

10/10/2023

# RECURSION CLASS2

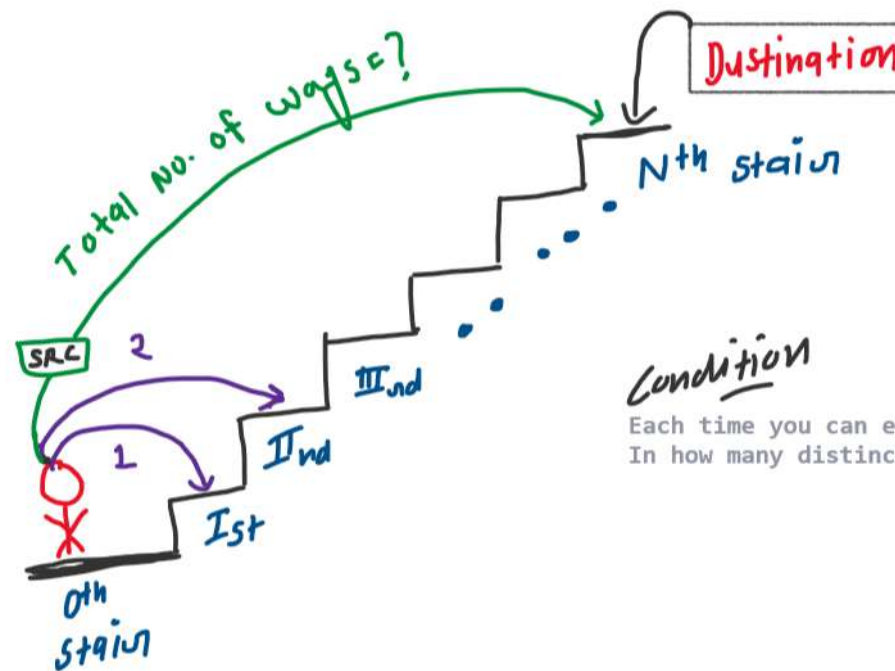
✓ Program 01: Climbing stairs (Leetcode-70)

Ex  $N=1$   
Output = 1

$N=0$   
Output = 1

$N=2$   
Output = 2

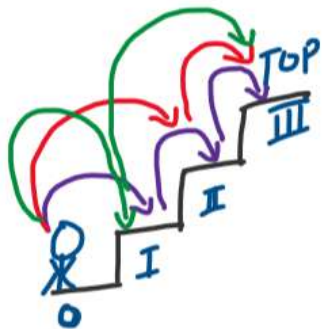
$N=3$   
Output = 3



Condition

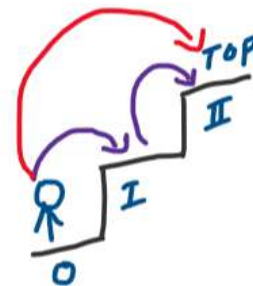
Each time you can either climb 1 or 2 steps.  
In how many distinct ways can you climb to the top?

Ex  $N=3$



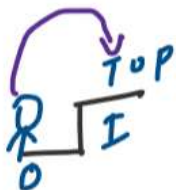
1	1	1	} Total 3 ways
2	1		
1	2		

Ex  $N=2$



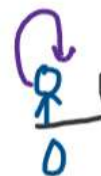
1	1	} Total 2 ways
2		

Ex  $N=1$



1	} Total 1 way

Ex  $N=0$



1	} Total 1 way

$N^{\text{th}}$  stair  $\longrightarrow$  Total no of ways

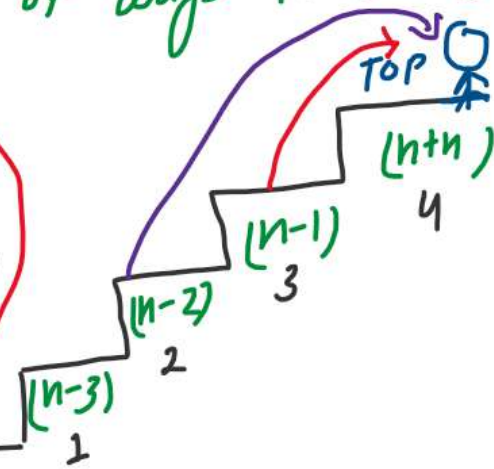
$f(N)$  = Total no of ways to  $n^{\text{th}}$  stairs

way 1  
JAB MU **stair no. 2** par  
tha TO MU **Two stair**  
EK SATH par kar Rha HU  
 **$n^{\text{th}}$  stair** Tak jane  
ki liye TO US  
TARIKE KO MU  **$(n-2)$**   
Bol Rha HU

