Department of Computer Science and Engineering

SCILAB

LINEAR ALGEBRA AND ITS APPLICATIONS -UE19MA251

Session: Jan-May 2021

NAME OF THE STUDENT : Abhishek Aditya BS

SRN : PES1UG19CS019

BRANCH : Computer Science and Engineering

SEMESTER & SECTION : 4th Semester, A1 Section

Contents

Gaussian Elimination

Question: Solve the following system of equations by Gaussian Elimination. Identify the pivots in each case.

$$5x + 2y = 2$$

$$2x + y - z = 0$$

$$2x + 3y - z = 3$$

Scilab Code:

```
clc;clear;close;
A=[5,2,0;2,1,-1;2,3,-1],b=[2;0;3]
Augmented_A=[A b]
a=Augmented_A
n=3;
for(i=2:n)
    for(j=2:n+1)
        a(i,j)=a(i,j)-a(1,j)*a(i,1)/a(1,1);
end
a(i,1)=0;
end
for(i=3:n)
    for(j=3:n+1)
        a(i,j)=a(i,j)-a(2,j)*a(i,2)/a(2,2);
end
a(i,2)=0;
end
x(n)=a(n,n+1)/a(n,n);
for(i=n-1:-1:1)
    sum_k=0;
    for k=i+1:n
        sum_k=sum_k+a(i,k)*x(k);
    end
    x(i)=(a(i,n+1)-sum_k)/a(i,i);
end
disp('The values of x,y,z are ',x(1),x(2),x(3));
disp('The pivots are ',a(1,1),a(2,2),a(3,3));
```

Output:

```
"The values of x,y,z are "

-0.2000000

 1.5000000

 1.1000000

"The pivots are "

 5.

 0.2000000

 10.000000
```

LU Decomposition of a Matrix

Question: Factorize the following matrix as $A = LU$.

$$A = \begin{bmatrix} 6 & 18 & 3 \\ 2 & 12 & 1 \\ 4 & 15 & 3 \end{bmatrix}$$

Scilab Code:

```
clear;clc;close();
A=[6,18,3;2,12,1;4,15,3];
U=A;
disp('The given matrix is A=',A);
m=det(U(1,1));
n=det(U(2,1));
a=n/m;
U(2,:)=U(2,:)-U(1,:)/(m/n);
n=det(U(3,1));
b=n/m;
U(3,:)=U(3,:)-U(1,:)/(m/n);
m=det(U(2,2));
n=det(U(3,2));
c=n/m;
U(3,:)=U(3,:)-U(2,:)/(m/n);
disp('The upper triangular matrix is U=',U);
L=[1,0,0;a,1,0;b,c,1];
disp('The lower triangular matrix is L=',L);
```

Output:

```
"The given matrix is A="

 6.   18.   3.
 2.   12.   1.
 4.   15.   3.

"The upper triangular matrix is U="

 6.   18.   3.
 0.   6.    0.
 0.   0.    1.

"The lower triangular matrix is L="

 1.          0.    0.
 0.3333333   1.    0.
 0.6666667   0.5   1.
```

Question: Solve the following system of equations by decomposing A as a product $A = LU$.

$$x + y + z = 1$$

$$4x + 3y - z = 6$$

$$3x + 5y + 3z = 4$$

Scilab Code:

```
clear;clc;close();
format('v',5);
A = [1,1,1;4,3,-1;3,5,3];

for l=1:3
    L(l,l)=1;
end

for i=1:3
    for j=1:3
        _sum_ = 0;
        if j>=i
            for k=1:i-1
                _sum_ = _sum_ + L(i,k)*U(k,j);
            end
            U(i,j)=A(i,j)-_sum_;
        else
            for k=1:j-1
                _sum_ = _sum_ + L(i,k)*U(k,j);
            end
            L(i,j)=(A(i,j)-_sum_)/U(j,j);
        end
    end
end

b = [1;6;4];
c = L\b;
x = U\c;
disp('Solution of the given equation is : ',x)

