

13 / 10 / 2023

RECURSION CLASS 3

✓ Program 01: Check array sorted or not

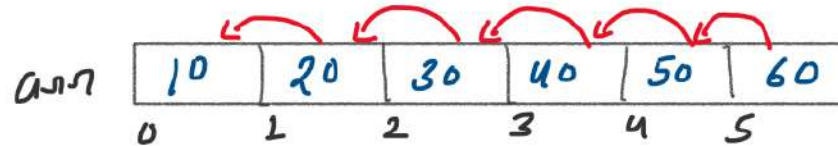
Base case

```
if (index == size)
    return true
```

Processing

```
if (arr[index] > arr[index-1])
{
    f(arr, size, index+1);
}
else
{
    return false;
}
```

Relation



1st step

20	>	10
30	>	20
40	>	30
50	>	40
60	>	50

→ index = 1

→ index = 2

→ index = 3

→ index = 4

→ index = 5

→ index = 6 X Ruk jao ---

$arr[index] \geq arr[index-1]$

Recursion
Dikh legi

```
// Program 01: Check array sorted or not
#include<iostream>
using namespace std;

bool checkSorted(int *arr, int size, int index){
    // Base Case
    if(index >= size){
        return true;
    }

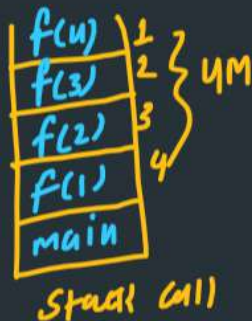
    // Processing
    if(arr[index] > arr[index - 1]){
        // Aage check karna padega to ab recursion dekh lega
        bool aageKaAns = checkSorted(arr, size, index + 1);
        return aageKaAns;
    }
    else{
        // Iska mtlb array sorted nhi hai
        return false;
    }
}
```

```
int main(){
    int arr[300] = {10,20,30,40,50,60};
    int size = 6;
    int index = 1;

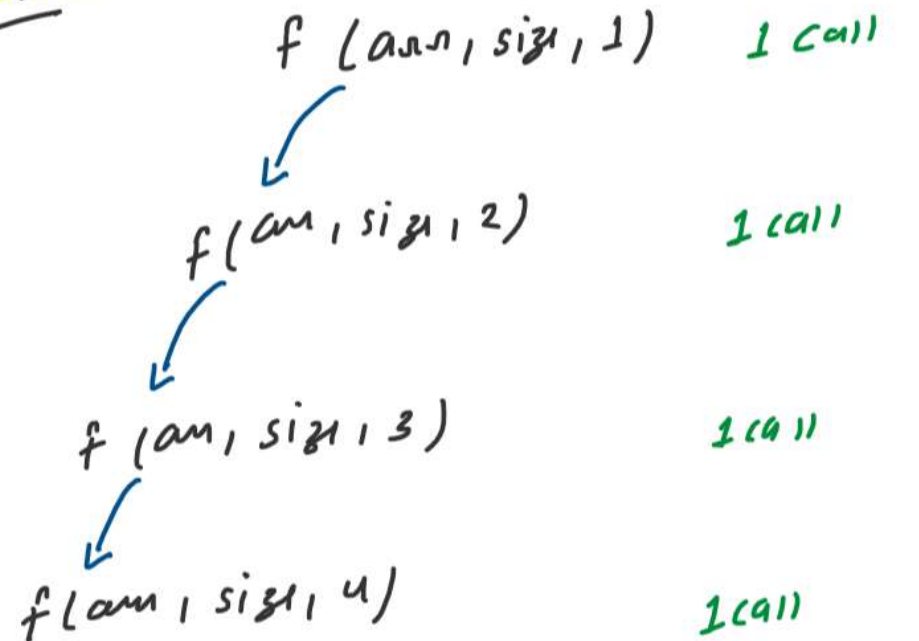
    bool ans = checkSorted(arr, size, index);

    if(ans){
        cout<<"Array is sorted"<<endl;
    }
    else{
        cout<<"Array is not sorted"<<endl;
    }

    return 0;
}
```



Size = N = 4



T.C. = $O(N)$

S.C. = $O(N)$

$\rightarrow O(4M) \Rightarrow O(NM) \rightarrow \text{Byte (constant)}$
 $\rightarrow O(N)$

