by the ML method is to find the solution of (14). ML estimator is known to be asymptotically normal and unbiased estimator.



(a) PSK

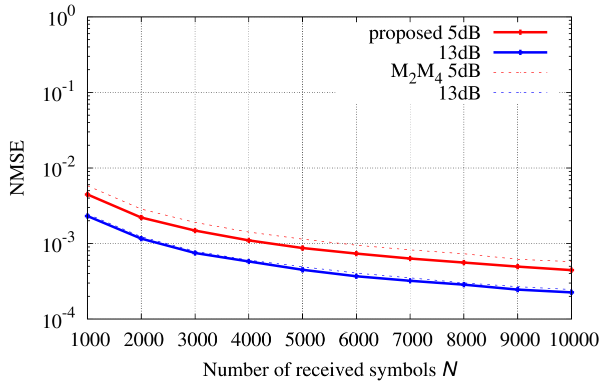


(b) 16QAM

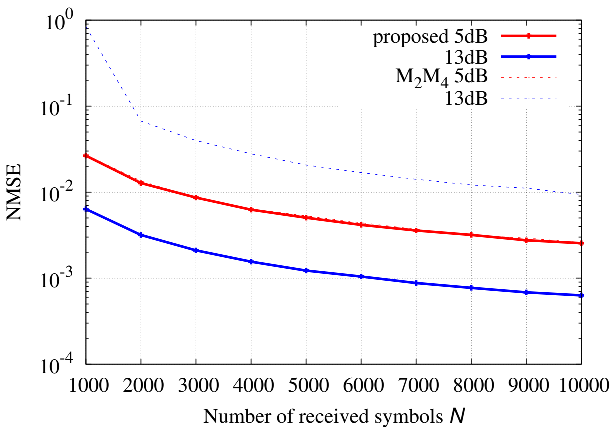


(c) 64QAM

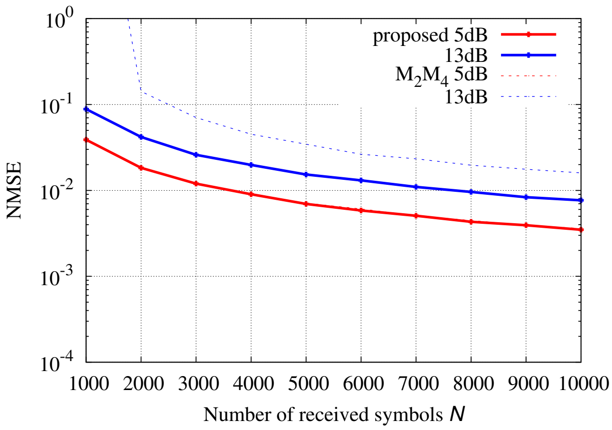
Fig. 3: NMSE for PSK, 16QAM and 64QAM



(a) PSK



(b) 16QAM



(c) 64QAM

Fig. 4: NMSE for PSK, 16QAM and 64QAM

III. Simulation Results

The proposed estimation method is evaluated by computer simulations. Normalized mean square error (NMSE) is obtained for PSK, 16QAM and 64QAM. The NMSE is defined as

$$NMSE = \frac{1}{K}\sum_{k=1}^{K}\frac{(\hat{\gamma}_k-\gamma)^2}{\gamma^2}, \tag{15}$$

where K is the number of experiments and $\hat{\gamma}_k$ is the kth estimation value of SNR.

Fig. 3 compares the NMSE of the proposed method with M_2M_4 [8]. M_2M_4 is the method based on the 2nd and 4th moment of received signal envelope. The proposed ML estimator maximizes of the log likelihood function (13) using Brent's algorithm [11]. Figs. 3(a), 3(b) and 3(c) show NMSE of PSK, 16QAM and 64QAM for $N = 5000$ of the number of samples respectively when K is 100000. For PSK, the proposed method is slightly better than M_2M_4 in the whole tested SNR range. For 16QAM and 64QAM, the performance of M_2M_4 degrades with increasing of SNR above 5dB. The performance of the proposed method for 16QAM does not improve well for 3dB $<$ SNR $<$ 9dB and the improvement is significant above 10dB. At high SNR, the proposed method is much better than M_2M_4 for 64QAM. In Fig. 3(b) of 16QAM, the increase of SNR does not reduce NMSE well in 3dB $<$ SNR $<$ 9dB. This is because the likelihood given by high SNR is sometimes larger than the likelihood given by the true SNR when some signal points A_q are close to other signal points $A_{q \neq p}$ due to the effect of noise. In other word, the distribution at the specific SNR range is similar to the distribution of the higher SNR and SNR is misestimated. Since 64QAM signal has more amplitude levels than 16QAM. This phenomenon becomes evident.

Fig. 4 compares the NMSE of the proposed method with M_2M_4 for the number of received symbols. Figs. 4(a), 4(b) and 4(c) show NMSE of PSK, 16QAM and 64QAM for $1000 < N < 10000$ respectively when K is 10000. For PSK, NMSE of the proposed method is slightly smaller than that of M_2M_4 in SNR $=$ 5dB and 13dB in the whole tested number of N. For 16QAM, the proposed methods of SNR $=$ 13dB give the much better performance than one of SNR $=$ 5dB, while M_2M_4 of SNR $=$ 13dB degrades NMSE compared with that of SNR $=$ 5dB. In SNR $=$ 13dB, M_2M_4 requires about 9000 more symbols for 16QAM and using about 2000 more symbols for 64QAM to obtain the same NMSE as the proposed method.

IV. Conclusion

In this paper, the maximum likelihood SNR estimation methods have been presented for PSK and QAM. The maximum likelihood estimation employs the pdf of received signal envelope. The methods can be used without the carrier synchronization under unknown signal power conditions. Simulation results have shown that the proposed methods yield the superior estimation performance than the moment methods especially in high SNR.

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