

Arrays

Array overview

- Collection of homogeneous data elements stored at contiguous memory location

For 1D array:

- Let **ARR[lb,ub]** be an array where **lb** and **ub** are values of lowest index and largest index .
- then size of array (n) = $ub - lb + 1$
- To access an element in array, we write

name_array[subscript] like ARR[i]

OR

name_array(subscript) like ARR(i)

OR

name_array_{subscript} like ARR_i

Memory representation of 1D Array

- Address of first element of an array **a** is known as base address – **base(a)**
- Address of k^{th} element is given by
 - **loc(a[k]) = base(a) + w * (k - lb)** where **w** is number of bytes per element
 - **w** can be determined by data types of the element.
 - E.g. for int $w=4$, for double $w=8$, for char $w=1$, and for long double $w = 10$
- If $lb=0$ (C ,C++ has array indexing 0 to $n-1$)
 - **loc(a[k]) = base(a) + w * k**
- **Note: To access any element in array constant time is needed because other elements scanning is not required.**

2D array (Matrices)

- Two subscripts are needed to access any element.

Let **m** and **n** are the number of rows and columns in array then,

Size of 2D array = m x n

- To access an element in array, we write

name_array[rsubscript][csubscript] like ARR[i][j] // C, C++ style

OR

name_array(rsubscript, csubscript) like ARR(i)[j]

OR

name_array_{rsubscript csubscript} like ARR_{i,j}

OR

name_array[rsubscript, csubscript] like ARR[i,j] used in PASCAL

Memory representation of 2D Array

- Recall that address of first element of array **a** is known as base address –
base(a)

Row major order:

- Let **w** be number of bytes per element, **Ibc** is lower bound of columns and **Ibr** is lower bound of rows.
- Address of element of i^{th} row and j^{th} column in an array of $m \times n$ size is given by,

$$\text{loc}(a[i][j]) = \text{base}(a) + w * [n * (i - Ibr) + (j - Ibc)]$$

$$\text{loc}(a[i][j]) = \text{base}(a) + w * [n * i + j] \quad \text{In C/C++}$$

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33
40	41	42	43

Let $\text{base}(a) = 2013$

$$\begin{aligned}\text{loc}[2][1] &= 2013 + 4 * [4 * 2 + 1] \\ &= 2013 + 4 * 9 \\ &= 2049\end{aligned}$$

NOTE: C/C++ uses row major order for 2D arrays.

Memory representation of 2D Array (Contd...)

Column major order:

- Let **w** be number of bytes per element, **lbc** is lower bound of columns and **lbr** is lower bound of rows.
- So, ij element address in array of mxn is given by,

$$\text{loc}(a[i][j]) = \text{base}(a) + w * [m * (j - lbc) + (i - lbr)]$$

$$\text{loc}(a[i][j]) = \text{base}(a) + w * [m * j + i] \quad \text{In C/C++}$$

00	01	02	03
10	11	12	13
20	21	22	23
30	31	32	33
40	41	42	43

Let $\text{base}(a) = 2013$

$$\begin{aligned}\text{loc}[2][1] &= 2013 + 4 * [5 * 1 + 2] \\ &= 2013 + 4 * 7 \\ &= 2041\end{aligned}$$

NOTE: Pascal, Fortran and Matlab use column major order for 2D arrays.

Problem

double percentage[50];

If this array **percentage** is stored in memory starting from address **3000**.
then what will be the address of **percentage[20]** element.

Ans: $3000 + 20 * 8 = 3160$

What will be size of array in bytes?

Ans: 400

Operations on arrays

Traversing – Visiting each element exactly once. Complexity is $O(n)$

Insertion – Insert an element at given position. Complexity is $O(n)$

Deletion – Remove an element from given position. Complexity is $O(n)$

Searching – 1. Linear Search 2. Binary Search

Sorting – Arrange the elements of array on some logical order

- 1. Bubble sort 2. Insertion sort 3. Selection sort
- 4. Quick sort 5. Merge sort

Operations on arrays

Traversing – Traversing an array A of size N.

Traverse(A,N)

1. Set $i=0$
2. **While($i < N$)**
3. **Print $A[i]$**
4. $i=i+1$
5. **STOP**

Time complexity in best case = $O(n)$

Time complexity in worst case = $O(n)$

Operations on arrays

Insertion – Insert an element P at index location k .

Insert(A[N],k,P)

1. Set i=N
2. While(i>k)
3. A[i]=A[i-1] //shift toward right
4. i=i-1
5. A[k]=P
6. N=N+1
7. STOP

Time complexity in best case =O(1) (When k is N i.e. insert at last)

Time complexity in worst case =O(n) (When k is 0 i.e. insert at first)

Operations on arrays

Deletion – Remove an element from index location k.

Delete(A[N],k)

1. Set D=A[k]
2. i=k
3. While(i<=N-2)
4. A[i]=A[i+1] //shift left
5. i=i+1
6. N=N-1
7. STOP

- Time complexity in best case = $O(1)$ (When k is N-1 i.e. delete last element)
- Time complexity in worst case = $O(n)$ (When k is 0 i.e. delete first element)

Sparse Matrix

- Most of the elements are zero
- If not sparse then it is called dense matrix
- As many of its elements are zero, we can save space by storing only non-zero elements.
- A sparse matrix can be represented by using following TWO representations:

1. **Triplet Representation or Array representation**
2. **Linked Representation (To be discussed later)**

Sparse Matrix representation using array

Example:

0	2	0	0
0	1	0	0
3	0	0	0
0	0	0	0

Number of non-zero entries = 3
Number of zero entries = 13
Elements to be stored = 16

Number of rows(m)

Number of cols(n)

Number of non-zero entries

4	4	3
1	2	2
2	2	1
3	1	3

Only non-zero elements:
<row, col, element>

Elements to be stored = 12

Let see one more example -

Number of non-zero entries = 16

Number of zero entries = 26

Elements to be stored = 42

0	5	0	9	0	0
0	45	0	0	8	0
8	0	0	0	0	9
67	0	0	5	5	0
0	5	0	30	0	0
8	0	0	8	0	0
8	0	0	7	0	31

Elements = 51



7	6	16
1	2	5
1	4	9
2	2	45
2	5	8
3	1	8
3	6	9
4	1	67
4	4	5
4	5	5
5	2	5
5	4	30
6	1	8
6	4	8
7	1	8
7	4	7
7	6	31

Number of non-zero entries = k

Number of rows = m

Number of columns = n

Condition: $3(k+1) < m \times n$

Advantages of Sparse Matrices

- Two advantages:
 - **Storage:** There are lesser non-zero elements than zeros and thus lesser memory can be used to store only those elements.
 - **Computing time:** Computing time can be saved by logically designing a data structure traversing only non-zero elements.

Is this Sparse Matrix? If YES then represent it.

0	0	0	0	3	0
0	9	0	0	0	2
0	0	1	8	0	0
0	0	0	0	4	0
3	0	0	0	0	0

YES

Here

Number of non-zero entries = k=7

Number of rows = m=5

Number of columns = n=6

Condition: $3(k+1) < m \cdot n = 3(7+1) < 30$

5	6	7
1	5	3
2	2	9
2	6	2
3	3	1
3	4	8
4	5	4
5	1	3