Total call = 4

$T(N) = O(N)$

$f(arr, 3, 1)$

```
bool checkSorted(int *arr, int size, int index){
    // Base Case
    if(index >= size){
        return true;
    }

    // Processing
    if(arr[index] > arr[index - 1]){
        // Aage check karna padega to ab recursion dekh lega
        bool aageKaAns = checkSorted(arr, size, index + 1);
        return aageKaAns;
    }
    else{
        // Iska matlab array sorted nhi hai
        return false;
    }
}
```

$f(arr, 3, 2)$

```
bool checkSorted(int *arr, int size, int index){
    // Base Case
    if(index >= size){
        return true;
    }

    // Processing
    if(arr[index] > arr[index - 1]){
        // Aage check karna padega to ab recursion dekh lega
        bool aageKaAns = checkSorted(arr, size, index + 1);
        return aageKaAns;
    }
    else{
        // Iska matlab array sorted nhi hai
        return false;
    }
}
```

$f(arr, 3, 3)$

```
bool checkSorted(int *arr, int size, int index){
    // Base Case
    if(index >= size){
        return true;
    }

    // Processing
    if(arr[index] > arr[index - 1]){
        // Aage check karna padega to ab recursion dekh lega
        bool aageKaAns = checkSorted(arr, size, index + 1);
        return aageKaAns;
    }
    else{
        // Iska matlab array sorted nhi hai
        return false;
    }
}
```

TRUE

SORTED
ARRAY

arr

10	20	30
0	1	2

$f(arr, size, index)$

MAIN

✓ Program 02: Binary search recursive solution

Target = 40

arr

	10	20	30	40	50	60	70
0	1	2	3	4	5	6	

MID

Base case

if (s > e)
return -1

Processing

if (arr[mid] > target)
return f(arr, target, start, mid-1);
else
return f(arr, target, mid+1, end);

Relation

start = 0 end = 6 mid = $\frac{start + end}{2}$

1st
STEP

if (arr[mid] == target)
→ return mid

Recursion

aur kya

if (arr[mid] > target)
→ return f(arr, target, start, mid-1);
else
→ return f(arr, target, mid+1, end);

```
// Program 02: Binary search recursive solution
#include<iostream>
using namespace std;
```

```
int binaryS(int arr[], int target, int start, int end){
    // Base Case
    if(start > end){
        return -1;
    }

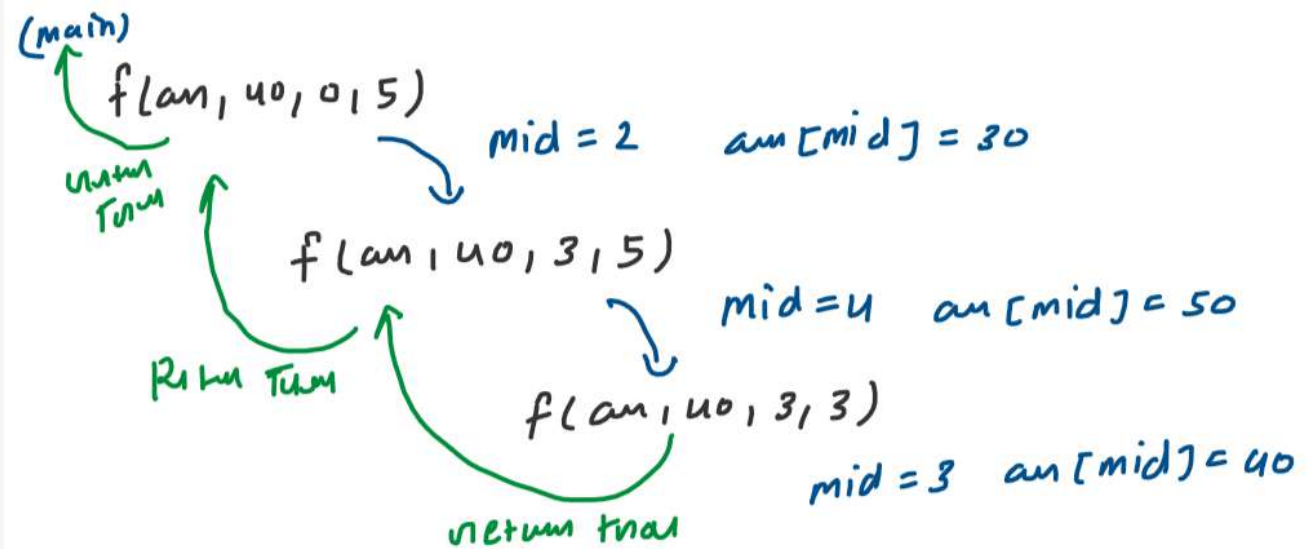
    // Processing -> Ek case me khud solve kar loonga
    int mid = start + (end - start)/2;
    if(arr[mid] == target){
        return mid;
    }

    // Baki ka recursion dekh lega
    else if(arr[mid] < target){
        // Right jao
        return binaryS(arr, target, mid + 1, end);
    }
    else{
        // Left jao
        return binaryS(arr, target, start, mid - 1);
    }
}
```

```
int main(){
    int arr[] = {10,20,30,40,50,60};
    int size = 6;
    int start = 0;
    int end = size - 1;
    int target = 40;

    int ans = binaryS(arr, target, start, end);

    if(ans > 0){
        cout<<"Target found at index "<< ans <<endl;
    }
    else{
        cout<<"Target not found"<<endl;
    }
    return 0;
}
```



