Examples of Linear Block Codes

Saravanan Vijayakumaran
sarva@ee.iitb.ac.in

Department of Electrical Engineering
Indian Institute of Technology Bombay

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Hamming Code

Hamming Code

- For any integer $m \geq 3$, the code with parity check matrix consisting of all nonzero columns of length m is a Hamming code
- For $m = 3$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

- For $m = 4$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Length of the code $n = 2^m - 1$
- Dimension of the code $k = 2^m - m - 1$
- Minimum distance of the code $d_{min} = 3$

Hamming's Approach

- Observes that a single parity check can detect a single error
- In a block of n bits, m locations are information bits and the remaining $n - m$ bits are check bits
- The check bits enforce even parity on subsets of the information bits
- In the received block of n bits the check bits are recalculated
- If the observed and recalculated values agree write a 0. Otherwise write a 1
- The sequence of $n - m$ 1's and 0's is called the checking number and gives the location of the single error
- To be able to locate all single bit error locations

$$2^{n-m} \geq n+1 \implies 2^m \leq \frac{2^n}{n+1}$$