(+) OR

JAB MU **3rd stair** par HU  
TO MU **one stair** EK TIME par  
chal Raha HU  **$n^{\text{th}}$  stair** Tak  
jane ki liye TO US way KO MU  **$(n-1)$**   
way 2 Bol Rha HU

= Total way



$N=4$

Relation

$$f(N) = f(N-1) + f(N-2)$$

BAS  
AHI

if  $(n == 0 \parallel n == 1)$   
return 1



```
// ✓ Program 01: Climbing stairs (Leetcode-70)
```

```
// Approach 1: Recursion ✗ TLE ✗
```

```
class Solution {
```

```
public:
```

```
    int climbStairs(int n) {
```

```
        // Base Case (Stop Knaha Par Hona Hai)
```

```
        if(n == 0 || n == 1){
```

```
            return 1;
```

```
        }
```

```
        // Relation Calls
```

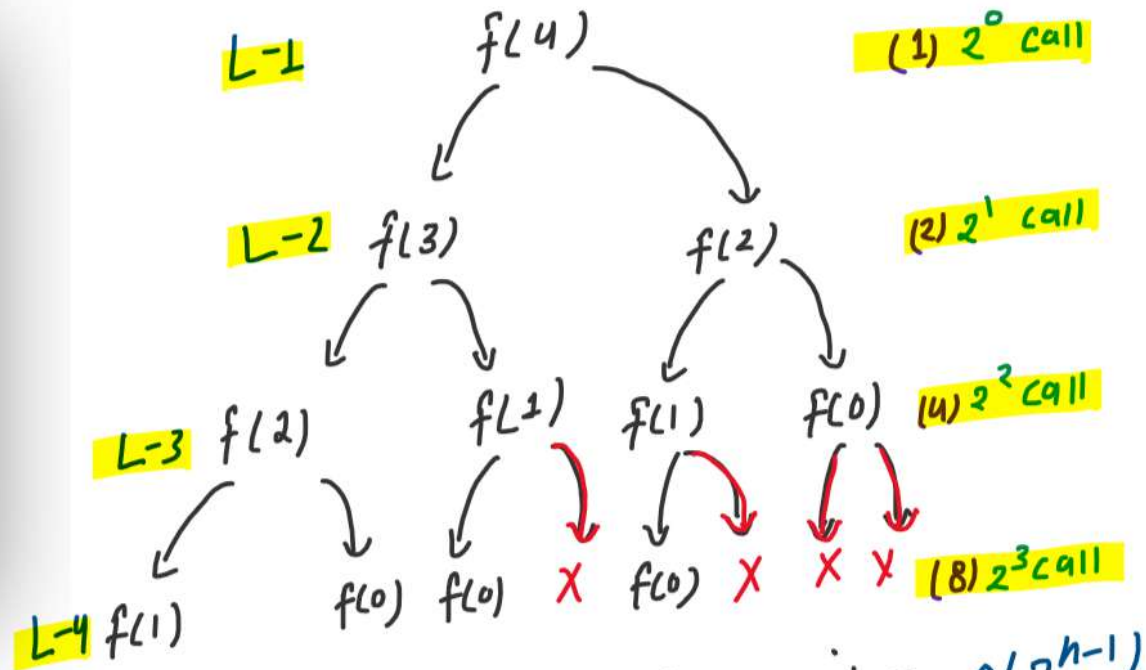
```
        int ways = climbStairs(n-1)+climbStairs(n-2);
```

```
        return ways;
```

```
    }
```

```
};
```