T.C. and S.C. $\Rightarrow O(\log N)$

✓ Program 03: Subsequence of string

Ex-1

String = "A B C";

✓ x x	= "A"
x ✓ x	= "B"
x x ✓	= "C"
✓ ✓ x	= "AB"
✓ x ✓	= "AC"
x ✓ ✓	= "BC"
✓ ✓ ✓	= "ABC"
x x x	= ""

Total sub-sequences string
ans 8 (2^3)

String size = $N = 3$

✓ ✗
Pattern 01: Include and exclude pattern

Ex-2

String = "X Y"

✓ x	= "X"
x ✓	= "Y"
✓ ✓	= "XY"
x x	= ""

Total SS = 4
= (2^2)

$N = 2$

$str = "ABC"$
 $output = ""$
 $index = 0$

2⁰ call

$f(str, output, index)$

\swarrow Inc A \searrow Exc

2¹ call

$f("ABC", "A", 1)$

$f("ABC", "", 1)$

\swarrow Inc B \searrow Exc

\swarrow Inc B \searrow Exc

$f("ABC", "B", 2)$

2² call

$f("ABC", "AB", 2)$

$f("ABC", "A", 2)$

\swarrow Inc C \searrow Exc

\swarrow Inc C \searrow Exc

$f("ABC", "AC", 3)$

$f("ABC", "A", 3)$

$f("ABC", "BC", 3)$

$f("ABC", "", 2)$

\swarrow Inc C \searrow Exc

$f("ABC", "C", 3)$

2³ call

$f("ABC", "ABC", 3)$

$f("ABC", "AB", 3)$

$f("ABC", "B", 3)$

$f("ABC", "", 3)$

RUK jao

RUK jao

RUK

RUK

$T.C. \Rightarrow O(2^3) \Rightarrow O(2^N)$
 $S.C. \Rightarrow ??$
 where N is size of string


```

// Program 03: Subsequence of string
#include<iostream>
#include<string>
using namespace std;

void findSubsequence(string str, string output, int index){
    // Base Case
    if(index >= str.length()){
        // Ans jo hai output string me build ho chuka hai to print
        kardo
        cout<<"-> "<<output<<endl;
        return;
    }

    // Processing
    int ch = str[index];

    // Include - Koi ek character ko include kardo Like "A"
    output.push_back(ch);
    findSubsequence(str, output, index + 1);

    // Exclude - Jis character ko include kiya hai ussi character ko
    // ek bar output me se remove bhi kardo Like "A"
    output.pop_back();
    findSubsequence(str, output, index + 1);
}

int main(){
    string str = "ABC";
    string output = " ";
    int index = 0;

    findSubsequence(str, output, index);
    return 0;
}

```

```

/*
    1      2
    AGAR PAHLE EXCLUDE AND BAD ME INCLUDE KAROGE TO OUTPUT YEH AANA CHAIYE
    -> C
    -> B
    -> BC
    -> A
    -> AC
    -> AB
    -> ABC
    1      2
    AGAR PAHLE INCLUDE AND BAD ME EXCLUDE KAROGE TO OUTPUT YEH AANA CHAIYE
    -> ABC
    -> AB
    -> AC
    -> A
    -> BC
    -> B
    -> C
*/