```

Output:

```
"Solution of the given equation is : "

 1.
 0.5
-0.5
```

The Gauss-Jordan method of calculating Inverse of a Matrix

Question: Find the inverse of the following matrix

$$A = \begin{bmatrix} 11 & 5 & 6 \\ 7 & 8 & 3 \\ 4 & 9 & 10 \end{bmatrix}$$

Scilab Code:

```
clc;clear;
A=[11 5 6;7 8 3;4 9 10];
n=length(A(1,:));
Aug=[A,eye(n,n)];

for(j=1:n-1)
    for(i=j+1:n)
        Aug(i,j:2*n)=Aug(i,j:2*n)-Aug(i,j)/Aug(j,j)*Aug(j,j:2*n);
    end
end

for j=n:-1:2
    Aug(1:j-1,:)=Aug(1:j-1,:)-Aug(1:j-1,j)/Aug(j,j)*Aug(j,:);
end

for (j=1:n)
    Aug(j,:)=Aug(j,:)/Aug(j,j);
end
B=Aug(:,n+1:2*n);
disp('The inverse of A is: ',B);
```

Output:

```
"The inverse of A is: "

 0.1106472   0.0083507  -0.0688935
-0.1210856   0.1795407   0.0187891
 0.0647182  -0.1649269   0.1106472
```

Span of the Column Space of A

Question: Identify the columns that span the column space of A in the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Scilab Code:

```
clc;clear;close();
disp('The given matrix is:')
a=[1,0,2,0;0,1,-1,1;1,1,1,1];
disp(a);
a(2,:)=a(2,:)-(a(2,1)/a(1,1))*a(1,:);
a(3,:)=a(3,:)-(a(3,1)/a(1,1))*a(1,:);
disp(a);
a(3,:)=a(3,:)-(a(3,2)/a(2,2))*a(2,:);
disp(a);
a(1,:)=a(1,:)/a(1,1)
a(2,:)=a(2,:)/a(2,2)
disp(a)
for i=1:3
    for j=i:4
        if(a(i,j)<>0)
            disp('column',j,'is a pivot column')
            break
        end
    end
end
```

Output:

```
"The given matrix is:"

 1.   0.   2.   0.
 0.   1.  -1.   1.
 1.   1.   1.   1.

 1.   0.   2.   0.
 0.   1.  -1.   1.
 0.   1.  -1.   1.

 1.   0.   2.   0.
 0.   1.  -1.   1.
 0.   0.   0.   0.

 1.   0.   2.   0.
 0.   1.  -1.   1.
 0.   0.   0.   0.

"column"

 1.

"is a pivot column"

"column"

 2.

"is a pivot column"
```

The Four Fundamental Subspaces

Question: Find the four fundamental subspaces of the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Scilab Code and Output:

```
clear;
close;
clc;
A=[1 0 2 3;0 1 4 5;0 0 0 0]
disp('A=',A);
[m,n]=size(A);
disp('m=',m);
disp('n=',n);
[v,pivot]=rref(A);
disp(rref(A));
disp(v);
r=length(pivot);
disp('rank=',r);
cs=A(:,pivot);
disp('Column Space=',cs);
ns=kernel(A);
disp('Null Space=',ns);
rs=v(1:r,:)';
disp('Row Space=',rs);
lns=kernel(A');
disp('Left Null Space=',lns);
```

```
"A="

 1.   0.   2.   3.
 0.   1.   4.   5.
 0.   0.   0.   0.

"m="

 3.

"n="

 4.

 1.   0.   2.   3.
 0.   1.   4.   5.
 0.   0.   0.   0.

 1.   0.   2.   3.
 0.   1.   4.   5.
 0.   0.   0.   0.

"rank="

 2.

"Column Space="

 1.   0.
 0.   1.
 0.   0.

"Null Space="

-0.5105888  -0.1656342
-0.5411189  -0.6854941
-0.4647936   0.6141557
 0.4800587  -0.3542257

"Row Space="

 1.   0.
 0.   1.
 2.   4.
 3.   5.

"Left Null Space="

 0.
 0.
 1.
```

