EE 605: Error Correcting Codes

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Endsemester Exam : **30 points** **Duration**: 180 minutes

1. Each of the parts in this question is worth 2 points.

   (a) Let C be a binary linear code with generator matrix

   $$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

   Find a systematic generator matrix for C.

   (b) Suppose a binary linear code has minimum distance 3. The coset leaders in the standard array correspond to the correctable error patterns. Prove that all weight one error patterns can be chosen as coset leaders in the standard array construction.

   (c) Find the number of codewords of weight 4 in a binary linear code with the following generator matrix. The list of codewords is not necessary but give some explanation of the procedure used to arrive at the number.

   $$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

   (d) Let C be a $(15, 11)$ binary cyclic code with generator polynomial $g(X) = X^4 + X + 1$. Find the systematic codeword in C corresponding to the message word $[0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$. Find the syndrome corresponding to the received vector $[0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1]$.

   (e) Let α be a primitive element in the field $\mathbb{F}_{81}$. If $p(X)$ is the minimal polynomial of α^5, list all the roots of $p(X)$.

2. Let C be a linear block code. [3 points]

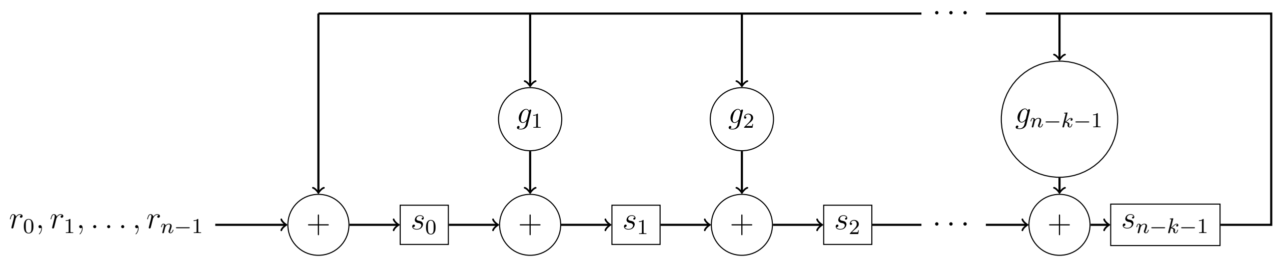
   (a) Prove that $(C^\perp)^\perp = C$.

   (b) Prove that $\mathbf{v} \in C$ if and only if $\mathbf{v} \cdot H^T = \mathbf{0}$ where H is the parity check matrix of C.

3. A linear code is said to be self-orthogonal if $C \subseteq C^\perp$. Prove that if every codeword of a binary linear code C has weight divisible by 4, then C is self-orthogonal.

   [4 points]

4. Let C be an (n, k) binary cyclic code with generator polynomial $g(X) = 1 + g_1X + g_2X^2 + \cdots + g_{n-k-1}X^{n-k-1} + X^{n-k}$. Consider the circuit below. If the received vector $\mathbf{r} = [r_0\ r_1\ r_2\ \cdots\ r_{n-1}]$ is shifted into this circuit with the initial contents of the registers $s_0, s_1, \ldots, s_{n-k-1}$ set to zero, prove that the final values of the register contents represent the syndrome of $\mathbf{r}$. Note that $\mathbf{r}$ is shifted into the circuit most significant bit first i.e. r_{n-1} first, then r_{n-2}, r_{n-3} and so on till r_0. [4 points]



5. Consider a $(2, 1)$ convolutional code with encoder matrix $G(D) = [1+D^2\ \ 1+D+D^2]$. [5 points]

   (a) Draw the trellis diagram corresponding to this encoder for an information sequence of length $h = 6$. Assume that the encoder is initially in the all zeros state and is forced to end in the all zeros state after the information sequence has entered it.

   (b) Using Viterbi algorithm on the above trellis, decode the received sequence $\mathbf{r} = [11\ 01\ 11\ 01\ 10\ 01\ 01\ 11]$.

6. What is the rate of a 13 error correcting binary BCH code of length 127? Use the table below where the integers have been reduced modulo 127. [4 points]

i	1	2	3	4	5	6	7	9	11	13	15	16	17	19	21	22	23	25
$2i$	2	4	6	8	10	12	14	18	22	26	30	32	34	38	42	44	46	50
$4i$	4	8	12	16	20	24	28	36	44	52	60	64	68	76	84	88	92	100
$8i$	8	16	24	32	40	48	56	72	88	104	120	1	9	25	41	49	57	73
$16i$	16	32	48	64	80	96	112	17	49	81	113	2	18	50	82	98	114	19
$32i$	32	64	96	1	33	65	97	34	98	35	99	4	36	100	37	69	101	38
$64i$	64	1	65	2	66	3	67	68	69	70	71	8	72	73	74	11	75	76
$128i$	1	2	3	4	5	6	7	9	11	13	15	16	17	19	21	22	23	25
$256i$	2	4	6	8	10	12	14	18	22	26	30	32	34	38	42	44	46	50