```

```

// Exclude - Koi ek character ko ignore karunga Like "A"
findSubsequence(str, output, index + 1);

// Include - Jis character ko ignore kiya hai ussi character ko
// ek bar output me include bhi kardo Like "A"
output.push_back(ch);
findSubsequence(str, output, index + 1);

```

✓ Program 04: Maximize the cost segment (GFG)

Pattern 02: Exploring all possible ways pattern

Example 01:

Input:

$N = 4$

$x = 2, y = 1, z = 1$

Output: 4

Example 02:

Input:

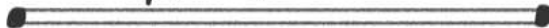
$N = 5$

$x = 5, y = 3, z = 2$

Output: 2

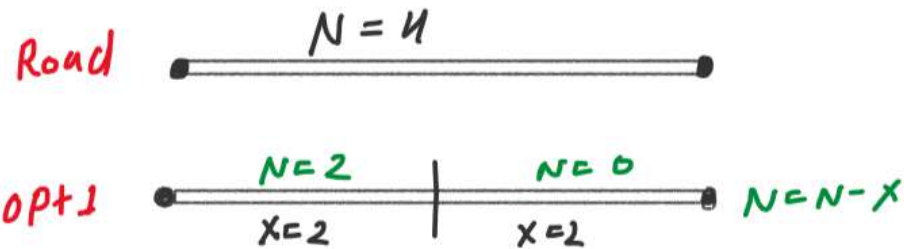
Road

$N = 4$

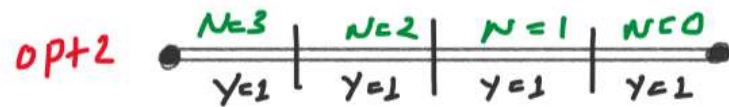


$N = 5$

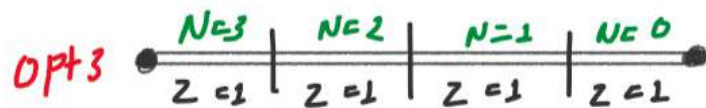




2 parts of the road

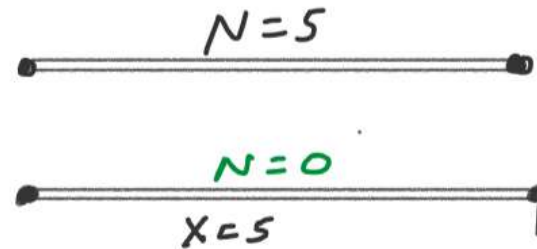


4 parts of the road

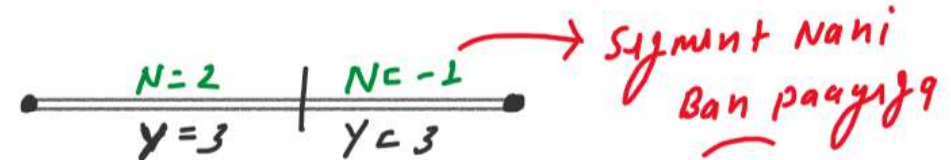


4 parts of the road

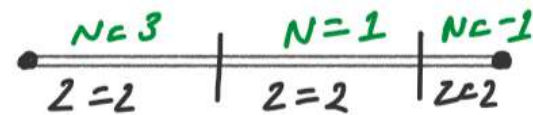
$$\text{Max}(\text{opt1}, \text{opt2}, \text{opt3}) = \text{Max}(2, 4, 4) \\ = 4 \text{ output}$$