Projections by Least Squares

Question: Find the line of best fit $Ax = b$ for the following system

$$\mathrm{A} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathrm{x} = \begin{pmatrix} C \\ D \end{pmatrix}$$

$$b = \begin{pmatrix} 4 \\ 5 \\ 9 \end{pmatrix}$$

Scilab Code:

```
clear;close;clc;
A=[1 -1;0 1;1 0];
disp('A=',A);
b=[4;5;9];
disp('b=',b);
x=(A'*A)\(A'*b);
disp('x=',x);
C=x(1,1);
D=x(2,1);
disp("C=",C);
disp("D=",D);
disp("The line of best fit is b=C+Dt");

```

Output:

```
"A="

 1.  -1.
 0.   1.
 1.   0.

"b="

 4.
 5.
 9.

"x="

 9.
 5.

"C="

 9.

"D="

 5.

"The line of best fit is b=C+Dt"
```

The Gram-Schmidt Orthogonalization

Question: Apply the Gram-Schmidt process to the following set of vectors and find the orthogonal matrix.

$$(1,1,0), (1,0,1), (0,1,1)$$

Scilab Code:

```
clear;close;clc;
A=[1 1 0;1 0 1;0 1 1];
disp('A=',A);
[m,n]=size(A);
for k=1:n
    V(:,k)=A(:,k);
    for j=1:k-1
        R(j,k)=V(:,j)'*A(:,k);
            V(:,k)=V(:,k)-R(j,k)*V(:,j);
    end
    R(k,k)=norm(V(:,k));
    V(:,k)=V(:,k)/R(k,k);
end
disp('Q=',V);
```

Output:

```
"A="

 1.   1.   0.
 1.   0.   1.
 0.   1.   1.

"Q="

 0.7071068   0.4082483  -0.5773503
 0.7071068  -0.4082483   0.5773503
 0.          0.8164966   0.5773503
```

Eigen Values and Eigen vectors of a given square matrix

Question: Find the Eigen Values and the corresponding Eigen vectors of the following matrix.

$$\begin{bmatrix} -26 & -33 & -25 \\ 31 & 42 & 23 \\ -11 & -15 & -4 \end{bmatrix}$$

Scilab Code:

```
clc;close;clear;
A=[-26 -33 -25;31 42 23;-11 -15 -4]
lam=poly(0,'lam')
lam=lam
charMat=A-lam*eye(3,3)
disp('The characteristic matrix is:',charMat)
charPoly=poly(A,'lam')
disp('The characteristic polynomial is:',charPoly)
lam=spec(A)
disp('The Eigen Values of A are:',lam)
function[x,lam]=eigenvectors(A)
    [n,m]=size(A);
    lam=spec(A)';
    x=[];
    for k=1:3
        B=A-lam(k)*eye(3,3);
        C=B(1:n-1,1:n-1);
        b=-B(1:n-1,n);
        y=C\b;
        y=[y;1];
        y=y/norm(y);
        x=[x y];
end
endfunction
get("eigenvectors")
[x,lam]=eigenvectors(A)
disp('The Eigen Vectors of A are:',x);
```

Output:

```
"The characteristic matrix is:"

-26 -lam  -33       -25
31         42 -lam  23
-11       -15       -4 -lam

"The characteristic polynomial is:"

270 -63lam -12lam² +lam³

"The Eigen Values of A are:"

 15. + 0.i
-6.  + 0.i
 3.  + 0.i

"The Eigen Vectors of A are:"

 0.4082483   0.8082904  -0.8164966
-0.8164966  -0.5773503   0.4082483
 0.4082483   0.1154701   0.4082483
```