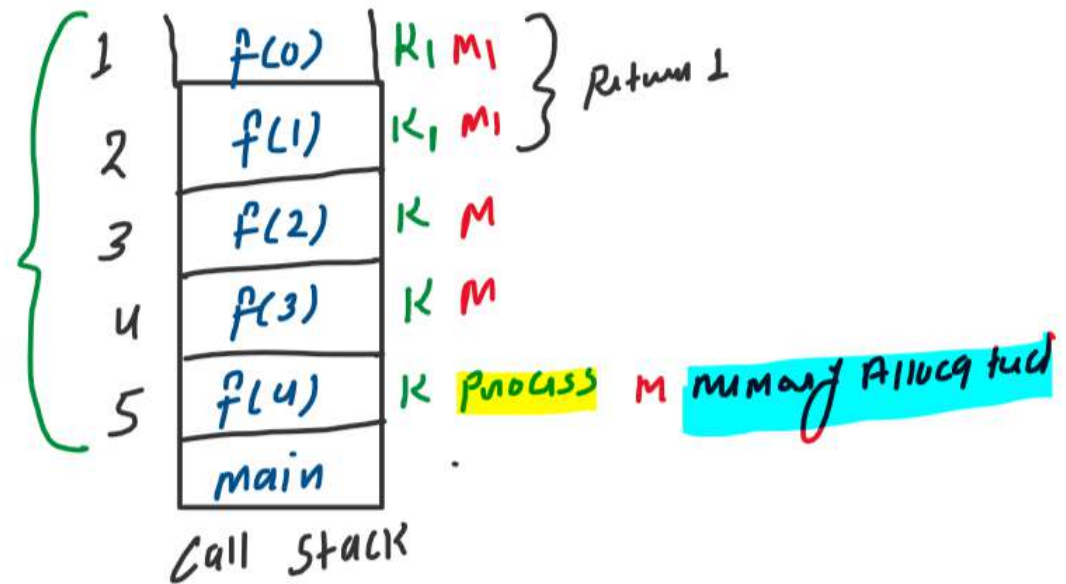
Bukan Code  
Hqī



Space Complexity  $N=4$

$$O(N+1) = \begin{matrix} \text{Total} \\ \text{ENTRY} \\ 5 \end{matrix}$$

$$S.O.C = O(N)$$



✓ Program 02: Print array

arr

10	20	30	40	50
0	1	2	3	4

Initial Index  
index = 0  
size = 5

Relation

f(arr, index+1, size)

Base case

if (index >= size)

return



Relation

$f(arr, index+1, size)$

Base case

if (index  $\geq$  size)  
return

```
// Program 02: Print array
#include<iostream>
using namespace std;

void printArray(int arr[], int index, int N){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    cout<<arr[index]<<" ";

    // Relation Call
    printArray(arr, index + 1, N);
}

int main(){
    int arr[500] = {10,20,30,40,50};
    int size = 5;
    int index = 0;

    printArray(arr, index, size);
    return 0;
}
```