1 part of the road

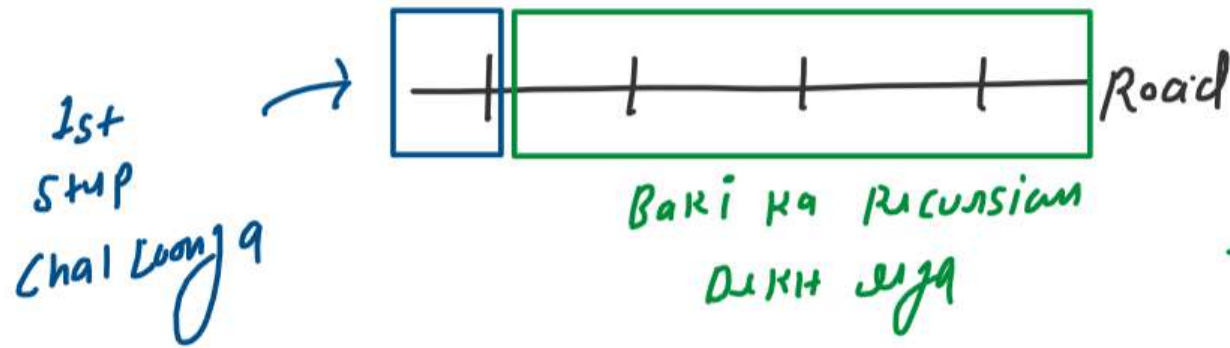


1 part of the road



2 part of the road

$$\text{Max}(1, 1, 2) = 2 \text{ output}$$



Relation

$$\text{option1} = 1 + f(N-x, x, y, z)$$

$$\text{option2} = 1 + f(N-y, x, y, z)$$

$$\text{option3} = 1 + f(N-z, x, y, z)$$

BASE CASE

if ($N == 0$)
return 0;

Agar $N == 0$ hai to zero segment
Ban GATE HONGI

if ($N < 0$)
return invalid no.

Agar $N < 0$ to iss case main koi bhi
segment nahi banana chahiye

```
// ☒ Program 04: Maximize the cost segment (GFG)
class Solution
{
    public:
        //Function to find the maximum number of cuts.
        int maximizeTheCuts(int n, int x, int y, int z)
        {
            // Base Case
            if(n == 0){
                return 0;
            }
            if(n < 0){
                return INT_MIN;
            }

            // Maine x length ka ek segment cut kar liya, baki ka recursion dekh lega
            int option1 = 1 + maximizeTheCuts(n-x, x, y, z);
            // Maine y length ka ek segment cut kar liya, baki ka recursion dekh lega
            int option2 = 1 + maximizeTheCuts(n-y, x, y, z);
            // Maine z length ka ek segment cut kar liya, baki ka recursion dekh lega
            int option3 = 1 + maximizeTheCuts(n-z, x, y, z);

            int finalAns = max(option1, max(option2, option3));
            return finalAns;
        }
};
```

I will Do latam
 DRY RUN = ??

Time = ??

S.C. = ??

✓ Program 05: Coin change (Leetcode-322)

Pattern 02: Exploring all possible ways pattern

Example 1:

Input: coins = [1,2,5], amount = 11

Output: 3

Explanation: $11 = 5 + 5 + 1$

Example 2:

Input: coins = [2], amount = 3

Output: -1

Example 3:

Input: coins = [1], amount = 0

Output: 0

way 1

coin = 1 takes 11 times $\Rightarrow 11 \times 1 = 11$ 11 coins

coin = 2 takes 5 times & coin = 1 takes 1 time $\Rightarrow (5 \times 2) + 1 = 11$ 6 coins

