MODULE CB

EXTENDS $Integers$

CONSTANTS $X,\ Y,\ Z$

ASSUME $\land\ X \in 0\,..\,100$
$\land\ Y \in 0\,..\,100$
$\land\ Z \in 0\,..\,100$
$\land\ X + Y + Z > 0$

```
**************************************************************************************
--fair algorithm CB{
    variable r = X, g = Y, b = Z ;
        { S: while ( TRUE )
                { either
                      { await  (r > 1) ;
                            r := r - 2 ;
                         } ;
                  or
                      { await (g > 1) ;
                            g := g - 2 ;
                             } ;
                  or
                      { await (b > 1) ;
                            b := b - 2 ;
                             } ;
                  or
                      { await (r > 0 ∧ g > 0) ;
                            r := r - 1 ; g := g - 1 ; b := b + 1
                             } ;
                  or
                      { await (g > 0 ∧ b > 0) ;
                            g := g - 1 ; b := b - 1 ; r := r + 1
                             } ;
                  or
                      { await (b > 0 ∧ r > 0) ;
                            b := b - 1 ; r := r - 1 ; g := g + 1
                             } ;
                }
        }
  }
**************************************************************************************
```

BEGIN $TRANSLATION$ − the hash of the $PCal$ code: PCal-4b438aebbd77830f73a3c8dc8ac321c0

VARIABLES $r,\ g,\ b$

$vars \triangleq \langle r,\ g,\ b \rangle$

$Init \triangleq$ Global variables
$\land r = X$
$\land g = Y$
$\land b = Z$

$$
\begin{aligned}
Next \triangleq\ & \lor \land (r > 1) \\
& \quad \land r' = r - 2 \\
& \quad \land \text{UNCHANGED } \langle g,\ b \rangle \\
& \lor \land (g > 1) \\
& \quad \land g' = g - 2 \\
& \quad \land \text{UNCHANGED } \langle r,\ b \rangle \\
& \lor \land (b > 1) \\
& \quad \land b' = b - 2 \\
& \quad \land \text{UNCHANGED } \langle r,\ g \rangle \\
& \lor \land (r > 0 \land g > 0) \\
& \quad \land r' = r - 1 \\
& \quad \land g' = g - 1 \\
& \quad \land b' = b + 1 \\
& \lor \land (g > 0 \land b > 0) \\
& \quad \land g' = g - 1 \\
& \quad \land b' = b - 1 \\
& \quad \land r' = r + 1 \\
& \lor \land (b > 0 \land r > 0) \\
& \quad \land b' = b - 1 \\
& \quad \land r' = r - 1 \\
& \quad \land g' = g + 1
\end{aligned}
$$

$$
\begin{aligned}
Spec \triangleq\ & \land Init \land \Box[Next]_{vars} \\
& \land \mathrm{WF}_{vars}(Next)
\end{aligned}
$$

END *TRANSLATION* − the hash of the generated TLA code (remove to silence divergence warnings): TLA-641e08bc6e7c95f1

$invariant \triangleq (r + g + b \geq 0)$
$termination \triangleq \Diamond(r + g + b \leq 1 \land r + g + b \geq 0)$

*nontermination*1 and *nontermination*0 are for checking the ending at all even constants or all odd constants
$nontermination1 \triangleq \Diamond(r + g + b = 1)$
$nontermination0 \triangleq \Diamond(r + g + b = 0)$

Invariant is $P = \{r + g + b \geq 0\}$ because: − $invariant \triangleq (r + g + b \geq 0)$; this invaraint doesn't throw an error while model checking
$stable(P)$
$P \land G \Rightarrow P(x,\ e)$

$(r + g + b \geq 0)\ \land\ (r > 1)\ \Rightarrow r - 2 + g + b \geq 0$
$(r + g + b > 1)\ \Rightarrow r + g + b \geq 2$ (This holds)-1st
$\land$
$(r + g + b \geq 0)\ \land\ (g > 1)\ \Rightarrow r + g - 2 + b \geq 0$ (This holds;similar to the 1*st* one)-2nd

$\wedge$
$(r+g+b \geq 0) \ \wedge \ (b > 1) \ \Rightarrow r+g+b-2 \geq 0$ (This holds;similar to the $1st$ one)-3rd
$\wedge$
$(r+g+b \geq 0) \ \wedge \ (r > 0 \wedge g > 0) \ \Rightarrow r-1+g-1+b+1 \geq 0$
$(r+g+b \geq 0) \ \wedge \ (r > 0 \wedge g > 0) \ \Rightarrow r+g+b \geq 1$
$(r+g+b \geq 2) \ \Rightarrow r+g+b \geq 1$ (This holds)-4th
$\wedge$
$(r+g+b \geq 0) \ \wedge \ (g > 0 \wedge b > 0) \ \Rightarrow r+1+g-1+b-1 \geq 0$ (This holds; similar to $4th$ one)-5th
$\wedge$
$(r+g+b \geq 0) \ \wedge \ (r > 0 \wedge b > 0) \ \Rightarrow r-1+g+1+b-1 \geq 0$ (This holds; similar to $4th$ one)-6th

Also from running the models using –fair algorithm; $i.e$ the algorithm cannot halt if it has at least one enabled action:-

if X, Y and $Z(declared\ constants)$, are all $even(includes\ 0)$ or all odd: model terminates at $r+g+b=0$ thus comes under $r+g+b \geq 0$

else: model terminates at $r+g+b=1$ thus comes under $r+g+b > 0$

Fixed Point is $0 \leq r+g+b \leq 1$ because: $-$
$(r > 1)' \wedge \ (g > 1)' \wedge \ (b > 1)' \wedge \ (r > 0 \wedge g > 0)' \wedge \ (g > 0 \wedge b > 0)' \wedge \ (r > 0 \wedge b > 0)'$
$(r \leq 1) \ \wedge \ (g \leq 1) \ \wedge \ (b \leq 1) \ \wedge \ (r = 0 \vee g = 0) \ \wedge \ (g = 0 \vee b = 0) \ \wedge \ (r = 0 \vee b = 0)$
$r+g+b \leq 1$
invaraint is given by $r+g+b \geq 0$ Thus fixed point is $0 \leq r+g+b \leq 1$

Progress: $-$ $termination \ \triangleq \ \Diamond \ (r+g+b \leq 1 \wedge r+g+b \geq 0)$; this property doesn't throw an error while model checking Taking $r+g+b = M$; M is bounded below by Zero To show this is a valid metric function; prove M doesn't increase by any action

$(r > 1) \ \rightarrow r := r-1$ (r decreases hence M decreases)
$(g > 1) \ \rightarrow g := g-1$ (g decreases hence M decreases)
$(b > 1) \ \rightarrow b := b-1$ (b decreases hence M decreases)
$(r > 0 \wedge g > 0) \ \rightarrow r := r-1;\ g := g-1;\ b := b+1$ (net effect is such that M decreases by 1)
$(g > 0 \wedge b > 0) \ \rightarrow g := g-1;\ b := b-1;\ r := r+1$ (net effect is such that M decreases by 1)
$(r > 0 \wedge b > 0) \ \rightarrow r := r-1;\ b := b-1;\ g := g+1$ (net effect is such that M decreases by 1)
metric function cannot go below 0 M eventually reaches 1 or 0, so this program terminates

* Modification History
* Last modified Sun Oct 18 19:05:35 EDT 2020 by $admin$
* Created Sat Oct 17 01:17:14 EDT 2020 by $admin$