$f(arr, 0, 5)$

Print 10  
index+1

$f(arr, 1, 5)$

Print 20  
index+1

$f(arr, 2, 5)$

Print 30  
index+1

$f(arr, 3, 5)$

Print 40  
index+1

$f(arr, 4, 5)$

Print 50  
index+1

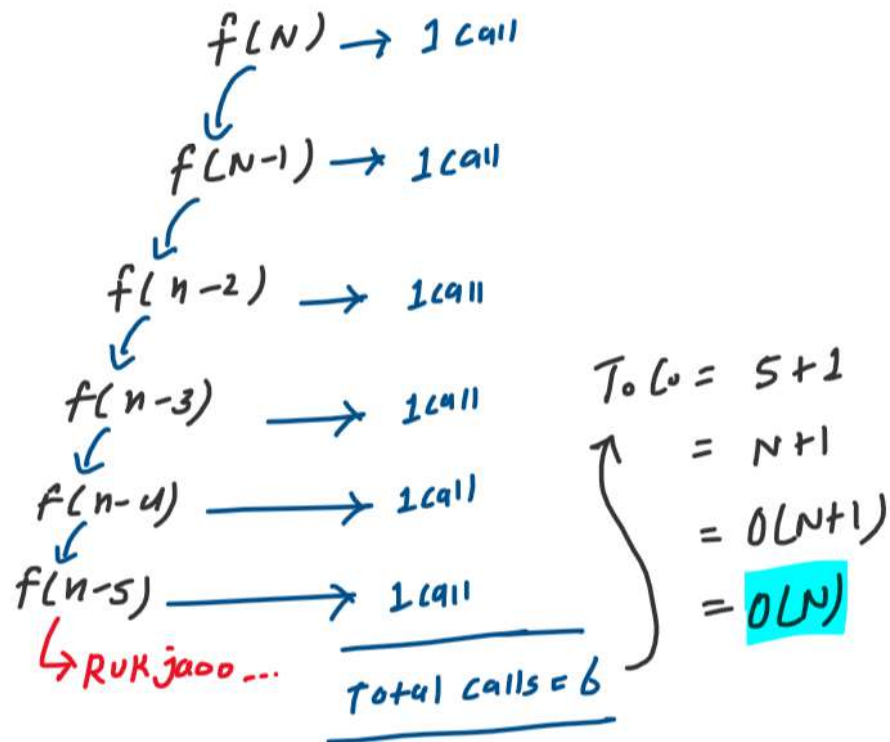
$f(arr, 5, 5)$

X Ruk jao



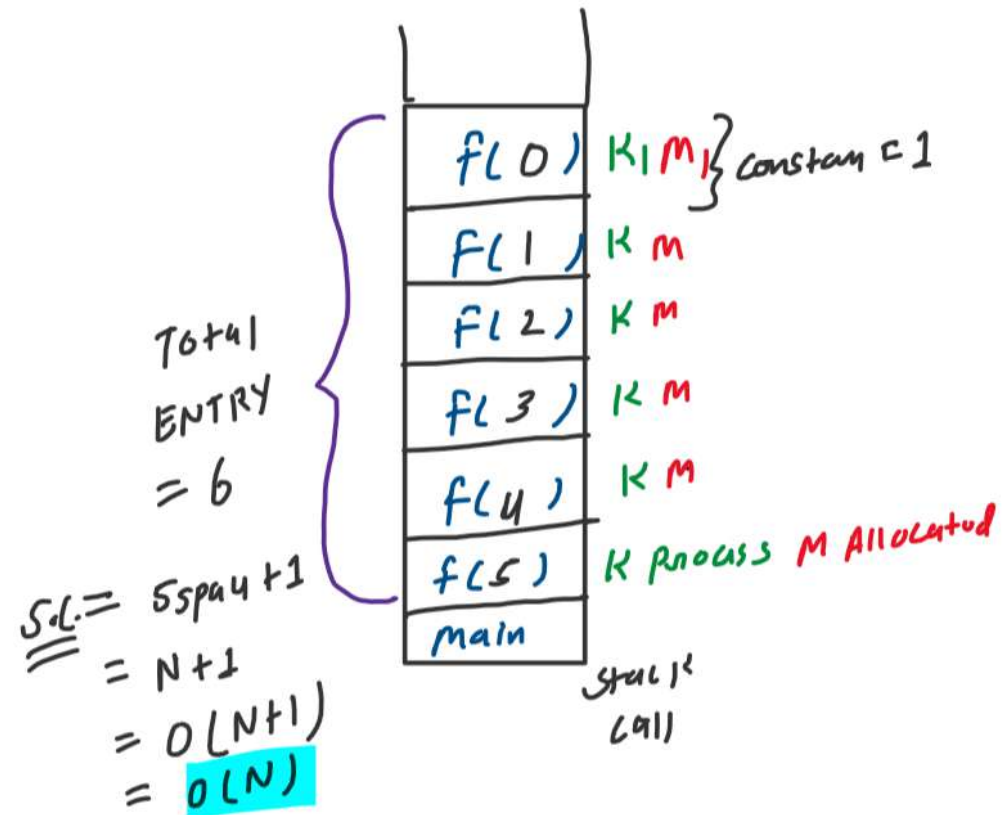
## Time Complexity

$N=5$   
↳ size of Array



## Space Complexity

$N=5$



### ✓ Program 03: Search in array

(Linear search)

YE RECURSION KAN MGA

Target = 50, 100  
Output = True, False

10	20	30	40	50
0	1	2	3	4

1 Step Chaina Mujhe  
Aha Hai

Relation

$f(arr, index + 1, size, target)$

Base  
Case

if ( $arr[index] == target$ )  
     $\rightarrow$  return True

if ( $index \geq size$ )  
     $\rightarrow$  return False

```
// ✓ Program 03: Search in array
#include<iostream>
using namespace std;

bool searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){
        return false;
    }
    if(arr[index] == target){
        return true;
    }

    // Recursive Relation/Call
    bool aageKaAns = searchArray(arr, index + 1, N, target);
    return aageKaAns;
}

int main(){
    int arr[500] = {10,20,30,40,50};
    int size = 5;
    int index = 0;
    int target = 50;

    cout<<searchArray(arr, index, size, target)<<endl;
    return 0;
}
```

Time Complexity

$$= O(N+1)$$

$$\begin{array}{l} \text{Time} = O(N) \\ \text{Comp} \end{array}$$

Like as Print Array

Space Complexity

$$\text{Solo} = O(N)$$

← Like as

$f(arr, 0, 3, 30)$

```
bool searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){ X
        return false;
    }
    if(arr[index] == target){ X
        return true;
    }

    // Recursive Relation/Call
    bool aageKaAns = searchArray(arr, index + 1, N, target);
    return aageKaAns;
}
```

$f(arr, 1, 3, 30)$

```
bool searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){ X
        return false;
    }
    if(arr[index] == target){ X
        return true;
    }

    // Recursive Relation/Call
    bool aageKaAns = searchArray(arr, index + 1, N, target);
    return aageKaAns;
}
```

$f(arr, 2, 3, 30)$

```
bool searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){ X
        return false;
    }
    if(arr[index] == target){ ✓
        return true;
    }

    // Recursive Relation/Call
    bool aageKaAns = searchArray(arr, index + 1, N, target);
    return aageKaAns;
}
```