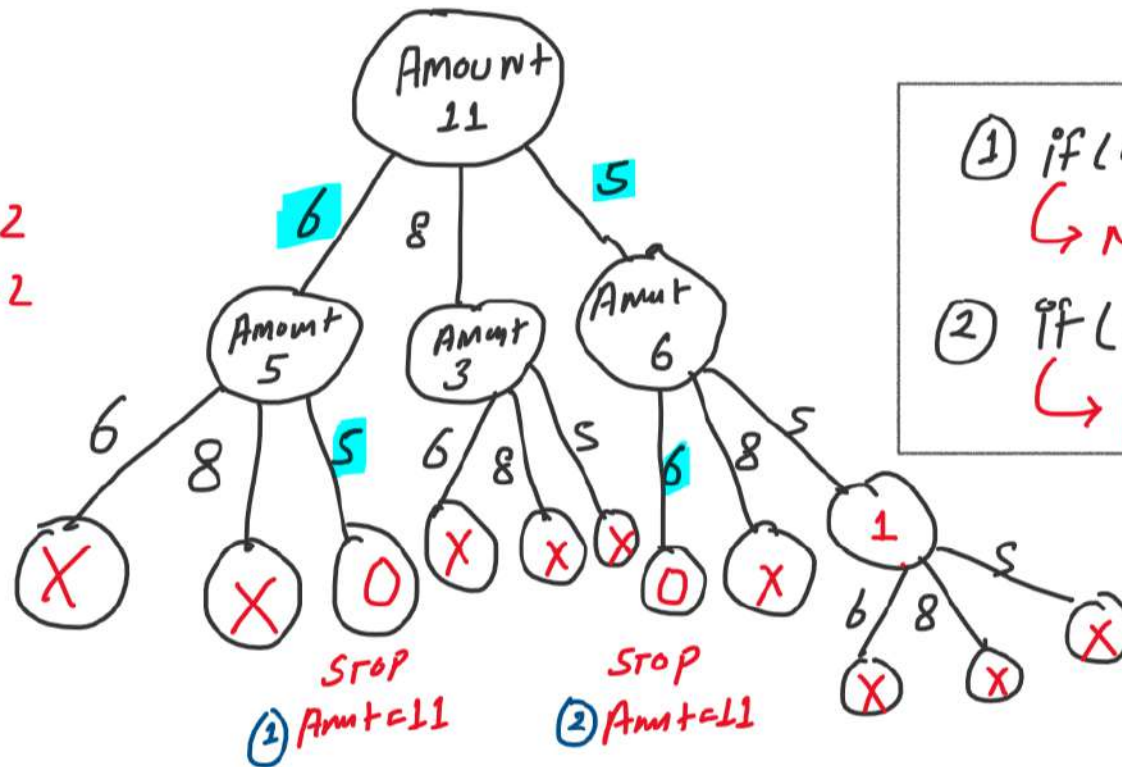
coin = 5 takes 2 times & coin = 1 takes 1 time $\Rightarrow (2 \times 5) + 1 = 11$ 3 coins

$\text{Min}(11, 6, 3) \Rightarrow$ 3 coins

Ways
 Amount = 11
 Coins = 6, 8, 5

- Call
 ① $6 + 5 = 11$ } coins = 2
 ② $5 + 6 = 11$ } coins = 2

$\min(2, 2) = 2$
Output



- ① if (coin > Amount)
 ↳ No Recursion call
- ② if (Amount == 0)
 ↳ Return 0 coin
 ↳ Base Case

Way 3
Amount = N = 11
coins [6, 8, 5]

$$f(N) = f(11) \xrightarrow{6 \text{ coin}} f(5) \Rightarrow \boxed{1 + f(N-6)}$$

1 STEP

$$f(N) = f(11) \xrightarrow{8 \text{ coin}} f(3) \Rightarrow \boxed{1 + f(N-8)}$$

$$f(N) = f(11) \xrightarrow{5 \text{ coin}} f(6) \Rightarrow \boxed{1 + f(N-5)}$$

{ But AISA zaroori
Nahi Hai ki Har Bar
3 types ki coin hi
Ho usse jada
Bhi ho skata
hai to
Har Ek coin ko
check karke ki
kya loop ka
use karunga. KE
(coin <= Amount)
hai ya nahi }

```

// Program 05: Coin change (Leetcode-322)
class Solution {
public:
    int solve(vector<int>& coins, int amount){
        // Base Case
        if(amount == 0){
            return 0;
        }
        if(amount < 0){
            return INT_MAX;
        }

        // Processing
        int mini = INT_MAX;
        for(int i=0; i<coins.size(); i++){
            int coin = coins[i]; // current coin is already used

            // Current coin ko tabhi use karenge jab uski
            // value <= amount hogi
            if(coin <= amount){
                // Relation
                int recAns = solve(coins, amount - coin);
                // agar recAns INT_MAX ke equal hai or usme 1 add kar diya to
                // ans ki value integer ki range se bahar hojayegi
                if(recAns != INT_MAX){
                    int ans = 1 + recAns;
                    mini = min(ans, mini);
                }
            }
        }
        return mini;
    }

    int coinChange(vector<int>& coins, int amount) {
        int ans = solve(coins, amount);

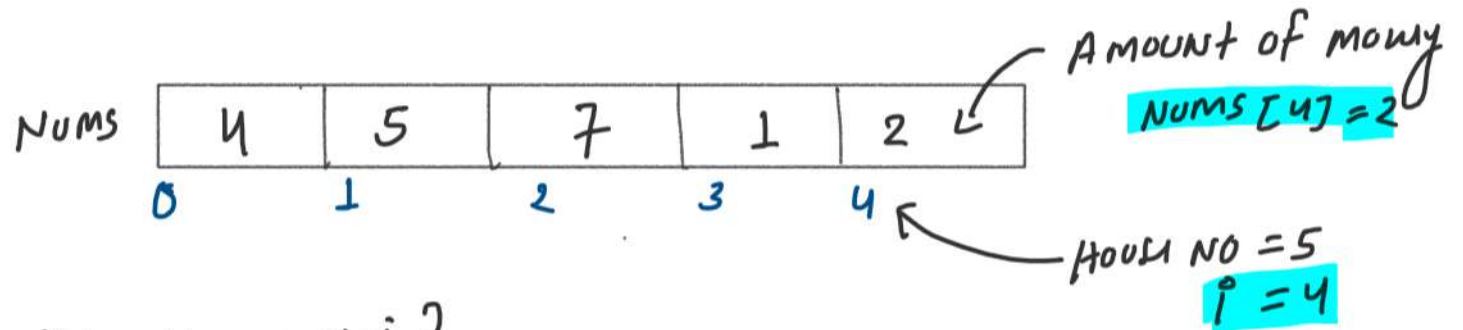
        // agar ans = INT_MAX aa rha hai to me coin change nhi kar skta hu
        if(ans == INT_MAX){
            return -1;
        }
        else{
            return ans;
        }
    }
};