**TRUE**  
ANS

$f(arr, 0, 3, 30)$

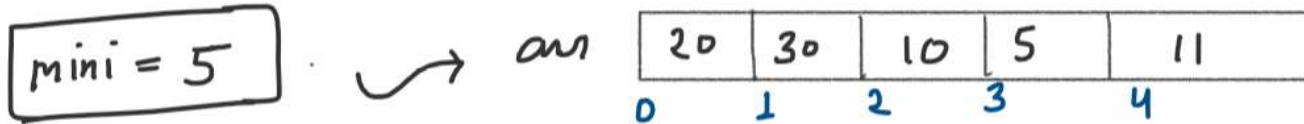
↑     ↑     ↑     ↑  
Array Initial Size target  
Index Value

arr

10	20	30
0	1	2

Target = 30

✓ Program 04: Minimum in array



BASE CASE

if (index == size)  
return

Processing

mini = min(arr[index], mini)

Relation

f(arr, index+1, size, mini)

{ when mini = ~~INT~~ MAX }

index = 0  
index = 1  
index = 2  
index = 3  
index = 4

20  
20  
10  
5  
5

→ index = 5 X stop BASE CASE

```
// Program 04: Minimum in array
#include<iostream>
#include<limits.h>
using namespace std;
```

```
void findMin(int arr[], int index, int N, int &mini){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    mini = min(arr[index], mini);

    // Recursive Relation/Call
    findMin(arr, index + 1, N, mini);
}
```

```
int main(){
    int arr[500] = {20,30,10,5,11};
    int size = 5;
    int index = 0;
    int mini = INT_MAX;
    cout<<"Before Calling findMin then mini: "<< mini<<endl;
    findMin(arr, index, size, mini);
    cout<<"After Calling findMin then mini: "<< mini<<endl;
    return 0;
}
```

→ CATCH  
GALTI  
HONE  
KE  
CHANN  
HAI

Best Case M Tolo  
 $O(N)$

Space Complexity  
 $O(N)$

✓ Program 05: Arrays **even** element store in vector

Base case

```
if (index == size)
    return
```

process

```
if (arr[index] % 2 == 0)
    v.push_back(arr[index])
```

Relation

```
f(arr, index + 1, size, v)
```

Input arr

10	11	12	13	14
0	1	2	3	4

Output vector

10	12	14
0	1	2



```
// Program 05: Arrays even element stored in vector
```

```
#include<iostream>
```

```
#include<vector>
```

```
using namespace std;
```

```
void solve(int arr[], int index, int N, vector<int> &v){
```

```
    // Base Case
```

```
    if(index >= N){
```

```
        return;
```

```
    }
```

```
    // Processing
```

```
    if(arr[index]%2 == 0){
```

```
        v.push_back(arr[index]);
```

```
    }
```

```
    // Recursive Relation/Call
```

```
    solve(arr, index + 1, N, v);
```

```
}
```

```
int main(){
```

```
    int arr[500] = {10,11,12,13,14};
```

```
    int size = 5;
```

```
    int index = 0;
```

```
    vector<int> v;
```

```
    solve(arr, index, size, v);
```

```
    for(auto even: v){
```

```
        cout<<even<<" ";
```

```
    }
```

```
    return 0;
```

```
}
```

SV  
Hi  
pass  
Kanna  
Other  
wise  
ANS  
WRONG  
Aayaz-

Upper Bound last m

$$T_{0C} = O(N)$$

$$S_{0C} = O(N)$$

$\text{solve}(\text{arr}, 0, 3, v)$

```
void solve(int arr[], int index, int N, vector<int> &v){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    if(arr[index]%2 == 0){
        v.push_back(arr[index]);
    }

    // Recursive Relation/Call
    solve(arr, index + 1, N, v);
}
```

*Handwritten annotations:* 'X' over the base case, 'True' next to the processing block, and a box containing '10' with a 'v' above it. A yellow arrow points from the recursive call to the next function call.

$\text{solve}(\text{arr}, 1, 3, v)$

```
void solve(int arr[], int index, int N, vector<int> &v){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    if(arr[index]%2 == 0){
        v.push_back(arr[index]);
    }

    // Recursive Relation/Call
    solve(arr, index + 1, N, v);
}
```

*Handwritten annotations:* 'X' over the base case, 'True' next to the processing block, and a box containing '10' with a 'v' above it. A yellow arrow points from the recursive call to the next function call.

$\text{solve}(\text{arr}, 2, 3, v)$

```
void solve(int arr[], int index, int N, vector<int> &v){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    if(arr[index]%2 == 0){
        v.push_back(arr[index]);
    }

    // Recursive Relation/Call
    solve(arr, index + 1, N, v);
}
```

*Handwritten annotations:* 'X' over the base case, 'True' next to the processing block, and a box containing '10 | 30' with a 'v' above it. A yellow arrow points from the recursive call to the next function call.

$\text{solve}(\text{arr}, 3, 3, v)$

```
void solve(int arr[], int index, int N, vector<int> &v){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    if(arr[index]%2 == 0){
        v.push_back(arr[index]);
    }

    // Recursive Relation/Call
    solve(arr, index + 1, N, v);
}
```

*Handwritten annotations:* 'True' next to the base case, indicating it is reached.

v [10 | 30] output

$\text{solve}(\text{arr}, 0, 3, v)$   
 ↑   ↓   ↑   ↑  
 arr   index   size   vector

I/P arr

10	21	30
0	1	2

O/P v

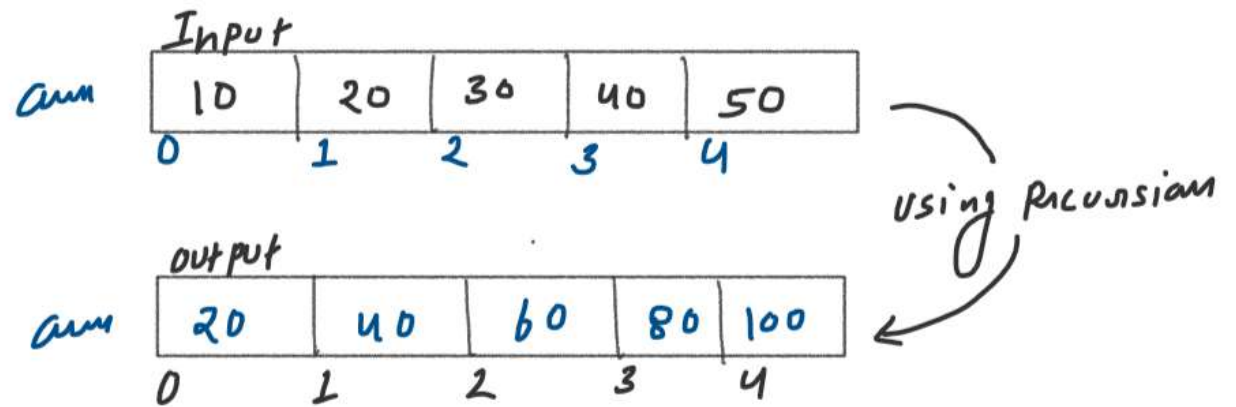
10	30
0	1


✓ Program 06: Double each element

```
if (indx == size)
    return
```

```
arr[indx] = arr[indx] * 2
```

```
f(arr, indx + 1, size)
```



```
//  Program 06: Double each element
#include<iostream>
using namespace std;
```

```
void doubleArray(int arr[], int index, int N){
    // Base Case
    if(index >= N){
        return;
    }

    // Processing
    arr[index] = arr[index] * 2;

    // Recursive Relation/Call
    doubleArray(arr, index + 1, N);
}
```

```
int main(){
    int arr[500] = {10,20,30,40,50};
    int size = 5;
    int index = 0;

    doubleArray(arr, index, size);

    for(int i=0; i<size; i++){
        cout<<arr[i]<<" ";
    }
    return 0;
}
```

$$\underline{T.C.} = O(N)$$

$$\underline{S.C.} = O(1)$$

## ✓ Program 07: Find in array

Linear Search

target = 50, 100  
Output = 4, -1

arr

10	20	30	40	50
0	1	2	3	4

Base Case

if (index >= size)  
return -1;

Processing

arr[index] == target  
return index

Relation

f(arr, index+1, size, target)

```
// ✓ Program 07: Find in array
#include<iostream>
using namespace std;

int searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){
        return -1;
    }
    if(arr[index] == target){
        return index;
    }

    // Recursive Relation/Call
    searchArray(arr, index + 1, N, target);
}

int main(){
    int arr[500] = {10,20,30,40,50};
    int size = 5;
    int index = 0;
    int target = 50;

    cout<<searchArray(arr, index, size, target)<<endl;
    return 0;
}
```

T.C. =  $O(N)$   
S.C. =  $O(1)$

✓ Program 08: Print index of all occurrence of target

Target 10

Output 0,2,3

arr 

10	20	10	10	30	40
----	----	----	----	----	----

  
0      1      2      3      4      5

Base case

if (index == size)  
return

Process

if (arr[index] == target)  
cout << index << " ";

Relation

f(arr, index+1, size, target)

```
// ✓ Program 08: Print index of all occurrence of target
#include<iostream>
using namespace std;
```

```
void searchArray(int arr[], int index, int N, int target){
    // Base Case
    if(index >= N){
        return;
    }
    if(arr[index] == target){
        cout<<index<<" ";
    }

    // Recursive Relation/Call
    searchArray(arr, index + 1, N, target);
}
```

*T.C. = O(N)  
S.C. = O(N)*

```
int main(){
    int arr[500] = {10,20,10,10,30,40};
    int size = 5;
    int index = 0;
    int target = 10;

    searchArray(arr, index, size, target);
    return 0;
}
```