```

Dof RUN
 T.C. = ?
 S.C. = ?

✓ Program 06: House Robber (Leetcode-198)

Ex¹⁰¹



FIND KYA KARNA HAI?
 → Return the maximum Amount of money

CASE:1
 Chori KARNA wala huan

✓ 1 st	1st
✓ 2 nd	3rd 1st
✓ 4 th	3rd

amount

4
7
2
<hr/>
Total = 13

CASE:2
 Chori NA KARNE wala huan

✓ 1 st	0th	2nd
✓ 3 rd	2nd	4th

Amount

5
1
<hr/>
total = 6

max Amount = 13
 output

Example: 02

Nums	1	2	3	1
	0	1	2	3

Case-1

Chori Kame wali Ghar	Amount
✓ 0th	1
✓ 2th	3
1th X 3th	<hr/> 4

Case-2

Chori Na Kame wali Ghar	Amount
✓ 1th	2
✓ 3th	1
X 0th X 2th	<hr/> 3

maxAmount = 4

output

nums

4	5	7	1	2
0	1	2	3	4

Case:1

0th index me
chahi karna

OR

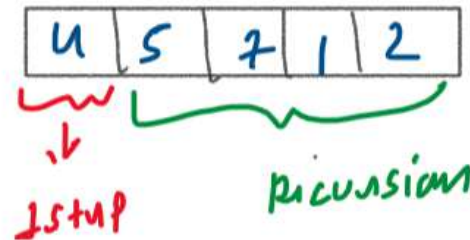
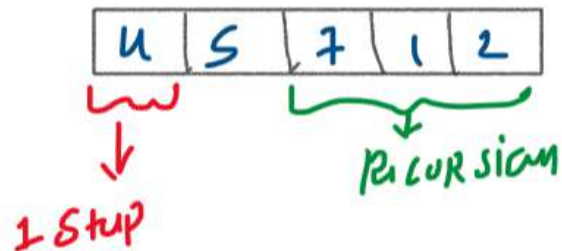
Case:2

0th index me
chahi nahi karna

$$4 + f(i+2, N-1)$$

$$0 + f(i+1, N-1)$$

} max Amount $\Rightarrow ?$



```

// ✅ Program 06: House Robber (Leetcode-198)
class Solution {
public:
    int solve(vector<int>& nums, int size, int index){
        // Base Case
        if(index >= size){
            return 0;
        }

        // Chori karlo --> ith index par
        int option1 = nums[index] + solve(nums, size, index + 2);

        // Chori mat karo --> ith index par
        int option2 = 0 + solve(nums, size, index + 1);

        // return the Maximum Amount
        int finalAns = max(option1, option2);
        return finalAns;
    }
    int rob(vector<int>& nums) {
        int size = nums.size();
        int index = 0;
        int ans = solve(nums, size, index);
        return ans;
    }
};

```

DRY RUN
T.C. = ?
S.C. = ?