✓ Program 09: Return vector with all occurrence of target

BASE  
Case

vector<int> ans;

if (index == size)  
return ans

Processing

if (arr[index] == target)  
ans.push\_back(index)

Relation

vector<int> f(ans, index+1, size, target)

Processing

for (auto occ : fAns) {  
ans.push\_back(occ);  
}

Target

10

arr

10	20	10	10	30	40
0	1	2	3	4	5

ans

0	2	3	
0	1	2	

→ output vector

$$T.C. = O(N)$$

$$S.C. = O(N)$$



```
//  Program 09: Return vector with all occurrence of target
#include<iostream>
#include<vector>
using namespace std;

vector<int> searchArray(int arr[], int index, int N, int target){
    vector<int> ans;
    // Base Case
    if(index >= N){
        return ans;
    }

    // Processing
    if(arr[index] == target){
        ans.push_back(index);
    }

    // Recursive Relation/Call
    vector<int> aageKaAns = searchArray(arr, index + 1, N, target);

    // Processing
    for(auto occ: aageKaAns){
        ans.push_back(occ);
    }
}

int main(){
    int arr[500] = {10,20,10,10,30,40};
    int size = 5;
    int index = 0;
    int target = 10;

    vector<int> v = searchArray(arr, index, size, target);
    for(auto occ: v){
        cout<<occ<<" ";
    }
    return 0;
}
```

✓ Program 10: Print digits of number

I/P value = 4215

O/P

function V

4	2	1	5
0	1	2	3

Base Case

if (value == 0)  
return

Processing

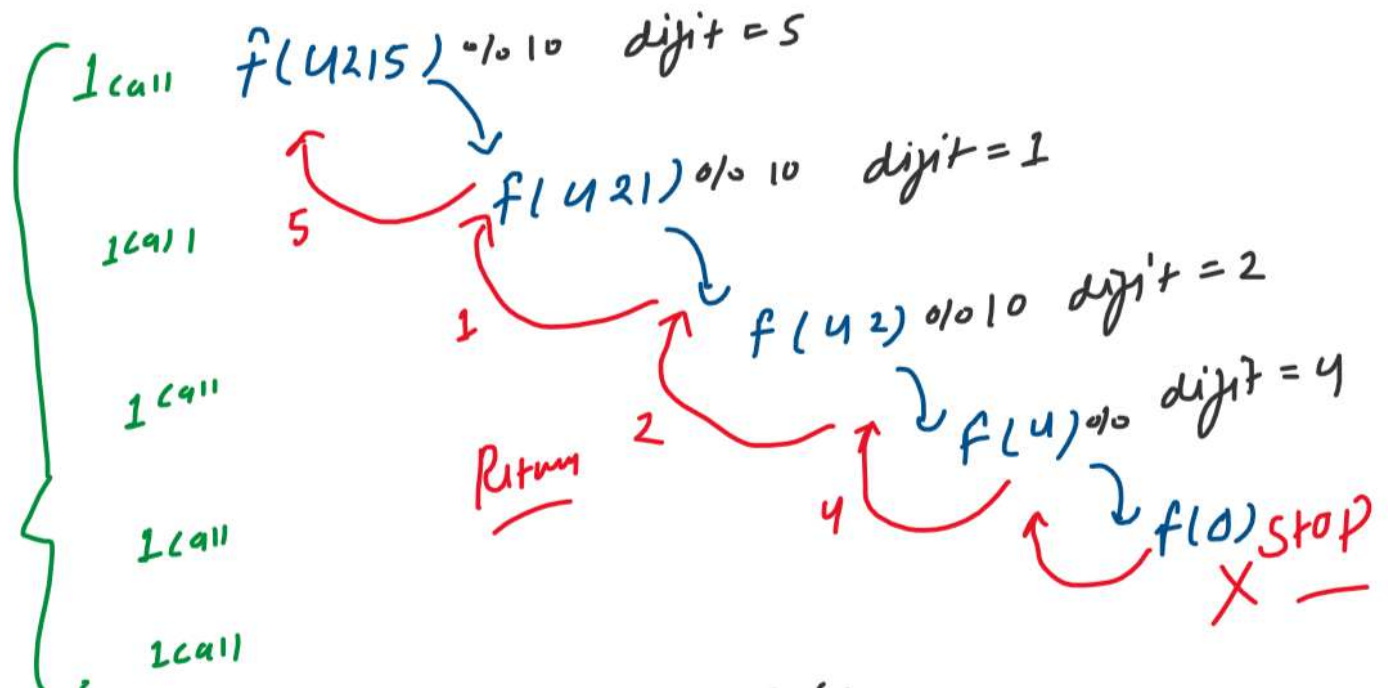
digit = value % 10  
value = value / 10

Relation

f(val, v)

Processing

v.push\_back(digit)



BASIC

`if (value == 0)  
return`

Recursion

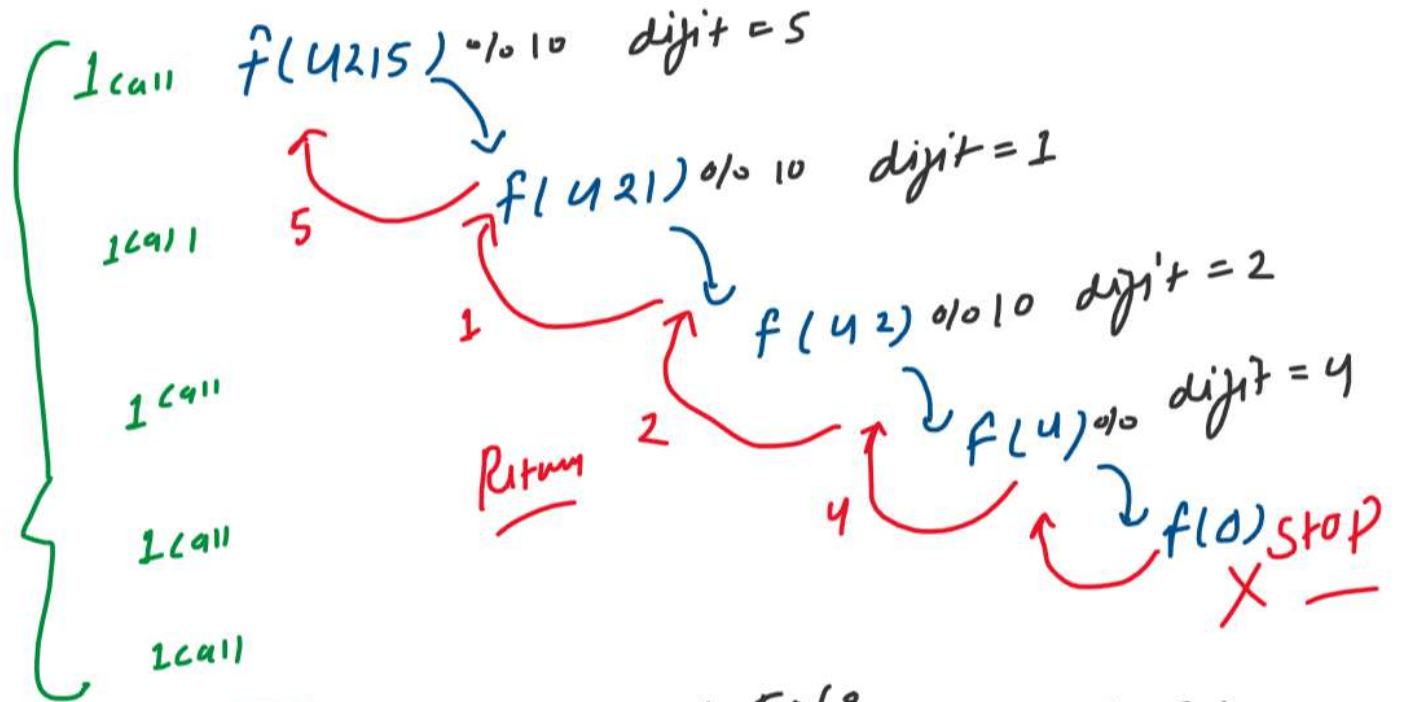
`digit = value % 10  
value = value / 10`

Relation

`f(val, v)`

Recursion

`v.push_back(digit)`



total calls = 5

$N = \text{No of digits of value}$

$= 4 + 1$   
 $= N + 1$

$\left| \begin{array}{l} \text{Time Complexity} \\ \Rightarrow O(N + 1) \\ \Rightarrow O(N) \end{array} \right| \left| \begin{array}{l} \text{Space Complexity} \\ \Rightarrow O(N) \end{array} \right|$

```
// Program 10: Print digits of number
```

```
#include<iostream>
```

```
#include<vector>
```

```
using namespace std;
```

```
void printDigits(int &value, vector<int> &v){
```

```
    // Base Case
```

```
    if(value == 0){
```

```
        return;
```

```
    }
```

```
    // Processing
```

```
    int digit = value % 10;
```

```
    // update the value
```

```
    value = value / 10;
```

```
    // Recursive Relation/Call
```

```
    printDigits(value, v);
```

```
    // Processing
```

```
    v.push_back(digit);
```

```
}
```

```
int main(){
```

```
    int value = 4215;
```

```
    vector<int> v;
```

```
    printDigits(value, v);
```

```
    for(auto digit: v){
```

```
        cout<<digit<<" ";
```

```
    }
```

```
    return 0;
```

```
}
```

→ v.push\_back(digit)

